A Model for Equilibrium Consumer Search in Markets with Correlated Products

by

Mason L. Wang

Daniel Fershtman
Adviser

Economics Department

Daniel Fershtman

Adviser

Kyungmin (Teddy) Kim

Committee Member

Andreas Züfle

Committee Member

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April 9, 2025

Abstract

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Keywords: Consumer search; Bertrand competition; Pricing policy

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†Emory University. Email: mason.wang@emory.edu

1 Introduction

This paper considers a simple duopoly model. Two differentiated competing products which possess uncertain match values to the consumer are sold. Both the firms and the consumer observe the fixed correlation between the match values. After observing the posted prices, the consumer conducts a sequential search on one or both products, and ultimately purchases one of the products searched. Through each costly search, the consumer observes the value of the product and may also infer the gain of the subsequent search. The paper characterizes both the optimal consumer search strategy and some typical equilibrium pricing strategies of firms.

The foundation of this paper builds on Wolinsky (1986), which uses consumer search behavior to explain the connection between product differentiation and monopolistic competition. The paper assumes that products have independent values and consumers' expectations about values of unseen products remain constant throughout their sequential searches. The effect of correlation between product values on market outcomes is not addressed in that paper and remains understudied today.

This paper differs from Wolinsky (1986) by two main assumptions. Firstly, the consumer can observe the prices posted by the firms before their search. Thus, the price of a product affects not only the consumer's decision on whether to purchase after they search, but also the order and probability of being searched. Secondly, this paper assumes that both the firms and the consumer can condition their strategies on the correlation between the values of products. After searching for one product, the consumer updates the expected value of the subsequent search and then decides whether to continue searching. For example, a consumer who dislikes one electric vehicle after a test drive might decide not to test drive another similar electric vehicle but an internal combustion engine vehicle.

Key findings in this paper suggest that product correlation influences both firms' optimal pricing strategies and market structure in equilibrium. Negative product correlation and low search costs can encourage the consumer to conduct an additional search and incentivize firms to set higher prices with greater variation. Fixing the level of correlation, decreasing search cost can increase equilibrium prices and decrease the consumer's expected utility. On the other hand, increasing search cost can encourage firms to price competitively, which contrasts with Wolinsky (1986) that shows equilibrium prices approach competitive levels when search costs are sufficiently small.

The remainder of this paper is structured as follows. Section 2 introduces the model. Section 3 derives the optimal consumer search strategy and the firm's payoff. Section 4 analyzes firms' pricing strategies in equilibrium through key properties and special examples. Section 6 summarizes the paper. All omitted proofs are in the appendix.

2 The Model

There are two firms and one consumer in the market.

The firms are denoted by $i \in \{1, 2\}$. Each firm sells a unique product priced at $p_i \in \mathbb{R}_+$ to maximize expected revenue. For both firms, the cost of production is normalized to zero.

The consumer determines the order of sequential search and whether to continue searching or purchase from the searched products after each search. The consumer designs a strategy to maximize expected ex-post utility. The ex-post utility is given by

$$U = \max \left\{ U(i, n) = v_i - p_i - ns, -\infty \right\},$$

where v_i is the match value of product i that is purchased, n is the number of searches conducted before purchasing, and s is the strictly positive search cost. Note that there is no outside option, so the consumer must search at least one of the products.

The possible values of v_i are h and ℓ, where $h > \ell$. The values of both products follow an identical probability distribution, given by:

$$\Pr(v_i = h) = \mu \quad \text{and} \quad \Pr(v_i = \ell) = 1 - \mu.$$

To avoid degenerate cases in which the consumer knows the value of products before searching, we assume $\mu \in (0, 1)$.

The correlation between the values of the two products is φ, where

$$\varphi = \frac{\Pr(v_1 = \ell, v_2 = \ell) \cdot \Pr(v_1 = h, v_2 = h) - \Pr(v_1 = h, v_2 = \ell) \cdot \Pr(v_1 = \ell, v_2 = h)}{\mu(1 - \mu)}.$$

Note that $\varphi \in \left[\max\left(-\frac{\mu}{1-\mu}, -\frac{1-\mu}{\mu}\right), 1\right]$. When $\varphi = 1$, the two products have the same value to the consumer with probability 1. When $\varphi = 0$, the values of the products are independent, so the realized value of a searched product does not give any information regarding the expected value of the unsearched product. Perfect negative correlation ($\varphi = -1$) can only be achieved when $\mu = 0.5$, since $\Pr(v_1 = h, v_2 = \ell) = \Pr(v_1 = \ell, v_2 = h) = (1 - \varphi)(1 - \mu)\mu$.

The game unfolds as follows. Before the game, both firms and the consumer observe $s, \varphi, \mu, \ell,$ and h. Nature decides v_i based on μ and φ, and the results are not observed by the firms or the consumer. Then, each firm sets a price $p_i \in [0, \infty)$ simultaneously. The consumer observes the prices and decides which product to search first. After the search, the value of the product becomes known to the consumer, who decides whether to purchase the product or continue searching. If the consumer searched both products, they can decide to purchase either product with costless recall. The game concludes once a purchase is made.

3 Consumer Behavior

3.1 Optimal Search Strategy

Given the prices p_i set by the firms, the consumer chooses to search one of the products at their first decision node. Since all features of both products are identical to the consumer except for the price, the consumer decides whether to search the lower-priced or higher-priced product first. Thus, a search strategy defines three decisions when facing any given prices and value realizations: conducting an initial search of either the lower-priced or higher-priced product, deciding whether to purchase or continue searching based on the first search's result, and selecting a product to purchase if a second search is conducted.

Proposition 1 *Let p_1 denote the price of the lower-priced product, and let p_2 denote the price of the higher-priced product. The optimal consumer search strategy in this game is as follows:*

1. *Search the lower-priced product.*
2. *Search the other product if the lower-priced product has a value of ℓ and $s < (1 - \varphi)\mu(h - \ell - p_2 + p_1)$. Otherwise, purchase the lower-priced product.*
3. *Purchase the higher-priced product if it has a value of h. Otherwise, purchase the lower-priced product.*

In step 3, the optimal strategy is to choose the product that maximizes $v - p$ when both products are searched. In step 2, the consumer can infer the conditional probability of realizing high or low value for the unsearched product. Then, they compare their first deal $v - p$ with the expected value of the maximum of two deals $v - p$ minus the search cost.

Fixing the strategies in step 2 and 3, we compare the expected utility of searching the lower-priced and higher-priced product first. Intuitively, because only the prices of the products differ at step 1, it is optimal for the consumer to secure a better deal first to reduce additional search costs. Rigorous proof is provided in the appendix.

3.2 Shopping Outcomes

Given the optimal search strategy in this game, we derive the expected revenue function of the firms.

The lower-priced product will be searched first. With probability μ, it results in a value of h and is purchased without a second search. With probability $1-\mu$, it results in a value of ℓ. Only when the lower-priced product results in a value of ℓ, and $s < (1-\varphi)\mu(h-\ell-p_2+p_1)$, the consumer searches the higher-priced product. Note that since $s > 0$, the condition implies $\varphi \neq 1$. This is equivalent to:

$$p_2 - p_1 < h - \ell - \frac{s}{(1-\varphi)\mu},$$

which indicates two scenarios. When the right-hand side is strictly positive, a small price difference may induce the consumer to search both products when the lower-priced product has a low value. Otherwise, the consumer searches only the lower-priced product, regardless of the price of the higher-priced product. Intuitively, a small difference between whether liking a product would discourage the consumer to conduct a costly additional search, even when the prices of the products are the same.

Lemma 1 *When $s \geq (1-\varphi)\mu(h-\ell)$, the revenue of firm i charging p_i when the other firm charges p_j is given by:*

$$\pi_i(p_i, p_j) = \begin{cases} 0 & \text{if } p_i > p_j, \\ 0.5p_i & \text{if } p_i = p_j, \\ p_i & \text{if } p_i < p_j. \end{cases}$$

The condition $s \geq (1-\varphi)\mu(h-\ell)$ is equivalent to $h - \ell - \frac{s}{(1-\varphi)\mu} \leq 0$. Thus, only the lower-priced product is searched and receives all demand. Note that perfect correlation ($\varphi = 1$) is a special case of this. Because the values of the products are guaranteed to be the same, the consumer always search the lower-priced product only.

The other case is when $s < (1-\varphi)\mu(h-\ell-p_2+p_1)$, the consumer searches the higher-priced product if realizing a low value for the lower-priced product. The firm posting the higher price receives demand when the consumer realizes $v_1 = \ell$ and $v_2 = h$, which has a probability of $(1-\mu)(1-\varphi)\mu$.

Lemma 2 *When $\varphi \neq 1$ and $s < (1-\varphi)\mu(h-\ell)$, the expected revenue of firm i charging p_i when the other firm charges p_j is given by:*

$$\pi_i(p_i, p_j) = \begin{cases} 0 & \textit{if } p_i > p_j + h - \ell - \frac{s}{(1-\varphi)\mu}, \\ (1-\mu)(1-\varphi)\mu p_i & \textit{if } p_j < p_i \leq p_j + h - \ell - \frac{s}{(1-\varphi)\mu}, \\ 0.5 p_i & \textit{if } p_i = p_j, \\ \left[1-(1-\mu)(1-\varphi)\mu\right] p_i & \textit{if } p_j - h + \ell + \frac{s}{(1-\varphi)\mu} \leq p_i < p_j, \\ p_i & \textit{if } p_i < p_j - h + \ell + \frac{s}{(1-\varphi)\mu}. \end{cases}$$

To simplify our notation, when $\varphi \neq 1$ and $s < (1-\varphi)\mu(h-\ell)$, we let

$$\theta = h - \ell - \frac{s}{(1-\varphi)\mu} \text{ and } \lambda = (1-\mu)(1-\varphi)\mu.$$

θ is the threshold price difference. If two firms post prices that differ by more than this threshold, the consumer would not search the higher priced product regardless of the result of the lower priced product, thus the higher price product would receive demand and profit of 0. When the prices differ by less than the threshold, with probability λ, the consumer would end up buying the higher priced product. Note that in the model the products are assumed to be ex-ante identical. Thus, it is natural that $\lambda \leq 0.5$ and lower priced product always get at least the same demand as the higher priced product.

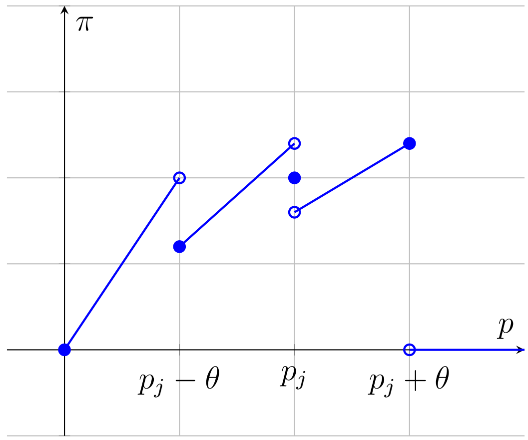


Figure 1: Graph of the expected revenue when setting a price of p while the other firm sets a price of p_j. Note that the graph is discontinuous at three points.

4 Market Equilibria

In this game, the competing firms set prices simultaneously first, and the consumer searches subsequently. We have already fully characterized the consumer's strategy in equilibrium. We also derived the payoff functions of firms by inferring the consumer's behavior in the equilibrium. Now we derive key properties of the firms' pricing strategies in the equilibrium.

Proposition 2 *When $s \geq (1-\varphi)\mu(h-\ell)$, both firms charge a price of 0 in equilibrium.*

Following the payoff function from Lemma 2, when the search cost is significantly big, the higher-priced product receives no demand. If the opponent chooses a strictly positive price, the best response is to undercut and become the lower-priced product. Following the argument in the proof of the pricing equilibrium in Bertrand competition, we can show that both firms charge a competitive price in the unique equilibrium.

Proposition 3 *When $s < (1-\varphi)\mu(h-\ell)$, there exists no pure strategy equilibrium.*

The best responses to the opponent's price p_j must be one of $p_j - \theta - \varepsilon$, $p_j - \varepsilon$, or $p_j + \theta - \varepsilon$, where ε is infinitesimally small and less than θ. Thus, it is impossible for two prices to be mutually best responses. Intuitively, there is still an incentive to slightly undercut the opponent to become the lower-priced product and win the majority of demand. However, his case differs with when $s \geq (1-\varphi)\mu(h-\ell)$ as 0 is never a best response. This means that when the opponent is sacrificing much price for the demand, it becomes profitable to post a high price for the minority of the demand. The proof is provided in the appendix.

Since there is no pure strategy equilibrium when $s < (1-\varphi)\mu(h-\ell)$, we restrict our attention to symmetric mixed-strategy Nash equilibria.

Several properties about the symmetric mixed-strategy Nash equilibria in this game should be noted. We can show that in equilibrium, no firm would choose a specific price with a strictly positive probability. If the support of the probability density function of mixed-strategy is continuous, we can also show that there exist a lower bound and upper bound for the length of support. However, we should note that we cannot easily rule out the possibility of a gap in support generally by using the methods used in classical equilibrium consumer search models.

Observation 1 *When $s < (1-\varphi)\mu(h-\ell)$, there is no symmetric equilibrium in which the firms choose a specific price with a strictly positive probability.*

We first consider when $\varphi \neq 0.5$. Let p_1 be the lowest price that is chosen with strictly positive probability. Then $p_1 - \varepsilon$ yields a profit that is higher than equilibrium profit. We can safely assume that $p_1 > 0$, because setting a price of 0 is never a best response. If p_1 is chosen with probability r, then we expect to earn an extra profit of $r((1-\lambda)(p_1-\varepsilon)-0.5p_1)$ as $p_1-\varepsilon$ yields more demand than p_1 when the opponent chooses p_1. If the opponent chooses $p_1+\theta$, choosing $p_1-\varepsilon$ may also yield more demand than p_1. When the opponent chooses any other price, $p_1-\varepsilon$ yields same demand as p_1 and the loss in profit converges to 0 as $\varepsilon \to 0$. As $\varepsilon \to 0$, we can see that profit earned by $p_1-\varepsilon$ converges to a value higher than expected profit earned by choosing p_1.

Then we assume that $\varphi = 0.5$. Let p_1 be the lowest prices that is chosen with strictly positive probability. If $p_1+\theta$ is not chosen with strictly positive probability, then choosing

$p_1+\varepsilon$ yields same demand as p_1. A higher price yields strictly more profit. If $p_1+\theta$ is chosen with strictly positive probability, then choosing $p_1-\varepsilon$ yields more demand than p_1 when the opponent chooses $p_1+\theta$. When the opponent chooses any other price, $p_1-\varepsilon$ yields same demand as p_1 and the loss in profit converges to 0. So as $\varepsilon \to 0$, $p_1-\varepsilon$ earns more than p_1.

Observation 2 *Assume that $s < (1-\varphi)\mu(h-\ell)$ and the support for mixed-strategy prices is continuous, then the minimum and maximum price on the support must differ by at least θ.*

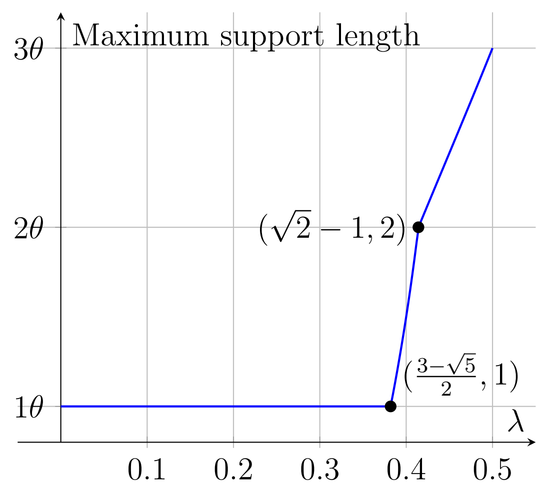
This follows the intuition that when the support is less than θ, then the highest price on the support always receive a demand of $1-\lambda$ because both products would be searched. It is then guaranteed to be profitable to increase price until reaching exactly θ above the lowest price on the support. Thus, a support length less than θ cannot form a symmetric mixed-strategy equilibrium.

However, when the support length reaches θ, posting a price higher than the maximum price on the support is not guaranteed to be profitable. If the opponent poses a price that is lower by more than θ, the higher priced product would not be searched and would receive a demand of zero.

Observation 3 *Assume that $s < (1-\varphi)\mu(h-\ell)$ and the support for mixed-strategy prices is continuous, then the minimum and maximum price on the support must differ by less than 3θ.*

The proof is in the appendix. The intuition we use is to compare the payoff of lower bound $\underline{p}$ and upper bound $\overline{p}$ with interior points $\underline{p}+\theta$ and $\overline{p}-\theta$ respectively. Since $\overline{p}$ earns zero profit against prices below $\overline{p}-\theta$, we know that $\overline{p}$ must yield higher profit than $\overline{p}-\theta$ against prices in $[\overline{p}-\theta, \overline{p}]$. Yet $\overline{p}$ earns less demand. Thus, the higher the $\overline{p}$, the more profit it loses compared to $\overline{p}-\theta$. We thus find the upper bound for $\overline{p}$ in terms of λ and θ.

We can conduct a similar analysis to calculate for the lower bound for $\underline{p}$. Taking the difference, we arrive at the following graph for maximum possible support length if the support is continuous.



In general, we cannot rule out the possibility of gaps in the support of mixed-strategies. In classical search models, when a gap exists in the opponent's mixed-strategy, the firm can increase some prices (for example the maximum price below the gap) without losing any demand. However, this is not true for the model in this paper. This is due to the three discontinuities of the payoff function. If the opponent would not set prices slightly above a specific price, it does not mean that increasing from that price can increase payoff without decreasing demand. Demand can decrease when the new price is no longer the significantly low price receives full demand, or becomes the significantly high price that will never be searched.

4.1 Special Equilibria

In this paper, we focus on symmetric mixed-strategy equilibria with continuous support. With three special cases, we can derive the unique equilibrium. The uniqueness allows us to directly compare the effects of search costs and correlation coefficient. Those special equilibria derived can be applied to construct equilibria in more general cases.

Proposition 4 *Suppose that the values of the products are perfectly correlated ($\varphi = 1$), both firms charge the competitive price in the unique symmetric equilibrium.*

This is a Bertrand competition, which is a special case of $s \geq (1-\varphi)\mu(h-\ell)$. Intuitively, when the products are guaranteed to have the same value, the consumer would only search the product with the lower price. Thus, the firms compete for the demand by charging a low price. In equilibrium, both firms charge the competitive price. We can see that search cost has no effect on the equilibrium, as no matter how low the search cost is, the firm posting the higher price would have zero demand. While the consumer minimize their search cost by conducting only one search, increasing search cost would decrease the consumer's expected utility.

Proposition 5 *Suppose that the values of the products are independent ($\varphi = 0$).*

If $s \geq \mu(h-\ell)$, then both firms charge the competitive price in the unique symmetric equilibrium.

Otherwise , there exists a unique symmetric mixed-strategy equilibrium with continuous support. Suppose $h = 1$, $\ell = 0$, $\mu = 1/2$, and $s < 1/2$, the cumulative density function of the mixed strategy is

$$F(p) = \begin{cases} \frac{3(2p+2s-1)}{4p}, & p \in \left[\frac{1}{2}(1-2s), \frac{3}{2}(1-2s)\right] \\ 0, & \textit{otherwise} \end{cases}$$

From Observation 2 and 3, we know that if symmetric mixed-strategy equilibrium with continuous support exists when $s \geq \mu(h-\ell)$, its support length must be θ. After we obtain the CDF of the mixed-strategy, we test for whether there exists any profitable deviations. The values $h = 1$, $\ell = 0$, $\mu = 1/2$ are only used to maintain the simplicity of the results and make further comparisons clear. The proof is provided in the appendix.

We observe that the search cost have continuous and negative effect on prices. As search cost increases from 0, the price level and support length both decrease. And as search cost becomes sufficiently big, the equilibrium price is 0.

While increasing the search cost directly decreases equilibrium price and revenue for the firms, it has mixed effects on the consumer's expected utility. Increasing search cost decreases the consumer's expected utility as they spend more on searching, but increases the utility lowering equilibrium prices. In this case of independent values, the two effects cancel each other, giving the consumer an equilibrium utility of 0 if $h = 1$, $\ell = 0$, $\mu = 1/2$, and $s < 1/2$. When $s \geq 1/2$, the consumer begins to experience expected negative utility.

We can generalize the approach to find equilibrium in other levels of correlation. Setting the support length to be θ, we can find the mixed-strategy equilibrium when $\varphi \geq -0.62$. When the correlation is more negative, the share of demand to the firm setting the higher price is more. It is then relatively more likely for the firms to set a higher price on the support. This gives the opposing firm incentive to charge a price that is slightly below the support. While sacrificing price by a small magnitude, the firms wins full rather than partial demand when the opponent chooses a high price.

Proposition 6 *Suppose that the values of the products are perfectly negative correlated ($\varphi = -1$).*

If $s \geq 2\mu(h - \ell)$, then both firms charge the competitive price in the unique symmetric equilibrium.

Otherwise , there exists a unique symmetric mixed-strategy equilibrium with continuous support. Suppose $h = 1$, $\ell = 0$, $\mu = 1/2$, and $s < 1$, the cumulative density function of the mixed strategy is

$$F(p) = \begin{cases} \frac{-2+\sqrt{2}p+2s}{\sqrt{2}(1+p-s)}, & p \in \left[\sqrt{2}(1-s), (\sqrt{2}+1)(1-s)\right] \\ \frac{2p+(3+\sqrt{2})(-1+s)}{-1+p+s}, & p \in \left[(\sqrt{2}+1)(1-s), (\sqrt{2}+2)(1-s)\right] \\ 0, & \textit{otherwise} \end{cases}$$

Combining observation 2 and 3, we know that if there exist symmetric mixed-strategy equilibrium with continuous support when $\varphi = -1$ and $s < \mu(h - \ell)$, it must have support length in $[\theta, 3\theta]$, where $\theta = h - l - \frac{s}{2\mu} > 0$.

Additionally, we observe that when $\varphi = 0.5$, the support length cannot be less than 2θ. For any price that is less than or equal to θ away from any price on the support, it always yields a demand of 0.5 either being search first or secondly. If there are multiple prices that satisfies this condition, the higher price would dominate a lower price. Thus, when $\varphi = 0.5$, support length must be greater or equal to 2θ.

We can also prove that the support length cannot be greater than 2θ. The proof is provided in the appendix. The intuition is that if the length of the support is between 2θ and 3θ, then the lowest price on the support would earn lower payoff than a slightly higher price except when the opponent would likely post a price in the middle of the support. However, if this is the case, the highest price on the support would not earn as much as a price slightly below that.

Setting the length of support to be 2θ, we can find the probability of the price to be from the lower half and the upper half. We find the equilibrium payoff and derive the CDF on the lower and upper half separately. We then test for whether are profitable deviations from the equilibrium. The proof is illustrated in the appendix.

The technique to construct the mixed-strategy equilibrium with support length of 2θ applies to all $\lambda > 0.414$ which is equivalent to $\theta > -0.657$. This corresponds to our Observation 3.

Again, we can observe a continuous and negative impact of search cost on equilibrium prices. The higher the search cost, the lower the price level and the shorter the support length will be. As search cost is sufficiently big, the equilibrium price becomes 0 and the firms do not earn expected profit.

While it is trivial that when $s \geq \mu(h - \ell)$ increasing search cost would decrease consumer utility due to a lack of outside options, it is crucial to note that when $s < \mu(h - \ell)$, increasing search cost increases consumer utility. When $s < \mu(h - \ell)$, increasing search cost would drive down the equilibrium prices. Even though the consumer spend more on searching, the consumer receives more net expected utility as the search cost increases. The consumer's expected utility with respect to the search cost can be viewed as a continuous concave function, with its maximum at $s = \mu(h - \ell)$.

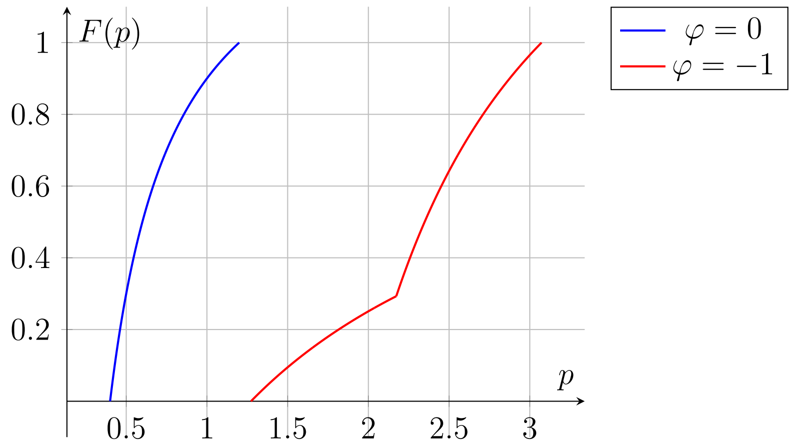


Figure 2: Cumulative distribution functions (CDFs) of the unique mixed strategy equilibrium for the cases where product values are independent ($\varphi = 0$) and perfectly negatively correlated ($\varphi = -1$), assuming a small search cost ($s = 0.1$).

Comparing the cases where product values are perfectly correlated, independent, and perfectly negatively correlated, we can see that in the price level is the highest and the support length is the longest is the case of perfect negative correlation. The support level and support length are both proportional to θ, which is increasing in μ and $h - \ell$, decreasing in s.

Overall, we can see that when the correlation between product values changes from perfectly negative correlation to perfect correlation, the maximum support length is more and more restricted. We can also see that if φ belongs to a range for which we have characterized a continuous mixed-strategy equilibrium, increasing φ would decrease both the support level and the support length.

5 Discussion

This paper identifies the optimal consumer search strategy in a duopoly market with complete information on prices and product correlation. We derive conditions on the search cost and correlation coefficient that determine whether market competition becomes perfectly competitive. Through several illustrative examples, we show that low search costs can enable firms to charge high prices in equilibrium, thereby reducing consumer utility despite lower expenditures on search and an increased likelihood of purchasing a preferred product. Furthermore, we find that high correlation between products limits firms' ability to randomize prices over a wide interval in equilibrium.

These findings contribute to a better understanding of market design. For instance, online recommendation systems that suggest similar products may induce firms to set more competitive prices, as the conditions for Bertrand competition may be met when product correlation is high. Conversely, certain features of online markets that reduce consumer search costs may inadvertently harm consumer welfare, as firms have an incentive to set high prices while still attracting consumers.

Nevertheless, this paper has several limitations. A key constraint built into the model is the absence of outside options. This restricts the analysis of how search costs affect consumer utility when such costs are significantly high and exceed the product's value.

Another limitation is that the paper does not characterize the full set of equilibria across all combinations of search costs and correlation levels. This limitation arises due to the discontinuities in the payoff function at $p \pm \theta$.

For example, we cannot rule out the existence of gaps in the equilibrium support by appealing to standard arguments, that some prices on the support can be increased without losing demand. In our model, the change in demand depends not only on the ε-neighborhood of p, but also on the ε-neighborhood of $p \pm \theta$.

The analysis also encounters challenges in constructing equilibria whose support lengths are not integer multiples of θ. For example, in attempting to construct an equilibrium with support length in $(\theta, 2\theta)$, a high probability of price realization near the midpoint of the support renders a downward deviation profitable, as charging a price just below the support has a high likelihood of undercutting the opponent by θ.

Future research may consider extending the model by introducing outside options. To address the discontinuities at $p \pm \theta$, future work may also introduce uncertainty in consumer search behavior, particularly when consumers face price differences near their indifference threshold for conducting additional searches.

6 Appendix

Proposition 1 *Let p_1 denote the price of the lower-priced product, and let p_2 denote the price of the higher-priced product. The optimal consumer search strategy in this game is as follows:*

1. *Search the lower-priced product.*

2. *Search the other product if the lower-priced product has a value of ℓ and $s < (1-\varphi)\mu(h-\ell-p_2+p_1)$. Otherwise, purchase the lower-priced product.*

3. *Purchase the higher-priced product if it has a value of h. Otherwise, purchase the lower-priced product.*

Proof. In step 3, the optimal strategy is to choose the product that maximizes $v-p$ when both products are searched. In step 2, the consumer searches if and only if the expected net utility of step 3 is greater than the net utility of purchasing at step 2. Finally, fixing steps 2 and 3, we compare strategies of searching the lower-priced product in step 1 (denoted as $S1$) versus searching the higher-priced product in step 1 (denoted as $S2$).

The expected utility of s_1 is $\alpha(h-p_1-s)+(1-\alpha)\max\{(1-\varphi)\alpha(h-p_2)+[1-(1-\varphi)\alpha](l-p_1)-2s, l-p_1-s\}$

The expected utility of s_2 is $\alpha\max\{h-p_2-s,(1-\varphi)(1-\alpha)(h-p_2)+[1-(1-\varphi)(1-\alpha)](h-p_1)-2s,(1-\varphi)(1-\alpha)(l-p_1)+[1-(1-\varphi)(1-\alpha)](h-p_1)-2s\}+(1-\alpha)\max\{l-p_2-s,[1-(1-\varphi)\alpha]l+(1-\varphi)\alpha h-p_1-2s\}$

define u_1, u_2, u_3, u_4 as following

$u_1 = \alpha(h-p_1-s)$

$u_2 = (1-\alpha)\max\{(1-\varphi)\alpha(h-p_2)+[1-(1-\varphi)\alpha](l-p_1)-2s, l-p_1-s\}$

$u_3 = \alpha\max\{h-p_2-s,(1-\varphi)(1-\alpha)(h-p_2)+[1-(1-\varphi)(1-\alpha)](h-p_1)-2s,(1-\varphi)(1-\alpha)(l-p_1)+[1-(1-\varphi)(1-\alpha)](h-p_1)-2s\}$

$u_4 = (1-\alpha)\max\{l-p_2-s,[1-(1-\varphi)\alpha]l+(1-\varphi)\alpha h-p_1-2s\}$

We want to show that $u_1 + u_2 > u_3 + u_4$.

Firstly, note that when $u_4 = (1-\alpha)(l - p_2 - s)$, $\implies u_1 > u_3$ and $u_2 > u_4$

Then, suppose $u_4 = (1-\alpha)([1-(1-\varphi)\alpha]l + (1-\varphi)\alpha h - p_1 - 2s)$, $u_4 - u_2 = \min\{(1-\alpha)(1-\varphi)\alpha(p_2 - p_1), (1-\alpha)(1-\varphi)\alpha(h-l) - (1-\alpha)s\}$

$u_1 - u_3 = \min\{\alpha(p_2 - p_1), \alpha(1-\varphi)(1-\alpha)(p_2 - p_1) + \alpha s, \alpha(1-\varphi)(1-\alpha)(h-l) + \alpha s\}$

Note that in $u_1 - u_3$, $\alpha(1-\varphi)(1-\alpha)(p_2-p_1)+\alpha s, \alpha(1-\varphi)(1-\alpha)(h-l)+\alpha s$ are strictly greater than $(1-\alpha)(1-\varphi)\alpha(p_2 - p_1), (1-\alpha)(1-\varphi)\alpha(h-l) - (1-\alpha)s$ respectively, since $s > 0$. Also, $\alpha(p_2 - p_1) \geq (1-\alpha)(1-\varphi)\alpha(p_2 - p_1)$

Thus, searching the lower-priced product first is the dominating strategy. ■

Proposition 3 *When $\varphi \neq 1$ and $s < (1-\varphi)\mu(h-\ell)$, there exists no pure strategy equilibrium.*

Proof. Let $\theta := h - \ell - \frac{s}{(1-\varphi)\mu} > 0$, and let $\varepsilon \in (0, \theta)$.

Assume that firms i and j choose prices p_i and p_j in a pure strategy equilibrium. Then p_i is a best response to p_j and must equal one of the following:

$$p_j - \theta - \varepsilon, \quad p_j - \varepsilon, \quad \text{or} \quad p_j + \theta - \varepsilon,$$

where ε is a infinitesimally small value and $\varepsilon < \theta$. Note that at least one of the three values would be strictly positive.

- If $(p_i = p_j - \theta - \varepsilon, p_j)$ are equilibrium strategies, firm j would receive expected revenue of 0 and could decrease the price to obtain positive demand.

- If $(p_i = p_j - \varepsilon, p_j)$ are equilibrium strategies, firm j could increase the price without losing demand.

- If $(p_i = p_j + \theta - \varepsilon, p_j)$ are equilibrium strategies, firm j could also increase the price without losing demand.

Thus, there exists no pair (p_i, p_j) such that p_i and p_j are mutual best responses. ■

Observation 3 *Assume that* $s < (1-\varphi)\mu(h-\ell)$ *and the support for mixed-strategy prices is continuous, then the minimum and maximum price on the support must differ by less than* 3θ.

Proof. Let $\underline{p}$ and $\overline{p}$ be the lowest and highest price on the support of the equilibrium mixed-strategy respectively.

First, let's assume that $\overline{p} - \underline{p} > 2\theta$.

We compare the profit of choosing $\underline{p}$ and $\underline{p} + \theta$. If the opponent chooses $p \geq \underline{p} + 2\theta$, choosing $\underline{p} + \theta$ yields strictly higher profit. For $\underline{p}$ and $\underline{p} + \theta$ to have equal expected profit, $\underline{p}$ should yield more profit than $\underline{p} + \theta$ either when opponent chooses between $(\underline{p}, \underline{p} + \theta)$ or $(\underline{p} + \theta, \underline{p} + 2\theta)$. We thus get the inequalities:

$$(1-\lambda)\underline{p} > \lambda(\underline{p} + \theta) \text{ or } \underline{p} > (1-\lambda)(\underline{p} + \theta),$$

which is equivalent to

$$\underline{p} > \frac{\lambda}{1-2\lambda}\theta \text{ or } \underline{p} > \frac{1-\lambda}{\lambda}\theta.$$

We then compare the profit of choosing $\overline{p}$ and $\overline{p} - \theta$. When the opponent chooses $p < \overline{p} - \theta$, choosing $\overline{p}$ yields 0. So, $\overline{p}$ must yield more expected profit than $\overline{p} - \theta$ when opponent chooses on $(\overline{p} - \theta, \overline{p})$. Since $\overline{p}$ only makes strictly positive profit when opponent chooses on $(\overline{p} - \theta, \overline{p})$, we also need to make sure it yields more profit than $\overline{p} - 2\theta$. We get the inequalities:

$$\lambda\overline{p} > (1-\lambda)(\overline{p} - \theta) \text{ and } \lambda\overline{p} > \overline{p} - 2\theta,$$

which is equivalent to

$$\overline{p} < \frac{1-\lambda}{1-2\lambda}\theta \text{ and } \overline{p} < \frac{2}{1-\lambda}\theta.$$

We can graph the function of minimum lower bound as $\min(\frac{\lambda}{1-2\lambda}\theta, \frac{1-\lambda}{\lambda}\theta)$ and function of maximum upper bound as $\min(\frac{1-\lambda}{1-2\lambda}\theta, \frac{2}{1-\lambda}\theta)$.

We can see that only when $\lambda > \sqrt{2} - 1 \approx 0.414$, interval length can be greater than 2θ.

Interval length can never be greater than 3θ.

Now, we assume that $\theta < \overline{p} - \underline{p} < 2\theta$. Using a similar analysis, we get

$$\underline{p} > \min(\frac{\lambda}{1-2\lambda}\theta, \frac{1-\lambda}{\lambda}\theta) \text{ and } \overline{p} < \frac{1-\lambda}{1-2\lambda}\theta.$$

To conclude, when $\lambda < \frac{3-\sqrt{5}}{2} \approx 0.38$, the only possible interval length is θ. When $\lambda \in (0.38, 0.414)$, the maximum interval length is between θ and 2θ. When $\lambda > 0.414$, the maximum interval length is between 2θ and 3θ. ∎

Proposition 5 *Suppose that the values of the products are independent ($\varphi = 0$).*

If $s \geq \mu(h - \ell)$, then both firms charge the competitive price in the unique symmetric equilibrium.

Otherwise , there exists a unique symmetric mixed-strategy equilibrium with continuous support. Suppose $h = 1$, $\ell = 0$, $\mu = 1/2$, and $s < 1/2$, the cumulative density function of the mixed strategy is

$$F(p) = \begin{cases} \frac{3(2p+2s-1)}{4p}, & p \in \left[\frac{1}{2}(1-2s), \frac{3}{2}(1-2s)\right] \\ 0, & \textit{otherwise} \end{cases}$$

Proof. We suppose $s < \mu(h - \ell)$. Otherwise, we apply Proposition 2.

Suppose $\overline{p} - \underline{p} = \theta$ and the mixed strategy follows a distribution defined by the CDF $F(p)$. Then, as prices in the interval yield the same expected payoff,

$$(1-\lambda)\underline{p} = \lambda(\underline{p} + \theta).$$

We can now express $\underline{p}$ as $\underline{p} = \frac{\lambda\theta}{1-2\lambda}$ and the equilibrium payoff π^* as $\pi^* = \frac{\lambda(1-\lambda)\theta}{1-2\lambda}$. Each price in the interval yields the same expected equilibrium payoff. Thus,

$$F(p)\lambda p + [1 - F(p)](1-\lambda)p = \frac{\lambda(1-\lambda)\theta}{1-2\lambda}.$$

We get

$$F(p) = \frac{(1-\lambda)p - \frac{\lambda(1-\lambda)}{1-2\lambda}\theta}{(1-2\lambda)p}.$$

Next, we test if firms have an incentive to deviate from this interval and CDF. When the competing firm mixes on an interval of length θ, the prices $\underline{p} - \theta$, $\underline{p} - \Delta$ (where $\Delta \in (0, \theta)$), and $\overline{p} + \Delta$ dominate other price candidates.

The price $p = \underline{p} - \theta$ yields

$$\frac{(3\lambda - 1)\theta}{1 - 2\lambda},$$

which is greater than the equilibrium payoff when $\lambda > \sqrt{2} - 1$.

The price $p = \underline{p} - \Delta$ yields

$$\underline{\pi}(\Delta) = F\left(\frac{\lambda\theta}{1-2\lambda} - \Delta + \theta\right)(1-\lambda)\left(\frac{\lambda\theta}{1-2\lambda} - \Delta\right) + \left[1 - F\left(\frac{\lambda\theta}{1-2\lambda} - \Delta + \theta\right)\right]\left(\frac{\lambda\theta}{1-2\lambda} - \Delta\right),$$

where $0 \leq \Delta \leq \lambda$.

Taking the difference with the equilibrium expected revenue, downward deviation by Δ gives extra expected revenue of

$$\underline{T}(\Delta) = \frac{\Delta(\Delta(1 - 5\lambda + 7\lambda^2 - 2\lambda^3) + \theta(-1 + 4\lambda - 5\lambda^2 + 3\lambda^3))}{(-1 + 2\lambda)(\Delta + \theta(-1 + \lambda) - 2\Delta\lambda)}.$$

Taking the derivative of $\underline{T}$ with respect to Δ, we get

$$\frac{\partial \underline{T}}{\partial \Delta} = \frac{\Delta^2(1-2\lambda)^2(1 - 3\lambda + \lambda^2) - 2\theta\Delta(1 - 6\lambda + 12\lambda^2 - 9\lambda^3 + 2\lambda^4) + \theta^2(1 - 5\lambda + 9\lambda^2 - 8\lambda^3 + 3\lambda^4)}{(-1 + 2\lambda)(\Delta + \theta(-1 + \lambda) - 2\Delta\lambda)^2}.$$

At $\Delta = 0$,

$$\frac{\partial \underline{T}}{\partial \Delta} = \frac{\theta^2(1 - 5\lambda + 9\lambda^2 - 8\lambda^3 + 3\lambda^4)}{(-1 + 2\lambda)(\theta(-1 + \lambda))^2}.$$

When $0 \leq \lambda < \lambda_1 = \text{Root}(-1 + 4\lambda - 5\lambda^2 + 3\lambda^3, 1) \approx 0.4056$, $\underline{T}(\Delta) < 0$ for any $\Delta \leq \theta$. Thus, downward deviation is not profitable.

Similarly, we derive the extra expected revenue of $\overline{p}+\Delta$ as

$$\overline{T}(\Delta)=\frac{\Delta\lambda\left(\Delta\lambda\left(-1+2\lambda\right)+\theta\left(-1+3\lambda-3\lambda^2\right)\right)}{\left(1-2\lambda\right)\left(\Delta+\theta\lambda-2\Delta\lambda\right)}.$$

Since the numerator is non-positive and the denominator is positive in the defined range of λ, there is no incentive for upward deviation.

Substituting θ and λ with original parameters, we derive the conditions under which this type of mixed-strategy equilibrium holds. ■

Lemma *When $\varphi=-1$ and $s<2\mu(h-\ell)$, if the symmetric mixed-strategy Nash equilibrium with continuous support exists, it cannot have a support length between 2θ and 3θ.*

Proof. Support that the lowest price on the continuous support is $\underline{p}$, we can express the highest price on the support as $\underline{p}+2\theta+t$, where $t<\theta$. We aim to show a contradiction.

Let a denote the probability that a price between $\underline{p}$ and $\underline{p}+\theta$ is chosen in the equilibrium. Let b denote the probability that a price between $\underline{p}+\theta+t$ and $\underline{p}+2\theta+t$ is chosen in the equilibrium. Then the probability that a price is chosen between $\underline{p}+\theta$ and $\underline{p}+\theta+t$ is $1-a-b$. Note that no price should can be chosen with strictly positive probability.

First we compare the expected payoff for $\underline{p}$ and $\underline{p}+t$. Only when the opponent chooses a price between $\underline{p}+\theta$ and $\underline{p}+\theta+t$, $\underline{p}$ can yield a greater profit than $\underline{p}+t$. We can set up and simplify a equation based on the equivalence of expected payoffs to get

$$0.5(1-a-b)(\underline{p}-t)=0.5at+bt.$$

Similarly, we compare the expected payoff for $\underline{p}+2\theta$ and $\underline{p}+2\theta+t$. Both earn zero payoffs against prices between $\underline{p}$ and $\underline{p}+\theta$. $\underline{p}+2\theta$ yields more payoff than $\underline{p}+2\theta+t$ when the opponent's price is between $\underline{p}+\theta$ and $\underline{p}+\theta+t$, and less payoff when the opponent's price is $\underline{p}+\theta+t$ and $\underline{p}+2\theta+t$. We can set up and simplify a equation based on the equivalence of expected payoffs to get

$$0.5(1-a-b)(\underline{p}+2\theta)=0.5bt.$$

The two equations contradict each other. Thus, the support length cannot be between 2θ and 3θ. ■

Proposition 6 *Suppose that the values of the products are perfectly negative correlated ($\varphi=-1$).*

If $s\geq 2\mu(h-\ell)$, then both firms charge the competitive price in the unique symmetric equilibrium.

Otherwise , there exists a unique symmetric mixed-strategy equilibrium with continuous support. Suppose $h=1$, $\ell=0$, $\mu=1/2$, and $s<1$, the cumulative density function of the mixed strategy is

$$F(p)=\begin{cases}\frac{-2+\sqrt{2}p+2s}{\sqrt{2}(1+p-s)}, & p\in\left[\sqrt{2}(1-s),(\sqrt{2}+1)(1-s)\right]\\ \frac{2p+(3+\sqrt{2})(-1+s)}{-1+p+s}, & p\in\left[(\sqrt{2}+1)(1-s),(\sqrt{2}+2)(1-s)\right]\\ 0, & otherwise\end{cases}$$

Proof. Mathematica is used for the derivation of the mixed-strategy equilibrium.

```
ClearAll["Global‘*"]
underlineppayoff = fofunderlinepplustheta (1 - \[Lambda]) underlinep +
  (1 - fofunderlinepplustheta) underlinep;
underlineppayoff = Simplify[underlineppayoff]

midpointpayoff = fofunderlinepplustheta \[Lambda] (underlinep + \[Theta]) +
  (1 - fofunderlinepplustheta) (1 - \[Lambda]) (underlinep + \[Theta]);
midpointpayoff = Simplify[midpointpayoff]
```

```
overlineppayoff = (1 - fofunderlinepplustheta) \[Lambda] (underlinep + 2 \[Theta]);
overlineppayoff = Simplify[overlineppayoff]

Reduce[underlineppayoff == midpointpayoff &&
  midpointpayoff == overlineppayoff && \[Lambda] > 0 &&
  \[Lambda] < 1/2 && \[Theta] > 0 && fofunderlinepplustheta > 0 &&
  underlinep > 0, {fofunderlinepplustheta, underlinep}]

fofunderlinepplustheta = Simplify[
  (1 + 2 \[Lambda])/(4 \[Lambda]) -
  1/4 Sqrt[(-1 + 7 \[Lambda] - 8 \[Lambda]^2 +
  4 \[Lambda]^3)/(\[Lambda]^2 (-1 + 3 \[Lambda]))]];

underlinep = Simplify[
  (\[Theta] - fofunderlinepplustheta \[Theta] - \[Theta] \[Lambda] +
  2 fofunderlinepplustheta \[Theta] \[Lambda])/
  (fofunderlinepplustheta + \[Lambda] - 3 fofunderlinepplustheta \[Lambda])];

profit = fofunderlinepplustheta (1 - \[Lambda]) underlinep +
  (1 - fofunderlinepplustheta) underlinep;
profit = Simplify[profit];
profitAtLambdaHalf = profit /. \[Lambda] -> 0.5

lowerhalfpayoff = fofp \[Lambda] p +
  (fofpplustheta - fofp) (1 - \[Lambda]) p + (1 - fofpplustheta) p;
lowerhalfpayoff = Simplify[lowerhalfpayoff]
```

```
upperhalfpayoff = (fofpplustheta - fofp) \[Lambda] (p + \[Theta]) +
  (1 - fofpplustheta) (1 - \[Lambda]) (p + \[Theta]);
upperhalfpayoff = Simplify[upperhalfpayoff]

Solve[lowerhalfpayoff == upperhalfpayoff, fofpplustheta]
fofpplustheta = (-fofp p - \[Theta] + p \[Lambda] +
  3 fofp p \[Lambda] + \[Theta] \[Lambda] + fofp \[Theta] \[Lambda])/
  (-p - \[Theta] + 3 p \[Lambda] + 2 \[Theta] \[Lambda]);
fofpplustheta = Simplify[fofpplustheta]

fofpSolution = Solve[
  -p (-1 + fofp - 2 fofp \[Lambda] + fofpplustheta \[Lambda]) == profit, fofp];
fofp = Simplify[fofp /. fofpSolution[[1]]];

fofpplustheta = Simplify[
  (-fofp p - \[Theta] + p \[Lambda] +
  3 fofp p \[Lambda] + \[Theta] \[Lambda] +
  fofp \[Theta] \[Lambda])/(-p - \[Theta] + 3 p \[Lambda] + 2 \[Theta] \[Lambda])];
fofpplustheta = Simplify[fofpplustheta];

fofsmallp = fofp
fofbigp = Simplify[fofpplustheta /. p -> p - \[Theta]]

exp6 = Simplify[fofsmallp /. p -> underlinep - \[CapitalDelta] + \[Theta]]
profitDownwardDelta = Simplify[
  exp6 (1 - \[Lambda]) (underlinep - \[CapitalDelta]) +
  (1 - exp6) (underlinep - \[CapitalDelta])]
```

```
exp7 = Simplify[fofbigp /. p -> underlinep + \[CapitalDelta] + \[Theta]]
profitUpwardDelta = Simplify[
  (1 - exp7) \[Lambda] (underlinep + 2 \[Theta] + \[CapitalDelta])]

profitDownwardDelta2[\[CurlyPhi]_, s_, \[CapitalDelta]_] = Simplify[
  profitDownwardDelta /. {\[Lambda] -> (1 - \[CurlyPhi])/4,
  \[Theta] -> 1 - 2 s/(1 - \[CurlyPhi])}]

Manipulate[
 Plot[profitDownwardDelta2[\[CurlyPhi], s, \[CapitalDelta]],
 {\[CapitalDelta], 0, 1.1}, PlotRange -> All,
 AxesLabel -> {"\[CapitalDelta]", "Expression"},
 PlotLabel -> "Adjustable Plot"],
 {{\[CurlyPhi], -1, "\[CurlyPhi] (phi)"}, -1, 1, 0.01},
 {{s, 0.4, "s"}, 0, 1, 0.01}]

phiRange = {-1, 0.99};
sRange[\[CurlyPhi]_] := {0.01, 0.5 (1 - \[CurlyPhi]) - 0.001};
DeltaRange[s_, \[CurlyPhi]_] := {0, 1 - (2 s)/(1 - \[CurlyPhi])};

phiSteps = 50; sSteps = 50; deltaSteps = 50;
phiValues = Subdivide[phiRange[[1]], phiRange[[2]], phiSteps];
results = {};

Do[
 If[\[CurlyPhi] < 1,
```

```
  sValues = Subdivide[sRange[\[CurlyPhi]][[1]],
                    sRange[\[CurlyPhi]][[2]], sSteps];
  Do[
   deltaValues = Subdivide[DeltaRange[s, \[CurlyPhi]][[1]],
                         DeltaRange[s, \[CurlyPhi]][[2]], deltaSteps];
   deltaCondition = AllTrue[deltaValues,
     With[{val = firstDeriv /. {\[CurlyPhi] -> \[CurlyPhi],
                               s -> s,
                               \[CapitalDelta] -> #}},
         Im[val] == 0 && val < 0] &];
   AppendTo[results, {\[CurlyPhi], s, If[deltaCondition, Green, Red]}];,
  {s, sValues}]], {\[CurlyPhi], phiValues}]

Graphics[{#3, Disk[{#1, #2}, 0.02]} & @@@ results,
 Axes -> True, AxesLabel -> {"\[CurlyPhi]", "s"},
 PlotLabel -> "Sign of First Order Derivative"]

profitUpwardDelta2[\[CurlyPhi]_, s_, \[CapitalDelta]_] = Simplify[
  profitUpwardDelta /. {\[Lambda] -> (1 - \[CurlyPhi])/4,
  \[Theta] -> 1 - 2 s/(1 - \[CurlyPhi])}]

Manipulate[
 Plot[profitUpwardDelta2[\[CurlyPhi], s, \[CapitalDelta]],
 {\[CapitalDelta], 0, 1.1}, PlotRange -> All,
 AxesLabel -> {"\[CapitalDelta]", "Expression"},
 PlotLabel -> "Adjustable Plot"],
 {{\[CurlyPhi], -1, "\[CurlyPhi] (phi)"}, -1, 1, 0.01},
```

```
 {{s, 0.4, "s"}, 0, 1, 0.01}]
```

```
firstDeriv = D[profitUpwardDelta2[\[CurlyPhi], s, \[CapitalDelta]], \[CapitalDelta]];
```

■

References

Asher Wolinsky. True Monopolistic Competition as a Result of Imperfect Information. *The Quarterly Journal of Economics*, 101(3):493, August 1986. ISSN 00335533. doi: 10.2307/1885694. URL https://academic.oup.com/qje/article-lookup/doi/10.2307/1885694.