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A Dynamic Optimization Model Incorporating the VIX Index to Predict Future Returns

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04/25/2010

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An abstract of
A thesis submitted to the Faculty of Emory College of Arts and Sciences
of Emory University in partial fulfillment
of the requirements of the degree of
Bachelor of Business Administration with Honors

Department of Economics

2010

Abstract

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This paper demonstrates that there is a significant relationship between a periods change in the VIX index and future asset returns for the data tested on six mutual funds. Each of the six funds has a negative relationship with the VIX index, indicating an increase in expected market volatility is associated with decreased future asset returns. However, as expected, there is a significant amount of variance in the intercept terms and the beta coefficients between the assets, indicating that some assets are more sensitive to a given change in the VIX index. Additionally, through several out-of-sample simulations, a portfolio that incorporates the VIX index to project future asset returns, while holding constant variances, co-variances, and expected returns outperformed both the standard mean-variance optimization using static estimates, as well as the equally weighted (1/N) portfolio. The VIX optimized portfolio outperformed the other two portfolios in several aspects, including total return as well as return/risk performance ratios. The benefits of the VIX index is especially useful during bear markets, as losses were substantially minimized during market downturns compared with the other two portfolios. In addition, the data in this paper supports the theory that large increases in expected volatility may be an indicator of an oversold market. This is demonstrated by the fact that the optimal VIX portfolio increased its positions in riskier assets and reduced its position in safer assets subsequent to a rise in expected implied market volatility represented by a change in the VIX index.

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I. INTRODUCTION

Since Markowitz (1952) introduced his mean-variance optimization model, it has been the most widely implemented method for portfolio selection. The Markowitz model requires several statistical parameters to be estimated in order for the model to return asset weights for a portfolio that will allow an investor to minimize portfolio variance given a level of expected return. Namely, the Markowitz model requires an estimate of the expected returns, variances of returns, and co-variances between assets for all assets in a portfolio. The most common method to estimating these parameters is simply using historical data to predict future results. For example, in order to estimate the expected return on asset *x*, a simple average of past results could be calculated to predict future returns. However, we have seen that historical data is not always an accurate predictor of future results, which was demonstrated during the past decade through the financial crisis and other periods of high volatility in the equity markets. Therefore, the purpose of this paper is to introduce and demonstrate a dynamic model which relies on the VIX index in order to predict future asset returns.

This paper will first explore the historical relationship between the VIX index and asset returns. Using simple econometric regressions, I will demonstrate that increased volatility as indicated by an increase in the VIX index is generally associated with negative returns across a spectrum of asset classes. Next, I will introduce an optimization model which attempts to predict expected returns as a function of the VIX index. This optimization model will be tested using an out-of-sample simulation against a static Markowitz standard optimization portfolio as well as an equally weighted (1/N) strategy, which has been demonstrated as a difficult benchmark to outperform (DeMiguel 2009)

II. Understanding the VIX Index

The Chicago Board Options Exchange's Market Volatility Index (VIX) is measure of the expected volatility of the S&P 500 index during the next 30 days. It is computed on a real-time basis throughout each trading day. The VIX index is similar to other indices, such as the Dow Jones Industrial Average, but instead of measuring the price of an assset, it is a measure of implied volatility. The VIX was introduced in 1993 in order to provide a benchmark of short-term market volatility and to provide an index in which futures and options contracts could be written. Although the index wasn't introduced until 1993, minute-by-minute values were computed using index options dating back to the beginning of January 1986 in order to provide a comparison to historical levels. It is important to emphasize that the VIX index is a *forward-looking* measure of volatility, and does not measure historical volatility. As Whaley (2009) indicates, VIX is similar to a bond's yield-to-maturity. Whaley states, "Yield to maturity is the discount rate that equates a bond's price to the present value of its promised payments. As such, a bond's yield is *implied* by its current price and represents the expected *future* return of the bond over its remaining life. In the same manner, VIX is *implied* by the current prices of the S&P 500 index options and represents expected *future* market volatility over the next 30 calendar days." Specifically, the VIX index is calculated by the following:

$$\sigma^2 = \frac{2}{T} \Sigma_i \frac{\Delta K_i}{K_i^2} e^{rt} Q(K_i) - \frac{1}{T} [\frac{F}{K_0} - 1]^2$$

Where:

- σ is $\frac{VIX}{100} \rightarrow VIX = \sigma * 100$
- T is the time to expiration

- F is the forward index derived from index option prices
- K_0 is the first strike price below the forward index level, F
- K_i is the strike price of ith out-of-the-money option: a call if $K_i > K_0$ and a put if $K_i < K_0$; both put and call if $K_i = K_0$
- ΔK_i is the interval between strike prices; half the difference between the strike on either side of K_i:

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$$

- R is the risk-free interest rate to expiration
- $Q(K_i)$ is the midpoint of the bid-ask spread for each option with strike K_i

Therefore, the components of the VIX index are both near and next term put and call options. The VIX is a function of the premium paid for the longer term options, which is justified by the implied volatility during the investment period.

Because of its interpretation, the VIX has assumed the nickname, "*The Investor Fear Gauge*". Although volatility technically means either a positive or negative change in the reference index, the S&P 500 index options market has historically been dominated by investors who are concerned with a potential drop in the stock, or buying "insurance" in case the market drops. Therefore, the VIX is an indicator of portfolio insurance and, as the demand for this insurance increases, the price of the insurance increases accordingly.

The relationship between the VIX index and the S&P 500 can be demonstrated by looking at their historical relationships, demonstrated by *Figure 1* below. As demonstrated by the figure, spikes in the VIX index are associated with negative returns in the S&P 500 index. The VIX reached its highest historical level during the October 19th, 1987 stock market crash. The second highest level of the VIX index was achieved during the most recent financial crisis.

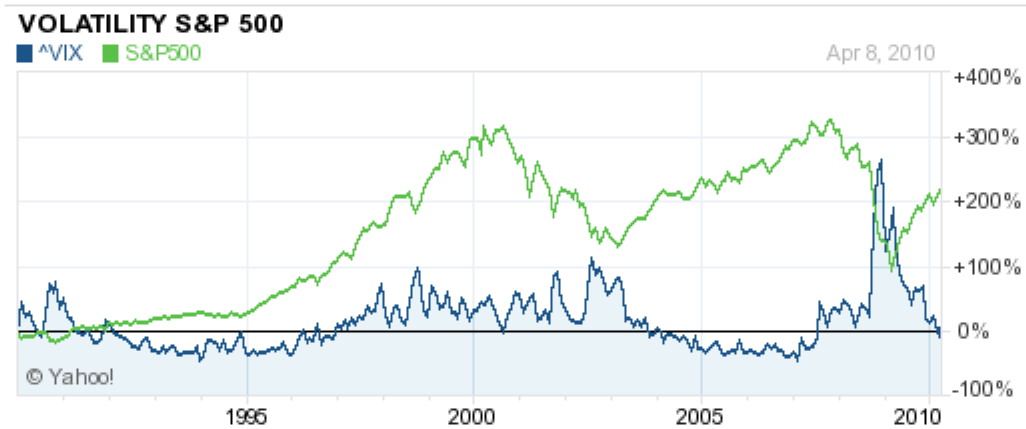

Figure 1: VIX and S&P 500 (Yahoo Finance)

III. Data Description

The historical relationship between six assets and the VIX index was determined by using data obtained from *Yahoo Finance.* My regressions and out-of-sample simulations were conducted on the VIX index and six *Fidelity* mutual funds. These specific funds were chosen arbitrarily, but were intentionally designed to represent a spectrum of riskiness. All data for these funds were obtained from *Yahoo Finance* daily data, and were transformed into monthly logarithmic returns. I used data from April, 1990 – January, 2000 to determine the relationship between the VIX index and the individual funds, and then used data from January, 2000 – April 2010 in order to run the out-of sample simulations. The first fund that was used is the *Fidelity Investment Grade Bond (FBNDX)*. This fund typically invests 80% of the fund's assets in investment-grade debt securities of all types and repurchase agreements. The fund is designed to have similar overall interest rate risk to the Barclays Capital US Aggregate Bond index, and invests in securities of various market sectors and maturities. The second fund chosen was the *Fidelity Capital & Income Fund (FAGIX)*. This fund principally invests in lower-grade debt securities in various market sectors both domestically and internationally. The

Fidelity Telecom and Utilities Fund (FIUIX) is the third fund chosen, which invests in the common stock of securities in the telecommunications service companies and utility companies. Another fund included in the simulation is the *Fidelity Select Technology Portfolio (FSPTX)*, which invests in common stock securities related to the technology industry. The *Fidelity Select Leisure Portfolio (FDLSX)* is the fifth asset included in the simulation. This fund normally invests in the common stock of companies engaged in the design, production, or distribution of goods or services in the consumer discretionary leisure industries. The last asset considered is the *Fidelity Select Consumer Staples Portfolio (FDFAX)*, which primarily invests in the common stock of companies principally engaged in the manufacturing, sale, or distribution of consumer staples.

IV. METHODOLOGY

In order to determine the relationship between the VIX index and the S&P 500, I have conducted a simple OLS regression. Namely, the regression equation will be defined as followed:

$$R_{i,t+1} = \alpha_i + \beta_i RVIX_t + \varepsilon_i$$

Where:

- $R_{i,t+1}$ is the monthly return on asset i over period t to $t+1$
- $RVIX_t$ is the monthly return on the VIX at the beginning of period $t-1$ to t

It is important to note that I used the prior month return on the VIX as of the first day of the month, and regressed this against the returns for the respective fund over the subsequent 30 days. The reason behind calculating the regression estimates as such is because the VIX index attempts to predict market volatility over the next 30 days, which should then be regressed against the actual return data over the next 30 days. This allowed me to determine if the VIX index is significant in predicting future asset returns.

In order to conduct the out-of-sample simulation, I began by downloading daily data for the six *Fidelity* funds as well as the VIX index over the period from April 1990 to January 2000. Using this data, I transformed the daily nominal prices into percent changes using the following logarithmic return formula:

$$r_x = \log \frac{P_t}{P_{t-1}}$$

Where:

- r_x is defined as the return over period *t*
- P_t is defined as the price of asset *x* at period *t*
- P_{t-1} is defines as the price of asset *x* at period *t-1*

From these logarithmic returns, I constructed "monthly returns" by summing the logarithmic returns, which, for the purpose of this paper, will be defined as 21 trading days. Next, for each asset, I calculated the variance of the returns over this sample period, denoted as σ_x^2. Subsequently, for each pair of assets, I calculated a variance-covariance matrix by using the single-index model, using the VIX as the market index, in order to reduce some of the computational complexities of calculating the matrix. The following equations comprise the single-index model:

$$R_{i,t+1} = \alpha_i + \beta_i VIX_t + \varepsilon_i$$

$$\sigma_{i,j} = \beta_i \beta_j \sigma_{VIX}^2 \ when\ i \neq j$$

$$\sigma_{i,j} = \sigma_i^2 \ when\ i = j$$

$$S = \begin{matrix} \sigma_{1,1} & \cdots & \sigma_{1,n} \\ \vdots & \ddots & \vdots \\ \sigma_{n,1} & \cdots & \sigma_{n,n} \end{matrix}$$

Standard Optimization and Equally Weighted Portfolio

Using the estimated parameter described above, I constructed a portfolio using Markowitz's standard optimization model. For various levels of a given expected return, I minimized the portfolio variance and determined the optimal portfolio weights. As an additional benchmark, I used a simple equally weighted portfolio (1/N). I chose to include the Equal Weight portfolio because it has been shown as a difficult benchmark to outperform (DeMiguel 2009). DeMiguel tested 14 models across seven empirical datasets, and determined that none of the models are consistently better than the 1/N strategy in terms of the Sharpe ratio, certainty-equivalent returns, or turnover. This led DeMiguel to conclude that the gain from optimal diversification is more than offset by estimation error. Using these portfolio weights for both the standard optimization model as well as the equally weighted portfolio, I simulated returns for various time periods from January 2000 to April 2010.

VIX Optimization Model

For the VIX optimization model, I used the same variance and covariance estimates as with the standard optimization model in order to isolate the VIX model's ability to predict future asset returns. Accordingly, the expected return for this model was calculated as a function of the VIX index. For each asset, I used historical data (1990-2000) to run regressions against the VIX to determine the relationship between changes in the VIX and *future* asset returns using the following regression:

$$R_{i,t+1} = \alpha_i + \beta_i RVIX_t + \varepsilon_i$$

Once the intercept and the slope of the regression for each asset was determined, I used this information to calculate the expected return for each asset every month, defined as:

$$E(r_{i,t+1}) = \alpha_i + \beta_i * RVIX_{(t)}$$

Where:

- $E(r_{i,t+1})$ is the expected return for asset *i* over period $t+1$
- $RVIX_{(t)}$ is the return on the VIX from period *t-1* to *t*

As a result, I am using the return on the VIX index over the past month to estimate returns on each asset for the future month. Using these monthly expected returns, I re-optimized the portfolio *at the beginning* of each month. Using the optimal portfolio weights at the beginning of each month, I would use the subsequent months' returns for the respective assets to determine the performance of the portfolio. To demonstrate, the first out-of-sample test month was January of 2000. I first would calculate the return on the VIX index from December 1999 – January 2000. Subsequently, I would plug this number into the expected return formula above to determine the implied expected return for each fund for the month of February 2000 as a function of the VIX. Using these expected returns, I would optimize the portfolio to determine the portfolio weights, and use these weights to determine the performance of the portfolio by multiplying each asset weight by their respective return. It is imperative to note that the estimation of expected returns and, therefore, the portfolio weights at the beginning of each month includes no bias of future results; no knowledge of future returns is incorporated in the optimization of the portfolio at the beginning of each month. This process of monthly optimization was conducted during the same period as the standard and equally weighted portfolios (January 2000 – April 2010). Since the variances, co-variances, assets, and expected returns are identical in both the standard and the VIX optimization model, this allowed me to isolate the effects of the VIX's ability to predict future asset returns and examine the results using an out-of-sample simulation.

Multiple simulation tests were ran in order to examine the results given different expected return preferences. I started with a global minimum variance portfolio, which resulted in approximately a 1% expected monthly return. I then increased this expected monthly return by 0.1%, while minimizing portfolio variance, until my expected monthly return was 2.0%. Although these expected returns perhaps seem high, my sample period (1990-2000), which is restricted by the inception of the VIX itself, was associated with relatively large returns, which resulted in relatively high monthly expected returns. Summary statistics for the sample period are presented in *Figure 2* below.

Summary Statistics: Monthly Returns (1990-2000)						
	FBNDX	FAGIX	FIUIX	FSPTX	FDLSX	FDFAX
Avg. Monthly Return:	0.627%	1.072%	1.391%	2.537%	1.609%	0.998%
Standard Deviation:	1.260%	2.254%	2.797%	7.026%	4.373%	3.184%

Figure 2

These optimizations were conducted imposing a short sale restriction of no greater than (0.2) for each asset. I examined the results in through various time periods, including total period (2000 – 2010), 2000 – beginning of the financial crisis (11/2007), beginning of the financial crisis (11/2007) – market bottom (3/2009), beginning of the financial crisis (11/2007) – April 2010, and the market bottom (3/2009) – April 2010. These time frames allowed me to compare the results of these three portfolios during different periods of returns and volatility. The portfolios were evaluated on total returns, standard deviations, Sharpe Ratios, Modified Sharpe Ratios, Treynor Ratios, and Sortino Ratios.

IV. RESULTS

VIX Index and Fund Regressions

Using simple OLS regressions, I established that there is generally a significant relationship between VIX returns and future period returns for the mutual funds chosen. As the return on the VIX increases, this is associated with a decrease in the return on the funds. For example, *Figure* 3 shows the relationship between the return on the VIX and the FAGIX mutual fund. The figure demonstrates that there is a negative relationship between the returns of the VIX index and FAGIX returns.

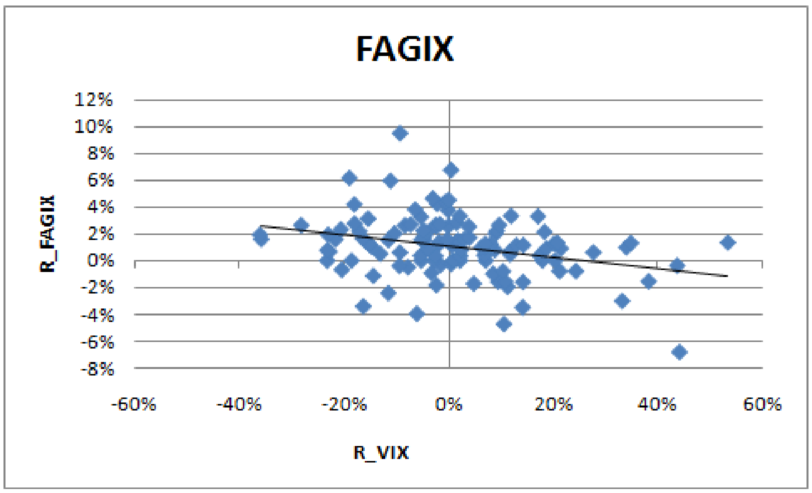

Figure 3

Of the six asset returns regressed against the VIX, each one of them had a negative beta coefficient and a positive intercept term, as expected. The funds varied in respect to their levels of significance as well as their coefficients and intercepts. The first fund, *FBNDX*, had an intercept term of 0.65%, a slope of (1.4%), and a t-stat of (2.01). *FAGIX*, the second fund tested, had an intercept term of 1.1%, a slope of (4.2%), and a t-stat of (3.40). The third and fourth funds, *FIUIX* and *FSPTX*, had intercepts of 1.4% and 2.6%, slopes of (2.5%) and (4.2%), and t-stats of (1.55) and (1.04), respectively. *FDLSX* and *FDFAX* had intercept terms of 1.7% and 1.0%, slopes of (4.5%) and (2.0%), and t-stats of

(1.80) and (1.09), respectively. The variability in these statistics has revealing results regarding the individual funds' relationships with the VIX index. For example, the fund with the largest, 2.6%, and smallest, 0.6%, intercept terms are *FSPTX* and *FBNDX*, respectively. This indicates that for a given period with an unchanged VIX index, the fund *FSPTX* returns a much larger future period return than the fund *FBNDX*. This makes intuitive sense as one would expect an investment grade bond fund to have a lower expected return than a fund comprised of the common stock of technology companies, given no change in the volatility of the markets. In addition, there is a large amount of variability across the slopes of the regressions.

The fund with the largest slope, (4.5%), and the smallest slope, (1.4%), was *FDLSX* and *FBNDX*, respectively. This indicates that for a given increase in the expected volatility of the S&P 500, represented by a change in the VIX index, the expected return for the subsequent period of *FDLSX* will decrease at a greater rate than *FBNDX*. For example, if the VIX index increases by 10%, this would be associated with a decrease in the expected return for the future period of *FBNDX* by 0.14%, while *FDLSX* would be associated with a decrease in the expected return for the future period by 0.45%.

The t-stat, or statistical relationships, between the funds and the VIX also varied. The fund with the largest significance, 3.40, and the smallest significance, 1.04, was *FAGIX* and *FSPTX*. Even though the fund FSTPX has only a t-stat of -1.04 representing a 70% confidence level, this could still potentially reveal some useful information regarding future asset returns, even if only for a fraction of the periods. The results of the regressions are summarized in *Table 1* below. Using these beta coefficients, the single-index model, explained above, returned the following variance-covariance matrix (*Table 2*).

fbndx	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
VIXRtn	-.0143716	.0071386	-2.01	0.046	-.0285104	-.0002328
_cons	.0064598	.0011486	5.62	0.000	.0041849	.0087347

fagix	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
VIXRtn	-.0420767	.0123899	-3.40	0.001	-.0666166	-.0175369
_cons	.0112686	.0019935	5.65	0.000	.0073203	.015217

fiuix	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
VIXRtn	-.0246822	.0159591	-1.55	0.125	-.0562912	.0069267
_cons	.0142295	.0025678	5.54	0.000	.0091437	.0193153

fsptx	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
VIXRtn	-.0418944	.0403171	-1.04	0.301	-.1217474	.0379587
_cons	.0259183	.0064869	4.00	0.000	.0130702	.0387664

fdlsx	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
VIXRtn	-.0447998	.0248671	-1.80	0.074	-.0940523	.0044528
_cons	.0166747	.004001	4.17	0.000	.0087502	.0245993

fdfax	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
VIXRtn	-.0198329	.0182642	-1.09	0.280	-.0560074	.0163416
_cons	.0102334	.0029386	3.48	0.001	.004413	.0160537

Table 1

Variance-Covariance Matrix

	FBNDX	FAGIX	FIUIX	FSPTX	FDLSX	FDFAX
FBNDX	0.0001587	0.0000157	0.0000092	0.0000156	0.0000167	0.0000074
FAGIX	0.0000157	0.0005078	0.0000269	0.0000457	0.0000489	0.0000216
FIUIX	0.0000092	0.0000269	0.0007822	0.0000268	0.0000287	0.0000127
FSPTX	0.0000156	0.0000457	0.0000268	0.0049364	0.0000487	0.0000216
FDLSX	0.0000167	0.0000489	0.0000287	0.0000487	0.0019127	0.0000230
FDFAX	0.0000074	0.0000216	0.0000127	0.0000216	0.0000230	0.0010139

Table 2: Variance-Covariance Matrix

Giot (2005) also established a significant negative relationship between the VIX index and the underlying market index. First, Giot established that there is a strong negative relationship between changes in implied volatility indices and the underlying stock indices for equivalent periods. In other words, there is a strong relationship between the change in the VIX index and a change in the underlying index during the same period. Additionally, Giot established that there is a relationship between future asset returns and large changes in the VIX index.

Out-of-Sample Simulations

Total Period Performance

In general, the out-of-sample simulations demonstrated consistently favorable results for the VIX optimization model over both the standard optimization model and the equally weighted portfolio. During the 11 optimizations for desired monthly expected returns ranging from 1.0% to 2.0%, the VIX optimization model did substantially better, on average. The following table (*Table 2*) represents the average statistics of all 11 simulations for the total test period (2000 – 2010):

	Total Period			
	VIX	Stand.	Eq Wt	S&P 500
Tot Rtn	96.05%	38.30%	49.67%	3.84%
Avg Rtn	0.79%	0.31%	0.41%	0.03%
Stdev	6.13%	4.91%	4.22%	5.14%
Sharpe	0.1464	0.0809	0.0964	0.0061
MSR	0.1464	0.0809	0.0964	0.0061
TNR	0.0103	0.0046	0.0053	0.0003
STNO	0.2492	0.1106	0.1313	0.0081

Table 3: Average Statistics

As you can see, the VIX portfolio, on average, returned 96.1% return over the total test period, while the standard optimization model and equally weighted portfolio only returned 38.3% and 49.7%, respectively. In addition both the Sharpe Ratio as well as the Sortino Ratio (which only accounts for downside risk) are substantially higher for the VIX optimization portfolio, with the VIX portfolio returning a Sortino Ratio of 0.249, while the standard and equally weighted portfolios had a Sortino Ratio of only 0.111 and 0.131, respectively. As DeMiguel (2009) demonstrated, the equally weighted portfolio performed better than the standard optimization portfolio. However, the VIX portfolio outperformed the standard optimization and equally weighted model in every trial in terms of both total return as well as performance ratios, demonstrated by *Figure 4* and *Figure 5*. *Figure 4* plots total returns against expected portfolio returns for the 11 simulations, while *Figure 5* plots the Sharpe Ratios for each of the 11 simulations. In many cases, the VIX portfolio had a higher total return than the standard optimization portfolio by double or triple.

In the standard optimization model, as the level of expected return increased, the portfolio actually realized a lower level of expected return. This is most likely attributable to the fact that the out-of-sample test period (2000 - 2010) was associated with exceedingly high levels of market volatility due to events such as 9/11 and the recent financial crisis, which caused the riskier portfolios to perform poorly, as these riskier assets were associated with larger negative returns during the bear markets. However, the VIX optimization model was able to keep a relatively stable level of realized return, even as the portfolio was weighted towards more risky assets. The Sharpe Ratios, however, decreased for both the standard and VIX models, indicating that the return/risk trade-off was diminished as the desired monthly returns increased.

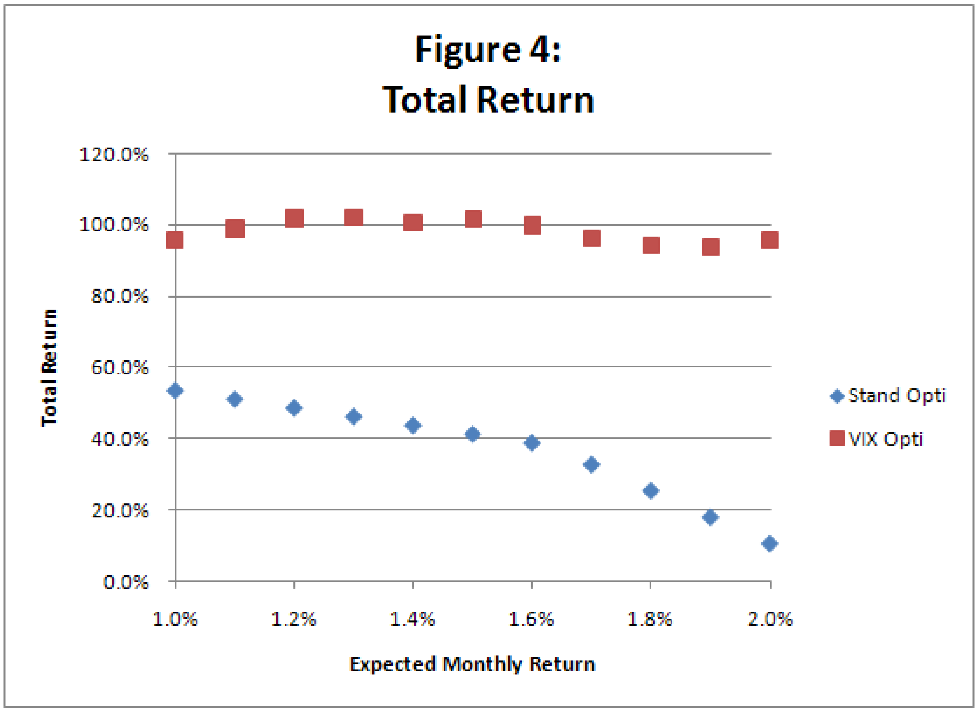

Figure 4

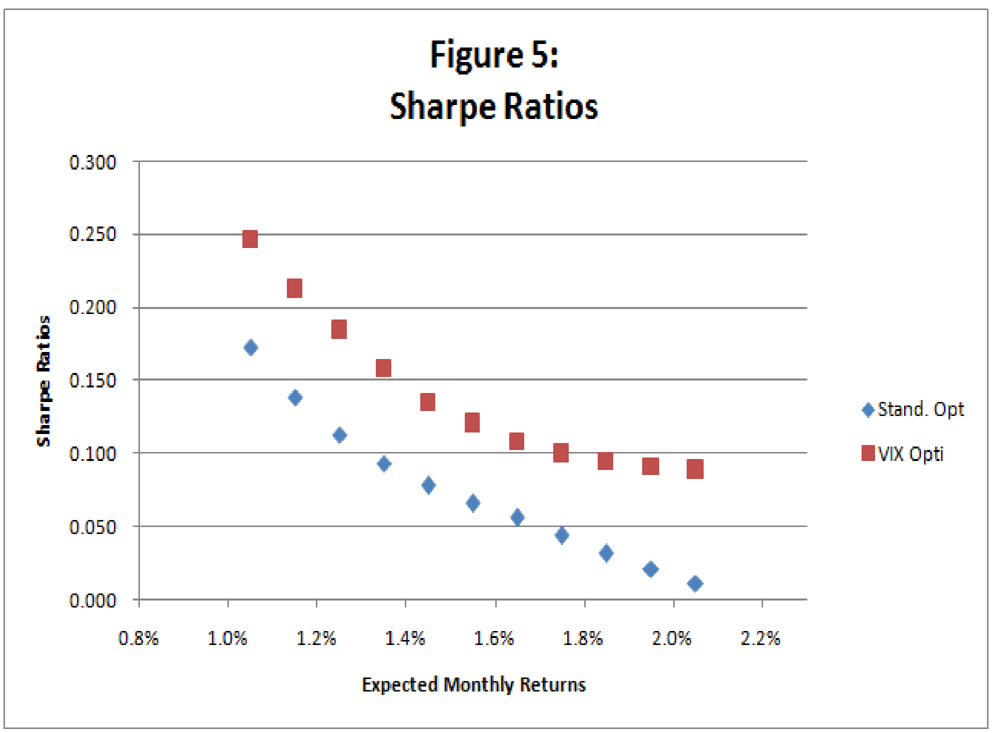

Figure 5

It is apparent from *Figure 6* (below), which represents the total return for the VIX model and the standard mean-variance portfolio optimized for an expected monthly return of 1.0%, the equally weighted portfolio, and S&P 500, that the VIX optimization was able to reduce overall variance and smooth returns. Losses during bear markets were minimized with the VIX portfolio relative to the standard and equally weighted portfolios. In addition, the VIX model was able to capture the upside during periods of bull markets as well. Performance during these various bull and bear markets will be explored more comprehensively in the next section.

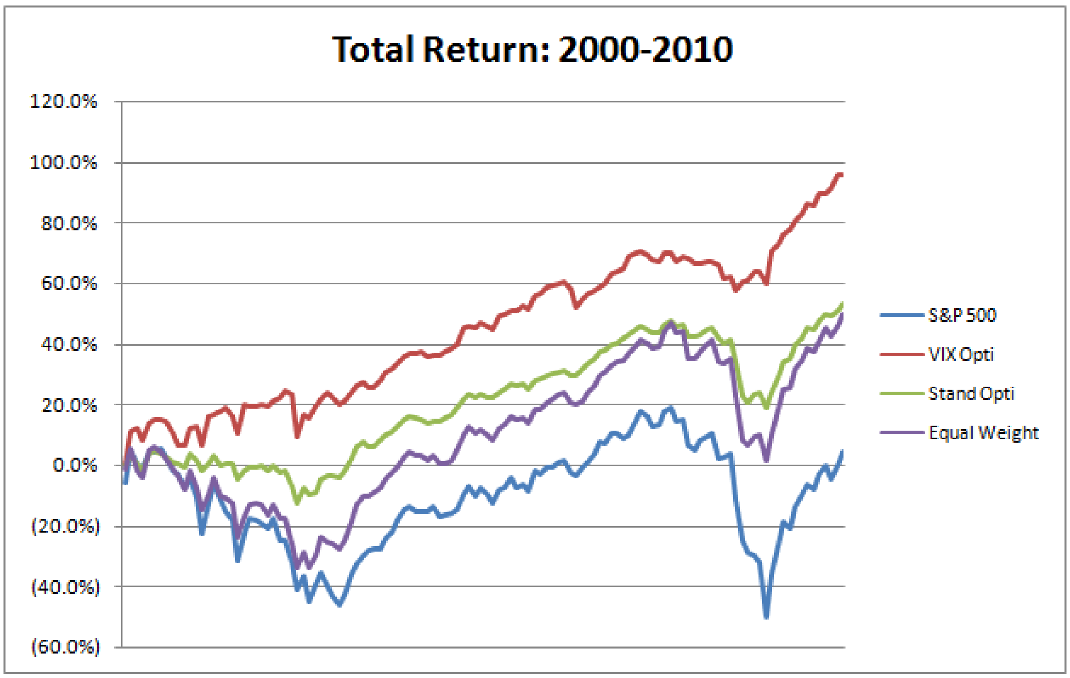

Figure 6: 1.0% Monthly Expected Return

However, the benefits of the VIX optimization model are minimized as I increased the desired monthly expected return. For example, *Figure 7* represents the total return of the portfolios for a monthly expected return of 2.0%. As you can see, the VIX portfolio becomes much more volatile relative to *Figure 6*. This is most likely because in order to achieve the desired 2.0% expected return, the portfolio was forced to weight more heavily towards the relatively riskier assets, which realized a higher level of negative returns during bear market periods.

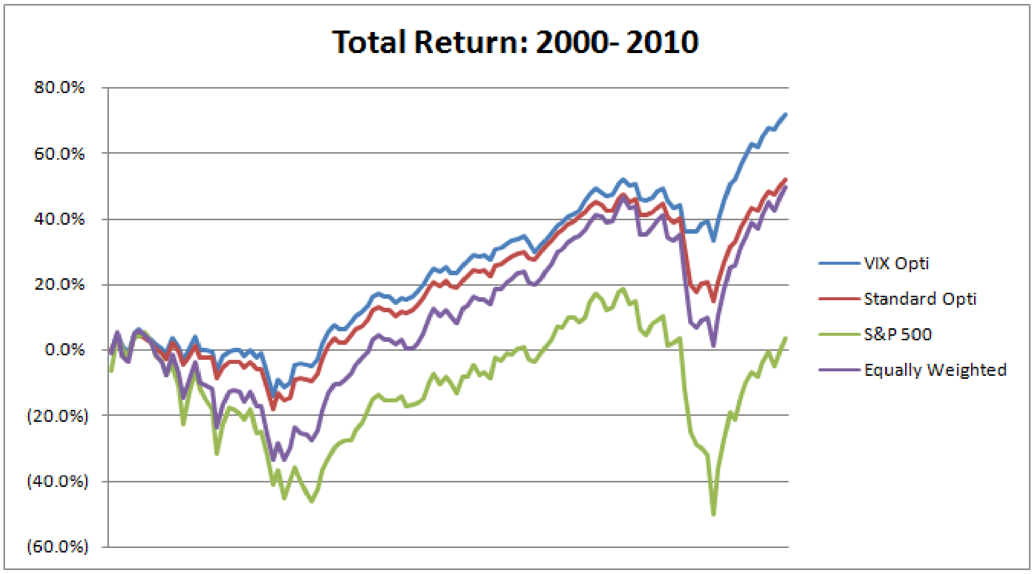

Figure 7: 2.0% Expected Monthly Return

Performance During the Financial Crisis

While the VIX optimization model performed better during the entire period, it also did better, on average, during various periods of the recent financial crisis. For example, the following the table represents the performance of the various portfolios, on average during the 11 simulations, during the beginning of the financial crisis (November 2007) to the market bottom (March 2009).

Beg. Of Crisis-Market Bottom				
	VIX	Stand.	Eq Wt	S&P 500
Tot Rtn	(34.9%)	(51.2%)	(45.3%)	(68.7%)
Avg Rtn	(2.183%)	(3.198%)	(2.830%)	(4.295%)
Stdev	4.8%	6.1%	5.6%	6.6%
Sharpe	-0.44176	-0.51871	-0.50815	-0.65452
MSR	-0.00118	-0.00212	-0.00158	-0.00282
TNR	-0.05848	-0.03978	-0.03880	-0.04581
STNO	-0.43169	-0.48217	-0.47436	-0.56505

Figure 8: Average Statistics

The VIX portfolio, an average during the 11 simulation scenarios, lost approximately 34.9%, while the standard optimization and equally weighted portfolios lost 51.2% and 45.3%, respectively. Additionally, the VIX portfolio had only a standard deviation of 4.8%, while the standard optimization and equally weighted portfolios had a standard deviation of 6.1% and 5.6%, respectively. Since the overall returns are negative, the most relevant ratio for comparison is the Modified Sharpe Ratio (MSR). The VIX portfolio had a higher MSR than both of the other portfolios.

The benefits of the VIX optimization are particularly beneficial when the portfolios are optimized on the lower end of the desired risk spectrum, as demonstrated by *Figure 9*, which represents the performance of the portfolios when the portfolios are optimized to achieve a 1.0% monthly return.

Beg Crisis-Market Bottm				
	VIX	**Stand.**	**Eq Wt**	**S&P 500**
Tot Rtn	(10.2%)	(28.9%)	(45.3%)	(68.7%)
Avg Rtn	(0.639%)	(1.806%)	(2.830%)	(4.295%)
Stdev	2.4%	3.7%	5.6%	6.6%
Sharpe	(0.26913)	(0.49298)	(0.50815)	(0.65452)
MSR	(0.00015)	(0.00066)	(0.00158)	(0.00282)
TNR	(0.05361)	(0.04025)	(0.03880)	(0.04581)
STNO	(0.30186)	(0.46376)	(0.47436)	(0.56505)

Figure 9: Expected Return of 1%

During this simulation, the VIX optimized portfolio lost only 10.2% during a time when the S&P 500 lost nearly 69%, and the standard optimized and equally weighted portfolios lost 28.9% and 45.3%, respectively. In addition, the VIX portfolio only realized a standard deviation of 2.4%, while the standard and equally weighted portfolios realized standard deviations of 3.7% and 5.6%, respectively. This resulted in the VIX portfolio having a substantially better Modified Sharpe Ratio (MSR).

In addition to minimizing the downside risk of the portfolio during the bear market, the VIX portfolio also outperformed relative to the other portfolios as the economy entered a bull market. *Figure 10* shows the average of the 11 simulations for the portfolios with an expected return ranging from 1.0% to 2.0% from the market bottom (March 2009) to April 2010.

March 2008 - April 2010				
	VIX	**Stand.**	**Eq Wt**	**S&P 500**
Tot Rtn	52.0%	49.7%	48.3%	53.9%
Avg Rtn	4.003%	3.824%	3.718%	4.150%
Stdev	4.1%	3.9%	3.4%	5.0%
Sharpe	0.999	1.063	1.088	0.827
MSR	0.999	1.063	1.088	0.827
TNR	0.065	0.062	0.061	0.045
STNO	7.835	5.119	4.454	2.739

Figure 10: Average Statistics

As the figure demonstrates, the VIX portfolio had a higher total return than both the standard and equally weighted portfolios. Although the VIX portfolio returned a higher standard deviation and lower Sharpe Ratio, the VIX portfolio had a higher Sortino Ratio, which only accounts for *downside* risk. Looking at the total return graph from the beginning of the financial crisis to the present also shows the VIX portfolio outperforming the other benchmarks (*Figure 11; 1.0% expected monthly return)*. Once again, market fluctuations are minimized while the returns are higher.

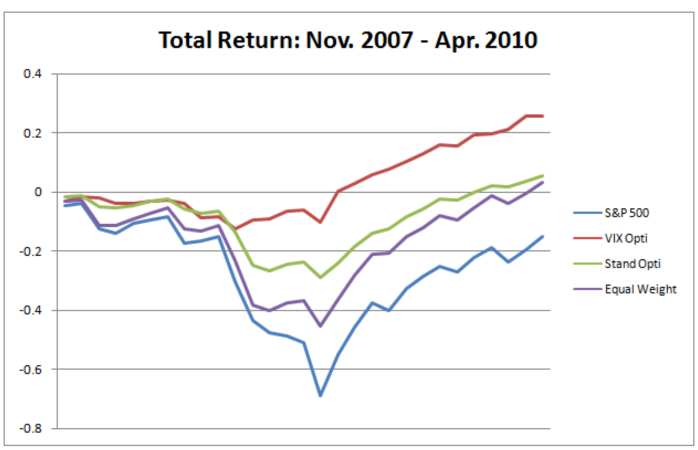

Figure 11

As demonstrated by the preceding statistics and graphs in this section, it is clear the VIX optimized portfolio substantially outperformed the standard optimized and equally weighted portfolios. First, standard deviation of portfolio returns were minimized in the VIX portfolio, especially at lower levels of expected return. Also, the VIX portfolio consistently returned a higher level of total return through the various test periods, and as much as 9.5 times more in one trial. In addition, not only did the VIX portfolio realize higher levels of return, but it also realized these returns on a superior return/risk criteria, as demonstrated by the various performance ratios. The downside protection during bear markets was particularly beneficial, as the VIX portfolio lost 32% less, on average, than the standard optimization portfolio, and as much as 65% less in one trial.

VIX Model Asset Weights

Plotting the prior period change in the VIX against the various asset weights revealed how the weights in the different assets changed as the VIX increased or decreased (*Figure 12*). As demonstrated by the figures, some of the funds became weighted more heavily than others as the VIX increased, and others decreased their weights as the VIX increased. The determinant of these changing asset weights are a function of the regressions that were conducted over the sample period. As shown in a previous section, some of the assets were more volatile given a change in the VIX, which is why the VIX optimization model allowed the weights to vary given different changes in the VIX index.

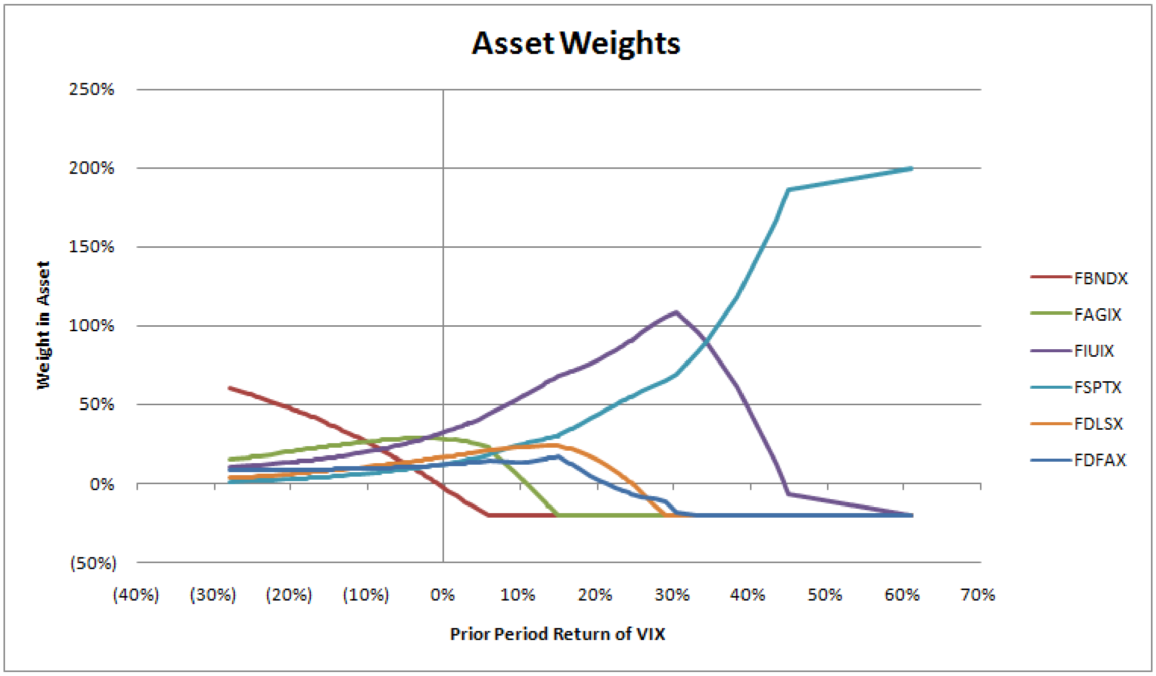

Figure 12

Examining the changes in the individual asset weights for various changes in the VIX reveals some interesting information. For the first asset, *FBNDX*, as the VIX increases, the weight of this asset in the portfolio decreases steadily until it reaches its lower constraint around a 5% increase in the VIX index (*Figure 13*). This relationship is somewhat counter-intuitive to what would be expected. Given that the VIX increases, I would have expected the portfolio to assign a larger weight to the *FBNDX* asset, since it is an investment grade bond portfolio. However, this data suggests that as market volatility increases in the prior period, an optimal portfolio should reduce the weight it has in the investment grade fund, the safest asset. Pierre Giot (2005) offers an explanation as to why this counter-intuitive relationship may exist. Giot suggests that large implied volatility levels may indicate oversold markets. The fact that large impled volatility levels may signal an oversold market, this may present an opportunity to buy relatively riskier assets to capture the market returning to its intrinsic value. This idea is supported by my data as well. For example, refer to *Figure 14,* which displays the asset weight for fund *FSPTX (Technology Sector)* for a given change in prior period return on the VIX. This figure shows that as the expected market volatility increases, the optimal portfolio should consistently increase its weight in a riskier asset to capture the oversold market opportunity.

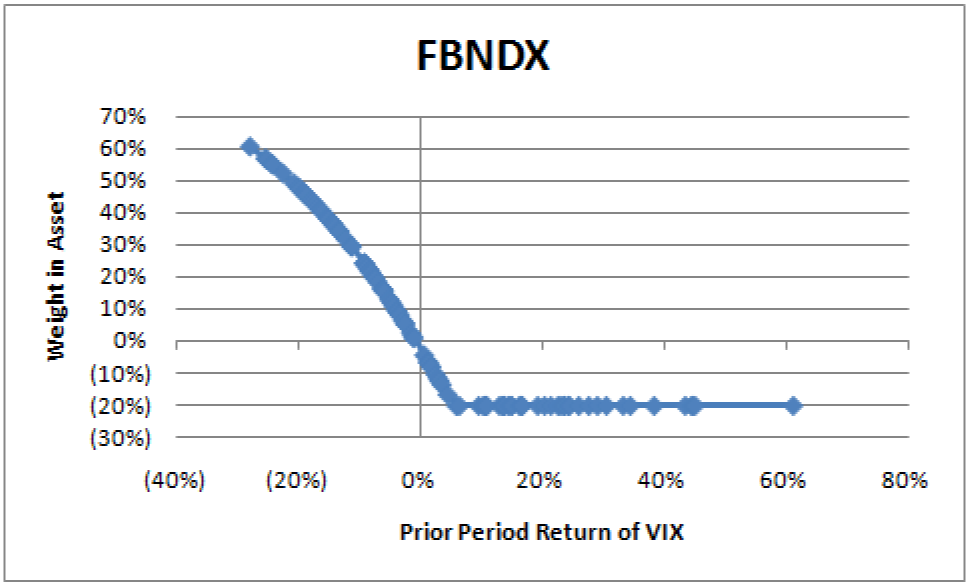

Figure 13

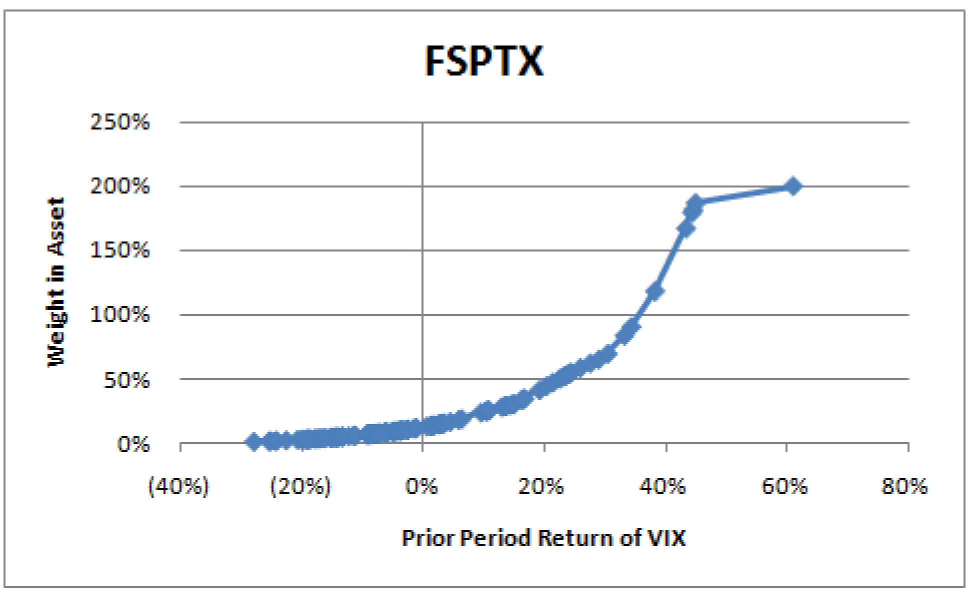

Figure 14

Standard Markowitz Asset Weights

I also recalculated the optimal standard mean-variance portfolio at three distinct points in time (1/2002, 12/2005, and 4/2010) to view how the portfolio weights changed as the markets went through the financial crisis. *Figure 15* reports the results of the funds respective weights in a mean-variance optimized portfolio with a monthly expected return of 1.5%. As demonstrated by the figure, the optimal portfolio included more of the relatively safer assets and less of the riskier assets as time progressed. However, this data could only be ascertained after the fact, unlike the VIX model, which has been demonstrated to have some predictive power of future asset returns.

	FBNDX	FAGIX	FIUIX	FSPTX	FDLSX	FDFAX
Jan-00	-0.15409	0.52982	0.56996	0.19740	-0.20000	0.05691
Dec-05	0.32155	0.12280	0.25457	0.15489	-0.10846	0.25465
Apr-10	0.68019	0.16602	-0.12702	0.06331	-0.01730	0.23479

Figure 15: Optimal Portfolio Weights

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V. Brief Conclusion and Further Research

This paper demonstrated there is a significant relationship between a periods change in the VIX index and future asset returns for the data tested on the six mutual funds. Each of the six funds had a negative relationship with the VIX index, indicating an increase in expected market volatility is associated with decreased future asset returns. However, as expected, there is a significant amount of variance in the intercept terms and the beta coefficients between the assets, indicating that some assets are more sensitive to a given change in the VIX index.

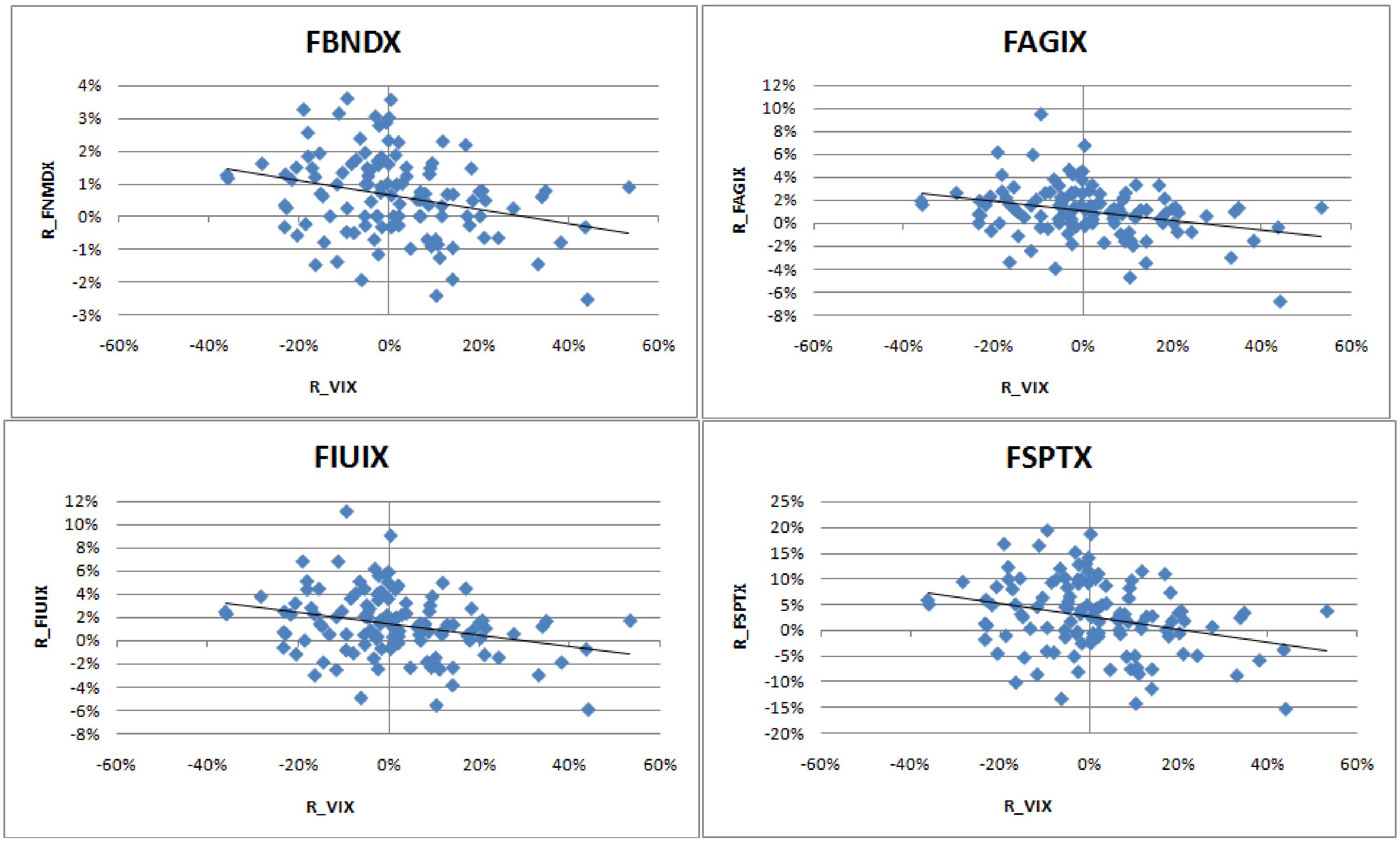
Additionally, through several out-of-sample simulations, a portfolio that incorporates the VIX index to project future asset returns, while holding constant variances, co-variances, and expected return, outperformed both the standard mean-variance optimization using static estimates, as well as the equally weighted (1/N) portfolio. The VIX optimized portfolio outperformed the other two portfolios in several aspects, including total return as well as return/risk performance ratios. The benefits of the VIX index were especially useful during bear markets, as losses were substantially minimized during market downturns compared with the other two portfolios.

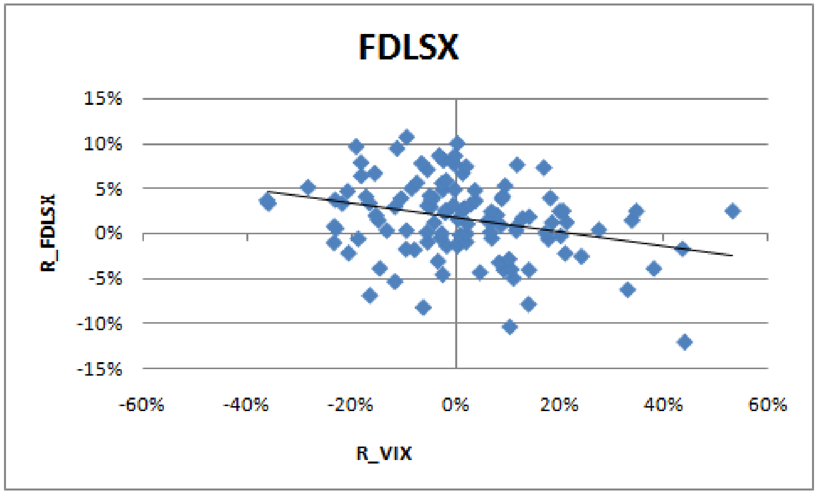
In addition, the data in this paper supports the theory that large increases in expected volatility may be indicative of an oversold market. This is demonstrated by the fact that the optimal VIX portfolio increased its positions in riskier assets and reduced its position in safer assets subsequent to a rise in expected implied market volatility represented by a change in the VIX index.

Additional research could be conducted in order to determine if the benefits of a VIX optimization portfolio are consistent with other assets and data sets. Additionally, it will be interesting to see if the benefits of a VIX optimization continue to be beneficial through a time period that is not characterized with such large market fluctuations, although the time-period subsets indicate that it would be. Another extension of this paper would be to incorporate not only a model where expected returns are a function of the VIX index, but the variance-covariance matrix is also a dynamic function that depends on a change in the VIX. As Jacquier (1999) demonstrated, periods of financial distress result in a significantly different covariance matrix relative to periods of normal economic data.

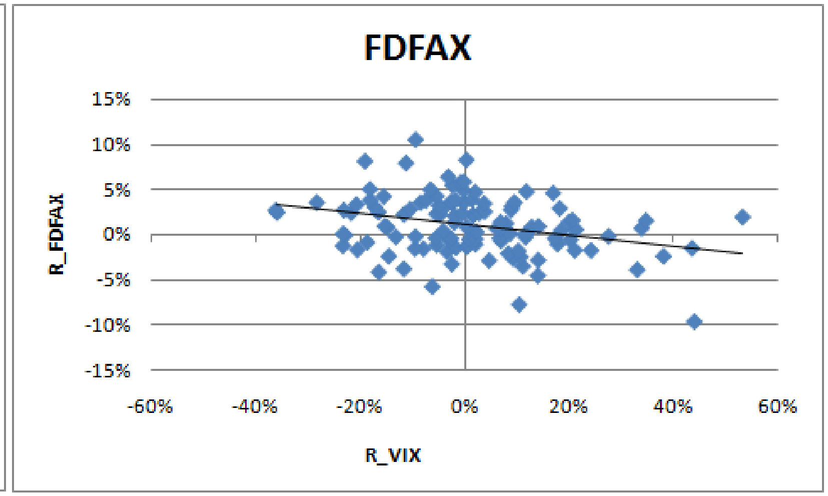
Additional Exhibits

Exhibit 1: Asset Returns vs. Change in VIX for Prior Period





FDLSX
R_FDLSX
R_VIX
15%
10%
5%
0%
-5%
-10%
-15%
-60%
-40%
-20%
0%
20%
40%
60%



FDFAX
R_FDFAX
R_VIX
15%
10%
5%
0%
-5%
-10%
-15%
-60%
-40%
-20%
0%
20%
40%
60%

Exhibit 2: Return Data: 11 simulations with monthly expected return ranging from 1.0%-2.0%

	Total Period				Pre-Crisis- Beg Crisis (2/2000-11/2007)				Beg Crisis-Market Bottm (11/2007-3/2009)				Beg Crisis-April 2010 (11/2007-4/2010)				End Crisis-April 2010 (3/2009-4/2010)			
	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500
Tot Rtn	95.76%	53.29%	49.67%	3.84%	70.16%	47.76%	46.62%	18.61%	(10.22%)	(28.89%)	(45.29%)	(68.72%)	25.60%	5.53%	3.05%	(14.77%)	35.82%	34.42%	48.34%	53.95%
Avg Rtn	0.78%	0.44%	0.41%	0.03%	0.75%	0.51%	0.50%	0.20%	(0.64%)	(1.81%)	(2.83%)	(4.29%)	0.88%	0.19%	0.11%	(0.51%)	2.76%	2.65%	3.72%	4.15%
Stdev	3.19%	2.53%	4.22%	5.14%	3.26%	2.04%	3.67%	4.32%	2.37%	3.66%	5.57%	6.56%	3.01%	3.71%	5.71%	7.22%	2.68%	1.87%	3.42%	5.02%
Sharpe	0.2461	0.1730	0.0964	0.0061	0.2314	0.2515	0.1365	0.0463	(0.2691)	(0.4930)	(0.5082)	(0.6545)	0.2935	0.0514	0.0184	(0.0706)	1.0280	1.4174	1.0883	0.8272
MSR	0.2461	0.1730	0.0964	0.0061	0.2314	0.2515	0.1365	0.0463	(0.0002)	(0.0007)	(0.0016)	(0.0028)	0.2935	0.0514	0.0184	(0.0004)	1.0280	1.4174	1.0883	0.8272
TNR	0.0189	0.0103	0.0053	0.0003	0.0139	0.0132	0.0064	0.0020	(0.0536)	(0.0402)	(0.0388)	(0.0458)	0.0335	0.0042	0.0014	(0.0053)	0.0815	0.0853	0.0613	0.0450
STNO	0.4043	0.2434	0.1313	0.0081	0.3694	0.4044	0.1982	0.0642	(0.3019)	(0.4638)	(0.4744)	(0.5650)	0.5609	0.0659	0.0235	(0.0888)	20.5549	16.1136	4.4537	2.7389
Exp Return	*1.00%*		*Max Short*	*(20.00%)*																

	Total Period				Pre-Crisis- Beg Crisis (2/2000-11/2007)				Beg Crisis-Market Bottm (11/2007-3/2009)				Beg Crisis-April 2010 (11/2007-4/2010)				End Crisis-April 2010 (3/2009-4/2010)			
	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500
Tot Rtn	98.82%	50.85%	49.67%	3.84%	74.75%	46.73%	46.62%	18.61%	(16.01%)	(34.08%)	(45.29%)	(68.72%)	24.07%	4.12%	3.05%	(14.77%)	40.08%	38.20%	48.34%	53.95%
Avg Rtn	0.81%	0.42%	0.41%	0.03%	0.80%	0.50%	0.50%	0.20%	(1.00%)	(2.13%)	(2.83%)	(4.29%)	0.83%	0.14%	0.11%	(0.51%)	3.08%	2.94%	3.72%	4.15%
Stdev	3.81%	3.01%	4.22%	5.14%	3.91%	2.53%	3.67%	4.32%	2.83%	4.19%	5.57%	6.56%	3.55%	4.27%	5.71%	7.22%	3.06%	2.28%	3.42%	5.02%
Sharpe	0.2124	0.1383	0.0964	0.0061	0.2056	0.1989	0.1365	0.0463	(0.3530)	(0.5087)	(0.5082)	(0.6545)	0.2339	0.0333	0.0184	(0.0706)	1.0061	1.2892	1.0883	0.8272
MSR	0.2124	0.1383	0.0964	0.0061	0.2056	0.1989	0.1365	0.0463	(0.0003)	(0.0009)	(0.0016)	(0.0028)	0.2339	0.0333	0.0184	(0.0004)	1.0061	1.2892	1.0883	0.8272
TNR	0.0154	0.0080	0.0053	0.0003	0.0117	0.0100	0.0064	0.0020	(0.0599)	(0.0401)	(0.0388)	(0.0458)	0.0252	0.0027	0.0014	(0.0053)	0.0740	0.0755	0.0613	0.0450
STNO	0.3551	0.1915	0.1313	0.0081	0.3406	0.3043	0.1982	0.0642	(0.3712)	(0.4745)	(0.4744)	(0.5650)	0.4132	0.0425	0.0235	(0.0888)	12.2489	7.6973	4.4537	2.7389
Exp Return	*1.10%*		*Max Short*	*(20.00%)*																

	Total Period				Pre-Crisis- Beg Crisis (2/2000-11/2007)				Beg Crisis-Market Bottm (11/2007-3/2009)				Beg Crisis-April 2010 (11/2007-4/2010)				End Crisis-April 2010 (3/2009-4/2010)			
	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500
Tot Rtn	101.86%	48.41%	49.67%	3.84%	80.34%	45.70%	46.62%	18.61%	(22.83%)	(39.27%)	(45.29%)	(68.72%)	21.52%	2.71%	3.05%	(14.77%)	44.35%	41.98%	48.34%	53.95%
Avg Rtn	0.83%	0.40%	0.41%	0.03%	0.86%	0.49%	0.50%	0.20%	(1.43%)	(2.45%)	(2.83%)	(4.29%)	0.74%	0.09%	0.11%	(0.51%)	3.41%	3.23%	3.72%	4.15%
Stdev	4.53%	3.53%	4.22%	5.14%	4.66%	3.03%	3.67%	4.32%	3.36%	4.74%	5.57%	6.56%	4.15%	4.84%	5.71%	7.22%	3.47%	2.71%	3.42%	5.02%
Sharpe	0.1844	0.1125	0.0964	0.0061	0.1853	0.1621	0.1365	0.0463	(0.4247)	(0.5182)	(0.5082)	(0.6545)	0.1789	0.0193	0.0184	(0.0706)	0.9825	1.1904	1.0883	0.8272
MSR	0.1844	0.1125	0.0964	0.0061	0.1853	0.1621	0.1365	0.0463	(0.0005)	(0.0012)	(0.0016)	(0.0028)	0.1789	0.0193	0.0184	(0.0004)	0.9825	1.1904	1.0883	0.8272
TNR	0.0131	0.0064	0.0053	0.0003	0.0104	0.0080	0.0064	0.0020	(0.0662)	(0.0400)	(0.0388)	(0.0458)	0.0186	0.0015	0.0014	(0.0053)	0.0688	0.0690	0.0613	0.0450
STNO	0.3193	0.1538	0.1313	0.0081	0.3260	0.2398	0.1982	0.0642	(0.4258)	(0.4811)	(0.4744)	(0.5650)	0.2967	0.0245	0.0235	(0.0888)	9.2327	5.3628	4.4537	2.7389
Exp Return	*1.20%*		*Max Short*	*(20.00%)*																

	Total Period				Pre-Crisis- Beg Crisis (2/2000-11/2007)				Beg Crisis-Market Bottm (11/2007-3/2009)				Beg Crisis-April 2010 (11/2007-4/2010)				End Crisis-April 2010 (3/2009-4/2010)			
	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500
Tot Rtn	102.22%	45.97%	49.67%	3.84%	84.48%	44.67%	46.62%	18.61%	(29.86%)	(44.46%)	(45.29%)	(68.72%)	17.74%	1.30%	3.05%	(14.77%)	47.60%	45.76%	48.34%	53.95%
Avg Rtn	0.84%	0.38%	0.41%	0.03%	0.91%	0.48%	0.50%	0.20%	(1.87%)	(2.78%)	(2.83%)	(4.29%)	0.61%	0.04%	0.11%	(0.51%)	3.66%	3.52%	3.72%	4.15%
Stdev	5.30%	4.05%	4.22%	5.14%	5.49%	3.55%	3.67%	4.32%	3.95%	5.30%	5.57%	6.56%	4.70%	5.43%	5.71%	7.22%	3.72%	3.16%	3.42%	5.02%
Sharpe	0.1581	0.0930	0.0964	0.0061	0.1654	0.1354	0.1365	0.0463	(0.4723)	(0.5239)	(0.5082)	(0.6545)	0.1300	0.0082	0.0184	(0.0706)	0.9836	1.1144	1.0883	0.8272
MSR	0.1581	0.0930	0.0964	0.0061	0.1654	0.1354	0.1365	0.0463	(0.0007)	(0.0015)	(0.0016)	(0.0028)	0.1300	0.0082	0.0184	(0.0004)	0.9836	1.1144	1.0883	0.8272
TNR	0.0113	0.0052	0.0053	0.0003	0.0093	0.0066	0.0064	0.0020	(0.0709)	(0.0399)	(0.0388)	(0.0458)	0.0133	0.0007	0.0014	(0.0053)	0.0660	0.0644	0.0613	0.0450
STNO	0.2790	0.1258	0.1313	0.0081	0.3037	0.1956	0.1982	0.0642	(0.4584)	(0.4852)	(0.4744)	(0.5650)	0.2011	0.0104	0.0235	(0.0888)	7.5159	4.2784	4.4537	2.7389
Exp Return	*1.30%*		*Max Short*	*(20.00%)*																

	Total Period				Pre-Crisis- Beg Crisis (2/2000-11/2007)				Beg Crisis-Market Bottm (11/2007-3/2009)				Beg Crisis-April 2010 (11/2007-4/2010)				End Crisis-April 2010 (3/2009-4/2010)			
	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500
Tot Rtn	100.67%	43.53%	49.67%	3.84%	86.49%	43.64%	46.62%	18.61%	(36.57%)	(49.66%)	(45.29%)	(68.72%)	14.18%	(0.11%)	3.05%	(14.77%)	50.75%	49.54%	48.34%	53.95%
Avg Rtn	0.83%	0.36%	0.41%	0.03%	0.93%	0.47%	0.50%	0.20%	(2.29%)	(3.10%)	(2.83%)	(4.29%)	0.49%	(0.00%)	0.11%	(0.51%)	3.90%	3.81%	3.72%	4.15%
Stdev	6.10%	4.59%	4.22%	5.14%	6.37%	4.07%	3.67%	4.32%	4.55%	5.88%	5.57%	6.56%	5.26%	6.03%	5.71%	7.22%	3.98%	3.61%	3.42%	5.02%
Sharpe	0.1352	0.0777	0.0964	0.0061	0.1461	0.1153	0.1365	0.0463	(0.5021)	(0.5275)	(0.5082)	(0.6545)	0.0929	(0.0006)	0.0184	(0.0706)	0.9808	1.0547	1.0883	0.8272
MSR	0.1352	0.0777	0.0964	0.0061	0.1461	0.1153	0.1365	0.0463	(0.0010)	(0.0018)	(0.0016)	(0.0028)	0.0929	(0.0000)	0.0184	(0.0004)	0.9808	1.0547	1.0883	0.8272
TNR	0.0097	0.0044	0.0053	0.0003	0.0084	0.0056	0.0064	0.0020	(0.0729)	(0.0399)	(0.0388)	(0.0458)	0.0093	(0.0001)	0.0014	(0.0053)	0.0638	0.0610	0.0613	0.0450
STNO	0.2414	0.1043	0.1313	0.0081	0.2763	0.1636	0.1982	0.0642	(0.4775)	(0.4878)	(0.4744)	(0.5650)	0.1366	(0.0008)	0.0235	(0.0888)	6.4542	3.6508	4.4537	2.7389
Exp Return	*1.40%*		*Max Short*	*(20.00%)*																

	Total Period				Pre-Crisis- Beg Crisis (2/2000-11/2007)				Beg Crisis-Market Bottom (11/2007-3/2009)				Beg Crisis-April 2010 (11/2007-4/2010)				End Crisis-April 2010 (3/2009-4/2010)			
	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500
Tot Rtn	101.62%	41.08%	49.67%	3.84%	88.57%	42.61%	46.62%	18.61%	(40.81%)	(54.85%)	(45.29%)	(68.72%)	13.05%	(1.52%)	3.05%	(14.77%)	53.86%	53.32%	48.34%	53.95%
Avg Rtn	0.83%	0.34%	0.41%	0.03%	0.95%	0.46%	0.50%	0.20%	(2.55%)	(3.43%)	(2.83%)	(4.29%)	0.45%	(0.05%)	0.11%	(0.51%)	4.14%	4.10%	3.72%	4.15%
Stdev	6.89%	5.13%	4.22%	5.14%	7.24%	4.60%	3.67%	4.32%	5.06%	6.47%	5.57%	6.56%	5.74%	6.64%	5.71%	7.22%	4.25%	4.07%	3.42%	5.02%
Sharpe	0.1208	0.0656	0.0964	0.0061	0.1316	0.0996	0.1365	0.0463	(0.5042)	(0.5297)	(0.5082)	(0.6545)	0.0784	(0.0079)	0.0184	(0.0706)	0.9746	1.0070	1.0883	0.8272
MSR	0.1208	0.0656	0.0964	0.0061	0.1316	0.0996	0.1365	0.0463	(0.0013)	(0.0022)	(0.0016)	(0.0028)	0.0784	(0.0000)	0.0184	(0.0004)	0.9746	1.0070	1.0883	0.8272
TNR	0.0088	0.0037	0.0053	0.0003	0.0077	0.0048	0.0064	0.0020	(0.0676)	(0.0398)	(0.0388)	(0.0458)	0.0077	(0.0006)	0.0014	(0.0053)	0.0619	0.0583	0.0613	0.0450
STNO	0.2198	0.0875	0.1313	0.0081	0.2557	0.1396	0.1982	0.0642	(0.4776)	(0.4895)	(0.4744)	(0.5650)	0.1126	(0.0100)	0.0235	(0.0888)	5.7160	3.2428	4.4537	2.7389
Exp Return	***1.50%***		***Max Short***	***(20.00%)***																

	Total Period				Pre-Crisis- Beg Crisis (2/2000-11/2007)				Beg Crisis-Market Bottom (11/2007-3/2009)				Beg Crisis-April 2010 (11/2007-4/2010)				End Crisis-April 2010 (3/2009-4/2010)			
	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500
Tot Rtn	99.94%	38.64%	49.67%	3.84%	87.78%	41.58%	46.62%	18.61%	(44.81%)	(60.04%)	(45.29%)	(68.72%)	12.16%	(2.93%)	3.05%	(14.77%)	56.97%	57.10%	48.34%	53.95%
Avg Rtn	0.82%	0.32%	0.41%	0.03%	0.94%	0.45%	0.50%	0.20%	(2.80%)	(3.75%)	(2.83%)	(4.29%)	0.42%	(0.10%)	0.11%	(0.51%)	4.38%	4.39%	3.72%	4.15%
Stdev	7.58%	5.68%	4.22%	5.14%	7.97%	5.13%	3.67%	4.32%	5.64%	7.07%	5.57%	6.56%	6.25%	7.25%	5.71%	7.22%	4.53%	4.54%	3.42%	5.02%
Sharpe	0.1081	0.0558	0.0964	0.0061	0.1184	0.0872	0.1365	0.0463	(0.4970)	(0.5310)	(0.5082)	(0.6545)	0.0671	(0.0139)	0.0184	(0.0706)	0.9666	0.9683	1.0883	0.8272
MSR	0.1081	0.0558	0.0964	0.0061	0.1184	0.0872	0.1365	0.0463	(0.0016)	(0.0027)	(0.0016)	(0.0028)	0.0671	(0.0001)	0.0184	(0.0004)	0.9666	0.9683	1.0883	0.8272
TNR	0.0079	0.0031	0.0053	0.0003	0.0069	0.0042	0.0064	0.0020	(0.0634)	(0.0398)	(0.0388)	(0.0458)	0.0065	(0.0011)	0.0014	(0.0053)	0.0602	0.0561	0.0613	0.0450
STNO	0.1965	0.0740	0.1313	0.0081	0.2316	0.1209	0.1982	0.0642	(0.4710)	(0.4907)	(0.4744)	(0.5650)	0.0941	(0.0175)	0.0235	(0.0888)	5.1246	2.9560	4.4537	2.7389
Exp Return	***1.60%***		***Max Short***	***(20.00%)***																

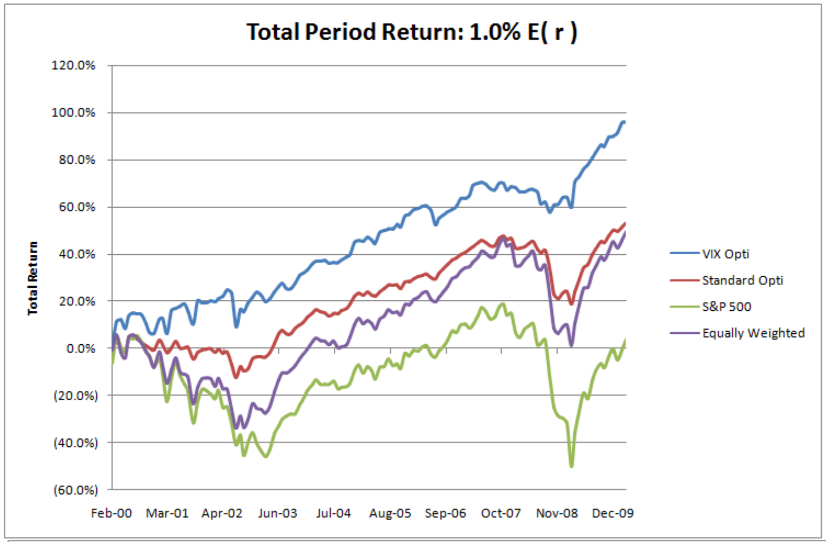
	Total Period				Pre-Crisis- Beg Crisis (2/2000-11/2007)				Beg Crisis-Market Bottom (11/2007-3/2009)				Beg Crisis-April 2010 (11/2007-4/2010)				End Crisis-April 2010 (3/2009-4/2010)			
	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500
Tot Rtn	96.06%	32.61%	49.67%	3.84%	83.45%	37.38%	46.62%	18.61%	(47.34%)	(63.45%)	(45.29%)	(68.72%)	12.61%	(4.78%)	3.05%	(14.77%)	59.95%	58.68%	48.34%	53.95%
Avg Rtn	0.79%	0.27%	0.41%	0.03%	0.90%	0.40%	0.50%	0.20%	(2.96%)	(3.97%)	(2.83%)	(4.29%)	0.43%	(0.16%)	0.11%	(0.51%)	4.61%	4.51%	3.72%	4.15%
Stdev	7.89%	6.16%	4.22%	5.14%	8.27%	5.64%	3.67%	4.32%	6.08%	7.50%	5.57%	6.56%	6.67%	7.68%	5.71%	7.22%	4.83%	4.91%	3.42%	5.02%
Sharpe	0.0998	0.0434	0.0964	0.0061	0.1085	0.0712	0.1365	0.0463	(0.4866)	(0.5286)	(0.5082)	(0.6545)	0.0652	(0.0215)	0.0184	(0.0706)	0.9556	0.9184	1.0883	0.8272
MSR	0.0998	0.0434	0.0964	0.0061	0.1085	0.0712	0.1365	0.0463	(0.0018)	(0.0030)	(0.0016)	(0.0028)	0.0652	(0.0001)	0.0184	(0.0004)	0.9556	0.9184	1.0883	0.8272
TNR	0.0071	0.0024	0.0053	0.0003	0.0062	0.0034	0.0064	0.0020	(0.0583)	(0.0396)	(0.0388)	(0.0458)	0.0062	(0.0017)	0.0014	(0.0053)	0.0587	0.0532	0.0613	0.0450
STNO	0.1760	0.0573	0.1313	0.0081	0.2053	0.0980	0.1982	0.0642	(0.4630)	(0.4895)	(0.4744)	(0.5650)	0.0907	(0.0269)	0.0235	(0.0888)	4.5567	2.6203	4.4537	2.7389
Exp Return	***1.70%***		***Max Short***	***(20.00%)***																

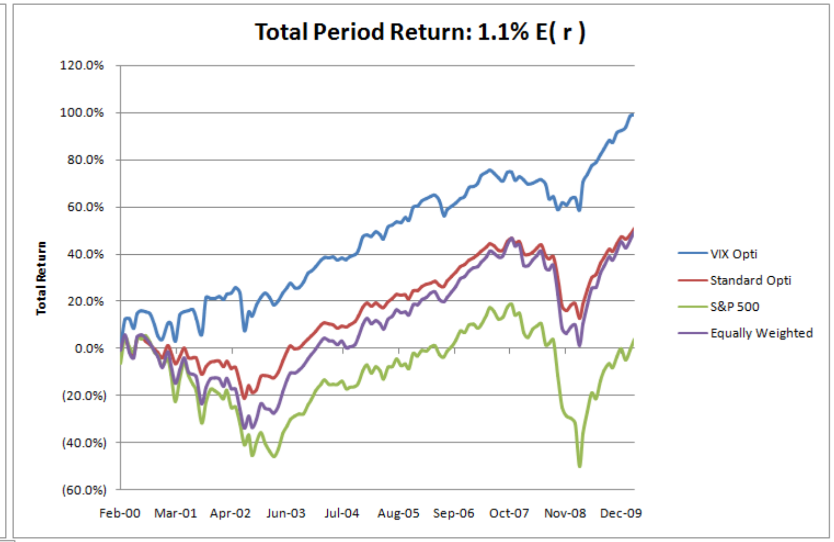
	Total Period				Pre-Crisis- Beg Crisis (2/2000-11/2007)				Beg Crisis-Market Bottom (11/2007-3/2009)				Beg Crisis-April 2010 (11/2007-4/2010)				End Crisis-April 2010 (3/2009-4/2010)			
	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500
Tot Rtn	94.21%	25.23%	49.67%	3.84%	80.75%	32.01%	46.62%	18.61%	(49.07%)	(66.21%)	(45.29%)	(68.72%)	13.46%	(6.78%)	3.05%	(14.77%)	62.52%	59.43%	48.34%	53.95%
Avg Rtn	0.77%	0.21%	0.41%	0.03%	0.87%	0.34%	0.50%	0.20%	(3.07%)	(4.14%)	(2.83%)	(4.29%)	0.46%	(0.23%)	0.11%	(0.51%)	4.81%	4.57%	3.72%	4.15%
Stdev	8.19%	6.63%	4.22%	5.14%	8.55%	6.17%	3.67%	4.32%	6.42%	7.89%	5.57%	6.56%	7.01%	8.04%	5.71%	7.22%	5.11%	5.27%	3.42%	5.02%
Sharpe	0.0943	0.0312	0.0964	0.0061	0.1015	0.0558	0.1365	0.0463	(0.4775)	(0.5245)	(0.5082)	(0.6545)	0.0662	(0.0291)	0.0184	(0.0706)	0.9411	0.8672	1.0883	0.8272
MSR	0.0943	0.0312	0.0964	0.0061	0.1015	0.0558	0.1365	0.0463	(0.0020)	(0.0033)	(0.0016)	(0.0028)	0.0662	(0.0002)	0.0184	(0.0004)	0.9411	0.8672	1.0883	0.8272
TNR	0.0066	0.0018	0.0053	0.0003	0.0057	0.0027	0.0064	0.0020	(0.0540)	(0.0395)	(0.0388)	(0.0458)	0.0061	(0.0023)	0.0014	(0.0053)	0.0572	0.0504	0.0613	0.0450
STNO	0.1628	0.0410	0.1313	0.0081	0.1871	0.0762	0.1982	0.0642	(0.4563)	(0.4870)	(0.4744)	(0.5650)	0.0918	(0.0363)	0.0235	(0.0888)	4.0893	2.3266	4.4537	2.7389
Exp Return	***1.80%***		***Max Short***	***(20.00%)***																

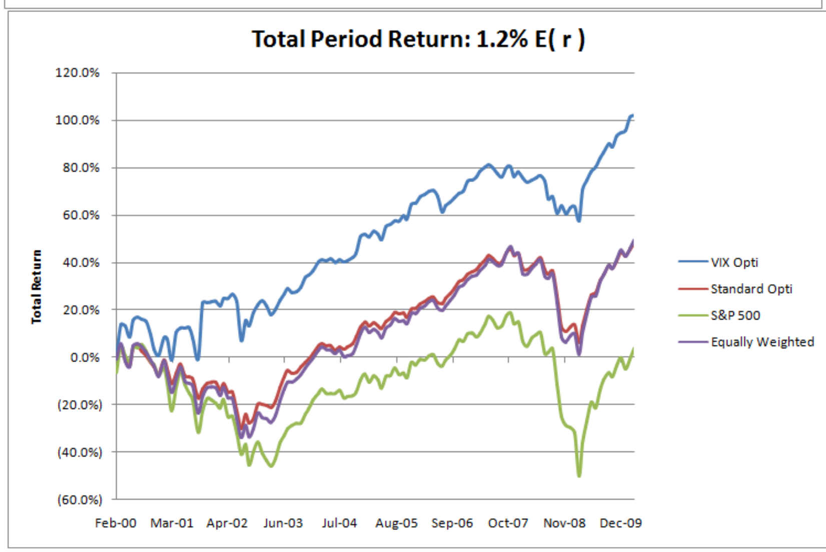
	Total Period				Pre-Crisis- Beg Crisis (2/2000-11/2007)				Beg Crisis-Market Bottom (11/2007-3/2009)				Beg Crisis-April 2010 (11/2007-4/2010)				End Crisis-April 2010 (3/2009-4/2010)			
	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500
Tot Rtn	93.82%	17.86%	49.67%	3.84%	79.31%	26.64%	46.62%	18.61%	(50.70%)	(68.96%)	(45.29%)	(68.72%)	14.51%	(8.78%)	3.05%	(14.77%)	65.22%	60.18%	48.34%	53.95%
Avg Rtn	0.77%	0.15%	0.41%	0.03%	0.85%	0.29%	0.50%	0.20%	(3.17%)	(4.31%)	(2.83%)	(4.29%)	0.50%	(0.30%)	0.11%	(0.51%)	5.02%	4.63%	3.72%	4.15%
Stdev	8.50%	7.12%	4.22%	5.14%	8.86%	6.70%	3.67%	4.32%	6.74%	8.29%	5.57%	6.56%	7.37%	8.42%	5.71%	7.22%	5.47%	5.64%	3.42%	5.02%
Sharpe	0.0905	0.0206	0.0964	0.0061	0.0963	0.0427	0.1365	0.0463	(0.4703)	(0.5200)	(0.5082)	(0.6545)	0.0679	(0.0360)	0.0184	(0.0706)	0.9173	0.8210	1.0883	0.8272
MSR	0.0905	0.0206	0.0964	0.0061	0.0963	0.0427	0.1365	0.0463	(0.0021)	(0.0036)	(0.0016)	(0.0028)	0.0679	(0.0003)	0.0184	(0.0004)	0.9173	0.8210	1.0883	0.8272
TNR	0.0062	0.0012	0.0053	0.0003	0.0054	0.0021	0.0064	0.0020	(0.0515)	(0.0393)	(0.0388)	(0.0458)	0.0062	(0.0028)	0.0014	(0.0053)	0.0553	0.0478	0.0613	0.0450
STNO	0.1535	0.0269	0.1313	0.0081	0.1735	0.0580	0.1982	0.0642	(0.4506)	(0.4842)	(0.4744)	(0.5650)	0.0944	(0.0447)	0.0235	(0.0888)	3.7367	2.0964	4.4537	2.7389
Exp Return	***1.90%***		***Max Short***	***(20.00%)***																

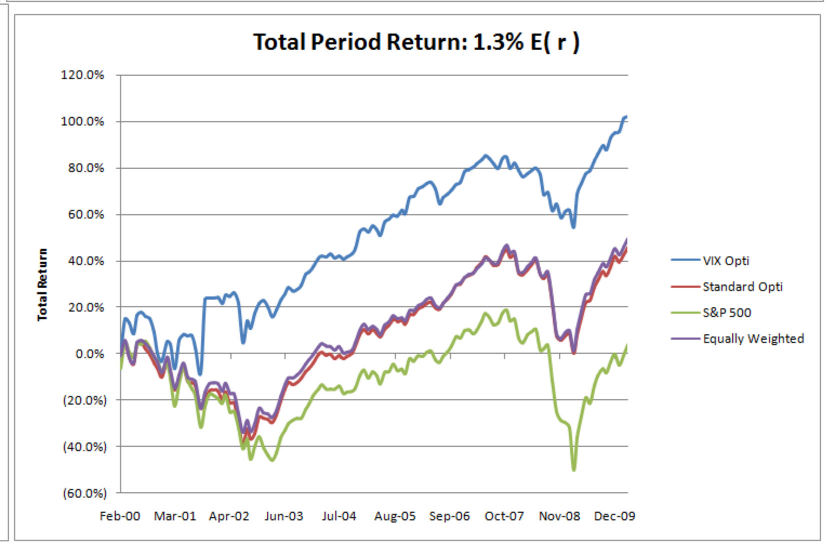
	Total Period				Pre-Crisis- Beg Crisis (2/2000-11/2007)				Beg Crisis-Market Bottom (11/2007-3/2009)				Beg Crisis-April 2010 (11/2007-4/2010)				End Crisis-April 2010 (3/2009-4/2010)			
	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500	VIX	Stand.	Eq Wt	S&P 500
Tot Rtn	95.79%	10.48%	49.67%	3.84%	79.30%	21.27%	46.62%	18.61%	(52.31%)	(71.72%)	(45.29%)	(68.72%)	16.49%	(10.78%)	3.05%	(14.77%)	68.80%	60.93%	48.34%	53.95%
Avg Rtn	0.79%	0.09%	0.41%	0.03%	0.85%	0.23%	0.50%	0.20%	(3.27%)	(4.48%)	(2.83%)	(4.29%)	0.57%	(0.37%)	0.11%	(0.51%)	5.29%	4.69%	3.72%	4.15%
Stdev	8.83%	7.62%	4.22%	5.14%	9.17%	7.25%	3.67%	4.32%	7.07%	8.70%	5.57%	6.56%	7.79%	8.81%	5.71%	7.22%	5.94%	6.01%	3.42%	5.02%
Sharpe	0.0889	0.0113	0.0964	0.0061	0.0930	0.0315	0.1365	0.0463	(0.4625)	(0.5152)	(0.5082)	(0.6545)	0.0730	(0.0422)	0.0184	(0.0706)	0.8910	0.7792	1.0883	0.8272
MSR	0.0889	0.0113	0.0964	0.0061	0.0930	0.0315	0.1365	0.0463	(0.0023)	(0.0039)	(0.0016)	(0.0028)	0.0730	(0.0003)	0.0184	(0.0004)	0.8910	0.7792	1.0883	0.8272
TNR	0.0060	0.0006	0.0053	0.0003	0.0051	0.0015	0.0064	0.0020	(0.0494)	(0.0392)	(0.0388)	(0.0458)	0.0066	(0.0033)	0.0014	(0.0053)	0.0536	0.0456	0.0613	0.0450
STNO	0.1489	0.0147	0.1313	0.0081	0.1645	0.0426	0.1982	0.0642	(0.4446)	(0.4812)	(0.4744)	(0.5650)	0.1024	(0.0523)	0.0235	(0.0888)	3.5034	1.9105	4.4537	2.7389
Exp Return	***2.00%***		***Max Short***	***(20.00%)***																

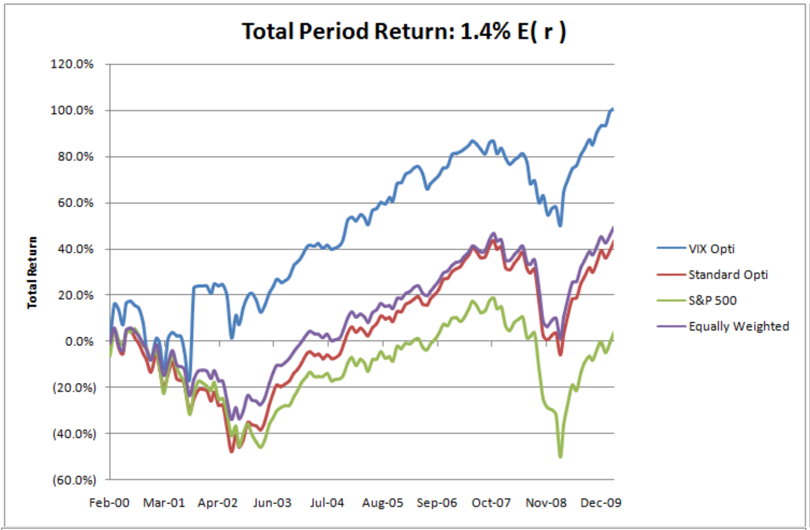
Exhibit 3: Total return graphs for 11 simulations with expected monthly return ranging from 1.0%-2.0%

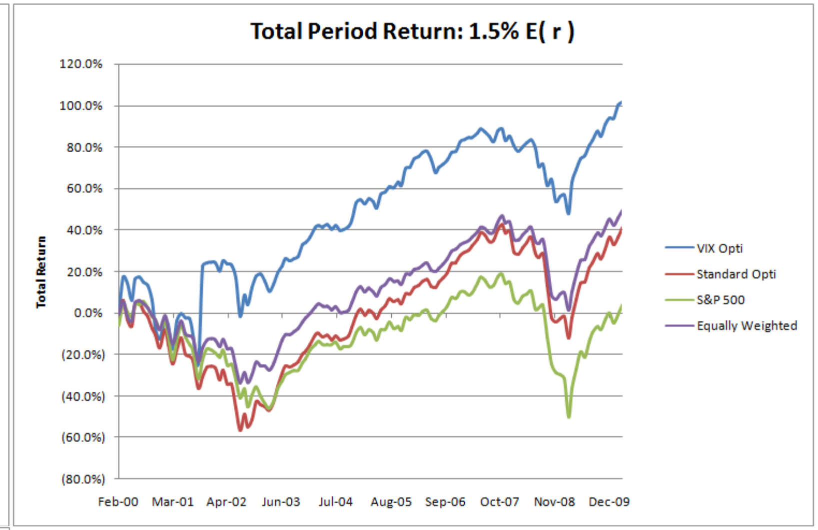


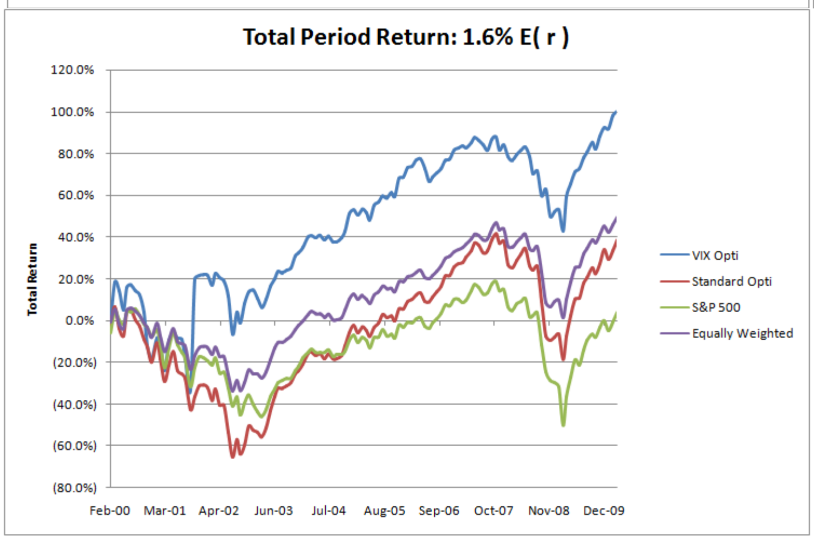


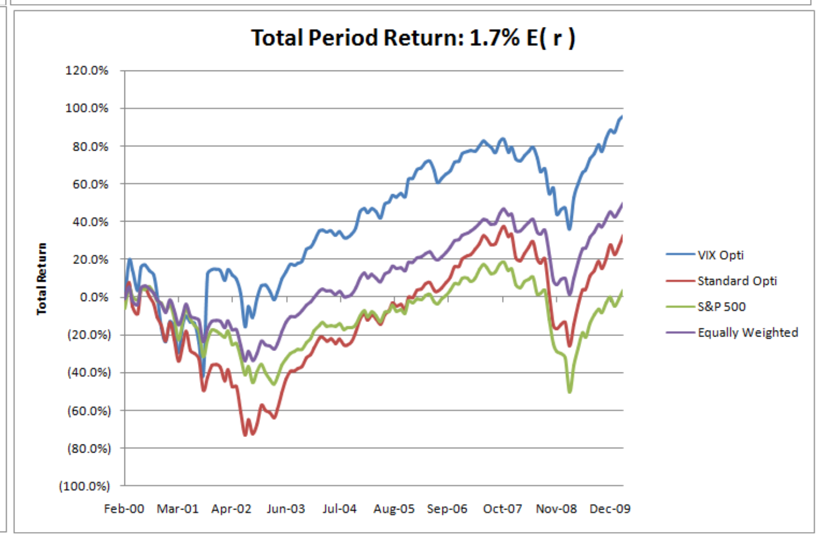


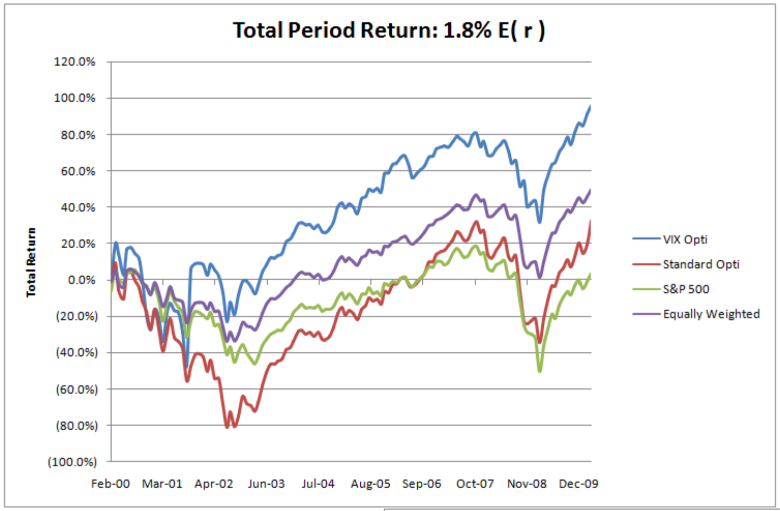


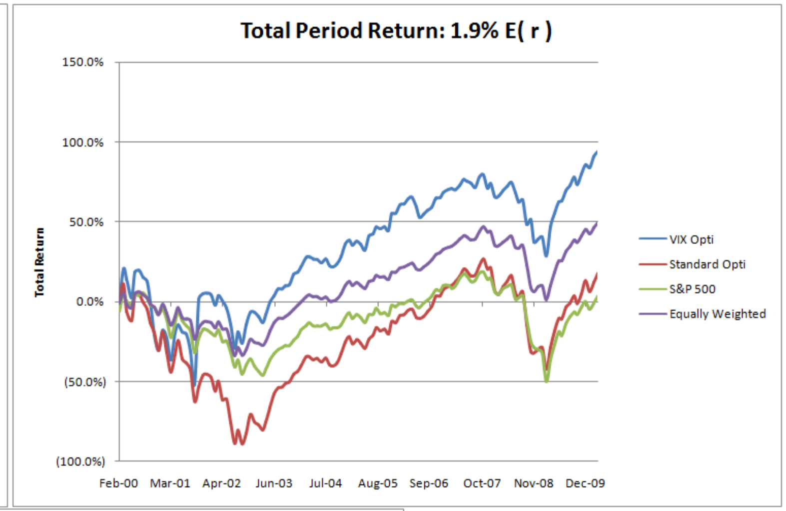

Total Period Return: 1.4% E(r)
Total Return
120.0%
100.0%
80.0%
60.0%
40.0%
20.0%
0.0%
(20.0%)
(40.0%)
(60.0%)
Feb-00 Mar-01 Apr-02 Jun-03 Jul-04 Aug-05 Sep-06 Oct-07 Nov-08 Dec-09
VIX Opti
Standard Opti
S&P 500
Equally Weighted


Total Period Return: 1.5% E(r)
Total Return
120.0%
100.0%
80.0%
60.0%
40.0%
20.0%
0.0%
(20.0%)
(40.0%)
(60.0%)
(80.0%)
Feb-00 Mar-01 Apr-02 Jun-03 Jul-04 Aug-05 Sep-06 Oct-07 Nov-08 Dec-09
VIX Opti
Standard Opti
S&P 500
Equally Weighted


Total Period Return: 1.6% E(r)
Total Return
120.0%
100.0%
80.0%
60.0%
40.0%
20.0%
0.0%
(20.0%)
(40.0%)
(60.0%)
(80.0%)
Feb-00 Mar-01 Apr-02 Jun-03 Jul-04 Aug-05 Sep-06 Oct-07 Nov-08 Dec-09
VIX Opti
Standard Opti
S&P 500
Equally Weighted


Total Period Return: 1.7% E(r)
Total Return
120.0%
100.0%
80.0%
60.0%
40.0%
20.0%
0.0%
(20.0%)
(40.0%)
(60.0%)
(80.0%)
(100.0%)
Feb-00 Mar-01 Apr-02 Jun-03 Jul-04 Aug-05 Sep-06 Oct-07 Nov-08 Dec-09
VIX Opti
Standard Opti
S&P 500
Equally Weighted


Total Period Return: 1.8% E(r)
Total Return
120.0%
100.0%
80.0%
60.0%
40.0%
20.0%
0.0%
(20.0%)
(40.0%)
(60.0%)
(80.0%)
(100.0%)
Feb-00 Mar-01 Apr-02 Jun-03 Jul-04 Aug-05 Sep-06 Oct-07 Nov-08 Dec-09
VIX Opti
Standard Opti
S&P 500
Equally Weighted


Total Period Return: 1.9% E(r)
Total Return
150.0%
100.0%
50.0%
0.0%
(50.0%)
(100.0%)
Feb-00 Mar-01 Apr-02 Jun-03 Jul-04 Aug-05 Sep-06 Oct-07 Nov-08 Dec-09
VIX Opti
Standard Opti
S&P 500
Equally Weighted

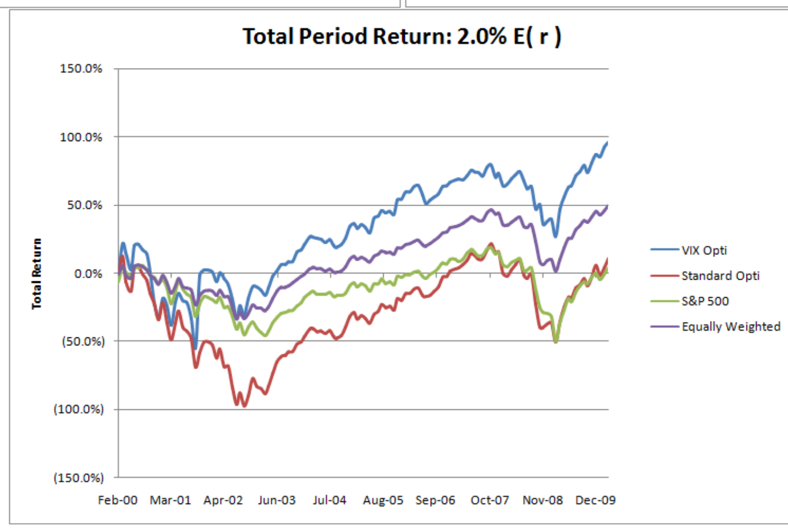
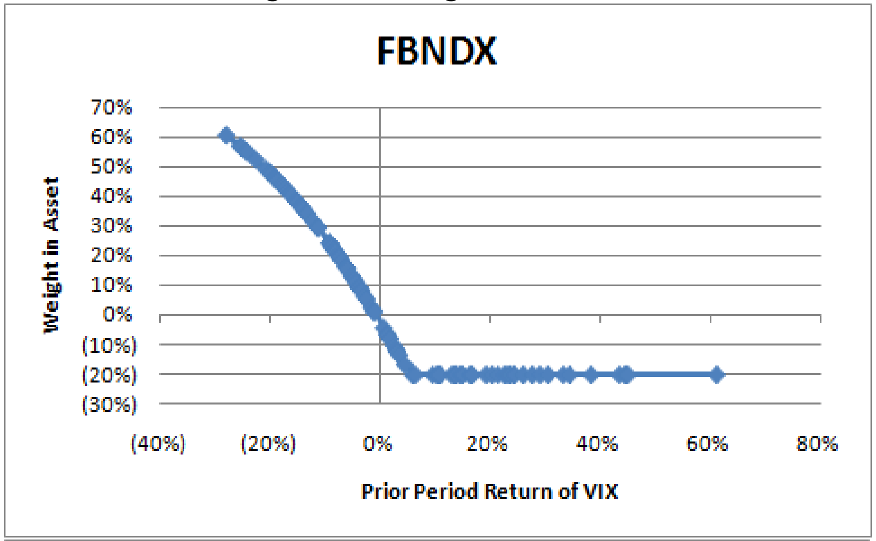
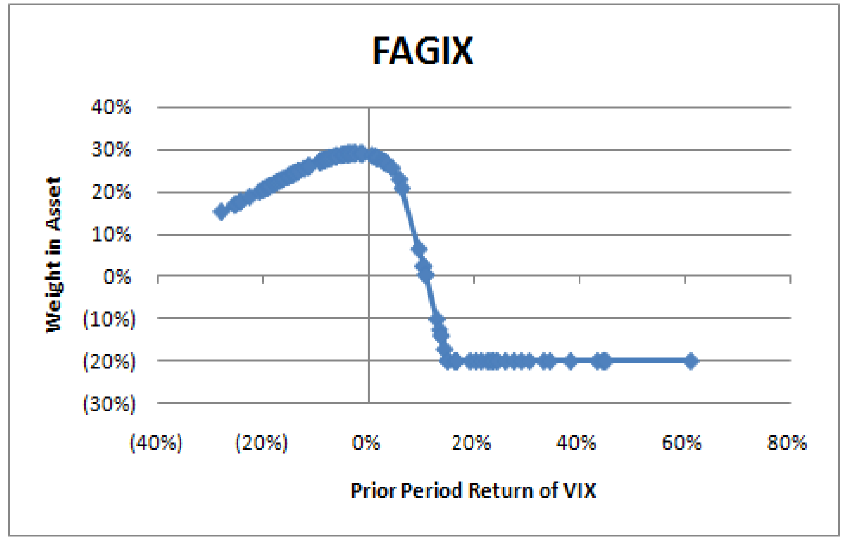
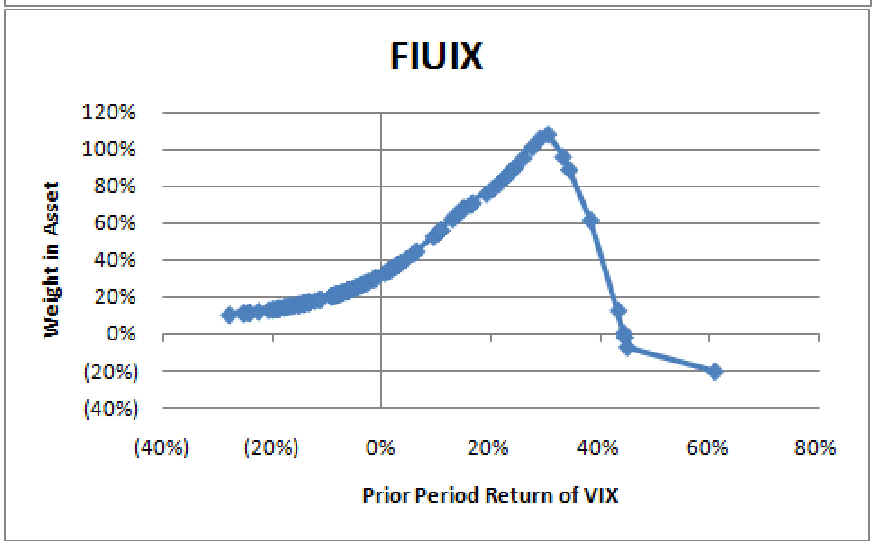
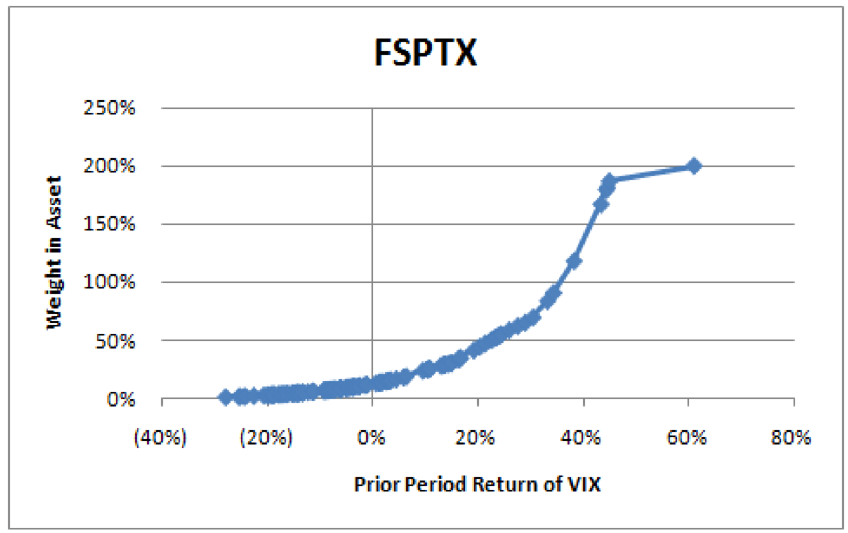

Total Period Return: 2.0% E(r)
Total Return
150.0%
100.0%
50.0%
0.0%
(50.0%)
(100.0%)
(150.0%)
Feb-00 Mar-01 Apr-02 Jun-03 Jul-04 Aug-05 Sep-06 Oct-07 Nov-08 Dec-09
VIX Opti
Standard Opti
S&P 500
Equally Weighted

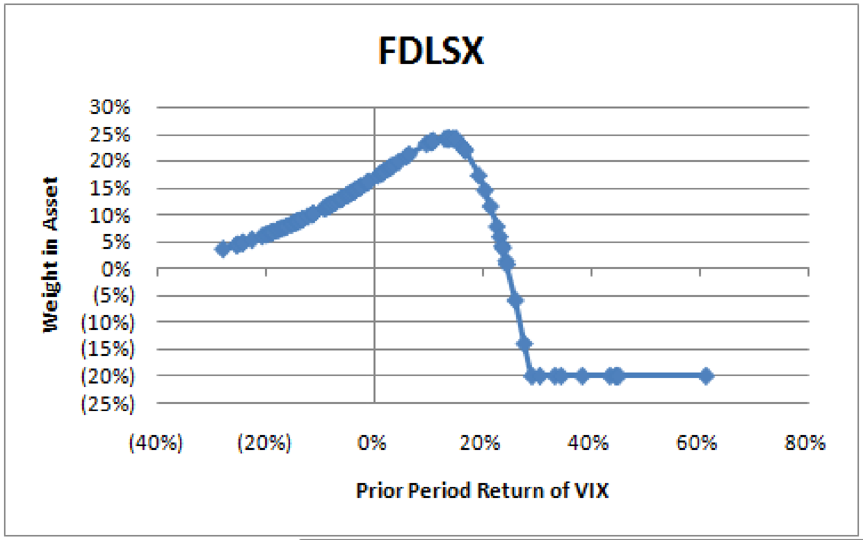
Exhibit 4: Asset Weights vs. Changes in VIX

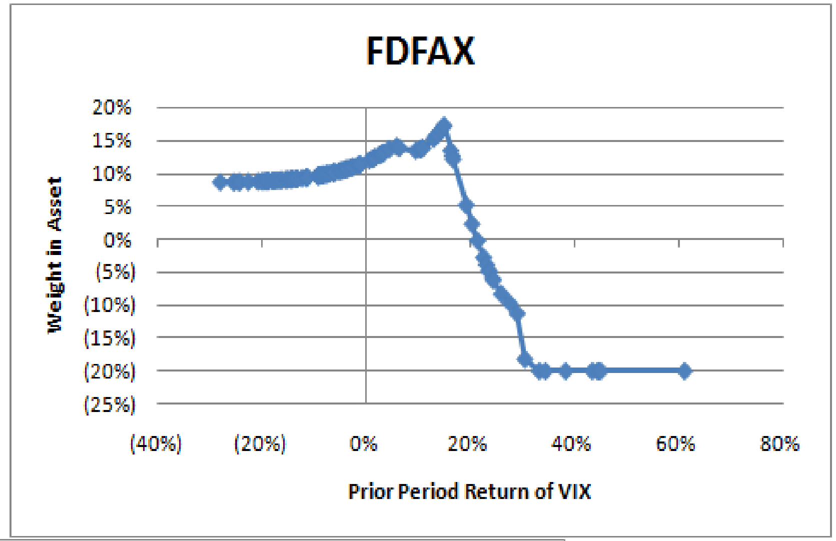


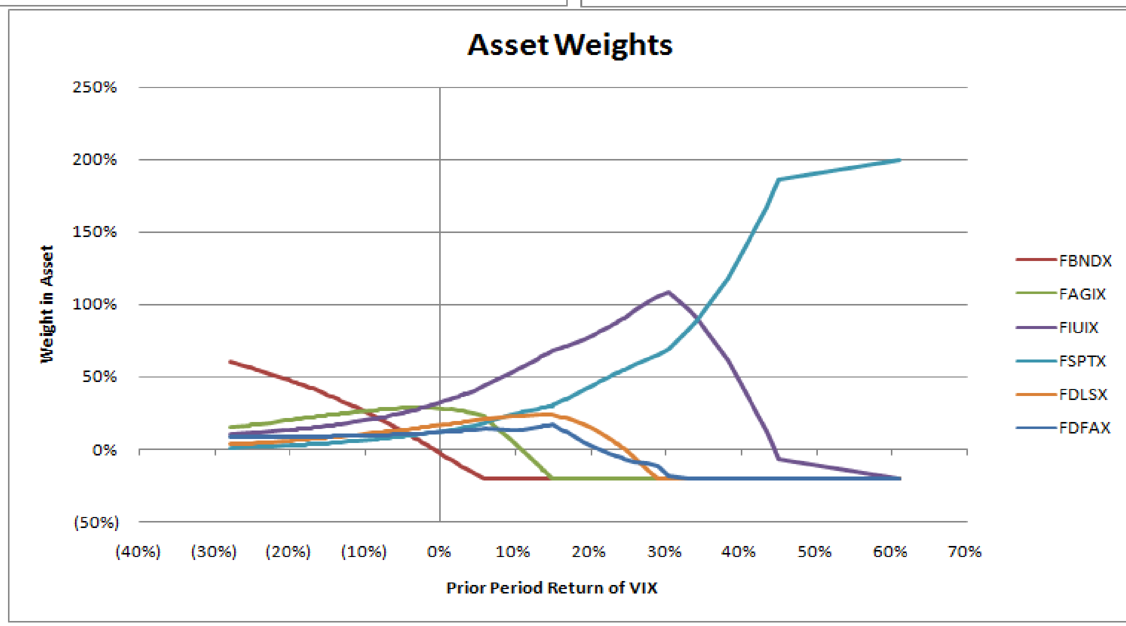







FDLSX
Weight in Asset
30%
25%
20%
15%
10%
5%
0%
(5%)
(10%)
(15%)
(20%)
(25%)
(40%)
(20%)
0%
20%
40%
60%
80%
Prior Period Return of VIX


FDFAX
Weight in Asset
20%
15%
10%
5%
0%
(5%)
(10%)
(15%)
(20%)
(25%)
(40%)
(20%)
0%
20%
40%
60%
80%
Prior Period Return of VIX


Asset Weights
Weight in Asset
250%
200%
150%
100%
50%
0%
(50%)
(40%)
(30%)
(20%)
(10%)
0%
10%
20%
30%
40%
50%
60%
70%
Prior Period Return of VIX
FBNDX
FAGIX
FIUIX
FSPTX
FDLSX
FDFAX

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