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A New Machine for Thinking: Historical Epistemology in Twentieth Century France

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Abstract

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The dissertation is an historical and philosophical examination of the epistemology of science in France between 1905 (the year Einstein [1879-1955] published the quartet of papers including the germinal formulation of the special theory of relativity) and 1971 (the year Georges Canguilhem [1901-1995] retired as the director of the Institut d'Histoire des Sciences et des Techniques), dates which I argue bookend the emergence of an epistemological tradition which Dominique Lecourt (1944 -) has dubbed "historical epistemology." I argue that, within this time frame (1905-1971), a methodological consensus emerges within French philosophy according to which the epistemological norms used to evaluate the validity of scientific knowledge must be continually revised in order to account for the historical transformation of scientific concepts and methods.

I argue that two consequences proceed from this basic methodological program which have largely escaped critical notice in the Anglo-American and Francophone scholarship devoted to the subject. The first concerns a transformation of the implicit norms guiding epistemological critique. Unlike the Positivist or Neo-Kantian projects of the nineteenth century, the tradition of historical epistemology systematically rejects predetermined theories of the developmental logic of science, on the one hand, (Positivist historiography of science), and the transcendental determination of epistemic normativity on the other (Neo-Kantian *Wissenschaftslehre.*) The second concerns a transformation of the epistemological status of the history of science itself. The tradition rejects the history of science as a stable configuration, indeed, I argue that a sophisticated historiographical method unites the historical epistemologists in a common project whereby the writing of the history of science becomes a task that must be perpetually renewed. The dissertation has two primary aims. 1) To demonstrate the theoretical consistency of the tradition by determining the nature and function of the normative epistemological commitments which organize its concepts and methodologies, and, in so doing, to contribute to the history of philosophy in twentieth century France and to the history of the philosophy of science. 2) To reconstruct the variable histories of philosophy and of science which the historical epistemologists produce as the result of the epistemological determination of the nature of scientific rationality.

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I dedicate this work to the memory of my father, Paul Cauvin, my grandfather, Robert Hallatt, and my aunt, Kimberley Hallatt-Jones.

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Introduction: What is Historical Epistemology? On the Normative Force of the Historicity of Science

> Twentieth-century science has liberated history, which was enslaved by the philosophy of history to an illusory imitation of the philosophy of nature – an imitation which we know to be illusory *because twentieth-century science itself has taken the form of a history.* Thanks to the intervention of the calculation of probabilities, irreversibility (which seemed a logical scandal to the first interpreters of the principle of Sadi-Carnot) has taken on a form which men are to-day agreed in recognizing as entirely satisfactory to reason. And the primary role attributed by Einstein to the velocity of light, treated as a constant, established the solidarity of the confirmation of cosmic space with the measure of time, itself conceived as a physical operation. In this respect the general appearance of contemporary science is clearly expressed by Eddington in these words from *The Expanding Universe*: 'Thus we can scarcely isolate the thought of vast extension from the thought of time and change; and the problem of form and organization becomes merged in the problem of origin and development.' And this movement is going so far, the brilliant advances which are brought to our attention by the correlation of the growing wealth of theories and the extraordinary precision of observation are extending our spatial horizons and our temporal perspectives to such an extent, that in the same work we find a formula which a few years ago would have seemed the height of paradox: 'The original radius of the universe has been given, but we are unable to calculate the present radius.'[1]
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> -Léon Brunschvicg, "History and Philosophy"

Léon Brunschvicg (1869-1944) here observes, in a brief essay presented to Ernst Cassirer (1874-1945) on the occasion of his sixtieth birthday, the liberation of history by twentieth century science. What can the liberation of history by science possibly signify? For Brunschvicg, as it will soon become clear, scientific objectivity and historical objectivity, though necessarily informed by different epistemological

[1] Léon Brunschvicg, "History and Philosophy," Trans. Marry Morris in *Philosophy and History, Essays Presented to Ernst Cassirer,* eds., Raymond Klibansky and H.J. Paton. (London: Oxford University Press, 1936), 32.
A note on translations. I have consulted available English translations from the French wherever possible. In the case of texts only available in French, the translations are my own.

criteria, find themselves mutually clarified in their relationship to one another by an event which was itself historical, and which Brunschvicg locates, still rather ambiguously, somewhere at the beginning of the twentieth century. If history is now liberated by science it must also be the case that it was not always so, and that the problem of the relationship between history and science itself has a history. In fact, the historical evolution of this relationship does not confine itself to the relatively narrow methodological problem of isolating something like the emergence of the history of science as an academic discipline. It is not the history of science which Brunschvicg intends to elucidate, either in its objects or in its methods, but *the historicity of science* which has somehow transformed and redeemed history.

For Brunschvicg, the historicity of science is at once an epistemological and a historical configuration. Science (here understood primarily as mathematical-physics) has brought about certain fundamental transformations of the categories of reason thanks to the calculation of probabilities (inspired by Carnot's [1796-1832] theorization of irreversible thermodynamic transformations) and the modified space-time of Einsteinian physics. Carnot and Einstein (1879-1955) are historical actors in a rational sequence (the rational system of scientific concepts) which finds itself indelibly transformed by their work. The boundaries of reason in the twentieth century have been re-drawn in accord with the "growing wealth of theories and the extraordinary precision of observation" with which Einstein and his colleagues have revolutionized physics. The relationship between science and reason (at least in this passage) seems quite clear: science issues new problems to reason and determines the boundaries of what is thinkable at any given moment. Space and time were absolute categories of thought and of being for Newton; for Einstein we know that they can only be relative to a system of measurement. The objectivity of physical space, as Cassirer elsewhere observes, is definitively dissolved by Einstein.[2]

Does the historicity of science then signify the mutability of reason? The relationship between science and history, so I will argue in this dissertation, is nowhere more fraught than at this junction of epistemological and historiographic vectors. For Brunschvicg, the claim that "twentieth century science itself has taken the form of a history" says as much about the epistemological organization of mathematical physics as it does about its historical development. The historicity of science *does* imply the progressive transformation of reason in Brunschvicg, but the historical factors motivating these transformations in no way warrant a simple opposition between epistemology and history. The historicity of science is not the negation of that which is rationally normative within science. The invocation of history, as I will argue, does not reduce scientific reason to a relativism of incompatible truth-claims distributed, more or less arbitrarily, by the contingencies of historical development.[3] On the contrary, *the nature of scientific reason is made*

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[2] See Ernst Cassirer, *Substance and Function and Einstein's Theory of Relativity* Trans. William Curtis Swabey and Marie Collins Swabey (Chicago: The Open Court Publishing Company, 1923), 432.

[3] On this point witness the programmatic debate between Thomas Kuhn (1922-1996) and Karl Popper (1902-1994) in Imre Lakatos and Alan Musgrave, eds., *Criticism and the Growth of Knowledge* (Cambridge: Cambridge University Press, 1970.)

manifest by history. The epistemology of science must not oppose reason and history, in fact, as the dissertation will demonstrate, *the claim that scientific reason is historically instantiated and transforms over time is not the negation of normative epistemological criteria, but the condition of possibility of epistemological normativity.*

In order to explain the epistemological significance of *science endowed with the normative force of historicity* Brunschvicg must reframe not only the history of science but the history of philosophy. In "History and Philosophy," Brunschvicg notes that "the guise in which history first appears to the philosopher" is not "properly historical" in any recognizable sense.[4] Instead, following Henri Bergson's (1859-1941) distinction between the "fabulatory" and "verificatory" functions of reason, the history of civilization discloses a horizon of religious revelation which always precedes historical consciousness as such.[5] The fabulatory function produces "the theogonies and cosmogonies" of a world-view which is religious and cosmological. It is not until the religious cosmologies of the fabulatory function are overturned by the verificatory function of reason founded "on the intrinsic virtue of demonstration" that a gap can open up in the immanence of religious and cosmological values. History, for Bergson and Brunschvicg, requires this minimum difference between *fabulation* and *verification*, a distinction which both philosophers claim first appears in the ancient Greek world. Brunschvicg writes:

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[4] See Brunschvicg 1936, 27.

[5] On the fabulatory and verificatory functions of reason in the history of religion and at the origins of historical consciousness see Henri Bergson, *Les deux sources de la morale et de la religion* (Paris: Presses Universitaires de France, 1969), 105-220.

> The opposition between these two functions, which was already so clear in the fragments of Xenophanes of Colophon, is fully established with Platonism. The rigour and probity of the use of the dialectical method, based on the positivity of mathematical methodology, put in their proper place the 'impure' histories which had been given credit by the genius of a Homer or a Hesiod.[6]

Brunschvicg celebrates the rigor of a method which can resist even the "genius" of Hesiod and Homer – the genius of ecstatic participation in the world of divine forces and in the cosmological time of pre-ordained destiny. The positivity of mathematical demonstration in Plato constructs a new criteria of truth which is inseparable from its verification. This is the epistemological innovation at the dawn of historical consciousness and the beginning of a rational sequence which Brunschvicg will trace in his own work throughout the history of philosophy. The historicity of twentieth century science is the culmination of Plato's dialectical method insofar as it extends and refines a procedure of *verification* which in turn transforms the nature of objectivity.

Plato himself, Brunschvicg affirms, is unable to master the relation between the fabulatory and verificatory powers of reason in the evolution of his own doctrine. In antiquity "the victory of truth over fable…was precarious and ephemeral."[7] The Platonic dialogues, in Brunschvicg's view, end up multiplying the fabulatory elements of myth in order to "palliate the rational insufficiency of his physics and enhance the political efficacy of his moral theory."[8] The struggle between fabulation and verification in Plato is paradigmatic of the historical

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[6] Brunschvicg 1936, 27.
[7] Brunschvicg 1936, 27.
[8] Brunschvicg 1936, 27-28.

emergence of reason in its normative aspect: the dialectic holds all false claimants to knowledge at bay. The dialectic is a method which both selects and rectifies rational norms. For Brunschvicg, however, "[t]he struggle between fabulatory history and verifying reason was not resumed till the beginning of the seventeenth century."[9] *The history of verifying reason* is itself discontinuous, it begins with Plato but does not truly resume until "the labors of Galileo." The intervening *millennia* are characterized by the erasure of the regulative epistemological criteria of the verificatory function of reason: the immanence of "the method of the intellect and of truth," or the refusal of what Brunschvicg calls "external causes" in the construction of epistemological norms. This is indeed the crux of the matter. The immanence of the epistemological norm can have no reference but itself. When Brunschvicg isolates the emergence of the epistemological norm historically and then *proceeds to write a history of the transformations of the intellect,* he reframes the epistemological significance of the history of philosophy and the history of science relative to this normative epistemological configuration. The significance of this transformation, as I will argue throughout the dissertation, is that it enables us to understand that the *normative force of reason can only be understood in the light of its historical transformations.* History is not the contradiction of what I call the normative epistemological force organizing the intelligibility and validity of science, it is the confirmation of a rational necessity which animates the history of science.

[9] Brunschvicg 1936, 28.

Gaston Bachelard (1884-1962), also writing in 1936, describes the epistemological novelty of scientific objectivity in contemporary mathematical-physics as follows:

> We have in fact attained a level of knowledge at which scientific objects are what we make of them, neither more nor less. We have mastery over objectivity. The history of the laboratory phenomena is very precisely that of its measurement. The phenomenon is contemporaneous with its measurement. Causality is in a way solidified by our instruments. Objectivity becomes the purer as it cases to be passive in order to become more markedly active, as it ceases to be continuous in order to become more clearly discontinuous. We *realize* our theoretical thought by degrees. We end up extracting complex phenomena from their own particular time – a time that is always vague and indistinct – in order to analyse them in an artificial time, a time we determine, the time of our instruments.[10]

Bachelard here echoes *and* modifies Brunschvicg's presentation of mathematical physics. In accord with Brunschvicg, objectivity enters into a new relationship with measurement by which the phenomenon becomes co-extensive with the act of observation. However, whereas Brunschvicg tended to emphasize the integration of the observer into the system under observation, Bachelard emphasizes the *activity* of the instruments of measurement in the *production* of the phenomenon. In the former case, the rectification of objectivity proceeded by the integration of the subject into the object of study, thereby dissolving the illegitimate disparity of an epistemology which would simply oppose subject and object, distorting rationality (on the part of the subject), and objectivity (on the part of the observed), simultaneously. In the latter case, the rectification of objectivity proceeds according to the discontinuous realization of the phenomenon in the "artificial time"

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[10] Gaston Bachelard, *The Dialectic of Duration* Trans. Mary McAllester Jones (Manchester: Clinamen Press, 2000), 77.

constructed by scientific instruments. Twentieth century science for Brunschvicg is the perfection of a rational ideal which began with Plato; it is the systematic elimination of "fabulation" and the construction of an autonomous and self-correcting rational mechanism. For Bachelard, the novelty of physics in the twentieth century is not the perfection of an ideal which already existed; it is the *realization* of a rational procedure which is entirely unprecedented in the history of science.

Is it necessary, based on this apparently fundamental disagreement regarding the epistemological significance of twentieth century physics, to drive a wedge between Brunschvicg and Bachelard as thinkers of the normative epistemological commitments underlying the physics of relativity? If the criteria by which Brunschvicg and Bachelard evaluate the significance of relativity are truly incompatible, then they must be relegated to different camps. If, however, their apparent disagreement reveals a *mobile perspective* within a shared normative continuum, then we must conclude that individual differences in *judgment* do not constitute an *epistemological contradiction* between world views. Beneath the opposing *letter* of the law in Brunschvicg and Bachelard, I would like to suggest the operation of a convergence of rational values which implies more than a shared *spirit,* and in fact points toward a coherent philosophical problematic shared by both thinkers.

Bachelard's highly programmatic text on the nature of scientific objectivity effectively condenses the central affirmation of his epistemological profile of modern science: "We *realize* our theoretical thought by degrees." Modern science, as I will show later in the dissertation, is the realization of the rational for Bachelard. In fact, this excerpt from Bachelard's *Dialectic of Duration* (1950) can function as a synecdoche for Bachelard's epistemological program in general. We can replace the relationship between *the phenomenon* and *measurement* with the relationship between *history* and *epistemology* and obtain the normative epistemological perspective which subtends all of Bachelard's work. Bachelard writes:

> We have mastery over objectivity. The history of the laboratory phenomena is very precisely that of its measurement. The phenomena is contemporaneous with its measurement.[11]

Just as the phenomena is contemporaneous with its measurement, *so history is contemporaneous with the epistemological norm that produces its legibility.* Bachelard and Brunschvicg agree that there is no contradiction in asserting that rational norms are *constructed* and are *historically variable.* Further, they are in agreement that the historical instantiation of scientific rationality also necessitates the *immanence* of the rational norm within science. Finally, they concur that the historical instantiation of an immanent rational norm does not preclude but *requires* the transformation of this norm.

This three-fold affirmation of *the historical instantiation of the rational norm, the immanence of the norm within science, and the necessarily progressive historical transformation of the norm* defines the profound epistemological solidarity uniting Brunschvicg and Bachelard. The fact remains that this shared set of epistemological

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[11] Bachelard 2000, 77.

commitments is itself the result of a historical provocation which constitutes a privileged locus of reflection for both thinkers: Einstein's theory of relativity. The theory of relativity, so I claim, is the historical occasion prompting the systematic re-organization of the relationship between the philosophy of science and the history of science in France at the beginning of the twentieth century. The historicity of science (Einsteinian physics) which Brunschvicg experiences with such clarity in 1936 is a relatively recent phenomenon which Bachelard confidently asserts can "be very precisely dated from 1905." Bachelard writes:

> …the era *of the new scientific mind* could be very precisely dated from 1905, when Einstein's relativity came along and deformed primordial concepts that we thought were fixed forever. From then on reason multiplied its objections, dissociating fundamental ideas and then making new connections between them, trying out the boldest of abstractions. Over a period of twenty five years, ideas appear that signal and amazing intellectual maturity, any one of which would suffice to shed luster on the century. Among these are quantum mechanics, Louis de Broglie's wave mechanics, abstract mechanics, and doubtless there will soon be abstract physics which will order all the possibilities of experience.[12]

For Bachelard, it is not an exaggeration to say that thought finally begins in 1905, after a long pre-history of half measures and tentative approximations of the real. The new scientific mind is exemplary of a new orientation of thought towards the real: mathematical-physics no longer attempts to model the natural phenomenon, it "determines the phenomena," it "constructs the phenomena," it "realizes the phenomena," so many iterations of the same "mastery over objectivity" which is without precedent in the history of science.

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[12] Gaston Bachelard, *The Formation of the Scientific Mind, A Contribution to a Psychoanalysis of Objective Knowledge* Trans. Mary Mcallester Jones (Manchester: Clinamen Press, 2002), 19.

Bachelard's epistemology, as I will demonstrate in the fourth chapter, is aggressively contemporary, that is to say, for Bachelard the novelty of relativity is such that there are no pre-existing concepts in the history of philosophy which can grasp the new theoretical and experimental possibilities of relativistic physics. By the same logic, the history of philosophy is a history which must be overcome and systematically refuted. Bachelard's thought is immediately polemical and constructive: he is constantly destroying the philosophical errors of the past in order to assemble a new battery of concepts which are without precedent in the history of philosophy or of science. Bachelard in fact concludes that in the age of Einstein philosophy must bow to physics, which empties it of its traditional content and supplies it with a *new* metaphysics.

Where Brunschvicg would speak of the historicity of science, Bachelard would speak of the unprecedented technical modernity of science. It is as if Brunschvicg is unable to see the epistemological break (to use Bachelard's term) which truly separates Newton from Einstein, just as Bachelard is unable to perceive the epistemological norm which links the "meta-mathematics" (to use Brunschvicg's term) of Platonic dialectic to the Non-Euclidean geometry underpinning the tensor calculus of special relativity. In highly schematic terms, Brunschvicg seems to describe an evolution of epistemological norms which culminates in Einstein, whereas Bachelard seems to describe the crystallization of an epistemological system which is a true singularity, conceptually and historically *sui generis.* Bachelard, perhaps Brunschvicg's most celebrated student, *sees an entirely different* *history of philosophy and of science* than does Brunschvicg. It is not a matter of affirming or denying the historicity of science. This has been given by an irrevocable epistemological fiat whose structure I will explain in the concluding section of this introduction. It is instead a matter of attempting to understand *what kind of history becomes legible through the normative framework of a determinate epistemological system.*

This dissertation addresses the complex re-organization of the relationship between philosophy and the history of science in twentieth century France as occasioned by the publication, in 1905, of the papers which would lead Einstein to formulate the special and general theories of relativity. An historical event is thus the precipitating factor of an epistemological awakening which will in turn generate new conceptual and methodological possibilities for the history of science. As Georges Canguilhem (1904-1995) writes concerning the emergence of the history of science as a literary genre in the eighteenth century:

> I find that insufficient attention has been paid to a significant fact about the emergence of this genre: it required no fewer than two scientific and two philosophical revolutions as its preconditions. One scientific revolution in mathematics, in which Descartes' analytic geometry was followed by the infinitesimal calculus of Leibniz and Newton: the second revolution, in mechanics and cosmology, is symbolized by Descartes' *Principles of Philosophy* and Newton's *Principia.* In philosophy, and more precisely in the theory of knowledge, that is, the foundations of science, Cartesian innatism was one revolution and Lockeian sensualism the other. Without Descartes, without a rending of tradition, there would be no history of science.[13]

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[13] Francois Delaporte, *A Vital Rationalist, Selected Writings from Georges Canguilhem* Trans. Arthur Goldhammer (New York: Zone Books, 1994), 27.

Relativity is the rending of the theory of knowledge at the beginning of the twentieth century in France. The nineteenth century was characterized by a strong tradition of positivist historiography of science (inspired by Auguste Comte [1798-1857]) and of neokantian *Wissenschaftslehre,* the former emptying science of internal epistemological norms, the latter pre-determining the form and content of the theory of science by invoking a fixed structure of the mind.[14] Relativity scandalizes both traditions by seeming to impose new criteria of the intelligibility of scientific concepts which radically depart from any intuitive concept of space or time. The dissertation reconstructs this transformation of the history and philosophy of science into what Emile Meyerson (1859-1933), writing in 1908, first called *épistémologie.*[15]

The precise signification of *épistémologie* is difficult to render into English. It is undoubtedly synonymous with the philosophy of science but also suggests a more or less comprehensive integration of the history of science into the epistemological analysis of scientific concepts. Dominique Lecourt (1944-) therefore refers to the epistemological tradition which developed in the wake of relativity theory as the school of "historical epistemology."[16] I retain Lecourt's designation because it accurately captures the conceptual reflexivity at the core of the tradition: the

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[14] For an excellent summary of the history of the philosophy of science in France see Anastasios Brenner, *Les origines françaises de la philosophie des sciences* (Paris: Presses Universitaires de France, 2003.)
[15] See Emile Meyerson, *Identity and Reality* Trans. Kate Lowenberg (New York: Dover Publications, 1962), 5.
[16] See Dominique Lecourt, *Marxism and Epistemology, Bachelard, Canguilhem, Foucault,* Trans. Ben Brewster (London: New Left Review Editions, 1975.)

relationship between the epistemological norms determining the intelligibility and validity of scientific concepts on the one hand, and the historiography of conceptual transformations in the history of science on the other. The dissertation has two primary aims. 1) To demonstrate the theoretical consistency of the tradition of historical epistemology by determining the nature and function of the normative epistemological commitments which organize its concepts and methodologies. In so doing I hope to contribute both to the history of philosophy in twentieth century France and to the history of the philosophy of science. 2) To reconstruct the variable *histories of philosophy and of science* which the historical epistemologists produce as the result of the epistemological determination of the nature of scientific rationality.

The central argument of the dissertation is that the normative epistemological commitments of the tradition unambiguously determine the *legibility* of conceptual transformations in the history of science. Accordingly, for each of the historical epistemologists I examine I first reconstruct *the normative epistemological program* which defines their theory of science before examining *the history of science* which then becomes legible through the interpretive framework of epistemology. As the epistemological configuration of rational norms changes, so too must the history of science. Therefore, the history of science is not a stable referent but an active site of contestation, transformation, and revision. One of the lessons of the tradition of historical epistemology is that there is no more contentious history than the history of scientific objectivity. The history of science is not only mutable with reference to different thinkers, it may be actively revised any number of times within the trajectory of the same research program. The relationship between epistemology and history at the center of the tradition is not a complimentary synthesis of two methodologies on equal footing, it is a determinate relationship of cause and effect. Epistemology is the productive cause of the history of science. The epistemologist stands behind the historian and determines, retroactively, what belongs to the history of science.

I argue that two consequences proceed from this basic methodological program which have largely escaped critical notice in the Anglo-American and Francophone scholarship devoted to the subject. The first concerns a transformation of the implicit norms guiding epistemological critique. Unlike the Positivist or Neo-Kantian projects of the nineteenth century the tradition of historical epistemology systematically rejects predetermined theories of the developmental logic of science on the one hand (Positivist historiographies of science) and the transcendental determination of epistemic normativity on the other (neokantian *Wissenschaftslehre.*) The second concerns a transformation of the epistemological status of the history of science itself. The tradition rejects the history of science as a stable configuration. Indeed, I argue that a sophisticated historiographical methodology unites the historical epistemologists in a common project whereby the very writing of the history of science becomes a task that must be perpetually renewed.

Finally, I argue that the catalyst of the epistemological tradition was itself historical – the publication, in 1905, of a quartet of papers by Albert Einstein in the

Annalen der Physik [“On a Heuristic Point of View about the Creation and Conversion of Light”, “Investigations on the Theory of Brownian Movement,” “On the Electrodynamics of Moving Bodies”, and “Does the Inertia of a Body Depend Upon Its Energy Content?”] which in turn precipitated the conceptual re-organization of mathematical-physics.[17] I follow Gaston Bachelard’s characterization of the history of physics between 1905 and 1930 as the beginning of “epistemological modernity,” a transformative threshold in the history of science which functions as a privileged locus of reflection for the members of the tradition of historical epistemology. Although relativistic physics is an epistemological object which functions as a recurrent center of debate and reflection for the tradition, it is not the sole unifying theme. It is accompanied by an equally intense pre-occupation concerning the objectivity of mathematics. I therefore argue that the tradition of historical epistemology is organized by the relationship between two epistemological controversies. The first concerns the nature of scientific objectivity in the wake of the physics of relativity. The second concerns the nature of mathematical objectivity.

The historical boundaries of the dissertation are set by Einstein on the one hand (the publication of the *Annalen der Physik* papers in 1905) and by the death of Gaston Bachelard in 1962, on the other. The dissertation focuses on four

[17] See Jürgen Renn, *Einstein’s Annalen Papers: The Complete Collection 1901-1922* (Weinheim: Wiley-VCH Verlag, 2005.) The literature on Relativity is enormous. An extremely well organized and useful bibliography of sources can be found in Jürgen Renn and Matthias Schemmel, *The Genesis of Relativity Volume 4, Gravitation in the Twilight of Classical Physics, The Promise of Mathematics* (Dordrecht: Springer Publications, 2007.)

epistemologists of science whose work is central to the tradition of historical epistemology and whose primary research programs concern the relationship between mathematics and physics on the one hand (as exemplified by relativity), and the nature of mathematical objectivity on the other. In Chapter One, "Emile Meyerson and the Philosophy of Intellect," I introduce the concept of *épistémologie* at the turn of the century in the work of Emile Meyerson, a chemist by training and an autodidact of prodigious erudition who produces a new synthesis of the history of science and philosophy. Although Meyerson enjoyed no institutional affiliation during his lifetime he exerted an enormous influence on the tradition of historical epistemology by coining a new epistemological vocabulary which was capable of unifying complex epistemological claims and sustained historical arguments within the same system of explanatory concepts.

Meyerson is the first philosopher of science to use the term *épistémologie* in this new simultaneously normative and historical sense. I introduce the basic concepts of Meyerson's epistemology and history of science as expressed by what Meyerson calls "The Philosophy of the Intellect." For Meyerson, the intellect is governed by a rational norm called the principle of identity which requires the formation of a causal postulate: the unity, in the mind, of the antecedent and the consequent in a temporal sequence as expressed by the concept of causality. This principle, upon which scientific intelligibility is founded (and by which we are persuaded) is accompanied by a principle which interrupts it. Meyerson calls this principle "The Irrational" and defines it as the eruption of a form of time irreducible to causality into the temporal uniformity of the causal postulate. I argue that Meyerson's epistemology discloses a theory of science which is self-regulating and self-transforming. The causal postulate is the invariant form by which the scientific mind seeks to reduce the diversity of experience to stable explanatory sequences of cause and effect. However, the very normativity of the causal postulate leads it to an inevitable encounter with phenomena it cannot reduce to causal relations: it cannot avoid the irrational. Science progresses by reconfiguring the causal postulate in response to the provocation of the irrational. Although the form of causality remains the same, the deductive sequence by which it is realized is constantly evolving. I argue that this allows the epistemological norm of the principle of identity to reconfigure itself historically, thus establishing the basic template of historical epistemology: the epistemological principle underpinning scientific intelligibility is resolutely normative, but it is also capable of historical transformation. I reconstruct three historical sequences from Meyerson's history of science and philosophy in order to demonstrate the "Philosophy of the Intellect" at work: Sadi Carnot's discovery of irreversible transformations in thermodynamics, Hegel's critique of mathematics in his *Philosophy of Nature* and *Science of Logic*, and the epistemological significance of the theory of relativity.

I analyze the work of Léon Brunschvicg, an influential historian of philosophy and a specialist in the philosophy of mathematics and physics, in the second chapter. "The Immanence of Intelligence: Léon Brunschvicg and the Progressive Intelligibility of the Real" reconstructs the normative theory of science which informs what Brunschvicg call his "Philosophy of the Intelligence." The Philosophy of Intelligence, unlike Meyerson's Philosophy of the Intellect, does not oppose reason and experience, but demonstrates that the rational and the real require one another in order to constitute a dual realization of *rational objectivity* and *objective rationality.* The Philosophy of Intelligence is also a philosophy of history: for Brunschvicg the history of intelligence is quite literally the history of science. Brunschvicg denies the possibility of any transcendental criteria determining the nature and limits of the mind. Instead, in order to understand the nature of intelligence and the nature of thought, we must look to the history of philosophy and of science. It is not just any history of ideas which can serve as the basis for the construction of a philosophy of intelligence; Brunschvicg interprets the history of philosophy and of science through the normative categories of a philosophical system he calls "the Philosophy of Judgment." For Brunschvicg, to think is to judge. The structure of judgment, following Kant and Fichte, always requires a determinate relation between experience and intelligibility, or in Brunschvicg's formulation, between a configuration of *possible forms of objectivity* and *possible forms of rationality.* The history of intelligence is the history of the changing relationships between these two constitutive poles of judgment. I argue that Brunschvicg's Philosophy of Judgment is a normative epistemological program which reconstructs the changing boundaries of reason and objectivity. Using the Philosophy of Judgment as an interpretive framework, I examine Brunschvicg's critique of mathematical intelligence in Plato and Aristotle, his critique of the foundations of analytic geometry in Descartes and Kant, and his presentation of relativity theory as the apotheosis of the Philosophy of Judgment: the realization of a truly rational objectivity and a truly objective rationality.

Chapter Three is a reconstruction of Jean Cavaillès' (1903-1944) philosophy of mathematical experience. Cavaillès, trained in mathematics and philosophy, was also a student of Brunschvicg and a friend and colleague of Bachelard. In "Mathematics as Experience: Jean Cavaillès and the Dialectical Philosophy of the Concept," I argue that Jean Cavaillès develops a philosophy of mathematical experience which offers a precise description of the conceptual mechanisms enabling the historical transformation of scientific concepts. Cavaillès confronts the apparent paradox of historical epistemology head-on: he demonstrates the necessity of the structure of mathematical concepts while also asserting the necessity of their transformation. The dual injunction of the theory of science, according to Cavaillès, is to demonstrate the rigor of the deductive sequences which determine the forms of mathematical concepts while simultaneously describing the thresholds of transformation which re-organize the intelligible contents of mathematical objects. I examine Cavaillès' epistemology of mathematical experience by reconstructing his critique of Kant's theory of mathematics, Bernard Bolzano's (1781-1848) theory of objective propositions, and Edmund Husserl's (1859-1938) phenomenological philosophy of mathematical objectivity.

In Chapter Four, "Gaston Bachelard's Normative Epistemological Program and the History of Science," I reconstruct the basic concepts of Bachelard's epistemology and trace the complexities of his relationship with the tradition that preceded him. In so doing I also uncover the normative epistemological commitments that govern his work and that unite him, despite his frequent polemical outbursts, with his predecessors. I argue that three epistemological theses inform Bachelard's theory of science. 1) The affirmation of a rationality immanent to scientific practice. 2) The necessarily progressive nature of scientific objectivity. 3) The recognition that the history of science is inseparable from the epistemology of science. The history of science can only be constituted in the light of epistemological norms which allow the historian to recognize the actual moments of conceptual transformation in the history of science.

I argue that the structure of epistemological normativity in Bachelard is deeply convergent with the epistemology of mathematics developed by Brunschvicg and Cavaillès. In fact, I demonstrate that Bachelard's determination of the nature of scientific objectivity requires a "hard core" of mathematical objectivity as the absolute immanence of the rational and the real. I argue that Cavaillès and Bachelard are thus mutually complimentary in their respective determinations of mathematical and scientific objectivity. Although Bachelard entertains a profound conceptual proximity with Cavaillès and Brunschvicg, he is resolutely polemical in his relations with Meyerson. My examination of Bachelard's reading of relativity theory traces his systematic critique and repudiation of Meyerson's own reading of relativity. I argue that Bachelard constructs many of the fundamental categories of his own epistemological project largely through his repudiation of Meyerson.

Finally, I turn to Bachelard's fundamental critique of the existing philosophies of science (circa 1940.) I argue that the core of Bachelard's life-long polemic against traditional philosophies of science stems from his conviction that philosophy is unable to recognize the immanence of rational norms within science. Bachelard claims that the denial of the immanent normativity of science is what allows philosophy to attempt to determine the epistemological foundations of science. The recognition of the immanent normativity of science, he therefore suggests, would oblige philosophy to construct its own concepts in accord with science.

In a brief conclusion I offer a schematic summary of the relationship between epistemology and the history of science for each of the figures considered in the dissertation. I also reconstruct two highly influential appropriations of the epistemological tradition by examining Louis Althusser's (1918-1990) epistemologically inflected philosophy of history, and Michel Foucault's (1926-1984) encomium to his teacher, Georges Canguilhem (1904-1995). I trace Foucault's presentation of the significance of the history of science for French philosophy in the nineteenth and twentieth centuries and examine his characterization of Canguilhem's historical epistemology. I conclude that the relationship between epistemology and the history of science in Canguilhem confirms the central thesis of the dissertation: the normativity of science is not contradicted by the historical transformations of scientific concepts, on the contrary, the history of science is the confirmation of an epistemological normativity immanent to science.

The philosophical and historical reconstruction of the epistemological tradition I defend in these pages corresponds to a theoretical problem which must be stated precisely. It concerns the relationship between the epistemological criteria governing the intelligibility of scientific concepts and the historiographic methodology informing the conceptualization of the history of science. I understand this relationship as normative, that is to say, as determinate and exhaustive. I argue that, with varying degrees of self consciousness, but without exception, epistemological criteria are primary in the configuration of the relationship between epistemology and historiography. Epistemological norms precede historiographic methods, or what amounts to the same, the novel conceptions of the history of science which the tradition affords us only become legible through the evaluative framework of the epistemological criteria. The existing scholarship in the Anglophone and Francophone traditions addressing the epistemological tradition in France have not clearly determined the normative nature of this relationship. Accordingly, the *normative theory of science* which subtends the historiography of science emerging from the tradition has remained largely unexamined. The dissertation systematically reconstructs this normative theory of science and diagrams its relationship to the historiography of science.

Three philosophical theses, so I argue, define the normative epistemology at the heart of the tradition. Thesis one: *science is rigorously autonomous.* The content and structure of scientific concepts and methods cannot be referred to any external authority. The epistemology of science must therefore determine the nature and structure of the rational procedures immanent to science. This immanence of the rational norm to the objects and concepts of science is ultimately identified with the objectivity of mathematics.

Thesis two: *science is a process rather than a fixed methodology.* The nature of the scientific concept does not reside in the accomplished form of a well established theoretical or experimental system; on the contrary, the concept emerges at the moment of transformation between two heterogeneous organizations of scientific rationality. The stability of scientific concepts is epistemologically less significant than the transformation of scientific concepts. The history of scientific concepts is therefore marked by successive ruptures and unforeseeable transformations. The "essence" of the scientific concept is to be found in the *novelty* of its unanticipated transformations.

Thesis three: *the historiography of science emerges as the effect of epistemological critique.* The legibility of the scientific concept is grounded in the unconditional affirmation of the autonomy of science. The moment this autonomy is upheld the scientific concept becomes intelligible as that which resists all external principles of legitimation. The historical perspective is conditioned by the epistemological structure of the scientific concept because the normative imperative of scientific rationality *is to bring about a change in the concept.* The affirmation of the autonomy of the scientific concept in no way implies a changeless image of reason. In fact, it discloses a conceptual dynamism which *requires the perpetual revision of the structure of scientific objectivity.* The normative force of epistemology is manifest in the transformation of the scientific concept, not the invariability of a given rational procedure. The historiography of science as informed by epistemological critique is the history of these conceptual transformations. The immanent rationality of science is the basis of the autonomy of science. This rationality is instantiated historically and transforms itself historically. The historicity of science and the autonomy of science are thus mutually enforcing, not contradictory. The autonomy of science requires the historical transformation of the epistemological norms of science.

These three philosophical theses are operative, with varying degrees of emphasis, in each of the figures examined in the dissertation. In Chapter One I turn to the work of Emile Meyerson in order to trace the emergence of *épistémologie* as a novel theory of scientific knowledge at the beginning of the twentieth century.

Chapter One - Emile Meyerson: The Philosophy of the Intellect

Meyerson's Project: Preliminary Outline of the Philosophy of the Intellect

Emile Meyerson (1859 – 1933) occupies a curious place in the intellectual history of twentieth century France. A chemist by training, a Polish émigré, and for at least a brief time the director general of the *Jewish Colonization Association*, a post which he accepted and then declined in 1897, Meyerson enjoyed no institutional support or affiliation within the official channels of the French academy.[18] Although he may have been unclassifiable institutionally, he was at the center of a vibrant network of historical and philosophical research which helped define the methods and objects of the tradition which is now called historical epistemology.[19] As is well known, "épistémologie" in France is synonymous with the philosophy of science.[20] The notion however that the French tradition of epistemology at some point grafted historical considerations onto its properly epistemological orientation is inaccurate. Whether or not one appends history to the title of epistemological research in twentieth century France, the fact remains that considerations of epistemology (understood as philosophical reflection on the forms of intelligibility proper to scientific concepts) inevitably involved considerable work in the history of science

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[18] See Y. Mayorek, "Un philosophe comme directeur general: Émile Meyerson et la *Jewish Colonization Association* en Palestine," in *De Bonaparte à Balfour. La France, L'Europe occidentale et la Palestine* 1799-*1917,* eds. D. Trimbur and A. Aaronson. (Paris: CNRS Editions Mélanges du CRFJ, 2001), 386.

[19] The term "historical epistemology" seems to have been coined in 1968 by Dominique Lecourt as the title of a Master's Thesis written under the supervision of Georges Canguilhem. See Lecourt 1975.

[20] For a canonical presentation of this view see Michel Fichant "L'épistémologie en France," in *Histoire de la Philosophie VIII: Le XXieme Siècle,* ed. François Châtelet. (Paris: Librairie Hachete, 1973), 135-176.

and often required substantial reflection on the historiographic considerations which then imposed themselves on the epistemologically literate historian of science.[21] Michel Fichant credits Meyerson with the introduction and popularization of the term into the academic lexicon, stating that it was probably an appropriation of its English cognate, itself a neologism resulting from the effort to translate the German *Wissenschaftslehre* into English.

Although Meyerson is probably responsible for the standardization of epistemology as the term designating a more general history and philosophy of science, Meyerson has a more precise designation for his own philosophical project. In an article published the year after his death Meyerson provides a synthetic and synoptic definition of his research under the heading of a "philosophy of the intellect."[22] The title of the essay is in fact "The Philosophy of Nature and The Philosophy of the Intellect," by which Meyerson means to signal the structure of the essential tension motivating his own work. Between the irreducible diversity of nature and lived experience on the one hand, and the principle of rational identity which rules the intellect on the other, Meyerson will trace the dynamic trajectories of science and philosophy. Meyerson's entire project can be conceived as an effort to overcome the metaphysical antinomy defined by Heraclitus and Parmenides. Meyerson characterizes the real in Heraclitean terms. The diversity of sensation,

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[21] See Enrico Castelli-Gattinara, *Les inquietudes de la raison: épistémologie et histroire en France dans l'entre-deux-guerres* (Paris: Librairie Philosophique J. Vrin, 1998.)

[22] Emile Meyerson, "Philosophie de la nature et philosophie de l'intellect," *Revue de metaphysique et du morale,* t. XLI. no. 2 (1934.) Reprinted in Emile Meyerson, *Essais*. (Paris: Librairie Philosophique J. Vrin, 1936), 59-105.

becoming and duration are unknowable in themselves. The intellect on the other hand is characterized in Parmenidean terms: the changeless sphere of Parmenides denies the reality of becoming and asserts that beneath the diversities of sensation there is an ontological bedrock of eternal being. The norm of intelligibility is fabricated in the image of Parmenides' changeless sphere. In order for experience to become intelligible it must be subjected to a process of rectification, abstraction and explanation which transforms flux into stability, diversity into unity, and becoming into being. Although Meyerson characterizes his own work as an epistemology of science, this would only be a subsidiary classification of its more general orientation as a philosophy of the intellect.

The great paradox of Meyerson's philosophy is that his research does not in fact end up confirming a static image of unchanging reason. There is in truth a dialectic of wonderful complexity and transformative power which constantly forces the intellect to renegotiate its relationship with experience. The philosophy of the intellect is an ambitious effort to diagram the network of tensions and the history of transformations which have driven the intellect to modify the contours of its norms of intelligibility according to the ceaselessly renewed demands of experience. Meyerson developed the theses of the philosophy of the intellect over the course of five painstakingly researched and documented volumes which combine great historical erudition with equally impressive philosophical expertise and scientific authority.

Identité et Realité, published in 1908, already contains the essential components of the philosophy of the intellect. Meyerson's first great work can be read as the historical vindication of a philosophical thesis. In this work Meyerson presents the principle of identity as the invariant procedure of the intellect and proceeds to demonstrate how this principle in turn has determined the structure of scientific theories from antiquity through the nineteenth century. *De l'explication dans les sciences,* published in 1921, reprises the thesis of *Identité et Realité* but against a much more ambitious philosophical and historical background. This work engages the most directly with the philosophies of nature. Meyerson attempts to demonstrate that the work of Hegel and Schelling is at once supremely admirable and supremely mistaken in so far as these philosophers perceive so clearly that the mind contains the absolute within itself while also failing to understand the true nature of the dialectic which thus subjects the mind to a series of transformations best explained historically, rather than metaphysically or through the resources of transcendental idealist philosophy. *La déduction relativiste*, published in 1924, is Meyerson's encounter with Einstein's general theory of relativity. The publication of the special theory of relativity in 1905 had already galvanized philosophical communities all over Europe to rethink the relationship between philosophy and science insofar as science now seemed to be redrawing the most fundamental categories of the mind according to its own requirements. Meyerson's most synthetic and ambitious work appeared in 1931, two years before his death. *Du cheminiment de la pensée* is a massive three volume work which is simultaneously a treatise on logic, an anthropology of knowledge, a history of science and philosophy, and a comparative psychology of the historical and geographical diversities of human intelligence. Finally, the last book published in Meyerson's lifetime was a kind of sequel to *La déduction relativiste* entitled *Réel et déterminisme dans la physique quantique,* published in 1933. In his last work Meyerson attempts to reconcile his epistemological analysis of relativity with the new models of quantum indeterminacy emerging from the work of Niels Bohr (1885-1962) and the Copenhagen school.

It is impossible to address all of these works in equal detail. Consequently, this chapter will develop the major concepts and themes of the philosophy of the intellect by focusing on three works, *Identité et Realité, De l'explication dans les sciences,* and *La déduction relativiste*. The chapter is divided into four sections. Each section focuses on a conceptual problem and an historical period or figure in order to demonstrate how Meyerson's philosophy of the intellect interweaves epistemological considerations and historical materials in order to arrive at a dialectical theory of the changing norms of intelligibility. The first section introduces the basic elements of Meyerson's philosophy of the intellect and locates his work relative the Positivism of Auguste Comte, the critical philosophy of Kant, and Herni Bergson's philosophy of metaphysical duration. The second section provides an analysis of Meyerson's reading of the work of Sadi Carnot (1796 – 1832), arguably the most important figure in the history of science for Meyerson, as the chapter will demonstrate. The third section is a summary and critique of Meyerson's reading of

Hegel's philosophy of nature. Finally, the fourth section is an examination of the implications of Einstein's relativistic physics for the philosophy of the intellect.

Law and Cause: Primary Elements of a Philosophy of the Intellect and a Theory of Science

In full accord with Meyerson's characterization of his project as a philosophy of the intellect, Meyerson's first book, *Identity and Reality* (1908), contains the basic elements of this philosophy; namely, a definition of the method to be followed (an analysis of the forms of reason), and the object (the history of science from antiquity through the nineteenth century.) In the introduction Meyerson describes his work as an epistemological analysis and as a contribution to the philosophy of science. In order to understand the real significance of the work, however, it is necessary to distinguish more sharply between the aims and methods of the philosophy of the intellect and the philosophy of science more generally. Meyerson's materials are drawn from the history of science, but his intent is not produce a novel or rectified history of science, nor does he claim to provide an original philosophy of science. The history of science is treated as material in the service of an analysis of the forms of reason, thus Meyerson's true object is the intellect and his analysis of scientific theories will always be in the service of fully explicating the *nature* of the intellect.

Identity and Reality is a book of philosophy which attempts to isolate the subtle mechanism of an imperative internal to reason. The philosophy of the intellect becomes necessary because the intellect has no conscious grasp of its own normative requirements. It mistakes the diversity of its interests for a diversity of methods, and it mistakes its historical transformations as evidence of a necessarily progressive development. In fact, the diversity of the objects of science and the historical complexities of its many revolutions belie an essential unity to which the procedures of reason must conform.

> Whatever we do, it is always with our reason that we reason. We do not know and we cannot know any other way to establish a bond between concepts other than that followed by our reason, a term which can only signify here 'conscious reason.' Even when we believe that we most radically deviate from it, it is always with the help of scraps of conscious reasoning that we try to create another which is different.[23]

The task of the philosophy of the intellect will be to recast the normative status of epistemology as it relates to the sciences. Epistemology will no longer be asked to evaluate the types of knowledge which science produces in order verify their accuracy or to aid science in the purification of its methods. On the contrary, what Meyerson hopes to contribute is an elemental clarification of the very nature of the intellect. The philosophy of the intellect does not hold up a mirror to nature. It performs an excavation of the layers of a rational procedure and shows how great a disparity exists between what the scientist says, and what the scientist actually thinks. Is Meyerson a Kantian or Neo-Kantian philosopher by dint of this normative insistence on a determining but obscured form of reason which has remained largely invisible owing to its very ubiquity? The proximity to Kant is undeniable but Meyerson is entirely bereft of the a priori categories of reason which articulate the theory of Knowledge in Kant. By contrast, as will be demonstrated, Meyerson's

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[23] Emile Meyerson, *Identity and Reality* Trans. Kate Lowenberg (New York: Dover Publications, 1962), 8.

philosophy is rigorously a fortiori without however failing to insist on the normative *form* of reason as largely determinate of the proper *contents* of reason.[24]

We can make no sense of Meyerson's work or his method if we think he is simply paraphrasing Kant. Meyerson's question is not, "what must knowledge be like in order for science to exist" (the Kantian question of the epistemological conditions of possibility of science), but "what must the structure of the intellect be like in order for the diversity of experience to be rendered intelligible" (an epistemological question without transcendental protocols conditioning the possibility for all rational procedures). In Kant, reason is the subject of a transcendental critique designed to eliminate the inevitable illusions and false problems of an overly ambitious rationalism. The purpose of Kantian critique is to delimit the forms of possible knowledge. Meyerson does not however intend to limit the possible forms or objects of knowledge from the outset. His problem is to identify the nature of the intellect. Meyerson is not concerned with the *criticism of knowledge* but with the *determination of the normative form of knowledge.*

What counts as a regulating norm of reason for Kant and for Meyerson is therefore very different. For Kant, as is well known, the forms of experience must determine the possible categories of the understanding. Experience and reason are therefore co-determined by a transcendental deduction of the necessary forms of knowledge *and* experience – for example, the intuition of space in the form of

[24] For a clear discussion of the development of Meyerson's epistemological commitments as incompatible with those of Kant, see J Lowenberg "Meyerson's Critique of Pure Reason," *The Philosophical Review*, vol. 41, no. 4 (1932): 351-367.

extension and the intuition of time in the form of succession. Kant's epistemology is normative in this sense, the a priori is given as the necessary form of experience. Meyerson's epistemology is normative in a different sense. He does not determine the forms of experience from the outset. He maintains instead that the intellect deploys certain operations *regardless of the nature of experience.* The philosophy of the intellect is a normative epistemology installed at the level of already formed ideas. Whereas Kant unites reason and experience, Meyerson denies the possibility of their ultimate identification. Instead, contrary to Kant's critical philosophy, *it is the nature of the relationship between reason and experience which is itself at stake.* Kant claims to have resolved this problem. Meyerson will demonstrate that the intellect constantly arrives at new configurations of the relationship between reason and experience.

Meyerson's philosophy of the intellect thus serves as the basis for an entirely novel theory of scientific knowledge. *Identity and Reality* occupies an important strategic position within the philosophical landscape at the turn of the century because it triangulates a critical position relative to three influential philosophies of science – the Positivist philosophy descending from Comte, the French Neo-Kantian philosophies of the nineteenth century, and Henri Bergson's dual ontology of science and metaphysics.[25] Against Auguste Comte, Meyerson repudiates Positivism as a suitable theory of science because, he claims, Positivism mistakenly asserts that

[25] On Meyerson's relation to the philosophical currents of his time, especially Bergson, see Frédéric Fruteau de Laclos *L'épistémologie d'Emile Meyerson. Une anthropologie de la connaissance* (Paris: Librairie Philosophique J. Vrin, 2009).

science is free from metaphysical presuppositions.[26] Meyerson's work, on the contrary, describes the unbreakable solidarity between scientific concepts of causality and the metaphysical postulate of an underlying changeless substance, which Meyerson ascribes to the sphere of Parmenides. Contrary to the assertions of Positivism, science must affirm an extreme metaphysical position in order to make use of causality as an explanatory principle.

Meyerson's work also positions itself at a strategic distance from the French school of Neo-Kantianism as represented by the work of Emile Boutroux (1845-1921), Octave Hamelin (1856-1907), Jules Lachelier (1832-1918) and Leon Brunschvicg.[27] Meyerson departs from the Neo-Kantians in at least three important respects. 1) He denies the co-constitution of experience and knowledge under the productive legislation of a transcendental subjectivity. Against transcendental theories of the constitution of knowledge and experience, Meyerson posits an irreconcilable parting of the ways between the structure of perception and of experience, and the progressive *denaturing of experience* which the scientific concept requires. Science emerges from experience but it does not and cannot return to it: its destination is a metaphysical ideal which negates the diversity of sensory experience, the sphere of Parmenides. 2) Meyerson locates the genesis of

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[26] On Meyerson's critique of Positivism, see Fichant 1973, and Georges Mourélos, *L'Épistémologie positive et la critique meyersonienne* (Paris: Presses Universitaires de France, 1962).
[27] On the relation between Neokantian philosophy and epistemology in France, see Laurent Fedi and Jean-Michel Salanskis, eds., *Les philosophies françaises et la science: dialogue avec Kant,* (Paris: ENS Editions, 2001). On the disputation between Meyerson and Brunshvicg concerning intuition and causality, see Laclos 2009.

logic in a metaphysical principle which requires the reduction of diversity to an underlying spatial homogeneity. He further dismisses appropriations of Kant which model the organization of the sciences on the subordination of mathematics to logic. 3) Above all, Meyerson does not find that science is epistemologically stable in its orientation toward experience. There can be no definitive epistemological foundation of scientific rationality because the intellect does not continue to encounter the same problem of formalization. Rather, science must continually emancipate itself from experience every time it encounters the problem of causality. For Meyerson, the problem of causality is normative and invariable but the *occasion* of its emergence is historically variable. It therefore requires a changing series of historical solutions. The philosophy of the intellect thus encounters a *series* of causal principles in the history of science, each of which represents an entirely distinct normative configuration of the relationship between reason and experience.

Finally, Meyerson's philosophy entertains a special relationship with the philosophy of Henri Bergson.[28] If Meyerson is the philosopher of the intellect, Bergson is no doubt the philosopher of intuition. In fact, the relationship between the two thinkers is far more intimate than this simple dichotomy suggests, as evidenced by the extensive correspondence between the two thinkers and Meyerson's own increasingly complex relationship with Bergson's texts throughout

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[28] On Meyerson's relation to the intellectual climate at the turn of the century and his proximity to Léon Brunschvicg and Henri Bergson, see Frédéric Worms, "L'Esprit et la Réalité: Meyerson et le Moment 1900 en philosophie," in *L'histoire et la philosophie des sciences á la lumière de l'oevre d'Émile Meyerson (1859 – 1933),* eds., Eva Telkes-Klein and Elehanan Yakira*.* (Paris: Honoré Champion, 2010), 215-223.

the twenties and the years before his death in 1933.[29] Bergson's *Introduction to Metaphysics* (1903) and *Creative Evolution* (1907) are the works which exert the most immediate influence on Meyerson. Bergson's *Introduction to Metaphysics* expounds the first clear formulation of a distinction in kind between a metaphysical philosophy founded on an intuition of lived duration and the irreducible multiplicities of sensation on the one hand, and a mechanist theory of science founded on quantitative reasoning and problem solving, on the other. This duality of experience and intellect will becomes canonical for Meyerson. *Creative Evolution* refines this distinction and forges a dynamic philosophy of life understood as the increasing complexity of material organization, and of science as the increasing rigidity and segmented distribution of the problem solving intelligence.

Creative Evolution, I claim, can be read as the positive metaphysical doctrine which underlies the negative profile of *Identity and Reality*. *Identity and Reality* proceeds by subtraction as it were, in contrast to Bergson's elucidation of the expressive content proper to intuition and duration. Meyerson, however, eventually disagrees with Bergson on exactly this problem of the relationship of metaphysics to science. For Meyerson, the two must form an insoluble block, whereas for Bergson, they must be held apart. Meyerson's two most ambitious works, *Explanation in the Sciences* (1921) and *Du cheminement de la pensèe* [*The Ways of Reason*] (1931), continue to develop at some length a metaphysics of science which is fully consistent with the image of reason postulated by the philosophy of the intellect.

[29] On the correspondence between Bergson and Meyerson, see Laclos 2009, 138-150.

Bergson is no doubt one of Meyerson's master thinkers, but as Bergson himself points out in a letter to Meyerson, the philosophy of the intellect displays a unity among its parts which evidences a deep philosophical integrity far in excess of a mere commentary on the development of the sciences. Whereas Bergson continued to pursue the implications of duration and intuition in heterogeneous domains, Meyerson refined the profile of reason first established in *Identity and Reality* throughout his entire career, finally concluding that metaphysics and science are not and cannot be treated as separate entities.

Although Meyerson will make more programmatic statements regarding this unity in his later works, the overlap of the two domains is already well established in his first book. *Identity and Reality* is premised on a conceptual distinction between lawfulness or legality, defined as the formulation of a rule governing the *perception of orderliness in nature,* and the causal principle, defined as a mode of scientific reasoning which purports to explain natural phenomena by imposing absolute equality between the antecedent and the consequent in a causal chain. The history of science and of philosophy typically confuse law and cause according to Meyerson, which results in a disastrous obfuscation of the true orientation of scientific rationality: to arrive at causal explanations. Science seeks above all to provide not merely an empirical description of objective experience (which remains within the realm of lawfulness and of Positive philosophy), but to furnish an order of explanation which satisfies the demands of the intellect. This entails the construction of a causal mechanism capable of producing, as its effect, any phenomenon taken as an object of rational investigation. In this regard Meyerson fully endorses Leibniz' principle of sufficient reason.[30] If there is to be any kind of scientific knowledge it must submit to a particular form: that of the causal postulate. Science of course begins in sensory experience, but it does not tarry with the diversity of perception, nor does it return to it. Once scientific reasoning takes hold of the intellect it is inexorably bound to look for causal relationships and to take whatever means are necessary in order to arrive at a satisfying causal principle. This is the dynamic the philosophy of the intellect discovers at work without exception in the history of science and philosophy.

How has it happened that law and cause are so frequently taken as interchangeable by philosophers, and what is the relationship between these two concepts? For Meyerson, the law is a duplicitous concept in its very nature. It presents itself to the mind as an empirical survey of events but it conceals an intellectual procedure which is, in fact, a partial idealization of states of affairs. The task for the philosopher of the intellect is to attempt a decoupling of the empirical *image* and the ideal *content* of the law. The image of lawfulness is a *naturalization of perception* beneath which must be discerned an entire history of cultivation and selective intellectual labor.

> In positing the existence of rules we postulate that they are knowable. A law of nature of which we are ignorant does not exist in the strictest sense of the term. Certainly, nature seems to us ordered. Each new discovery, each realized anticipation, confirms this opinion in us. So much so that nature itself seems to proclaim its own orderliness; this idea appears to enter our

[30] On Meyerson's conception of Logic in relation to Leibniz, see Joseph LaLumia, *The Ways of Reason* (New York: Humanities Press, 1966), 48-50.

> minds from outside, as it were, without our doing anything but receiving it passively; in the end the orderliness appears to us as a purely empirical fact, and the laws formulated by us appear as something belonging to nature, as the "laws of nature," independent of our intelligence. This is to forget that we were convinced in advance of this orderliness, of the existence of these laws. All the acts of our daily lives witness to it. This is to forget also how we arrived at these laws…The law which governs the action of the lever considers only the "mathematical level"; but we know very well that we shall never encounter anything like it in nature.[31]

Nature seems to proclaim its orderliness such that the idea of the law or the perception of natural succession passes seemingly without effort into the mind. This orderliness is taken as an empirical fact but Meyerson's claim is that the appearance of order cannot be situated on the grounds of any purely empirical survey of experience. Order *as such* is always arrived at by an intellectual predisposition which Meyerson will explain presently. The impression of passive receptivity by which we receive the forms of natural law is in reality an active construction. The theoretical objects of natural laws are just that: theoretical or artificially pure models of reality. What requires analysis is the moment of conversion at which the intellectual nature of the law is replaced by the impression of a natural immanence which is itself a consummate artifice. The philosophy of the intellect attempts to define the mechanism by which scientific realism becomes the spontaneous ideology of the scientific mind.[32] Meyerson cannot accept on faith that the theoretical objects of natural laws automatically assume the status of natural objects. His whole project consists in the identification of the moments at which the

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[31] Meyerson 1962, 30.
[32] On this point Meyerson is very close to Gaston Bachelard, despite Bachelard's own protests to the contrary. See Chapter Four for a discussion of Bachelard's critique of Meyerson on exactly this point.

mind finds itself predisposed to accept as empirical, natural, and real, cognitive objects which are ideal, artificial, and theoretical. Speaking of the "mathematical lever," Meyerson affirms that:

> [i]n the same way we shall never encounter the ideal gases of physics nor the crystals of which we possess crystallographic models. But even when we affirm that sulphur [sic] has such and such properties we are not thinking of some particular piece of well known yellow substance. Sometimes what we affirm applies to a sort of average of the pieces we are apt to encounter in industry, and sometimes even (when we speak of "pure sulphur") it applies to an ideal matter to which we can approach only after many experiments; the qualities of a piece of sulphur taken at random may differ widely from those of the substance in question. [33]

Meyerson rejects the notion that science postulates general essences of things which can be said to wait for us with mute obedience in the exterior world. The opposite must be the case, for scientific objectivity is not defined by passively receiving the truth of things "as they are in themselves," but by striving to realize an artificial purity illustrative of an intellectual principle. In this case, the active production, synthesis, and laborious manufacture of artificially pure substances. Substances like a chemically pure sulfur, which can in no way be said to exist in nature but which are instead the product of industrial applications and of experimental reification.[34] Meyerson is an acute critic of what might be called,

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[33] Meyerson 1962, 30.

[34] Bernadette Bensaude-Vincent develops this theme persuasively in an article devoted to the relationship between Meyerson's training as an industrial chemist and his philosophical convictions regarding epistemology. She writes: "From his training as a chemist Meyerson inherits at least three philosophical theses: In the first place, there is no science without ontology: positivism is no more than a superficial rhetorical position in this regard. Secondly, there exists an internal tension, an irreducible polarity in the human intellect between identity and diversity. Finally, all scientific statements are inserted in a more general framework

following Alfred-North Whitehead, the fallacy of misplaced objectivity. A naïve objectivity gives to science the power of discovery, while entirely covering over the power of invention. To say that science invents its objects does not mean it fabricates them capriciously, or that it must abandon all claims to rigor. What is at stake is the nature of scientific rigor, which for Meyerson consists in the denaturing of experience, the elimination of diversity, and the promotion of an idealizing tendency which strives for purity, consistency, and axiomatic closure. Meyerson believes there is a persistent illusion at work in the laboratory and locates it in the appearance of lawfulness itself, according to which idealities appear as realities. Insofar as the philosophy of the intellect is also a critical endeavor, it aims to show how ideal constructs in turn become the objects of concrete experience. Meyerson wishes to show that a specific form of work is required to pass from one domain to the other so seamlessly, and that this very slippage is indicative of a fundamental characteristic of the intelligence which must be examined if we are to understand the deep structures motivating the very intelligibility of scientific theories.

The law is permeated by artifice. It grasps the phenomenon not as it is, but as the law requires it to be. In this sense, the law is what remains after all falsifying instances have been systematically removed. The law is a purification of the phenomenon:

> …we only attain laws by violating nature, by isolating more or less artificially a phenomenon from the whole, by checking those influences which would

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of presuppositions which are often tacit and frequently non-verbal." See Bernadette Bensaude-Vincent, "Émile Meyerson, Chimiste Philosophe," in Telkes-Klein and Yakira 2010, 67-90.

> have *falsified* the observation. Thus the law cannot directly express reality. The phenomenon as it is envisaged by it, the "pure" phenomenon, is rarely observed without our intervention, and even with this it remains imperfect, disturbed by accessory phenomenon. Experiments performed in academic lectures designed to illustrate certain laws, claim sometimes to show us this pure phenomenon. We know with what minute care these experiments must be previously regulated in order to succeed. Even then they make the impression upon the spectator of something profoundly artificial; the professor appears as a sort of prestidigitator.[35]

Previously, Meyerson used the language of artifice to describe the fabrication of the law, but now the very *lawfulness* of the law is characterized as a violence done to nature. By analogy, the laws of nature are a violent departure from reality, which the law cannot express. If the law cannot express reality then science must continue to elaborate its concepts until it encounters a concept supple and powerful enough to deal more adequately with the real. This is the formidable contradiction at the heart of the law and the reason why science cannot content itself with lawfulness alone. Science must seek a superior concept. Nevertheless, the very persistence of what Meyerson calls "the legalist tradition" in the history of science must indicate that the law serves some necessary purpose.

Meyerson has already established that the law has a profoundly pedagogical aspect in that it aims to demonstrate the nature of a reified or artificially purified substance, as in the demonstration in the lecture hall. If the concept of the law applied only to the fabrication of theoretically pure substances in the laboratory or the lecture hall, then it would really be a purely synthetic affair; a method or

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[35] Meyerson 1962, 31.

procedure which would allow for the synthesis of artificial substances. The law would literally be applied chemistry. The law, however, goes a step further than this insofar as it has a predictive value. It discloses another function which has to do with the orderly behavior of the contents and objects of experience.

> The law is an ideal construction which expresses, not what happens, but what would happen if certain conditions were to be realized. Doubtless, if nature were not ordered, if it did not present us with similar objects, capable of furnishing generalized concepts, we could not formulate laws. But these laws themselves are only the image of this ordering; they correspond to it only in the measure that a projection can correspond to a body of *n* dimensions. They can only express it in so far as a written word expresses the thing, for in both cases we must pass through the medium of our intelligence. [36]

The law expresses the hypothetical order of nature. Some minimum degree of organization is required as the extra-intellectual substrate upon which the law comes to be formulated. However, Meyerson will emphasize that the law really only captures a projection of this order. The law is an image of order, or as Meyerson says, a projection of "a body of *n* dimensions." The medium of the intelligence is not appropriate to the thing but to the image or the word, that is to say, the thing itself does not enter the intelligence except as a concept bounded on all sides by the stipulations and qualifications of a hypothesis.

If the law is therefore circumscribed on all sides, the question which imposes itself on the intellect is how the image of lawfulness persists. The persistence of the law is only intelligible because of its successive iteration in time. Therefore,

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[36] Meyerson 1962, 32.

lawfulness is the concept science employs in order to impose uniformity on the form of time:

> Since time passes unceasingly (this is the *unique independent variable* of Newton) laws, if they are to be knowable can only be so as a function of the changing of time. It will suffice then, strictly speaking, for nature to seem ordered to us, that we know the form of this function – that is to say, that we know how laws are modified as times advances.[37]

The law itself is modifiable according to the function of time. However, it is only through this *constancy of change* that the law persuades the mind that nature is ordered. The law surpasses the empirical observation of phenomena insofar as it attributes a stability of relations which experience *seems* to confirm, but which experience cannot itself guarantee. On the one hand, the law is a system of hypothetical judgments, on the other, it is a system of prediction. Between hypothesis and prediction the law assumes its full grasp of the uniformity of time. It must not only delimit the state of affairs as they are, it must be predict what they are going to be. The law must obey the orientation of time. Laws, Meyerson writes:

> have prediction as their end, it is just as interesting for us to know *when* things will take place as to know *what* takes place. If the dog to whom I throw a piece of meat wants to catch it, he must be able to calculate at what precise moment the morsel will attain the height of his jaws. Thus it is our conviction of nature's regularity which intervenes; nature lends itself to it, this is incontestable, but this conviction, as we have noted, exceeds the limits of direct observation; it is absolute and guarantees the future.[38]

Meyerson establishes a solidarity between the measurement of time and the laws of nature. The law itself might be subject to change, but the impression the law gives us of time is itself constant. Therefore, the law is convincing to the intellect because

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[37] Meyerson 1962, 32.
[38] Meyerson 1962, 33.

it ultimately refers to the standardization of time. The laws of nature are lawful insofar as they obey the regularity of time. The form of change (as in Kant) must be constant with respect to time.

Here again the limitation of the law becomes pressing because "the postulate of the law in no way implies that objects themselves remain immutable in time." For Meyerson, the law always acts at a distance from the object. It does not take its bearing from objects, but from an idealization of time. What requires explanation is this belief in the uniformity of time and the equally compelling belief in the uniformity of space. The analysis of the law provided some insight into the homogeneity of time insofar as Meyerson was able to deduce that laws come into being in order to serve the practical necessities of prediction. Spatial uniformity proves to be of a somewhat different nature. Whereas the law upholds the uniformity of relations among objects, another principle is required if we desire that objects themselves should remain constant. This second principle is causality, and it differentiates itself from the law by constructing an absolutely homogenous form of time.

In order to account for the superior abstraction of homogenous time that causality produces, Meyerson must first compare the provisional homogeneity of time and space. Time and space for Meyerson are not equally empty. Time is provisionally emptied of diversity by the concept of lawful succession, but this lawfulness is always provisional. It is constrained by the narrowness of hypothetical judgments on the one hand, and by the practical orientation of prediction, on the other. Can the same be said of the homogeneity of space as it must serve the scientific intellect?

> …geometry proves to us equally that our belief in the homogeneity of space implies something more than the persistence of laws. We are, indeed convinced that not only laws – that is, the relations between things – but even things themselves are not modified by their displacement in space. This is what geometry plainly postulates. And one of the masters of contemporary scientific thought [Henri Poincaré] has very aptly said geometry would not exist if there were no solids displacing themselves without modifications. Now it is very essential to state that geometry (and in general those sciences which we include under the term "pure mathematics"), though dealing with the abstract concepts of thought, enjoys evidently the advantage of application to reality in an absolute manner.[39]

By invoking geometry and the authority of Henri Poincaré, Meyerson intends to cut through everything that would be provisional or derived from experience regarding the homogeneity and ideality of space. It must be remarked that Meyerson includes geometry among the sciences of "pure mathematics" (a gesture which his contemporary Léon Brunschvicg would not be able to endorse, as I will show in the next chapter.) It is, however, perfectly consistent with the philosophy of the intellect to identify as "pure" those sciences which constitute themselves out of the immanence of the undifferentiated. What Meyerson understands by purity is clearly expressed in this passage: the conceptualization of reality "in an absolute manner." Regardless of the number of displacements an object might experience in its spatial orientation, its constancy as an object is assured. Therefore, the sense in which space provides the template or model of constancy seems to far exceed that described by natural laws. Causality emerges as a concept when this homogeneity of

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[39] Meyerson 1962, 37.

space is simply transferred to the concept of time. According to the form of the law time is imperfectly constant. According to the form of causality time is able to apply itself as an absolute category of thought, and therefore to secure what the intellect desires: an explanation which preserves the constancy of the object. Causality is able to secure the constancy of the object by forging a perfect equality between the antecedent and the consequent in a logical sequence.[40]

> The law states simply that, conditions happening to be modified in a determined manner, the actual properties of the substance must undergo an equally determined modification; whereas according to the causal principle there must be equality between causes and effects – that is, the original properties plus the change of conditions must equal the transformed properties.[41]

It is understandable why lawfulness and causality have so frequently been mistaken for one another once it becomes clear that they are relative and absolute forms of the standardization of time. The danger of mistaking the one for the other proceeds from the fact that the intelligibility of science is defined as the absolute standardization of time for Meyerson. Therefore, to confuse the form of the law with the form of causality is to misrecognize the standard by which scientific intelligibility must be evaluated. For Meyerson, the distinction between law and cause is the foundation upon which it becomes possible to construct a normative epistemology of the scientific intellect. Insofar as science claims to distinguish itself from other forms of knowledge by adhering to the norm of logic it must submit all of

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[40] For a critical appraisals of Meyerson's attempt to equate causality with the homogeneity of time, see Owen N. Hillman, "Emile Meyerson on Scientific Explanation," *Philosophy of Science,* vol. 5, no. 1 (1938) :73.
[41] Meyerson 1962, 41.

its concepts to the oversight of the causal principle. Science forges an alliance between logic, as a method of investigation, and causality, as a principle of explanation. The identity of objects in time establishes the principle of causality as the maximum degree of certainty to which scientific intelligibility can aspire. Causality is what the scientific intelligence extracts from experience and the standard by which it legitimates its own knowledge.

> It is moreover easy to establish the union between the notion of the rational and that of the persistent throughout time. *The principle of identity is the true essence of logic,* the real mold into which man puts his thought. [my italics] "I agree," says Condillac in the *Langue des Calculs,* "that in this language, as in all others, one only makes identical propositions when the propositions are true," and in his *Logique* he affirms that "the evidence of reason consists solely of identity."[42]

The principle of identity is the soul of logic. If the rational is the real, it only becomes so by way of causal explanation. Meyerson does not deny the heterogeneity of scientific practices or of scientific concepts. Not all concepts which the scientist makes use of are directly convergent with the principle of causality. Nevertheless, science is a normative intellectual activity and for Meyerson this means that it is constrained by a necessity which compels it to organize its concepts in the service of providing explanations. Identity is the soul of logic, but reality is an unending complication of experience. The dynamic which emerges at the site of the encounter between the principle of identity and reality is what Meyerson calls science. Science attempts to respond to the exigencies of reality according to the forms of logic. Because we are bound to use our reason, *and our reason is invariably*

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[42] Meyerson 1962, 43.

guided by the principle of identity, science will manifest an overwhelming obedience to explanatory forms which repeat the principle of causality.

The difference between lawfulness and causality is also the difference between the practical and the theoretical forms of intelligence. It is often necessary to distinguish Meyerson's philosophy of the intellect from Kant's epistemology because Meyerson makes so much use of the standardization of time in order to demonstrate the coherence and necessity of the scientific intellect. Lest the comparison be taken too far, it should be noted that Meyerson simply rejects Kant's affirmation that time and space are equally "pure and empty forms."[43] Insofar as time achieves this purity and constancy it is only because the causal principle has arrogated to itself a stability which truly belongs to space. As will be demonstrated as the chapter progresses, time remains an exceedingly complicated problem for thought. The epistemological problem of time in relation to causality is ultimately that which forces thought to continually reformulate the procedures by which experience becomes tractable for the intellect. This dynamic captures the entire drama of the philosophy of the intellect, as well as the conceptual urgency which drives the historical turbulence of scientific transformations. *For Meyerson the*

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[43] With regard to the ideal homogeneity of space versus the provisional standardization of time established by lawfulness, Meyerson writes: "What proceeds shows us that it would be vain to attempt for space the deduction which we carried out for time. The postulate of lawfulness alone will not suffice, for, as we have just seen, we attribute to space more uniformity than lawfulness would strictly demand for it…Objects are not modified by the action of space as they are changed by the action of time; the very expression seems paradoxical, startling in the first case, trite in the second. Space (the same thing had been affirmed of time, but wrongly) is really a "pure form" void of all content." Meyerson 1962, 37-38.

history of science and the philosophy of the intellect converge at the point defined by the confrontation of the upsurge of time and the concept which seeks to bind it according to a theory of causality. The history of science is the history of the forms of intelligible duration, just as the history of the intellect is the perpetual deformation and reinvention of the forms of spatial intelligibility. Without this interlocking and mutually correcting relationship of history, science, and the epistemological forms of intelligibility, Meyerson's entire project remains incompletely understood.

The next section develops some of the implications of this dynamic by examining a moment in the history of science which seems to overturn the principle of causality: Carnot's principle. The philosophy of the intellect proves to be a more subtle and capacious instrument than some of Meyerson's readers, such as Bachelard, would be prepared to admit. The historical moment defined by Sadi Carnot's work (1796 – 1832) prompts Meyerson to develop a concept of *the irrational* as the internal limit of the forms of intelligibility. The dialectic of the intelligible and the irrational is the motor proper to the philosophy of the intellect. It is what integrates the historical and epistemological facets of Meyerson's philosophy, subjecting the epistemological norms to revisions occasioned by historical transformations, and equally casting light on historical sequences whose empirical diversities conceal a more essential integrity.

Carnot's Principle and The Irrational

Identity and Reality might be said to replace the Kantian antinomies with a new antinomy of law and cause. Lawfulness corresponds to the practical necessity of prediction but fails to ground the necessity and universality of science. Causality provides the form of absolute homogeneity required to guarantee the constancy of the object as represented by the scientific concept without however actually encountering this unchanging object in experience. This antagonism is endemic to Meyerson's metaphysics: a flux which repudiates knowledge on the one hand, and an unchanging form which renounces diversity, on the other. The history of epistemological programs would seem to amount to little more than the consolidation of one of these two positions. Genuine knowledge is impossible because being is flux – the Heraclitean fire; or, in the contrary formulation, the appearance of diversity itself is illusory for only the intelligible is real – the Parmenidean Sphere and the Platonic Forms. There is no question that the philosophy of the intellect intends to solve this metaphysical stalemate and that the history of science itself will be mobilized as the evidence of a successful renegotiation of the status of intelligibility in a world of flux. The novelty of Meyerson's solution is that the status of reason is at once *normative* and *historically variable.* How can this be?

In the previous section the causal principle was shown to be the principle of logical identity established between the antecedent and the consequent in a logical sequence. Only causality can authorize the intelligibility and legitimacy of scientific explanation because causality alone is capable of referring the sequence of modifications informing temporal and spatial diversities to an underlying uniformity which reconciles them to an invariant form. It is important to emphasize that Meyerson has yet to introduce any specific considerations regarding the *content* of scientific theories, all he intends to demonstrate at this point is the *necessary form* scientific intelligibility must assume in order to persuade the intellect. Meyerson intends to resolve the epistemological stalemate of Heraclitus and Parmenides by demonstrating the dialectical transformation of the *contents of intelligible theories* on the one hand, while also affirming the necessary *rediscovery of the principle of identity* internal to the changing forms of causality, on the other. The philosophy of the intellect upholds that reason is one: it is the principle of identity. The image of the intelligible is the changeless sphere of Parmenides. At the same time however, the encounter between the principle of identity and reality is ceaselessly renewed and reason encounters certain internal boundaries which force it to reformulate its own normative status. Meyerson calls these internal boundaries the limits of *the irrational*, which should not be read as a pejorative but simply as a description: that which is beyond or outside the sphere of intelligibility.

Bearing these considerations in mind, the philosophy of the intellect can be said to be doubly normative: it affirms that there must be a normative form of intelligibility, but it also affirms that this norm can and must change in accord with varying circumstances. Every successful scientific theory confirms this thesis, and *Identity and Reality* is in part an extended historical vindication of a philosophical thesis. With more of the general structure of the philosophy of the intellect in place it is possible to enumerate some of the essential divisions within the system. All scientific propositions can be ascribed to one of three varieties. Two have already been introduced. Statements described as lawful are hypothetical and describe the outcome of a state of affairs following certain well defined criteria. These statements are concerned with *prediction* and as such are of a *practical* orientation. They cannot be demonstrated logically and they do not form a system of axiomatic closure. Statements of causality on the contrary do not entertain a direct relationship with the objects of experience. They define an absolute identity between the poles of a logical sequence and demonstrate the uniformity of an unchanging reality which subtends the transformative sequence uniting antecedent and consequent. There are also statements or propositions of a third type, which Meyerson calls *plausible.* Owen Hillman offers the following summary of statements defined as plausible by Meyerson.

> *Plausible* propositions, that is statements of identity, are neither *a priori* nor *a posteriori*. They are not *a posteriori* since they elicit a conviction of their truth for which their empirical evidence alone does not account, and they are not *a priori* since they are not characterized by apodictic certainty. They stand, therefore, intermediate between the *a priori* and the *a posteriori* different from either yet having something of the character of each.[44]

As Hillman notes, all statements of identity are plausible, but plausibility itself "increases in proportion as it approximates the more closely to being a statement of simple self identity." Therefore, as Hillman explains, statements which begin to approximate the form of a tautology are also the most plausible. Plausibility is required in order to mediate between statements of law and causal principles.

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[44] Hillman 1938, 78.

Meyerson also claims that the formation of the law is a necessary first step on the way to the formation of the causal principle. It is therefore equally necessary to affirm that the progressive rectification of the law can proceed indefinitely toward an increasingly plausible explanation without yet offering a genuinely causal mechanism. Plausibility is a function of the progressive correction of law like statements in accord with the ideal of the causal postulate. That which is plausible makes itself amenable to reason.

The organization of epistemological norms at any given moment in the history of science must be the simultaneity and overlap of these statements, their combination and recombination according to historical circumstance and the stages of theoretical development. At the same time however, the philosophy of the intellect recognizes that the system of lawful, plausible, and causal propositions constitutes something like science in its "non-revolutionary state" in the Kuhnian sense. When a system of concepts enjoys relative equilibrium it proceeds with a kind of serenity from law to cause by way of increasing plausibility. The intellect never finds itself disturbed or interrupted in the accumulation of its certainties or in the verification of its theses. This has tended to be the image of reason which many of Meyerson's commentators have subjected to the most intense scrutiny (most significantly Bachelard). It is not difficult to understand why, for it hardly seems to be the case that the history of science in fact constitutes an even fabric of inexorable progress and thematic continuity.[45]

The philosophy of the intellect is not, in point of fact, an apologetics of uninterrupted conceptual development. Meyerson's concept of the *irrational* is the necessary interruption of the causal principle. Where causality posits a perfect equality and a changeless substance beneath appearances, the irrational heralds a return to flux and an immersion in complexity, even a radical discord. The reconfiguration of the epistemological norm is instigated by the irrational, which introduces a radical heterogeneity and discontinuity into the system of norms among themselves, making their unity at a more general level impossible. If Meyerson is the philosopher of the principle of rational identity, he must also be the apologist of a dynamic and historically variable heterogeneity. These two sides of his epistemology are inseparable. They require one another in a radical immanence which is all to often neglected in summaries of his work.

The history of thermodynamics, and the work of Sadi Carnot in particular, constitute the site of Meyerson's most extended reflections on the implications of

[45] Here it is necessary to note that Meyerson's ubiquity in the early decades of the twentieth century is matched only by his obscurity after the second world war, owing largely to Bachelard's successful appropriation of the methods and vocabularies of epistemology. Bachelard's *La valeur inductive de la relativité* (1929) is a direct rebuttal of Meyerson's *La deduction relativiste* (1925) and effectively misconstrues Meyerson as a thinker of unbroken epistemological development. In fact, as this chapter will demonstrate, it is more appropriate to characterize Meyerson as a thinker of non-communicating and independent sequences of epistemological normativity. See Gaston Bachelard, *La Valeur Inductive de la relativité* (Paris: J. Vrin, 1929). A brief summary of Bachelard's critique of Meyerson is included in section four of this chapter.

the irrational for the history of science and for the governing norms of the intellect. Sadi Carnot published a monograph bearing the inauspicious title *Réflexions sur la puissance motrice du feu* in 1824, to which Meyerson introduces the reader following Pierre Duhem's apt summary of Carnot's principle:

> The transformation value of a modification is equal to the diminution that a certain magnitude, connected with all the properties which fix the state of the system, but independent of its motion, undergoes through this modification.[46]

The magnitude in question will subsequently be reformulated in terms of the entropy of a given system. Carnot's principle is however most concisely formulated by Clausius in 1868: "Heat cannot be made to pass from a cold body to a hot one."[47] Henri Poincaré therefore often refers to the fundamental contribution of Carnot under another name as the "axiom of Clausius," as Meyerson notes. The Carnot-Clausius axiom is central to the modern conceptual system of thermodynamics because it teaches us something important about the behavior of heat in bodies; namely, that unlike other physical systems which strive for a state of equilibrium (as Meyerson documents in his chapters on the principles of the conservation of matter and energy and the principle of inertia), differences in heat strive for equilibrium by dissipation or transference of energy from the warmer body to the cooler body.[48] According to Meyerson's summary of the Carnot-Clausius principle it is conceptually

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[46] Meyerson 1962, 60.
[47]Meyerson 1962, 287.
[48] On Meyerson and the history of physics, see Sophie Roux, "Histoire de la Physique Classique et Historicité des Sciences chez Meyerson," in *L'histoire et la philosophie des sciences á la lumière de l'oevre d'Émile Meyerson (1859 – 1933)* Eds., Eva Telkes-Klein and Elahanan Yakira. (Paris: Honoré Champion, 2010,) 91.

inaccurate to designate this movement of heat as a striving for equilibrium because the direction of modification in the system is constant. Therefore, we cannot speak of an equilibrium *of change itself* since the entire system is subjected to a progressive modification of its energy.

> We saw that not only the principles of conservation, but also scientific statements in general, are conceived with the constant, though perhaps latent, prepossession of causality and identity in time. Hence the tendency to give to the rule which determines the modalities of change a form which emphasizes that which endures throughout the change, a tendency which manifests itself externally by the equation, in which the change is, so to speak, suppressed and conjured away. On the contrary, Carnot's principle is clearly not of conservation, but of change. It affirms not even an apparent identity, but a diversity. Given a state, this principle establishes that it *ought* to modify itself, and in what direction.[49]

Carnot's principle is part of the foundational elements of the second law of thermodynamics, which requires that the entropy in any closed system increase until the system evolves into a state of maximum entropy, sometimes called thermodynamic equilibrium. Part of the fascination of Carnot's principle for Meyerson must be that this principle which so troubles the principle of identity also postulates the eventual uniformity of the distribution of matter in the universe. This would of course be the physical instantiation of the Parmenidean ideal. The Carnot cycle is an idealized description of the movement of heat in a theoretical machine or heat engine whose mechanical motion is powered by the quantity of heat in the system. Thus, physical systems can be described very precisely in terms of the

[49] Meyerson 1962, 265.

dissipation of energy in the form of heat.[50] Carnot thus introduces time into considerations of heat or molecular motion, however, as Meyerson observes, the implications of this new mechanical theory of heat and of the heat engine extend all the way to cosmology.

> In opposition to the illusions of identity to which mechanical theories, the principles of conservation, and even the form of laws in general give rise, Carnot's principle stipulates that the whole universe is modifying itself in time and in a constant direction. Clausius formulated this with great clearness: 'We frequently hear that everything in the world has a circular course. Whereas some transformations take place in one direction, in a determined place and at a certain time, other transformations are accomplished in an inverse direction, in another place and at another time, so that the same conditions are generally reproduced and the state of the world remains invariable, when we consider things broadly and in a general manner. The world, therefore, may continue to exist eternally in the same fashion. When the first fundamental principle of the mechanical theory of heat was announced, it might perhaps have been considered as a striking confirmation of the aforesaid opinion…The second fundamental principle of the mechanical theory of heat contradicts this opinion in the most explicit manner. From this it results that the state of the universe much change more and more in a determined direction.'[51]

The world cannot subsist unchanged beneath the diversity of its modifications when it becomes clear that the universe itself is in a state of continuous transformation, and in a determined direction. Carnot thus interrupts the causal principle at the height of its powers because after Carnot a new kind of

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[50] For a capable summary of the conceptual development of thermodynamics, and for a thoughtful survey of the debt Clausius and Helmholtz in particular owe to Sadi Carnot, see P.M. Harman, *Energy, Force, and Matter: The Conceptual Development of Nineteenth –Century Physics* (New York: Cambridge University Press, 1983). On Carnot, 45-50. On Clausius , 64- 8. On thermodynamics, 128-129. See also Rene Taton, *Science in the Nineteenth Century* (London: Thames and Hudson Limited, 1965), 246-250. For a survey of thermodynamics inspired by Meyerson but critical of his position, see Isabelle Stengers, *Cosmopolitics I* Trans. Robert Bonnono (Minneapolis: The University of Minnesota Press, 2010), 15-16.
[51] Meyerson 1962, 265-266.

transformation is introduced into the conceptual vocabulary of physics : irreversible transformations. The origins of thermodynamics fit very uneasily within the rational and celestial mechanical systems which preceded them. At the heart of thermodynamics, as Meyerson explains, there is a new procedure of quantification which allows two previously distinct domains to enter into a new theoretical unity. Heat and motion are inseparable in the molecular statistics of the thermodynamic universe, but for the cosmology of rational mechanics this subtle world of the kinetics of molecules is unfathomable. Between the macroscopic scale of observable movement and the microscopic scale of perception there obtains an equally canonical distinction between quantities and qualities.

Theories of motion in the universe of rational mechanics are ideally reversible: their intelligibility is indexed to space and not to time. Carnot brings about a change in the orientation of the concept's intelligibility when it is indexed to time, an irreversible variable. The universe described by rational mechanics can be productively imagined as a clock work of unfathomable complexity. Nevertheless, time exists in this clockwork as the perpetuity of its motion. It has no conceivable origin and no conceivable end, not unlike the unmoved mover of Aristotelian cosmology. Carnot is the architect of a new unity of time and movement, after which the two concepts become linked inextricably. This unity is brought about by a transformation of scale: time is no longer indexed to macroscopic bodies, but to the microscopic movement of molecular bodies. Time is symmetrical and reversible before Carnot: it is simply movement in space. Time is asymmetrical and irreversible after Carnot and Clausius: it is the statistics of heat gradients and the distribution of probabilities.

Following Hans Blumenberg's work on the rhetorical devices which often form the substrate of conceptual transformations in the history of science, it is possible to point to a profound transformation of the "background figures" of time which inform Carnot's work.[52] For the rational mechanics of the seventeenth century time was only thinkable according to the image of the clock-work. The clock-work, as image of time, both is and is not a figure (clocks tell time, they are not time itself, and yet the concept of time *is* the eternal movement of the parts.) With Carnot, time is perhaps closer to an hour-glass, except we must subject the image of this hour glass to continual revision for we cannot know its shape, volume, or rate of change with equal precision and stability from moment to moment. Carnot enacts dual orders of mutually interacting change such that the *rate of change* itself is subject to change. So much then for the equivalence of the metaphor and the concept of time once Carnot ruins the stability of their mutual definition. *Meyerson identifies the irrational element in Carnot's work for the scientific community in the very form of its reasoning: what has changed, and according to an incommensurable transformation, is the form of change itself.*

> …we notice that as soon as Carnot's principle was solidly established in science by Clausius, attempts were made to escape from the consequences which it entails. Not that one could deny the fundamental phenomenon, nor even the propositions that are deduced from it with absolute rigour. But a sort of secret repugnance was felt to the idea of a continual change of the

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[52] See Hans Blumenberg. *Paradigms for a Metaphorology* Trans. Robert Savage (Ithica: Cornell University Press, 2010), 99-114.

> universe in the same direction. Helm very clearly observed this mental attitude of the contemporaries of Clausius and saw that it had its source in the concepts of conservation. But we can follow the manifestation of the same current of thought at a much later date and even in our times. A very characteristic example is found in Haeckel: 'If this theory of entropy were true, we should have a 'beginning' corresponding to this assumed 'end' of the world. Both ideas are quite untenable in the light of our monistic and consistent theory of the eternal cosmogenetic process; both contradict the law of substance…the second thesis of the mechanical theory of heat contradicts the first and so must be rejected.' Carnot's principle can only be applied 'to distinct processes,' but 'in the world at large quite other conditions obtain.' Declarations of the same kind are found in Arrhenius. This scientist considers that it is indispensable to *free* ourselves from the difficulties of the theory of Clausius. 'If the ideas of Clausius were true, this thermal death ought to have been realized in the infinite time of the existence of the world.' Moreover, we cannot suppose that there was a beginning, since energy cannot be created. Consequently, 'this is totally incomprehensible to us.' It is clearly seen that it is a question both with Haeckel and Arrhenius of an *a priori* tendency, anterior and superior to all experience. *What shocks them is that on Carnot's principle we must suppose ourselves to be at a precise moment of a continuous development. Lack of identity between the antecedent and the consequent necessitates the admission of a datum which must necessarily appear to us as inexplicable, like the diversity which matter engenders in space and from which we are trying to free ourselves by assimilating matter to ether and to space itself.*[53] [my italics]

As Meyerson notes, Arrhenius repeats an objection to Clausius' theory which was already part of Stoic and Epicurean physics. If thermal death is possible, if energy can actually dissipate without remainder, then the universe would already have destroyed itself. This chain of reasoning pre-supposes that an infinite duration must already have elapsed, it forbids in its very organization the idea which makes entropy thinkable as a model of time: a quantitative limit on the heat/information saturation of the universe. Carnot's contemporaries are scandalized not because they misunderstand Carnot, but because they understand him all too well: if change

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[53]Meyerson 1962, 268-269.

cannot be eliminated from the order of explanation, then change itself must realign the contour of thought rather than disappearing beneath it. Two concepts combine to make this paradoxical *permanence of change* theoretically possible in Carnot. On the one hand, the idea of *a universal duration* or the assimilation of the cosmos to time (rather than time as a moving image of eternity as in some of the Neo-Platonic natural philosophies still at work in the mechanical theories, for example, the shadow which Nicholas of Cusa casts even over Newton), and on the other, the "inadmissible datum" of a temporal relation between the two sides of the changing universe. The inadmissible feature of this datum, which is none other than the changing significance of time, is that if the universe is itself changing moment to moment, then we cannot secure the antecedent and consequent proportions of our reason when we attempt to conceive of the universe according to any model of totality authorized by causality. There is no identity between antecedent and consequent in Carnot's understanding of the dissipation of heat in the universe because the universe can never be resolved into equal unchanging halves.

Accordingly, it is perhaps necessary to propose a more suitable "background image" of the figure of time in Carnot's work, something closer to a vortex or spiral of atoms orbiting and draining into a void. This is an image of time as the inevitable transformation of the system in which it is being measured. The reliable symmetry of causality is dispersed or distributed along a line which circles a void and consequently places the entire system of reference into movement. Carnot's principle is therefore a fact unlike the other facts of science. It seems to overturn the program of scientific rationality which would explain all transformations according to an invariant symmetry of causal equality.

> Carnot's principle is a fact, and by far the most important fact of all science. Indeed it is enough to regard reality without prejudice to be convinced that that which is permanent is but little compared with what is changing. It is solely that causal illusion which impels us to exaggerate the importance of the first at the expense of the second; what has remained is the essential, the "substance," whereas what is modified is only the "accident."[54]

It may seem that as Meyerson develops his analysis of Carnot he is providing something like a vindication of the Heraclitean universe against the Parmenidean sphere of being. Science finally breaks the transcendental illusion of causality by theorizing the principles of change without reference to unity. If, as Meyerson maintains, the immutable structure of the mind is the postulate of identity, this still does not preclude the theorization of a domain which refuses to submit to the intelligibility of the identical. *What is at stake with Carnot is a clear determination of the relationship between the empirical and explanatory registers of science, and accordingly, the possibility of the changing intelligibility of nature.*

With Carnot we are forced to account not only for a transformation of the conceptual intelligibility of time, but for an entirely different metaphysics. Unlike the Positivists, Meyerson does not think that science can ever rid itself of metaphysical notions. Therefore Meyerson's claim that Carnot's principle is "the most important fact of all science" must also carry with it a host of metaphysical implications. From what perspective does Meyerson distinguish among the facts of science, if not from the critical perspective of his own philosophy of the intellect?

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[54] Meyerson 1962, 278.

Between the lived reality of the phenomenon and the scientific explanation which renders it intelligible there is no possible mediation. We do not represent the world as it is, we construct the rules by which it becomes intelligible. In so doing we must lose the phenomenon itself. The domain of the plausible overlaps this gap in order to bring experience in line with the requirements of our intellect. The domain of the irrational overlaps this gap in order to reinstate, within science itself, the rights of reality.

> Identity is the eternal framework of our mind. We can only find it again, therefore, in all that it creates, and we have in fact noted that science is penetrated by it. But this does not make up the totality of science. On the contrary, Carnot's principle is an integral part of science. It is not quite fair to state, as Hannequin does, that 'science penetrates no part of real Becoming,' or else we must give to the verb *penetrate* the meaning of *render intelligible*, *rational*. Indeed, this proposition is only correct for explicative science. On the contrary, the task of empirical science is to penetrate the becoming; change in time is its proper domain. And that is why science – which includes both one and the other – is more and more dominated by Carnot's principle.[55]

The role of the intellect would seem to be that it adjudicates, based on what it is willing to accept as persuasive, between two functions or versions of science. That part of science which we call rational, we must understand in the restricted and specialized sense of Meyerson's philosophy of the intellect, according to which the rational is truly what is intelligible. The rules of intelligibility are the by now familiar processes of the formation of identity by the exclusion of time and diversity. *Explicative science* on this account proposes identity anytime it claims to understand the true cause of natural phenomena. What common sense rejects is precisely this

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[55] Meyerson 1962, 284.

disappearance of becoming. That which is “common” to common sense is indeed sense perception and the flux of appearances or *becoming,* which Meyerson assigns to the empirical. *Empirical science* seeks to penetrate becoming. Change is its proper domain and by change Meyerson understands *that which in time offers itself to thought*. That which is thinkable, Meyerson affirms, may not yet be that which is intelligible. The paradox of this *unintelligible thought* is not situated between the rational and the irrational, but between the irrational and the intelligible. Indeed, science is increasingly dominated by the *irrational thinking of time.*

> Science itself, therefore, reinstates reality in its rights. It proves that, contrary to what causality postulated, it is not possible to eliminate time, since this elimination would have reversibility as its preliminary condition, and reversibility does not exist in nature. The reversible phenomenon is purely ideal. It is only a limiting case of real phenomena, all irreversible at bottom. Antecedent and consequent are not ‘interchangeable,’ as we say in speaking of the pieces of a machine; they cannot be equivalents. The effect is not equal to the cause, contrary to what the scholastics affirm, because it cannot ‘reproduce the entire cause or its like,’ as Leibniz postulated.[56]

Only now is it possible to determine the corrective force of Meyerson’s critique of causality. In effect, Meyerson’s philosophy of the intellect demonstrates that science itself produces something much like a transcendental illusion every time it refers the intellect to causality. Indeed, science cannot help but think it can *undo* the flux of time because it constructs a reversibility of time when it equates antecedent and consequent in the postulate of identity. If science has these two fundamentally irreconcilable tendencies, one which serves the intellect, and one which at the same time corrects this excess by its empiricism, then it also seems as if

[56] Meyerson 1962, 284-285.

science is self critical and self optimizing, and this in spite of the tendencies of the mind. Science enters its truly critical phase only with Carnot. Empiricism finally refutes the totalizing ambition of the intellect. Here Meyerson's philosophy of the intellect proposes an alliance with the philosophy of Bergson.

> According to Bergson's profound formula great discoveries have frequently been made by 'soundings cast into duration.' The master sounding – the final discovery – is Carnot's principle, because it renders precise what is the very foundation of our concept of the sensible world which we, however, only feel obscurely: notions of time, of change, and irreversibility.[57]

This "master sounding" of duration whose scientific threshold comes into being with Carnot also has a metaphysical equivalent in Bergson, indeed, metaphysics itself changes into the intuition of duration. At the same time, a new precision becomes possible with Carnot which far surpasses the lived experience of qualitative change and transformation. This lived duration (the hallmark of Bergson's opposition between metaphysics and science) nevertheless points toward what Carnot unleashes with his empirical sounding of time. Perhaps it is better to reverse the formula: it is time that makes empiricism possible on the plane of thought. Time, as duration, introduces new possibilities of quantification inaccessible to the intelligibility of explanation. On this point Meyerson departs from Bergson. As we will see, Meyersonian empiricism is like the uncanny twin of Bergsonian duration in many respects. Even though there is no epistemological

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[57] Meyerson 1962, 286. The problem of inter-textual relations between Meyerson and Bergson is a substantial one. According to Meyerson's own account, Bergson's *Creative Evolution* is the metaphysical correlate to *Identity and Reality.* It is interesting to note in passing that Bachelard's *Dialectic of Duration* (1936), explicitly directed against Bergsonian duration, can also be productively read as the liquidation of Meyersonian philosophy of science.

continuum between explanation and empiricism (although there is the graduated field of increasingly plausible statements of law), there is still something like a shared but crucially non-communicating overlap of territories in the cognitive domains of scientific thinking which Meyerson has been trying to develop under the name of the irrational, on the one hand, and the plausible, on the other.

To maintain the philosophical lines of filiation and antagonism we have been pursuing with respect to Positivism and Bergsonian duration, it is possible to say that Meyerson's concept of the irrational is leveraged against the unhelpful critique of metaphysics offered by the Positivists (according to which metaphysics offers a clear image of being but at the cost of sacrificing quantitative precision), and in favor of Bergson's advocacy for a metaphysics of duration (according to which the intellect pursues tactics of quantification at the cost of sacrificing any understanding of being), but with the crucial caveat that the irrational is a kind of boundary internal to reason which forces the intellect to create new norms of intelligibility.

> We now clearly see how wrong we would have been to attribute to science the progressive elimination of reality, which is the consequence of successive identifications. We carry this idealistic theory with us before we constitute science, since it is with its aid that we have constituted science. 'Human intelligence,' says Bacon, 'is given to abstraction by its very nature; and it pretends to find constant those things which are in flux.' It is we who try to establish identity in nature, who bestow it upon her, who 'suppose' it hers, if we may give to this word the meaning it has in the expression 'suppositious' child. And that is what we call understanding and explaining nature. It yields itself to a certain extent, but it also resists. Reality rebels and does not allow us to deny it. Carnot's principle is the expression of the resistance which nature opposes to the constraint which our understanding, through the principle of causality, attempts to exercise over it.[58]

[58] Meyerson 1962, 286.

Meyerson's singularity as a thinker (a philosopher of the intellect) is to align empiricism and abstraction according to a novel methodology I would like to call irrationalism in keeping with the concept in its precise definition. Meyerson attempts to diagram the relations between the internal regulations of thinking and the pattern of resistance empirical science seems to offer to these regulating tendencies. It is important to note that for Meyerson empiricism is actually more abstract than the conventional rationalism of causality: more abstract because the real sign of abstraction is not disembodiment or ideality, but the modeling of time since time is the concept underlying change. As soon as science departs from its traditional task of explanation it begins to describe change according to a set of tactics which do not admit causality. This modeling of time becomes in effect an *unintelligible thinking of time;* duration without causality.

As we have seen, Carnot's principle is the forced recognition of the irreducible phenomena of change. The empirical obviousness of change, which Meyerson finds confirmed in the reality of our senses, also has an abstract and descriptive counter-part in the thermodynamics of heat and information exchange. Empirical science in this sense would constitute a boundary zone between the irrational and the constitution of new norms of intelligibility. It then becomes possible to qualify the philosophy of the intellect as a dialectical philosophy of epistemological norms. In order to explore the implications of the dialectical production of new norms intelligibility which Meyerson isolates as the very heart of scientific activity, it is necessary to examine Meyerson's encounter with the master of dialectical thought: Hegel.

Hegel and the Subordination of Mathematical Reason

The philosophy of the intellect demonstrates that when we reason correctly we have done so according to a common and invariant procedure. The clearest case of this procedure, as has been demonstrated previously, consists in the reduction of diversity in space and time in order to arrive at the principle of identity. The procedures by which reason empties diversities in space and time are uniform in their results. They always lead us to the principle of identity. However, the paths reason follows and the contents it encounters over the course of its operations differ fundamentally. This was the central insight of Meyerson's dialectical philosophy of the intellect and it prompted him to develop the theory of the irrational: the irreducible resistance of the diversity of experience to the principle of identity. Thus, in compiling a history of scientific reason, Meyerson inevitably compiles a history of the procedures of the reductions of diversity. The dialectical motor of Meyerson's theory of science is that while the results of this reduction of the diverse always serve explanation, the tactics, means, and methods of explanation are constantly changing. Mathematics is perhaps the most powerful method in the service of explanatory science for it represents the most rigorous means of deducing identity. It cannot however be mistaken for a mere tautology. Joseph LaLumia summarizes Meyerson's thinking regarding the procedures of identification in mathematics as follows:

> Are mathematical equations tautologies? Meyerson answers: the only real tautology is the principle of identity itself since no *cheminement de la pensée* [course of thought] is necessary for understanding it. The fact is that actual instances of thinking never involve statements precisely like 'A=A.' They involve some initial perception of differences, succeeded by an effort to substitute an identity. The principle 'A is A' is an ideal of the mind, but, if this ideal were ever realized, no thinking could ever take place afterward…This might be expressed in another way by saying that, although the mathematical equation may be tautological, the actual reasoning involved in understanding it to be true is not tautological.[59]

The philosophy of the intellect must always discriminate between the *products* of reason and the *procedures* of reason. The failure to discriminate between the products and the procedures of reason is perhaps most disastrous in the domain of mathematics, which constantly runs the risk of appearing tautological when in truth it is the most elegant and powerful means of manipulating a diversity of mental contents in order to produce identity. Meyerson cites Poincaré as an authority on "the art of mathematical reasoning:"

> …all mathematical deduction is composed of a series or (to use an expression that Henri Poincaré applied to his schema of reasoning by recurrence) a *cascade* of equations, and each time we write an equal-sign it goes without saying that what is placed to the right and to the left of it cannot really be identical, that it cannot be the actual repetition of the same formula – for then reason would not progress – but that what we have is an identity that will be recognized if we consider things from a particular point of view. The whole art of mathematical reasoning consists precisely in choosing what can be ignored, eliminated – either in virtue of unchanging mathematical conventions, as for example the convention recognizing the invariability of a geometric figure transported in space, or in virtue of previously demonstrated *identities* – so that what remains will provide us with a proposition bringing us closer to what we are trying to demonstrate. The

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[59] LaLumia 1966, 52.

> larger the step we take, that is, the more unexpected the proposition at which we finally arrive, the more felicitous, the more *elegant* the reasoning will be.[60]

Meyerson identifies the characteristic of mathematical elegance with the magnitude of the leap between the conventional starting place, the unconventional itinerary, and the return to the familiar proposition of equality. What is of interest in mathematics for Meyerson is the perhaps uncanny impression that something is being forced on us *in the name of the familiar or self evident* which is not at all familiar and self evident. The startling percolation of difference beneath identity and of diversity beneath equality is what Meyerson and Hegel both seek to understand in the dialectical motor of the principle of identity. The difference between them being that for Meyerson mathematics is an exemplary case of the identical emerging from the diverse, whereas for Hegel mathematics is beneath the concern of philosophy because it cannot break with the nullity of the tautological proposition. It should then come as no surprise that Hegel figures so prominently in Meyerson's second great work, *Explanation in the Sciences* (1921), which takes up and expands the theses developed in *Identity and Reality* in order to more fully explore the dialectic of the intelligible and the irrational in the history of science and philosophy.

Explanation in the Sciences stages two direct encounters with Hegel. The Fifth Chapter, "Identity and Identification," essentially reprises the thesis of *Identity and Reality* but from the perspective of the history of mathematics in order to

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[60] Emile Meyerson, *Explanation in the Sciences* Trans. Mary-Anne Sipfle and David Sipfle (Dordrecht: Kluwer Academic Publishers, 1991), 110-111.

demonstrate that mathematical reasoning does not proceed according to *self evident equivalences* but must rather construct or invent a system according to which the dissimilar can become the similar. Here Hegel is at once an ally and a foil for Meyerson. Hegel recognizes the dialectical power of synthetic judgments while also apparently denying this power to mathematical reason. Meyerson will maintain, on the contrary, that mathematical reason is the exemplary case of dialectical synthesis. The Eleventh Chapter, "Hegel's Attempt," examines Hegel's writings on the natural sciences in order to critique Hegel's rejection of the sciences as procedures of explanation in favor of notional or speculative explanations furnished by the philosophy of the absolute. Meyerson claims that because Hegel is unable to appreciate the dialectical resources of mathematical thought, he is equally unable to understand the nature of the dialectic governing the form and the development of scientific concepts. For Hegel, only the speculative power of the concept can overcome the contradiction internal to all natural laws: the qualitative *content* of natural phenomena subjected to the quantitative *form* of mathematical equivalence. Here the concrete power of reason must correct the abstraction of the intellect, which mistakes the intelligible for the real. Meyerson will critique Hegel's dialectic of concrete and abstract reason on the grounds that it attempts to render the irrational rational through the intermediate concept of *becoming.* Becoming, or temporal duration, is the domain of the empirical for Meyerson. Hegel thus distorts the true dialectic of scientific intelligibility, which is that of the intelligible and the irrational (the empirical), rather than the abstract and the concrete. The consequences of this error, as Meyerson will demonstrate, are profound.[61]

Meyerson claims that Hegel is fundamentally unable to appreciate the nature of mathematical thinking. Hegel's mistake is to identify mathematical reasoning with the form of tautological propositions:

> Generally, in mathematics, 'knowledge advances along the lines of bare equality,' and 'herein consists the formal character of mathematical evidence.' But of course herein also lies its chief defect.
> For what is lifeless, not being self-moved, does not bring about distinction within its essential nature; does not attain to essential opposition or unlikeness; and hence involves no transition of one opposite into its other, no qualitative, immanent movement, no self movement. (*Phänomenologie,* 2:35 [Baillie 103])[62]

The lines of bare equality are not really the lines of thought. They do not advance beyond what is essentially an immediacy of perception. Hegel reduces mathematics to "the formal character of mathematical evidence" as if it were a stable and inert self identity incapable of "immanent movement." Hegel's mistake is canonical: for him mathematics is a body of evidence and not a progress of reason. It is a tautological proof rather than a method by virtue of "not being self moved." This is of course to neglect the founding principle of the philosophy of the intellect: do not confuse the end and the means of a rational procedure.

> It must be added that although Hegel sometimes deals at length with certain mathematical concepts in the two *Logics*, anything having to do with the internal mechanism of mathematical demonstration is treated more or less in passing and without great rigor. This is apparently because the way thought

[61] For an admirable survey of twentieth century perspectives on the possible implications of Hegel's philosophy of nature for contemporary research in the philosophy of science see Robert S. Cohen and Marx W. Wartofsky, *Hegel and the Sciences* (Boston: Riedel Publishing Company, 1984).
[62] Meyerson 1991, 284.

> proceeds there seems almost insignificant to him, at least in comparison to the *dialectical* process, which is the one he believes conscious thought must use in order to arrive at results having a real value from the standpoint of comprehension.[63]

Insofar as Hegel devotes any time and attention to specifically mathematical concepts, it is without any dynamic appreciation of mathematical demonstration as a genuinely *creative* task. The intellectual labor of mathematics must be defended against Hegel's dismissal because it is just as challenging and rewarding as the dialectical process. What is most distressing about Hegel's subordination of mathematics to dialectics is that Hegel is uniquely positioned to appreciate the dialectical resources hidden beneath apparently serene statements of equality. On this point Hegel distinguishes between logic and mathematics, for whereas mathematics is a lifeless inertia of bare equality, logic contains an element of internal difference which makes it an ally of "the concrete philosophical sciences:"

> Mathematical reasoning must not be imitated in any other science; in particular 'in the concrete philosophical sciences philosophy must take the logical element from logic, not from mathematics' (*Wiss. Der Logik,* 3:250 [Miller 216]). 'The evidence peculiar to [mathematics]…rests solely on the poverty of its purpose and the defectiveness [*Mangelhaftigkeit*] of its material, and is on that account of a kind that philosophy must scorn to have anything to do with' (*Phanomenologie,* 2:34 [Baillie 102.])[64]

What is the specific defect by which mathematics becomes unsuitable as an ally to philosophy? Here we encounter the gap between the order of quantitative determination and qualitative determination. Philosophy cannot accept the poverty

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[63] Meyerson 1991, 284.
[64] Meyerson 1991, 284-285.

of exclusively quantitative determinations, especially as these come into being in the laws of nature which have as their ends the explanation of qualitative realities:

> For example, speaking of the Newtonian demonstration of Kepler's laws, he [Hegel] states that
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> > Mathematics is altogether incapable of proving quantitative determinations of the physical world in so far as they are laws based in the qualitative nature of the moments [of the subject matter]; and for this reason, that this science is not philosophy, does not start from the Notion, and therefore the qualitative element, in so far as it is not taken lemmatically from experience, lies outside its sphere.[65]

Mathematics is incapable of joining the qualitative and quantitative together in the determination of the laws of nature. For the philosophy of the intellect, the gap or interval separating the order of quantitative determination from qualitative nature is called the irrational. It is exactly at this point that Meyerson and Hegel pursue radically opposed itineraries. Hegel will seek to overcome the irrational by way of the concept of *becoming*. Meyerson will insist on the irreducibility of the irrational in order to uphold his theory of scientific explanation. For Meyerson, scientific explanation *can only come into being as the bracketing and exclusion of the irrational.* If one seeks to reason about the irrational, that is to say, to treat it as the intelligible, then it becomes impossible to grasp the specificity of the epistemological norms which govern the movement and development of science. The elemental form of the irrational as such is the non-applicability of quantitative determinations to the world of qualitative experience. It is crucial, where science is concerned, that the irrational remain irrational. One does not want to philosophize if one is concerned

[65] Meyerson 1991, 280.

with explaining phenomena. It is necessary to break with experience, not to seek to do it justice.[66]

The issue at stake here is the role of mathematical explanations in the natural sciences, which would apparently cross the divide of the rational and the real by deducing the nature of physical reality from physical laws. The difference between the philosophy of the intellect and Hegel's philosophy becomes increasingly aggravated from the perspective of an epistemological imperative which Pierre Duhem (1861-1916) has called "saving the phenomena."[67] Does that part of science which purports to explain the phenomena in fact betray the phenomena, as Hegel maintains, or does the phenomena itself bend the scientific concept around its own contour? Meyerson thinks that science is an effort to explain reality but he has no intention of grounding the phenomena itself in a scientific world view. The phenomena must remain groundless *if it is to be explained by science.* The philosophy of the intellect hails as a virtue what Hegel's philosophy of nature condemns as a distorting abstraction. The philosophy of the intellect and the philosophy of nature therefore confront each other with incompatible dialectical procedures. Meyerson's theory of science asserts that science is governed by the perpetual transformation of epistemological norms driven by the conflict of the irrational and the intelligible. The irrational becomes intelligible by means of mathematical labor. Hegel affirms that there is something fundamentally awry with

[66] On this point Meyerson is actually very close to Bachelard.
[67] See Pierre Duhem, *To Save the Phenomena, An Essay in the Idea of Physical Theory from Plato to Galileo* Trans. Edmund Dolan and Chaninah Maschler (Chicago: The University of Chicago Press, 1985.)

scientific explanations of nature, and that philosophy must confront natural laws with a dialectic of concrete and abstract forms of reason in order to correct the waywardness of exclusively quantitative determinations of phenomena. The mathematical excess of natural law must be corrected by logic, which is a more supple mechanism of thought than mathematics. Meyerson summarizes Hegel's dialectic of the forms of reason as follows:

> …the faculty invoked to direct our thoughts, or in other words *reason*, actually turns out to be twofold: on the one hand, there is *abstract reason*, which, in its search for perfect identity, is led to deny diversity and, on the other hand, there is *concrete reason*, which, by going beyond this diversity, brings about reconciliation…From the standpoint of form, therefore, logic presents a triple aspect: the abstract or rational, the dialectical or negatively rational, the speculative or positively rational. Obviously the first of these aspects conforms to abstract reason, the second is the one in which the inner contradiction of thought conceived according to that reason is revealed, while the third is destined to resolve the contradiction.[68]

Logic is capable of resolving contradictions (owing to its three forms) which are intractable for mathematics. Hegel's *Encyclopedia* elaborates a breathtaking deduction of concepts following this procedure. Meyerson must confront Hegel's philosophy because Hegel proposes to do with logic what Meyerson seeks to accomplish with the philosophy of the intellect; namely, to overcome the contradiction of being and becoming which confronts the epistemologist. Meyerson finds that Hegel attempts to deduce a mediating concept between the diversity of sense experience and the identity of abstraction by following the dialectical progress of the concrete and the abstract in their logical unfolding. Hegel's concept

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[68] Meyerson 1991, 264-265.

of *becoming* in short plays the same role in the philosophy of nature that Meyerson's concept of the *irrational* plays in the philosophy of the intellect.

> Becoming (which is the speculative concept) is the unity of being (the abstract concept) with nonbeing (the dialectical concept.) No doubt it is generally claimed that we are unable to arrive at a representation of the unity of being and nonbeing, but this is not correct; on the contrary, each of us has an infinite number of these representations:
>
> > The readiest example of it is *Becoming*. Every one has a mental idea of Becoming, and will even allow that it is *one* idea: he will further allow that, when it is analyzed, it involves the attribute of Being, and also what is the very reverse of Being, viz. *Nothing:* and that these two attributes lie undivided in the one idea. (*Enc., Logik,* 6:174 [Wallace 165-166])
>
> Furthermore, 'pure being and pure nothing are… the same …Their truth is…this movement of the immediate vanishing of the one in the other: becoming.' Thus Heraclitus' doctrine surpasses that of Parmenides while preserving it (this is the Hegelian concept of *aufheben…)*[69]

The foundation of logic itself is at issue here because Hegel thinks it is possible to escape the tautological emptiness of the mathematical proposition by elaborating a *transcendental logic* capable of producing new knowledge by putting the forms of reason into motion. The *becoming* of the speculative concept allows Hegel to unify the abstract concept (the proposition of logical identity) with the nonbeing of the dialectical concept. Hegel wants to surpass Parmenides by preserving the principle of identity he assures while also affirming the nonbeing or flux of Heraclitus. Hegel wants to restructure the relations between empirical observation, "mathematical description," and notional content (the body of natural laws), in order to secure the intelligibility of natural laws while also preserving the qualitative fullness of the phenomena. Hegel does not want intelligibility to come at the cost of sacrificing being, and he does not want being to be an unintelligible

[69] Meyerson 1991, 266.

continuum of transformation. The speculative concept of becoming allows Hegel to surpass Parmenides while also preserving him.

Meyerson is wary of the concept of becoming because it unifies the rational and the real (the irrational in Meyerson) in a way that the philosophy of the intellect must reject: the *diversity* of the irrational cannot be compromised or surpassed without also compromising the integrity of the forms of reason which emerge in contradistinction to it. Meyerson admires Hegel's ambition, but he finds that Hegel underestimates the irrational by making it internal to identity as contradiction. This is in fact to standardize the irrational: it is the attempt to reason about that which must always escape reason. The irrational is indeed the diversity of sensation which naturally opposes the principle of identity, *but this is "only an initial manifestation of the irrational:*"

> The element of contradiction whose presence Hegel establishes in each application of the principles of identity, that is to say in all reasoning and particularly in scientific reasoning, is certainly only an initial manifestation of the irrational. It is the discovery that *diversity* exists, while our reason would prefer that there be none. Of course, Hegel asserts that this reason is not the right one, that it is only abstract reason and that above it is another reason, concrete reason, which understands diversity perfectly and makes use of it in order to explain things…what can be seen immediately is that by enlarging the scope of his explanation in this way, Hegel includes at least something of the irrational element constituted by the existence of diversity.[70]

Hegel wants to "enlarge" the scope of explanation by enlarging reason. He wants to incorporate the diversity of experience into reason by making the irrational serve reason. By reasoning about the irrational (by treating it as a dialectical moment in the universal movement of the intelligible) Hegel distorts the complexity of the

[70] Meyerson 1991, 277.

irrational. The speculative concept of becoming is far too general, it passes over without regard or interest that which Meyerson wants to pay close attention to; namely, the resistance which the irrational throws up in the face of reason. The resistance of the irrational forces reason *to interrupt its own procedures of causal explanation and to produce a new norm of intelligibility.* Hegel thinks he has solved the dilemma of Parmenides and Heraclitus with the concept of becoming. In fact, he has distorted the relation between the principle of identity and the real, which Meyerson's dialectic of the irrational and the intelligible attempts to preserve.

> …first of all it should be noted that Hegel himself practically nullified any possible advantages his irrational might provide by the fact that, having included it in his explanation, he therefore had to consider it explicable or *reasonable*. He *reasoned* about this irrational, reasoned exactly as if it had been rational (for when all is said and done there is only one way of reasoning). Thus he rationalized it, disciplined it so to speak, attempting through pure reasoning to assign it its share and its limits.[71]

The difference between Meyerson and Hegel is apparently irresolvable where explanations of natural phenomena are concerned. In Hegel there is no truly resistant irrational element in experience. There is thus no need no develop a theory of scientific explanation which takes into account the very necessity by which scientific explanation *must* bracket or leave out *sensation, diversity,* and all deductions from experience. Indeed, for Hegel, the opposite is the case: deductions proceed from experience and not the other way around. The role of logic and mathematics is curtailed in the interests of philosophical deductions from the Notion, which is qualitative rather than quantitative. The problem is certainly not

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[71] Meyerson 1991, 279-280.

that Hegel doesn't understand the limits of logical deduction. The problem is rather that he cannot tell why those limits are necessary, not just for the natural sciences, *but for the constitution of intelligibility itself*. The intelligible, for Meyerson, does not proceed toward knowledge of the Absolute but ceaselessly constructs a local or provisional absolute in the form of the principle of identity. There is no possible unity among the forms of intelligibility in Meyerson. For Hegel, as is well know, the successive forms of reason continue to be surpassed and preserved in the dialectic of the Absolute as a Science of science.

Meyerson can make his case at an even more fundamental level by calling into question Hegel's very opposition of the concrete and the abstract. Is it not the case that this distinction (abstract/concrete) is itself hopelessly abstract (or meaninglessly concrete) in the absence of the determinate difference which truly gives sense and urgency to the distinction in the first place, namely, the perspective authorized by a given system of scientific explanation in the service of a determinate system of causality? We do not simply abstract from the concrete. It only makes sense to distinguish between the abstract and the concrete when there is a principle of identity already in place, and thus a more fundamental opposition between the intelligible and the irrational. Meyerson would replace Hegel's abstract/concrete distinction with an identity/difference distinction always already in the service of causal explanation.

Meyerson's central disputation with Hegel concerns the unity of reason, on the one hand, and the multiplicity of the irrational, on the other. Where Hegel insists on reasoning about the irrational (sublating all that which would resist thought by pursuing a concept of becoming), Meyerson will insist on a strict division between phenomenal becoming and epistemic identity. At the core of Meyerson's disagreement with Hegel, as we have seen, is the anomalous status of mathematical deduction in the Hegelian system. By denying dialectical transformation to mathematical deduction Hegel misconstrues the nature of scientific explanation, rejecting it as tautological when in fact it does produce something new in the content of its constantly renewed procedure of rationality. Meyerson wishes to retain Hegel's vital insight that all thought is crossed by contradiction, but he cannot accept Hegel's affirmation of a superior synthesis uniting and surpassing the contraries of thought. At exactly the point where Hegel dismisses mathematics for pursuing abstract reason in the face of the diversity of sense experience, Meyerson wishes to extract a regulative procedure for science itself:

> What must be retained from the Hegelian *process* is, first of all, the very important fact of the existence of a contradiction, an antinomy, in every instance of reasoning; and then it is the way in which reason goes beyond this conflict, putting it aside, so to speak, since, on the whole, although we are aware, if we ask ourselves, that there is a contradiction, the rest of the time, we do not hesitate to act as if it did not exist, as if everything were perfectly rational. And Hegel was also correct in believing that this process of *going beyond* is indispensable for our scientific knowledge, that it is applied quite generally in the domain of science. But this generality is even greater than he supposed. For there is more uniformity in the procedures of our understanding than follows from his system. Indeed, for him the *going beyond* took place by means of two essentially different procedures: on the one hand – in mathematical reasoning – by simply setting diversity aside and, on the other hand – in all other reasoning, including that of the physical sciences – by the 'dialectical' process properly speaking, which enriches the concept by means of negation. Now, such is not the case, and our reasoning – mathematical, physical or otherwise - is entirely analogous in this respect. For the alleged 'dialectical' process actually amounts quite simply to the

> same *setting aside* that Hegel did indeed observe in mathematics but disdained, as it were, declaring it unsuitable for application elsewhere. In reality it is to mathematical reasoning that science conforms, or at least tries to conform, in all its parts.[72]

For Meyerson, mathematics provides the very form of the authority of reason. This does not however mean reason is obedient to a tautology without content. Reason must prove to itself, by following the paths of rational deduction, that it has the right to explain natural phenomena by *learning* how to reduce phenomena to causal mechanisms. Although reason must arrive at this result the *method* is never determined in advance, nor is the itinerary. For Meyerson, the problem is precisely *not to overcome the contradiction between the rational and the real* but to constantly find new ways of stating this very contradiction. *The enumeration of contradictions is the prolegomena to any new science in Meyerson's philosophy of the intellect.* Reason is one. Science remains multiple. What Meyerson's theory of scientific rationality is able to do is to indicate how a unitary form of reason can also produce new concepts and rationalities. This is just what Hegel's *Science of Logic* must disavow. When Hegel embraces becoming he turns his back on the authentic dialectic of the intelligible and the irrational. Let the dogma of science be causality and identity: let the dogma of experience be change and becoming. They do not oppose each other in the mirror of the absolute. The real seduces the intellect. It conjures the forms of intelligibility in ceaseless varieties which do not themselves compose a unity. This would be the abdication of thought.

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[72] Meyerson 1991, 290-291.

It was necessary for Meyerson to confront the philosophy of Hegel because Hegel's dialectic represents an alternative effort to overcome the epistemological antagonism of being and becoming, of Parmenides and Heraclitus. The philosophy of the intellect found itself confirmed in its thesis that the intellect requires the irrational, and that any effort to overcome or to surpass the irrational by internalizing it as a part of a transcendental logic can only result in an intolerable distortion of the image of reason. In the next and final section, Meyerson's philosophy of the intellect will encounter the most prestigious and theoretically ambitious scientific theory of its time, Einstein's general theory of relativity. Relativity would seem to call into question the central conviction of the philosophy of the intellect by presenting a theory of space-time which makes both time and space relative to a third domain: a curved space-time. Is Meyerson able to provide a persuasive interpretation of relativity without mangling the specificity of Einstein's work (as was the case with Bergson's ultimately unsuccessful debate with relativity), and without abandoning his thesis that reason must obey the principle of identity?[73] Does relativity really bear out the thesis that physics amounts to a geometrization of time? Or does time again evade the intellect and remain inconceivable beneath a haze of contradictory intuitions?

The Progressive Rationalism of Deduction: Meyerson Reads Einstein

[73] See Henri Bergson, *Duration and Simultaneity* Trans. Leon Jacobson (Manchester: Clinamen Press, 1999.)

As we have seen, Hegel's philosophical system was of great interest to Meyerson because it attempted to overcome the opposition of the real and the rational. Meyerson however was critical of Hegel's dismissal of mathematics as an empty tautology. Consequently, Meyerson found that Hegel was unable to appreciate the real relationship between mathematics, as the form of the principle of identity, and the necessary bracketing of experience which this principle entails. Rather than attempting to overcome this boundary between the rational and the real, as Hegel attempted to do with his transcendental logic and the concept of becoming, the philosophy of the intellect upholds the irrational in all its diversity and unintelligibility. It maintains that mathematical reason is not tautological from the perspective of its operations. Only its products conform to the principle of identity. Mathematics then must be affirmed in its own right as a dialectical and creative activity. The creativity proper to science is driven not by the overcoming of the irrational, as Hegel attempted to show in his philosophy of nature, but by the provisional bracketing of the irrational as the result of the principle of causality. The causal principle and the irrational continually confront one another: the intellect demands that reason conform to the principle of identity but the paths by which this identity is achieved are necessarily multiple. As a dialectical theory of the norms of intelligibility, the philosophy of the intellect is at once unitary and multiple. It is unitary in so far as we can only reason according to the principle of identity. It is multiple insofar as the encounter with diversity (the empirical aspect of science) always selects for a new set of criteria with which to assemble the causal principle.

Meyerson's encounter with Hegel is premised on the latter's dismissal of mathematics. His encounter with Einstein, on the contrary, attempts to understand the rational procedure which subtends the claims of relativistic physics. This will involve a rational reconstruction of the deductive chain defined by the geometrical theories out of which Relativity emerges. Einstein and Hegel are complimentary but inverted figures in the philosophy of the intellect. Hegel's intellectual adventure is of interest to Meyerson insofar as it attempts to bypass mathematics. It is defined by the principle that mathematics is incapable of addressing the real. Einstein's intellectual adventure is interesting because it extends the deductive sequence of mathematics further than any previous physical theory. It is defined by the spatialization of time itself according to a geometrical theory of space-time. Mathematics becomes the ideal of a new form of objectivity defined by an unprecedented combination of abstraction and precision, a thesis Meyerson attempts to demonstrate conclusively in *The Relativistic Deduction* (1924).[74]

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[74] Gaston Bachelard defines his epistemological method against Meyerson's by pursuing a systematic polemic against the latter's *Relativistic Deduction* over the course of his own book on relativity, *La Valeur Inductive de la relativité* (1929). Bachelard's critique of Meyerson is as follows. Bachelard claims that Meyerson is too concerned with the application and verification of relativity. For Bachelard, this means that Meyerson is affirming a parity of thinking and being: "[Meyerson] demonstrates that thought really touches the real, that that which is coherent in the mind is equally coherent among things, that the consequences of a geometrical ideation are the very consequences of experience." (Bachelard 1929, 201). It must be noted that this characterization entirely distorts Meyerson's determination of the relation between experience and geometry in relativity, as this section will demonstrate. Bachelard's objection is that (contra Meyerson) before one can explain one must construct: *Mai avant d'expliquer, il faut construire.* (Bachelard 1929, 202). Meyerson's epistemology of explanation is committed to a form of realism which Bachelard wishes to avoid for two reasons: realism is always derived rather than a

How does Meyerson understand the role of mathematics in physics?

> …the role mathematics plays in physics tends to take a quite specific form. It is easily seen that, even though the mathematics of abstract magnitude, that is, algebra, is regularly used in physics, still, whenever it is a question of representing reality, one resorts exclusively to the mathematics of spatial magnitude, namely geometry… geometrical properties can be transformed into algebraic properties by means of the physical operation of measurement and thereafter treated according to the rules established for algebraic operations. Conversely, results obtained by strictly geometrical methods, where spatial intuition plays an important role, can be translated into purely algebraic language. The parallelism between algebra and geometry strikes as us soon as we enter the intellectual realm.[75]

Physics, far from being a homogenous idealization of space already reveals itself to be composed of heterogeneous regimes. Meyerson calls this the "parallelism" of algebra and geometry. Physics is a discourse premised on the translation between two domains, the domain of "abstract magnitudes" described by algebra, and the domain of "spatial magnitude," described by geometry. At issue is the mutual immanence of two systems of magnitude. Abstract magnitudes must be translatable

starting point (and Meyerson, in truth, agrees with this.) The epistemological question cannot be (as it is in Meyerson) where is the real? But rather "in which direction and according to what organization of thought can one securely approach the real?" (Bachelard 1929, 203). For Bachelard, this second orientation reveals epistemological critique as a *moving perspective*, not a static starting point or transcendental perspective. Epistemology is not a location or a topoi for Bachelard, it is a movement or a traversal between at least two points. Following Andre Lalande's criticism of Meyerson, Bachelard wants to insist on the procedures of correction by which the mind clarifies its ideas about itself (which he calls corrections of principle), and its ideas about reality (which he calls corrections of things.) The relativistic rectification addresses principles exclusively, so this requires a different starting point than Meyerson's. For Bachelard, this means that Meyerson's realist epistemology can never understand what relativity actually does (the objectivity it constructs) because a strict realist must *find* or *uncover* reality rather than constructing it. (Bachelard 1929, 204). For Bachelard, essence must be a function of relation (Bachelard 1929, 208), as I will explain in Chapter Four.

[75]Emile Meyerson, *The Relativistic Deduction* Trans. Mary-Alice Sipfle and David A. Sipfle (Boston: D. Riedel Publishing Company, 1985), 28-29.

into spatial magnitudes and vice versa. Physics is perpetually astride these two domains. Its purely theoretical aspect treats "results obtained by strictly geometrical methods," i.e. empirical measurements, by "the rules established for algebraic operations." Any point in space can be defined algebraically according to the conventions of the Cartesian grid by which it is assigned numerical values corresponding to the three dimensions of space. Geometry is in turn related to what Meyerson calls "spatial intuition," which refers to the metaphysical presuppositions which subtend our intuitions of the properties of space.

Although mathematical physics is only possible because of the parallelism of algebra and geometry, Meyerson notes that geometry "seems to contain elements that cannot be reduced to number, not even to concepts derived from number by extension…"[76] The element of spatial magnitude which cannot be reduced to number is what Meyerson calls spatial or geometric intuition:

> The special quality of space which resists being entirely translated into its algebraic equivalent is what would seem to account for the advantage referred to above that makes the spatial, unlike the purely algebraic, appear to be endowed with a certain reality, and consequently suitable for representing the real.[77]

Physics embodies with peculiar clarity the tension internal to all norms of intelligibility. The abstract must assert itself in all the rigor of its abstraction but it must also take its departure from the real. It must assert its right of "representing the real." In physics this tension is generated by the feature of geometric magnitudes which bear the imprint of what Meyerson calls "the spontaneous

[76] Meyerson 1985, 31.
[77] Meyerson 1985, 32.

metaphysics of our perception." Part of Meyerson's critique of Positivism is that it fails to appreciate how metaphysical notions arise spontaneously in all our cognitive acts. Even when we reflect on our spontaneous perception, we cannot help entertain certain metaphysical convictions. In the case of perception, Meyerson claims that we spontaneously generate a concept of space because we have the impression that our physical sensations have causes outside themselves. Meyerson thus concludes that perception contains the germinal form of a theory of bodies in relation to space, for either:

> we are dealing with the relations of one body to another – as in the case of impact or gravitational attraction – or else we are dealing with the relations of a body to space – as for example in all cases involving the motion of a single body in space.[78]

The metaphysical aspect of perception is the conviction that bodies must either interact with one another or with the medium in which they are immersed. This conviction of a minimum degree of relation, either between bodies or between a body and space, constitutes the element of geometrical intuition which cannot be expressed algebraically. From the perspective of the philosophy of the intellect, physics thus composes itself out of two different methods of explanation which do not entirely overlap, even though they enjoy a formal equivalence with one another.

Physics cannot therefore claim to be entirely divorced from some minimum degree of qualitative entanglement because it carries within itself this trace of spatial intuition. The objectivity of physics then comes to represent an interesting problem for the philosopher of the intellect because the principle of identity is

[78] Meyerson 1985, 32.

premised on the exclusion of all relations with the phenomenal world. Objectivity as such is not the clear comprehension of the object. It is instead the supreme separation of the phenomenon from the observer. The phenomenon becomes not the object of perception but the object of science – a rational construction. How does physics claim to bring about this separation between the observer and the phenomenon, and how does relativity in particular advance the progressive abstraction of the principle of identity?

> …what the Einsteinian physicist seeks is a way of representing physical phenomena valid not only for an observer on earth or on the sun, or even for a super observer placed somewhere in space and assumed to be immobile (this is, of course, more or less Newton's point of view), but for all possible and imaginable observers at once. Obviously he can do this only by separating the phenomena from the observer more completely than science had ever done before, that is, by locating the phenomena outside him in a more absolute way…It is indeed in this form that Einstein himself first presented this theory to the scientific world, insisting on the fact that general relativity furnishes a system of representation of the phenomenon independent of any system of reference (that is to say, of anything referring to a particular observer), whatever the motion of the system.[79]

Relativistic physics achieves a new level of abstract representation. Its superiority over the Newtonian system ultimately consists in that fact that it is able to remove the phenomenon from any system of reference related to a particular observer. Meyerson thus has a peculiar understanding of how representation operates in the domain of scientific explanation. The ontological status of the representations of science is not called into question by the increasing separation of the phenomenon from the observer. Rather, the ontology of the phenomenon consolidates itself to the

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[79] Meyerson 1985, 48-49.

same degree that it is able to progressively emancipate itself from lived experience:[80]

> …one would be mislead by appearances if one believed that relativity theory connects the phenomenon any more strongly to the observer, or (to use the philosophical term) the self. Quite on the contrary, the phenomenon is more detached from the observer, as we have seen, and the reality of relativity theory is certainly an ontological absolute, a veritable being-in-itself, even more absolute and ontological than the things of common sense or pre-Einsteinian physics.[81]

It is necessary to reverse the typical distribution of the idealist and realist poles of philosophy when it comes to relativity theory. The philosophy of the intellect forges a counter-intuitive relation between the increasingly abstract representation of the phenomenon and the ontological status of the theory itself. Relativity surpasses the ontological certainty of Newtonian physics because it embraces and *explains* the entire Newtonian system. Einstein's physics also adds a supplementary layer of theoretical complexity which relativizes the invariants of the Newtonian universe, time, space, and gravity.

Relativity accomplishes this ontological envelopment of the Newtonian universe by completely dissolving the intuitive organizing principles of time and space. Between the spatial intuition immanent to perception and the *a priori* geometry of relativity there can be no mediation. The physics of relativity thus effects a break with pre-relativist physics by transforming the epistemological contour of all of its concepts. Relativity no longer entertains a simple parallelism of

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[80] Bachelard will develop an almost identical critique of the nature of objectivity in relativistic physics. See Chapter Four.
[81] Meyerson 1985, 55.

algebra and geometry. The distinction between the abstract and the spatial magnitude is destined to be overcome by a continuum of relations which describes an absolute fabric of space-time. The explanatory rigor of the geometry describing this space-time manifold surpasses Newtonian physics by accomplishing a true immanence of space and time, rendering both concepts meaningless. It is no longer possible to uphold the reality of time and space as distinct variables:

> 'Henceforth,' Minkowski says, in setting forth the fundamentals of his conception, 'space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.' A little later in the same fundamental exposition of his theory he repeats that 'space and time are to fade away into shadows, and only a world in itself will subsist.'[82]

The distressing objectivity of relativistic physics dissolves the embodied spatial and temporal co-ordinates of an embodied observer in favor of a "world in itself" which can have no intuitive correlate in our experience or imagination. The ontology of relativity is more absolute and more real than the ontology of lived experience exactly to the degree that it is able to effect a complete separation from experience while also providing a mathematical explanation of the nature of experience.

The more relativity estranges us from out intuitions, the more it secures its hold in a superior objectivity which the physicist cannot render in the language of customary experience:

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[82] Meyerson 1985, 70. See also Elie Zahar, "Einstein, Meyerson and the Role of Mathematics in Physical Discovery,' *British Journal of the Philosophy of Science*, no. 31 (1980):1, for a rigorous and clear account of the differences between the special and general theories of relativity from the perspective of what Zahar calls "*a priori* geometry."

> Einstein himself asserted that in relativity, 'from a 'happening' in three-dimensional space, physics becomes, as it were, an 'existence' in the four-dimensional 'world'.' Eddington maintains that in relativity theory 'the continuum formed of space and imaginary time is completely isotropic for all measurements; no direction can be picked out in it as fundamentally distinct from any other.' Thus, 'events do not happen; they are just there, and we come across them [as we follow our world line]. 'The formality of taking place'…of the event in question…has no important significance.'[83]

We are immersed in the three dimensions of space and one of time, consequently, events seem to befall us and the direction of time seems irrevocable and necessary. Relativity, however, allows us to think the totality of space-time and therefore achieves a perspective external to the Newtonian universe. From this perspective, 'events do not happen' but are themselves features of a landscape which displays time and space as the curvature of a unified super-symmetry of space-time: a hyper-dimensional still life of all the events internal to space-time. An n-dimensional geometry can describe the tendencies of a dynamic system as a series of periodic attractors and singularities determining the topological features of a phase space. This is the mathematical equivalent of the spatial representation of changes in time.

Is this not the scientific ontology which most perfectly realizes the ideal of the Sphere of Parmenides?

> It should be noted that if time and space are henceforth to be more or less merged into a single continuum, this change will clearly work to the advantage of space. This is apparent from the evidence we have just cited, for if all becoming is to be transformed into being (according to Einstein), so that the act of taking place becomes a simple unimportant formality for an event (according to Eddington); if succession is only an illusion (according to

[83]Meyerson 1985, 71. The editor to the English translation of *The Relativistic Deduction* points out that in the full context of the citation Eddington in fact rejects the position Meyerson here assigns to him. See Arthur Eddington, *The Nature of the Physical World* (Ann Arbor: University of Michigan Press, 1958), 50-52, 55-58.

> Marais), and if every physical system constitutes a changeless whole (according to Cunningham) - this can only mean one thing: the abolition or disappearance of time. Therefore, Cunningham does not hesitate to speak of 'the timeless universe of Minkowski…' Let us observe, moreover that this already follows from the very fact that the construction at which one arrives is a *geometry.* And one need only open an exposition of the doctrine to note that, where time is concerned, the writer always speaks of one dimension, obviously conceived as spatial, while no attempt is ever made to represent the properly spatial dimension in terms of time.[84]

Being must be without internal differentiation in order to be thinkable. Parmenides knew this and so dismissed the reality of change. The *more geometrico* reproduces this dogmatic image of being in a more persuasive form in the deductions of relativity. The disappearance of time from the universe is equally the assertion that time, *ontologically*, must be an illusion. The objectivity of the relativist is indeed a geometrical construction, it is nothing less than the geometry of space-time. However, it is equally certain that time does not exhibit degrees of freedom in the same way that space admits the reversibility of motion. This is the irreducible form of the irrational which science self consciously ignores and excludes from the chain of deductions that proceed from the forms of spatial explanation.

> Is it accurate to say with Eddington that in physical reality 'the continuum formed of space and time is completely isotropic for all measurements…' it is nonetheless certain that the ability to move in time is extremely limited, even according to the new concepts themselves…Thus Einstein himself uses the argument that we 'cannot telephone into the past,' and Eddington states that the notion of entropy has survived the Einsteinian revolution and is (along with the principle of least action) one of the two generalizations towards which physics is converging. Now the notion of entropy, which grew out of Carnot's principle, is nothing more than the expression of the irreversibility of phenomena, that is, of continual progress in time.[85]

[84] Meyerson 1985, 72.
[85] Meyerson 1985, 72-73.

Entropy is the constant erosion of the sphere of Parmenides. It is the undeniable refutation of changeless reality or the insertion of time into thought. The isotropism of relativity cannot, in point of fact, treat time as a reversible variable. If contemporary physics (according to Eddington) is tending toward a convergence of relativity and the "irreversibility of phenomena," then the status of time in relativity discloses an immediate limit to the theory. Physics can remain indifferent to changes in time by treating it as a magnitude. The manipulation of time at the level of representation treats it like a substance which can be increased or diminished. Time, however, is not a magnitude in this regard, as experience plainly indicates.

The schematism of reason, in Kant, is designed to bring about the impression that there is a profound agreement between thought and reality (this is what prevents Meyerson from being Kantian in the strict sense.) This system worked very well in classical mechanics until Carnot, at which time it became clear that in order to maintain the agreement between thought and reality, a special dispensation was required to mediate between causality and entropy.

> In pre-Einsteinian mechanics, as we said, irreversibility seems to be an anomaly, because mechanics, as a 'rational' science, demands that things remain what they are and that, given the effect, the cause can be reproduced. Consequently, if one wishes to approximate reality, one is forced to formulate a special principle. This is Carnot's principle, which constitutes a clear-cut expression of the so called anomaly; for this very reason it was only belatedly and reluctantly accepted by the scientific community. Admittedly, it has been possible to give this principle a mechanical interpretation by means of a device introducing statistical methods. However, there remains something of the original character of the observation underlying this principle, something incommensurable with reason, for one must allow a fundamentally

> improbable state 'at the beginning of time,' without trying to make this state arise (Svante Arrhenius to the contrary) from another more probable one.[86]

Irreversibility is what distinguishes time from space as a concept. Spatial intuition, however, always interprets change as an anomaly, or at the limit, as a meaningless formality, as was the case in Minkowski and Eddington. Carnot's principle is the formulation of this anomaly itself. Meyerson interprets the development of statistical methods of analysis as a hybrid mechanics of the micro-physics of change. Statistical approximation is still not enough to rationalize Carnot's principle, which itself refutes the image of a static and unchanging being. Carnot's principle revises the image of being by replacing the necessity of equilibrium with the necessity of asymmetrical transformation. The briefest expression of this ontological maxim would be that the only necessity is that of change or asymmetry. Being itself is "a fundamentally improbable state" after Carnot. In the conceptual system of relativity, as in the case of rational mechanics, the irrational returns in the form of a discontinuity. What can discontinuity signify in the relativistic space-time of isotropic measurement?

> …what is revealed by discontinuity, whether of atoms or quanta, is, as in the case of Carnot's principle, the component of reality that cannot be deduced from the fundamental principles of reason. Moreover, since in the theory of relativity the rational appears to be truly spatial – geometrical – in nature, one can say that what prevents us from making everything rational in this case is the properly physical, as opposed to mathematical, component of reality.[87]

[86] Meyerson 1985, 105.
[87] Meyerson 1985, 109.

Meyerson demonstrates that the rational is the unwitting servant of the real because it cannot help but encounter the irrational. Spatial deduction is the chief instrument behind the mathematization of nature in the relativistic deduction. It is also the surest means of uncovering, in equally precise forms, what is irrational in nature. Discontinuity is rediscovered in the form of irreversibity. *The interest and value of Meyerson's philosophy of science would seem to reside in the progressive dialectic of this rediscovery of reality, in new forms, out of the very abstractions which so assiduously tried to dispel it in the service of the rational.*

Ultimately, the fate of the scientific concept is bound up with the clarity and rigor of its definition. The acuity of the scientific concept discloses, in the most aggravated sense of the word, an ambivalent regard for reality. Science, as we learned from Meyerson's encounter with Hegel, excels in bracketing the irrational. This is ultimately what grounds the ontological structure of science as a deductive sequence. As soon as the scientific concept begins to explain phenomena by absorbing them into the unchanging background of spatial intelligibility, the intellectual priority of this continuum asserts itself. Relativity theory is the supreme accomplishment of a deductive sequence authorized by the spatialization of time. However, this translation of time into space rediscovers the discontinuous due to the irreversibility of time. In order to push its explanatory sequence to the limit, science inevitably produces an ontology which must destroy the reality of the irrational; namely, the diversities of sense, experience, time and becoming. The form of intelligibility itself progressively destroys its own foothold in reality, even as it consolidates an increasingly powerful explanatory system. Meyerson calls this tendency of explanatory science to dissolve reality its "acosmism:"

> Science is realistic; we know, however, that if explanation is pushed to its limit it can only end up at acosmism – the destruction of reality. In relativism, precisely because it is a very advanced, very perfect form of explanation, these two extremes of existence and nonexistence are found to be quite close to each other. This leads to a painful sort of conflict for the physicist.[88]

The dialectical transformations of the intelligible and the irrational place before us "the extremes of existence and non-existence." Scientific explanation in its most advanced and perfect forms, while being incredibly precise and powerful, also tends to dissolve the world by dismantling the last vestiges of intuitive representation. These are the aporias of scientific realism. The greatest achievement of scientific deduction for Meyerson is the relativist deduction. The continuum of space-time can explain the phenomena of the Newtonian universe in rigorously objective terms. Objectivity, however, is that unique form of certainty which ruthlessly undermines itself by its effort to eliminate time. Upon realizing the threshold of a maximally abstract representation of the phenomenon, reason again encounters the irrational in the form of the discontinuous or irreversible nature of time. Physical reality again becomes of concern to science, which must then renew its efforts by reformatting its epistemological normativity.

> Why does reason nevertheless agree to the very difficult task of changing its own nature? Why does it resign itself to a sacrifice it obviously considers extremely painful? There can be no doubt on this point: it is because this sacrifice seems to be more than offset by what reason takes to be a gain on another side; it is because it considers this sacrifice the inevitable price of a victory. Reason is by nature absolute; it does not allow anything at all in

[88] Meyerson 1985, 140.

> reality to escape it. Everything must be subject to it, everything must appear rational. This is the fundamental postulate of human knowledge and consequently serves as the basis for both science and philosophy… Reason resigns itself to the necessity of changing its own nature because it becomes convinced that by so doing it will be able to master a side of reality that hitherto eluded it.[89]

Reason changes its nature because its desire to rationalize the real is absolute. The ambition of reason is such that it leaps over its own provisional forms in order to better comprehend the real. Meyerson's philosophy of the intellect demonstrates that the very dogmatism of the principle of identity is also the motor of its successive revolutions.

[89] Meyerson 1985, 211-212.

Chapter Two - The Immanence of Intelligence: Léon Brunschvicg and the Progressive Intelligibility of the Real

Introduction: A Philosophy of Critical Idealism Astride History and Science

"*One must never substitute the transcendence of the intelligible for the immanence of the intelligence."*[90] With these words Jean Hyppolite (1907-1968) opened a special meeting of the *Société française de philosophie* dedicated to the fiftieth anniversary of the publication of Léon Brunschvicg's *Les étapes de la philosophie mathématique* (1912). Léon Brunschvicg (1869-1944) was at the center of French academic life for nearly fifty years but remains largely unknown to most contemporary readers, even in France. In the words of Howard E. McElroy:

> When Léon Brunschvicg died in 1944, a victim of the Nazis who had burned his books and manuscripts on entering Paris, French philosophy lost one of its greatest figures, a man who deserves to rank along with Henri Bergson and Emile Meyerson as one of the leaders of the generation just past.[91]

Brunschvicg died in Exile, fleeing Paris for Aix-En-Provence before settling in Aix-les-Bains. Brunschvicg was born in Paris and attended the prestigious Lycée Condorcet where he was classmates with Célestin Bouglé (1870-1940), who would later become an important sociologist, Marcel Proust (1871-1922), whom Brunschvicg often referred to with great affection, Xavier Léon (1868-1935), and Élie Halévy (1870-1937), with whom Brunschvicg founded the influential *Revue de*

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[90] Jean Hyppolite, "Commémoration du cinquantenaire de la publication des *Étapes de la philosophie mathématique* de Léon Brunschvicg," *Bulletin de la Société française de philosophie*, vol. 57, no. 1 (1963): 1.
[91] Howard C. McElroy, "Brunschvicg's Interpretation of Pascal," *Philosophy and Phenomenological Research,* vol. 11, no. 2 (1950): 200.

métaphysique et de morale in 1893. Brunschvicg was accepted to the *École normale supérieure* in 1888 along with his classmates Bouglé and Halévy. While taking classes at the Sorbonne Brunschvicg met Émile Boutroux, one of the most prominent Neo-Kantian philosophers in France and also a respected historian of science and philosophy. Brunschvicg also took courses from Victor Brochard (1848-1907), an important classicist. Brunschvicg married Cécile Kahn (1877-1946), a prominent French feminist and politician in 1899, with whom he had four children. Kahn later became the first female undersecretary of education (1936-1937). Brunschvicg was appointed *Maître de conferences* at the Sorbonne in 1909 and became the chair of the History of Modern Philosophy in 1927.

Brunschvicg was additionally the president of the *jury d'agrégation* between 1936 and 1938 and an important member of the Société Française de Philosophie, founded in 1901 by André Lalande and Xavier Léon. A specialist in the history of modern philosophy, Brunschvicg wrote many works of criticism on Spinoza, Descartes, Montaigne, and most notably Pascal, whose complete works he edited and published in fourteen volumes between 1904 and 1914. His first book on Spinoza (aptly titled *Spinoza*), published in 1894, is often credited with inaugurating a new approach to Spinoza and to the history of rationalism in turn of the century France. Brunschvicg was also instrumental in modifying the requirements for the *aggrégation* in philosophy. Following his reforms, all students intending to pursue degrees in philosophy were required to obtain a certificate of qualification in mathematics or the natural sciences. Brunschvicg's pedagogical reform of the educational system reflected his own scholarly trajectory, which combined the history of philosophy and science seamlessly. Finally, as the teacher and mentor of Gaston Bachelard, Jean Cavaillès, Jean Hyppolite, and Albert Lautman (1908-1944), the pedagogical legacy of Brunschvicg should not be underestimated.[92]

Brunschvicg's work defies easy characterization but like his contemporary Emile Meyerson, he relies on materials from the history of science and philosophy in order to develop an exhaustive presentation of the stages of human intelligence and the fundamental characteristics of scientific rationality. Like Meyerson, as this chapter will argue, Brunschvicg makes use of historical materials in order to illustrate an epistemological thesis concerning the nature of human intelligence. Unlike Meyerson, however, Brunschvicg does not defend a categorical distinction between the domain of knowledge and the domain of experience. His philosophy is not a philosophy of the intellect. However, the parallel with Meyerson remains compelling insofar as Brunschvicg begins to characterize his work in the twenties as a *philosophy of thought.* Meyerson's philosophy of the intellect and Brunschvicg's philosophy of thought do not, however, share the same fundamental epistemological commitments. For Brunschvicg, the relationship between reason and experience can never be determined in advance. Additionally, neither thought nor being can be said

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[92] For further biographical details see "Brunschvicg, Léon." *Complete Dictionary of Scientific Biography.* http://www.encyclopedia.com/doc/1G2-2830900682.html (accessed March 25, 2013). See also Dominique Lecourt, "Brunschvicg, Léon," in *Dictionnaire d'histoire et philosophie des sciences,* ed. Dominique Lecourt. (Paris: Presses Universitaires de France, 1999), 154-155.

to exist independently of the other. The philosophical dualism which is most anathema to Brunschvicg is therefore that of idealism and realism. For Brunschvicg, pure *a priori* intelligibility is as absurd as the transcendental realism of nature. Neither pure rationalism nor pure objectivity can have any meaning for the thinking being. Rather, the proper orientation of thought is characterized by what Brunschvicg calls a *critical idealism* developed largely out of the resources of Kantian critique but bracketing the schematism of the *a priori* intuitions of space and time, which represent unwarranted predeterminations of the possible forms of experience according to Brunschvicg.

Critical idealism is above all a philosophy of judgment, as Brunschvicg argues in his first work, *La modalité du jugement* (1897), also the most systematic presentation of his philosophy. For Brunschvicg, the primary evidence of thinking is always a judgment. In fact, there is no thought which is not also a judgment. To think is to judge. The modalities of judgment are the orientations of thought. These modalities are always mixtures of two ideal forms, judgments of interiority (which express "the internal unity of ideas"), and judgments of exteriority (which express the shock of a reality that resists thought.) Every judgment is the affirmation of the copula "is" which joins two terms. For judgments of interiority, the copula expresses the connection between two ideas. Brunschvicg's preferred example of a judgment expressing the form of interiority is "the sum of the interior angles of a triangle is two right angles." Gary Gutting provides a useful summary of the form of interiority as follows:

> Such a judgment expresses the unity that the mind finds between two notions that are only verbally separated and are in themselves mutually implicated. Here Brunschvicg will say that judgment takes the 'form of interiority,' since its 'is' expresses the internal unity of ideas.[93]

The form of interiority reveals a *rational unity* which the mind experiences as a fully autonomous and necessary reality. Judgment may also express "brute facts" or "shocks of reality" which, again following Gutting's formulation, "the mind must simply accept without understanding." As Brunschvicg writes, the form of exteriority "…is the impossibility of the intellect's penetrating to the interior of what it represents in order to analyze and understand it."[94]

The form of exteriority thus interrupts the progress of the intellect in order to affirm the resistance of the real. The copula of being here expresses the resistance of reality rather than the rational unity of the intelligible. As Martial Gueroult notes in his study of Brunschvicg as a historian of philosophy, the "shock of the real" is adapted from Fichte and allows Brunschvicg's critical idealism to embrace objectivity as a form of resistance to interiority.[95] The form of exteriority does not violate Brunschvicg's essentially idealist theory of knowledge, for it is in fact the form of *the immanent presentation of the object to thought* or the *disposition of the consciousness of reality as such.* It must be repeated that both forms of judgment are ideal cases. They are the boundaries of a continuum of judgment which is, in

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[93] Gary Gutting, "Thomas Kuhn and French Philosophy of Science," in *Thomas Kuhn*, ed. Thomas Nickles. (New York :Cambridge University Press, 2003), 47.
[94] Leon Brunschvicg, *La modalité du jugement* (Paris: Presses Universitaires de France, 1964), 88. Cited in Nickels 2003, 47.
[95] Martial Gueroult, *Dianoématique. Livre I, Histoire de l'histoire de la philosophie, Tome 3, En France De Condorcet À Nos Jours* (Paris: Aubier, 1988), 878.

practice, always a mixture of interior and exterior forms. This leads Brunschvicg to characterize his own work as a *critique of experimental judgment.* It is therefore impossible to claim, as Gutting does, that:

> there is an unbridgeable epistemological gap between what can be known simply through the mind's internal reflection and what requires the jolt of external experience.[96]

No doubt this is the perennial epistemological distinction. It is, however, precisely what Brunschvicg's entire oeuvre sets out to dismantle through the historical examination of the doctrines founded on this very error.

The singularity and importance of Brunschvicg's work, I claim, proceeds from the denial of the very epistemological dualism which Gutting suggests Brunschvicg embraces, as this chapter will demonstrate. On this point, a brief comparison with Meyerson's philosophy of the intellect is again instructive. Both Meyerson and Brunschvicg make use of Einstein's relativistic physics as an example of the culmination of a developmental logic of scientific progress. Meyerson's project attempts to demonstrate that relativistic physics is in perfect continuity with the normative form of deduction tending toward identity in the concept of causality. The progressive abstraction of relativity, however, leads Meyerson to diagnose the "acosmism" of relativity as symptomatic of science in general. Science can only achieve the furthest levels of abstraction by sacrificing reality itself. Experience and science thus form oscillating poles of a tendency which corrects itself but which can

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[96] Nickles 2003, 48.

never be resolved or surpassed in the form of a superior synthesis, or located at the level of a more fundamental unity of intelligence and experience.

In short, Meyerson is a strict realist where experience is concerned and a strict idealist where knowledge is concerned. The antagonism of the idealist and realist poles which science thus internalizes is paradigmatic of the epistemological fallacy Brunschvicg wishes to critique on the part of what he calls the false psychologies of scientific reason. Brunschvicg maintains that it is impossible to begin the project of epistemological critique from a starting point of transcendental certainty. We have access neither to the ascesis of a strict philosophy of the intellect, nor to the realism of a philosophy of nature. If we are to eliminate the distortions in our thinking which arise from vagaries in perception or from transcendental illusions of reason, we cannot default to pure forms of realism or idealism. In reality, Brunschvicg claims, we are in a state which is always defined by the reciprocal presupposition of both positions. Realism and idealism exist only as the defining poles of a continuum of the acts of intelligence which Brunschvicg calls by turns *the probable form of judgment* and *experimental judgment.*

Experimental judgment is always a mixture of the two ideal forms of pure interiority and pure exteriority. If Kant is the philosopher with whom Brunschvicg has the closest and the most contentious relationship, then Fichte is the philosopher who allows Brunschvicg to cut through the rational and cosmological antinomies of transcendental idealism and realism by refining the modes of reflective interiority which furnish the complete system of a theory of determinate judgment. This is why

Brunschvicg will eventually claim that the metaphysics of science does not *determine* scientific knowledge but only *reflects* on the form of experimental judgment proper to contemporary science. Brunschvicg's idiosyncratic theory of judgment is a corrective of Kantian critique, against which Brunschvicg affirms the primacy of judgment over the concept. Brunschvicg disavows the role of the pure *a priori* intuitions of space and time as determinate of all possible forms of experience. Brunschvicg is equally critical of Bergson, against whom he affirms the mutually reinforcing requirements of a methodological *idealism* which defines the form of interiority proper to analytic judgment, and an empirical *realism* which defines the constancy of the object *for the successive acts of judgment*.

The philosophy of judgment thus forms the substrate of Brunschvicg's critical idealism because the immanence of intelligence within experience discloses a dynamic relationship between the constituting poles of the continuum of possible judgments. Science is the progressive reconciliation of the realist pole with the idealist pole. *The rigor of our mathematical idealism is what furnishes the necessity and universality of our realism.* Brunschvicg ultimately concludes that rationality and objectivity are mutually reinforcing and that Einstein's relativity allows us to understand the epistemological structure of a truly *objective rationalism,* on the one hand, and a truly *rational objectivity,* on the other. The relationship between mathematics and physics dramatizes the stages of this progressive movement, as this chapter will demonstrate.

Brunschvicg's philosophical output was extremely varied but three works in particular form the essential triptych of the critique of experimental judgment. *Les étapes de la philosophie mathématique* (1912) is an epistemological history of the forms of mathematical intelligence from Pythagoras to Russell and Whitehead. *L'expérience humaine et la causalité physique* (1922) is a history of the concept of causality from antiquity to the relativistic physics of Einstein. *Le progrés de la conscience dans la philosophie occidentale,* published in 1927 (dedicated to Bergson no less) extends the epistemological reflections of the previous studies and attempts to plot the course of the general emancipation of human intelligence from all forms of dogmatic philosophy.[97] This emancipation of the intellect is the leitmotif of Brunschvicg's philosophy. The fundamental argument of this chapter is that Brunschvicg's critique of experimental judgment is premised on the *immanence* of

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[97] The *Complete Dictionary of Scientific Biography* gives a comprehensive overview of Brunschvicg's bibliography to date as follows: *La modalité du jugement* (Paris, 1897), 3rd ed., enl., entitled *La vertu métaphysique du syllogisme selon Aristote* (Paris, 1964); *Introduction à la vie de l'esprit* (Paris, 1900); *L'idéalisme contemporain* (Paris, 1905); *Les étapes de la philosophie mathématique* (Paris, 1912); *Nature et liberté* (Paris, 1921); *L'expérience humaine et la causalité physique* (Paris, 1922); *La génie de Pascal* (Paris, 1924); *Spinoza et ses contemporains* (Paris, 1924); *Le progrès de la conscience dans la philosophie occidentale* (Paris, 1927); *De la connaissance de soi* (Paris, 1931); *Pascal* (Paris, 1932); *Les âges de l'intelligence* (Paris, 1934); *La raison et la religion* (Paris, 1939); *Descartes et Pascal, lecteurs de Montaigne*(Neuchâtel, 1942); *Héritage des mots, héritage des idées* (Paris, 1945); *L'espirt européen* (Neuchätel, 1947);*Agenda retrouvé, 1892–1942* (Paris, 1948); and *La philosophie de l'espirt* (Paris, 1950). His complete articles are collected in *Écrits philosophiques*, 3 vols. (Paris, 1949–1958). See "Brunschvicg, Léon," *Complete Dictionary of Scientific Biography,* http://www.encyclopedia.com/topic/Leon_Brunschvicg.aspx (accessed March 26, 2013). The bibliography however mistakenly identifies the 3rd edition of *la modalité du jugement* , which is in fact a French translation of Brunschvicg's secondary latin thesis *Qua ratione Aristotles metaphysicam vim syllogism inesse demonstaverit,* 1897.

intelligence within experience, the necessarily progressive nature of all scientific rationality and the open ended structure of science itself, which does not possess a transcendental foundation in any predetermined form of reason or experience.

Accordingly, this chapter will focus on four moments from Brunschvicg's critique of experimental judgment which exemplify this thesis. The perspective of the chapter will necessarily be synthetic and synoptic, drawing on a broad array of Brunschvicg's books and articles but focusing primarily on *Les étapes de la philosophie mathématique* and *L'éxperience humaine et la causalité physique.* The first section addresses the historical emergence of what Brunschvicg calls "the perennial mathematical philosophy:" the identification of mathematical reason with the highest form of truth. This section addresses the transformations of mathematical intelligence in the doctrines of Pythagoras, Plato and Aristotle. The Pythagorean doctrine identifies being with number and brings about what Brunschvicg calls a synthesis of dialectic and cosmology. Plato's philosophy will develop a dual methodology of analytic regression and dialectic ascension which makes mathematical knowledge subordinate to a realm of ideas which are the intelligible itself. Accordingly, mathematics can no longer be identified with the essential structure of being, as was the case with Pythagoras. Ultimately however Plato recognizes the need to reconcile the intelligible with the sensible, although Brunschvicg claims he fails to provide an adequate account of how this might be accomplished.

Aristotle crafts his own logic and metaphysics in the space opened up by the Platonic impasse and develops a metaphysics of substance which Brunschvicg calls a "qualitative physics of essences." Aristotle's metaphysics is designed to bring about a continuity between the order of being and the order of knowledge, thereby resolving the Platonic antinomy. For Brunschvicg, Aristotle commits the cardinal sin of western philosophy: he *naturalizes perception and intelligence* by modeling the forms of logic on perception and common sense. In so doing he undermines the constitutive dynamism of the intellect which is not the stability of reason or the mute exteriority of the phenomenon but the mutual implication of both in a progressive dialectic of rational objectivity.

The second section addresses the mathematical philosophy of Descartes. Descartes transforms the qualitative physics of Aristotle into a quantitative science of structured relations. He thereby reorients the development of science in the west by dismantling the Aristotelian scholasticism of the middle ages. The third section addresses Brunschvicg's reading of the *Critique of Pure Reason* and Kant's effort to replace the Cartesian order of purely analytic reason with a synthetic theory of mathematical knowledge based on the a priori intuitions of space and time. Brunschvicg will divide the critical philosophy in two, affirming Kant's discovery that *reason itself constitutes an experience and cannot be held apart from the spontaneity of our representations* but dismissing the transcendental unity of apperception which, as was the case with Aristotle, models all possible forms of knowledge on a ready made form of experience.

The fourth section examines Einstein's special and general theories of relativity which for Brunschvicg are not only of scientific interest but represent an apotheosis of the form of experimental judgment, definitively transforming the relationship between rationality (the form of interiority) and objectivity (the form of exteriority), such that they become simultaneous and mutually correcting perspectives of a new scientific intelligence. The implications of Brunschvicg's critical idealism only emerge when the wager of the immanence of intelligence within experience is taken seriously, for then it becomes necessary to affirm that science proceeds *without a stable epistemological foundation* but instead *immanently constructs* the epistemological consistency it requires by continually reformulating the mutual verification of the objective boundaries of reason and the rational limits of objectivity.

The Ambivalence of the "Greek Miracle:" From Meta-Mathematics to the Logical Syllogism

For Brunschvicg, the history of mathematical intelligence is a veritable history of the truth. Philosophy is called upon to reflect on the transformations which this form of truth passes through, not to determine from the outset the necessary boundaries which it must observe. *Les Étapes de la Philosophie Mathématique* proceeds historically but its goal is to uncover the behavior of the intelligence across some two and a half thousand years of human activity. Brunschvicg's work was hailed by historians of mathematics and science but he himself politely declined this honor:

> It has happened that we have been classified, with a very kind intention, among historians of mathematics. We have every reason to decline such an honor…Our task is not to know how the *nature of things* is made, but to tell how *man's mind* is made.[98]

A similar reception awaited his next major work, *L'Experience Humaine et la Causalité Physique* (1922), causing Dominque Parodi to lament what appeared to him as Brunschvicg's whole sale conversion to "a history of scientific concepts."[99] Again, nothing could be further from the truth. Both works, although addressing different conceptual problematics (the first the historical transformations of mathematics as the governing norm of epistemological intelligibility, the second the history of the concept of causality as determining the relationship between rationality and objectivity) are firmly located on the ground of epistemological critique, seeking to uncover the *history of experimental judgment* immanent to the history of science. This section, and those which follow, combine materials from both of Brunschvicg's major works in order to trace, as precisely as possible, not the historical progress of science but the epistemological concepts and problems which are at the center of Brunschvicg's philosophy of thought and his ambitious critique of experimental judgment.

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[98] Leon Brunschvicg, *L'experience humaine et la causalité physique* (Paris: Librairie Félix Alcan, 1922), xiii. Cited in McElroy, 1950, 201.

[99] Dominque Parodi and Léon Brunschvicg et al. "Histoire et Philosophie. Séance du 31 mai 1923", *Bulletin de la Société française de Philosophie* no. 23 (1923): 145. Cited in Cristina Chimisso, *Writing the History of the Mind: Philosophy and Science in France, 1900 to 1960* (Hampshire: Ashgate Publishing, 2008), 71. Chimisso provides an excellently documented reception history of Brunschvicg's work among philosophers and historians of science in France, especially during the inter war period.

For Brunschvicg, the history of mathematical intelligence cannot be given a distinct point of origin because it is ultimately immanent to human experience, nevertheless, the profound entanglement of cosmology, philosophy and mathematics reaches a point of maximal intensity in the doctrine of Pythagoras (c. 570 B.C – c. 495 BC). Brunschvicg situates the Pythagoras school among the early natural philosophers of the Ionian peninsula alongside Thales and Anaximander. The Pythagorean doctrine relies heavily on the Astronomical system developed by the Chaldeans, who had already constructed an elaborate table of constellations and a corresponding astronomical calendar. The astronomical component of the Pythagorean doctrine is crucial for Brunschvicg because:

> observed with the naked eye, in effect, a constellation has two characteristics, the number of stars which constitute it and the geometric figure they draw in the sky.[100]

According to Brunschvicg, the act of observing the constellations and of describing their annual progress across the regions of the sky establishes the possibility of an intellectual association between the "natural necessity" of this orderly movement and a more general conception of the universe itself as well ordered.

Whether or not the origins of Pythagoreanism are actually historically derived from astronomical observations is of secondary interest to Brunschvicg. He is more interested in attempting to understand the sources of the Pythagorean theory of numbers, which as Brunschvicg will show, are not simply abstract ideas but elements of a complexly organized cosmology. An examination of the

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[100] Leon Brunschvicg, *Les étapes de la philosophie mathématique* (Paris: Librairie Félix Alcan, 1912), 33.

relationship between Chaldean astronomy and Pythagorean number theory reveals a close connection between the observation of the constellations and the association of the figure thus constructed with the number of stars of which it is composed.[101] The intellectual identification thus established results in the association of the constellation with a number. In analogous fashion, the historian must attempt to understand the conceptual universe of the Pythagoreans as one in which *the knowledge of a thing is inextricable from the knowledge of number.* Brunschvicg pushes the relationship between knowledge and number even further: "Not only do all things *possess* numbers, but further still, all things *are* numbers."[102]

Brunschvicg will take great pains to clarify the conceptual order within which the Pythagorean doctrine brings about this identification of being and number. It is of the utmost importance to avoid caricaturing or distorting the nature of the Pythagorean numerical doctrine, for it is in fact an important intellectual achievement on the order of a *theory of truth* constructed out of a *theory of numbers*, and so of immediate concern to Brunschvicg's own critical idealist project which attempts to understand the internal exigencies driving the progress of knowledge. Part of Brunschvicg's task will then involve dispelling some of the more influential mischaracterizations of Pythagoreanism which have since obscured or distorted the significance of the tradition. On this point, Brunschvicg must confront Aristotle's *Metaphysics*, which perhaps more than any other book in the history of ideas has

[101] Brunschvicg 1912, 33.
[102] Brunschvicg 1912, 34.

cast an unfavorable light on Pythagoras and his followers. As Aristotle writes in *Metaphysics* Book N:

> …the Pythagoreans, because they saw many attributes of numbers belonging to sensible bodies, supposed real things to be numbers – not separable numbers, however, but numbers of which real things consist. But why? Because the attributes of numbers are present in a musical scale and in the heavens and many other things. But those who say that mathematical number alone exists cannot according to their hypothesis say anything of this sort [here Aristotle refers to the Platonic tradition]; indeed, they used to say that those numbers could not be objects of the sciences. But we maintain that they are, as said before. And it is evident that the objects of mathematics do not exist apart; for if they existed apart their attributes would not have been present in bodies. The Pythagoreans on this point are open to no objection; but in that they construct natural bodies out of numbers, things that have lightness and weight out of things that have not weight or lightness, they seem to speak of another heaven and other bodies, not of the sensible.[103]

Brunschvicg here identifies the argument which will come to characterize Aristotle's objection to the Pythagorean and Platonist mathematical philosophies. Aristotle maintains that "critical reflection" (Brunschvicg's phrase) must separate that which is proper to the "order of intelligence" and that which is proper to the "order of reality." "Number is a concept, things are objects or substances. How can it be that the number is then taken in turn as an object or a substance?"[104] The problem with this objection, according to Brunschvicg, is that it imposes a category distinction on the Pythagoreans which, although perfectly justified from the perspective of Aristotle's well regulated division of the sciences of being, can only distort our

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[103] Brunschvicg 1912, 34. Brunschvicg cites Aristotle, *Metaphysics* Book N, 1092 A, 32 and gives the Greek without translation into French. Consequently I have consulted an English translation and have included lines 20-35 in order to provide more context for the criticism which Brunschvicg will subsequently raise against Aristotle. See Jonathan Barnes ,*The Complete Works of Aristotle, Volume Two* (Princeton: Princeton University Press, 1995), 1722-1723.

[104] Brunschvicg 1912, 34.

understanding of the Pythagoreans if we assume that the Pythagorean theory of numbers is exclusively conceptual.

Brunschvicg maintains that we are mistaken if we think we can attribute to the Pythagoreans our own abstract conception of number. Intead, we must try and approach "the intuitive application" of number among the Pythagoreans by noting the inseparability of the point, the line, and the figure in the Pythagorean theory of number:

> …before saying that things are numbers the Pythagoreans began by conceiving numbers as things, the expressions of squared or triangular numbers are not metaphors, these numbers are effectively, before the eyes and the mind, squares and triangles.[105]

As Brunschvicg notes, the Pythagorean "squared or triangular number" is in accord with a Cartesian conception of knowledge capable of satisfying the imagination and the understanding simultaneously. The difference between the rationalist and the Pythagorean understanding of number in fact hinges on this distinction, for the separation of faculties which is necessary for the moderns is inconceivable for the Pythagorean. The analytic geometry of the Cartesians which will be examined in the next section, is premised on the deliberate synthesis of the "*algebraic idea*" and the "*object extended in space.*" The mathematical modernity of Descartes which established the conceptual foundations of mathematical physics will elaborate the synthesis of these two domains at great length. On the contrary, what the Pythagoreans attest to, is the spontaneous unity of the idea of number and the idea of space according to a scheme which will only become truly problematic in the

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[105] Brunschvicg 1912, 34.

seventeenth century. The Pythagorean doctrine presents us with a spontaneous form of mathematical intuition which:

> …rather than proceeding from abstract necessity to concrete application, envelops all the functions of the mind in a sort of integral intuition from which it derives its equilibrium and its plenitude. Number is presented immediately as a sum of points-figures in space, and the figures – lines, surfaces or volumes – which are traced by these points are immediately given as numbers.[106]

The theory of number truly defines the Pythagorean doctrine because "numbers are not axillaries of calculation, 'logistical' intermediaries, but natural realties…"[107] The Pythagorean number has characteristic properties which are in turn defined by the "consequences of the operations by which the number itself is attained and defined."[108] These operations refer to the number and organization of points which then define a figure. The number of points and the figure constitute an essence with invariable properties. Consequently, Brunschvicg will sometimes refer to Pythagorean numbers as *points-figures.* The most basic mathematical operation in Pythagorean number theory is addition or multiplication, by which process a point-figure can be multiplied into a greater number and a more complex volume (a square which becomes a cubic volume), or diminished from a more complex "volume figure" to a "surface figure" of lesser complexity.

The Pythagorean philosopher Philolaus (470 -385 B.C approx.) additionally divided numbers into odd and even categories, to which there corresponds "the fundamental opposition" of Pythagorean philosophy: the "cosmological antithesis of

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[106] Brunschvicg 1912, 35.
[107] Brunschvicg 1912, 35.
[108] Brunschvicg 1912, 35.

the *limited* and the *unlimited.*"[109] Virtues are associated with different combinations of odd and even points-figures which thus establish a complex system of numerical correspondences, as in the figure of the *decade* which, Brunschvicg says, Philolaus in particular seemed to exalt because it was "the sum of the first four numbers -1+2+3+4" and thus exhibited in the order of its progress the oscillation of the even and the odd as well as the first *perfect square* number, itself the termination of the series. In a fashion which Brunschvicg extends to Greek mathematics in general, the Pythagoreans investigated the internal structure of numbers from the perspective of the mathematical operations which produced them. This analysis is particularly significant in the case of square numbers which the Pythagoreans subjected to arithmetical and geometrical description. The fact that the sum of uneven numbers in an ordinal sequence always produces a square number (1+3=4, 1+3+5=9 etc.), leads the Pythagoreans to develop a geometric understanding of the proportional relationship between odd numbers and square numbers which again corresponds with the distinction of the limited and the unlimited. Most notable for Brunschvicg, however, is the Pythagorean correlation of sound and number, out of which the Pythagoreans develop a complex scale of harmonics.

The Pythagoreans extend the compositional order of the number-figure to the harmonic or tonal scale by noting the correspondence between numbers and the intervals of musical harmony. Pythagorean numbers are thus also complex bodies

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[109] Brunschvicg 1912, 36.

which address the senses at multiple levels. The theory of numbers includes the rules of numerical production, the geometric properties of the figure, and the divisions of the tonal scale. A number produced by being raised to its own power two times or cubically is also a harmonic tone:

> Geometric harmony and musical harmony correspond with one another: that which is intelligible for the mathematician is also visible to the eyes and sensible to the ears.[110]

Aristotle's complaint that the Pythagoreans confuse number and being would seem to overlook the complex harmonies of numerical correspondence which make the intelligibility of number simultaneously visible and audible. The reality of number, for the Pythagorean, appeals to the senses and proceeds by increasing the orders of correspondence between the observable degrees of order at all the levels of intellectual and sensible intuition.

Brunschvicg claims that the Pythagoreans invent a new form of the relationship between ideas of order and the perception of nature which allows the *idea of order itself to become an idea of nature.* The necessary accord of the figure, the tone, and the number is taken as evidence of a universal order which transforms the universe into a coherent and intelligible harmony of mathematical correspondences:

> One understands how the speculative imagination of the Pythagoreans, surrounded by the luminous nucleus formed by the positive theories of arithmetic, geometry, and acoustics – and one must include here the germinal form of Copernican astronomy – entered a zone infinitely more

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[110] Brunschvicg 1912, 40.

> expansive in which, without hesitation, solutions could be found for all matters both human and divine.[111]

For the Pythagoreans, as Brunschvicg will show, the number is a "principle of truth" because it is incapable of deception. The mathematical cosmos of the Pythagoreans is also a moral cosmology of harmonic correspondences which necessarily embrace and complement one another in successive orders. The Pythagorean doctrine of points-figures embraces an entire theory of cosmogenesis according to which the observable order of nature is referred to the intelligible development of the point, the line and the volume. Here Brunschvicg cites Diogenes Laertius:

> Alexander in his *Successions of Philosophers* says that he found in the Pythagorean memoirs the following tenets as well. The principle of all things is the monad or unit; arising from this monad the undefined dyad or two serves as material substratum to the monad which is cause; from the monad and the undefined dyad spring numbers; from numbers, points; from points, lines; from lines, plane figures; from plane figures, solid figures; from solid figures, sensible bodies, the elements of which are four, fire, water, earth and air; these elements interchange and turn into one another completely, and combine to produce a universe animate, intelligent, spherical, with the earth at its centre, the earth itself too being spherical and inhabited round about.[112]

The unbroken development from the monad to the production of the elements and their successive combinations proceeds according to the rational progress of the number as it passes through the point and into more complexly embodied figures. The governing trope of Pythagorean cosmology is thus the *naturalization* of the number in such a way that it is able to support increasingly

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[111] Brunschvicg 1912, 40.

[112] Brunschvicg 1912, 41. Brunschvicg cites Diogenes Laertius, Book VIII but again cites the Greek without translation into French. I have consulted the English translation as follows. Diogenes Laertius, *The Lives of Eminent Philosophers, Diogenes Laertius Book II The Loeb Classical Library Vol. 185* Trans. R.D. Hicks (Massachusetts: Harvard University Press, 2005), 341-343.

elaborate forms. Against this profound intuition of the rational unity of the number, from which proceed its generative powers, the infinite appears as that which escapes natural and intelligible boundaries. "The advent of harmony in the world is the victory of number over the infinite."[113] The infinite disrupts this sensible and intelligible unity because it cannot be identified with a harmonious or completed form. Brunschvicg will develop the implications of the tension between rational unity and the infinite when he turns his attention to the Platonic interpretation of Pythagorean number theory.

The Pythagorean doctrine is therefore as much a moral and religious system as it is a mathematical or geometrical theory. The order of the universe thus established is a safeguard against the threat of an unorganized chaos of appearances which would in turn lack any essential significance. The very multiplicity of the significance of number which the Pythagoreans develop, perhaps deliriously, is often held against them as a sign of undisciplined excess. Brunschvicg, however, upholds the splendor of what he calls "the rational experience of numerical harmony" which he further claims the Pythagoreans develop with great rigor. The virtue of this rational experience of numerical harmony is that it allows synthetic correspondences to be established between multiple previously unconnected domains of experience. The religious and moral universe of the Pythagoreans is above all the identity of the number and a complex system of values: it is a clear

[113] Brunschvicg 1912, 40-41.

expression of the master trope of Brunschvicg's perennial mathematical philosophy, the identification of truth with a theory of number.

Far from a superstitious or capricious projection of mathematical idealism onto an otherwise resistant or unintelligible natural reality, Brunschvicg sees no reason not to apply the regulative principles of positive science to the Pythagorean doctrine:

> {I}f the ideal of positive science is to repose on principles which are at once clear and delimited, developed out of principles which have already been established, and to maintain contact with experience which confirms and controls these principles, then the Pythagorean mathematicism already satisfied this ideal.[114]

"Mathematicism" thus defined is not the free or unregulated mysticism of number. It is already a system of the methodological verification of experience by reason and vice versa. Further, it is the careful construction of correspondences which extend and deepen the syntheses of which reason is capable. This is as much a redemption of Pythagoras as it is a critique of positivism. In the case of the Pythagorean doctrine, it becomes clear that what is essential are not the finer points of the system of belief thus engendered but the implications of the rational method which produced them. In the case of the positivist doctrine, it becomes clear that the relationship between experience and the foundational principles of order cannot be determined from the outset according to an invariant procedure but must instead be referred to the network of correspondences which at any given moment define the boundaries of rational experience. The rational experience of number which the

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[114] Brunschvicg 1912, 41.

Pythagoreans develop is capable of establishing a powerful epistemological norm which imposes an equivalence between the universe and the idea: "The harmony of the universe finds itself reflected in the harmony of ideas…"[115] There is a harmony among things and a harmony among ideas but what is primary in both cases is the unitary cause which underlies them: a harmony defined by the internal structure of the numerical idea and its laws of production. There is a power internal to the numerical idea by which it generates itself and the universe. It is at this point of maximum intensity between a doctrine of number and a theory of being that Brunschvicg brings Platonism and Pythagoreanism into contact with one another.

Brunschvicg claims that Platonism and Pythagoreanism must be defended against the Aristotelian reading which would make them simply interchangeable with one another. For Aristotle, the Pythagorean and Platonist philosophies are founded on the same confusion between being and number. It makes little difference if one affirms that "being imitates number," the Pythagorean view, or if one affirms that "being participates in number," the Platonic view, for *imitation* and *participation* are different expressions of the same confusion between the order of things and the order of concepts. As Brunschvicg will show, Aristotle's entire logical program is installed in the gap which Plato's philosophy leaves open between a *meta-mathematical domain of numerical ideas* and the domain of sensibility and experience. The Platonist and the Pythagorean may commit the same category error but Aristotle does not hesitate to discern on the side of the Platonist a tendency to

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[115] Brunschvicg 1912, 42.

make numbers *transcendental* and *determining of being*. For the Pythagoreans, on the contrary, there is an immanence of number and being.

Before turning to Plato, Brunschvicg again returns to the *Metaphysics* in order to elaborate the notion of *imitation* among the Pythagoreans. Aristotle writes:

> …the Pythagoreans, as they are called, devoted themselves to mathematics; they were the first to advance this study, and having been brought up in it they thought its principles were in all things. Since of these principles numbers are by nature the first, and in numbers they seemed to see many resemblances to the things that exist and come into being – more than in fire and earth and water (such and such a modification of numbers being justice, another being soul and reason, another being opportunity – and similarly almost all other things being numerically expressible); since, again, they saw that the attributes and the ratios of the musical scales were expressible in numbers; since, then, all other things seemed in their whole nature to be modeled after numbers, and numbers seemed to be the first things in the whole of nature, they supposed the elements of numbers to be the elements of all things, and the whole heaven to be a musical scale and a number.[116]

Aristotle's presentation of the Pythagoreans is here governed by the notion of all things "being numerically expressible," from which Brunschvicg concludes that the Pythagoreans tend toward an immanence of being and number. The Pythagoreans collapse "numerical reality" and "natural reality" onto the same plane by stating that the attributes of numbers are themselves the causes of the order of nature.

Plato, by contrast, elaborates a sharp division between "the elements of number" and "the whole of nature." Brunschvicg cites the *Philebus* in order to demonstrate the transcendental tendency of Plato's "mathematicism:"

> SOCRATES: Whatever seems to us to become 'more and less', or susceptible to 'strong and mild' or too 'too much' and all of that kind, all that we ought to subsume under the genus of the unlimited and its unity. This is in compliance

[116] Brunschvicg 1912, 44. Brunschvicg cites *Metaphysics* A 5, 985 b, 27 but does not give the text. I have consulted the English translation. See Barnes 2005, 1559.

> with the principle we agreed on before, that for whatever is dispersed and split up into a multitude, we must try to work out its unifying nature as best we can, if you remember.
> PROTARCHUS: I do remember.
> SOCRATES: But look now at what does not admit of these qualifications but rather their opposites, first of all 'the equal' and 'equality' and, after the equal, things like 'double," and all that is related as number to number or measure to measure: If we subsume all these together under the heading of 'limit.' we can seem to do a fair job.[117]

Socrates distinguishes between two orders of relation, a "more and less" of sensible comparisons which are dispersed and which must be gathered together under a "unifying nature," and a properly numerical order of relations which does not admit of oppositions at the level of experience (as in the case of the "more and the less") but which is concerned with relations of "number to number" and "measure to measure." In the order of sensible comparisons the same object may be subjected to different forms of determination. A man is large in comparison to an infant but small in comparison to an oak tree. Brunschvicg says that Plato draws our attention to the *types of determination themselves* in order to distinguish the "form of smallness" from the "form of largeness." Although the forms of largeness and smallness may be spoken of in changing combinations relative to the objects of experience, they maintain a stable and fixed relationship to each other on the superior plane of the intelligible. The purely conceptual relations unique to this plane are defined by the connections of "number to number," or as Brunschvicg prefers to say, the rational experience of the connections between ideas.

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[117] Brunschvicg 1912, 44. Brunschvicg cites the Greek text of *Philebus*, line 25 A. I have consulted an alternate English translation as follows. John M. Cooper and D.S. Hutchinson, eds., *Plato. Complete Works* (Indianapolis: Hacket Publishing Company, 1997), 412.

This constitutes one version of the transcendental experience of the rationality of number in the Platonic tradition which Brunschvicg will attempt to reconstruct in order to understand how Plato uses the *experience of mathematical reason* to construct the ur-form of mathematical idealism: "the science of the connections between ideas."[118] Between the Pythagorean and the Platonic philosophies of number Brunschvicg claims there is an intermediary evolution of mathematical technique which forbids Plato from entertaining the fusion of mathematical and concrete realities which so entranced Pythagoras. The paradoxes of Zeno express the contradiction which Plato inherits and which the Pythagoreans were unwilling or unable to conceptualize: how can the plural discontinuity of points be made continuous with the concrete "continuous reality of space in which things are located?"[119] Brunschvicg claims that what remains unthinkable for the Pythagorean becomes provocative for Plato, for the very discontinuity between the ideal unity of the point and the experiential plenitude of the spatial continuum will force Plato to develop a new theory of knowledge and a corresponding model of truth.

Brunschvicg states that the discovery of irrational numbers subtends the problem of the relation between the numerical and the geometrical in Greek mathematics. In a rapid survey of several Platonic dialogues Brunschvicg indicates that Plato repeatedly makes use of this problem in order to highlight the necessity of distinguishing between numerical intelligibility and that which is intelligible in

[118] Brunschvicg 1912, 45.
[119] Brunschvicg 1912, 48.

itself. In the *Theaetetus,* Plato makes reference to the mathematical writings of one of his own teachers, Theodorus, who established the irrationality of the square root of prime numbers. In book VII of the *Laws,* Plato refers to the "distinction between commensurable and incommensurable magnitudes," which Brunschvicg again identifies with the discovery of irrational numbers. Finally, it is the *Meno* which develops at length the epistemological theory of *reminiscence* by which the uneducated slave is induced by Socrates to expound the principles of geometry. Brunschvicg claims that what thus comes to our attention in all three dialogues is the model of Plato's philosophical method: regressive analysis.

Analytic regression or "the method of division into parts" serves one purpose according to Brunschvicg: it introduces a precise distinction between mathematics as a form of knowledge and knowledge itself. It places the *idea* between the number and the intelligible itself. The Platonic repurposing of the Pythagorean doctrine dissolves the ontological identity of being and number by installing between the order of ideas and the order of things a complex hierarchy of mediating relationships. Brunschvicg states that one of Plato's many philosophical inventions is the idea itself. The Platonic idea has a very precise technical function relative to the Pythagorean doctrine of numbers: the idea intervenes in the order of Pythagorean numerical being in order to disrupt what is an immanent production of the real out of the internal harmony of number. The idea has a paradigmatic function in Plato. The things of sensation are copies of a paradigm. Sensible things are imitations of ideal things. Thanks to the intercession of the idea the place of number in the order of being assumes a new and more technical function. Number is no longer the immediacy of being but paradigmatic of a form of truth. Plato not only distances himself from Pythagoras on this point, he also distinguishes himself from Parmenides, from whom he had previously borrowed the opposition between the unity of being and the discord of appearances.

Brunschvicg therefore claims that Plato poses the problem of the relation of number to being in an entirely unprecedented manner. Neither Parmenides nor Pythagoras can pose the problem of this relation correctly. Pythagoras overdetermines the ontological importance of number just as Parmenides liquidates the generative power of number by proposing an absolutely static ontology. What is common in both cases is an *incorrect mixture of number and being*. Plato will attempt to unravel this complicated knot by clearly distinguishing among the forms of knowledge and thereby also establishing a rigorous distinction between philosophy and mathematics. As is the case in the *Republic,* one studies mathematics, geometry and astronomy as precursors to dialectics. After Plato's demarcation of philosophy from all the other divisions of knowledge it is no longer possible to mistake mathematics for being.

As Brunschvicg states, the Pythagoreans had located mathematics and philosophy "on the same plane." Plato, through the intercession of the idea, will make mathematics the science which participates in the form of number and not the immediate identity of being and number. There is unquestionably a profound relationship between mathematics and truth but mathematics is not and cannot be the immediacy of the true itself. Rather, mathematics participates in certain features of the true. The dialectical method thus attempts to pass from the conditioned elements of mathematical knowledge (that which participates) to the unconditioned truth of mathematics (that which is participated in.) Dialectics then encounters a problem of acute difficulty, for it must

> forge an uninterrupted chain of ideas, which, suspended in the form of an absolute principle, constitutes a world completely independent of the sensible… [120]

This world of ideas, one might add, would also be independent of the participating forms of intelligibility. In short, everything in Plato becomes what C.S. Pierce calls an *indexical sign* which points beyond itself and which can only refer the interpretive intelligence indefinitely toward a horizon which recedes as rapidly as one approaches it.[121]

For Brunschvicg, Plato's philosophy must be affirmed as the true inversion of positivism. Platonic idealism is the system of thought which demonstrates the autonomy of philosophy and the inevitable multiplicity and subservience of science. Even mathematics, which is the most beautiful of the sciences "remains a kind of dream without direct purchase on reality," in Brunschvicg's paraphrase of Plato.[122] Brunschvicg cites the *Republic* in order to illustrate how Plato distinguishes among dialectic and the various applied domains of knowledge or "craft:"

> ...no one will dispute it when we say there is no other inquiry that systematically attempts to grasp with respect to each thing itself what the

[120] Brunschvicg 1912, 55.
[121] Brunschvicg 1912, 56.
[122] Brunschvicg 1912, 55.

> being of it is, for all the other crafts are concerned with human opinions and desires, with growing or construction, or with the care of growing or constructed things. And as for the rest, I mean geometry and the subjects that follow it, we described them as to some extent grasping what is, for we saw that, while they do dream about what is, they are unable to command a waking view of it as long as they make use of hypotheses that they leave untouched and that they cannot give any account of. What mechanism could possibly turn any agreement into knowledge when it begins in something unknown and puts together the conclusion and the steps in between from what is unknown?[123]

Brunschvicg states that in this passage Plato "directs mathematical philosophy in an entirely novel direction" which departs from the method of regressive analysis that was concerned with the proper division and categorization of things. The new philosophical method of *dialectic* is capable of referring what is merely hypothetical even among the postulates of geometry to their truly unconditioned principles. In this fashion dialectic is the inverse practice of regressive analysis, uniting the parts of being in a grand design which tends toward the unconditional. However, as Brunschvicg immediately clarifies for the reader, this grand doctrine which Brunschvicg proposes to call *metamathematical* (as against the Aristotelian metaphysics which will supplant it) and which would constitute the rational experience of mathematical ideality at the very core of Platonism, must remain at best a speculative doctrine because Plato was unable or unwilling to develop it "in a complete and systematic exposition."[124]

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[123]Brunschvicg 1912, 55. Here Brunschvicg cites *Republic* Bk. VII, 533 B on the superiority of dialectic. I have consulted the English translation in Cooper and Hutchinson 1997, 1148-1149.
[124] Brunschvicg 1912, 56.

Despite the lack of a complete system the broad outline of the metamathematical doctrine can be reconstructed from the available dialogues. Platonic metamathematics would have been primarily concerned with *ideal numbers* and *ideal figures*. Brunschvicg will rely on books M and N of the *Metaphysics* (for despite his hostility to Aristotle the *Metaphysics* remains an invaluable document for Brunschvicg's work on the history of mathematical intelligence in antiquity) in addition to a broad selection of the dialogues in order to sketch on outline of the esoteric experience of mathematical reason subtending Plato's philosophy. For Brunschvicg, the *Phaedo* is perhaps the richest source of a dialogue offering an example both of the theory of participation and the theory of ideal numbers. In this dialogue there is little doubt that Socrates speaks of numbers as if they were ideas:

> …would you not avoid saying that when one is added to one it is the addition and when it is divided it is the division that is the cause of two? And you would loudly exclaim that you do not know how else each thing can come to be except by sharing in the particular reality in which it shares, and in these cases you do not know of any other cause of becoming of two except Twoness, and that the things that are to be two must share in this, as that which is to be one must share in Oneness, and you would dismiss these additions and divisions and other such subtleties, and leave them to those wiser than yourself to answer.[125]

The "subtlety" of arithmetic appears insufficient as the real cause of numbers when placed against the backdrop of the theory of participation, for it is necessary to align each number with the "particular reality in which it shares" and not simply to treat

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[125] Brunschvicg 1912, 57. Brunschvicg cites *Phaedo* 101 B. I have included line C in order to clarify the argument Brunschvicg will pursue in subsequent readings of the *Phaedo.* See Cooper and Hutchinson 1997, 87.

numbers as elements of calculation which have no essence beyond themselves. "Oneness" or the *monad* is therefore responsible for all that has a share in "being one," just as "twoness" or the *dyad* is responsible for all that has a share of "being two." In short, the idea of number replaces the *act* of calculation with the *being of the idea*. Just as the Pythagoreans spoke of points-figures, Platonic metamathematics speaks of *numerical ideas* which are in turn responsible for sensible and intelligible manifestations of number. The idea of the number upholds the internal unity and specificity of a given number. According to book N of the *Metaphysics,* Plato held that there were specific ideas for each of the first ten numbers and that each was defined by an entirely unique internal structure.[126]

Brunschvicg discerns in the *Philebus* and the *Sophist* a fusion of the theory of numerical ideas with a more general tendency of Greek cosmology by which the unity of the One opposes and contains the multiplicity and disunity of the infinite. In these dialogues the *One* or a principle of self identity confronts a diversity or multiplicity of disorderly transformations in order to correctly divide appearances into their resolute categories. Brunschvicg wishes to emphasize that the numerical ideas are thus often made to serve a theory of cosmogony which is driven by the confrontation of the One with "the perpetual heterogeneity which characterizes the infinite."[127] The numerical ideas seem to have as much to do with the properties of mathematical knowledge as with the requirements of a cosmological theory called

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[126] See *Metaphysics* M. 1083a 1 – 1084b 1.

[127] Brunschvicg 1912, 58. Brunschvicg also cites the *Sophist* 253 C and following on this point.

on to account for the apparent organization of the world and the heavens. As was the case with the Pythagoreans, Plato's ideal numbers are complexly interwoven into a fabric of moral and political considerations. In both traditions the One is at times interchangeable with the eminently theological concept of the Good, just as the infinite, in its cosmological aspect, seems to convey not just chaos or disharmony but an equally disturbing and even evil fascination.

The problem of locating the place of mathematical knowledge within Plato's philosophy is therefore intimidatingly complex, as Brunschvicg readily admits. In the dialogues concerned with cosmogony the relationship between philosophy and mathematics appears to be more complex than that of the definitive superiority of dialectic over all other forms of knowledge. Rather than simply serving the dialectician as a *paradigm* of knowledge, the ideal number and the ideal form seem to be engaged in a process of general cosmological significance. According to Brunschvicg, the *Timaeus* is the dialogue which best exemplifies the complexity of the relationship between mathematics and geometry, on the one hand, and dialectic and cosmology, on the other. Brunschvicg references the following passage in the *Timaeus* in order to illustrate the mathematical cosmogony of Plato's demiurge:

> Now that which comes to be must have bodily form, and be both visible and Tangible, but nothing could ever become visible apart from fire, nor tangible without something solid, nor solid without earth. That is why, as he began to put the body of the universe together, the god came to make it out of fire and earth. But it isn't possible to combine two things well all by themselves, without a third; there has to be some bond between the two that unites them. Now the best bond is one that really and truly makes a unity of itself together with the things bonded by it, and this in the nature of things is best accomplished by proportion. For whenever of three numbers which are either solids [here the editor notes that a solid is a cubic number 2x2x2 = 8

> etc.] or squares the middle term between any two of them is such that what the first term is to it, it is to the last, and, conversely, what the last term is to the middle, it is to the first, then, since the middle term turns out to be both first and last, and the last and the first likewise both turn out to be middle terms, they will all of necessity turn out to have the same relationship to each other, and, given this, will all be unified.[128]

In the elemental distribution of forms there is at once complementarity and antagonism. Fire and earth are the extreme ends of a relation defining the visible and the tangible, water and air (though not mentioned in this passage) are the mediating elements between these extremes. Brunschvicg finds that the play of the elements as governed by the forms of proportional relation come to exemplify the highest exigency of a Platonic science of order. The relationship between forms must itself obey the strictest principle of order. The combination of unlike elements requires a third term which "makes a unity of itself together with the things bonded by it." Brunschvicg recognizes in this uniting principle the proportional relationships which are proper to the series of Platonic solids. The chain of correspondences which the demiurge imposes on the elements is modeled on a rigorous sequencing of mathematical relations. The demiurge accordingly fulfills a very delicate and precarious function, for the *Timaeus* is a mythological expression of an epistemological problem; how is it that the intelligible order imposes itself on the visible order of created things?

This is the unresolved crisis of metamathematics. The very form of the *Timaeus* is cited by Brunschvicg as evidence that Plato was aware of the problem.

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[128] Brunschvicg 1912, 58. *Timaeus* 31 C. Brunschvicg cites only the line number from the *Timaeus* and does not give the text. I have consulted the English translation, see Cooper and Hutchinson 1997, 1237.

Timaeus invokes the demiurge and describes a process of cosmogenesis which is exactly what Plato requires if he is to secure the passage between the world of the forms and the world of appearances. For Brunschvicg, the *Timaeus* is less an exposition of the actual elements of the metamathematical doctrine and closer to a dramatic presentation of Plato's own philosophical ambition. *The Timaeus is the elaboration, using the elements of mythological description, of a philosophical problem, namely, how are we to understand the connection between the science of ideas (dialectic) and the structure of the universe (cosmology?)* The demiurge governs the creation of the universe by remaining "perpetually attentive to the system of ideas" which stipulate that the compositional relations of the elements must follow geometrical principles of internal symmetry, self containment, and proportional complementarity. These rules in turn compose a harmony of relations.[129]

As has often been remarked, the mathematical cosmogony of the *Timaeus* appears to mathematize nature according to an elaborate multiplicity of organizational principles which may or may not testify to a coherent order among themselves. Brunschvicg is exceedingly cautious about any attempt to interpret this element of Plato's thought as an anticipation of the mathematical science which will underpin the sequence of modern scientific revolutions in the seventeenth century. The experience of mathematical orderliness which underlies the concept of the lawful succession of phenomena in the seventeenth century is itself constructed

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[129] Brunschvicg 1912, 59.

immanently out of an experience of reason which, for Brunschvicg, is just as powerfully present to Plato and Pythagoras as it is to Descartes, Leibniz, or Einstein. For the modern era, however, it is enough to render the phenomenon intelligible by pursuing a mathematical transformation of qualities into quantities.

Plato, however, could never be satisfied with such a *restricted* used of mathematics. The numerical idea is not a means to an end in the progressive elucidation of nature, it belongs to a qualitatively different order of reality. For Brunschvicg, it is this *autonomy of the idea* as that which breaks with common sense which constitutes the veritable trans-historical but progressive identity of science. In order to understand the history of science in the Occident it is therefore necessary not only to chart the progress of the idea, but also its shortcomings, regressions, and inevitable failures. The failure of the Platonic tradition to overcome the gap between dialectic and cosmology as dramatized by the *Timaeus,* sets the stage for Aristotle's own effort to bridge this gap with a veritable science of being.

What remains of metamathematics after Plato? Brunschvicg writes:

> The equilibrium which Plato attempted to establish, at least at one stage of his work, between the Socratic tradition and the Pythagorean tradition, between *conceptualism* and *mathematism,* will be broken. The successors of Plato will clearly perceive the impossibility of following at the same time the 'path' of mathematics and the 'path' of concepts. Aristotle makes this choice; Speusippus and Xenocrates do as well.[130]

Brunschvicg introduces the neologism "conceptualism" to refer to the ascendancy of the concept over the applied forms of knowledge, including mathematics. Aristotle, however, is the true authority of the *concept* as indeed is Kant, and they will both

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[130] Brunschvicg 1912, 68.

radically circumscribe the place of mathematics in the system of the sciences in order to secure a powerful regulative mechanism by which knowledge enters into conformity with the categories of experience. The unity of knowledge and experience pursued by Aristotle and Kant is directed against the Platonic doctrine of metamathematics which always seeks to distinguish the superiority of connections between ideas from the entanglement of the forms of knowledge and the forms of experience.

Even in Plato the entanglement of knowledge and experience is perhaps nowhere more vexing than in the difficult relationship between numerical ideas and properly mathematical operations. The numerical ideas which seemed to play such an important role in late speculative dialogues such as the *Philebus*, the *Sophist* and the *Timaeus*, and which might even represent in Brunschvicg's words "a dynastic change" by which "the scepter is passed from dialectics to mathematics," still fail to give an account of how the transition between the realm of numerical ideas and mathematical objects might actually operate. As Brunschvicg writes:

> Platonism suspends the technical aspect of mathematics, the positive domain of this science, in favor of a dialectic which surpasses it and which is estranged from it.[131]

The unresolved status of the numerical idea casts into question the legitimacy of their transcendence. The doctrine of metamathematics is at an impasse. On the one hand, it still entertains a partial fusion of identities with Pythagoreanism, by which the dialectic of mathematics verges on an occultism or

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[131] Brunschvicg 1912, 70.

theosophy of number. On the other hand, if the principles of mathematics are constrained they no longer become paradigmatic of an ideal form of knowledge. They become incisive in their own right, no longer referring to a transcendental reality but delimiting a "positive science," as Brunschvicg writes. The speculative ambition of a universal *organon* based on mathematical ideas and aspiring to a knowledge of the essence of number gives way to a very precise determination: the science which deals with abstract quantities. The conflict between Plato and Aristotle is incisive on this point. For Brunschvicg, Aristotle designs the philosophical method by which Plato's mathematical idealism must give way to a "grammatical intuition" founded on the description of relations between the entities of this world.

Aristotle very clearly describes the difficulties which seem to obstruct the formation of a Platonic doctrine of numerical ideas. Brunschvicg divides Aristotle's objections into two main categories. The first concerns the lack of a clear determination of the relationships between the ideas themselves. How does the idea of the Good or the Beautiful relate to the One, etc.?[132] The second broad class of objections isolates a series of logical problems which seem to be implied by the categorical distinction between numerical ideas and "sensible numbers." How can there be any relationship between the idea of number and mathematical numbers? Plato himself suggests that there is an intermediate number between these two orders which he refers to as "the arithmetical number." If this is the case it only

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[132] Brunschvicg 1912, 71. Brunschvicg cites *Prior Analytics*, I. 22-83b 6 and *Metaphysics* M. 8 1084a 12.

compounds the difficulty, for a seemingly infinite regress is then installed between the ideal number and the arithmetical number, on the one hand, and the arithmetical number and the mathematical number, on the other.[133]

In order to resolve these issues Aristotle will establish a "determinate" system of logic. Aristotle must repudiate all that seems "mythological or metaphorical" in Plato, while also redeeming and perfecting that which is dialectical and logical. The relationship between being and number must be definitively resolved. Mathematics is expressive of a particular category of being: quantity. This in turn enables a distinction between mathematics and physics. Physics is concerned with qualities and founded on an intuition of substance. Mathematics is exclusively concerned with numerical quantities. The metaphysics of substance which Aristotle develops will accordingly give priority to a *physics of qualities* rather than to a *mathematics of quantities.*

> Aristotle's oeuvre will constitute a system of thought which is at once determinate [*défini*] and universal. Accordingly he will reject all that which he judges to be mythological or metaphorical even among the dialectical and logical elements of Plato's oeuvre. Numbers are restricted to their proper mathematical usage and the domain of mathematics restrained to a category which is a particular relation of the affirmations of being to the category of quantity; the conception of which is linked to the fundamental notions of the doctrine: the definitive independence of the category of quality, whose superiority is recognized in the intuition of substance which Aristotle will establish as the foundation of his first philosophy, finally, the constitution of a technical methodology adequate to the exigencies of a qualitative physics and an intuitive metaphysics.[134]

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[133] Brunschvicg 1912, 72. Brunschvicg cites *Metaphysics* A 9, 991a 20.
[134] Brunschvicg 1912, 72.

Although Aristotle borrows many techniques from Plato's method of regressive analysis (the isolation of examples of particular cases which are then classed under a general category, the identification of traits, functions, or signs common to a class, the reduction to absurdity in order to demonstrate the presence of contradictory categories etc.), they must be adapted to serve a *logic of classes* which Brunschvicg claims express an attentiveness to the *reality of individuated being* which has no precise analogue in Plato. According to Brunschvicg, the origins of Aristotle's logic are to be found in the biological discrimination of the different forms of living beings. Indeed, Aristotle intends to construct a metaphysical system capable of accounting for the determinations of individual beings, of which the living being is the most compelling example.

According to Brunschvicg's reading of the *Timaeus,* this is exactly what Plato proved incapable of accomplishing because of the gap between the dialectical and cosmological aspects of his philosophy. Whereas the method of regressive analysis in Plato was complimented and indeed surpassed by a dialectical ascent toward the idea, in Aristotle the same method of division into classes separates itself from the ascendant or unifying path of dialectic in order to concentrate on the reality of individuated beings. The individuated being becomes thinkable in Aristotle through the concept of causality (individuated beings are known according to their causes) and the logical form of the syllogism (which organizes individuated beings into classes of belonging.) Aristotle thus attempts to locate the causes of individuated being in the observable order of nature. For Plato, by contrast, individuated beings owed their individuation (by way of participation) to a transcendental and determining *idea.*

The difference between the schools of Plato and Aristotle can be expressed schematically as follows: In Plato the epistemological program is a dialectics modeled on mathematical knowledge. In Aristotle the epistemological program is a logic modeled on biological classes. Brunschvicg writes:

> At the *Lyceum,* the emergent science of biology replaces mathematics as the central discipline, from which proceed the generalizations of philosophy. The role of formal logic will be to define the processes the naturalist has recourse to in the encounter with living beings.[135]

The classification of species and the attributes shared by different species are the building blocks of the logical proposition in Aristotle. Brunschvicg cites a celebrated passage from the *Prior Analytics* which relates three species (mules, horses, men) according to a common property (the lack of gall or bile) in order to conclude that *all animals lacking gall are long lived.*[136] What Brunschvicg intends to isolate in this example is the structure of the logical proposition in Aristotle.

> There is between the two terms of the proposition [mules, horses and men are animals that lack gall; animals that lack gall are long lived] the same equivalence as between the two terms of a mathematical equality: it is permissible to attribute to animals lacking gall the property that will be affirmed at the same time of the man, the horse and the mule; we obtain from the proposition the conclusion: *all animals lacking gall are long lived.*[137]

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[135] Brunschvicg 1912, 74.
[136] *Prior Analytics*, II, 23, 68b 17.
[137] Brunschvicg 1912, 75.

Brunschvicg in turn concludes that inductive reasoning in Aristotle accomplishes the passage between *objective observation* and *logical necessity:*[138]

> the relation defined in the conclusion becomes the principle of a progressive synthesis, it constitutes in effect a total truth which it is impossible to divide into a series of particular truths, as the genus is divided into species.[139]

The syllogism *overwhelms* reason by compelling assent not through a purely rational deduction from first principles (Descartes' attempt to remake science on a priori grounds will however follow this procedure, as the next section will demonstrate) but by forcing the correspondence between two propositions which must themselves already be treated as exhaustively determined.

This is only half of the logical system. The syllogism not only determines relations and attributes at the general level of species and genus (all animals lacking gall are long lived), it also determines how individuals relate to the species they compose and what characteristics they therefore possess. Brunschvicg gives the example of Bucephalus: " *All horses are devoid of gall. Bucephalus is a horse. Bucephalus is devoid of gall."*[140] Here the sequence of propositions presents the same characteristics as in the previous example, "the conclusion is implied with the same necessity" and the objective correlation between the species and the individual is taken as a logical necessity. The difference is that now the syllogistic form "descends to the level of the individual," determining the individual at the level of fully concrete particularities.

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[138] This is the subject of Brunschvicg's secondary Latin dissertation, *Qua ratione Aristotles metaphysicam vim syllogism inesse demonstaverit,* 1897.
[139] Brunschvicg 1912, 75.
[140] Brunschvicg 1912, 76.

The structure of the logical syllogism therefore manifests "an astonishing degree of conformity with the observable order of things."[141] For Brunschvicg, this will become the defining feature of Aristotle's logic. It is also the sign of a definitive break with the rational experience Brunschvicg calls the *mathematicism* or the *intellectualism* of Plato, by which we must understand a model of science which pursues the connections between ideas. Aristotle's logic, by contrast, models the logical order on the objective order. The logical form of the syllogism establishes a solidarity between logic and common sense. The syllogism models the rules of logic on the domain of the living. For Brunschvicg, this *naturalization of logic according to the forms of the living* amounts to a displacement of knowledge from its proper element, the order of ideas, and leads to inevitable distortions of the understanding. The Aristotelian syllogism comes to reflect an *order of being* rather than an *order of ideas*:

> …the syllogism ignores the order of knowledge, placing it instead in the order of being. In the eyes of the biologist which Aristotle assuredly is, it seems as if the two premises unite as living beings, and by virtue of their generative power give birth to the conclusion. The system of three terms and three propositions constitutes a sort of organic life which is parallel to the existence of things and which furnishes the means to comprehend their genesis.[142]

The "parallel life" of the syllogism presents itself as a purely formal system but it is, in fact, inseparable from Aristotle's metaphysics as a whole, as Brunschvicg will demonstrate. More precisely, the form of the syllogism draws its persuasive force *not from its coherence as a logical procedure but from its deployment of Aristotle's*

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[141] Brunschvicg 1912, 77.
[142] EPM. p. 79.

four-fold theory of causality. The form of the syllogism and the theory of causality must be interpreted as part of the same conceptual system. The syllogism does not only define logical relations, it does so within a broader framework according to which form and matter enter into a relation of "intelligible unity." The logical form of the syllogism determines the relationship between form and matter in order to define the intelligibility of individuated beings.

According to Brunschvicg, Aristotle conceives the formal relations of his propositional or predicate logic within the framework of a metaphysics of substance which itself has been formulated in response to an earlier metaphysical contradiction proper to the philosophies of Plato and Democritus. The contradiction in question corresponds to the doctrines of mathematicism and atomistic materialism. Aristotle's four causes (formal, material, efficient and final) integrate and supplement the incompatible forms of intelligibility proper to Plato and Democritus, who provide the foundational elements of formal and material causality respectively. Democritus provides the model of *material causality* with the theory of atomism:

> According to the doctrine of atomism the world is rendered intelligible by an elementary analysis similar to that which we call chemical analysis. This elemental analysis is a decomposition into parts which is taken beyond the boundaries of sense perception to the very limit of an ultimate term whose durability makes it indivisible: the *atom.* Atoms are indivisible by reason of their smallness. It is remarkable that although he opposes the infinite divisibility of matter, Democritus admits without difficulty atoms in infinite numbers. Their juxtaposition and their entanglements give rise to bodies.
>
> This fundamental thesis is completed by the very precise distinction between two planes of reality: on the one hand, the fundamental properties of the atom as determined by their spatial characteristics of size,

> configuration, orientation… on the other, sensible qualities such as sweetness or bitterness or heat and cold. These qualities, so says a celebrated text of Democritus preserved by Sextus Empiricus, exist only as conventions… [143]

It may seem remarkable to the contemporary reader, says Brunschvicg, that Democritus is able to suspend the order of perception in order to arrive at a fundamental theory of matter out of which all observable phenomena are in turn assembled. This is, however, exactly what Aristotle objects to in the atomistic doctrines. By renouncing any contact with experience and thereby any experimental procedure of verification, atomism becomes a purely speculative theory, indeed one of the most abstract in the ancient world, as Brunschvicg claims, following his teacher Emile Boutroux.

Atomism also introduces a distinction between the purely geometrical relations which define the organization and orientation of the atoms, on the one hand, and the sensible qualities of the bodies they thus compose, on the other. The purely spatial determination of atoms is the underlying cause of all observable phenomena, whose sensible qualities thus become artifacts of our experience. The difficulty with this theory, according to Aristotle, is that it does not provide any account of the source or origin of the movement which causes the atoms to mingle and to "entangle" with one another in the void:

> Aristotle's profound grievance with Democritus is that in supposing the spontaneous movement of atoms in the void he refuses to investigate the cause of this movement.[144]

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[143] Leon Brunschvicg, *L'expérience humaine et la causalité physique* (Paris: Librairie Félix Alcan, 1922), 124.
[144] Brunschvicg 1922, 125.

Democritus provides an elegant and powerful cosmological theory based on the principle of atoms interacting and giving rise to the material diversity of the visible cosmos, but at the great cost of leaving the initial cause of this productive interaction entirely unaccounted for.

The mathematicism of Plato is the exact antithesis of atomism from this perspective. In the Platonic doctrine, it is the formal principle of intelligibility which is developed at the cost of its apparent inability to interact with matter. Democritus has a working cosmological theory but fails to supply a dialectical method which explains the internal organization and intelligibility of the cosmos. Plato furnishes a compellingly beautiful dialectic of ideas without however bringing it into contact with cosmology, except in the most speculative manner:

> In Plato as in Democritus *analysis* typifies intelligibility. However, following the pleasing terminology of Leibniz, Democritean analysis is the *resolution into parts* where as Platonic analysis is the *resolution into notions.* The first sets aside the whole in order to retain only the constitutive parts, the second, on the contrary, attaches itself to the whole in order to understand that which determines it in its totality. Democritus assigns to geometry nothing more than the external image of the spatial juxtaposition of atoms, Plato however turns toward the internal connections proper to the intelligence. Accordingly, that which will become the principal object of Platonic mathematicism is exactly that which for atomism remained inexplicable: the order and proportion by which the object receives its aesthetic form and its harmony.[145]

Plato's conception of mathematics is what Brunschvicg calls an *ultra-quantitative* principle of order, harmony, and beauty. Following Leibniz, the ultra-quantitative principle of order refers not to discrete and non-decomposable units of matter but

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[145] Brunschvicg 1922, 129.

to orders of determination which govern the totality of experience.[146]

Brunschvicg admires Leibniz's distinction between the forms of analysis in Democritus and Plato because it effectively conveys the problem underlying the relationship between dialectic and cosmology which Aristotle wants to unify in his own logic and metaphysics. The atomism of Democritus (a theory of material causality) is a material theory which addresses itself to the *part* at the cost of sacrificing the intelligibility of the *whole*. The mathematicism of Plato (a theory of formal causality) addresses itself to the compositional *unity* of the cosmos as an ideal of intelligibility at the cost of neglecting the particularities of *individual entities.* The relationship between the part and the whole is also the relationship between matter and form in Democritus and Plato. In order to secure the intelligibility of the individuated being Aristotle must unite what Democritus and Plato hold apart. The logical form of the syllogism and the metaphysics of substance is Aristotle's attempt to unite the material and formal theories of causality developed by Democritus and Plato.

Aristotle's system of four causes adapts and refines the theories of material and formal causality which were already present in Democritus and Plato but seeks to make them necessary complements of one another rather than antithetical systems of cosmology and dialectics:

> Form is that which enters matter in order to determine it, it is that by which…Bucephalus is a horse and not a steer. The objects which are given in human experience are constituted by a matter without which one could not

[146] Brunschvicg 1922, 129.

> conceive their substantial reality and a form without which one could not conceive their intelligible unity.[147]

Brunschvicg renders the consequences of this mutual implication of form and matter explicit: it is no longer possible to conceive of a material aspect of the cosmos which would be radically distinct from the principle of its organization. It is equally impossible to uphold the unity of form without also upholding its determination in matter. Aristotle reformats the relationship between form and matter and thus attempts to overcome the ontological fracture which had divided the material and formal theories of causality from one another:

> In the history of human thought the appearance of the syllogism is a decisive moment for it marks the introduction of an instrument adequate to the analysis of the relation of matter and form. The Platonic dialectic claimed to go beyond ομσια (which is substantive of being, at the same time essence and existence) because it aimed to achieve the pure unity of the intelligible. Aristotelian speculation, on the contrary, is based entirely on the notion of ομσια which is the pivot of formal logic and which includes being under its triple aspect: *form* and *matter*, which when composed generate the *individual*. By putting this triple signification of ομσια into play Aristotle claims to resolve the crisis of being and becoming in Hellenistic philosophy, the same crisis which lead Plato in the *Timaeus* to depreciate the value of his own physics. For Aristotle, the composite of matter and form corresponds to becoming. That which is waiting on becoming and change, that which will become the particular, is matter. That which has become the particular, that which is determined, is that which has received form.[148]

The relationship between matter and form conceived as the *becoming* of the individual obeys a temporality which is quite intricate. Matter *will become* the particular, it is a zone of potential and of a power of becoming. Form is that which *has become* determined, it is the hither-side of individuation, no longer the

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[147] Brunschvicg 1922, 138.
[148] Brunschvicg 1922, 139.

individuating act itself but the determinate individual. Brunschvicg in turn reads the relationship between form and matter as representative of another prototypical conceptual antagonism in the systems of Greek philosophy: the relationship between being and becoming. The essential function of the syllogism is to define the determinations of form and to render the becoming of matter.

Brunschvicg wants to understand the implications of the logic of individuated beings which Aristotle thus seems to establish. The triple signification of ομσια necessarily includes the individuated being as such. Brunschvicg, however, claims that it is still possible to refer the specificity of a *materialized form* or an *informed matter* to a specific power of individuation which would still privilege either form or matter. Brunschvicg cites two moments in Aristotle which do not seem to be capable of reconciliation with one another. The first concerns form as that which individuates being. Brunschvicg cites the *Metaphysics*:

> One might raise the question, why woman does not differ from man in species, female and male being contrary, and their difference being a contrariety; and why a female and a male animal are not different in species, though this difference belongs to animal in virtue of its own nature, and not as whiteness or blackness does; both female and male belong to it *qua* animal. This question is almost the same as the other, why one contrariety makes things different in species and another does not, e.g., 'with feet' and 'with wings' do, but whiteness and blackness do not. Perhaps it is because the former are modifications peculiar to the genus, and the latter are less so. And since once element is formula and one is matter, contrarieties which are in the formula make a difference in species, but those which are in the compound material thing do not make one.[149]

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[149] Brunschvicg 1922, 140. Brunschvicg cites the Greek text at *Met.* 1059a 37. I have consulted Barnes 2005, 1672 and I include lines 1058 a 29 – 1058 b 2.

"Modifications peculiar to the genus" are modifications of form ('with feet" or "with wings"), while variations within the species are already included by the form of the species itself which admits of some degree of plasticity (male and female, black and white etc.) Some differences belong to the animal "by virtue of its own nature," and these are necessarily the set of modifications already authorized by the form itself. A species is set of variations on a form: a table of morphological characteristics which are all centered on a common template. Between man and woman we posit only a material difference but the same form. If, however, a difference is introduced into "the formula," then there must also be a correspondingly different set of possible morphological characteristics: all possible varieties of horse, all possible varieties of bird, etc. Brunschvicg suggests that according to the theory of formal individuation, differences between individuals of the same species are insignificant. The difference that *makes a difference* is not to be found at the level of the particular individual but at the more general level of the form which itself already encompasses a plasticity of matter. The very plasticity of matter is suppressed in order to serve a formal or compositional unity of the species. The species, as a formal template, is endowed with an aesthetic consistency capable of supporting morphological variations which would already encompass a predetermined variety of possible material modifications.

If this schema is inverted it is not form which individuates matter according to the distribution of a variable morphology of species but matter which gives rise to the developmental singularity of a unitary being. This unitary being is the result of a unique developmental history. Differences between individuals cannot be accounted for by the *generic form* of the species but refer to the *material development* or process of maturation and cultivation proper to an individual, material life:

> But all things that are many in number have matter. (For one and the same formula applies to *many* things, e.g., the formula of man; but Socrates is *one).*[150]

Socrates is *one* man among the multitudes of men and if we wish to understand his nature we must not refer to his form but to his *material formation.* The logic of material individuation cannot be explained by the commonality of form, which will only refer us to the unity of the species. The unity of the individual is the result of a historical or developmental series of material transformations which unfold within the boundaries of form but which cannot themselves be reduced to form.

Brunschvicg finds here two entirely autonomous intuitions of individuation, two separate logics of the individuated being, and two approaches to the relationship between form and matter:

> Here we arrive at the point of inevitable divergence between the two dominant tendencies of the philosophy of Aristotle: *artificialism* and *naturalism.* Aristotle speaks by turns as a *sculptor* and a *biologist*: the sculptor and the biologist cannot but interpret the relationship between matter and form in contradictory fashions.[151]

The logic of formal individuation is that of the sculptor. The logic of material individuation is that of the biologist. When Aristotle speaks of the individuated

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[150] Brunschvicg 1922, 140. Brunschvicg cites the Greek text at *Met.* 1074 a 33. I have consulted Barnes 2005, 1697-1698.
[151] Brunschvicg 1922, 140.

being he oscillates between these two incompatible registers. Brunschvicg maintains that this is not an incidental divergence between two causal tendencies which might otherwise be united in a superior synthesis or grounded in a common metaphysics. *It is the metaphysics of substance itself that is radically inconsistent*. Aristotle's competing logics of individuation attest to this inconsistency. The relationship between form and matter is the central element of Aristotle's logic and metaphysics but this relationship itself is not a coherent or stable unity. The very definition of matter will change depending on the perspective from which it is interrogated, just as the implications of form will vary according to the use that is made of it:

> What is matter for the sculptor? A homogenous bloc, unformed; the role of the sculptor is to give form to the marble, and by the same gesture to confer individuality upon it. Praxiteles sculpts an Eros or a Hermes; this Hermes in particular, the Olympian Hermes, not to be confused with the images of Hermes made by other sculptors, nor with other Hermes sculpted by Praxiteles himself, nor with the replicas which might be made of this statue: the sculpture is characterized, for the artist, by what makes it unique, and by all evidence it is form which is the principle of individuality here.[152]

From the perspective of the sculptor (the perspective of form) matter is that which is "unformed," it is that which receives individuality. The aesthetic singularity of the sculpture is something the sculptor imposes on matter and which cannot be mistaken even for other generic types within a shared continuum of forms. This is because what the sculptor understands as form is individuation itself. Form is not shared between the model and the copy. The model is individuated, the copy is merely a repetition of this individuation: no power of formation has acted upon it.

[152] Brunschvicg 1922, 140-141.

Nor can the same aesthetic form be common to all members of a class of artworks. All sculptures of Hermes are not modified expressions of the same transcendental form of Hermes. Each would instead be an entirely singular individuation. Brunschvicg has here pushed the individuating logic of form all the way to the limit of an absolutely singular aesthetic purposiveness (a topic which Kant will reprise in his own fashion, as with so much else in Aristotle, in the *Critique of Judgment*). The authenticity of form is that which cannot be repeated but which can only be enacted by the creation of a singular being. Every artwork is thus an affirmation of form as that which individuates being. From the perspective of what Brunschvicg calls *artificialism,* form is the supreme aesthetic intuition of the artist confronting the inertia of matter and imposing the unique outline of a singular being upon it.

The logic of material individuation forgoes the singularity of form suddenly and irrevocably imposed on matter in favor of a developmental logic which affirms the plasticity of matter. Material individuation obeys a temporality which is entirely distinct from that of formal individuation. The individuality of matter is expressed historically: it is the specificity of a life and a developmental trajectory. Brunschvicg locates the logic of material individuation in “the spectacle of living nature:”

> From the studio of the sculptor we pass to the spectacle of living nature. The scientist is the one who discerns the form of being in the individual, who in seeing Callias has the immediate intuition of humanity…in other words, form here is species. For a plurality of individuals of the same species the form is homogenous, their differences arise from matter which makes each of them a particular subject out of common predicates. In the domain of biology it is evident that matter is the principle of individuality. While aesthetic curiosity is only satisfied by seizing the work of art according to its proper and

> individual characteristics, that which is instructive before a herd of domestic animals is, on the contrary, to know the species to which these animals belong in order that the repetition of experiences common to the species can be put to use in breeding [*élevage*].[153]

The singularity of matter is not the totalizing presence of the artwork which cannot be repeated but the continuous development of the tendencies immanent to matter. For the sculptor, form is absolutely singular and matter is accordingly exhausted once it has received form. For the biologist, the naturalist, or the breeder of animals form is generic, it is a template upon which the plasticity of matter will express itself. In order to understand the diversity of individuated living beings one must have the eye of the naturalist rather than the eye of the sculptor. The logic of material individuation perceives the form of the species as homogenous and approaches the diversity of the living not from the perspective of their static morphological characteristics but from the individual developmental histories which describe the becoming of matter. Matter is the domain of temporal development: the successive modification of a form whose unity is defined by the continuity of matter rather than the consistency of form.

The dual logics of individuation oscillate between the domination of matter by form or the subordination of form to matter. The metaphysics of substance, therefore, finds itself internally divided between these two contradictory determinations of the individuated being. For Brunschvicg, Aristotle is not unaware of this difficulty. *Efficient* and *final* causality are the necessary complements to an otherwise underdetermined metaphysics of substance: "The causes which must be

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[153] Brunschvicg 1922, 141.

added to matter and to form to account for the process by which being is individuated are *the efficient* and *the final.*[154]" The marble which will become a statue of Hermes or of Eros must receive this form from outside itself. Form is doubly exterior to matter when filtered through efficient and final causality. The efficient cause is that which can be grasped by the immediacy of observation and intuition. It is the series of direct acts on an object which bring about a transformation of the object. Efficient causality, however, is just the means to an end. It is a method in the service of a *telos* which governs the entire procedure of individuation. The final cause is not immediately given. It cannot be grasped in the instantaneous transformation of an action but must be referred to the entire developmental continuum of the individuated being:

> The blows of the sculptor's chisel give rise to the statue and furnish the immediate cause of change: the motive, efficient, or poetic cause. This cause is *for the eyes* the cause par excellence. But it does not suffice *for the mind*; it is incapable of explaining what is essential: how the blows of the chisel were lead to discover a definitive form, an Eros or a Hermes. The form which appears in the completion of the artwork had to be conceived before the series of efficient acts which in turn owe their meaning and succession to it. That which is the *end* of the process, despite or rather because it is the *end* must be considered as being the *cause.*[155]

Efficient and final cause furnish formal causality with "two inverse orders" of explanation. Efficient causality is an "exterior order of knowledge" which accords with the accidents which can befall substance. Wood can be chopped with an axe and marble can be chipped with a hammer and chisel. Final causality is an "intimate

[154] Brunschvicg 1922, 143.
[155] Brunschvicg 1922, 143-144.

order" which refers to the "internal production of the real."[156] The final cause is not an accident which befalls substance, it is the *purposiveness of transformation* and necessarily refers to a telos. In some sense it is internal to the individuated being even if it cannot be discerned in it until the final outcome of the process of individuation has been achieved. The same doubling of exterior and intimate orders of explanation is also applied to material causality:

> The growth of the child toward humanity is not intelligible from his apparent point of departure, for the child himself lacks the characteristics which constitute and define the man. That which gives the child his own true nature is the fact that he is in the process of acquiring the proper nature of the man, that he is oriented towards the form of the man… This development cannot be explained integrally except from its final point of arrival, that is to say by its final cause. Man is the final cause of the child… It is the father, the man who has engendered the child, or more precisely, who has communicated to the child the capacity of finality.[157]

The sculptor possesses the capacity to impart form to the marble by virtue of an aesthetic sensibility. The father, however, has the capacity to impart finality to the child by sharing a material continuity with him. As the father is so shall the child become. The work of art and the growth of the living being must be grasped by the triple determination of the relation between form and matter, the exterior order of efficient causes, and the intimate order of final causality. It is impossible to truly separate form and finality from material causality, just as it is impossible to separate efficient and final causality from form. Therefore, for Aristotle, the art work possesses an organic unity just as the living being embraces a telos and an aesthetic destiny.

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[156] Brunschvicg 1922, 144.
[157] Brunschvicg 1922, 144-145.

Aristotle's metaphysics thus thoroughly saturates his theory of causality. Following the work of Pierre Duhem, Brunschvicg notes that Aristotle's theory of causality must be considered pre-scientific precisely because of the connection it systematically establishes between what Brunschvicg calls "abstract ontology" (the metaphysics of substance and the logic of individuation) and "certain knowledge." The most abstract theory is simultaneously the most certain and the most authoritative because it clarifies the essence of being: "the physicist alone possesses the power to know causes, and only he possesses truth with the apodictic character which must define it."[158] Brunschvicg is especially critical of the solidarity Aristotle establishes between the form of the logical syllogism and the logic of individuation. For Brunschvicg, logic in Aristotle is simultaneously a matter of *the proper discretion of the categories of being* and the *identification of the category of being according to its cause.* Certain knowledge is knowledge of cause. Logic describes the organization of causes in a coherent chain of being. *The order of knowledge confirms the order of being and vice versa.* This is the fundamental affirmation of the Aristotelian system and it is also the problem which Brunschvicg claims Descartes must subject to the most radical critique. The theory of causality in Aristotle must unite the diversity of individuated beings within a shared metaphysics of substance. Following the metaphysical agon of Platonic philosophy it must reconcile dialectic with cosmology, the intelligible with the sensible, and the objective order with the rational order.

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[158] Brunschvicg 1922, 173.

Descartes' project will also attempt this reconciliation but by an entirely different method. Indeed, method and philosophy become synonymous in Descartes. The diversity of individuated beings is of little concern to Descartes. What is paramount is the relationship between two forms of science; a science of measure which generates the positive sciences, and a science of order which secures the apodictic certainty and unity of all science, a *mathesis universalis.* Brunschvicg is wary of the readiness with which the history of philosophy has assimilated Descartes to an unproblematic vindication of the Platonic tradition over the scholasticism of the middle ages. Plato and Descartes are necessary references in the history of mathematical intelligence. However, the Cartesian method itself is as much a critique of Plato as it is a return to the foremost ambition of the human intellect for Brunschvicg: the affirmation of the experience of reason.

The Order of Reasons: Analytic Geometry as Foundation of the Intelligible Order in Descartes

Against the intellectual background of Neo-Thomist scholasticism the philosophy of Descartes signifies a fundamental reorientation of the intellect. According to Brunschvicg, Descartes eschews both "the illusory ontology of the peripatetics" and the "sterile empiricism of the skeptics" favoring instead a new philosophical persona, the natural philosopher as the student of mathematics.[159] In

[159] Brunschvicg presents this reading of Descartes in a review of Étienne Gilson's new edition of the *Discours de la méthode*. See Léon Brunschvicg, "Mathématique et

a review of Etienne Gilson's (1884-1978) critical edition of the *Discourse on Method* , Brunschvicg remarks that between the *Regulae ad directionem ingenii* (1629) and the *Discourse* of 1637 Descartes develops the technical expertise required to conceptualize a new scientific problem: the relationship between the domains of *pure mathematics* (in Descartes' case an "algebra of structured numerical relations"), and *universal mathematics* (physics.) Here again the tropes of Brunschvicg's philosophical history of the intelligence are in full effect. What Descartes seeks to negotiate in the name of an order of reasons is precisely the relationship between dialectic and cosmology which Aristotle also attempted to construct, albeit in the form of a qualitative physics of individuated being and a metaphysics of substance.

Brunschvicg will examine how Descartes reformulates both the content and the form of this relation. It is not simply the case that Descartes is a repetition of Plato or a vindication of the theory of ideas over the logic of common sense represented by Aristotle's philosophy. Descartes is a necessary reference in the history of the mind because he invents a new form of truth indexed, as was the case with Pythagoras and Plato, to the experience of mathematical intelligibility. Unlike his predecessors, however, Descartes attempts to derive the point of contact between the order of reasons and the order of things based on an entirely immanent experience of the apodictic self evidence of mathematical truths. Consequently,

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Métaphysique Chez Descartes," *Revue de metaphysique et de morale,* vol. 34, no 3, (1927): 277. Reprinted in Léon Brunschvicg, *Écrits Philosophiques Tome I* (Paris: Presses Universitaires de France, 1951), 11.

Brunschvicg's extensive writings on Descartes, despite their great diversity, do tend to return to a stable constellation of three general problems. 1) How is the rationalism of Descartes a vindication of the epistemological status of geometric demonstration ? 2) What constitutes "pure analysis" in Descartes ? 3) How is the domain of physics transformed from a qualitative discourse (as in Aristotle) to a "science of structured relations?''

For Brunschvicg, the philosophical significance of the *Discourse* can be reconstructed "solely on the terrain of mathematics" by following the progressive order of Cartesian analysis which had been perfected by the time of the *Discourse* but which was already present, albeit in a more technical aspect, as early as the *Regulae* of 1629. Against the general consensus of the history of philosophy, and departing from Etienne Gilson's commentary, Brunschvicg claims that it is possible to reconstruct the entire philosophical edifice of Descartes' system based on an examination of his mathematical works and as his contributions to physical theory. Brunschvicg claims that by 1637 the Cartesian mathematical philosophy is fully formed because Descartes had arrived at a consistent method for constructing algebraic solutions to geometric problems. In this light the *Geometry* (1637) is the most important text in the Cartesian corpus and articulates the epistemological framework of modern rationalism more generally:

> The secret of modern rationalism in its entirety is present at the beginning of the third book of the *Geometry*, strangely ignored or misunderstood by historians of philosophy. There in fact thought is characterized in opposition to the logical deduction by which thought inevitably deteriorates as it

> pursues its object, in favor of an ascending process of thinking which is a continuous enrichment of truth.[160]

The *Geometry* is the crystallization of the rationalist project insofar as it reasserts the necessary identity of thought with a "continuous enrichment of truth" whose very body is the progressive order of mathematical reason. The systematic dismantling of the Scholastic and Thomistic hierarchy of being is above all a rejection of the union of thought and being enforced by these systems. Descartes will reaffirm the perennial mathematical philosophy by once again making mathematics the index of truth. Unlike the Pythagorean and Platonic philosophies, the Cartesian order of reasons will not pursue a metamathematical doctrine but will affirm the ontological primacy of a rational order entirely immanent to mathematical reason. Therefore, while acknowledging the proximity of Plato and Descartes, Brunschvicg also wishes to avoid an over simplified identification of their respective mathematical philosophies:

> It is true that Descartes, as a philosopher… resumes the Platonic tradition; but that which makes this resumption triumphant is that Descartes the mathematician and physicist has resolved, according to the progress of positive knowledge, the problems which Plato could only pose in the *Republic* and in the *Timaeus,* and to which the Peripatetics could give only illusory responses.[161]

Descartes' philosophy is above all enriched by the novelties of its contributions to an entirely "positive knowledge" which has advanced, by methodical and technical elaboration, well beyond the purely theoretical

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[160] Brunschvicg 1951, 16.

[161] Léon Brunschvicg, "Platon et Descartes," in *Écrits Philosophiques Tome I* (Paris: Presses Universitaires de France, 1951), 86.

speculations of the *Republic* and the *Timaeus.* Descartes must be distinguished from Plato according to three important criteria. The first principle of demarcation concerns the ambivalence of mathematical knowledge in Plato, which is by turns the model of truth and an image of truth, both the form and the copy, depending on where one intervenes in the development of the Platonic doctrine. The relationship between mathematics and the dialectic is at the center of this contentious issue. Descartes is unequivocal in his rejection of any extra-mathematical dialectic governing the elaboration of mathematics. The second principle concerns the relation of the rational order to the sensible order, or of the intelligible to the domain of experience and becoming. In Plato, Brunschvicg affirms, it is impossible to account for the genesis of the sensible world except "by recourse to myth" as in the Timaeus. In Descartes, the problem of cosmogenesis is transformed into the relationship between two domains of mathematical reason: a domain of pure analysis (defined algebraically as *pure mathematics* by Descartes) and a domain of universal analysis (defined by the laws of physics, also referred to as *universal mathematics* by Descartes.) The relationship between pure analysis (algebra) and universal analysis (physics) also determines the relationship between the intelligibility of mathematics and the world of appearances. The third principle of demarcation refers to the entirely transcendental relation of the Platonic ideas to the elements of experience. In Descartes there is a very rigorous development of the immanence of mathematical reason to the structures of thought. Brunschvicg therefore wishes to distinguish Descartes from Plato in three important respects:

the self sufficiency of mathematical knowledge, the relationship of the intelligible to the sensible, and the immanence of mathematical reason to the intellect.

Brunschvicg claims that in order to understand the full significance of Descartes in the history of philosophy and of science, it is necessary to affirm that although he closely engages with the mathematical and philosophical traditions of antiquity, he does so from the perspective of an entirely original philosophical intuition:

> Cartesian intuition does not correspond to an element of the sensible or to an analog of the sensible, as did the intuition of the atomists, nor to the abstraction of a concept or a principle, as was the case with the intuition of the dialecticians… Cartesian intuition is, or tends to be, an intuition not of the thing, but of thought. The reality of thought consists of an act. This act is above all the act of judgment.[162]

Brunschvicg had previously compared the atomistic physics of Democritus to the dialectical mathematicism of Plato in order to highlight the difference between the traditions of material and formal causality which informed Aristotle's metaphysics of substance. The intuition of the atomist suffers from an excessive reductionism. It dismantles the sensible world but can give no account of the structure of the cosmos. The intuition of the Platonist suffers from an excessive formalism. It raises the idea to a self sufficiency beyond being and therefore severs the intelligible connection between the idea and the cosmos. The malady of intuition seems to be an excessive determination of the idealist and realist poles of philosophy, a vacillation

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[162] Léon Brunschvicg, "La Pensée Intuitive chez Descartes et chez Les Cartesians," in *Écrits Philosophiques Tome I* (Paris: Presses Universitaires de France, 1951), 56.

which Brunschvicg glosses as the antagonism between dialectic and cosmology. Descartes appears to reconfigure this relationship by seeking to ground intuition in its proper element, the reality of the acts of judgment in the form of pure interiority or *pure analysis*. For Brunschvicg this is above all the discovery that thought, guided by intuition, discovers certain necessary structures:

> The fundamental point is this: *atomistic intelligence lacks structure; Cartesian intelligence,* constituted by the analytic composition of equations, degree by degree, *is a structure.*[163]

The systematic elaboration of the necessary structures of thought (which will become the order of reasons) is revealed by the examination of the intelligible structure of *pure analysis*. For Brunschvicg it is the transformation of geometry into analytic geometry that allows Descartes to discover the autonomy and rigor of pure mathematics, and therefore to affirm the necessary idealism of the order of reasons:

> …Descartes is led to take a step of decisive importance for philosophy. He decisively breaks the correspondence between the dimensions of space and the degrees of the equation. *He creates a geometry which is above all an algebra.* Undoubtedly the 'proportional relations' sufficient for the constitution of pure analysis are capable of being represented by line segments; but the characteristic properties of these lines are no longer inherent to the structure of space, as in the case of intuitive geometry, which our perception apprehends…*The Space of geometry, in 1629, seemed to be the indispensable support for the realism of a universal mathematics, the space of analytic geometry, in 1637, is by contrast a simple auxiliary support for the idealism of pure mathematics.*[164]

Here the transformation of intuition becomes a necessary consequence of the procedures of analysis. Between 1629 and 1637 Descartes constructs a new order of intuition by effecting the transition between universal mathematics and pure

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[163] Brunschvicg 1951, 19.
[164] Brunschvicg 1951, 19.

mathematics. The entire diagnostic configuration of Brunschvicg's critical idealism is at play in this transformation. In 1629 geometry is the science of extended bodies, it is founded on an intuition of lived space and it is immanent to the nature of sensible experience. Universal mathematics is thus what Brunschvicg calls a "mathematical realism" by which mathematics becomes the measure of experience. Realism as such is the eternal antagonist of the order of ideas because it ignores the autonomy of the intelligible order. A geometry founded on an intuition of the sensible (an intuition of things rather than of thought) will always be an organ of mathematical realism. The analytic geometry of 1637 (whose conceptual genesis Brunschvicg will describe shortly) is not reducible to spatial representation. It breaks entirely with the order of representation and opens onto a properly idealist domain of the order and connection of ideas. It is important to note that Brunschvicg will always associate realism with representation in this manner and therefore also assign to philosophical critique the task of transforming *representations* into *ideal relations*. For Brunschvicg, as for Descartes, ideal relations "are the acts of thought." Representation, Brunschvicg claims, is the cardinal vice of realism and the invariable symptom of an improperly configured philosophical intuition.

How does Descartes himself arrive at this break with the representative realism of geometry? Pierre de Fermat (1601 -1665) is an important precursor to Descartes and in fact established the principles of analytic geometry in the posthumously published *Isagoge ad locos planos et solidos,* which appeared in 1679 but which was probably composed in 1636, therefore predating Descartes' *Geometry.* If Fermat seems to have established with all due rigor and clarity the operations of plane and solid geometry, what then does the *Geometry* contribute to the history of mathematics and philosophy? Brunschvicg distinguishes the *Isagoge* from the *Geometry* based on the underlying conception of science which informs the composition of both works. The *Isagoge* is "the work of a technician who is at the same time a scholar" and who was able to elevate the practical techniques of geometry "to their highest point of elegance and simplicity."[165] The *Geometry,* by contrast, is a work not only of technical expertise but of methodological elaboration "which proceeds according to a universal conception of science and bequeaths to its successors an original notion of scientific truth."[166]

Brunschvicg treats the *Regulae* and the *Discourse* as the historical bookends of a continuous conceptual development in which Descartes constructs a new image of science. In the *Regulae* and the *Discourse,* Descartes returns to the elementary forms of arithmetic and geometry, citing Pythagoras and Euclid as the exemplary models of science in the ancient world. The science of the Greeks, however, lacks the methodological consistency Descartes is looking for because it remains mired in the elements of representation. In order to discover the foundations of a universal method it is necessary to reorganize the relationship between arithmetic and geometry. Universal mathematics is Descartes' methodological proposal for a unification of the branches of science according to a common intuition by which the

[165] Brunschvicg 1912, 101.
[166] Brunschvicg 1912, 101.

simplicity of number becomes the foundation of the principles of knowledge. However, as Brunschvicg notes, universal mathematics seems to follow two different trajectories in Descartes, one which is proper to the "extension of the mathematical method to the universality of cosmological problems," and one which is proper to "the reduction of the problems of geometry to the problems of algebra."[167]

Both trajectories undoubtedly follow the same inspiration but "it would be incorrect to conclude that they can be superimposed on one another."[168] For Brunschvicg, the first is equivalent to "a reform of physics by mathematics," while the second is a "reform of mathematics itself." Underlying these divergent trajectories of universal mathematics are differing notions of space:

> Space plays in the physics and mathematics of Descartes two different roles. In the physics the reduction of quality to quantity consists in retaining nothing from the sensible phenomena save those determinations which can be measured with the help of the dimensions of extension. In the Geometry, by contrast, spatial figures appear as qualities which will be reduced to the purely abstract and intellectual forms of quantity, by the degrees of the equation. In short, the *Principles of Philosophy* are a physics of geometry; the *Geometry* is a geometry of analysis. This explains how proceeding from one work or the other one arrives at two different quite distinct conceptions of the mathematical philosophy.[169]

The first conception of universal mathematics is a "physics of geometry" because it is informed by a new concept of science which Descartes formulates explicitly in the *Principles of Philosophy.* Here "the fundamental idea is that science is essentially a unity because it is the work of human intelligence and there is but one means of

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[167] Brunschvicg 1912, 107.
[168] Brunschvicg 1912, 107.
[169] Brunschvicg 1912, 107.

understanding."[170] Science follows the contour of human intelligence which alone furnishes the principles of understanding and of true knowledge. From this perspective mathematics must be the "science of *order* as well as the science of *measure;* and it includes, in addition to arithmetic (or algebra) and geometry, astronomy, music, optics and mechanics."[171] As Descartes writes:

> When I considered the matter more closely, I came to see that the exclusive concern of mathematics is with questions of order or measure and that it is irrelevant whether the measure in question involves numbers, shapes, stars, sounds or any other object whatever. This made me realize that there must be a general science which explains all the points that can be raised concerning order and measure irrespective of the subject-matter, and that this science should be termed *mathesis universalis* – a venerable term with a well established meaning – for it covers everything that entitles these other sciences to be called branches of mathematics.[172]

The deficits of the individual sciences are overcome in the general science of the *mathesis universalis* from which, in turn, they borrow their warrant as sciences in the first place. The *form* of deductive rigor present in all the sciences of measure points to a common origin in universal mathematics. It then becomes possible to conceive of a *definitive method* proper to universal mathematics which would be adequate to the *form of intelligibility proper to all the objects which universal mathematics describes:*

> This definitive method is founded on the notion of space in so far as space is adequate to the reality of things. This adequation will be effectively obtained on the condition that space has sustained an elaboration which has at once simplified and generalized the notion. Space, as it had been envisaged by the ancient geometers, is a system of figures susceptible of being measured

[170] Brunschvicg 1912, 108.
[171] Brunschvicg 1912, 108.
[172] John Cottingham, Robert Stoothoff and Dugald Murdoch, eds., *The Philoosphical Writings of Descartes Vol 1.* (Cambridge: Cambridge University Press, 1985), 19.

> following three dimensions; thanks to the enumeration of these dimensions the problems of geometry can be easily determined. On the other hand, the magnitudes in space represent the first three degrees of mathematical or algebraic magnitudes: "the simple quantity (called the first degree in modern algebra) is called *the root*; the second is called the *square*, the third *cube*, the fourth *bi-squared* etc.'[173]

The space proper to measurement and the rational space of order must not be confused with one another. The geometry of the Greeks is above all a "system of figures." The determination of these figures according to the degrees of algebraic magnitude is no longer to engage in the measurement of figures, but to define the very notion of the figure as a relation among magnitudes. The science of order encompasses the science of measurement from every conceivable dimension by transforming the intuitive resources of representation into clear and distinct ideas. In place of the mental image of the line segment universal mathematics defines the clear and distinct ideas of figure, magnitude, and movement as the plotting of a point, a line, and a surface (or volume) in the coordinate system of the Cartesian grid.

The idea of a universal science thus becomes possible based solely on the notions of *extension* and *movement*, ensuring an epistemological homology between the domain of geometry and mechanics and thus the thorough rationalization of physics based not on measurement (which remains a necessary tool) but on the rational principles surpassing and legitimizing the sciences of measurement:

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[173] Brunschvicg 1912, 110. Brunschvicg cites *Regulae* XVI*, AT*, X, 456.

> the principles of the *mechanics* being homogenous with those of the *geometry* insofar as the idea of movement contains no element which is not already implicated in the idea of space.[174]

Universal mathematics as the "physics of geometry" culminates in the principles of Descartes' *mechanism,* by which "matter is defined as that which is extended in length, depth, and width." Combined with the principle of movement defined as a magnitude susceptible to the same algebraic determinations as spatial figures, Descartes has all the elements he requires to furnish "the fundamental equations of mechanics." Brunschvicg notes that once Descartes establishes universal mathematics as the foundation of a no less universal mechanics, he is able to claim that all the phenomena of nature (conceived mechanistically) become, by default, intelligible in their very nature. Universal mathematics therefore becomes the principle of the intelligibility of nature.

The universal mathematics of Cartesian mechanism is the algebraic determination of the three dimensions of spatial extension plus motion. As Brunschvicg writes: "… in this sense universal mathematics is an extension of geometric methods to the universality of the problems of mechanics and physics."[175] The *Geometry* offers a different formulation of universal mathematics by "operating a transformation of the technical methods of Geometry and Algebra."[176] This transformation will consist of a reexamination of the nature of geometric demonstration. For the ancients, Descartes claims, the methods of geometry were

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[174] Brunschvicg 1912, 113.
[175] Brunschvicg 1912, 113.
[176] Brunschvicg 1912, 114.

synthetic: "they examined causes by their effects." In the *Replies to the second set of objections,* Descartes proposes the following distinction between the analytic and synthetic methods in geometry:

> It was synthesis alone that the ancient geometers usually employed in their writings. But in my view that was not because they were utterly ignorant of analysis, but because they had such a high regard for it that they kept it to themselves like a sacred mystery.
>
> Now it is analysis which is the best and surest method of instruction, and it was this method alone which I employed in my *Meditations.* As for synthesis…it is a method which may be very suitable to deploy in geometry as a follow up to analysis, but it cannot so conveniently be applied to these metaphysical subjects.
>
> The difference is that the primary notions which are presupposed for the demonstration of geometrical truths are readily accepted by anyone, since they accord with the use of our senses… In metaphysics by contrast there is nothing which causes so much effort as making our perception of the primary notions clear and distinct. Admittedly, they are by their nature as evident as, or even more evident than, the primary notions which the geometers study; but they conflict with many preconceived opinions derived from our earliest years, and so only those who really concentrate and meditate and withdraw their minds from corporeal things, so far as is possible, will achieve perfect knowledge of them.[177]

The synthetic method does not depart from the evidence of the senses and in fact is in conformity with them. In the domain of geometric demonstration synthesis would seem to be the natural ally of the geometer. In the imagination, as in perception, the lines which compose the geometric figure seem to be the elementary components of the problems of geometry. As Brunschvicg writes, they "naturally represent the *absolute.*" The analytic approach involves a revision of perception. Analysis does not confirm experience. It departs from experience and converges with the effort to achieve a clear and distinct knowledge of primary notions.

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[177] John Cottingham, Robert Stoothoff and Dugald Murdoch, eds., *The Philosophical Writings of Descartes Vol. 2* (Cambridge: Cambridge University Press, 1984), 111.

Accordingly, it is not the figural element of the line (so easily grasped by synthetic manipulation) but the analytic notion of the *metrical relation* which subtends the geometric figure:

> …these lines, which are for the imagination the elementary terms of the problem and which naturally represent the *absolute* are in reality *effects* because they depend upon metrical relations which are contained in the statement of the problem; it is the metrical relations and not the lines which constitute the veritable *absolute*, if we understand by *absolute*, not the object which seems, to the eyes, to detach itself with the appearance of independence, but the simple principle, the *cause*, which for the mind commands and generates all determinations.[178]

As Descartes had already indicated in the *Regulae,* the absolute of geometry (the primary notion which "generates all determinations") can be conceived as a rational relation. The *Geometry* and the *Reply to the second objection* make this explicit by referring all effects to their causes. The line is therefore not the primary notion of geometry but the effect of a principle of reason which determines all its possible relations. It is not so much that analysis is opposed to synthesis as that it operates at a different level. The analytic solution to a geometric problem consists in assigning a numerical value to all the line segments of a given figure (known or unknown) and defining the relationship between the line segments algebraically rather than synthetically. The analytic solution perceives the set of all possible relations as already determined, and in this respect Descartes' metaphysical project aligns perfectly with his analytic method. The *Geometry* is a careful exposition of the algebraic expression of geometrical relations. What is of paramount importance, however, is not the translation between the domains of algebra and geometry but

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[178] Brunschvicg 1912, 117.

the newly affirmed autonomy of the algebraic expression. The algebraic equation, as such, is no longer a place holder for synthetic realities but the absolute metric of the geometric figures themselves: a causal principle.

The *Geometry* appears as a somewhat disordered book from the perspective of a practicing mathematician, as Brunschvicg admits, but this disorder is the sign of an ongoing investigation which Brunschvicg calls the first modern *theory of the equation:* "The algebraic equation expresses the fundamental relation which constitutes magnitude, it is the *absolute."*[179] The constitution of magnitudes following the method of analysis leads Descartes to "transform the conception of pure mathematics and the fundamental notion of *quantity."*[180] In the *Regulae* mathematics played the role of a universal organon, it stipulated the proper method by which the human intellect could pose and resolve problems. Universal mathematics as the general science of order and measure establishes a parallelism between arithmetic and geometry. Algebra may correct the procedures of geometry, but it remains unchanged in its own nature because it is confined to the definition of the working methods of mechanism. The *Geometry,* however, no longer asserts the parallelism of the sciences of order and measure but asserts the *hierarchy of analytic functions over synthetic representations*:

> With the *Geometry* the juxtaposition [of analytic and synthetic methods] transformed itself into a hierarchy; quantity, which was submitted to the restriction of spatial representation [in the systems of mechanics] becomes something composed out of relations uniquely defined by means of arithmetic operations, expressed by the symbolic systems of algebra… from

[179] Brunschvicg 1912, 121.
[180] Brunschvicg 1912, 121.

> this proceeds the consequence that the limits of algebraic science determine the limits of geometric science.[181]

The parallelism of algebra and geometry which was necessary in order to secure the unity and intelligibility of the sciences of measure and order, gives way to a *determinate order of analytic reason* which establishes the limits of synthetic reason and makes geometry the *effect* of an algebraic universal mathematics which underlies it and which surpasses it. The transformation of quantity from the formal designation of an empirical measurement to the structural relation of the symbolic systems of algebra establishes quantity as an *absolute*, that is to say, as part of "the uninterrupted movement of the mind" proper to primary notions.

The two planes of universal mathematics are cosmological and intellectual. The first extends geometry into the universe by transforming space into a system of orderly and structured relations. Here universal mathematics is understood as physics and cosmology. The second plane of universal mathematics attempts to surpass the synthetic reason of geometry by arriving at an intellectual solution to the sensible intuitions of geometry. The analytic determination of quantity cannot be captured by any imaginative representation of space or of measurement. Analytic determinations are purely theoretical constructs which are accessible only to the mind. They are objects of meditation rather than measurement:

> …the idea of the science of mathematics is transformed: quantity is no longer, as it was in Euclid, a determination extracted by abstraction from the observation of objects; the science of quantity is no longer comparable to a natural science. The notion of quantity is purely intellectual, it is established

[181] Brunschvicg 1912, 121.

> *a priori* solely by the capacity of the mind to direct and to follow to the infinite "the long chains of reasons."[182]

Descartes' inestimable contribution to the history of science is to have invented the possibility of a science which is not a science of nature put purely of reason. This is the origin of what Brunschvicg calls "the double ascesis of rationalism:" the rational determination of the boundaries of knowledge and the discovery of an entirely new intellectual domain. Analysis is the foundation of synthesis because it cannot itself be reduced to an anterior order. Descartes transforms the synthetic geometry of the Greeks into a derivation of the analytic order of reasons. In so doing he transforms analysis into a genuinely creative procedure. The unity of science is premised on the primacy of analysis. At the same time, the necessity and universality of science is referred not to the objective condition of the sensible world, but to the self evidence of simple notions. It is the irrecusable self evidence of a genuinely intellectual intuition which secures the connection between the order of things and the order of reasons.

The parallelism of geometry and algebra allows Descartes to construct a mechanical philosophy which is itself descriptive of the intelligible structure of the universe. The mechanist *mathesis universalis* does not however contain its own principle of legitimation. This principle must be discovered in the order of reasons disclosed by meditation on the contents of mathematical apodictic self evidence. The rationalist philosophies of Malebranche, Spinoza and Leibniz will each develop provocative variations on the ontological and metaphysical implications of the

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[182] Brunschvicg 1912, 123.

newly autonomous order of reasons. However, in Brunschvicg's presentation of the historical unfolding of the forms of mathematical and scientific intelligibility, it is Kant's critical philosophy which once again establishes the necessity of a new inquiry into the boundaries of reason and experience. The primacy of analysis in Descartes becomes for Kant not the legislator of the necessity of science but a transcendental illusion of reason itself.

The development of mathematical physics after Descartes dramatizes a tension which Descartes thought he had resolved definitively by making the synthetic sciences of measure the products of the analytic science of order. Is geometry a science of nature or of the mind? Does science describe things as they are or must things be as science describes them? From what source and according to what necessity does science derive its authority? Descartes thought he had discovered the immanent source of this authority in the surveillance of mathematical thinking. From Kant's perspective, however, this would be to mischaracterize the nature of experience and of thought. Experience and thought, Kant affirms, cannot simply be opposed to one another as an order of representation is opposed to an order of reasons. Experience is the beginning and the end of thought. It is an all encompassing domain which cannot be reduced to a purely intelligible domain of analysis.

The problem of the relationship between synthesis and analysis is therefore internal to experience and does not permit the division of science into two domains; one which would be purely intellectual and one which would be purely practical.

Kant cannot accept the dogmatic solution of rationalism to the problem of the nature and necessity of scientific knowledge. Nor can he endorse the empirical rejection of science as nothing more than a habitual association of ideas. What is required is a fundamental critique of reason and a reconstruction of the nature of experience. The distortions of dogmatic metaphysics must be corrected so that the rational principles of natural science might be established on secure foundations.

Mathematics as Schematism: Kant and the Critical Philosophy

Kant's philosophy, like Plato and Descartes before him, invents a new form of rational experience indexed to a new form of truth. Kant is situated at the cross roads of simultaneous and overlapping revolutions in mathematical physics and philosophy. The critical philosophy is designed to overcome the deadlock of two overdetermined forms of the relationship between experience and reason. On the one hand, empiricism, especially in the philosophy of Hume, makes experience itself an absolute by foreclosing the possibility of any form of knowledge which does not restrict itself to the immediacy of sense impressions. This is the skeptical liquidation of the order of reasons. On the other hand, the rationalist tradition makes the experience of reason into its own dogma, cultivating a form of pure analysis and pure mathematics which entirely breaks with the order of lived experience. The relationship between mathematics and physics in the seventeenth century embodies these discordant philosophical projects.

Kant inherits a burgeoning natural science with wildly different empirical and analytic procedures. Newtonian physics internalizes, simultaneously, a purely deductive image of science (incorporative of the *a priori* geometry of Descartes and Leibniz) while also eschewing a priori deductions from first principles. Brunschvicg admits that this distinction between the deductive and the empirical is far too cumbersome to capture the conceptual development of mathematical physics in the seventeenth century. It does however allow him to extract the philosophical problem which is at the core of Kant's critical project. How can the "norm of truth which mathematics furnishes man" be reconciled with "concrete reality" in the domain of lived experience?

If the mathematical physics of the seventeenth century is the historical occasion of this problem, it does not exhaust its implications. Indeed, the doctrine of the antinomies is Kant's effort to establish not just the epistemological ground of science, but the boundaries of all possible forms of knowledge. Brunschvicg will reconstruct Kant's presentation of the nature of mathematical knowledge in order to demonstrate how Kant makes the objectivity of geometry depend on the subjective experience of space and time. Kant's singular contribution to the history of the mind is the seemingly paradoxical discovery that the structure of objectivity depends on the nature of the *a priori* forms of intuition.

A germinal form of the doctrine of the antinomies is present as early as 1756. In the *Metaphysicae cum geometria junctae usus in philosophia naturali, cujus specimen primum contient Monadologiam physicam* Kant writes: "It is easier to harness griffons to horses than it is to bring transcendental philosophy and geometry together."[183] Transcendental philosophy here refers primarily to the presentation of pure mathematics as a strictly analytic science. The refutation of this philosophy of mathematics will become the foundation of the *Critique of Pure Reason.* In 1756 Kant attempts what Brunschvicg calls a "dogmatic synthesis" of the Newtonian and Leibnizean systems of physics by reformulating "Newtonian conceptions in Leibnizean language."[184] This synthesis is ultimately unsuccessful, but it does lead to an important realization when Kant recognizes that the conflict between Newton and Leibniz "is not only an empirical disagreement between results obtained by different methods" but in fact concerns "two types of truth." One type of truth is proper to mathematics, the other is proper to physics.

As Brunschvicg writes: "Following Descartes and following Leibniz physics is an extension of mathematics."[185] Cartesian science is premised on the strictest possible distinction between sensibility and imagination and the truths of analysis. Descartes reformulates the ideal of truth in accord with this strict analytic procedure and makes the universe itself "the object of a speculative geometry." If anything, Leibniz surpasses Descartes by enriching the abstract resources of analysis with the invention of the calculus:

> Leibniz, who thought he had found in the analysis of the infinite the means of mathematically attaining the fugitive multitude of movements of which sensible qualities are the manifestation, required the order of abstract truths

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[183] Brunschvicg 1912, 253.
[184] Brunschvicg 1912, 254.
[185] Brunschvicg 1912,. 254.

> to furnish a principle of discrimination capable of distinguishing 'the well founded phenomena' from the illusions of the immediate.[186]

In Leibniz, as in Descartes, the analytic foundation of philosophy, constructed using the vocabularies of mathematics, is called upon "to justify not only the accord of the intelligible and the sensible, but further, the existence of the sensible." The analytic incontrovertibility of pure *a priori* mathematics secures epistemological authority and ontological certainty. The order of certain knowledge subtends the structure of being.

Kant discovers the basis of the doctrine of the antinomies when he realizes that Newton constructs his physics based on an entirely different understanding of the relationship between mathematics and experience. Newton's philosophy is described as "experimental" precisely because it is "experience which appears to found and to justify the formulas of mathematics."[187] In fact, experience is capable of discovering and elaborating "types of relations" which pure reason is incapable of generating on its own. For Kant, it is Hume who first realizes the full implications of the experimental method in mathematical physics for the philosophies of dogmatic rationalism. If the necessity and universality of reason are not intrinsic to thought but are the results of experience, then there is no a priori connection between the order of reasons and the order of things, much less a foundational primacy of the rational over the sensible. In fact, what is called for is a *relativizing* of reason, for all that it is possible to affirm is a certain association between ideas which are no more

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[186] Brunschvicg 1912, 254.
[187] Brunschvicg 1912, 255.

than states of consciousness. The principle of the association of ideas replaces the order of reasons. Empiricism makes experience an absolute which is itself beyond transcendental legitimation.

A strange consequence proceeds from this, as Kant notes, for what Newtonian science introduces into the structure of knowledge is the germ of a radical skepticism regarding the necessity of knowledge: "the extension of Newtonian science to the domain of the mind casts into doubt the capacity of man to establish science."[188] Kant retains from Hume's skeptical undermining of the analytic foundations of science the necessity of a distinction between the *truths of reason* and the *truths of experience.* Reason and experience now stand opposed to one another in such a way that the very possibility of science seems imperiled. How can the rules of apodictic reason apply themselves to sense experience if the connection between reason and experience is no longer internally derived from reason itself? Brunschvicg claims that once Kant defines the problem in this way, he then turns to the theory of mathematical knowledge in order to formulate a new relation between reason and experience.

The role of mathematical knowledge in the evolution of Kant's thought is thus of primary significance for Brunschvicg, especially between 1770 (the year Kant publishes his Dissertation, *de mundi sensibilis atque intelligibilis forma et principiis*) and 1781, when Kant publishes the first edition of *Critique of Pure Reason.* Kant's genius, Brunschvicg claims, is to be found in the procedure by which he

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[188] Brunschvicg 1912, 256.

derives a new foundation for the sciences of nature. Kant discovers this foundation "as the corollary of the solution to an analogous problem," namely, the problem of the relation of mathematics to experience. Kant reformulates this relation as *internal* to mathematics itself. Descartes and Leibniz both pushed the boundaries of mathematical technique in order to further advance the claim that the truths of mathematics were entirely self sufficient. The autonomy of mathematics would thus be capable of founding a science which could grasp the intelligible structure of the universe. Kant, by contrast, seems indifferent to the subtleties of analytic geometry and calculus. He focuses his attention on the most elemental forms of mathematical reason, arithmetic and geometry. Kant has no need to look beyond the addition of whole numbers and the "first propositions of Euclid" in order to secure all that he requires for his novel theory of knowledge.

In this regard, both Kant and Aristotle perform strikingly similar reformulations of the mathematical philosophies which preceded their own work. For Brunschvicg, both philosophers understand this task as consisting above all in delimiting the proper scope of mathematics relative to experience: "For him [Kant] arithmetic and geometry have the same character of perfection which he recognizes in the logic of Aristotle."[189] The perfection of the logical syllogism is the confirmation of the logical order by the experiential order. As Brunschvicg had previously shown, Aristotle's logic is also a response to the failure of Plato to properly delimit the possible applications of mathematics. Kant inherits a similar

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[189] Brunschvicg 1912, 257.

speculative excess from Descartes and Leibniz, and similarly wonders how best to circumscribe the problem of mathematical knowledge. Brunschvicg thus wonders if Kant's treatment of mathematical knowledge is not already situated in an Aristotelian framework according to which there can be no science of pure mathematics, but only a quantitative natural science of *numbered things*. Brunschvicg needs to determine at the outset of his investigation if Kant, at any point in his career, was ever prepared to admit the possibility of a strictly analytic mathematical science, or if, on the contrary:

> the idea of mathematics in Kant has been subjected to a kind of unconscious slippage [un sorte de glissement inconscient] which has resulted in the demonstrations of arithmetic and geometry being brought to bear directly on numbered things or on the outlines of figures. Later in his career, undoubtedly, Kant believes that he makes the passage from 'pure mathematics' to physics; but the question is whether he does not begin by substituting for the notion of pure mathematics a conception of *applied arithmetic* and of *applied geometry*, in such a fashion that the passage from arithmetic or geometry to physics will in fact be the passage from a simple form to a more complex form of applied mathematics.[190]

Brunschvicg isolates Kant's *Attempt to Introduce the Concept of Negative Magnitudes into Philosophy* (1763) as particularly instructive in this regard. In this text Kant seems to deny to negative magnitudes anything but a strictly representative value. Insofar as negative magnitudes "intervene in calculation in order to modify the result" it is only inasmuch as "they have an efficacy of opposition, they exert a positive action, just as a screen is a positive obstacle to the transmission of light."[191] Brunschvicg writes that Kant here transposes problems of the logical order into

[190] Brunschvicg 1912, 257-258.
[191] Brunschvicg 1912, 258.

problems of the physical order. The problem of causality in Leibniz underlies the entire debate, but what is immediately pressing for Brunschvicg is the systematic reordering of the problem of negative magnitudes into a representative scheme which designates transformations in a physical system. The negative magnitude, Kant seems to claim, is in effect the formal artifact of a technical procedure of calculation which must ultimately refer to a domain of synthetic positivity.

By 1763 Kant already seems to have concluded that mathematics can have no autonomous analytic foundation in an order of reasons, but must instead take its bearings from the objects and relations of experience:

> Arithmetic is no longer the science of numbers as ideal objects, it is the science of *numbered things* and it is the nature of the relations among things themselves which determines the relations among numbers. In fact, if in the dissertation of 1770… Kant presents the number as 'an intellectual concept in itself' it is only to add immediately that it cannot concretely actualize itself without the aid of the notions of time and space; a doctrine which contains in germinal form the idea of the *transcendental schematism.*[192]

Arithmetic must confine itself to the formal representation of "the science of numbered things" because Kant does not seem to allow for the possibility of a pure mathematics. What is the necessity of this structuring relation which applies equally to things and to numbers? Brunschvicg points to the "aid of the notions of time and space," however, Kant has not yet elucidated the nature of our spontaneous representations, so the problem of the relationship between mathematics and experience cannot yet be formulated in terms of a fundamental theory of the nature of experience.

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[192] Brunschvicg 1912, 259.

Kant's definitive response to this problem is given in the *Transcendental Aesthetic* and the *Transcendental Analytic* of the first critique. However, in order to draw attention to the singularity of Kant's treatment of mathematics, a brief examination of the *Metaphysical Foundations of the Natural Sciences* (1786) is necessary. As mentioned previously, Kant is relatively uninterested in the mathematical *avant garde* of his age, since, as will be shown, the critical philosophy has all the mathematical resources it requires in the elementary forms of mathematics and geometry. Nevertheless, Kant does briefly turn his attention to the calculus in order to distinguish mathematical knowledge from the metaphysical requirements of the natural sciences:

> To be sure, mathematics in its internal use can be entirely indifferent with regard to the chicanery of a misguided metaphysics, and can persist in the secure possession of its evident claims as to the *infinite divisibility of space,* whatever objections may be put in its way by a sophistry splitting hairs on mere concepts. However, in the application of its propositions governing space to the substance that fills it, mathematics must nonetheless accede to an examination in accordance with mere concepts, and thus to metaphysics…For it does not necessarily follow that matter is physically divisible to infinity, even if it is so from a mathematical point of view, even if every part of space is a space in turn, and thus always contains [more] parts external to one another.[193]

The mathematician is free to pursue the "internal" complications of the mathematical idea of infinite divisibility but, contra Leibniz, the calculus is not the general structure making possible a science of the infinite as the subtle and penetrating universal logic of the cosmos. The solidarity of analysis and *mathesis universalis* is broken apart by Kant, who assigns to metaphysics and the *a priori*

[193] Immanuel Kant, *Metaphysical Foundations of the Natural Sciences* Trans. Michael Friedman (Cambridge: Cambridge University Press, 2004), 42-43.

intuitions of space and time the role of determining the categories of the understanding. The mathematician might come up with an elegant proof of the infinite divisibility of an abstract space but this principle of infinite divisibility will never be able to establish the intelligibility of the natural sciences. Infinitesimal analysis defies the nature of physical space, which does not admit of infinite division and which therefore imposes an obstacle between the logical propositions of calculus and the causal propositions of physics. Kant does supply a metaphysical refutation of the infinite divisibility of space based on the impossibility of matter being composed of an infinity of parts, but this line of argument is already supplementary to a more fundamental characterization of the nature of space. It is ultimately Kant's understanding of space that leads him to set aside the proliferating mathematics of the infinite in favor of the intuitive and synthetic geometry of the ancients.

The indispensable "auxiliary of mechanics" is geometry and not calculus in Kant's revision of the contours of science. Newton's *Mathematical Principles of Natural Philosophy* furnishes the best example of the relationship between mathematics and natural philosophy in Kant's estimation because, in the words of Gaston Milhaud (1858-1918), in Newton's philosophy "mathematics… proceeds by the representation of concrete intuition and not according to the abstractions of analysis."[194] Space is not a purely geometrical ideal which can be grasped according to the intrinsic principles of logic. It is geometry which must be conceived according

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[194] Brunschvicg 1912, 260.

to the nature of space. In 1768, two years before the inaugural dissertation, Kant publishes a brief essay on the differentiation of the parts of space, “Concerning the Ultimate Ground of the Differentiation of Directions in Space,” which Brunschvicg claims represents a theory of mathematical knowledge which is already compatible with the critical philosophy. In a brief analysis of the essay, Brunschvicg writes:

> The intrinsic nature of space as it is manifest in the geometry of three dimension does not allow itself to be transposed into a system of purely intellectual relations…there is in space something inherent to the universe as *given,* which appears connected to the parts of our body and which attests, in the final analysis, to the absolute and original relation to space.[195]

Kant has now defined space relative to an experience which cannot be captured by “purely intellectual relations.” Space is therefore an absolute insofar as it is “irreducible to an ideal system,” apparently inverting Descartes’ effort to define a purely analytic conception of numerical magnitudes which would in turn be the absolutes of analysis. The science purporting to describe spatial relationships is therefore also irreducible to an ideal system, referring as it does to “an original relation to space,” rather than to an anterior science of order. The philosophical examination of arithmetic and geometry must now serve as “references for the establishment of a theory of scientific knowledge” rather than as possible foundations of science themselves. Arithmetic and Geometry are somehow inextricable from the nature of experience rather than pointing beyond experience to a realm of pure intelligibility.

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[195] Brunschvicg 1912, 262.

The brilliance of the *Critique,* according to Brunschvicg, is that while mathematics and geometry are not allowed to become *a priori* sciences in their own right, the experience of space which seems to underlie them is also barred from becoming a transcendent entity. Space, in itself, cannot be taken as an object of experience because it is what structures experience. It is this bracketing of space as that which structures the intelligibility of experience which leads Kant to formulate the doctrine of the pure intuition of space. Accordingly, if space is an intuition, it cannot be a concept, as Brunschvicg elaborates:

> …nothing is more repugnant to the nature of space than the proper function of the *concept* in the sense with which Kant employs the word, and which he specifies by the epithets *discursive* and *universal*. The determinations of space are its parts and not its specifications, there is but one space [il n'y a donc qu'un espace] in which is comprised the totality of things.[196]

We are accustomed to the "traditional antithesis of the empirical given and the abstract concept," but the a priori intuition of space does not conform to either of these possibilities, as Kant will make clear in the dissertation and later in the first critique.

Although Kant admires Newton's circumspection where mathematical knowledge is concerned, he repudiates the transcendental realism of space and time which accompanies Newton's empiricism. Newton is not alone in making space and time absolute conditions for any possible existence whatsoever. Berkeley, whose idealism would seem to undermine the Newtonian system, also treats space and time "as the necessary condition for the existence of all things," as Kant writes:

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[196] Brunschvicg 1912, 262.

> For if we regard space and time as properties which, if they are to be possible at all, must be found in things themselves, and if we reflect on the absurdities in which we are then involved, in that two infinite things, which are not substances, nor anything actually inhering in substances, must yet have existence, nay, must be the necessary condition of the existence of all things, and moreover, must continue to exist, even though all existing things be removed, - we cannot blame the good Berkeley for degrading bodies to mere illusion. Nay, even our own existence, in being made thus dependent upon the self-subsistent reality of a non-entity, such as time, would necessarily be changed with it into sheer illusion – an absurdity of which no one has yet been guilty.[197]

The hypostatization of space and time as real entities produces a series of metaphysical absurdities including "two infinite things, which are not substances" which, even if the rest of the universe were devoid of content, would have to remain. It is this uncomfortable remainder of space and time as entities in their own right which Kant cannot accept, and which he subjects to criticism in both the empirical and idealist formations of the doctrine of the independent existence of time and space.

Space cannot be a purely empirical concept because it is not something encountered as a totality in experience. At the same time it cannot be a transcendental entity because it does not exist outside of experience. It is instead the indispensable element of what Kant calls *pure a priori intuition.* Intuition is thus called upon to become the condition of possibility of experience, the determining forms of which are space and time. The *Transcendental Aesthetic* forges a new relationship, internal to experience, between the forms of intelligibility and the contents of experience. Brunschvicg thus finds that Kant has managed to establish,

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[197] Henry E Allison, *Kant's Transcendental Idealism. An Interpretation and Defense* (New Haven: Yale University Press, 1983), 12.

within the phenomenal order, an "a priori activity" furnished by intuition capable of determining the unity of reason. Although Brunschvicg does not provide a detailed summary of the *Transcendental Aesthetic,* two elements from Kant's presentation of the pure *a priori* intuition of space seem especially significant for Brunschvicg's purposes:

> … Space is a necessary representation, *a priori*, which is the ground of all outer intuitions. One can never represent that there is no space, although one can very well think that there are no objects to be encountered in it. It is therefore to be regarded as the condition of the possibility of appearances, not as a determination dependent on them, and is an *a priori* representation that necessarily grounds outer appearances.
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> …The apodictic certainty of all geometrical principles and the possibility of their *a priori* construction are grounded in this *a priori* necessity. For if this representation of space were a concept required *a posteriori,* which was drawn out of general outer experience, the first principles of mathematical determination would be nothing but perceptions. They would therefore have all the contingency of perception, and it would not even be necessary that only one straight line lie between two points, but experience would merely always teach that. What is borrowed from experience always has only comparative universality, namely through induction. One would therefore only be able to say that as far as has been observed to date, no space has been found that has more than three dimensions.[198]

Kant attempts to demonstrate that the forms of *a priori* intuition are the conditions of possibility of experience. We do not learn about space from the empirical surveillance of our experience, rather, the *a priori* sensible intuition of space is the ground of all appearances. Geometry, therefore, cannot be the empirical discourse of observations, its claim to apodictic certainty is in turn established on the foundations of *a priori* intuition. Newton's absolute space and time no less than

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[198] Immanuel Kant, *Critique of Pure Reason* Trans. Paul Guyer and Allen W. Wood (Cambridge: Cambridge University Press, 1998), 158.

Leibniz's integral analysis cannot be understood as referring to entities which exist in their own right. Kant can thus claim that:

> We therefore assert the empirical reality of space (with respect to all possible outer experience), though to be sure at the same time its transcendental ideality, i.e, that it is nothing as soon as we leave out the condition of the possibility of all experience, and take it as something that grounds things in themselves.[199]

Transcendental idealism and empirical realism are reconciled by the forms of pure a priori intuition which define the forms of possible experience rather than the pure analytic of all possible knowledge or the transcendental realism of things in themselves. Brunschvicg claims that Kant's accomplishment with the *Transcendental Aesthetic* is to have given to phenomenal experience all the necessary resources to answer the fundamental epistemological question of the first critique: how are a priori synthetic judgments (e.g. the postulates of Euclidean geometry) possible? Kant must short circuit the skepticism of the empiricists and the dogmatism of the rationalists in order to found the necessity and universality of science, not against or beyond experience, but in accordance with the *a priori* possible forms of experience.

Kant has shown that space is not a transcendental reality or an empirical postulate but that it is a condition of possibility of all experience. The same must be said of time:

> …Time is a necessary representation that grounds all intuitions. In regard to appearances in general once cannot remove time, though one can very well take the appearances away from time. Time is therefore given *a priori.* In it alone is all actuality of appearances possible. The latter could all

[199] Kant 1998, 160.

> disappear, but time itself (as the universal condition of their possibility) cannot be removed.
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> …This *a priori* necessity also grounds the possibility of apodictic principles of relations of time, or axioms of time in general. It has only one dimension: different times are not simultaneous, but successive (just as different spaces are not successive, but simultaneous.) These principles could not be drawn from experience, for this would yield neither strict universality no apodictic certainty. We would only be able to say: This is what common perception teaches, but not: This is how matters must stand. These principles are valid as rules under which alone experiences are possible at all, and instruct us prior to them, not through it.[200]

The apodictic certainty of temporal synthesis governs all possible forms of experience. The a priori intuitions of space and time ensure that we experience the simultaneity of spatial extension and the succession of temporal duration. Time cannot be grasped as an empirical concept or as a transcendental entity. Although it organizes empirical experience it is not given by concepts drawn exclusively from experience. Time is not empirical but nor is it transcendental. It cannot exist separately from our sensible intuitions insofar as "no object can ever be given to us in experience that would not belong under the condition of time." Time is not, however, an autonomous reality but rather a transcendental ideality

> according to which it is nothing at all if one abstracts from the subjective conditions of sensible intuition, and cannot be counted as either subsisting or inhering in the objects in themselves.[201]

Time and space having been established as pure *a priori* forms of intuition, the question of mathematical knowledge and the foundation of the principles of natural science become indexed to the a priori conditions of possibility of experience. Here Kant breaks with all "ordinary psychology" which would refer to

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[200] Kant 1998, 179.
[201] Kant 1998, 181-182.

"the observation of individual consciousnesses" and instead determines the outline of all possible rational science by a reflection on the determinate features of experience. The universality and necessity of science is indexed not to the transcendental order of an analytic chain of reasons but to the very forms of sensible intuition. The diversities of sense experience and the unity of reason can be reconciled if one asks not *what is science*, or *what is reason*, but rather, *what must experience be like in order for there to be science?* Kant's real question then, Brunschvicg maintains, is how does objectivity arise within the subjective conditions of experience?

> The *subjectivity* of *space* is capable of founding the *objectivity* of *geometry*. By this means Kant is capable of resolving the problem he will deal with in definitive form in the *transcendental aesthetic: how are a priori synthetic judgments possible in mathematics?* Ten years later, it appears that the solution to this problem had prepared the solution for a problem posed in analogous terms: *how are a priori synthetic judgments possible in rational physics?*[202]

In the first critique, the principles of natural science are shown to be the a priori synthetic judgments of mathematics, which, by definition, find their apodictic certainty transcendentally synthesized by intuition. The basic elements of mathematical knowledge are not self contained analytic truths or empirical concepts. The notion of number itself is not a concept but rather, in the Kantian system, a *scheme*, which mediates between the categories of the understanding and the productive imagination: "The notion of number is not properly speaking a

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[202] Brunschvicg 1922, 267.

concept; it is a 'monogram of the pure *a priori* imagination,' a *scheme."*[203]

Relative to the problem of mathematical knowledge it is necessary to distinguish between "the synthesis of apprehension in intuition" (by which the sensible is given) and "the synthesis of recognition in the concept" (by which knowledge is given). As Kant writes:

> It must first be remarked that properly mathematical propositions are always *a priori* judgments and are never empirical, because they carry necessity with them, which cannot be derived from experience. But if one does not want to concede this, well then, I will restrict my proposition to pure mathematics, the concept of which already implies that it does not contain the empirical but merely pure *a priori* cognition…The concept of twelve is by no means already thought merely by thinking of that unification of seven and five, and no matter how long I analyze my concept of such a possible sum I will still not find twelve in it. One must go beyond these concepts, seeking assistance in the intuition that corresponds to one of the two…That 7 **should** be added to 5 I have, to be sure, thought in the concept of a sum = 7+5, but not that this sum is equal to the number 12. The arithmetical proposition is therefore always synthetic; one becomes all the more distinctly aware of that if one takes somewhat larger numbers, for it is then clear that, twist and turn our concepts as we will, without getting help from intuition we could never find the sum by means of the mere analysis of our concepts.[204]

Any act of thought must presuppose the synthetic unity of apperception "thanks to which the terms of judgment are referred to the same consciousness and form part of a unique act of affirmation."[205] The relation between the terms of mathematical judgment, in this case 5 and 7, cannot be found in the terms themselves, that is to say, "in the mere analysis of our concepts," but must be referred to intuition, for the term 12 is not an empirical concept naturally contained in 5 and 7 but a synthetic

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[203] Brunschvicg 1922, 268.
[204] Kant 1998, 144.
[205] Brunschvicg 1912, 266.

judgment which implies a category of the understanding, namely, the concept of *quantity.* Kant must invoke the "pure productive power of the imagination" between intuition and understanding because the synthesis of 5 and 7 occurs in the imagination. Kant calls this the synthesis of reproduction in the imagination. The imagination is thus the mediating stage between *apprehension* in intuition and *recognition* in understanding:

> Now if we can demonstrate that even our purest *a priori* intuitions provide no cognition except insofar as they contain the sort of combination of the manifold that makes possible a thorough going synthesis of reproduction, then this synthesis of the imagination would be grounded even prior to all experience in *a priori* principles, and one must assume a pure transcendental synthesis of this power, which grounds even the possibility of all experience (as that which the reproducibility of the appearances necessarily presupposes). Now it is obvious that if I draw a line in thought, or think of the time from one noon to the next, or even want to represent a certain number to myself I must necessarily first grasp one of these manifold representations after another in my thoughts. But if I were always to lose the preceding representations (the first part of the line, the preceding parts of time, or the successively represented units) from my thoughts and not reproduce them when I proceed to the following ones, then no whole representation and none of the previously mentioned thoughts, not even the purest and most fundamental representations of space and time, could ever arise.[206]

Brunschvicg is particularly fascinated by the transcendental operations of the mind in Kant (the synthesis of apprehension in intuition, the synthesis of reproduction in imagination, and the synthesis of recognition in understanding) because they betray "a veritable interruption in the reflexive course of critique."[207] Kant must show that the synthetic unity of the concept of quantity arises not in empirical experience but as a result of the transcendental operations of the mind

[206] Kant 1998, 230.
[207] Brunschvicg 1922, 266.

and in accord with the a priori structure of the representations of experience. This is not, however, an elucidation of the nature of mathematical intelligence for Brunschvicg. It is instead evidence that the schematism now truly rules Kant's thinking and has completely subjected mathematical intelligence to a predetermined form of experience. The first critique might just as well be called the *schematism of mathematical reason* as far as Brunschvicg is concerned, for once *a priori* synthesis becomes the ground of all mathematical judgment (and by extension the principles of natural science) the autonomy of mathematical intelligence is not only compromised but treated as a transcendental illusion of reason.

Nowhere is this more clearly on display than in Kant's theory of numbers:

> The pure image of all magnitudes (*quantorum*) for outer sense is space, for all objects of the sense in general, it is time. The pure schema of magnitude (*quantitatis*), however, as a concept of the understanding, is number, which is a representation that summarizes the successive addition of one (homogenous) unit to another. Thus number is nothing other than the unity of the synthesis of the manifold of a homogenous intuition in general, because I generate time itself in the apprehension of an intuition.[208]

Numerical magnitudes are synthesized transcendentally according to the determining features of time as an *a priori* intuition, that is to say, according to the principle of a homogenous succession which finds unity in the concept of *quantity.* The number is a pure a priori scheme or "monogram" of the productive imagination: it is the unity of temporal synthesis. Mathematics is thus entirely obedient to the a priori intuition of time, which is itself produced "in the apprehension of intuition." A

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[208] Kant 1998, 274.

priori synthesis is thus not only responsible for the synthesis of mathematical judgment but for the creation of numbers themselves as *representations of the concept of quantity*:

> The place of *a priori* synthesis is not in the bond between terms of judgment or in the demonstration of any particular 'numerical formula;' it is to be found in the general process which derives all particular numbers, in the creation of the notions themselves.[209]

Kant's effort to ground mathematics not in the order of analysis but in the order of synthesis first seemed to be a refutation of a dogmatic rationalism which was unable to confirm its own epistemological foundations. By the time Kant has fully developed the doctrine of a priori intuition, the synthetic unity of mathematical judgments does not refer to the *a priori synthesis of mathematical operations* but to the *constitution of number itself out of the temporal synthesis of intuitive apprehension.*

Kant's dismantling of the Cartesian analytic order of reasons finds itself profoundly confirmed in the synthetic nature of number itself. The relationship between algebra and geometry at the heart of Descartes' distinction between pure mathematics and universal mathematics is also transformed. The analytic purification of algebra cannot be the foundation of geometry because space and time are now constitutive of arithmetic and geometry, respectively, and do not entertain any possible translation into one another as a priori forms of intuition. Kant is very clear on this point: space is the simultaneity of homogenous extension, time is the succession of heterogeneous instants. Any translation between the two orders

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[209] Brunschvicg 1912, 270.

would confound the unity of transcendental apperception and violate the forms of possible experience. It is also necessary to affirm that arithmetic and geometry cannot be the respective sciences of time and space in Kant because "[t]ime is not an *object*; it is a condition of arithmetic, or more exactly of mathematics in general."[210]

Kant's critical philosophy thus assumes a profound ambivalence for Brunschvicg . On the one hand, the deadlock of empiricism and rationalism had been shown to be based on a misunderstanding of both reason and experience, the double critique of which Brunschvicg completely endorses. On the other hand, the creative impulse of human intelligence is subjected to a profound reification by the a priori synthesis of intuition and the categories of the understanding, which together constitute an invariable table of the possible forms of knowledge. Brunschvicg cannot accept this conclusions because he finds it refuted by the history of science and mathematics. Kant's own place in this history is equally ambivalent. Like Descartes, he recognizes that intuition and judgment are the life of the mind. Unfortunately, the critical philosophy which set out to dismantle the dogmatic unity of the Self, the World, and God, rediscovers an even more authoritative transcendence in the predetermination of the unity of science based on the possible forms of experience. As Gilles Deleuze (1925-1995) has said, if the transcendental is made in the image of the possible, then we can only hope to rediscover what is already present in experience.[211] The ideal and real transcendental conditions of

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[210] Brunschvicg 1912, 269.
[211] See Gilles Deleuze, *Kant's Critical Philosophy* Trans. Hugh Tomlinson and Barbara Habberjam (Minneapolis: University of Minnesota Press, 2003), 11.

experience have escaped our criticism. Brunschvicg leverages exactly this critique *against* Kantian critique, but not in the name of the transcendental: it is the history of science itself which militates against the uniformity of the intellect as determined by the schematism of the faculties.

Kant discovers something profound about the nature of reason when he realizes that the mind must give to itself the epistemological criteria capable of legitimizing the necessity and universality of science. Brunschvicg appropriates this element of the critical philosophy as the very core of his own normative epistemology. Critical idealism, as Brunschvicg understands it, is a philosophy of judgment irreducible to rationalism or empiricism. It is premised on the affirmation that reason itself is an experience and can have no predetermined condition of possibility or teleological outcome. The mind advances by the progressive verification of the bonds it establishes between the objective contours of being and the rational sequence of ideas. Rationality and objectivity, however, are both systems of judgment, as Brunschvicg attempted to demonstrate in *La modalité du jugement.* The history of science no less than the history of philosophy is always an uneven mixture of objectivity and rationality, which, although they pre-suppose one another, enjoy no guarantee of the free and creative construction of intelligible norms. For Brunschvicg, the construction of the intelligible norm, astride rationalism and objectivity, is the destiny of thought fulfilling its own nature.

That Kant should be the first to formulate the necessity of reflection on reason as itself constituting a domain of experience is to his enduring credit. There is no critical philosophy of the intelligence which is not Kantian in this respect. At the same time, the architectonic of the critical philosophy which Kant constructs on the basis of this discovery makes the forms of pure a priori intuition false idols of reason. Neither experience nor knowledge come to us in predetermined forms. The task of the intelligence and the destiny of the human in Brunschvicg's philosophy is to constantly produce new forms of truth and new *experimental judgments determining the internal bonds of objectivity and rationality*. The exigency of reason is not to rediscover the invariant form of truth but to postulate new truths. It is progress alone which is necessary. Because of this there can be no theory of truth which would not also be a history of truth, a history of reason, and the inevitable *relativization of all transcendental claims to the contrary.*

Einstein's relativistic physics is therefore a kind of apex of intelligence in Brunschvicg's philosophy. It is a revolution of intelligence itself because it redraws the relationship between objectivity and rationality. It collapses the last prejudice of transcendental metaphysics: that the mind can distance itself from being and freely create the metric by which the universe becomes an object of inquiry. It is the mind itself which must be set in motion. The mind must be immersed in time in order to become capable of a truly *rational objectivity* and truly *objective rationalism.* This will involve a the three fold revolution of perception, experience, and reason. In the Einsteinian universe there is no stable observer, there is no pre-existing form of experience, and there can be no distinction between the acts and objects of measurement.

Einstein and The Relativism of Reason: Remaking a Metaphysics Worthy of Science

The concept of the relativity of movement is foundational to modern mathematical physics and cannot be attributed to Einstein alone. Descartes had already repudiated the notion in his own formulation of the principle of inertia, "which implies the perpetuation of uniform rectilinear movement."[212] In fact, as Brunschvicg notes, citing a well known work by Pierre Duhem (*Le Mouvement absolu et le mouvement relatif* [1909]), the problem of the epistemological status of absolute and relative movement has been central to physics at least since Pierre Simon Laplace (1749-1827) fully integrated calculus into celestial mechanics. On both sides of the Newtonian revolution the fate of the relativity of motion seems to be clearly established. The Cartesians dismiss the possibility because it calls into question the absolute metric of space and time which they attempt to formalize according to the principles of the mechanical theory of motion. After Newton, by a neat inversion of the principle of absolute space and time (thanks largely to Kant), the relativity of motion becomes a common place in the probabilistic calculations of nineteenth century statistics and thermodynamics. However, both sides of this apparently decisive transformation in the working methods of physics conceal a more essential bias which has remained largely unexamined. Brunschvicg turns to Laplace, the founder of probabilistic physics and one of the most sophisticated mathematical minds of his generation, in order to diagnose the status of the relativity of motion before Einstein:

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[212] Brunschvicg 1922, 401.

> What could be more naïve than the fact that Laplace, in the very text seeking to illustrate relativity…introduces observers for whom the universe would be a spectacle: 'The universe thus reduced to the successively smallest space imaginable always offers the same appearance to its observers?' One cannot mediate enough on this passage: it perfectly expresses what the generations previous to Einstein made of the idea of relativity. On one side there was the universe, that is to say all that fills the totality of space and all the succeeds in the totality of time; on the other side, the observers who speculate on the constitutive relations which legislate the universe. When these observers decide to make a measurement of time they are free to choose the form of the unity of time according to their convenience…in effect the choice is an *intemporal decision*…it is the work of beings who are anterior to the birth of time, capable of creating time, like the Demiurge in the Platonic myth of the *Timaeus.*[213]

The final pages of Laplace's *Exposition du système du monde* (1808) directly engage the theological underpinnings of Newton's absolute space and time. The Newtonian *Sensorium Dei* posits the absolute uniformity and emptiness of space and time as they would appear to an observer paradoxically located at some ideal vantage point transcendent to space and time themselves. From such a perspective, the totality of creation would appear like a perfectly harmonious geometrical clock-work, or so Newton claims. In the divine understanding of time and space, temporal duration and spatial extension are absolute because they are totalizing, they are the boundaries by which creation is established. They are ontological markers before they are abstract concepts of temporal or spatial uniformity. Against this understanding of time and space as the ideal coordinates of exteriority (exemplified in the final analysis by God as the totalizing synoptic observer), Laplace posits the problem of indiscernibles: the infinitesimally small divisions of time and space which the probabilistic physicist invokes in order to serve the requirements of his

[213] Brunschvicg 1912, 405.

calculations. For the Newtonian, time and space are the effects of divine contemplation: they are the coordinates of being, realities unto themselves. For Laplace time and space are infinitely divisible variables of calculation: they are the coefficients of a differential equation.

In the Newtonian system the ultimate observer is God. In the Laplacean universe of rational mechanics all that is required is a physicist with sufficient mathematical training. The relativity of movement would thus be the realization that time and space are not predetermined from the outside by a transcendental authority but local dynamic relations indexed to a particular observer. Brunschvicg, however, is not convinced that the difference between Newton and Laplace is as profound as it seems. What remains identical in both systems is the stability of the relation between the observer and the universe. The Newtonian *Sensorium Dei* does not have to be explicitly formulated to coordinate the relationship between the observer and the universe as long as the basic parity of observation and experience remains unquestioned. As Brunschvicg says, the power of the observer, be it God or the physicist, amounts to an "intemporal decision" which itself occurs in a time anterior to time. *The observer chooses the frame of reference as if his own perspective were immobile, like that of the Demiurge*. Brunschvicg thus makes Laplace and Newton the immediate contemporaries of the Platonic myth. The paradox of the immutable frame of time and space which subtends the revolutions of modern mathematical physics is that this very rigidity is not the guarantee of objectivity but is itself the result of a subjective distortion: the projection of a single observer's frame of reference onto the structure of the universe itself. Laplace does not surpass Newton. He confirms the metaphysical presupposition of the Newtonian system by fully naturalizing the position of the observer.

The relativity of motion must also be the relativization of the observer. The scientific and philosophical significance of Einstein's relativity theory is to have finally provided experimental and theoretical resources capable of displacing the distorted image of an immutable relationship between the observer and the observed:

> Before M. Einstein, and by an unconscious postulate, physics posed *a priori*, and maintained for all occurrences, the immutable unity of human processes of measurement… granting to nature the absolutism of movement required to justify each definitive act of measurement. With Einstein, physics turns its attention to the way in which we make our measurements and it asks if we must not also take into account the conditions in which man finds himself, if not its imperfections then at least the diversity of means his circumstances impose on him.[214]

Brunschvicg here poses in historical terms what Kant established as a transcendental condition of the possibility of experience: the *a priori* unity of "human processes of measurement." The philosophical implications of the *a priori* presuppositions of the physicist far outstrip anything the critical philosophy is capable of entertaining, for the unity of measurement ensured by the immutability of the observer is also the absolutization of nature. Einstein's relativity, not unlike the critical philosophy of Kant, does not take the stability of appearances for granted. The relativity of Einstein is not the symmetrical division of the observer and the totality of the universe, it is the relativization of all points in space and the

[214] Brunschvicg 1922, 404.

dismantling of the "intemporal decision" which freezes the disposition of the cosmos into a stable object of measurement.

The relativity of Einstein locates the observer within the system being observed. The long tradition of a *mathesis universalis* which dedicated itself to the systematic emancipation of the sciences of order from the sciences of observation, finds itself drawn back within the orbit of space-time and irrevocably integrated into the fabric of the cosmos:

> With Einstein it is possible to say, and this time to the letter, that he makes philosophy descend from heaven to the earth. A God can choose at his discretion a planet to be his time piece, being himself independent of all movement, but man cannot do the same without being placed within his time piece, attached to his hour-glass. Our method of measurement cannot but connect us to the movement of the earth because our instruments participate in this movement. We are not transcendent observers relative to the universe, we are occupants in space living in a constant current driven and displaced by the mobile course of time.[215]

Einstein is the first to have posed a *relativized relativity* (*relativité relatée*) as against the *absolutist relativity* of the transcendent observer. The absolutist relativity of pre-Einsteinian physics is an *a priori* standardization of space and time as absolute entities, entirely distinct from the observer. This is a physics based on truly *a priori* principles. Brunschvicg sometimes calls pre-relativistic physics a "physics of principles" premised on a radical separation between the sciences of order and the sciences of measure. Physics thus conceived on purely analytic grounds is not

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[215] Brunschvicg 1922, 405.

disqualified as science: "it will not be false…but it will refuse any chance to become true."[216]

The dialectical transformation of physics which Brunschvicg thus implies is accomplished by Einstein. A relativized relativity does not construct the instruments of measurement as absolute metrics constituted "independently from that which is measured." Unlike Newtonian physics, Einsteinian relativity does not posit space, time and movement as a priori abstract realities in their own right. The metric frame ordering the relations of space and time is not given independently from space and time. As against the "physics of principles," the metric of space-time proper to Einstein's relativity is established experimentally and is always relative to a given system:

> Einstein's cosmology does not regard as fundamental either the definition of the concept, from which deduction proceeds, nor the data of experience, upon which induction rests. Science begins with an operation of measurement in which the formal and concrete occur simultaneously…in such a manner that one cannot exist without the other. The world of M. Einstein is a world of numbers; this world of numbers does not presuppose an *a priori* truth as the condition of its formal expression nor an intuitive image as the condition of its signification in physics. At the same time it must be understood that these numbers are not fictions, they are not even abstractions, they correspond to the coefficients which reality furnishes…Physics thus conceived is nothing other than *geometry* (or better *cosmometry*) following Descartes' formulation, but in a non-static sense which surpasses the Cartesian method. It no longer refers to a representation of space which is already established, already given. Space is not anterior to measurement, it is born with the act of measurement which constructs it piece by piece, following the procedures of Gauss and Riemann, which is to say that the space of the physicist is not the *a priori* representation presupposed by Descartes…it is the texture of reality, realized step by step…in relation with experimental coefficients. The general theory of relativity substitutes for the Newtonian law of gravity (which remains true as

[216] Brunschvicg 1922, 406.

> a first approximation) the determination of a gravitational field which presents the characteristics of a *non Euclidean continuum.*[217]

Neither the concept nor the data of experience are primary in Einstein's cosmology. This is the true significance of the relativization of relativity: the relationship between reason and experience is no longer that of a forced choice between the analytic primacy of pure deduction divorced from experience or the experimental empiricism of observation devoid of apodictic rigor. The history of science might seem to be constrained by this very antinomy, but this is to misunderstand the nature of science itself, which achieves a plateau of self understanding with Einstein insofar as it escapes the dead lock of an empty formalism, on the one hand, and an unintelligible materiality, on the other.

As with all apparent philosophical dualisms (according to Brunschvicg's philosophy of judgment) the truth must lie in the simultaneity of the opposition. The measurements of the relativistic physicist are simultaneously formal and concrete, theoretical and experimental. Einstein's cosmos is not an impenetrable thickness of qualities against which the abstraction of mathematics represents a departure from the real in the service of a schematic profile of intelligibility. On the contrary, any such presupposition of a domain of experience which subtends the act of measurement would fall prey again to the naturalist fallacy which establishes the determinate stasis of the observer and his "intemporal decision" to contemplate the totality of the observable universe. Such an observation will always be a distorted image of reality, not entirely false, but not yet true until the system is placed into

[217] Brunschvicg 1922, 429-430.

motion in real time. Physics must proceed at least at the speed of the cosmos itself, which requires it to furnish minimum and maximum constants. Einstein's cosmos is a mathematical fabric furnished by "experimental coefficients." What is most important, according to Brunschvicg, is that this mathematical fabric itself is not an *a priori* presupposition of an ideally empty and homogenous space but a patchy, piece-meal, Riemannian geometry of local spaces integrated into a non Euclidean continuum. The force which supplies the integrating unity of this continuum is gravity, precisely what the Newtonian universe had to admit as a law but without being able to offer an explanation for what appeared to be action at a distance. Einstein unifies space and time by making gravity the curvature of space-time. The Newtonian universe finds itself coherently absorbed by Einstein's general theory of relativity, which also adds cosmological depth and subatomic texture to Newton's cosmology.

Relativity after Einstein does not permit the epistemic distinction between a pre-analytic domain of experience, on the one hand, and a pure order of reasons, on the other. Space, time, and movement, considered as realities unto themselves, must be set aside as the dogmatic presuppositions of the Newtonian system. At the same time, Relativity also forbids Kant's solution to the relationship between reason and experience, which models all possible forms of experience on pre-determined *a priori* intuitions of space and time. Neither Kant nor Newton escape the fallacy of the unitary observer capable of contemplating the universe according to the ready made categories of the understanding. Objectivity after Einstein does not suffer any loss of certainty, it dispels the illusions which cloud its actual relations to the observable universe. The Kantian system of the sciences is above all a pre-determination of the possible forms of knowledge: “a pre-established harmony between the forms of knowledge and the content of experience.”[218] Relativity, however, does not discover a cosmological schematism which is in conformity with the representations of our concepts. Our intuition does not allow us to grasp the curvature of space-time in the immediacy of an apprehension. Neither apprehension in intuition, reproduction in imagination, or recognition in the concept, are capable of describing the experience of reason proper to Einstein’s relativity. On the contrary, the *experimental dismantling of the Newtonian absolutes of space and time must also result in the dismantling of the Kantian schematism*:

> Kant was right and not Comte: the theory of the science of the universe is inseparable from speculation on the relation which is established internal to science between the intellectual functions of man and the objective reactions of nature. Only this speculation can no longer have as its basis the perfection attributed to mechanics: a system in stable equilibrium with definitively fixed boundaries which one can obtain by a sort of instantaneous cut and which is supposed to crystallize for all time the becoming of human thought. It is to this form of knowledge that Kant claimed to grant a metaphysical foundation, which, confident in the indestructible solidity of the consequences which it claimed to legitimate, then presented itself as science…The idea of this knowledge appears to us today as an illusion…The metaphysics which physics actually implies renounces the pretention of being anterior to science: this is not because of a forced humility or provisional resignation, it is because in reality it is contradictory to seek, by reflection on science, the antecedent conditions capable of delimiting, *a priori*, all past and future knowledge within a static schema.[219]

[218] Brunschvicg 1922, 553.
[219] Brunschvicg 1922, 557.

Kant is right insofar as he insists that reason itself is an experience and must be submitted to critique. The theory of science will necessarily also be a theory of experience, and the structure of science will in turn express commensurate structures of experience. Kant is, however, incorrect when he makes the point of convergence between epistemology and phenomenology the a priori intuitions of space and time. Intuition in Kant is the transcendental justification of the pre-relativistic bias of a stable and unitary observer for whom time and space are necessary and invariant features of experience, if not of things in themselves. After Einstein it is necessary to reverse the relationship between metaphysics and physics: philosophy cannot determine in advance the nature of scientific understanding, it is physics which obliges us to modify our metaphysics. Brunschvicg's critique of the critical philosophy is quite concise: the examination of science cannot yield *a priori* conditions of knowledge because science is not dependent on *a priori* conditions of possibility. What post relativistic science reveals is not the ready made intuitions of space and time, but an increasingly complex mathematical topology which can have no correlative intuition in quotidian experience.

Einstein occupies a point of strategic significance in Brunschvicg's work because the relativistic physics which achieves its mature form in the general theory of relativity obliges philosophy to change its methods when attempting to reflect on science. Brunschvicg's argument is at once historical and philosophical. It is undeniably an historical transformation which has occurred in the conceptual development of physics which now obliges the philosopher to forge new tools of epistemological criticism. It is also, however, a philosophical claim, for what is at stake is the epistemological status of the historical progression of science itself, which had previously entertained a developmental simultaneity with philosophy, but which now no longer operates on the same plane, according to the same methods, or in response to the same problems. This obliges Brunschvicg to divide the Kantian critical philosophy in two, retaining the element which makes science and human experience mutually reflective of one another, but rejecting the dogmatic element which attempts to legitimate science according to a predetermined formatting of experience:

> Reflection must be born out of science itself, illuminated by its light…following the terminology we have already alluded to, the metaphysics of science is *reflection* on science and not the *determination* of science… In other words, in place of the deduction of principles after the fashion of the *Transcendental Logic* we propose as the foundation and culmination of our work an experimental *critique of judgment.*[220]

Philosophy is no longer in a position to determine from the outset the necessary structure and order of scientific knowledge. The relationship between philosophy and science is historically variable, but it is only beginning in the nineteenth century that Brunschvicg claims a difference "of more than degree" between the two disciplinary hierarchies. The dogmatic element of philosophy seeks to dictate the nature and necessity of scientific knowledge from the outside, by predetermined methods and according to pre-existing metaphysical systems. The inability of contemporary philosophical systems to integrate the *actual intellectual*

[220] Brunschvicg 1922, 557.

procedures of relativity into thought is in Brunschvicg's estimation the sign that the dogmatic oversight of science by philosophy has come to a definitive end. In antiquity "the dogmatism of a Democritus or an Aristotle did not have to reckon with the data of positive science," the rule of metaphysics in whatever form was inevitable and the rational procedures proper to contemporary science are invoked only in violation of the historical and philosophical materials.[221] In modernity the situation is more complicated, for science and philosophy seem to entertain a relationship of indisernibility with one another: "the dogmatism of a Descartes or a Newton occurs on the same level as scientific knowledge."[222] Descartes and Newton are scientists just as much as they are philosophers, and the two domains must in fact be read as part of a more complex continuum of thought which cannot be reduced to a simple opposition. Modernity for Brunschvicg is in fact the highest achievement of philosophy precisely because it entertains this identity with science. Beginning in the eighteenth century, however:

> the concepts of philosophy and the results of science can no longer be placed on the same plane, nor do they appear as part of the same order. The manifestation of human knowledge, seized in its totality, has been radically modified.[223]

Brunschvicg invokes a *critique of experimental judgment* which would be neither a *philosophy of nature* (the critique of which Brunschvicg borrows from Meyerson), nor a *philosophy of science,* but what Brunschvicg calls *a philosophy of thought (une philosophie de la pensée .)* The philosophy of thought comes into being

[221] Brunschvicg 1922, 563.
[222] Brunschvicg 1922, 563.
[223] Brunschvicg 1922, 563.

when philosophy is forced to abandon the dogmatic perspective which is that of the predetermination of the categories of experience and reason. The emancipation of reason is a counter-dogmatic procedure, moreover, it must renounce both the philosophy of nature and the philosophy of science. The philosophy of nature, exemplified for Brunschvicg by Hegel, is the perfection of dogmatic philosophy. The philosophy of science is no better, it is exemplified by the Positivism of Comte, who no less than Hegel or Kant expends a tremendous amount of energy stabilizing the boundaries between positivist knowledge and metaphysical speculation. The irrefutable lesson of relativity is that the demarcation of experience and reason is not legitimated *a priori* according to determinate forms of experience. On the contrary, the authentic progress of the mind brings about an increasingly subtle interpenetration and mutual presupposition of objectivity and intellect. The philosophy of thought is designed to interrogate the nature of this subtle dialectic without recourse to the dogmatic procedures of a philosophy of nature or of science.

Neither the a priori determination of the forms of experience or the skeptical realism of positivist objectivity are capable of following the procedures of scientific rationality. Following the lessons of Einstein's relativity, the philosophy of thought is critical of all dogmatic philosophy. It is necessarily an historical doctrine, but not for all that an historicist philosophy. It attempts to grasp thought in all the dynamism of its historical turbulence without prejudging the trajectory or the necessity of scientific development:

> Our survey has not lead us to fix the tableau of scientific knowledge in its perfected state, distributed along routes traced for it in advance in order to

> satisfy a taste for symmetry, or a mania for regularity. We offer something entirely different, which is, in our estimation, far richer: the course of knowledge with its sinuosities and sudden divergences, its tranquil waters and its sudden rapids, its natural rivers…Nature, considered independently from the mind which knows it, is, from our perspective, an abstraction, and similarly with science considered independently of its progress. The profile, already so complicated, which would draw the contemporary configuration of our scientific universe, expresses only an instantaneous cut in the chain that unites, across the periods of human evolution, the different profiles that themselves correspond to the successive ideas diverse generations have had regarding the universe.
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> The philosophy of thought proposes precisely to draw this concatenation of constitutive elements which would be the successive visions of the universe, with the hope of achieving anything other than a aggregate of disparate and divergent opinions, rather, turned towards an immanent critique which would be at once *mutual* and *progressive…*to present…a philosophy of human history, on the condition of course that this philosophy of history does not form a system modeled on the ancient systems of dogmatic natural philosophy, but follows the suppleness and complexity of a rhythm of progress which manifests the wealth and fecundity of the intelligence.[224]

The mutual correction of the "successive visions of the universe" is precisely the act of placing this sequence or montage of the configurations of the scientific universe in motion. It is the effort to grasp the history of science in its progressive movement. The philosophy of thought is a philosophy of history, but of a very particular kind of history constituted by the ever changing relationship between rationality and objectivity. The philosophy of thought is counter to all dogmatic philosophy and all predeterminations of thought and of being, which from Brunschvicg's perspective represent the transcendental illusions of idealism and realism respectively. The history of scientific rationality, on the contrary, is best conceived as a dialectical evolution of rationality and objectivity. The history of science, unlike other histories,

[224] Brunschvicg 1922, 570-571.

is unique insofar as it is the history of the experience of reason. It is progressive without being pre-determined, objective without being dogmatic, and rational without being *a priori.* The philosophy of thought attempts to render this history of the experience of reason in its very movement, not as a series of synchronic cuts but from the relativistic perspective of an immanent critique which is itself in motion.

The dismantling of dogmatic philosophy, in tandem with the dismantling of the pre-relativistic hegemony of the synoptic observer, requires a new conception of the relation of rationality to objectivity. Brunschvicg again turns to relativity in order to construct a theory of rational experience worthy of contemporary science. Relativity must be understood as a kind of convergence "between the destinies of mathematics and physics."[225] The critique of experimental judgment which Brunschvicg inaugurates with the philosophy of thought discovers that the form of objectivity which relativity pursues is at once mathematical and experimental. Experimental judgment is the form of judgment proper to science: it is necessarily an affirmation of objectivity but it is not a dogmatic reification of a stable object, on the one hand, and a stable form of intelligibility, on the either. It defaults neither to rationalism nor to empiricism. The mathematical element of relativity cannot be reduced to a unitary rational intuition, just as the experimental element cannot be reduced to a unitary intuitive content of experience. In order to arrive at an understanding of experimental judgment in relativity, Brunschvicg reconstructs what might be called a history of the forms of intuition in mathematical physics.

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[225] Brunschvicg 1922, 604.

At the most elementary stages of the development of science, physics "remains at the level of sensible intuition," just as mathematical intelligence remains at the level of rational intuition. There is no overlap between the order of experience and the order of intelligence, moreover, "progress consists only in correcting that which in the immediate data of experience remains incomplete or illusory."[226] Brunschvicg gives the example of the correction of immediate perception by experimental demonstration, as when light passes through a prism to be divided into the seven colors of the visible spectrum. This model of objectivity neatly divides experience into two symmetrical halves:

> on one side, there is the world of subjectivity, the images or the ideas that we carry within ourselves; and on the other, the objective world of things situated outside of us. Experience is the revelation of this objective world, necessarily *circumscribing* the subjective world, and thereby permitting the existence of the object as it exists beyond the image or the idea.[227]

In contemporary mathematical physics, however, a different order of experience comes to light which does not oppose the mathematical and physical orders but which unites, in an experience internal to mathematics:

> *rationality* and *objectivity* as reciprocal functions in solidarity with one another which cannot be separated from each other, because, contrary to the dream of dogmatic realism, rationality cannot transcend itself in the absolute of reason, in the form of pure evidence, no more than objectivity can in the absolute of an object, in the immediacy of apprehension.[228]

Brunschvicg calls this a second order or degree of the experience of reason and unlike the objectivity which *circumscribes* subjectivity, second order rationality is

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[226] Brunschvicg 1922, 604.
[227] Brunschvicg 1922, 605.
[228] Brunschvicg 1922, 606.

inscribed in the course of thought. In the Cartesian philosophy, the judgments of physics rested on the dogmatic metaphysics of "intelligible essences" or primary notions defined by the order of pure analysis. In Kant reason and experience are no longer opposed, but the forms of possible experience are pre-determined by *a priori* intuition. The physics of relativity does not distinguish between the science of order and the science of measure, nor does it assume the pre-existence of time and space as invariable features of reality.

Brunschvicg presents relativity as the definitive overcoming of the classical antinomies of form and matter, reason and experience, and the intelligible and the sensible. Nature as revealed by relativistic physics is neither "a substantial reality" or an" *a priori* intelligibility." Theory and experience compose a new relation of *rational objectivity*. In classical mechanics, Brunschvicg writes, "reason can flatter itself" that the fundamental concepts of mass or movement, force or energy, are in some sense directly tied to the nature of substantial reality or to the apodictic certainty of pure intelligibility:

> But the fundamental *invariants* of relativistic physics are situated at the junction of theory and experience. Their objectivity participates in the subtle complications of calculation just as much as in the technical minutiae of the laboratory; and in such a fashion that it is no longer possible to operate the separation between mathematical coordination and experimental facts; calculation and experience are no longer at the elementary stage at which the mathematician referred to simple ideas, capable of being defined distinctly and directly, and the physicist no longer has recourse to objects given in the isolation of immediate intuition.[229]

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[229] Brunschvicg 1922, 609.

The objects and concepts of relativity do not divide themselves into truths of reason and truths of experience. The curvature of space-time is experimentally and mathematically verifiable, but it is not resolvable into stable categories of form and content. It emerges and is verified as the result of a synthesis of "rational construction" informed by experimental observation and mathematical prediction and confirmation. The Einsteinian universe is a *cosmometry* as Brunschvicg writes, a "quadrimensional continuum the curvature of which characterizes the real."[230]

Objectivity becomes truly rational when it is no longer immobilized by the a priori differentiation of the observer and the observed. Relativity is the rectification of the apparent stability and universality of measurement. It puts the observer in the midst of a dynamic cosmos and forces the demiurge to contend with the speed of light and the force of gravity. It is the introduction of movement into cosmology according to the gravitational topography of a four dimensional curved space time: the forced recognition that all of our measurements occur in space-time and are part of an irreducible concatenation of cosmological speeds. Rationality becomes truly objective when the science of order and the science of measure become inseparable functions of a unified rational construction of the real. Relativity is the dismantling of the *a priori* intuitions of space and time as well as the expulsion of transcendental realism from the history of physics. There is no *Sensorium Dei* for which space and time could be apprehended in the spontaneity of intuition. Relativity dispels the predetermination of experience and of reason. The philosophy of thought affirms

[230] Brunschvicg 1922, 609.

that nature and science are part of an inseparable rational synthesis. Nature, however, does not cease to transform the aspects by which its objectivity resists our presuppositions and the smallness of our imagination, just as the experience of reason refuses to conform to the spontaneity of our untutored perceptions.

Chapter Three - Mathematics as Experience: Jean Cavaillès and the Dialectical Philosophy of the Concept

Jean Cavaillès (1903-1944) was a mathematician, historian and philosopher of rare synthetic ability, fully in command of the history of mathematics and philosophy and not unlike his teacher and mentor Léon Brunschvicg in this respect. Cavaillès was active in the leadership of the *Libération* movement and was executed by the Gestapo in 1944. His role in the French resistance and his subsequent execution have made Cavaillès a well known figure in histories of the resistance. Georges Canguilhem memorialized his friend and colleague as follows.

> There is something terrifying about Cavaillès' tenacity. He is a singular figure. A philosopher-mathematician bearing explosives, an audacious lucidity, resolute without optimism. If that's not a hero, what is a hero?[231]

Cavaillès is equally well known for the notorious density and rigor of his work. In the words of Gaston Bachelard, "to read Cavaillès is to work."[232] The diverse network of epistemological research which developed in France at the turn of the century and with particular intensity during the interwar period reaches a zenith of complexity and ambition in the thought of Cavaillès. In keeping with Canguilhem's vocabulary, the work of Cavaillès develops a truly singular epistemology of mathematical experience and an entirely original theory of science with an audacious lucidity and tenacity.

The previous chapters have demonstrated that for both Emile Meyerson and Léon Brunschvicg the history of science serves a diagnostic function: it furnishes the

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[231] Gabrielle Ferrières, *Jean Cavaillès un philosophe dans la guerre, 1903-1944* (Paris: Éditions du Félin, 2003), 13.
[232] Ferriéres 2003, 239.

material for an epistemological analysis of the nature of human intelligence. Meyerson's philosophy of the intellect and Brunschvicg's philosophy of thought, despite their many divergences, are unified from the perspective of the *reflexive critique of the internal normativity of science*. For both thinkers science is necessarily progressive but also continuously rectifying itself, revising its own foundations and redrawing its own frontiers in accord with the exigencies of a dialectic of reason and experience. Jean Cavaillès' philosophical project is in perfect accord with both Meyerson and Brunschvicg in this respect. All three thinkers affirm that science defines its regulative norms internally, that it is by nature progressive, and that in order to understand the nature of scientific intelligence it is necessary to turn to the history of science *because the normative content of science itself is subject to continuous transformation.* The three fold affirmation of historical epistemology is therefore that of the *immanent production of the normativity of science out of science itself, the continuous transformation and progressive movement of this intelligible norm, and the necessarily historical rather than a priori nature of scientific knowledge.* Cavaillès will develop a theory of science which pursues the implications of each of these affirmations in turn. Unlike his predecessors, however, Cavaillès will break with all the vestiges of what he calls *the philosophies of consciousness*, which refer the progressive dynamism of science to the agency of an intentional consciousness.

In place of a philosophy of consciousness (Kant and Husserl are the great antagonists of Cavaillès in this regard) Cavaillès will attempt to construct a dialectical philosophy of the concept which fully respects the autonomy of science, liberating it from the oversight not just of transcendental subjectivity and intuition (as Brunschvicg had done) but from any subjectivity whatsoever. The disappearance of the subject is also the disappearance of the transcendental origin of the legitimacy of science, as this chapter will demonstrate. Cavaillès thoroughly historicizes the most abstract domain of knowledge, namely, mathematical experience, in order to construct a theory of science capable of affirming the pure immanence of the scientific concept (which cannot refer to anything outside itself to justify its authority, not even the nature of the subjectivity which operates its concepts) while simultaneously upholding the historical transformation of the normative contents of scientific concepts according to a necessity which cannot be predetermined. Cavaillès would seem to have set himself an impossible task, for he requires that science be at once a strange mixture of immanence and progress, of history and necessity, and of formal rigor and experimental verification, all without reference either to an external (material or natural) authority or to any form of subjectivity. The paradoxes of a *strictly immanent and autonomous* theory of science seem to beg the question both of the *dynamism* of science (how does science progress without the incursion of empirical experience or the agency of a subject?) and the *authority* of science (how does science demonstrate its normativity in the absence of a theory of external verification or an internal system of apodictic self evidence?) It does so, Cavaillès claims, by the endogenous evolution of a conceptual dialectic of *formal contents* which cannot be reduced to a stable division of *intelligible form* and *sensible content*. The historical reality of science is the reality of a progressive dialectic which is *always in process*, always in the middle of a transformation by which one system of formal contents integrates and deepens the rational synthesis of another system of formal contents.

Cavaillès thus emphasizes the *process* of scientific rationality (more precisely, the processes of an ongoing procedure of mathematical demonstration) over any of its concrete results or instantiations. In truth, however, the objects of science, its particular moments and conceptual manifestations, are inseparable from a superior dynamism which traverses them and transforms them, resulting in what Cavaillés calls the *indisociability of posited and positing meaning*. Accordingly, every scientific concept can be taken as the object of another scientific statement, which explains what remains merely provisional or unaccounted for in the antecedent concept. Science thus internalizes an infinite capacity of recursive self reference which is not a mere formalization of already existing contents but the production of new contents based on the *reflexive thematization* of an always historically well defined state of affairs. Cavaillès explains the procedures of the progressive dialectic of the auto-development of science in terms of a theory of scientific demonstration (following Bernard Bolzano all science is demonstration for Cavaillès, the science of science would be a general theory of demonstration) which is the result of the relationship between two conceptual procedures: that of *paradigmatic demonstration* (which is the formalization of the rules implicit in any rational procedure, exemplified for Cavaillés by David Hilbert's [1862-1943] axiomatization of the postulates of geometry) and *thematic demonstration* (which is the production of a new conceptual field based on the generalization of a problem, what Cavaillès refers to as the vertical 'super-position' of horizontal conceptual planes in such a way that, for example, the Einsteinean cosmos not only encompasses the Newtonian cosmos but also introduces entirely new theoretical problems and conceptual entities.)

Jean Cavaillès was a philosopher of mathematics with an exhaustive technical and historical erudition of his chosen subject: the history of mathematics from roughly the beginning of the nineteenth century through the "foundational crises" of the early twentieth century. He is best known for three highly technical works in the philosophy of mathematics, *Remarques sur la formation de la théorie abstraite des ensembles*, published in 1938, *Méthode axiomatique et formalisme* also published in 1938, and *Sur la logique et la théorie de la science,* written in 1942 when Cavaillès was imprisoned in Montpellier under the Vichy regime but published posthumously in 1947 by his colleagues Georges Canguilhem and Charles Ehresmann. Cavaillès' first two books were written as his primary and secondary doctoral theses under the supervision of Léon Brunschvicg, whose well known affirmation that the history of science should serve as the "laboratory" of philosophical reflection also characterizes the working method of Cavaillès, though as this chapter will show Brunschvicg himself is frequently the subject of sustained criticism and revision in Cavaillès' work.

Cavaillès addresses almost the entirety of modern mathematics over the course of his three major works and a dozen or so articles. Of primary significance are the major movements at the end of the nineteenth century and the beginning of the twentieth which attempted to provide either logical, formal, or axiomatic foundations for arithmetic; the logicist program of Bertrand Russell (1872-1970) and Alfred North Whitehead (1861-1947), the formalist program of David Hilbert (1862-1943), and the intuitionist program of Luitzen Egbertus Jan Brouwer (1881-1966). It would be a mischaracterization, however, to claim that Cavaillès was concerned only with the broadly systematic debates regarding the very nature of mathematical objects. Cavaillès was also an exacting historian of mathematics and devoted exhaustive analyses to the history of projective geometry, non Euclidean geometry, abstract algebra, set theory, number theory, the arithmetization of analysis, probability theory and general topology. Cavaillès was also well versed in contemporary developments in logic and was an early commentator on Kurt Gödel (1906-1978) and Alfred Tarski (1901-1983) whose metamathematical theories profoundly informed Cavaillès' posthumous masterpiece, *Sur la logique et la theorie de la science.* As Henri Cartan remarks in his introduction to the re-edition of *Méthode axiomatique et formalisme* in 1981

> The reflections of the young Cavaillès of 1937 have lost none of their interest today: they help us point to the history of the evolution of ideas in an epoch which was fertile with controversies. Cavaillès was possibly the only one who was capable of presenting the complete picture of this evolution because he

> combined his philosophical training with a solid mathematical formation; he had also studied the logicians and assimilated the substance of their work.[233]

The abundant controversies of the nineteenth and twentieth centuries are the formative context of Cavaillès own entirely original philosophy of mathematics, which, as this chapter will demonstrate, is nothing less than a philosophy of the immanent creativity of mathematics, bound neither to the operations of subjectivity or to any transcendental program of legitimation. Although Cavaillès is critical of the major debates in the foundations of mathematics, his work entertains a special affinity with Hilbert and Brouwer. Cavaillés' work can be considered as an extended meditation on the nature of mathematical experience, and, in this regard, he is undeniably a student of Léon Brunschvicg. Unlike Brunschvicg, however, Cavaillés does not find that a philosophy of judgment is capable of accounting for either the genesis or the conceptual development of mathematical concepts. Instead, drawing on Hilbert's work on the nature of mathematical signs as the irreducible intuitive content of mathematical thinking, Cavaillés evolves a novel and powerful theory of *formal content* (a concept created by Gilles Gaston-Granger in reference to Cavaillès' epistemology of mathematical experience) as the unified *efficacy* and *demonstrative reality* of mathematical concepts, thereby eschewing the tendency either to idealize mathematics (making them rigorous fictions of the mind) or to subject them to the *logical realism of classes*, in which case mathematics is made subordinate to logic, as

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[233] Jean Cavaillès, *Ouevres complètes de philosophie des sciences* (Paris: Herman, 1994), 9.

Russell and Whitehead attempted to demonstrate in their monumental *Principia Mathematica* (1910.)

Hilbert supplies an innovation at the level of the theory of mathematical signs which will prove to be central to Cavaillès epistemological program, but it is Brouwer, in conjunction with Brunschvicg, who raises the possibility of "an epistemology of immanence" which Cavaillés will attempt to realize in a more perfect form, finally freeing mathematics from all specters of external legitimation and establishing on entirely immanent grounds the necessary unity *and* progressive transformation of the formal contents of mathematics. Brouwer's so called "intuitionism" is the necessary complement to the theory of the mathematical sign, for it allows Cavaillés to perform a double deterritorialization (as Deleuze and Guattari might say) of the intelligibility of mathematical experience dissociating, on the one hand, the signifying relations of mathematical signs from anything other than their "chains of reasons", and denaturing, on the other, the self evidence of intuition as the internal illumination of the contents of thought to itself.

For Cavaillès mathematics do not take place "in the mind;" they are not grounded in an intuition which produces them (his critique of Brouwer and Kant echoes Brunschvicg's, but as will be shown he goes much further than Brunschvicg) out of an a priori synthesis of time as the form of internal representation; rather, the reality of mathematical acts, objects, and operations (which will be given their technical definitions later in the chapter) is immanent to the manipulation of mathematical signs themselves. There is an intuitive content to the mathematical sign but it is captured *by the simultaneity of reference* of which mathematical signs are capable. The double reference of the mathematical sign is located simultaneously on two different levels of generality which Cavaillès calls *paradigmatic* and *thematic*, the former referring to the procedure of axiomatization which makes *explicit* the set of rules exhaustively determining any mathematical system (Cavaillès also calls this the *horizontal extension of a rational sequence*), the latter constituting a new level of generality which takes a whole domain of mathematical operations in turn as the *object* of a new mathematical theory (which Cavaillès calls the *vertical* super-position of a new more general level of reference on top of an already constituted domain which it thus encloses and treats, in turn, as an object.) Mathematical intuition can have no stable content for Cavaillès because it is always changing the levels of its reference, passing through subsequently more abstract or general planes of thematization which make intuition, like the mathematical sign, not only differential, but capable of a recursive self reference which is in principle infinitely extendable (the intuitive content of a mathematical object at one level is the abstract operation of a more general thematic at a higher level, and so on ad infinitum, in a series without definitive origin or termination.)

This chapter will primarily address Cavaillés' *Sur la logique et la théorie de la science* but will also make reference, where necessary, to lines of argumentation developed in Cavaillès' other major works. *Sur la logique et la théorie de la science* is an incomplete text, or rather, it is as complete as circumstances allowed, given that it was written during one period of confinement (sometime in 1942 while Cavaillès was in military custody in Montpellier), slightly modified in a brief respite between Cavaillès' duties as an officer in the resistance, and then published posthumously, three years after his execution, by his friends and colleagues (in the academy and in the resistance), Georges Canguilhem and Charles Ehresmann (1905-1979). The title itself is an editorial artifact, chosen by the editors of the text in an effort not so much to capture the spirit of the work but to give a summary of its ambition, which itself remains a matter of scholarly debate. As Canguilhem and Ehresmann note in their preface Cavaillès remarked to them in conversation that he envisioned the work as a "treatise on logic" and that it would require a substantial introduction which, of course, he did not have the opportunity to write. Pierre Cassou-Noguès remarks that in a letter to Brunschvicg Cavaillès wrote that he intended to write a book on *mathematical experience,* which leads Cassou-Noguès to wonder if the editors of the text perhaps unjustly emphasized the logical element of the work, overlooking the possibility that it is primarily an examination of a particular form of rational experience.[234] It must also be noted that *ontology* is a recurrent theme in the text, perhaps as frequently mentioned as logic or experience, in which case the dialectical knotting of all three domains in a rational synthesis becomes even more pressing.

Cavaillès' last work possesses an undeniable internal unity, but for every remarkable new synthesis it achieves it simultaneously announces a new aporia. This chapter will not attempt to reconcile what may appear unevenly developed in

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[234] Pierre Cassou-Noguès, *De l'experience mathematique, essai sur la philosophie des sciences de Jean Cavaillès* (Paris: Librairie Philosophique J. Vrin, 2001), 249.

Cavaillès' thought but rather to provide a synthetic reading of the posthumous work against the background of Cavaillès' previous books and articles. The *logique*, despite its occasionally enigmatic formulations and relative brevity, is arguably the most important work in the corpus of texts which define the literature of historical epistemology, and at least for two reasons. 1) The thought of Cavaillès is the esoteric center of historical epistemology insofar as his work is the purest expression (albeit incompletely developed) of the three inseparable theses which define the conceptual consistency of the tradition: *the immanent production of epistemological norms out of science itself, the necessity of the progressive transformation of the epistemological norms themselves, and the fundamental historicity of science.* 2) The philosophy of the concept which Cavaillès attempts to outline provides a perfectly systematic and technical description of the conceptual system of scientific knowledge defined *immanently* and characterized by a power of *self differentiation.* The philosophical problem at the core of the tradition of historical epistemology is precisely the discovery that the epistemological description of science always arrives at the same enigmatic formula: IMMANENCE=SELF DIFFERENTIATION.

I characterize the foundation of the normative force of epistemology in this dissertation as the result of the immanence of intelligibility and objectivity in mathematics. The concept of immanence is what guarantees the autonomy of the theory of science. Cavaillès' work provides the most thorough and systematic account of epistemological normativity conceived as the immanence of the intelligibility and objectivity of the concept. However. each of the figures considered in the dissertation articulate a variation of this philosophical thesis. The three philosophical theses with which I introduced the normative framework underpinning the epistemological commitments of the tradition (the autonomy of science, the procedural variation of the scientific concept, the historical instantiation of the scientific rationality) are conjoined in Cavaillès with remarkable clarity. The recognition of the autonomy of science is also the recognition of the immanence of the rational norm within science. The problem which then confronts the epistemologist is to determine how the rational norm thus transforms itself historically. The formula IMMANENCE=SELF DIFFERENTIATION expresses the realization of the epistemological ideal by which the autonomy of science discloses the conceptual differentiation of its own rational contents.

The recourse to history is not, therefore, a mere supplement to philosophical speculation; rather, to historicize science is to historicize the very normativity of knowledge. This is not at all a capitulation to an uncritical relativism by which science would be one "truth procedure" among others. On the contrary, the affirmation of the fundamental historicity of science is not to equivocate or displace the problem of normativity but finally to grasp the conceptual structure of the epistemological norm as self-positing, auto-critical, and self-transformative. Cavaillès remains the most ambitious and sophisticated theorist of historical epistemology conceived in this manner.

The aim of the chapter is four-fold. 1) To define the nature of Cavaillès' rigorously immanent epistemology of mathematics. 2) To define the necessity and significance of the historical nature of mathematics in Cavaillès' epistemology. 3) To give an account of the nature of mathematical *progress* in Cavaillès' epistemology. 4) To outline the enigmatic "dialectical philosophy of the concept" so often invoked in discussions of Cavaillès. The chapter is divided into four sections. The first section addresses Cavaillès' critique of Kant, Brouwer, and Brunschvicg, all of whom come under the heading of a philosophy of consciousness which, as will be shown, cannot but distort the nature of mathematical reality for Cavaillès. The second section addresses Cavaillès' reading of Bernard Bolzano's (1741-1848) *Theory of Science* which in many ways provides a useful model for Cavaillès insofar as Bolzano is the first to insist on the absolute autonomy of the scientific concept, thus prefiguring Cavaillès. The third section addresses what I will claim are Cavaillès' most significant and original contributions to the tradition of historical epistemology, the conceptual procedures which Cavaillès calls *paradigm* and *thematic.* These concepts, I argue, require a careful technical exposition if Cavaillès' epistemology is to be understood. The fourth section addresses Cavaillès' reading of Edmund Husserl's *Formal and Transcendental Logic.*

Kant and the Philosophies of Consciousness

In his introduction to the second edition of *The Logic and Theory of Science* (hereafter LTS) which appeared in 1960, Bachelard notes that Cavaillès, like Husserl, identifies an opacity proper to science and to logic which permits scientific inquiry to perfect and refine its formal methodologies without for all that having any consciousness of its “original meaning.” Bachelard writes:

> In a sense, modern logic shares the fate of all the sciences in being a discipline which can be perfected without endeavoring to inquire radically into its original meaning. Husserl recognized that what makes science possible is fortunately not the reflection which penetrates to the essence of things but the scientific instinct. The scientist, in order to build, need not concern himself in the preliminaries of philosophical critique.[235]

The explanatory efficacy of science can proceed perfectly well without “the preliminaries of philosophical critique” to guide it. What role then does a theory of science, conceived as a meta-discourse of science or a science of science, have to play in what already appears to be a smoothly functioning scientific methodology? Husserl’s answer is well known: the sciences are in a crisis of their own devising as a result of the increasing specialization and abstraction of the regions of knowledge. Science no longer recalls its origins or its foundational meaning and threatens to become divorced from the very interests of the civilization which produced it. Cavaillès cannot be assimilated to Husserl in this respect. Bachelard cites a letter written to Albert Lautman (1908-1944), another philosopher-mathematician, in which Cavaillès remarks: “I have tried to define myself in relation Husserl, somewhat in opposition to him.”[236] The opposition to Husserl is based on a profound disagreement regarding the nature and significance of logic, on the one

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[235] Gaston Bachelard, “Preface to Logic and Theory of Science” Trans. Theodore J. Kisiel, in *Phenomenology and the Natural Sciences,* eds. Theodore J. Kisiel and Joseph J. Kocklemans (Evanston: Northwestern University Press, 1970), 353-354.
[236] Jean Cavaillès, “On Logic and the Theory of Science” Trans. Theodore J. Kisiel, in *Phenomenology and the Natural Sciences,* eds. Theodore J. Kiesiel and Joseph J. Kocklemans. (Evanson: Northwestern University Press, 1970), 354.

hand, and the nature and significance of the history of science, on the other. For Husserl, then, a science of science (as transcendental phenomenology) is necessary in order to *correct science itself*, to reorient it according to the foundational intuitions of a life world which in the final analysis it both clarifies and redeems. Against Husserl's archaeology of sedimented forms of meaning latent within the contemporary formalisms of logic and mathematics, Cavaillès will propose a dialectical philosophy of the concept which discloses a mobile but normative *historicity of the forms of scientific demonstration* which continually reintegrate and transform the very significance of history for the living present of scientific knowledge. For Cavaillès Husserl misunderstands science insofar as he misunderstands the nature and necessity of scientific progress itself, which is not and cannot be the expression or explication of lived experience but which obeys and expresses a purely immanent and autonomous developmental logic. There is therefore no lost "originary significance" which must be phenomenologically disclosed because science does not evolve in the sphere of human necessity but follows its own endogenous sequences of necessary complication and evolution, as will be shown.

Cavaillès' encounter with Husserl will be detailed in the final section of this chapter, what Bachelard's introductory remarks point to, however, is a useful device for framing the scope and ambition of Cavaillès "philosophical testament," which is that logic must be understood not only as a *method* but as *the theory of science*, and therefore that it is not simply a means to an end (scientific explanation) but, in a technical register which this chapter will develop, an *essence.* The simplest presentation of the LTS is that it is a critique of various philosophical doctrines of logic beginning with Kant and ending with Husserl. In the interim Cavaillès also addresses the intuitionist program of Brouwer, the immanent epistemology of Brunschvicg, Hilbert's theory of mathematical signs, the theory of logical demonstration developed by Bolzano and the logicism of Carnap and Tarski. In each case Cavaillès will ask; what does this thinker take logic to be? What is the relationship of logic to the formal structure of science? What is the relationship between mathematics and logic? Does mathematics constitute its own domain of investigation and if so, does logic describe the structure of this domain? The philosophical context of Cavaillès investigation is, however, Kant's critical philosophy and the relationship between mathematics and logic which Kant attempts to establish in the *Critique of Pure Reason.*

In Kant's *Course on Logic* it is already perfectly clear that to invoke psychology in matters of logic is a disastrous category error. However, the problem which persists for Cavaillès is the transcendental theme of *consciousness* in Kant, which unlike psychology cannot be reduced to the particularities of subjective experience. The relationship of logic to the structure of science occurs against the background of a consciousness which is transcendental both to logic and to science, leading Cavaillès to affirm that

> [t]he necessity of rules, i.e., their unconditioned normative character, thus remains subordinate to the absolute of a consciousness whose presence and

> essential structure – which is consciousness in itself – are an irreducible which no rational content defines. There is nothing prior to consciousness.[237]

The entirety of Cavaillès' critique of Kant appears here in crystalline form: the normative character of logic (the rules which reason must follow) are not anchored in any "rational content" but suspended in consciousness itself. Consciousness supplies the "essential structure" of logic and so becomes the irreducible stratum of all rational procedures. The problem which Cavaillès then raises is that of determining the "intelligible content" for a philosophy of consciousness of apodictic rules of reasoning. Otherwise stated, if logic is denied a foundation in properly "rational contents" (a theme which will persist throughout the LTS) what, precisely, does logic illuminate and what stratum of experience does it describe?

For Cavaillès Kant and the logicians of Port-Royal construct similar solutions to this problem for in each case "logic is no longer a reflection of the mind upon its acts" but becomes a vehicle by which the *faculties* of reason and understanding come to be known through the analysis of *form.* Logic then is not an autonomous domain but a projection of the rational unity of the faculties of reason and understanding. Cavaillès isolates the problem of form as particularly instructive in the elucidation of the faculties because form as such comes to constitute the evidence of the operations of the rational faculties within an "irreducible consciousness." The most important feature of this consciousness (from the perspective of a theory of logic) is that it is "characterized by the property of an

[237] Cavaillès 1970, 358.

internal self-illumination."[238] Consciousness is transparent to itself, it sees and is co-present to all its acts. For Cavaillès this means the faculties of reason and understanding are able to determine themselves at every moment insofar as they require nothing beyond the introspection of the acts of consciousness to become actual – the co-presence of consciousness and the "actualization of logical contents" is achieved by form, which testifies to this very illumination of consciousness. Cavaillès writes:

> By recourse to form, the philosophy of consciousness strengthens itself and likewise illuminates itself, at least the way Kant presents it. In fact, the ambiguity of the formulas in which the nature of the invocation of our thought intervenes then becomes inoffensive. The starting point is indeed the actual, the immediate experience of an actual consciousness with its empirical accidents, but the absolute finds itself isolated as a result of a double process of elimination.[239]

The philosophy of consciousness looks to logic as a form which consciousness presents to itself following the legislative oversight of the faculties. By observing the forms of logic, their hierarchy, succession, and rules of transformation, consciousness also discovers the regulative procedures of the faculties and, therefore, not only the principles of logic but the necessary forms of knowledge. This is the sense in which Cavaillès discloses "the ambiguity of the formulas" which must govern thought, or, in Kant's own presentation, the relation between the empirical datum and the transcendental synthesis which unifies it in intuition. Intuition is ambiguous for Cavaillès because it claims to present the empirical according to the forms of what Cavaillès calls "the absolute," the a priori forms of

[238] Cavaillès 1970, 359.
[239] Cavaillès 1970, 359.

space and time which govern all representations. This formula becomes "inoffensive" not by actually resolving the problem of the nature of experience (which Cavaillès will attempt to do by directly developing a theory of mathematical experience over the course of the LTS) but by attempting to eliminate the problem of the relation of the empirical to its transcendental determination in the forms of intuition.

As Cavaillès signals in the above passage, a "double process of elimination" is required in order to bring the empirical contents of experience in line with the totalizing grasp of transcendental consciousness. Cavaillès wants to know how Kant displaces all possible forms of experience, including any possible forms of logical or mathematical experience, into the exhaustive interiority of a transcendental consciousness which then must become the ultimate bedrock and unity of all forms of knowledge. In order to achieve this liquidation of all other possible forms of experience Kant must very clearly determine how the passage from the empirical to "the pure or a priori" operates within transcendental consciousness. Cavaillès claims Kant brings this about in two ways. First, "positively," the a priori determination of experience is set apart from all acts of consciousness such that the a priori intervenes in a fixed form in all possible forms of experience. This is especially problematic in the case of mathematics where the synthesis of time itself in intuition is the productive origin of mathematical experience. Here Cavaillès remains quite close to Brunschvicg's reconstruction of Kant's mathematical philosophy in *Les étapes de la philosophie mathématique.* Second, "negatively," empirical experience is imprinted with the lack of order or, as Cavaillès writes, "the empirical manifests its essential fragility in the unpredictability of its characters, in the inclusion of that which to some degree is unlawful."[240]

Kant wishes to reconstruct the edifice of the sciences from the perspective of their legitimation in the procedures of a transcendental consciousness. The methodology of science, whether logical or mathematical, must be shown to be in conformity with the faculty of reason. If Kant begins on this transcendental ground, then everything which science subsequently discovers must already be contained in the productive intuition which shapes all possible forms of experience. Cavaillès is a faithful student of Brunschvicg in his critique of Kant as a philosopher of logic and mathematics insofar as he rejects this totalization and simplification of the relationship between reason and experience. Kant's understanding of logic betrays a simplification of the actual developmental process of science because it nullifies the actual role that experience plays in science. Cavaillès writes:

> …if the subsequent development of science is deductive, therefore analytic, all the empirical elements to come must already be found concentrated in the synthesis which produces these notions. A particular experience either offers no interest or overturns the edifice – unless the demonstration is of a mathematical order, through a construction of concepts. *But then the physical concepts must be represented completely within mathematics, which Kant does not admit and which moreover raises the difficulty of the specific character of physical construction in relation to mathematical construction* [my italics].[241]

If the guiding methodology of scientific demonstration proceeds according to the principles of logic, everything in rational experience must already be contained,

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[240] Cavaillès 1970, 359.
[241] Cavaillès 1970, 365.

virtually, in the syntheses of intuition. Once again the Kantian theory of experience reduces the theory of science to a confirmation of the regulatory requirements of the faculties. Science, in the Kantian system, is destined to discover what it already knows: the unity of the faculties and the pre-determination of the forms of experience. Cavaillès points to another problem in the passage cited above, namely, the difficulty of understanding the relationship between different domains of scientific activity. How are we to understand the relationship between the "constructions of physics" and "mathematical construction?" As Cavaillès will remark later in the essay, one of the most profound problems facing any theory of science is the relationship between the different domains of scientific theory construction. The foremost difficulty from this perspective is indeed understanding the basis from which it would be possible to reconstruct the concepts of the physical sciences entirely according to the conventions of mathematics.

Kant's philosophy of consciousness has forced an artificial wedge between logic and mathematics, reducing the former to the Aristotelian table of the categories of experience and the latter to the spontaneous production of intuition. In both cases Kant's theory of science is guilty of what Cavaillès describes as a profound "indifference to the object." For Kant there is no ontology of the formal objects of science, this would amount to a transcendental realism of scientific objectivity. The critical philosophy, in contrary fashion, attempts to define the contours of objectivity according to the structure of a transcendental subjectivity. Accordingly, mathematical constructions through concepts no less than logical deductions have their primary reference in a transcendental consciousness. Everything in Kant returns to an insufficient intuition which effectively makes science the mirror of the operations of the faculties. This is the ultimate sense of the analogy which Cavaillès perceives between the logic of Port Royal and Kant's epistemological program. Reflection on logic can only disclose transcendental operations of reason. As Cavaillès had previously remarked, in a philosophy of consciousness logic is transcendental or it is not at all.

This leads Cavaillès to conclude that Kant reduces the problem of method in science (conceived logically or mathematically) to a highly problematic formality "which does not even have an irreducible simplicity to guarantee it," since this formality of the forms of experience would be produced in intuition and would not be allowed to claim any autonomous authority for itself.

> The authority of method, as logical canon as well as mathematical organon, is thus reduced to the preeminence of a formality which does not even have an irreducible simplicity to guarantee it. Transcendental Analytic can indeed procure for science its framework, since 'all of the empirical laws are only particular determinations of the pure laws of the understanding,' so that 'understanding is not only a power of making rules for itself through the comparison of phenomena, but is itself the legislation of nature.' The type of science, its striving toward a classical mathematical type as well as its mathematical extension, completely neglects the bearing of the object on the structure of the theory. The theory is given in advance once and for all. The indifference to the object… is thus represented here by the subordination of matter to a form which tends to absorb it, even in the case of mathematization. There is no science as an autonomous reality and characterizable as such, but a rational unification, according to a fixed type, of a manifold already organized by the understanding, or a survey of a mass of evidence without plan or discovery.[242]

[242] Cavaillès 1970, 366.

Here all the themes of Cavaillès' project to construct a theory of science are on display, albeit in polemical form. The theory of science cannot be reduced to the authority of a transcendentally determined methodology without dangerously oversimplifying the objects of science. Cavaillès finds that Kant's effort to secure the logical and mathematical structure of the sciences on transcendental grounds only ends up liquidating the necessary autonomy of science. Kant's theory of science is incapable of addressing the reality of the objects of science, much less of posing the problem of the dialectic driving the transformations of scientific objects. On the contrary, Kant's theory of science is given by fiat and in fact requires an "indifference to the object" which denies "science as an autonomous reality and characterizable as such." What Kant's theory of science affirms is "a rational unification according to a fixed type" of the diversities of experience. The theory of science in Kant is really a theory of the form of experience, but this form of experience unfortunately is too rigid to encompass the *rational* experience of science. In neglecting the complexity of experience which science itself describes in the progressive elucidation of its own contents, Kant denies the very possibility of genuine novelty in scientific discovery. Once the scientific method is secured on a priori grounds, all that remains to the historical development of science is a kind of systematic fleshing out of the details. The scientific field is saturated the moment it is constituted in transcendental subjectivity. This is why Cavaillès and Brunschvicg find the Kantian theory of science ultimately so unsatisfying; it is incapable of even posing what is most interesting about science from an epistemological perspective: the perpetual transformation of the scientific field.

Cavaillès claims that because the Kantian theory of science is primarily a philosophy of consciousness, it is destined to discover the self-enclosed mechanisms of the Kantian faculties rather than disclosing anything about the nature of science. Historically, however, so Cavaillès claims, two different epistemological traditions have attempted to resurrect a theory of science along Kantian lines which nevertheless attempt to respect the autonomy of science. One tradition adheres to the theory of logical demonstration in Kant and attempts to develop a robust theory of logical demonstration in tandem with the theory of science, making the description of the scientific object inseparable from the theory of knowledge. Cavaillès claims that this tradition is inaugurated by Bernard Bolzano and is taken up again in the twentieth century by Edmund Husserl's work on logic and transcendental phenomenology. The other tradition attempts to reform the status of mathematical knowledge in Kant by subjecting the forms of intuition in Kant's philosophy to critique in order to construct a more supple theory of intuition capable of describing the developmental sequences of the history of mathematics. Cavaillès refers to this tradition as "the epistemology of immanence" and cites Brouwer and Brunschvicg as its most sophisticated theorists. Before turning to Bolzano's theory of science Cavaillès finds it necessary to pass through the "immanent epistemologies" of Brouwer and Brunschvicg in order to diagnose the limits of any theory of mathematical intuition.

Luitzen Egbertus Jan Brouwer (1881-1966) and Léon Brunschvicg are invoked as representatives of an epistemology of immanence which is meant to provide an alternative theory of science to Kant's philosophy of consciousness while still upholding the Kantian imperative of a rigorous description of the relationship between the forms of experience and the possible forms of knowledge. Brouwer and Brunschvicg can be classified as faithful to the Kantian tradition insofar as they both develop a sophisticated theory of the origins of scientific knowledge which rely on a theory of intuition. The invocation of immanence here refers to the inseparability of intuition and mathematical intelligence which Cavaillès claims is at the center of Brouwer's work on the foundations of mathematics and likewise of Brunschvicg's work on the history of the forms of scientific intelligence. Further, the epistemologies of immanence share a profound distaste for the logicist program in the philosophy of mathematics and science. Brouwer and Brunschvicg both reject, albeit for different reasons, any theory of science which claims to be able to reduce the content and necessary structure of mathematics to a system of logic. Mathematics cannot be reduced to logic for these thinkers. Indeed, there is something duplicitous about logic which makes it unsuitable as a theory of science for Brouwer and Brunschvicg, as this section will demonstrate. Additionally, neither Brouwer nor Brunschvicg accept language as the necessary form of rational intelligence; rather, as the reference to intuition confirms, the transformations of the intelligence occur at a level which can never be fully determined or thematized by linguistic expression. Finally, mathematics is the privileged domain of the intelligence for Brouwer and Brunschvicg; it is the epistemological foundation of the sciences. For the epistemologists of immanence, the secret to the purposive unity of the sciences is hidden in the evolutionary dynamics immanent to the transformations of mathematics.

Primordial Mathematical Intuition – Brouwer's Intuitionism

In his primary thesis *Méthode Axiomatique et Formalisme* (1938) Cavaillès explicitly locates Brouwer's project relative to the "essential themes of Kantianism," which however Brouwer subjects to careful revision.

> The originality of Brouwer's intuitionism is that its direct effort to resolve the actual problem of the foundation of mathematics leads it to revise the essential themes of Kantianism: the immediate intuitive character of mathematical knowledge for which truth is found in an experience *sui generis;* the definition of its development as an unforeseeable construction, independent of logic; and finally, the primacy of arithmetical constructions over geometrical constructions, a transposition of the priority of the schematism of number over the syntheses in space. However, intuition as it is here no longer has its own proper content: the form of space is eliminated entirely.[243]

Brouwer calls his foundational program "intuitionism" because it identifies the essential activity of mathematical thinking with an experience irreducible to logic or indeed to any linguistically mediated form of expression. The reference to Kant is indispensable insofar as Kant describes the form of temporal intuition by which the numerical scheme of the number appears as a synthesis of the instant in imagination, a procedure which Brunschvicg had also subjected to careful scrutiny

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[243] Jean Cavaillès, *Méthode axiomatique et formalisme* (Paris: Hermann, 1938), 32-33.

in *Les étapes de la philosophie mathématique.* The identity of the numerical scheme and the form of time in the representation of the imagination is the basic solidarity Brouwer entertains with Kant. Beyond this, however, important differences immediately appear. Brouwer rejects the predetermination of mathematical or scientific knowledge based on the form of foundational intuition. Brouwer also rejects the assimilation of mathematics to logic, which, no less than mathematics itself, must be reconstructed on the basis of a corrected understanding of mathematical intuition. Finally, the complementarity of spatial intuition as the form of external representation and of temporal intuition as the form of internal reflection is broken. Brouwer must subordinate space to time in order to make the temporal synthesis of number the productive cause of geometric constructions. The final reorientation of Kant as a philosopher of mathematics, then, is the subordination of geometric constructions to numerical constructions. The intuitive basis of mathematics is not to be found in the self-evidence of spatial relations but in the temporal experience of number understood as a synthesis of the instant. It is the intuition of time which makes mathematics as a science possible.

In the *Logic,* Cavaillès identifies "Mathematics, Science and Language," an article from 1929, as the definitive presentation of the philosophical commitments informing Brouwer's intuitionism. For Brouwer "Mathematics, Science and Language form the main functions of human activity, by means of which mankind rules Nature and maintains order in its midst."[244] The title of the article presents these "three forms of action of individual man's will to live" in the order of their genetic priority. Mathematics is the primary reference of all practical activity and the first form of the "will to live," as Brouwer understands it. Mathematical attention is the form of consciousness which corresponds to this will. There are two phases of mathematical attention. The primary phase is defined as

> nothing but the fundamental intellectual phenonomena of the falling apart of a moment of life into two qualitatively different things of which one is experienced as giving way to the other and yet is retained in an act of memory.[245]

The division of the "moment of life" into a memory trace in the mind and the continual perception of the passage of time into the next moment gives rise to a reflexive awareness of "the perceptional world" as that which escapes immediate self-presence in memory. There is a co-constitution both of the initial consciousness of the perceptible exterior world and what Brouwer calls "temporal two-ity," the initial awareness of the time sequence perpetually dividing itself into a present and a past. This initial awareness of time as "temporal two-ity" is the basis, in intuition, of all possible mathematical intellection because

> [t]emporal two-ity, born from this time awareness, or the two membered sequence of time phenomena, can itself be taken as one of the elements of a new two-ity, so creating temporal three-ity, and so on. In this way, by means of the self-unfolding of the fundamental phenomena of the intellect, a *time*

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[244] Luitzen Egbertus Jan Brouwer, "Mathematics, Science and Language," Trans. Walter P. van Stigt in *From Brouwer to Hilbert: The Debate on the Foundations of Mathematics in the 1920's.* Ed. Paolo Mancosu. (New York: Oxford University Press, 1998), 45.

[245] Brouwer 1998, 45.

> *sequence of phenomena is created or arbitrary multiplicity* [italics in original.][246]

The first phase of mathematical attention consists of the fundamental awareness of the temporal sequence which in turn is the basis of a possible numerical sequence. *The sequence of well ordered numbers is thus anchored in the intuition of temporal succession.* This is Brouwer's attempt to unite the continuum of natural numbers with the irreducible intuition of temporal succession.

The second phase of mathematical attention is called *causal attention* or *causal viewing* because it "consists in the act of the will identifying different time sequences of phenomena that extend over the past as well as the future."[247] Different temporal sequences are identified as part of a common causal sequence by the will which thus gains the ability of projecting a stable reference onto the objects of perception.

> A special case of causal attention is the construction in thought of *objects,* that is, of persistent, permanent things (simple or compound) of the perceptional world, so that at the same time the perceptional world becomes stabilized.[248]

Brouwer is careful to note that neither phase of mathematical attention should be construed as a passive receptivity on the part of consciousness but rather as a deliberative act of the will. For Brouwer there is no transcendental warrant for the form of time consciousness and by extension no transcendental justification of the

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[246] Brouwer 1998, 45.
[247] Brouwer 1998, 45.
[248] Brouwer 1998, 45.

temporal sequence or the number sequence which arises from it. Mathematical attention is instead justified by

> the 'expediency' of '*the mathematical act*' that follows from it and that can be described as follows: Causal attention enables man to force into being *indirectly* and by cool calculation a particular event that appears later in the sequence of phenomena, one that is desired instinctively but cannot be brought about by direct impulse and is known as *the aim.* He does that by bringing about an event known as *the means,* an event that appeared earlier in the sequence, and is perhaps not desirable for its own sake but draws the desired event in its wake as a *consequence.*[249]

Brouwer's mathematical attention is truly a *mathematical will* which secures causality as a supplement to the desires of this will. At no point is the structure of knowledge a matter of transcendental legitimation for Brouwer. His account is thoroughly pragmatic insofar as the initial forms of mathematical abstraction are produced by a problem-solving will which isolates and identifies those temporal sequences which allow it to satisfy its aims. The question of the objective reality of the causal sequence is thus referred to the "tendency of the human will toward mathematical acting; there is no question of an existence of a causal coherence in the world independent of man."[250]

Man makes causality in the service of his own interests, a thesis which resembles Meyerson's causal postulate. The causal sequence is utilized by human agents in the world to expand their sphere of influence. An anthropological and historical thesis thus accompanies the initial psychological description of mathematical attention. The efficacy of mathematical attention culminates (under

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[249] Brouwer 1998, 46.
[250] Brouwer 1998, 46.

sufficiently developed historical circumstances) in the system of *mathematical abstraction* which

> strips two-ity of its material content and retains it as an empty form, the common substratum of all two-ities. This common substratum… forms the *Primordial Intuition of Mathematics*, which in its self-unfolding also introduces the infinite as a thought reality and produces the collection of natural numbers… as well as the real numbers, and finally the whole of pure mathematics.[251]

Mathematical abstraction is a useful tool in Brouwer's account because it enables the mathematical will to more easily control and manipulate causal sequences by projecting them onto what Brouwer calls "the subsystems of pure mathematics," which can be understood as simplifications of the causal sequence. Brouwer posits a hierarchy of conjoined temporal, numerical, causal and mathematical systems all of which fall under the purview of the *primordial intuition of mathematics.* Mathematical abstraction receives its impetus from this projection of causal sequences into the empty forms of pure mathematics which are easier to manipulate because they have been stripped of their material contents. The causal sequence in turn receives clarification and systematic stability by being projected into mathematical abstraction. Natural laws and scientific hypotheses emerge as a result of the formalization of causal sequences by the mathematical system. The global unity of science and mathematics is also founded on this projective mechanism.

> The effectiveness of mathematical abstraction is based on the fact that many causal sequences are much easier to control if they are *projected* on subsystems of pure mathematics, that is, if their empty abstractions are

[251] Brouwer 1998, 46.

> embedded in more extended pure mathematical systems. In this way the relations existing within the extended system can also be used to provide an overview of the more restricted system, which often results in a drastic simplification of the latter. This is the way *scientific theories* come into being, where not only the elements of the causal system appear as a *"hypothesis"* but also the pure mathematical system that played a centralizing role in the overview.[252]

The progressive abstraction of the mathematical system is also a simplification of the "restricted systems" of causal attention or causal viewing. The crucial moment of the relation between causal attention and mathematical abstraction is the reaction of the mathematical "overview" on the perception of the causal sequence. *It is from the perspective of the mathematical subsystem that the scientific theory comes into being and becomes in turn a heuristic device which can then guide observation.* Brouwer, as Cavaillès observes, draws our attention to the development and becoming of the "intelligible sequence" of mathematical abstraction which finds itself engaged in a productive dialectic with the causal sequence. Mathematical intelligibility is not only the form of scientific intelligibility in general; it is the progressive motor of its development. Those scientific theories which are considered to be *exact*, that is, perfected as descriptions of causal mechanisms, are characterized by three features.

> These theories first of all relate to especially stable causal sequences (perceived as natural laws or called into being artificially as technical facts); second, their hypotheses produce a considerable simplification, and third, the causal sequences in their case correspond to special values of numerical parameters whose full value domain belongs to the mathematical system on which they are projected.[253]

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[252] Brouwer 1998, 46.
[253] Brouwer 1998, 46-47.

An exact scientific theory is a hybrid of “an especially stable causal sequence” and “the special values of numerical parameters” proper to a given mathematical system. The three parts of a scientific theory are 1) the causal sequence (which can be a natural observable sequence or an artificial technical construct;) 2) a simplified expression, in mathematical notation, of the observable causal sequence; and, 3) an autonomous mathematical domain which codifies the organization of the physical variables according to the laws of algebraic transformation. Brouwer’s primordial mathematical intuition has the merit of constructing a more nuanced presentation of the relationship between mathematics and natural science than Kant’s philosophy would seem to allow. Although Brouwer clearly makes mathematics the organon of the sciences (and, in the final analysis, of all problem-solving activity), Cavaillès is not convinced that primordial mathematical intuition differs in any significant way as a theory of science from the underlying philosophical commitments of the philosophy of consciousness.

Brouwer’s intuitionist program seeks not only to remake mathematics on the basis of primordial mathematical intuition but also to reframe the significance of logic relative to mathematics. Brouwer must amputate that part of the Kantian legacy which models the primary forms of reasoning on logic. The result is that logic in the intuitionist program must be subservient to mathematical constructions. Logic is equivalent to the description of an already accomplished cognitive act. It is not, however, able to describe the nature or necessity of this act. Mathematics must instead be reformulated on the grounds of intuition (mathematical attention and the mathematical act as extensions of the problem-solving will.) Most importantly for Cavaillès is the theory of science which proceeds from Brouwer's intuitionist doctrine. What is immediately apparent is the lack of a truly autonomous mathematical or scientific domain in Brouwer's epistemology. Everything in Brouwer proceeds from what might well be described as a mathematical will to power. Brouwer's intuitionism remains a refined version of the philosophy of consciousness insofar as mathematical and scientific developments are referred to the elementary forms of time consciousness and mathematical attention.

> …the reflection on the essence of mathematics – which founds the intuitionist norm – must be situated in relation to the spontaneous movement of its becoming, since it demands an amputation or rather a redress of classical mathematics. It is a question of knowing whether there is a reference to an absolute consciousness characterizable in other respects, to the content of concepts submitted to a dialectic which itself can be apprehended, or finally to the irreducible specificity of the mathematical movement.[254]

There is a specificity to mathematical movement in Brouwer, but it is not derived internally from mathematics. Primordial mathematical intuition is based on the awareness of time and is developed in order to extend the influence of man. Mathematics is a goal-directed behavior constantly interacting with anthropological and historical factors which drive the forms of its abstraction and which solicit its problem solving capabilities. Accordingly, there can be no truly immanent theory of mathematics or of science. On the contrary, what Brouwer's intuitionism establishes is the very form of the cognitive effort to make the world rational. For Brouwer primordial mathematical intuition is generated by a problem-solving orientation

[254] Cavaillès 1970, 368.

towards the world. The fundamental orientation of mathematics and science is not a matter of securing the objective rationality of the world or of experience (as in Brunschvicg's epistemology) but of refining and expanding the sphere of man's influence through the continued development of his technical, scientific, and mathematical tools.

Mathematical attention is the phenomenological foundation of techno-science and mathematics for Brouwer. However, as was the case with Kant, the primacy of consciousness in Brouwer fails to attend to the formal ontology of mathematics and science. More precisely, Brouwer's identification of mathematics with "the ordering of the world" unfortunately seems to take both "the world" and rationality for granted. By making mathematics and science practical extensions of the problem-solving will, Brouwer too hastily assimilates the developmental sequences of mathematical reason to the requirements of subjectivity. As a result of this unwarranted integration of the theory of science within the general sphere of a theory of practical action, Cavaillès concludes that Brouwer is unable to sufficiently differentiate the rational procedures of mathematics and the natural sciences.

> …then the second question poses itself: how to distinguish the mathematical movement within the general advance of science or even of culture? 'Mathematics,' says Brouwer, 'is the ordering of the world, rational thought of the world' – without submitting the terms 'world' and 'rationality' to any further critique. The relation to physics in particular remains vague. If every science is defined as a type according to Brouwer's rule, it is because again mathematics serves as an organon, but to such an extent that it absorbs the rest.[255]

[255] Cavaillès 1970, 368.

Mathematics may well be the organon of the sciences but this cannot be affirmed at the cost of reducing one set of rational procedures to the requirements of another. Cavaillès' theory of science is fundamentally opposed to genetic hierarchies of the forms of scientific rationality. Indeed, in Cavaillès' estimation the theory of science is called upon to account for the immanent specificity of each branch of science. A truly immanent theory of science, as Cavaillès' critique suggests, does not presuppose the ready-made unity or uniformity of scientific rationality. The relationship between mathematics and physics is a particularly vexing problem from this perspective and it must be noted that Cavaillès never offers a robust account of how he understood this relation. The relative simplicity of Brouwer's unified image of science, therefore, arouses Cavaillès' suspicion, as when he writes:

> It has been observed that intuitionist mathematics is particularly useful in modern physics. Is it not because the intuitionist attitude is first of all the attitude of the physicist, because the representation of a world, in which the mathematical act is performed and which determines it by furnishing it with at least a point of departure and a point of application, remains a primordial and perhaps a dominating element? The independence of a direct theory of science thus finds itself rather gravely handicapped.[256]

The intuitionist program lends itself so readily to the "attitude of the physicist" because the nature of the mathematical act is already understood to be oriented towards the performance of an act which solves a problem in the world, namely, extending the ability to frame and manipulate causal sequences. When Cavaillès speaks of a primordial and dominating element in the intuitionist program he means to highlight the representative aspect of the theory. Mathematics takes its

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[256] Cavaillès 1970, 368.

place between a primordial intuition which generates it and a technological over-determination which reifies it. It is entirely at the service of an instrumental will, and consequently, can claim no strictly internal motive or developmental dynamism. Intuitionism, in the final analysis, is a poor candidate for a theory of science because it is unable to grant any autonomy to the rational development of mathematics or science. Does Brunschvicg's philosophy fare any better?

The Progress of Consciousness: Brunschvicg's Critical Idealism

The previous chapter argued that for Brunschvicg a transcendental determination of the nature of scientific reason is impossible. By the same gesture the profile of scientific rationality itself must be subject to a constant and progressive revision in accord with the historical development of scientific intelligence. The philosophy of science, as Brunschvicg understands it, attempts to reflect the modality of experimental judgment proper to science. Philosophy can be an organ of reflection for science, but it cannot claim to dictate the nature of science. Brunschvicg's commitment to the immanence of the epistemological norms of science would seem to make his philosophy a better candidate for a suitable theory of science than Brouwer's excessively instrumental account. As demonstrated in the previous chapter, Brunschvicg subjects the critical philosophy itself to critique by bracketing the a priori forms of intuition in favor of an immanent description of the forms of intelligence. Brunschvicg attempts to replace Kant's pure forms of a priori intuition with a "relativized" epistemology fully conscious of the place of the observer in the system of experimental judgment which defines scientific intelligence at any given moment in its historical development.

According to Hourya Benis-Sinaceur, Cavaillès inherits a great deal from Brunschvicg's critical appropriation of Kant.[257] Both thinkers acknowledge the necessity of modifying the Kantian epistemological framework in order to elaborate a properly historical epistemology of mathematics which is capable of accounting for the necessarily progressive nature of mathematics. Both thinkers also emphasize that transformations of mathematical content are unpredictable but nevertheless grounded in a conceptual system which is anything but contingent.[258] The crucial element of any epistemology of mathematics will therefore be an interrogation of *the developmental sequences* of mathematical concepts. The emphasis on development necessitates the dismantling of Kant's schematism which would predetermine the possibilities of mathematical creativity from the outset. Brunschvicg rejects the a priori forms of intuition but maintains, so Cavaillès claims, an unnecessary reference to a creative consciousness constantly interrogating experience. As Sinaceur notes, both thinkers also share a method of "critical reflexivity" by which materials from the history of mathematics are subjected to epistemological criticism in order to outline a system of internal necessity highlighting the "internal rationality" of the historical sequence. Finally, Cavaillès

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[257] Hourya Benis Sinaceur, "From Kant to Hilbert: French Philosophy of Concepts in the Beginning of the Twentieth Century," in *The Architecture of Modern Mathematics: Essays in History and Philosophy,* eds. José Ferreirós and Jeremey J. Gray. (New York: Oxford University Press, 2006), 312.

[258] See Cavaillès 1938, 176, on the distinction between historical sequence and rational unity.

shares Brunschvicg's programmatic rejection of the reducibility of mathematics to logic.

Pierre Cassou-Noguès, however, claims there is a basic incompatibility between Brunschvicg and Cavaillès at the level of their fundamental epistemological commitments. For Brunschvicg, philosophy is called upon to interrogate the form of reason immanent to science, a programmatic ambition Cavaillès fully endorses. The problem, however, is that Cavaillès and Brunschvicg do not share the same understanding of the relationship between science, experience and reason. Cassou-Noguès claims that for Brunschvicg "science is the process of the objectification of experience by reason" and the analysis of the previous chapter largely supports such an interpretation.[259] For Cavaillès, however, it is impossible to characterize science (here understood primarily as mathematics) as a merely reflective organ which objectifies experience through reason. Mathematics must instead be approached as its own experience and not as a subservient undertaking which participates in a more general construction of objectivity. Cavaillès does not intend to deny objectivity to mathematics, but he cannot with Brunschvicg incorporate the objectivity proper to mathematics within a more general elucidation of experience. In a meeting of the Société Francaise de Philosophie in 1912, Brunschvicg, in response to a question from Emile Meyerson, presented his view of the essential unity of mathematics and physics as codetermining the progressive objectification of experience:

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[259] See Cassou-Nogués 2001, 259.

> For me, there is not a rupture between mathematics and physics as one would imagine between an abstract and formal discipline on the one hand and the study of the concrete on the other; there is not even a rupture between analysis and geometry… Arithmetic prolongs the empirical practices which construct step by step the norm of truth which makes science.[260]

Brunschvicg's critical idealism grasps mathematics and positive knowledge as part of the same progressive elucidation of experience. Arithmetic need not be opposed to empirical practices but is in fact part of the same construction of the norm of truth which science elaborates.

Cavaillès cannot accept this smooth integration of the necessity of mathematical becoming into the progressive rationalization of experience. Objectivity is not sufficiently differentiated in Brunschvicg's critical idealism, leading Cavaillés to a critique of Brunschvicg which is very similar to the objection leveraged against Brouwer.

> The being of the world, of a world posited externally and which it is the vocation of consciousness to reduce to terms of interiority, subsists as a determining condition of science. To know the world, to comprehend the world – here is a program which already represents an abandonment of creative autonomy, a surrender to a necessity which is related to nothing other than itself.[261]

Brouwer and Brunschvicg locate the developmental necessity of science and mathematics in an impulse which proceeds from a desire to know and to manipulate the world. For Cavaillès, in both cases the theory of science (despite its claims to immanence) reveals itself to be a fundamentally instrumental account of the genesis

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[260] Léon Brunschvicg, "L'idée de la Vérité Mathématique" in *Écrits philosophique tome III* (Paris: Presses Universitaires de France, 1958), 106.
[261] Cavaillès 1970, 369.

and development of mathematical objectivity. For Brunschvicg, mathematics is a rational procedure which is part of the objective elucidation of experience and, above all, a testament to the progressive development of rational consciousness. Cavaillès, however, is committed to the "creative autonomy" of the development of mathematics and cannot entertain a philosophy of the open ended "objectification" of experience.

The unification of the parts of experience is also the unification of science in the name of the objectification of experience through the epistemological norms of a progressive consciousness. Brunschvicg understands the history of science as the history of a progressive consciousness. Indeed, the history of science itself is the purest expression of the normative element of consciousness: the necessity of progressive transformation. In the closing pages of *Les Étapes de la Philosophie Mathématique,* Brunschvicg offers a compelling vision of the rational unity of mathematics and physics which begins to emerge in the nineteenth century:

> The unity of mathematics will be established here not with the help of a pre-existing concept, such as the concept of quantity, which will necessarily be too general and so almost void of meaning, but by a play of effective relations which make of a series of distinct disciplines an organic whole, by a *conjunction* of systems of *connections*. The scholastic monism which was modeled on Aristotle, the dream of which had been revived by Leibniz, is definitively replaced by the monism of immanence*, for which the intelligence appears to be a power of indefinite unification* [my italics].[262]

The reality of mathematics is to be found in the indefinite power of rational unification of which it is the expression: *mathematics is the sign of the intelligence unifying experience according to the norm of an increasingly rational objectivity and*

[262] Brunschvicg 1912, 550.

an increasingly objective rationalism. The objective and the rational do not differ essentially for Brunschvicg although they are mutually correcting in order to avoid the pitfalls of an uncritical realism (or an objectivity without internal rational consistency) or a transcendental idealism (a rationalism without possibility of progressive transformation.) For Cavaillès, this is the highest achievement of the philosophy of consciousness. At the same time, it is not a suitable theory of science because it fails to recognize the autonomy of the creative development of mathematics *by instead nominating consciousness as "the power of indefinite unification."* Cavaillès can therefore write:

> The progress of consciousness is undoubtedly detected not externally through an increase in the volume of its results, but through an intensity by means of the spiritual enrichment that refined concepts attain or through a unification of disciplines previously separated. It remains that progress by its very essence appears [only] after it is realized, that as a consequence the science which as such detects it on the one hand locates itself with difficulty, and on the other cannot claim to dominate science. This is moreover not Leon Brunschvicg's wish: rational immanence eludes every grasp outside the idea.[263]

Brunschvicg and Cavaillès pursue intimately convergent epistemological projects but they part ways regarding the objectivity of mathematics. For Cavaillès, mathematical objectivity is objective precisely insofar as it makes no reference to consciousness or lived experience. Following Brunschvicg mathematics constitutes its own domain of experience but this cannot be construed in the manner of a progressive rationalization of lived experience, as Brunschvicg's philosophy of consciousness affirms. In identifying the core of Brunschvicg's philosophy with the

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[263] Cavaillès 1970, 369.

progress of consciousness (also the title of Brunschvicg's last major work), Cavaillès points toward the "intensity" of "spiritual development" which characterizes the progressively deeper syntheses of rationality and objectivity brought about by consciousness. The immanence of the epistemological norm is also the immanence of a consciousness transforming itself according to a dynamism which cannot be reduced to the enlightened progress of science, which would be its symptom and not its cause. The causal orientation of Brunschvicg's philosophy points away from a theory of science and toward a "rational immanence" which must escape "every grasp outside the idea." For Cavaillès, Brunschvicg's philosophy of consciousness ultimately discloses a Spinozist epistemology of the third kind of knowledge (the system of relations between ideas of ideas, which can in turn only produce adequate notions) resulting again in an ideally unified system of the sciences immanently derived from the totalizing grasp of a progressive consciousness.

> The terms of spirituality and immanence presuppose the possibility of an asceticism or of a deepening of consciousness other than simply scientific comprehension. What is needed is either the absolute of intelligibility which legitimizes the Spinozist superposition of the idea of an idea, or the reference to a generative consciousness which has the property of grasping itself immediately in its authentic acts. In both cases an ontological analysis seems necessary. On the other hand no answer is given to questions of epistemology. Here again all the sciences are intertwined in the same movement. In particular, it is impossible to distinguish mathematics and the science of nature.[264]

Cavaillès claims that the philosophy of consciousness must in turn ground itself in "the absolute of intelligibility" (the idea of the idea) or in an immanent surveillance

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[264] Cavaillès 1970, 369.

of "its authentic acts" if it is to remain legitimate in its claim to be the origin of the developmental process of mathematics and science. The first option is modeled on Spinoza's metaphysics of Substance as described in the *Ethics* and Spinoza's epistemology as briefly outlined in *The Emmandation of the Intellect.* The second option is modeled on Husserl's transcendental logic and phenomenology as developed in the *Logical Investigations* and *Formal and Transcendental Logic*. Cavaillès will develop a theory of conceptual transformation in mathematical systems which is closely modeled on "the Spinozist super-position of the idea of the idea" in the third section of the essay. He will subject Husserl's transcendental logic to a thorough critique in the fourth and final section. It therefore seems fitting that while Cavaillès presents the philosophy of consciousness as unsuitable as a theory of science in its contemporary variations in the first half of the essay, he points toward possible resources for its rehabilitation in the second half.

Kant, Brouwer and Brunschvicg suffer from an inability to pose the problem of an ontology of science in terms which do not reduce the objectivity of science to a set of claims about the intelligibility of science ultimately rooted in a philosophy of consciousness. Additionally, neither Brouwer nor Brunschvicg is able to answer the epistemological questions raised by the efficacy of mathematical objectivity. They do not seize the autonomy of mathematical development by the contours of its true objectivity, nor do they provide adequate descriptions of the regional variations of scientific rationality, defaulting instead to an intuitively appealing but disingenuous unity of science which proposes to understand the generation of all mathematical and scientific knowledge either instrumentally (Brouwer) or as the result of a creative consciousness objectifying experience through the application of experimental judgment (Brunschvicg.) Cavaillès thus claims to have demonstrated the inadequacy of the philosophies of consciousness descending from Kant which present mathematics as the organon of the sciences. Cavaillès encounters a theory of science which seems to avoid the pitfalls of the philosophy of consciousness in the work of Bernard Bolzano (1781-1848), a mathematician, logician, and philosopher whose ambitious reform of logic directly contests Kant's prohibition that logic cannot be the organon of the sciences. For Bolzano, the theory of science is a theory of the logical proposition, a novel synthesis which will prompt Cavaillès to propose his own theory of rational sequences as the indispensable motor of the theory of science.

Wissenschaftslehre: Bernard Bolzano and the Autonomy of the Theory of Science

Bernard Bolzano's *Wissenschaftslehre* (1837) provides Cavaillès with a theory of science which has the merit of explicitly asserting the autonomy of science. Bolzano also has the distinction of offering an alternative configuration of the relationship between logic and science, one which eschews the delimitation of scientific objectivity according to the a priori forms of possible experience as furnished by Kantian intuition. Bolzano is concerned with elaborating a theory of scientific objectivity which avoids any reference to intuition because, as this section will show, the recourse to intuition un-necessarily forecloses the possibility of a form of mathematical objectivity entirely independent of the thinking subject. Accordingly, when Cavaillès reconstructs Bolzano's theory of science he will primarily emphasize Bolzano's characterization of the conceptual autonomy of scientific knowledge and the *theory of logical or rational sequences* Bolzano constructs in order to define the procedural dynamics of this entirely independent domain. Bolzano also serves a strategic function in Cavaillès' essay because he marks a decisive threshold in the text, bracketing the critical first half of the essay, marked by its almost exclusively polemical orientation, from the second half of the essay, characterized by the compact presentation of the conceptual mechanisms driving theoretical progress in mathematics.

Cavaillés recognizes in Bolzano an important precursor to his own effort to present an autonomous theory of science which would also be immanent to science. Of particular importance in Cavaillès' reconstruction of Bolzano's work is the identification of logic with the theory of science. Bolzano's *Contributions to a Better-Grounded Presentation of Mathematics* (1810) and *Theory of Science* (1837) constitute the two primary sources Cavaillès will draw upon in his work. Between 1810 and 1837 Bolzano gradually arrives at a conception of scientific objectivity which is unique in that it establishes the necessity of scientific objectivity without recourse to Kantian intuition. Bolzano models the autonomy proper to science on his understanding of the objectivity of mathematical constructions. Therefore, before turning to Cavaillès' presentation and critique of Bolzano it will be necessary to briefly summarize how Bolzano transforms the accepted sense of the objectivity of mathematics, what he understands by the theory of science, what he understands by logic, and finally, what he understands by the *proposition in itself* as the fundamental unit of the theory of science.

Bolzano's *Contribution to a Better-Grounded Presentation of Mathematics* notes that even for Euclid a clear presentation or definition of the nature of mathematical science did not seem to be of any immediate concern, for the *Elements* proceeds with its demonstrations without troubling itself about irrefutable analytic foundations. The modern characterization of mathematical science, on the contrary, is perhaps excessively conscious of the need to apply to mathematics clear definitions of an intuitive character, for example, delimiting mathematics as the science of quantities. Kant, as Bolzano well knew, found fault with this definition of mathematics "because in it, as he says, *'no essential characteristic* of mathematics is stated, and *the effect* is also mistaken *for the cause."*[265] The uncritical definition of mathematics is not yet a knowledge of the cause of mathematical objectivity, or what amounts to the same, it is not yet a presentation and defense of the possibility of a priori synthetic judgments, upon which the edifice of the concepts of natural science will be constructed. Bolzano shares with Kant the desire to better understand the nature of mathematical objectivity both for its own sake, and eventually, in the *Theory of Science*, in order to delimit the nature of scientific objectivity. For Bolzano, however, the Kantian philosophy and its adherents make

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[265] Bernard Bolzano, "Contributions to a Better-Grounded Presentation of Mathematics," in *The Mathematical Works of Bernard Bolzano* ed. Steve Russ. (New York: Oxford University Press, 2004), 91.

the mistake of claiming to be able to construct all the concepts of mathematics in pure intuition.

> How dubious the Kantian theory of intuition is, becomes particularly clear if it is extended to other propositions outside *geometry*. The principle of sufficient reason, and the majority of propositions of arithmetic are, according to *Kant's* correct observation, *synthetic propositions*. But who cannot feel how contrived it is, that *Kant*, in order to carry through his theory of intuitions generally, has to assert that even *these* propositions are based on intuition, indeed (for what else should it be?) the *intuition of time.* Yet the *principle of sufficient reason* also holds where there is no time, and (according to a remark that has already been made) it was only as a result of *this* proposition that *Kant* himself accepted the existence of the *noumena* which are not in time. The propositions of *arithmetic* do not require the intuition of time in any way.[266]

According to Bolzano, nothing less than the principle of sufficient reason refutes the intuition of time as the foundation of mathematical constructions. The objectivity of mathematics cannot have its origin in pure intuition because the principle of sufficient reason itself does not abide by this constriction and unless we are willing to exclude the principle of sufficient reason from mathematics we must correspondingly enlarge the domain of mathematical objectivity. The propositions of arithmetic are not dependent on time because they are ultimately irreducible to the experience of *cognitive acts*, as Bolzano will show. There is in fact "an intrinsic contradiction" in the very notion of *pure intuition* in Bolzano's reading of Kant because the first critique fails to provide convincing criteria of demarcation between *empirical* and *a priori* concepts. Bolzano writes that for Kant the following is evident:

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[266] Bolzano 2004, 135.

> [a]ll objects must have a *form*, but they need not possess *colour, smell,* and the like. I would answer that not *all* objects which may *appear* to us must possess a *form*, but only those which appear to us as *external*, i.e., in *space*. But even these must then also have something which *occupies* [*erfullt*] this form, and this, due to the particular nature of our perceptive faculties, can only be one of the following five things, either a *colour*, or a *smell* etc. Therefore, colour, smell, etc., are also *a priori* forms in the same sense of the word as space and time, except that the *range* to which the former relate is narrower than that of the latter, just as the form of *space* has a narrower *range* than that of *time.* Our conclusion is that among *concepts* there is no justifiable distinction according to which they could be divided into *empirical* and *a priori*: instead they are all *a priori.*[267]

Bolzano refuses Kant's distinction between empirical and a priori concepts based on the observation that even the diversities of sense data are pre-determined in the same way that the pure forms of intuition are pre-determined. The only issue is the *range* of objects to which these a priori determinations of experience can be applied. All objects which are extended in space must have both form and "something which occupies this form." Objects which do not appear extended in space but are rather representations in our imagination cannot be said to obey the same restricted range of possible forms. In fact, in opposition to Kant's doctrine, if all concepts are a priori then the concepts of our external representations are *more restricted* than the concepts of our internal representations. Kant's philosophy would circumscribe the objectivity of mathematics by limiting it to the internal form of representation according to the a priori intuition of time, but even this is too narrow a definition of *form* and of *representation* for Bolzano, who will instead offer a provisional definition of mathematics as the *general science of the conditions of possibility of form.*

[267] Bolzano 2004, 136 – 137.

Rather than locating mathematics within the transcendental determination of the possible forms of experience as given through intuition, Bolzano will propose a radical reversal of perspective: mathematics is the general science of forms "to which things must conform in their existence."

> I therefore think that mathematics could best be defined as a *science which deals with the general laws (forms) to which things must conform in their existence.* By the word '*things*' I understand here not merely *those* which possess an *objective existence* independent of our consciousness, but also those which simply exist in our *imagination*, either as *individuals* (i.e. *intuitions*), or simply as *general concepts*, in other words, *everything which can in general be an object for our capacity of representation [Vorstellungsvermögens*]. Furthermore, if I say that mathematics deals with *the laws to which these things conform in their existence,* this indicates that our science is concerned not with the proof of the *existence* of these things but only with the *conditions of their possibility.* In calling these laws *general*, I meant it to be understood that mathematics never deals with a single thing as an *individual* but always with whole *genera* [*Gattungen*]. These genera can of course sometimes be higher and sometimes lower, and this classification of mathematics into *individual disciplines* will be based on this.[268]

Mathematics is a general science of forms which at its most elementary level does not need to distinguish between different *modes* of existence. At this level of supreme generality the only requirement is the demonstration of logical possibility, not the actual existence of the object in cognition or in experience. This science is concerned with the most general capacity of representation regardless of the "proof of the existence of these things." Mathematics, as Bolzano states elsewhere, is concerned with *hypothetical necessity* rather than the *absolute necessity* treated by metaphysics, which must also entail proofs of the existence of the objects it deems necessary, as in the ontological proofs of the necessary existence of God.

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[268] Bolzano 2004, 94.

Mathematics on the contrary, at its foundational level, need not prove the existence of its objects, only the logical possibility of its objects. Therefore, like Kant, Bolzano pursues a critique of the conditions of possibility of mathematics. Unlike Kant, however, he does not think the *logical possibilities of that which can be represented* are given in any way by intuition. The structure of that which is logically possible in the most general sense describes the fundamental domain of mathematical science. Bolzano calls this science a *general mathesis* and claims it defines the logical structure of things in general, regardless of their objective or imaginary existence. General mathesis is followed by the domains of the particular mathematical disciplines, each of which describes *unique logical structures* proper to different kinds of logical objects. The mathematical disciplines in turn are divided into many sub branches among which Bolzano includes the theory of time (chronometry) and the theory of space (geometry). Taken together, the joint theories of time and space furnish the logical possibility of a theory of sensible things. The theory of sensible things *in concreto* includes a general theory of *sensible things in time*, and finally, at the most concrete deductive level of the logical architecture of mathematics, the *theory of pure natural science* or the theory of sensible things in time and space.

Figure 1. <u>The Logical Structure of Mathematical Objectivity</u>

General Mathesis
(logical structure of things in general)
⌄

Individual Mathematical Disciplines
(logical structure of particular things)
⌄

Abstract Theory of Sensible Forms
⌄

Theory of Time (chronometry)	**Theory of Space (geometry)**

⌄

Theory of Sensible Things in *Concreto*
⌄

Theory of Sensible Things in Time (sensible things in time)	**Theory of Pure Natural Science (sensible things in time and space)**[269]

As the above figure demonstrates, Bolzano does not locate time and space as the necessary and sufficient conditions of possibility of arithmetic or geometry. On the contrary, Kant's a priori forms of intuition occur at the *fourth level of the articulation of a general mathesis* of logical possibilities. Kantian intuition would be a sub-category of the abstract theory of sensible forms in Bolzano. Entire mathematical disciplines *which make no reference to space or time* precede the abstract theories of sensible forms and encapsulate them as *possibilities*. Mathematical objectivity in Bolzano comes to depend on the logical connections between different levels of generality and particularity rather than on a

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[269] For a more detailed table of Bolzano's presentation of the hierarchy of mathematical objectivity see Bolzano, 2004, 102.

transcendental and pre-formatted correspondence between the logical possibilities of representation and the possible forms of knowledge, as in Kant. The general presentation of the structure of mathematical objectivity as Bolzano understands it "does not deal at all with what actually *takes place* but with the *conditions* or *forms* which something must have *if it is to take place."*[270] Objectivity and actuality cannot be confused with one another. There is a logical structure of objectivity which remains valid, that is to say, *objective as such* even in the absence of all empirical states of affairs. For Bolzano the problem of the objectivity of mathematics thus comes to impinge on traditional notions of truth, for in considering the order and connection of the levels of mathematical objectivity Bolzano discovers that the connections between "true judgments" at a determinate level of mathematical abstraction constitute the only truly rigorous grounds of a purely scientific theory of truth.

> …in the realm of truth, i.e. in the collection of all true judgments, a certain *objective connection* prevails which is independent of our accidental and *subjective recognition* of it. As a consequence of this some of these judgments are the grounds of others and the latter are the consequences of the former. Presenting this objective connection of judgments, i.e., choosing a set of judgments and placing them one after another so that a consequence is represented as such and conversely, seems to be the real *purpose* to pursue in a scientific exposition. Instead of this, the purpose of a scientific exposition is *usually* imagined to be the greatest possible *certainty* and *strength of conviction.* It therefore happens that the obligation to prove propositions which, in themselves, are already completely certain, is discounted. This is a procedure which, where we are concerned with the practical purpose of certainty, is quite correct and praiseworthy; but it cannot possibly be tolerated in a scientific exposition because it contradicts its essential aim.[271]

[270] Bolzano 2004, 100.
[271] Bolzano 2004, 103.

Truth, rigorously conceived, becomes an effect of the correct presentation of the objective connections which obtain between judgments. The connection between objectivity and truth is nothing other than the logical validity of this connection "which is independent of our accidental and *subjective recognition* of it." The connection between the logical structure of mathematical objectivity and the theory of science is also clearly marked by Bolzano in this passage. The presentation of the "objective connection of judgments…seems to be the real *purpose* to pursue in a scientific exposition." Although Bolzano has not yet fully developed his theory of science in this text from 1810. he has already clearly identified its *purpose* - the theory of science will be concerned with the proper exposition of the objective connection obtaining among true judgments in a given scientific discipline.

The problem of the correct presentation of the objectivity of mathematics serves as the occasion for the discovery of another problem: the difference between the "real purpose" of a scientific exposition and the "imagined purpose." The former concerns the presentation of the objective connection between judgments in their proper order, the latter concerns "the greatest possible *certainty* and *strength of conviction."* If the true purpose of a scientific exposition is the presentation of the objective relations which obtain among propositions rather than the *persuasiveness* or *certainty* of the truths themselves, it is quite natural to wonder how one then demonstrates the truth of a given judgment. Bolzano here turns to the problem of *mathematical proofs.* The *objectivity of proof* has a general and a restricted definition, as Bolzano writes:

> …what should really be understood by the *proof* of a truth? One often calls any sequence of judgments and deductions [*Schlüsse*] by which the truth of a certain proposition is made generally *recognizable and clear*, a *proof of the proposition.* In this *widest* sense, *all* true propositions, of whatever kind they may be, can be proved. We must therefore take the word in a *narrower* sense and understand by the *scientific proof* of a truth the representation of the *objective dependence* of it on *other truths*, i.e., the derivation of it from those truths which must be considered as *the ground for it* – not fortuitously, but *actually and necessarily* - while the truth itself, in contrast, must be considered as their consequence [*Folge*].[272]

In the most general sense a proof of a proposition is any series of deductions which makes the proposition "recognizable and clear." Proof in the widest sense is simply the presentation *of one logical possibility leading to the deduction of the proposition.* The narrower or restricted sense of proof which Bolzano claims is proper to *scientific proof* duplicates the real purpose of scientific exposition: scientific proof is the demonstration of the objective dependence of true propositions among themselves. The truth itself is pre-supposed but not demonstrated by proof in the general sense, for as Bolzano writes "*all* true propositions, of whatever kind they may be, can be proved." Scientific proof, however, does not need to pre-suppose the truth because it in fact *produces it as a consequence*. The final sense of mathematical objectivity is therefore this restricted definition of truth as the consequence of the *objectivity of relations obtaining among judgments.*

The *Theory of Science* (1837) will explicitly take up the problem of the true purpose of the exposition of science in precisely these terms. For reasons which should now be clear, Bolzano does not propose a foundational account of science because he is not concerned with proving the certainty of science from an external

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[272] Bolzano 2004, 110.

or transcendental perspective. He is exclusively concerned with the *objectivity* of science in the narrow sense of truth as the necessary consequence of a logical sequence. The theory of science does not pre-suppose the truth in itself – it diagrams the objective relations which must obtain between propositions in order that the truth may be expressed, in turn, as an effect of the logical order of propositions. The theory of science is concerned with the *exposition* and *representation* of scientific truths. It points toward a codification of the truths of science into coherent categories which are implied by these very scientific truths, but which science itself may not have recognized as such.

> …by theory of science I mean the aggregate of all the rules which we must follow when we divide the total domain of truths into individual sciences, and represent them in their respective treatises, if we want to do a competent piece of work. Actually, it is evident that a science which is to teach us how we should represent the sciences must also teach us how to divide the total domain of truths into individual sciences, since a science can be properly represented in a treatise only if its boundaries are well defined. Hence we can express our definition of the theory of science more briefly by saying that it is the science which instructs us in the representation of sciences in adequate treatises.[273]

Bolzano distinguishes three levels of organization proper to the systematic exposition of scientific knowledge. 1) A primary level of truths themselves which would seem to correspond in the theory of mathematical objectivity to *General Mathesis* or the logical structure of things in general. 2) A secondary level of the initial differentiation "of the total domain of truths into individual sciences." 3) The theory of science itself as that "which instructs in the representation of sciences in

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[273] Bernard Bolzano, *Theory of Science* Trans. Rolf George (Berkeley: University of California Press, 1972), 2-3.

adequate treatises." Bolzano repeatedly indicates the significance of the theory of science in terms of *exposition, representation, and division.* The emphasis on the *discursive representation* of science is at once methodological and ontological. The method of the theory of science will be the logical representation of the objective relationships obtaining between propositions. The theory of science is an ontological affirmation of a special kind because Bolzano does not think scientific truths are reducible to actual states of affairs or donations from intuition. Finally, Bolzano's motivation is eminently pedagogical. He is looking for a method of instruction which preserves the objectivity of science without distorting its nature or invoking philosophical obscurities as explanations of logical necessity. In the end it is a matter of ensuring that the scientist has access to "adequate treatises" in order that he or she can "do a competent piece of work."

Because the presentation of the objective or logical structure of science is itself a scientific endeavor (thus making the theory of science a science of science), Bolzano entertains the possibility that the reader might reject at the outset the very possibility of the theory of science on the grounds that (as a science itself) it seems to presuppose or produce itself spontaneously. The problem of the relation of the theory of science to the actually existing state of science will also be a touchstone of Cavaillès' analysis, but Bolzano thinks he can provide a suitable response to this objection:

> One may perhaps doubt that the proposed theory of science is *possible.* Since, according to the above definition, the theory of science is to teach us how the sciences ought to be represented, and since it is itself a science, one might ask how it can be produced, since one does not know how any science is to be

> represented as long as there is no theory of science. This doubt can be easily removed. One can proceed according to the rules of the theory of science and thus generate any number of sciences (or rather their written expositions), among others the theory of science itself, without being clearly aware of these rules; one can find through reflection many or all of these rules without ordering and connecting them, as it must be done in the treatise of a science. Once these rules are known, every science, including the theory of science itself, can be further elaborated and represented in writing. This amounts to no more than arranging certain known truths in an order and connection which they themselves prescribe.[274]

The theory of science can be constructed at any point from the existing state of science "through reflection" on the rules of science. The rules which science obeys are precisely those which govern the objective validity of the connections among judgments or propositions. Science need not cultivate a doctrine of these rules themselves in order to obey them, consequently, science can do its work without quite understanding the conditions of its own validity. Here the profoundly *anti-foundational* nature of Bolzano's work again presents itself. He does not claim to reform science in its deepest nature but rather to clarify the exposition of its judgments. Bolzano, like Brunschvicg, believes in a deeply motivated convergence between the normative epistemological force of scientific rationality and the theory of judgment. The clarification of the latter explains the necessity of the former. Bolzano does not doubt that the rules of science are everywhere present and determinate of the concrete forms of scientific reason. The role of a theory of scientific exposition is to sharpen the logical contours of these rules and to affirm the objectivity of logic itself. This clarifies the significance of representation in Bolzano's epistemological vocabulary, for the theory of science does not mean to

[274] Bolzano 1972, 3.

oppose representation to reality as an image would mask or distort or an essence; on the contrary, representation is equivalent to logical exposition or the discursive presentation of the logical structure of science.

The theory of science is a discipline which concerns the proper exposition of the truths proper to science. It is a science of structure and order, *an organon.* When Bolzano identifies the theory of science with logic, he immediately addresses the objection that logic cannot serve as the organon of the sciences because logic does not concern itself with first principles. The theory of science, as Bolzano understands it, is not concerned with first principles because it is not intended to be a theory of the foundational principles of science. The theory of science is not a deduction of the existing truths of science from first principles. It provides an analytic framework which distinguishes the objective necessity of logical relations from all other forms of necessity, metaphysical, theological etc. In identifying the theory of science with logic, Bolzano means only to describe the rules which prescribe the organization of the truths of science.

> …the assertion that logic is a theory of science, or even an organon, is not in the least objectionable, if it is taken to mean that it establishes the rules according to which the total domain of truth is to be divided, and each science has to be treated. It would only be false if one were to imagine that this science had to contain the first principles upon which the edifice of all the other sciences is to be erected. Logic does not treat of the principles which lie at the base of each science, but it deals with the procedure which has to be followed in their exposition.[275]

The exposition of the truths of science and the discursive presentation of the structure of the sciences is assigned to logic. Bolzano will generally call the theory of

[275] Bolzano 1972, 4-5.

science logic because it is concerned with the rules which science follows in the presentation of its own contents. Bolzano acknowledges that for Kant logic is presented as the "the mere form of thinking" or "the necessary laws of understanding and reason in general," but this is too broad a definition because it lacks what Bolzano requires of the logic and theory of science: orientation toward "the recognition of truth." Logic, as Kant affirms, is certainly concerned with the rules of thinking, but Bolzano will amend this too general definition in favor of the definition offered by Wolff: "Logic is the science of directing a faculty toward true cognition."[276]

Although Wolff's logic is superior to Kant's, it still fails to give an account "of why the rules of scientific exposition should constitute such an extremely important part of logic, even its ultimate goal."[277] Wolff does not appropriately identify the objects which constitute "the field of logical enquiry." Wolff's mistake and that of virtually every other philosopher and logician is to have restricted the field of logic to pure ideation, to thought in itself, but for Bolzano logic encompasses truths or propositions which are true whether or not "they have been thought by anyone." Thus, Bolzano can write:

> [i]f it were the case that the validity of logical rules extends not merely over propositions that are thought, but over all propositions, irrespective of whether they have been thought by anyone, then the subject of logic would be too narrowly circumscribed if it were not extended beyond thoughts to propositions in general. I hope to demonstrate in the sequel that this is indeed the case, and that the source of most errors in logic has been the lack of distinction between thought truths and truths in themselves, and between

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[276] Bolzano 1972, 7.
[277] Bolzano 1972, 7.

> thought propositions and thought concepts on one hand, and propositions and concepts in themselves on the other.[278]

Bolzano's theory of science is premised on an ontological equivocation "between thought truths and truths in themselves." The validity of logic must extend beyond thought. Science is not the conformity of our thoughts and representations; it is the conformity of our propositions to the objective validity of logic, which is true with or without us. Logic requires neither the world nor thought, but it determines the rules which both must follow if they are to be included in a logically meaningful progression, that is to say, if they are to become the objects of propositions which are true. Logic encompasses the rules which guide thinking toward propositions which are true but it also encompasses propositions *which are true in themselves.* Bolzano characterizes logic as formal because it is concerned not only with propositions which can be confirmed or refuted experientially but with propositions which entail *truth contents* which are valid independently of the experience of the thinking being. The distinction between thought propositions and propositions in themselves entails a further distinction between the *objects* of science (the objective domain which science treats) and the *contents* of science (which are the independent truth contents of propositions in themselves.)

> I don't think its necessary to guard against the misunderstanding that logic, being called formal, has no contents, and hence, since only fully determinate propositions can be true, does not contain any truths. This would be so outlandish an error that it is unlikely to be committed. Although determinate propositions are not the *objects of* the theorems of logic, yet these theorems themselves *are* propositions. One must distinguish between the *objects* of a science (which it deals with or treats) and the *content* of a science (its

[278] Bolzano 1972, 13.

> theories.) For example, the object of geometry is space, the contents of geometry are propositions about space.[279]

Historically determinate propositions are not the objects of science, although they are of course that which science is composed of at the level of its practical activity. The object of science is the domain of truth proper to a given scientific discipline; in Bolzano's example, the object of geometry is space. The proposition in itself, understood as expressing a *truth content* irreducible to practical or subjective experience, is the content of science. *The theory of science secures its claim to autonomy by applying itself to the structure of the truth contents of propositions in themselves.*

Bolzano distinguishes between the plane of the object (that which science addresses) and the plane of the contents of science (that of which science speaks) in order to secure the logical possibility that propositions might have truth contents independently of their treatment by the mind or by actually existing scientific domains. The theory of science affirms the autonomy of the objects of science and, therefore, the independent truth content of the propositions which affirm this content. The proposition is an instrument of logic, but logic in turn refers to an objective domain which is irreducible to actually existing states of affairs or mental acts. Additionally, the relationship between the proposition and the objective domain defines the logical structure of all possible truths, as Bolzano will attempt to demonstrate. First, Bolzano will clarify what he understands by the proposition in itself and truth in itself. The proposition in itself:

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[279] Bolzano 1972, 14.

> … by proposition *in itself* I mean any assertion that something is or is not the case, regardless whether or not somebody has put it into words, and regardless even whether or not it has been thought…[280]

A proposition in itself is not defined by the medium of its representation. Bolzano says it can be spoken, written, or cognized, but it need not be any of the above; all that matters is that the proposition is an assertion of truth "that something is or is not the case." The proposition in itself is not the proposition of *"something actually proposed"* (which, as Bolzano notes, requires both an object and a subject) nor is it an idea, a belief, or a judgment (although these may all express written, spoken or cognized propositions). The proposition in itself has a valid truth content because the object of science in itself constitutes an autonomous domain, a domain which the proposition affirms or denies independently of any cognitive act. Although the proposition in itself is not reducible to ideas, beliefs or judgments, it can contain all of these "as one of its parts." The proposition in itself cannot belong to any determinate proposition, but any determinate proposition may be included in the proposition itself.

The truth in itself is a kind of proposition in itself, as Bolzano will affirm, since a truth must either affirm or deny an objective domain. Bolzano thus makes that which is true a feature of the relationship between scientific propositions and objects (or between the contents and objects of science.) Bolzano also uses this distinction to distance himself from charges of Platonism and idealism, refuting the former by refusing to grant any kind of "extra-real" being to the truth contents of

[280] Bolzano 1972, 20-21.

propositions, and refuting the latter by refusing to include the proposition in itself within the possible forms of judgment or the concept.

> … I shall mean by a truth in itself any proposition which states something as it is, where I leave it undetermined whether or not this proposition has in fact been thought or spoken by anybody. In either case I shall give the name of a truth in itself to the proposition, whenever that which it asserts is as it asserts it…I shall state the following fairly obvious theorems about truths in themselves.
>
> a. Truths in themselves are a kind of proposition in itself.
> b. They do not have actual existence, i.e. they are not something that exists in some location, or at some time, or as some other kind of real thing. *Recognized* or *thought* truths have indeed a real existence at a definite time in the mind of the being that recognizes or thinks them…but to truths themselves, i.e. the contents of these thoughts, no existence can be ascribed…the logician must have the same right to speak of truths in themselves as the geometrician speaks of spaces in themselves…[281]

Bolzano's insight is to have realized the necessity of treating the *contents* of propositions in themselves separately from attributions of the actual existence of their objects. Truth contents do not need to be confirmed by actually existing things or states of affairs (although every actually existing thing or state of affairs can be expressed by a proposition in itself.) Some states of affairs will be true without ever actually existing, as in the domain of objective rational structures described by mathematics. Because the content of a proposition in itself can be true without needing to be actualized in material or cognitive acts, Bolzano is able to reframe the problem of the objective validity of science, making its authenticity the result of the connections which are logically necessary between propositions in themselves. The theory of science describes the logical exposition of these truth contents without reference to actual states of affairs. This is the ultimate sense of the autonomy of the

[281] Bolzano 1972, 32-33.

theory of science*, it is the irreducibility of objective truth contents to actually existing states of affairs or to mental or cognitive acts.* The theory of science reprises in a more radical form what Bolzano's earlier investigations into the nature of mathematical objectivity had disclosed: objectivity is a relation among logical propositions and the truths of science are truths of logic. The theory of science is a reform of the idea of truth, which, if it is to have any meaningful or objective purchase in thought or in the world, must be referred to its conditions of logical validity.

Cavaillès' understanding of the autonomy of the theory of science borrows this dual qualification of the *objective truth content* of logical propositions from Bolzano. The theory of science must bracket both the world and cognition, neither of which can be the source of its objectivity. Cavaillès ultimately rejected the "epistemologies of rational immanence" represented by Brouwer and Brunschvicg because they betrayed the "creative autonomy" of science, locating scientific objectivity as part of the progressive rationalization of experience. Bolzano's theory of science offers a compelling alternative to this view by attempting to reframe the objectivity of science in terms of the radical immanence of logical propositions to objective domains *which are themselves logical propositions.* Bolzano seems to provide Cavaillès with an epistemological framework capable of finally extricating the theory of science from the oversight of the philosophy of consciousness. The transformation of the theory of science into an autonomous field is premised upon a novel presentation of the necessity of science which now draws its claim to objectivity from an internal logical framework and which also transforms science into what Cavaillés calls "demonstrated theory."

> Emphasis on the necessary character of science is precisely Bolzano's overriding concern. Science is above all demonstrated theory. Bolzano in the *Reine Analytischer Beweis* took issue with the overly facile recourse to evidence, an evidence that Descartes invoked only for the simple natures and which for Kant regulated all geometric reasoning. The crisis of [the] growth of algebra then required a change in the type of evidence, as in shown in the *Stages of Mathematical Philosophy* [Brunschvicg]. For Bolzano, it was a matter of the radical transformation of verification into demonstration.[282]

The *Reine Analaytischer Beweis* (1817) attempts to provide a purely logical proof of two propositions from the theory of equations which Bolzano claims still lack "a perfectly correct proof."[283] The first proposition asserts "between every two values of the unknown quantity which gives results of opposite sign there must be at least one real root of the equation." The second proposition asserts "every algebraic rational integral function of one variable quantity can be decomposed into real factors of first or second degree[284]." Bolzano claims that the existing proofs of these propositions are inadequate because they rely on evidence based on geometrical intuition:

> The *most common* kind of proof depends on a truth borrowed from *geometry, namely: that every continuous line of simple curvature of which the ordinates are first positive and the negative* (or conversely), *must necessarily intersect the abscissae-line somewhere at a point lying between those ordinates.* There is certainly nothing to be said against the *correctness*, nor against the *obviousness* of this geometrical proposition. But it is also equally clear that it is an unacceptable breach of *good method* to try to derive proofs of *pure* (or general) mathematics (i.e. arithmetic, algebra, analysis) from considerations which belong to a merely *applied* (or special) part of it, namely,

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[282] Cavaillès 1970, 370.
[283] Bolzano 2004, 253.
[284] Bolzano 2004, 253.

> *geometry…*anyone who considers that scientific proofs should not merely be *confirmations* [*Gewißmachungen*], but rather *groundings* [*Begründungen*], i.e. presentations of the objective reason for the truth to be proved, realizes at once that the strictly speaking scientific proof, or the objective reason of a truth, which holds equally for *all* quantities, whether in space or not, cannot possibly lie in a truth which holds merely for quantities which are in *space.*[285]

When Cavaillès identifies "the radical transformation of verification into demonstration" he means the transformation of verification by intuitive evidence proper to the applied domains of mathematics into the logical or objective demonstration of the *necessity* of a proof operating at the level of general mathesis. Bolzano's entire project is condensed in this passage; the rectification of the objectivity proper to mathematics is joined to the methodological exigency of the theory of science. If the theory of equations is to be brought into conformity with the logical exposition of the theory of science, it must be *re-written* in the language of the grounded presentation of objective reason. For Bolzano a geometric proof is not yet a scientific proof because geometry itself is a derivation or consequence of more general logical structures. For proof to be rigorous and not merely obvious it must in fact be *denatured* and removed from the realm of intuition and *reconstructed* at the level of "*axioms*, or *basic truths* [which are]…the *basis* for other truths and are not themselves consequences."[286]

"The overly facile recourse to evidence" Cavaillès speaks of must be replaced with an entirely axiomatic presentation of the necessity of the entire set of propositions which operate at the applied or derived levels of mathematics,

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[285] Bolzano 2004, 254.
[286] Bolzano 2004, 254.

including geometry and pure natural science. Bolzano is a necessary reference for Cavaillès because his attempt to conceive the objectivity of science on purely logical grounds is also an attempt to remove science from the oversight of any external authority. Bolzano allows science to be understood "as a being *sui generis"*, as Cavaillès writes:

> …the very being of science is submitted to critique [by Bolzano], in order to determine both what constitutes a science as such and the motive for its development…Perhaps for the first time science is no longer considered as a simple intermediary between the human mind and being in itself, depending as much on one as on the other and not having its own reality. Now science is regarded as a being *sui generis,* original in its essence, autonomous in its movement.[287]

Bolzano must be praised for demonstrating that science cannot be a simple interface "between the human mind and being in itself," which, as Cavaillès writes, can only result in the disappearance of the reality proper to science. There are two features of Bolzano's theory of science which Cavaillès will appropriate while simultaneously subjecting to critique. The first is the ontological assertion of the autonomy of science. The second is the "motive for its development." As Cavaillès' subsequent critique of Bolzano will make clear, both features are in the final analysis inseparable from one another.

Cavaillès focuses his critique of Bolzano on the difficulties which seem to be inherent in the "developmental motive" of science. Cavaillès fully endorses the notion that science articulates its own objectivity by a process of internal development and of the successive differentiation and variation of "objective levels."

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[287] Cavaillès 1970, 371.

In Bolzano this is simply the hierarchy of mathematical science. The system of logical proof described by the theory of science operates like a *chain of logical evidence* leading in every case back to the axiomatic presentation of truth contents at the level of general mathesis. Cavaillès wants to understand the implications of this inevitable reference to a pure and self subsisting logical architecture which underscores the objective structure of the sciences. The objective aspect of science which is entirely independent "of the universe of cultural objects"

> …requires unity, i.e., it cannot be reconciled with an actual multiplicity of singular realizations. There are not different sciences or different moments of one science, not even the immanence of a single science in various disciplines. But these disciplines are conditioned among themselves in such a way that the results as well as the meaning of one science require, insofar as it is a science, the utilization of others or the common inclusion within a system. A theory of science can only be a theory of the unity of science.[288]

As Cavaillès clarifies in a footnote, the unity of science Bolzano proposes is also a hierarchy "with a subordinated science using the results of the preceding ones as regulative principles."[289] Science has a hard core of objective truth contents which admit no "actual multiplicity of singular realizations" or local configurations of objective relationships valid only for a well defined region of the sciences. Instead, although science is differentiated in its parts, these parts themselves obey a coherent development or what may even be described as an instantaneous emanation from a common source in general mathesis. Each level of objectivity requires the guiding principles of its antecedent science and the developmental specificity of its successor science. The "meaning" of science is derived, in the fullest

[288] Cavaillès 1970, 371-372.
[289] Cavaillès 1970, 372.

sense of totality, from the mutual implication of all the parts of science in the whole of a uniform logical architecture.

It would, however, be a grave mischaracterization to think that the unity of science is simply reducible to a *logical content* described in terms of structural relationships among the levels of scientific objectivity. On the contrary, the problem which Cavaillès intends to address resides not in the static image of science which might be constructed based on a purely *ideal* presentation of science but in the very *real* quandary of the nature of scientific concept formation presented by *actually existing science.* The *historical reality* of "realized science" necessarily includes "incompletion and the requirement of progress [as] parts of its definition." The unity of science which Bolzano proposes is best captured, according to Cavaillès, by a novel conceptualization of *unity as movement.*

> This unity is a movement. Since it is a question here not of a scientific ideal but of realized science, incompleteness and the requirement of progress are part of its definition. Simply an autonomous progress, a self-enclosed dynamism, without absolute beginning or end, science moves outside of time – if time signifies a reference to the lived experience of consciousness. In it, change is growth in volume through the spontaneous generation of intelligible elements.[290]

The theory of science must bracket lived experience as a meaningful source of the conceptual development of the sciences. Referred only to the autonomous truth-contents of the proposition in itself, the *becoming* of science is "a self-enclosed dynamism" without determinate origin or teleological conclusion. Science has an invariable objective validity but no discernible historical origin. There is a

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[290] Cavaillès 1970, 372.

dynamism unique to science and even constitutive of its essence but it must be grasped as a generative capacity which operates outside the time of lived experience. The unity of science and the generative becoming of science must be grasped as part of the same movement. Following Bolzano's work on the structure of mathematical objectivity and his characterization of scientific proof as the logical demonstration of necessity, Cavaillès claims that the procedure of logical demonstration is itself generative of science. As Pierre Cassou-Noguès observes, in a science of pure conceptual becoming such as mathematics Bolzano's propositions in themselves and their autonomous truth contents inevitably enter into a spontaneous relationship of logical cause and deductive consequence. A consequence must be referred to its logical ground

> by a criteria of deducibility (the consequence is deducible from the reason), a criteria of generality (the reason is more general than the consequence), [and] a criteria of simplicity (the reason involves fewer simple concepts than the consequence.)[291]

The very movement of the reason and the consequence entails an inevitable movement of increasing complexity by which the different levels of scientific objectivity are generated. The unity of science is established on the necessary objectivity of its logical foundations and the inevitable consequences which follow.

> …every concept or system of concepts, through that which it poses to itself, is at once exclusion and need of the Other. In this sense, the representation of an absolutely simple infinitude of all knowledge is an image without any relation to the reality of militant science other than pushing to the limit a property of movement , namely, the absorption of the prior by the posterior which justifies it and to a certain extent suppresses it. But the road should not be abolished, if we want it to be followed. The true sense of a theory is

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[291] Cassou-Nogués 2001, 266.

> not in an aspect understood by the scientist himself as essentially provisional, but in a conceptual becoming which cannot be stopped.[292]

The scientific concept poses its own objectivity to itself; it demands a logical ground and a logical consequence. Thus, it spontaneously brackets itself and indicates both its origin and its successor. Cavaillès isolates "the property of movement" inherent in the scientific proposition as "the true sense of a theory" or what amounts to the same "a conceptual becoming which cannot be stopped." The very *sense* of science (mathematics) consists in this "conceptual becoming which cannot be stopped." The autonomy of science is secured not only on the basis of its logical foundations and their objective truth-contents but by the unstoppable conceptual movement of this objectivity which is auto-complicating, self-differentiating, and self-surpassing. Science is at once inherently expansive and progressive but also, by the same gesture, inherently self-contained.

> …scientific autonomy is simultaneously expansion and closure, negative closure in its refusal to borrow from or terminate in what is external to it… *Science is a Riemannian volume which can at once be closed and without anything external to it* [my italics.] No method issuing from this theory can recover the Cartesian decision: *sub scientia non cadit.*[293]

The autonomy of science is this *immanent differentiation* of logical possibilities which Cavaillès refers to as a Rimeannian volume, constantly enriching itself by the generation of new intelligible elements but never breeching its own boundary, never importing content from the world or from subjectivity. Bolzano's theory of science, Cavaillés observes, rebounds on a theory of logical demonstration which

[292] Cavaillès 1970, 372.
[293] Cavaillès 1970, 372-373.

alone seems capable of upholding both the autonomy of science and the incessant movement of its conceptual becoming.

> The structure of science not only is demonstration but is identified with demonstration. In it the essential traits are again encountered: unity, necessary and indefinite progression, finally, closure upon itself. The internal rule which directs it posits each of its steps; it is entirely in them and impossible to grasp *ne varietur* in any of them…Demonstration, simply by the fact that it posits the goal, extends and ramifies the domain created by means of combinations which it establishes as soon as they are possible. Finally, it cannot be allied with the undemonstrated. We again meet the Platonic reserve against the *dianoia* which lends its hypothesis to the visible world, an arrangement without justification from which nothing necessary can follow.[294]

Science and logical demonstration, in their deepest objective autonomy, are one and the same. Cavaillès indicates the knotting or inter-twinning of three features which define the essential elements of Bolzano's theory of science: the unity of science in a shared logical armature, the necessary movement or progression of science in the explication of this logic through its successive causes and consequences, and the self-closure or containment of science within a Riemannian volume. Demonstration is the simultaneity, at the level of the logical proposition, of possibility and actuality: "it extends and ramifies the domain created by means of combinations which it establishes as soon as they are possible." There is no gap between a logical possibility and a logical actuality. If a logical structure is possible within the Riemannian volume of science it is also actual. There is no gap between logical possibility and the necessity of its demonstration: the scientific proof or exposition of its objectivity.

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[294] Cavaillès 1970, 373-374.

It is with reference to the theory of demonstration that Cavaillès parts ways with Bolzano and constructs his own theory of the autonomous differentiation of the logical possibilities of mathematics. It is "the principle of generating movement" which must be excerpted from Bolzano's theory of science and subjected to a careful critique and revision. Cavaillès embraces Bolzano's three-fold affirmation of the theory of science: it must uphold the autonomy of science, the necessary conceptual becoming of science, and the self sufficiency and self-enclosure of science. Cavaillès differs from Bolzano in his evaluation of the role that logical demonstration plays within the closed but dynamic system of science. Science does possess an autonomous creative dynamism, but this creative capacity is located in a domain of *scientific experience* which cannot so easily be segregated from the ready made domain of a pure logicism, on the one hand, and an unregulated chaos of the experiential world, on the other. This is precisely the point for Cavaillès: the creative transformation of the boundaries between intuition and logic is itself constitutive of conceptual creativity in mathematics and science.

Bolzano's theory of science as a theory of logical demonstration ultimately duplicates the feature of *self-illumination* which Cavaillès had identified as characteristic of the philosophies of consciousness. In both cases science is made somehow perfectly coincident with itself and all its possibilities. There is no unforeseeable element in the transcendental subjectivity or the logical architecture which dictates the progressive becoming of the contents of science as well as the necessary form of its exposition.

> In defining a structure of science which is only a manifestation to itself of what it is, we specify and justify the preceding characteristics, not by an explication which would have its proper place and would in its turn be an object of reflection, but by a revelation which is not distinct from the revealed, present in its movement, a principle of its necessity. The structure speaks about itself.[295]

What Bolzano requires is an explication of the varying structures of science, but because he takes the structure itself to be invariant and given all at once, the true outline of the problem cannot emerge. Instead, there is a revelation of self-explication which is the very movement of science, the unity which passes through all of its parts in a passage of self-surveillance and self affirmation. The necessity of the structure of science is present in its self-explication. The structure speaks about itself, but for Cavaillés it has nothing to say.

A crucial element is missing from Bolzano's theory of science: a viable account of the unpredictable nature of transformations even within perfectly rigorous logical domains. Rigor must not exclude novelty. Based on his appropriation of Bolzano's theory of logical demonstration, Cavaillés will propose a more supple *theory of rational sequences* designed to account for the transformative capacity inherent in the nature of mathematical experience. In Bolzano, the transformations of logical propositions amounted to a self contained tautology of logical structures nested within one another. Between antecedent and consequent the same principle can be discovered dictating the progression of knowledge at every level of scientific objectivity.

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[295] Cavaillès 1970, 373.

> There is a twofold difficulty with Bolzano's solution. If it avoids subordination to a historical existent or to the absolute of consciousness, it must itself posit the totality of what it attains, to discern afterwards, if it can, the permanent essential element of what is mobile through it. It is, on the one hand, a construction of a pure theory of rational sequences, and on the other a relationship to science as it has developed. Without the resolution of these problems , scientific epistemology cannot constitute itself directly as primary, which is its ambition.[296]

Bolzano emancipates science from history and from consciousness, two absolutes which rob it of its specificity and its autonomy. In freeing itself from historical and transcendental determination in the philosophy of consciousness, the theory of science discovers the immanent dynamism of what Cavaillès calls the rational sequence. Bolzano proposes a pure theory of rational sequences but does not appropriately identify the relationship of the rational sequence to the existing body of scientific concepts. This relationship is necessarily historical for Cavaillès. It cannot be captured or predetermined by logic or consciousness, because it involves the transformation of both domains, as well as the very boundaries which are supposed to provide them with clear criteria of demarcation. In the place of "a structure which speaks of itself" Cavaillès will propose a theory of rational sequences which is also a theory of mathematical experience and experimentation.

Paradigm and Thematic: The Theory of Rational Sequences

Bolzano's theory of science furnishes Cavaillès with the necessary elements of his own theory of science. Bolzano managed to revise the very concept of science in such a way that its autonomy becomes intelligible without needing to make

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[296] Cavaillès 1970, 374.

reference to history or to consciousness. Instead, the logical structure of objectivity becomes the internal consistency of all scientific and mathematical propositions. The conjoined necessity of the unity of science as the progressive movement and transformation of concepts follows from this. Cavaillès rejects the pure theory of rational sequences which Bolzano proposes in order to account for the self-differentiation of scientific concepts out of a uniform logical structure. This *uniformity of transformation* at the level of the concept is unacceptable for Cavaillès. In *La pensée mathématique* (1939) Cavaillès summarizes his understanding of mathematical *experimentation* as a process which does not lack very precise rules of transformation but which still asserts a radical independence from its immediate antecedents:

> By experiment, I understand a system a moves, governed by a rule and under conditions independent of the moves…I mean that each mathematical process is defined in relation to an earlier mathematical situation on which it partially depends, and in relation to which it maintains an independence such that the result of the action must be observed in its accomplishment…That is to say…the act having been accomplished, by the very fact of it appearing, takes its place in a mathematical system extending the earlier system.[297]

Against the total determination of the structure of science by an invariable logical progression (as in Bolzano and the later logicism of Carnap and Tarski), the situation of mathematical experimentation which Cavaillès describes is at once partially determined by the "earlier mathematical situation" and partially independent of it, that is to say, irreducible to a fixed program of pre-determined cause and consequence. The theory of logical demonstration which Bolzano

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[297] Huisman 1994, 601-602. Cited in Sinaceur 2006, 320.

identified with science is not supple enough to grasp what is genuinely creative in mathematical experimentation: the liberation of a new meaning from a rational sequence.

In order to understand the nature of mathematics it is necessary to construct a more nuanced theory of mathematical experience. This will entail a more differentiated theory of demonstration, on the one hand (including a revised theory of rational sequences and a theory of mathematical objects and operations), and a theory of the mathematical sign, on the other. As Sinaceur remarks, Cavaillès discovers within the procedures of mathematical abstraction the motor of mathematical creativity itself.[298] Bolzano's theory of science ultimately proposed a unity of the structure of science (the logical architecture of its truth contents) and the progress of science, collapsing the developmental sequence into the logical order in a single act of intelligible necessity. Cavaillès rehabilitates the irreducible singularity of the progress internal to rational sequences in order to avoid the sterility of a tautological identity of logic and science. Cavaillès must show that while mathematical novelty is the expression of an internal necessity which cannot be evaded, it is nevertheless *genuinely novel*, that is to say, unforeseeable from within the existing rational sequence. This is because, contra the philosophies of consciousness, the rational content of a mathematical object or operation cannot be exhausted from a single and unitary perspective. There is an internal differentiation

[298] Hourya Benis-Sinaceur, " Structure et concept dans l'épistémologie mathématique de Jean Cavaillès," *Revue d'histoire des sciences,* vol. 40 no 1 (1987): 25.

at work within the rational sequence which prevents any mathematical object or operation from being exhaustively defined.

The rational sequence possesses a power of self-differentiation which Cavaillès will attempt to capture by modifying the theory of demonstration developed by Bolzano. Paul Cortois provides a useful summary of the philosophical theses informing Cavaillès' theory of rational sequences[299]. Cavaillès endorses a species of *mathematical holism* which holds that the unity of mathematics must be understood as

> an organic whole of concepts, methods and theories crossing each other and interdependent in such a way that any attempt to isolate one branch, even for the sake of providing a foundation for the whole body, would be in vain; nor would it make sense to break this organic solidarity by looking for an external foundation (be it natural science or logic or some psychological reality.)[300]

Mathematics is a rational unity which cannot be encompassed by any exterior theory. Here Cavaillès is in explicit agreement with David Hilbert against Bolzano; logic is, in fact, an elementary part of mathematics and not vice versa. The unity of mathematics also includes what Cavaillès calls its "technical artifices," the changing systems of notation which provide the substrate of all practical mathematical work. Mathematics is, at once, internally coherent and capable of radical transformation, which results in what Cavaillès refers to as polymorphism internal to the rational sequence. For Cavaillès, as Paul Cortois writes, the history of mathematics expresses

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[299] Paul Cortois, "The Structure of Mathematical Experience According to Jean Cavaillès," *Philosophia Mathematica,* vol. 4 no 3 (1996): 24.
[300] Cortois 1996, 24.

> a domain where it makes sense to combine a thorough developmental view with the denial of real contingency, even in the face of real novelty. If we have to have a privileged example of a knowledge, whose *stages*, at least, follow each other with necessity (whereas all other illusions of apodicticity have been unmasked), such knowledge cannot after all depend for its justification on things external to itself.[301]

The dual imperative of the theory of rational sequences is to uphold the holism and autonomy of mathematics as a coherent unity while also continually demonstrating the genuine novelty it is capable of producing out of its own resources.

This apparent dualism leads Gilles Gaston-Granger to underline the complexity of Cavaillès' historiography of mathematical concepts.[302] On the one hand, the theory of rational sequences, as a method of "internalist" or conceptual historiography, is superbly capable of registering and describing in detail the ruptures and discontinuities of a developmental sequence. The conceptual universe of Hilbertian axiomatics encompasses Euclid's geometry, but it would be an impossible absurdity to attempt a reconstruction of Hilbert's formalist project using only the postulates of Euclid. The theory of rational sequences attests to the *irreversible progress* of abstraction inherent to mathematics. On the other hand, the rational sequence is necessarily emplotted in a linear sequence which presupposes a continuity of what Granger calls the *formal contents* of mathematics. Cavaillès writes:

> …progress itself may not be augmentation of volume by juxtaposition, in which the prior subsists with the new, but a continual revision of contents by deepening and eradication. What comes after is more than what existed

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301 Cortois 1996, 25.
302 Gilles Gaston-Granger, "Jean Cavaillès et l'histoire," *Revue d'histoire des sciences,* vol. 49 no 4 (1996): 574-575.

> before, not because it contains it or even because it prolongs it, but because it departs from it and carries in its content the mark of superiority, unique every time…[303]

As Gaston-Granger notes "[t]he new contents necessarily issue from the preceding contents" and yet they are also surpassed by the "continual revision of contents by deepening and eradication." There is a continuity of formal content which runs through the entirety of mathematics as a "great chain" of rational development but which also renews itself as a genuinely singular content, a real event which deepens and alters the rational content of the preceding sequence. This leads Granger to observe that for Cavaillès the continuity or discontinuity of the rational sequence of mathematics is not the real issue. What is at stake is a perpetual revision of the relationship between the form and content of mathematical expression where form is provisionally equivalent to a theory of mathematical operations and content is provisionally equivalent to a theory of mathematical objects. This perpetual oscillation results in the progressive expansion of the domain of mathematics as well as the increasing complexity of its abstraction.

Mathematics is essentially holistic, as Cortois observes, but the theory of rational sequences is also specifically attuned to novelty. Cortois and Granger both emphasize *an epistemological principle of duality* which seems to inform Cavaillès' philosophy of the concept at a fundamental level. Mathematics is a domain which excludes all contingency, but it is also characterized by successive and irrevocable revolutionary transformations of its own content. When Cavaillès invokes a

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[303] Cavaillès 1970, 409. Cited in Gaston-Granger 1996, 574.

dialectical philosophy of the concept, he intends to bracket the generative mechanism of this transformative capacity of mathematics without invoking any authority which would be external to mathematics. The principle of duality refers to a basic pattern of alternation which underscores this transformative mechanism. Drawing on the work of Granger, Cortois identifies:

> an indissoluble correlation between mathematical operation and mathematical object: any object is only given as the projected result of an operation (or system of operations), and conversely, every operation requires an object domain on which to act and in which to project its results…the dialectic of concepts, seen from this perspective, consists in the mutual dependency of operational acts and the objects they reach.[304]

The conceptuality proper to mathematics finds itself distributed according to this mutual dependence of the object and the operation. The objective existence of mathematics is affirmed relative to the actualization of new mathematical objects and operations. As Cortois observes, this objectivity cannot be construed as purely instrumental because "the set of means of actualization and effective attainment is never bounded or definable in terms of fixed principles (such as predicativity) or restrictions."[305] On the contrary, objectivity is defined by the relationship between the operation and the object but this relationship itself is at once determinate and open-ended. Determinate insofar as mathematical objects behave like natural objects to the degree that they offer resistance to the set of possible operations and contain well-defined problems of a very particular nature; open-ended insofar as the objects themselves are brought about by operations or generating acts which

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[304] Cortois 1996, 26.
[305] Cortois 1996, 26.

are themselves “characterizable only in terms of the objects generated.” The mutual interpenetration of the object and the operation is theoretically endless. Moreover, what constitutes an object at one level may become an operation at another, and vice-versa, according to a conceptual dynamic which Cavaillès will define in terms of two basic demonstrative procedures: the paradigm and the thematic.

The basic principle of an interpenetrating duality of mathematical experience lends itself to a pragmatic definition of mathematics as a procedure which can be written schematically as follows: Operation A > , Object A >, Operation A1 >, Object A1 etc. The creations of mathematics are all immanently derived from the set of operations already internal to the rational sequence. There is no foundational intuition or donation from an experience external to mathematics (as a theory of rational sequences) which can be isolated as the origin of the sequence. The mutual necessity of the object and operation is always already in place. Mathematics in this sense, qua rational experience, cannot be grasped according to the principle of a definitive origin. It can only be grasped in its becoming, it “is” insofar as it is in process. By this means Cavaillès wishes to uphold the autonomy and self arising legitimacy of mathematical authenticity without for all that defaulting to a Platonism of independently existing essences. What is paramount for Cavaillès is to give an account of the productive mechanism internal to the rational sequence or what amounts to the same to affirm at once the immanence and the multiplicity of objects and operations.

Granger has gone the farthest in developing the outline of a robust theory of mathematics as a science of formal contents based on Cavaillès theory of rational sequences.[306] The science of formal contents is intended to replace Kantian a priori synthesis in the epistemological critique of mathematics. The concept of formal content is intended above all to displace the stability of the relationship between form and content as a feature of the intelligibility of objects. Just as operation and object are mutually pre-supposing in the production of new operations and objects, so form and content are similarly correlated in the organization of mathematical concepts. As will become clear in the exposition of Cavaillès' theory of rational sequences, the content of a mathematical theory can be abstracted into a theory of the form of the object, in turn giving rise to a new class of objects. Cortois writes:

> …every stage of formalization is at the same time a way of progressing in the articulation of conceptual content, and every contentual innovation goes together with the development of new formal means (in case they do not simply coincide, as when a certain form or formal level is just the theme of a further elaboration…)[307]

Here again the sense of the mechanism at work in Cavaillès' proposed dialectic of concepts conjoins both determinate rigor and genuine novelty. Content at one level provides the point of departure for formal innovation at another level, which then becomes the contentual ground for another stage of formalization. Just as there can be no initial or final object or operation initiating or terminating the rational sequence, there can be no initial or final form or content initiating or terminating

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[306] See in particular Gilles Gaston Granger, *Formes, operations, objets* (Paris: J. Vrin, 1994).
[307] Cortois 1996, 27.

the development of mathematics. The separation of form and matter at any level of mathematical development will only be legitimate for that level, any attempt to generalize a distinction between form and content must be repudiated as artificial and, indeed, arbitrary. The nature of the mathematical sign becomes especially pertinent when inserted into the theory of formal content, for it too takes on this oscillating nature, sometimes acting as the formal operator on a class of objects, sometimes becoming the determinate object of a formal operation. Formalization is inevitable in mathematics, as Cavaillès affirms throughout his work, but it is always necessary to plot the relationship of formalization "to a given stage of conceptualization; far from reaching some absolute core, to mistake a given stage for the ultimate is to hypostatize what is only transitory."[308]

Mathematics then is not simply an abstract "play with symbols;" it is not the fabrication of formal rules out of nothing: it is an objective development of the relationship between form and content or the self-explication of formal contents in the dialectical progress of the concept.[309] The progress of mathematics constitutes its authentic "rational history," the chain of rational necessity by which the historicity of mathematics resists all forms of contingency. For Cavaillès,

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[308] Cortois 1996, 27-28.

[309] Gerhard Heinzmann offers a suggestive summary of the coherent dualities which recur in Cavaillès' epistemology of mathematics. The theory of rational sequences describes the dialectical correlation and oscillation of two poles of mathematical experience which Cavaillès designates variously as "discourse and experience," "language and work," "symbolization and intuitive operation," "sign usage and experience of symbols," "thought and action," "necessary and contingent," and "multiplicity and singularity/actualization." See M. Gerhard Heinzmann, "La position de Cavaillès dans le probléme des fondements en mathématiques, et sa difference avec celle de Lautman," *Revue d'histoire des sciences,* Vol. 40 no. 1 (1987): 41.

mathematics is never without its history. Indeed, the progressive consolidation of its own formal content demands that it bear within itself the totality of its history at every moment. How though does this generation, consolidation, and deepening of the formal content of mathematics occur? Cavaillès distinguishes two dimensions internal to the rational sequences of mathematics.

> …within the identity of its essence the demonstrative can assume many aspects. Two dimensions in particular can be distinguished: a realm of singularities where the demonstrated adheres to the demonstration to the point of being indiscernible from it and characterizing a single moment of science; and the set of sequences in the strict sense which, although they are indispensable only to the extent that they actually demonstrate something and as a result cannot be renewed in their integrity, present a kinship of type according to groups, a mark of the unity of movements which can be manifested in the abstract. Thus emerges the notion of logical form indispensable to a theory of demonstration. The process of separation is two fold: longitudinal, co-extensive with the demonstrative sequence; vertical, establishing a new system of conjunction that uses the old as a base from which to start, no longer as a stage traversed by a movement but as an object of reflection in its true bearing. In each case, a property constitutive of the essence of thought (or of intelligible sequences) manifests itself: the *paradigm* and the *thematic.*[310]

The relationship between the paradigm and the thematic is the dialectical motor of mathematical experience. Paradigms, as Cavaillès remarks, are demonstrations of singularities, or in the vocabulary favored by Granger, the paradigmatic movement of mathematics is identified with the description of mathematical objects – the longitudinal extension of a rational sequence. Then there are "the set of sequences in the strict sense" which are "a mark of the unity of movements which can be represented in the abstract" from which a vertical dimension emerges which does not concern itself with objects but rather with an abstract theory of operations: the

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[310] Cavaillès 1970, 374-375.

rules by which a system of objects can be represented in the absence of all extraneous features. The thematic dimension is thus the super-position of a new rational sequence on top of the previous sequence – a novel iteration of the intertwining of object and operation.

The paradigm and the thematic are the interlocking procedures by which Cavaillès attempts to explicate the movement of mathematical abstraction. Granger's notion of formal content provides a useful clarification of the special precautions which must be taken when seeking to account for the nature of mathematical abstraction. Mathematical experience is the experience of a progressive abstraction. This abstraction, however, cannot be modeled on the abstraction away from the empirical contents of lived experience. As Cortois and Sinaceur observe, the mode of thought particular to mathematics requires a break from experience in the quotidian sense and even from the modes of rational experience proper to other branches of science.[311] Mathematical abstraction is characterized by its internal procedure of differentiation, the two aspects of which (paradigm and thematic) act upon each other in a helical interlacing of mutual explication and the progressive deepening of formal contents. It is tempting to

[311] Cavaillès continually underlines the centrality of the relation between the different forms of rational experience which compose the open ended and progressively transforming system of the sciences. The relationship between mathematics and physics is perhaps the most aggravated example of a correlation which requires a profound epistemological analysis. Cavaillès, however, does not offer this analysis. Bachelard's epistemological determination of the structure of scientific objectivity might point towards a possible reconciliation of the problem Cavaillès raises regarding the integration of mathematics and physics but a separate study would be required in order to draw out the complexities of this relation in Bachelard and Cavaillès.

identify the paradigm univocally with the object and with the content plane of mathematical abstraction, and the thematic with the operation and the formal plane. What is most vital, however, is the oscillation of both moments, the reciprocal exchange of perspective by which the object becomes the operation and the operation furnishes a plane of new objects. Cavaillès wishes to underscore that the synchronic isolation of the object and the operation is itself an entirely artificial act: the "essence" of mathematics is not the discrete identity of one element but the movement of their progressive unfolding.

Paradigm and Thematic

Nevertheless, this unity of movement which would describe the unique procedure of mathematical abstraction does offer distinct moments for analysis. The paradigm is the "singularity of realization of the sequence" as Cavaillès writes; it posits an object whose meaning cannot be captured in a single discrete act. Therefore, even at the level of the paradigm a considerable effort of elaboration is required. Cavaillès often speaks of this horizontal elaboration as the extension of the rational sequence. The mathematical object is perhaps better grasped as a domain with objective but indefinite boundaries. The paradigmatic movement is the delimitation or explicit rendering of this domain. The elaboration of the object domain is the well known *generalization* of the object, which Cavaillès refers to as a certain "dissociation" of meaning within a given mathematical concept which isolates the essential elements and excludes the contingent features. One of

Cavaillès' favorite examples of the generalization of a concept which occurs in the paradigm is the generalization of the concept of number in 19th century number theory. Citing Dedekind's Habilitation thesis Cavaillès asks how the number system can be subjected to a process of generalization. Dedekind writes:

> The enlargements of definitions leave no room for any arbitrariness but follow with necessity from the original definitions if one applies the principle that the laws, proceeding from them and characterizing the concepts they introduce, hold universally.[312]

The uniqueness of mathematical abstraction consists in the dissociation of meaning which enlarges the object domain. As Cortois writes:

> Where 'bound associations' of concepts held us captive, we arrive, one could say, at a 'free dissociation' instead, and come to a more general and abstract concept both of the object and the operation acting on it.[313]

The dissociation of meaning is the unfolding of the "virtual" dimensions of the object, rendering explicit that which was present only implicitly in the "bound associations" of the immediate object.

Against those who would see in the movement of axiomatization only the stultification of mathematics, Cavaillès finds an authentically generative impulse at work.[314] Because axiomatization is such a powerful tool of generalization it cannot help but lend itself to the further differentiation of formal contents. The generalization of the object is also the vivacity of the concept. The concreteness of

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[312] R. Fricke., E. Noether and O. Ore, eds., *Gesammelte mathematische Werke III* (Braunschweig: Vieweg, 1932), 32. Cited in Cortois 1996, 31.
[313] Cortois 1996, 31.
[314] Gian Carlo-Rota refers to the axiomatic ambition of Hilbert's program in the philosophy of mathematics as a "necrology." See Mark Kac, Gian-Carlo Rota and Jacob T. Schwarts, eds, *Discrete Thoughts: Essays on Mathematics, Science, and Philosophy* (Boston: Birkhauser, 1992), 16.

immediate intuition is fractured or interrupted in favor of a developmental sequence which explores the hidden complexity of what was self-evident in intuition. The crucial movement here is that of differentiation: generalization unfolds complexity while also adding detail to the definiteness of the object domain.

> The *paradigm* is characteristic of the actualization, not of an actualization *hic et nunc* in the lived experience from which the extrinsic is eliminated, in the simple seizure of thought by itself, but of an actualization required by the sense of what is posited, i.e., a relationship which as such is affirmed only in the singularity of realization of the sequence, but which calls for this singularity only in an indefinite way; it therefore suppresses it in positing it and thereby reveals an internal principle of variation. From arithmetical reasoning on the finite integer to the most abstract sequences, the same liberating dissociation of meaning occurs.[315]

Generalization actualizes the sense of what is posited. Mathematical objects are characterized by the excess of their own meaning, which cannot appear in a single moment but which call for sustained development. Cavaillès is dramatically opposed to Kant and Husserl on this point (as the final section of the chapter will demonstrate) because he insists that neither intuition nor the static data of the concept at any given moment can ever fully account for all the dimensions of the mathematical object-domain. That which is given in mathematical experience is not given *simultaneously* to the transparency of consciousness. Against the self-illumination of consciousness, mathematical experience requires a staggered development of formal contents.

In the domain of mathematical experience, concepts are objects and vice-versa. This is the reflexivity proper to the paradigmatic development of the rational

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[315] Cavaillès 1970, 375.

sequence. Every object is pregnant with latent formal contents which are developed according to concrete acts. As Cavaillès writes, every mathematical object both *suppresses* and *elaborate*s its own meaning: "[f]rom arithmetical reasoning on the finite integer to the most abstract sequences, the same liberating dissociation of meaning occurs." Cavaillès connects the "liberating dissociation of meaning" with an "internal principle of variation" – the extension of the rational sequence is the elaboration of a meaning proper to the object which is already given but which cannot appear in any simple totality. The *effectiveness* of the mathematical object requires the elaboration of its proper domain – the explicit writing of its latent meaning in what Cortois calls "distinct symbolical concatenations."

> …paradigmatic abstraction has to do with the fact that an actualization is required – no mathematical idea can be effective without being expanded in explicit and distinct symbolical concatenations; what is thus actualized and given a force of "reality" is a (cluster of) meaning constituent(s) somehow generated from or present in the singular. What is typical for this mechanism is the enlarging of the universe of objects considered up to then by erasing the differences which don't matter at a certain level of investigations (just as the class of geometrical objects 'equivalent' to a certain given object, say a sphere, is enlarged when the properties considered are just the topological ones.)[316]

Mathematical objects are "unfolded" by paradigmatic abstraction – everything is already present in the object, and simply requires actualization. As in the Bergsonian or Deleuzian language of virtual but real singularities, the mathematical object is entirely real (albeit partially virtual) and fully objective. It must, however, become actual by the liberation of formal contents which adhere in it as latent aspects of an expressive singularity. It is additionally impossible to simply oppose

[316] Cortois 1996, 33.

the concrete and the abstract in mathematical experience – abstraction is always occurring through concrete acts (for example, the algebraic determination of a new symbolic system capable of expressing the logical possibilities of an object domain [the set of finite integers] more fully than the set of natural numbers.) The mathematical object is progressively unfolded by concrete acts which are at once generalizations and elaborations.

The paradigm is the elaboration of the mathematical object-domain through "symbolical concatenations" which extend the operations of the object. In this sense the paradigm is the *horizontal correlation of the object and the operation.* However, what remains primary is the *posited meaning of the object,* which functions as the privileged reference of all the operations. Under paradigmatic abstraction, the rational sequence must be conceived as a linear sequence with a definitive origin in the singularity of the object. At the same time, however, the sequence is extended horizontally by adding operations linearly to the object. The meaning of the object is continually and explicitly unfolded by what Cavaillès calls "the suppression of the singularity," the abstraction or generalization of the object. Cavaillès presents this relation between object and operation using the vocabulary of *meaning* (object or posited meaning) and *act* (operation).

> What is unified is really the elaboration of the acts insofar as each of them, forgetting and realizing itself at once within a meaning, posits its own being only as an element of a set recognized as a plurality and at once as a basis from which to initiate new acts.
>
> And so the synthesis is coextensive with the production of what is synthesized…What is important here is the disengagement brought about with each suppression of the singularity. This is what is represented in logical calculus by the rule of substitution, i.e., the possibility of replacing in

> the new element that from which it actually proceeds with something else equivalent to it from the new point of view.[317]

This notoriously difficult passage expresses, so I claim, the horizontal relation of the object and the operation as well as the motive driving their relationship. *Paradigmatic abstraction explains how a rational sequence is capable of self-differentiation – it is an attempt to grasp the internal multiplicity of the mathematical object.* The suppression of the singularity is the suppression of a unitary identity in favor of a linear and expansive multiplicity. There are no self contained essences in mathematics, and certainly no tautologies. Mathematical singularity is the basis of all operations which in turn come into being by suppressing this singularity in the service of abstraction and generalization.

Cavaillès points to the rule of substitution in the logical calculus as paradigmatic of the suppression of a singularity. The object domain under the paradigm differentiates itself by suppression and substitution. Cavaillès writes:

> This is the *moment of the variable:* in replacing the determination of acts by the place vacated for a substitution, we progressively raise ourselves to a degree of abstraction which gives the illusion of an irreducible formality. This is what Leibniz tried in passing to the absolute through the mirage of an infinite whose simplicity makes conditions and conditioned simultaneous. Such are the attempts toward the algebraic or geometric art of characteristic, the theory of determinants, the symbolism of the infinitesimal calculus, where in each instance the desire to preserve only what is essential manifests itself.[318]

The moment of the variable is characteristic of a tendency internal to mathematical abstraction which makes formalization seem both inevitable and natural. It is

[317] Cavaillès 1970, 376.
[318] Cavaillès 1970, 376-377.

indeed characteristic of the object to differentiate itself by generalizing its own efficacy through the set of operations it gives rise to. What must be borne in mind, however, is what Cavaillès calls "the illusion of an irreducible formality." The theory of rational sequences is designed to demonstrate that form and content in mathematical abstraction are always relative terms. The paradigm is the instantiation of a relationship between form and content. This relation is given with, or determined by, the positing of the object itself. It is unique to the object and cannot be generalized. Between act (object, content etc.) and meaning (operation, form etc.) there is always a moment of suppression – the bracketing of the intuitive object (an integer, a geometric property etc .) in order to create, by a liberating dissociation, an operation of higher abstraction designed to "preserve only what is essential." The "essential" in mathematics can be indefinitely post-poned, displaced, suppressed and elaborated. The essential is, in fact, this very movement of conceptual elaboration unleashed by the suppression of the singularity.

The notion of formal content becomes invaluable from this perspective, because it guards against a misunderstanding which Cavaillès is keen to avoid and which perniciously distorts the progressive nature of mathematical abstraction. The suppression of the singularity is what allows a distinction to be introduced between form and matter. There is nothing spontaneous or natural about the distribution of this relationship. It is always the effect of concrete operations. When Granger prefers to speak of mathematics as the science of formal contents, it is because he is choosing, like Cavaillès, to define mathematics by the mode of conceptual creativity proper to it. The distinction between form and matter is itself a mathematical operation. As Cavaillès writes, the suppression of the singularity is itself

> …the source of the distinction at each step between matter, the singularity as origin, and form, the actual meaning. This is likewise the source of a distinction which, when projected into the absolute, would extend from one end of the general sequence to the other, like a continuous flaw through the nodes of the particular distinctions. It is an image favored by language, sometimes the consequence of an ontology as well. The illegitimacy of it is evident, since it misunderstands what it generalizes, namely, the motive of the necessarily progressive passage from the act to its meaning.[319]

The programs of formalism in mathematical epistemology are, unbeknownst to themselves, naturalizing and reifying the moment of the variable. They make an essence out of what is only an operation, and, therefore, inseparable from an object-domain. The relationship between form and matter proper to mathematical abstraction cannot be modeled on that of linguistic signification. The signs of natural language are far too unilateral in their organization to capture the constantly shifting boundaries of mathematical signification. The history of ontology is no less suspect as a reliable source for the clarification of this relationship. It too favors an absolute distinction between form and matter which would literally be incapable of making sense, that is to say, of elaborating the meaning and effects of mathematical objects.

When the distinction between form and matter is projected into the absolute, the motive of mathematical progress suffers a mischaracterization. There is no simple origin of the mathematical sequence in intuition or sensibility. As a science of formal-contents, mathematics is always at once intellectual and concrete, partially

[319] Cavaillès 1970, 376.

sensible and partially intelligible, and given to the steady alteration of this ratio. The "passage from the act to the meaning" is itself inexhaustible because it has no determinate origin and no fixed culmination. There is a surplus of meaning and its inseparable "symbolical concatentations" proper to mathematics which adheres entirely in the inter-mezzo of object and operation. When Cavaillès speaks of a certain formalist tendency which would misunderstand what it generalizes, his blanket criticism encompasses any position which reifies the relationship between form and matter. Here almost the entire contemporary philosophical landscape becomes complicit in an unhelpful simplification of the nature of mathematical abstraction.

At the same time, this misunderstanding is motivated by the nature of the paradigm itself, which explicitly takes the point of view of what Cavaillès calls "the final meaning of the sequence."

> There is no meaning without an act, and no new act without the meaning which produces it. It remains that we can, for a certain length of the rational chain, by placing ourselves immediately at the last term, consider its meaning as the unique form all along the successions of the acts. We proceed in this way in the systematic exposition of a mathematical theory, where the initial inventory of the proceedings springs both from the analysis of the final meaning of the sequences and from the artifice which consists in considering these chains as preparations for this meaning without intermediate or ultimate renewal.[320]

Act and meaning pre-suppose one another (this is the basis of the horizontal correlation of the object and the operation), but it is still possible to attempt a rational reconstruction of the sequence from the perspective of the accomplished

[320] Cavaillès 1970, 376.

meaning in such a way that the entire development seems both inevitable and natural. Such a procedure deduces the final product from stable initial conditions *after having already accomplished the elaboration of the object domain.* As Bachelard will affirm in another context, we can only formalize what we already know to be true. In taking the formalization as more real than the object we end up discounting the procedure by which we arrived at new knowledge. Once the paradigm has been established, it is definitive. The movement of elaboration which actualizes the object domain is by definition irreducible to the stable sequence it gives rise to. The productive dialectic of the concept is effaced by the "systematic exposition of a mathematical theory," which necessarily assumes the perspective of the accomplished meaning of the sequence.

If the paradigm is the extension of the rational sequence horizontally, the thematic is the vertical dimension of depth upon which the super-position of mathematical abstractions can be projected. As Granger remarks in *Pour la connaissance philosophique* (1988), Cavaillès' spatial metaphors imply that the horizontal rational sequence, an irreducible singularity in its own right, appears in turn as a procedure in active collaboration with other processes which relativize it from the perspective of thematization, the transversal rational sequence.[321] Granger affirms that the paradigm and the thematic are two different ways of generating the mathematical object and that they are fundamentally irreducible to one another because they correspond to different methods of demonstration. Although the

[321] See Gilles Gaston-Granger, *Pour la connaissance philosophique* (Paris: Éditions Odile Jacob, 1988), 70.

paradigm is capable of an indefinitely progressive abstraction, it does not yet ascend to a genuine formalization of its own contents because it extends its effects through concrete operations which share a stable 'retro-active" reference to the object as an already accomplished meaning. Cavaillès presents the distinction between paradigm and thematic as follows:

> ...formalization is realized only when the rules which regulate the structures are systematically superimposed on their outline. *Thematization* takes its start in the sequence seized en route, a trajectory which moves toward meaning. Thought no longer goes toward the created term but starts from the modes of creating in order to provide its principle through an abstraction of the same nature as the other one, but directed transversally…the most numerous examples appear in mathematics: theory of linear operations, of matrices, topology of topological transformations.[322]

The rule which regulates the structure *becomes an object in its own right* through super-position on the outline of the object. Thematization breaks the horizon of the paradigm and announces a rupture with the lateral immanence of the rational sequence. The trajectory of meaning, (the consistency of the object domain) is no longer grasped from the perspective of an accomplished meaning, but is "seized en route," or as Cavaillès attempts to clarify, is no longer referred to the "created term" but to the "creating modes." A Spinozist aura has been introduced into the conceptual dialectic, for it is not enough to give an account of mathematical objects as singularities (*natura naturata*) but as *veritable styles of creation* (*natura naturans.*) The mathematical object as singularity appears as a simple datum. It is given without reference to any generic domain, it is only with the introduction of the

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[322] Cavaillès 1970, 377.

thematic which stands over and against the paradigm that the singularity becomes a member of an entire rational universe.

How does one move from the object as singularity to the rule which gives the form of the object? How does one arrive at the perspective from which it is possible to generalize the entire object domain and not simply to go on extending the operations correlative of the object? Granger gives two examples from the history of mathematics which illustrate the procedures of demonstration proper to each perspective. Granger cites Eutocius' (c. 480- c. 540) commentary on the conics of Apollonius as an example of concept formation under a paradigm.

> Just as the ancients, investigating each species of triangle separately, proved that there were two right angles first in the equilateral triangle, then in the isosceles, and finally in the scalene, whereas the more recent geometers have proved a general theorem, that *in any triangle the three internal angles are equal to two right angles,* so it has been said for sections of the cone…Apollonius of Perga proved generally that all the sections can be obtained in any cone, whether right or scalene, according to different relations of the plane to the cone.[323]

For the ancient geometers "each species of triangle" appears *sui generis* and the possible forms of the conic section are likewise investigated as if they were individual and irreconcilable objects. Apollonious elaborates a mathematical paradigm when he suppresses the individual singularity of each conic section with reference to a more general operation defined by the intersection of a cone and a plane capable of producing any conic section. Here the individual accident of a given conic section is not what merits investigation; rather attention must be turned

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[323] Ivor Thomas, *Greek Mathematical Works, Vol. II* (Cambridge: Harvard University Press, 1941), 279. Cited in Gilles Gaston-Granger, *Pour la connaissance philosophique* (Paris: Éditions Odile Jacob, 1988), 71.

toward a cause capable of producing any conic section whatsoever. As Granger notes, when Descartes subjects the synthetic construction of Apollonius to algebraic representation, he does not break from the object domain, he simply elaborates the concrete acts capable of extending its efficacy by introducing (through algebraic notation) the moment of the variable and its associated liberation of meaning through the suppression of singularity. Under the paradigm the algebraic notation still serves the concrete object.

It is not until Descartes reflects on the algebraic system itself as a generic method of construction that is he is capable, as Granger writes, of "operating a thematization." In the second book of the *Geometry* (much beloved by Brunschvicg in his own reconstruction of Descartes' mathematical philosophy), Descartes speaks of a compass which would draw not simply a given curve "but an indefinite multiplicity of the laws of construction," in Granger's paraphrase. The "algebraic curve" or "curve of the second degree" is a rule "which generates the rules of construction" for the algebraic definition of any spatial representation[324]. Beyond the intuitive clarity of the geometric figure, the *idealization of space* appears as the thematization of all Euclidean geometries whatsoever. As discussed in the previous chapter, Descartes' effort to found geometry on purely analytic grounds, that is, to transform the intellectual foundation of spatial intelligibility into pure analysis constitutes, alongside the dual inventions of the integral and differential calculus, one of the great mathematical thematizations of seventeenth century rationalism.

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[324] Granger 1988, 72.

Granger also cites the projective geometry of Pascal and Desargues as examples of a comparable displacement of the geometric object in favor of a *variable rule of production* which does not rely on any “contingent content.” The conceptual style of mathematical modernity consists of this general transcendence of the paradigm by the novel clarity and asceticism of the thematic.

Thematization is not simply the rule which governs the object; *it is the rule by which the rule is given*. The thematization of the object relativizes the object, uproots it from its tranquility and makes it in turn an *effect* of a more comprehensive cause. Although the great texts of rationalism furnish an abundant landscape of conceptual thematization (Leibniz in particular is singled out for ambivalent praise by Cavaillès, who remained skeptical of the *mathesis universalis*), it is really the revolutions of nineteenth century mathematics which preoccupy Cavaillès. Both Hermann Hankel (1839-1873) and Hermann Grassmann (1809 -1877) bring about a profound “denaturing” of number theory, and Hankel’s work in particular provides an abundant stimulus for Cavaillés’ reflection on the motor of mathematical progress as essentially driven by the projection of the object into a novel abstract field which relativizes its parameters. For Granger Hankel is significant because he

> detaches the familiar operative processes of classical algebra from their originary domain, characterizing the notion of “number” no longer as a determined object but as an operative system, “ no longer combining magnitudes but the elements of thought.”[325]

[325] Granger 1988, 76. Granger cites Hermann Hankel, *Theorie der komplexen Zahlensystem* (Leipzig: Leopold Voss, 1867.)

The transversal movement of thematization is characteristically expressed by "the abstracting mechanisms of modern algebra" and number theory. Cortois writes:

> Here one takes an operation – after the accomplishment of its paradigmatic generalization – itself as the object of an operation of a higher type. One takes for example the abstract additive and/or multiplicative operation as a subject of investigation in its own right, thus uncovering its abstract working principles or properties (associativity,commutativity.) The original operation thus gives rise to new objects of a higher type: associative structures, *etc.* In this way the chain is not extended along one and the same line, but a new chain is created on a different level of concept/object formation, and superposed on the previous one. To extend the example just mentioned: one *thematizes* the idea of associative, commutative structures with two internal operations one of which is distibutive with respect to the other *etc…* and takes this idea as the origin of theorizing about all possible structures that fulfill the properties under consideration.[326]

The conceptual tools of abstract algebra and modern number theory are exemplary of the vertical rational sequence of thematization because they no longer point toward determined objects but toward fields of operations which are "the elements of thought" in Hankel's formulation – not merely dissociated meanings which have been abstracted from the singularity of the object but abstract structures generated without reference to a given singularity. As Sinaceur notes, the epistemology of mathematical novelty and the historical description of mathematical progress are immanently conjoined in Cavaillès. Indeed, the entirety of his work can be located within the affirmation that the history of mathematics is singularly expressive of the polymorphous elaboration of rational sequences along the horizontal and vertical vectors of the paradigm and the thematic. Sinaceur claims that Cavaillès' sensitivity to the different levels of rational organization

[326] Cortois 1996, 34-35.

proper to the conceptual development of mathematics as a historical sequence owes much to his historical research under the tutelage of Emmy Noether (1882-1935) at the University of Gottingen.[327] Noether's work on abstract algebra captured the extreme inventiveness of mathematics in the 1930's almost completely under the sign of algebraic relations. As P.S. Alexandroff writes in his tribute to the memory of Noether addressed to the Moscow Mathematical Society in 1935 : "Today the development of mathematics occurs under the sign of algebrization, that is to say, the penetration of algebraic ideas and methods into mathematical theories of the most diverse kinds."[328]

Sinaceur points to a small but influential convergence of mathematical expertise in the twenties and thirties in Gottingen and Hamburg which emphasized the heuristic value of abstract algebra as the intelligible formulation of the structures under-pinning many different mathematical fields. Emily Noether, David Hilbert and Emil Artin were the central figures in the intellectual landscape Cavaillès would have been immersed in as he pursued his education in higher mathematics. Claude Chevalley, one of Noether's students and a contemporary of Cavaillès, presents the significance of algebra within the architecture of contemporary mathematics as follows:

> Algebra is not only a part of mathematics, it also plays within mathematics the role which mathematics itself played for a long time with respect to physics. What does the algebraist offer to other mathematicians? Occasionally, the solution of a specific problem; but mostly a language in

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[327] Sinaceur 1987, 11.

[328] Sinaceur 1987, 11. Sinaceur cites P.S. Alexandroff, "In Memory of Emmy Noether," address to the Moscow Mathematical Society, September 5th 1935.

> which to express mathematical facts and a variety of patterns of reasoning, put in a standard form. Algebra is not an end in itself: it has to listen to outside demands issued from various parts of mathematics.[329]

Here the complementary dialectic of problem and solution is dissolved in favor of a far more general and ambitious project: that of establishing a common "language" of mathematical inquiry. Chevalley circumscribes the procedure of thematization effectively, albeit unwittingly, when he designates algebra as the language proper to the notation of "mathematical facts and a variety of patterns of reasoning, put in a standard form." If Sinaceur is correct in her thesis that Cavaillès' theorization of the rational sequence of mathematics is deeply influenced by the abstract algebraists of the twenties and thirties, then this would also seem to shed helpful light on the relation between the paradigm and the thematic. The thematic level of mathematical abstraction does not unify what was previously inchoate or anarchic among the paradigmatic elaborations of different objects. Its standardization is not the suppression of genuine differences but only the erasure of trivialities in order that "a standard form" might emerge within which it is possible to define intelligible relations between objects, and, by the same gesture, the objects themselves. The paradigm simply receives the object as a singularity. The thematic takes up the object as an element among others in a sequence it generates out of its own operations. *Thematization re-writes the singularity of the object by translating it into a universalist language of operations.* It is this capacity of mathematics to return to itself from a novel perspective, or to fall back into itself as into a renewable origin

[329] Claude Chevalley, *Fundamental Concepts of Algebra* (New York: Columbia University Press, 1956), V. Cited in Sinaceur 1987, 12-13.

which makes it at once self-complicating and self-differentiating. If thematization constitutes the moving threshold at which mathematics tentatively writes its own universality, it is always with the proviso that this universality itself will be the ground of new concrete operations, new singularities, and new objects which will in turn require a thematic deepening.

Consequently, although Cavaillès uses a language abundantly stimulated by Hegelian dialectical machinery, there is good reason to suspect the image of reason motivating his conceptual mechanisms is closer to a profoundly Spinozist affirmation of reason than to a Hegelian passage from objective to absolute reason, the very irreversibility of which would be offensive to Cavaillès' mathematical training and philosophical convictions.[330] The Spinozist aura of unlimited rational

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[330] In *La pensée mathematique* (Cavaillès 1994, 619) Jean Hyppolite, in discussing the theses of Albert Lautmann, diagnoses three apparently conflicting uses of the word "dialectic": the first is common to Cavaillès and Lautmann and refers to the necessity of mathematical development which must be reconstructed conceptually beneath the evident contingency of its historical becoming. Lautmann uses the term in two additional senses, to designate the kind of mathematical research which discovers and incarnates new problems and possibilities of mathematical thought, and finally, dialectics is used in the sense "most often used by philosophers", as Hyppolite writes, in order to designate the oscillation, internal to mathematics, between form and content, local and global etc. Hyppolite claims that of the these three senses only the first is legitimate because it captures the becoming of a given historical sequence. Hyppolite remarks that he is entirely in accord with Cavaillès' use of dialectics in this sense because it seems to eschew the open ended complexity which Lautmann's use of the term incorporates. See *La Penseé Mathematique* in Cavaillès 1994, 619-621. However, in the LTS Cavaillès also seems to explore, in considerable detail, the expanded senses of the internal dialectic of mathematical experience which Hyppolite had previously censured in Lautmann. In *Logic and Existence* (1952) Hyppolite suggests that the strictly internal dialectic of concepts Cavaillès ascribes to mathematics is not foreign to Hegelian logic, but that Cavaillès also seems to describe a process without a subject and is therefore closer to Spinoza than to Hegel. Hyppolite writes: " Cavaillès speaks of a deductive structure, creator

development is everywhere at work in Cavaillés' epistemology, even if it is only very rarely explicitly marked as such.[331] Thematization would seem to be an epistemological mechanism which closely resembles the "idea of the idea" or the method guaranteeing the passage from true idea to true idea. In Cavaillès, however, the idea of the idea does not return at the level of thematization in order to integrate what would have remained groundless as a purely paradigmatic development. The object does not require a rational integration into a more general schema in order to extend its effects; precisely the opposite is the case. Nor is it the case that the object is exhausted by the efficacy it authorizes through the horizontal extension of its operations. Instead, as in Spinoza, the issue is what kind of cause is capable of generating knowledge which is both true and infinite. The dialectic of the paradigm

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of the content that it attains… As in Hegelian dialectic, there is therefore an internal progression from singular content to singular content…If a dialectic proper to mathematics exists in this way, where would it fit in a Logic of being like that of Hegel? Perhaps however, in Hegel, the self is more immanent to the content than in Cavaillès; on this point, the rapprochement of Cavaillès with Spinoza would be more precise than the rapprochement of Cavaillès with Hegel. Cavaillès makes us think less of the unity of the subject and object resulting in sense than of God's infinite understanding in Spinoza and of the passage from true idea to true idea." See Jean Hyppolite, *Logic and Existence* trans. Leonard Lawlor and Amit Sen (Albany: State University of New York Press, 1997), 52-53.

[331] See Jean-Charles Augendre, "Cavaillès et l'ontologie Spinoziste," *La pensée,* no. 317 (1999): 105-117 and Gilles-Gaston Granger, "La montée vers Spinoza", *Les Études Philosophiques*, no. 3/4 (1947): 271-279 and Bruno Huismann, "Cavaillès et Spinoza" , in *Spinoza au XXe siècle*, ed. Olivier Bloch. (Paris: Presses Universitaires de France, 1993), 71. See also Cassou-Nogués 2001, 317-319. On the influence of Spinoza in Brunshcvicg as formative of Cavaillès see Didier Gil, "Le Vrai Spinoziste de Brunschvicg a Bachelard." in *Spinoza au XXe siècle*, ed. Olivier Bloch. (Paris: Presses Universitaires de France, 1993), 41. Finally, Canguilhem's encomium to Cavaillès situates the epistemological and political actions of his friend under the sign of a Spinozist logic of necessity, see Georges Canguilhem, *Vie et mort de Jean Cavaillés* (Paris: Éditions Allia, 2004.)

and the thematic authorizes what Cavaillès describes as "an unlimited intelligible complication:"

> The idea of the idea manifests its generative power on the lower level [the paradigm or object as object of an operation] which it defines without prejudice to an unlimited superposition…Here we have an unlimited intelligible complication which at once shows how the image of the separation between matter and form in the rational sequence was simplistic and how a systematic study of forms is indispensable. There is no formalism without syntax, no syntax without another formalism which develops it.[332]

Thematization shares with the idea of the idea "its generative power;" it is the method by which true ideas give rise to true ideas. In the domain of mathematical experience, this constitutes, on the one hand, the de-naturing of the object domain (suppression of its singularity) and the passage to a new formalism (the re-writing of the singularity in a universalist language), on the other. Most crucially, however, the passage from the paradigm to the thematic illustrates the essential nature of the constitution of mathematical meaning which can never be definitively reduced to a stable distinction between matter and form. Mathematical abstraction is irreducible to any abstraction from lived experience. Both form and content are internal to mathematical experience.

Here again the theory of formal contents seems indispensable if Cavaillès' theory of rational sequences is to be understood in its proper aspect as *the theory of the necessity of mathematical progress.* Mathematical progress must here be understood from two perspectives. 1) From the perspective of the relationship between form and content which is articulated horizontally by the elaboration of the

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[332] Cavaillès 1970, 378-379.

object (in which case content determines the forms of the object's efficacy). 2) From the perspective of what Cavaillès calls the difference between "posited meaning" – the mathematical act as given in itself (the singularity of the object) and "positing meaning" which is articulated vertically by the super-position of one rational sequence on top of another – the mathematical act as generative of a field of operations (in which case it is a matter of a formalism describing all possible structures of a pre-determined class of objects.) The paradigm concerns itself with the "posited meaning" of the mathematical object – it gives primacy to the content of mathematical experience. The thematic concerns itself with the "positing meaning" of the mathematical operation – it gives primacy to the form of mathematical experience. Mathematical progress however is defined by the dialectical entwining of these two procedures in a process of conceptual genesis which can only artificially be arrested. The theory of rational sequences is above all a theory of mathematical becoming and therefore a theory of the necessity of the transformation of the paradigm into the thematic, and of the crystallization of new paradigms out of thematic formalizations.[333]

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[333] The fundamental non-closure of the rational sequence is its truly essential characteristic: the saturation of a paradigm is not the inertia or obsolescence of a conceptual field, it is the premise to a formalization which super-poses a new rational sequence on top of the previous sequence. The relation between thematic elaboration and paradigmatic elaboration is not simply that of a cause embracing an effect which would otherwise have appeared as an anomaly lacking rigorous foundations, instead, contrary to all foundationalist projects, thematization is the invention of new mathematical possibilities, not the unearthing of an architectonic already pre-supposed but only imperfectly grasped by its effects. Here the Spinozism of Cavaillès is very close to the Spinozism of Brunschvicg, for whom the accomplished forms of rationalism are not the starting points of thought but the

The theory of rational sequences has profound implications for the logicist and formalist programs in the foundations of mathematics. On the one hand, Cavaillès will contest the reduction of mathematics to logical syntax (especially in the work of Carnap), arguing that unlike a logical proof, the essence of mathematical knowledge cannot be derived from a fixed number of axioms. On the other hand, in the case of Hilbert's formalist presentation of mathematics, Cavaillès will make a related criticism, with the added novelty of a fundamental critique of the mathematical sign, which Hilbert reduces to an empty formalism. For Cavaillès the nature of mathematical experience as a science of formal contents requires a more sophisticated approach to the problem of the writing systems of mathematics than that furnished by the arbitrary or conventional theory of the sign. If mathematics is a variety of rational experience irreducible to the requirements of a transcendental consciousness or the problem solving exigencies of the interrogation of empirical experience, then its writing system must be simultaneously sensible and intelligible, idealizing and concrete, as dictated by the reciprocal transformation of the paradigm and thematization.

Logic as the Theory of Science: The Limits of Logicism

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dialectical horizons of a thought which must increase both its subtlety and its differentiation if it is to be capable of constructing a truly rational contour for objectivity. The yield in complexity which thematization produces at one level is also a simplicity or an elegance at another level – the inevitable progress of mathematical intelligibility is marked by this to and fro of complexity and elegance which Cavaillès captures with the mechanism of "an unlimited intelligible complication."

When Cavaillès invokes the inseparability of formalism and syntax he gathers together the separate moments of the paradigm and the thematic into the elemental movement of their mutual explication: there is no final stability of form and content, no final distinction of posited and positing meaning, only the progressive elaboration of object domains and their attendant formalisms, and of formal operations and their attendant syntactic invariance. The dialectic of concepts proper to mathematical experience describes an indefinitely progressive rational sequence which would be constitutive of science itself. Cavaillès' critique of logicism proceeds from the failure of the logicist programs to grasp the essentially dynamic and progressive nature of the rational sequence. The history of this error has two phases. The first phase is inaugurated by Bolzano and passes through Frege, Russell and the early Carnap. The classical definition of logicism before Gödel is the construction of the logical principles from which all the theorems of mathematics can be deduced. Logicism in this early phase of its development is the thorough subordination of mathematics to logic. The second phase of the logicist program is marked by Cavaillès as beginning with Carnap's *Logical Syntax of Language* (1934.) Logicism in its mature form (after the publication of Gödel's two incompleteness theorems in 1933) no longer proposes an absolute architectonic of mathematics but instead, in keeping with the ambition of Bolzano, attempts to establish the logical basis underpinning any given system of demonstration. Cavaillès points to Carnap's principle of tolerance as indicative of the new logicism: "In logic there is no canon but the unlimited possibility of choice among the canons."[334]

Carnap is not concerned with the exhaustive presentation of the logical architecture of mathematics, as Bolzano, Russell and Whitehead were. Instead, as Cassou-Noguès observes, Carnap is interested in the "context of exposition" as an important feature of logic.[335] The idea of a logical syntax is the attempt to formalize or to render explicit the implicit rules of a logical system, be it a system of natural language or of mathematical theorems. Carnap's logicism does not presuppose a hierarchy of the forms of reason; it only presupposes that it is possible to model by the construction of a meta-logic (the syntax of a logical system) the rules by which a given logical system constructs its propositions. From this perspective it must remain an open question how the logical systems proper to the world, to language, and to mathematics would enter into relation with one another. The relation between logic and mathematics is, therefore, determinate but not exhaustive, or as Xavier Sabatier writes: "Logic enumerates the rules which can be used by mathematicians without deducing, on the basis of these rules, the totality of mathematical theorems."[336] As Sabatier remarks, by the time Cavaillès turns his attention to Carnap he has already presented his own theory of the dialectical movement of reason in the form of the theory of rational sequences and so cannot accept even the more modest claim of a mathematical logic which would purport to

[334] Cavaillès 1970, 379.
[335] Cassou-Nogués 2001, 275.
[336] Xavier Sabatier, "La logique dans la science: Place et statut de la logique dans la philosophie de Jean Cavaillès," *Revue d'histoire des sciences,* vol. 52, no. 1 (1999): 95.

define, axiomatically or syntactically, the logically valid forms of mathematical demonstration.

The fundamental critique of logicism is identical to the fundamental critique of formalism for Cavaillès – the logicist and the formalist do not understand how abstraction works relative to mathematical knowledge. Like Kant, they model the process of abstraction on the bracketing of empirical experience in order to construct an artificial domain of form which captures the intelligible profile of empirically given content. In Carnap this procedure takes place when, in Cavaillès' paraphrase, Carnap attempts to construct the "syntax of syntax." Carnap attempts to separate logical syntax from semantics, or the operative plane from the object-domain, a procedure which is always possible but which can only capture a synchronic moment of the rational sequence and not what is most essential – the perpetual unfolding of new logical vocabularies and syntactic orders which correspond to new objects, and vice versa. The syntax of syntax is the grand ambition of logicism; for Cavaillès, however, it is also a grand distortion of the intelligible.

> That all may not take place in a single stroke has nothing to do with history but is the characteristic of the intelligible. To misunderstand it is to leave the security of its immediate presence for a projection whose internal emptiness as well as the lapse into the historical appears in connection with the actualizations of a time and their accidental veneer. There is no attainment of an absolute, but a hypostasis of systems and procedures which exist only as transitory. To abstract from the type is not fix but to arrest the essence.[337]

[337] Cavaillès 1970, 380-381.

The intelligible is in no way pre-determined to appear in lucid self-evidence relative to the subject that contemplates it. The notion that mathematical logic should ultimately be reducible to evidence (in the form of a self evident identity) was already subjected to thorough critique by Bolzano, who attempted to transform the Cartesian order of evidence into a procedural hierarchy of demonstration based on propositions in themselves. "The characteristic of the intelligible" as Cavaillès writes is not stasis but active self-differentiation. The notion of a logical absolute must be contradictory: abstraction from a fixed type does not yield a genuine thematization of the object but only a hypostasis and an artificial clarity born of stasis.

Sabatier notes that, as Cavaillès' text proceeds, "logic and the doctrine of science are no longer conceived as synonyms but as clearly distinct notions."[338] Kant's transcendental logic and Carnap's overly hasty formalism are equally incapable of respecting simultaneously the autonomy of science (irreducible to consciousness or the verification of external states of affairs) and the necessary transformation of its concepts and its norms of intelligibility. The problem of a theory of science cannot easily be assimilated to logic even if logic is clearly an indispensable element of the structure of science. The critique of logicism, even in its post-Gödelean form, strongly indicates that for Cavaillès the minimum condition of a theory of science must be to respect what Cortois calls "mathematical holism" - the affirmation of the processual or emergent unity of mathematics irreducible to

[338] Sabatier 1999, 95-96.

any of its components or fields and equally irreducible to any of the multiple forms of logic it is capable of generating. The theory of science must defend the chiasmus of mathematics and logic; it cannot, however, turn to logic itself as the theory of science. What then is the proper signification of logic within Cavaillès' epistemology if it can no longer stand in as the discipline proper to the theory of science itself?

Cavaillès rarely addresses logic directly in his work with the exception of two articles *L'École de Vienne au congrés de Prague* (1934) and *Loqique mathématique et syllogisme* (1937). The former introduced the Francophone philosophical community to the work of Wittgenstein, Schlick and the logical positivism of the Vienna school; the latter addressed the substantive difference between traditional logic and formal or mathematical logic. If, as seems to be the case, logic cannot be the framework of the theory of science, then logic itself must be located within this theory. The concluding pages of *Méthode axiomatique et formalisme* certainly support this conclusion, as when Cavaillès writes of Brouwer's intuitionist logic:

> The true meaning of logic seems to have been definitively specified by Brouwer: it is the translation, in the syntax of language, of the general expression of finite systems; its authority is that of a primary stage through which it is always necessary to pass, an authority identical to that which arithmetic and analysis hold for posterior theories.[339]

The reference to finitude is primary here, as is the notion of a translation of a "finite system" into the syntax of language, that is to say, an order of propositions. Logic does not have the same relationship to mathematics as the theory of rational

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[339] Cavaillés 1994, 188.

sequences, because logic itself is a rational sequence, and of a very particular kind, a necessarily finite sequence presumably incapable of the indefinite elaboration proper to mathematical experience. Cavaillès explicitly aligns logic with finitude and not, as in the theory of rational sequences, with indefinite development and necessary progress. Logic is a science of finitude; mathematical experience, as Cavaillès affirms throughout his work, begins with the infinite.[340]

Carnap's logical syntax, Cavaillès claims, proves incapable of capturing the dynamic genesis of mathematical novelty in its historical transformations. Mathematical logic presumes that the rules of logical inference have analytic priority over properly mathematical notions such as number, thus requiring the subordination of mathematics to logic as the science which would encompass it. Cavaillès dismisses this possibility on the grounds that while mathematics and logic are inter-penetrating domains (logic makes use of mathematical notions just as mathematical procedure makes use of logical rules), mathematical experience can never be reduced to a determinate logical syntax. This would be to mistake a particular historical sequence in the rational chain of mathematical development for the definitive architecture of mathematical reason. Logic, following Brouwer, issues from determinate experience. Logic is itself the rigorous presentation of the syntax of "finite systems." The theory of rational sequences demonstrates, on the contrary, that mathematics is a system which thrives on the transformations of the relationships between its concepts (object domains and fields of operations.) It is

[340] See also, Sabatier 1999, 106.

impossible to retain the essence of mathematics as progress and transformation if the relationship between the object and the operation is crystallized in the form of an immutable syntax for operations and an inflexible semantics for objects.[341]

The Impasse of Formalism: Toward a Theory of the Mathematical Sign

Logicism fails the minimum conditions required by Cavaillès' understanding of the nature of mathematical experience and the theory of rational sequences. It cannot model the progress of mathematical concepts and it cannot perceive the necessarily historical transformation of the rational procedures internal to mathematical experience. David Hilbert's formalist approach, however, provides

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[341] Granger notes that the theory of formal content, inspired by Cavaillès, and immanent to the theory of mathematical experience, also has profound implications for mathematical logic and a special relevance in the wake of Gödel's incompletion theorems. See Granger 1994, 42. The fixity of the relationship between form and content can be understood as equivalent to the formation of a proof of the consistency of a formal system. The formalisation of a propositional calculus would consist in the exhaustive determination of a given configuration of form (as field of operations) to content (as object domain and semantic field.) Gödel's work however would seem to imply the discovery of an irreducible mobility of formal content, exactly what Cavaillès is so keen to uphold in the face of logicist and formalist reductions of mathematical experience to a syntax of syntax or to axiomatic consistency. The theory of formal content seems to be entirely compatible with a theory of the undecidability of formal systems – there can be no definitive correlation of the object plane and the operative plane, no natural unity of form and content, and no irreducible priority of syntax over semantics. The appearance of the complete determination of a formal system as necessarily consistent would then be something like a transcendental illusion foisted on mathematics by the consolidation of the paradigm. For Granger, the theory of formal contents results in the generalization of Gödel's theorem for all the symbolic vocabularies of human intelligence, resulting in the need to reformulate a general theory of semiosis on the basis of a theory of formal contents. See Granger's *Forme, opération, objet* especially Chapter Two, *La notion de contenu formel,* and Chapter Five, *Conditions protologiques des langues naturelles.*

Cavaillès with several important clarifications of the nature of mathematical experience, especially with reference to the nature of the mathematical sign. Is Hilbert's work in the conceptual foundations of mathematics capable of providing the elements of a theory of science which will satisfy Cavaillés' stringent conditions? On the one hand, Hilbert is the source of an important clarification of Kant regarding the role of intuition in the construction of mathematical experience; on the other hand, Hilbert remains proximate to Carnap in maintaining the arbitrariness of the mathematical sign, which Cavaillès will contest in order to locate the mathematical sign as already "interior to the mathematical act." What Hilbert's formalism reveals for Cavaillès is a definitive example of the need to reformulate the theory of the mathematical sign in order to account for the partially sensible, partially intelligible nature of mathematical experience.

In his lecture "On the Infinite," David Hilbert makes reference to Kant in order to secure the autonomy of mathematics against the incursions of logic. On this point Kant, Hilbert and Cavaillès would seem to be in at least temporary agreement, for there is a shared perception in each case that if logic is allowed to dictate to mathematics the entirety of its proper form and content, then something unique to the nature of mathematical knowledge will be lost. Hilbert writes:

> …we find ourselves in agreement with the philosophers, especially with Kant. Kant already taught – and indeed it is part and parcel of his doctrine – that mathematics has at its disposal a content secured independently of all logic and hence can never be provided with a foundation by means of logic alone; that is why the efforts of Frege and Dedekind were bound to fail. Rather as a condition for the use of logical inferences and the performance of logical operations, something must be given to our faculty of representation, certain

> extra logical concrete objects that are intuitively present as immediate experience prior to all thought.[342]

Logic cannot be the science encompassing mathematics because it can give no account of the concrete basis upon which mathematics, as an activity in time and space, must depend. On the contrary, in order to secure the coherence and reliability of logical inferences, "certain extra-logical concrete objects" must supply the foundational elements of thought in "immediate experience." For Hilbert this concrete grounding of thought is required for all understanding. In the case of mathematics "what we consider is the concrete signs themselves, whose shape, according to the conception we have adopted, is immediately clear and recognizable."[343]

As Cassou-Noguès remarks, it is important to clarify how Hilbert understands his debt to Kant in this passage.[344] As discussed in the previous chapter, Kant's project to found mathematics in intuition calls upon the transcendental functions of all the faculties. The representation of a concrete object in space and time requires the transcendental determination of our sensibility according to space as the exterior form of representation and time as the interior form of representation. The unity of the object is secured by the synthesis of intuition in the imagination, after which the recognition of the object as conforming

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[342] David Hilbert, "On the Infinite," in *From Frege to Gödel – A Source Book in Mathematical Logic, 1879-1931,* ed. Jean van Heijoonort. (Cambridge: Harvard University Press, 1967), 376. Cited in Pierre Cassou-Nogués, "Signs, Figures and Time: Cavaillès on 'intuition' in Mathematics", *Theoria*, no. 55 (2006): 92.
[343] Hilbert 1967, 376.
[344] See Cassou-Nogués 2006, 92-93.

to the categories of the understanding can occur. The concretion of the mathematical sign in Hilbert plays the same role as the determination of representation in Kant. The sign is the sensible foundation of mathematical practice and is the substrate upon which the scaffold of mathematical thought is built. Hilbert's formalist project must be understood in light of the theory of the mathematical sign upon which it rests, in which case a parallelism emerges between the visibility of the sign as the sensible foundation of mathematical practice and formalization as the explicit rendering of the axioms which subtend mathematical reason. Mathematical intelligibility is consistent with a set of rules which do not appear in the explicit writing of mathematics. Formalization is thus a re-writing of mathematical intelligibility in terms of this explicit formulation of the rules or "proof-figures," in Hilbert's expression, which authorize the deductions of the mathematician.

Cassou-Nogués refers to Hilbert's formalization as a "rectification of mathematical practice" by which any assumption of a specifically mathematical content is replaced by "an inventory of formulas."[345] Formalization denudes mathematics of any supposed "essence" underlying its procedures and lays bare the principles of its operations. What is at stake in the formalist project is a rectification of mathematical reason which destroys the illusion of mathematical content in order to illuminate the structure of the relationships which obtain between mathematical propositions. Formalization can be conceived as what Hilbert calls a "formula game"

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[345] Cassou-Nogués 2006, 93.

which, like the mathematical sign, exposes the elemental mechanisms of mathematics. Hilbert writes:

> This formula game enables us to express the entire thought content of the science of mathematics in a uniform manner and develop it in such a way that, at the same time, the interconnections between the individual propositions and facts become clear.[346]

The formula game is not only a rewriting of mathematics such that the "thought content" of mathematics achieves a systematic expression in a suitable system of signs; it is also the revelation of new knowledge in the form of "interconnections" between the parts of mathematical practice which, although implicit, may have remained unformulated. Formalization is not simply an abstract armor which would cover the body of "natural mathematics" in order to shield it from contradiction; it is in fact mathematics become self-conscious, a truly rigorous mathematics and thus a different order of knowledge entirely.

Formalization, as Cassou-Noguès remarks, is "[t]he correct expression of mathematical thoughts…to formalize a theory is to make explicit the different thoughts that constitute a proof, their relationships and the rules they obey."[347] Mathematical thinking is to be found, in unadulterated form, in the correct expression of mathematical proofs or "proof figures," as Hilbert says. Hilbert's formalist program establishes the identity of thinking and a certain form of writing – the formal exposition of mathematical rules. Hilbert claims that formalization is the

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[346] David Hilbert, "The Foundations of Mathematics," in *From Frege to Gödel – A Source Book in Mathematical Logic, 1879-1931,* ed. Jean van Heijoonort. (Cambridge: Harvard University Press, 1967), 475. Cited in Cassou-Nogués 2006, 93.
[347] Cassou-Nogués 2006, 93.

method that expresses "the activity of our understanding" and "the rules according to which our thinking actually proceeds. Thinking, it so happens, parallels speaking and writing: we form statements and place them one behind the other."[348] There is a rational identity of thinking, speaking and writing which is expressed, formally, in the presentation of the order of operations each must follow. The mathematical sign is already a thought; conversely, the depths of mathematical intelligence reveal a play of signs which obey a structure that can be explicated.

This is the ambition of Hilbert's formalism – to correctly express the rules of the understanding in the system of signs of which it is composed. This is also the point at which Cavaillès notes the profound separation of Kant and Hilbert, for contrary to Kant Hilbert endorses no pure faculty of understanding, since sensibility (the mathematical sign) and understanding (mathematical intelligibility) are co-constitutive. In *Méthode axiomatique et formalisme,* Cavaillès notes that as early as 1900 Hilbert already presents the mathematical sign as encompassing both the written notation of mathematical formulas and the figures of geometrical representation. Cavaillés paraphrases Hilbert: "mathematical signs are written figures."[349] Cavaillès insists that Hilbert seems intent to unify all the representative capacities of mathematical writing into a single theory of the mathematical sign which would then include symbolic notation, figures and diagrams, and the implicit organization of the formulas themselves. The unifying capacity of the mathematical

[348] Hilbert 1967, 475. Cited in Cassou-Nogués 2006, 93.
[349] Cavaillès 1994, 101. Cassou-Nogués identifies the passage in Hilbert and offers a more complete translation: "The arithematical signs are written figures, and the geometrical figures are designed formulas." Cited in Cassou-Nogués, 2006, 94.

sign appears in the efficacy of the formalizations it makes possible, uniting through the agency of the correct expression of mathematical intelligibility fields that appear disparate but which actually enjoy an intelligible unity. In the case of algebra and geometry Hilbertian formalism reveals that geometrical figures can be translated into axiomatic rules or "proof-figures" which are in no way bound to their spatial representations. There is a common plane of sign manipulation proper to the rules which govern mathematical intelligibility in all fields. Formalism is the explicit writing of these rules. Cavaillès defines the "correlative notions of the sign and the axiomatic" in Hilbert as follows:

> The essence of mathematics itself is the well ordered play of symbols which are not merely aides for memory but which define a sort of abstract space with as many dimensions as there are degrees of freedom in the concrete operation…[350]

Mathematics is an ordered play of symbol manipulation. The sign or symbol (interchangeable terms in Cavaillès' discussion of Hilbert) is not simply a place holder for intelligible contents but is rather expressive of what Cavaillès calls an "abstract space" not to be confused with space as the exterior form of representation in Kant. Hilbert's space is not Kantian space but a space of intelligence which is ordered according to very precise rules. Hilbert collapses the Kantian distinction between the sensible and the intelligible by locating the mathematical sign not at the axis or hinge between sensibility and intelligibility, but as a concrete mixture of both, *an intelligible sensibility or a sensible intelligibility*. This is the degree zero of mathematical practice, a concrete-intelligible sign or a

[350] Cavaillès 1994, 101.

writing of formal contents. The abstract space of mathematics (which Cavaillès will also refer to as a "combinatorial space") has all the dimensions required by the operations of mathematics; it therefore cannot be constricted by a rigorous analogy to the space of lived experience.

Hilbert's mathematical signs thus express a partial fidelity to Kant in refusing to subordinate mathematics to logic and by insisting on the necessity of a locus proper to mathematical acts. The mathematical sign is also a tacit repudiation of Kant on this point, for the locus of the mathematical act is not located in the intuitive forms of representation but in the mathematical sign itself, which is correlated with an abstract "combinatorial space" rather than an intuitive space. Hilbert does not need to separate sensibility and understanding into discrete faculties as Kant does because there is no pure element of thought which would escape the manipulation of signs. As Cavaillès remarks in *Réflexions sur le fondement des mathématiques* (1937):

> Mathematics does not break from the sensible world – from which is derived the partial truth of the Kantian theory and its modification in Hilbert's theory of the sign – but is an analysis without end of the nucleus of sensible gestures. It is only by ontological bias or psychological error that one could define as separate that which is immanent in this thought.[351]

Mathematics is a thought of immanence – the immanence of the sensible within the intelligible in the case of the mathematical sign, and the immanence of form and content as a science of formal-contents. Hilbert's formalism partially grasps the

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[351] Jean Cavaillès, "Réflexions sur le fondements des mathématiques" in *Oeuvres complétes de Philosophie des sciences* ed. Bruno Huisman (Paris: Hermann Éditeurs des sciences et des arts, 1994), 579.

necessity of rethinking mathematical intelligibility in terms of the immanence of the mathematical sign rather than the transcendental determination of understanding by categories of representation given prior to experience. Hilbert's mathematical signs are only partial recognitions of immanence because although he manages to collapse the distinction between the sensible and intelligible, he still defaults to a Kantian position by locating the sign in an "immediate experience prior to all thought."[352]

Hilbert is right to make the sign irreducible within mathematical experience as in the famous utterance "At the beginning, there is the sign." He is wrong, however, to suggest that it is a simple starting point and not already part of a historical process of the genesis of mathematical concepts. Hilbert fails to grasp the nature of mathematical development from the perspective of its historicity, which militates against any static theory of the genesis of the mathematical act. The mathematical sign is not arbitrary but expressive of a very precise necessity – the dialectical enchaining of the rational sequences of mathematics. The mathematical sign is expressive of both historical and normative forces which themselves are expressions of formal content; therefore, contra Hilbert, the sign cannot simply be the sensible origin of mathematics. Rather, as Cassou-Nogués writes: "[t]he sign, in mathematics, is not at the beginning but in the middle, a part and a product, of a historical development."[353] Cavaillès attempts to redress Hilbert's theory of the sign

[352] Hilbert 1967, 376.
[353] Cassou-Nogués 2006, 99.

as the determinate origin of mathematical experience by properly locating it within the rational sequences of mathematics.

> The sign is not an object of the world, but if it does not refer to another thing that it would represent, it refers to acts which utilize it, the indefinite character of the regression being essential here. All the comparisons of mathematics with a spatial manipulation run afoul of this fundamental character of the mathematical symbol, cipher, figure, even a stroke of the pen, of simply being there as an integrating part or basis of application for an activity which is already mathematical. The symbol is internal to the act; it can neither be its starting point nor its actual result (which is a production of other acts.) The definition of a complete formalism therefore cannot claim such a sensory barrier. What it assumes to be an absolute beginning is only the surreptitious recall of prior acts and sequences.[354]

The mathematical sign is not an inert datum given to sensibility. As Cavaillès writes, the sign is not simply "an object in the world" but part of an autonomous conceptual development internal to mathematical experience. The mathematical sign refers to mathematical acts and is, as Cavaillès affirms, internal to these acts, not prior to them. The problem of the causality of the sign (does the sign proceed from properly mathematical idea or give rise to mathematical ideas?) must be reinterpreted from within the theory of rational sequences, in which case, just like the relationship between object domains and operations, the form of the sign and the content it expresses can only be provisionally dissociated. The mathematical sign is a concrete mixture of form and content, or just as well, an actualization of virtual singularities. Formalism, no less than the critical philosophy of Kant, fails as a suitable theory of science because it seeks to impose a fixed (if modified) relationship of the sensible and the intelligible at the origin of mathematics. Hilbert may be a more

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[354] Cavaillès 1970, 383.

sophisticated theorist of the mathematical sign than Kant, but he still falls into the unwarranted Kantian prejudice of a philosophy of representation and of determinate origins which is too rigid to grasp the dialectical movement of formal contents. Mathematics does not begin with the sign; the possibility of the mathematical sign is latent within an experience which was already mathematical.

Mathematical experience fully encompasses the sign as part of its historical development, consequently the sign can be neither the origin nor the result of a rational sequence. Hilbert's formalism, based on a parallelism of thought and writing, mistakenly isolates the sign as the origin of mathematical experience rather than fully realizing the consequences of immanence in the genesis of mathematical concepts: the mathematical sign is located in a historical development whose prior acts are already mixtures of the sensible and the intelligible. In keeping with the theory of formal contents the sign cannot be a simple "immediate datum" upon which to construct a well ordered game of symbolic manipulation. There is no immediate datum in mathematics, intuitive, sensible or otherwise, but an always provisional co-ordination of form and content negotiated by the intersection of the object and the operation in the interlacing of paradigm and thematic. Formalism is a foundational project, as such it has a taste for absolute origins, but this is contrary to the nature of mathematical experience for Cavaillès, which requires, on the contrary, an analysis which begins in the middle of already constituted acts, and which is in the process of constituting new acts. Hilbert thinks he has found the sensible basis of mathematics in formalizing the language of "proof-figures", but this is a "surreptitious" reference to prior acts whose genesis arises in a space of conceptual dynamism constituted historically. There is no irrevocable gesture at the origin of mathematics; indeed, as Cavaillès repeats throughout his work, the search for origins is identical with the distortion of the intelligible.

It is, however, exactly this orientation toward a clarification of science by reference to the originary intention guiding its development that Husserl proposes to illuminate in his transcendental phenomenology. Husserl's *Formal and Transcendental Logic* (FTL)[1929] is, in Husserl's own words, the result of "decades of reflection" on the phenomenological clarification of logic as a theory of science.[355] Husserl's project in the FTL resumes the legacy of Kant insofar as it attempts to secure the foundational legitimacy of logic as internal to the nature of reason, and therefore, part of the armature of a transcendental subjectivity. In the economy of Cavaillès' essay Husserl and Kant represent symmetrical formulations of the same problem: the necessarily transcendental characterization of logic which results when logic is taken as the theory of science. Throughout the LTS Cavaillès has carefully documented the distortions of mathematical experience which inevitably follow this subordination of mathematics to logic as the theory of science. What remains constant throughout the historical sequence passing through Kant, Bolzano, Brouwer, Carnap, Hilbert and Husserl is the refusal to grant mathematics its own content. Husserl's project represents a particularly powerful challenge to Cavaillès'

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[355] See Paul Cortois, "From Apophantic to Manifolds: The Structure of Husserl's Formal Logic," *Philosophia Scientiae,* Tome 1, no. 2 (1996): 16, for a brief overview of the significance Husserl assigned to *Formal and Transcendental Logic.*

epistemology of mathematical experience because it canonizes both a transcendental subjectivity and an ontology of logical relations which subtend the intelligibility of all possible worlds. The singularity of mathematics then finds itself undercut on both fronts by what amounts to a radical consolidation and repurposing of the theory of science under the auspices of a transcendental phenomenology of logic. It is no wonder then that the third and final section of the LTS, dedicated to Husserl's logic and phenomenology, is at once the longest and perhaps the most difficult section of an already formidable work. Cavaillès' will contest Husserl's claim to have definitively clarified the origins of logic and to have located the proper orientation of the sciences within the horizon of intention described by transcendental phenomenology.

Husserl and the Evidence of Reason: From a Phenomenology of Logic to a Philosophy of the Concept

Cavaillès' *Logic and Theory of Science* can be read as a persistent critique of foundationalist projects in the history of mathematics and logic. For Cavaillès the irony of foundational accounts is that the desire to uncover the normative bedrock upon which the scaffolding of mathematics can be erected inevitably produces a distorted image of what is actually normative in mathematical experience: the progressive transformation of a rational sequence whose formal and material elements can be distinguished only provisionally. Mathematics as the science of formal contents is incapable of being translated into a meta-logic or a system of axioms. Nor is mathematics a pure game of sign manipulation. There is indeed a powerful creativity at work in mathematical writing systems but the nature of this creativity is a far cry from Hilbert's "proof-figures." Mathematics is not simply a chain of rational evidences in need of a material origin as in Hilbert's theory of the mathematical sign. Nor can mathematics be an exclusively idealist realm of necessary truths beyond the vagaries of transformation. There is something of the eternal at work in mathematics but it does not show itself by way of an immutable structure so much as through the endlessly renewed requirement to recompose the entire body of mathematics from the perspective of each historical transformation. Mathematics is a totalizing enterprise which cannot be arrested in the progressive movement of its transformation. It would be a totality incapable of entertaining fantasies of the Absolute. What Cavaillès wishes to propose in the place of logicism, formalism, and as this section will demonstrate, phenomenology, is a theory which takes seriously the singular content of mathematics as a rational experience and the unique procedures by which this rational experience elaborates itself without reference to external conditions or to a subject, transcendental or otherwise.

In the design of Cavaillès' essay, Husserl's project explicitly takes up the challenge of providing an adequate theory of science in the wake of two particularly acute failures: the inability of the philosophies of consciousness to admit the autonomy and necessary progress of science, on the one hand; and the equally paralyzing monolith of a logical architectonic encompassing and enervating mathematical becoming on the part of logicism in the wake of Carnap and Russell, on the other. Kant was unable to convincingly make logic the transcendental origin of mathematical knowledge because he restricted mathematics to the possibility of a priori synthetic judgment, thus locating it in the functions of a transcendental subjectivity. This led Cavaillès to conclude that for the philosophies of consciousness logic could only be the effect of a transcendental operation and incapable therefore of integrating "contributions from the object", or from the rational sequences of mathematics. Relative to the problem of mathematical experience, Kant and the philosophers of consciousness make the objects of mathematics disappear. For transcendental philosophies of consciousness all forms of knowledge express the functions of the faculties. For the philosophies of consciousness, epistemologically speaking, there is nothing "new" that can be learned from the history of mathematics once the nature of the faculties and the syntheses of intuition have been worked out. On the other hand, Carnap's mature logicism represents something like a tyranny of logic over mathematics as well as a massive over-determination of the objects of mathematics which become reduced to syntactical rules of formulations, ordinal sequence, and transformation. The first position corresponds to a privileging of the "acts of consciousness" over the objects of mathematics, while the second corresponds to a stultification of the powers of mathematical novelty by reducing it to an invariant logical architecture – an object without possibility of becoming. This then is the dead-lock Husserl claims to be able to cut through with a phenomenological analysis of logic.

Cavaillès enters into a fraught proximity with Husserl relative to the problem of mathematics conceived as *experience.* Husserl wants to clarify the nature of logic by phenomenological analysis in order to erect thereby a theory of science which would be in conscious possession of its motivating principles. Phenomenology, as Cavaillès understands it, is not only about intention; it is about originary experience and, therefore, about the search for the most radical of foundations: the orienting sources of experience. Husserl and Cavaillès cannot however be said to share anything like a common understanding of any of the notions which they would seem to hold in common; neither logic, nor experience, nor mathematics really overlap at any point in their relative expositions. Husserl, like Kant, cannot perceive that the generative force of mathematical experience is immanent to the rational sequence and is not the result of a donation from experience or consciousness.

In the *Formal and Transcendental Logic*, Husserl includes within the domain of logic all extant logical programs as well as mathematics, which Husserl calls "formal mathematics." The goal of the FTL is to present the structure of formal science in the light of its phenomenological orientation, which is to say, to correct the one sided impression logic as the theory of science has regarding its own vocation. Logic must be returned to its dual theme, for it has both an objective component (comprising a theory of the structure of judgment and a theory of the formal structure of logically possible worlds) and a subjective component (comprising a theory of truth – a 'truth logic', which reinstates logic as an intentionality toward objects in the world.) The two sides of this logical theme are designated by Husserl as formal apophantics and formal ontology, the former comprising a theory of the structure of judgment and the latter (taking mathematics as its model) comprising an ontological theory, a theory of the necessary logical structure of all possible worlds. At the very outset of his project, Husserl characterizes mathematics as formal ontology, as the theory of the structure of possible worlds.

The FTL is organized around a series of distinctions which conceal a deeper unity, for in the final analysis the theory of judgment and the theory of formal ontology will be united at the highest level of the system, the truth logic, which will disclose a bracketed "truth interest" in the case of apohantic judgment and an operative "truth interest" in the case of ontological forms.[356] The most strident division of Husserl's logic, that of the apophantic and the ontological, must not be taken as definitive; although the two theories of logic can be treated separately, ultimately they will be shown to converge on the same necessity, that of the clarification of the meaning of logic, which must ultimately be a theory of truth. According to Paul Cortois, there are three "basic dimensions" of Husserl's logic. The first dimension is the plane of fundamental logic, which has two sides, that of the apophantic and the ontological. The second dimension is the plane of the functions of logic as the theory of science. At this level, formal science can be understood abstractly as a theory of the valid forms of knowledge (irrespective of their truth) and, conversely, as a theory of the truth function of logic – the elaboration of the phenomenological horizon within which science operates (the system of latent and

[356] For a schematic presentation of the structure of Husserl's FTL, see Cortois 1996 "Husserl", 20-22.

operative truth interests.) Finally, three levels of differentiation can be distinguished within formalities of any kind; this differentiation of the formal is the third dimension of Husserl's FTL. Formal systems must be determined at the level of their grammatical organization, the structure of the rules for valid inference, and finally, at a meta-level, they must be located within a theory of logical systems. At this final level of abstraction, the apophantic logic encounters the logic of formal ontology, for the theory of systems is, in turn, part of ontology. As in geometry, none of these dimensions has an essential priority over the others. It is necessary to distinguish among them in order to arrive at phenomenologically clarified forms of eidetic evidence, but they cannot be conceived independently. Formal and transcendental logic ultimately constitute a unity and it is the job of the phenomenologist to disclose this unity within the proper horizon of phenomenological significance.

Husserl and Cavaillès both acknowledge the nineteenth century as an historical turning point in the history of science with particular significance for mathematics. Husserl agrees with Cavaillès that the invention of abstract algebra and the successive efforts to reconstitute the foundations of analysis in the nineteenth century finally enabled the construction of concepts which necessarily departed from the spontaneous agreement of traditional logic and intuition. Mathematics could no longer be confined to the science of "number and quantity" but opened onto a generalized problematic of forms and objects. For Husserl the nineteenth century represented a dual revolution in logic and mathematics which could no longer be accounted for using the predicate logic of Aristotle and Kant. Aristotle and Kant provide access to a very restricted class of logical forms. Moreover, this restriction of the forms of logic is quite arbitrary from the perspective of a theory of science, for it does nor correspond with any sound epistemological criteria except for that of lived experience. It is necessary then to remake logic while bracketing intuition as the legislator of the valid forms of logic. On the other hand, by an equal and simultaneous necessity, logic loses sight of its subjective orientation in taking itself to be merely a theory of judgment. The theory of judgment must be complemented by an ontology, a description of the world within which judgment as such operates in order to orient itself toward horizons of meaning. The regulation of judgment within the ontology it presupposes is the task of what Husserl calls truth logic.

That part of formal logic which is concerned with the theory of judgment is called apophantic in accord with the Greek *apophansis* (assertion) in order to emphasize the theory of judgment *as judgment*, which is to say, a theory of a restricted class of meanings which pass through some element of material mediation (usually linguistic.) One pole of logic is concerned with judgment; the other pole is concerned with objects. In order to complete the theory of judgment a theory of the object is required. The theory of objects (as a problem of logic) is given by formal ontology or mathematics.

> When one considers the naturally broadest universality of the concepts set and number, and considers also the concepts element and unity which respectively determine their sense, one recognizes that the theory of sets and the theory of numbers relate to the empty universe ‘any object whatever’ or

> 'anything whatever'…the *formal mathematical* disciplines are formal in the sense of having as fundamental concepts certain *derivative formations of 'anything-whatever.'*[357]

All judgments implicitly refer to objects. Logic fails to understand itself when it entertains the fantasy that it has no necessary relation to the object. Formal mathematics is the pure description of the form of objects; as such, it entertains an identity with apophantics insofar as it is a description "of forms pertaining to any object of our epistemic intendings."[358] In order to establish the true structure of science based on logic as the theory of science, both the object pole and the judgment pole of logic must be clarified and brought into relation with one another. Husserl takes mathematics to be "an apriorical theory of objects (*Gegenstandslehre*)" and, therefore, the discipline which describes the logical structure of objects and their relation to each other. Logic can claim to be the theory of science because it encompasses both a theory of judgment and of systems of judgment and a theory of objects and of systems of objects.

Logic as theory of science discloses this division between latent and operative "truth concerns" at every level of its development. It is therefore possible to "practice" logic in two very different ways. Husserl calls logic which is concerned with pure analytics *consequence logic,* because it brackets its relation to truth and simply determines the rules of consistency and contradiction. Logic which is concerned with "the premliminary conditions of possible truth" is called formal

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[357] Edmund Husserl, *Formal and Transcendental Logic,* Trans. Dorian Cairns (The Hague: Martinus Nijhoff, 1978), 77. Cited in Cortois "Husserl" 1996, 22.
[358] Cortois "Husserl" 1996, 22.

logic of truth because it self-consciously assigns itself to the elucidation of the object. At the heart of this distinction is a differentiation of the types of evidence mobilized by logic. As developed in the *Logical Investigations* (Vol 1. 1900, Vol 2. 1901) and the *Cartesian Meditations* (1931) evidence can be characterized by *distinctness* (*Deutlichkeitsevidenz*), proper to consequence logic and apophantics, and by *clarity* (*Klarheitevidenz*), proper to the formal logic of truth. Logic demands a hierarchy of forms because there is a hierarchy of evidence. Husserl claims to be the first to discern within eidetic evidence a genuine differentiation of levels of priority corresponding to different possible forms of truth.[359] The differentiation of the forms of evidence is for Husserl equivalent to the intentional activity of a subject oriented toward different kinds of epistemic goals. Evidence of distinctness indicates orientation toward a truncated theory of truth, for all that can be grasped from this activity are the rules of consistency. Evidence of clarity indicates orientation toward a more capacious theory of truth, which in addition to distinctness is capable of integrating a concern for clarity. Evidence of distinctness is restricted to verification of pure symbol manipulation – it is a matter of determining the content of valid inferences within a given symbolic chain. Evidence of clarity is preparatory to the realization of knowledge that is true, that is, beyond distinctness it is concerned with the elucidation of objects or object systems which may not be

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[359] See Cortois "Husserl" 1996, 23, and Suzanne Bachelard, *A Study of Husserl's Formal and Transcendental Logic*, Trans. Lester E. Embree (Evanston: Northwestern University Press, 1968), 101-105.

purely symbolic. It engages a richer domain of objects and requires a correspondingly richer system of evidences.

The first dimension of Husserl's logic addressed the distinction between the theory of judgment and the theory of objects or formal ontology, or between the syntax of symbol manipulation and the semantics of objects. The second dimension addressed two orientations toward logic – logic as exclusively concerned with consistency (consequence logic) and logic as preliminary to the construction of true knowledge (formal truth logic.) The third dimension of Husserl's logic concerns the differentiation of the levels which can be discerned within the formation of scientific theories. Here logic as the theory of science assigns itself the task of determining all possible forms of scientific knowledge by distinguishing an essential level of logical grammar, a subsequent level of relations of derivability proper to judgments and to mathematical concepts and, finally, a meta-theory or theory of theories (sometimes also called by Husserl a system of systems) which grasps the morphological character of the derivability relations – a theory of the forms of systems and how they are organized hierarchically. This final level would seem initially to correspond with thematization in Cavaillès, but as we will see the analogy does not hold. Cortois refers to these layers as the *grammatical,* the *inferential* and the *systems* layers and presents them schematically as follows:

F. Apophantics.	**F. Ontology**
(3) theory of deductive systems	(3') theory of manifolds
(2) derivability apophantics	(2') formal math. theories

(1) logical grammar (1') ontological grammar[360]

For Cortois, the third independent dimension of Husserl's logic presents a clear picture of the content and ambition of Hussserl's phenomenological reformation of logic. As the diagram attempts to make explicit, the hierarchy of apophantic and ontological forms discloses the extraordinary range of logic as a phenomenological theme. The primary level of logical and ontological grammar already implies the existence and operation of what Husserl calls "apriorical rules" which would be immanent in the forms of natural language even if the problems of logic have yet to be explicitly stated. At this level of logic, Husserl is concerned with well formed judgments, that is to say, judgments endowed with a possible sense. The phrase *"All one cats then"* for example does not express a "cognitively meaningful unit;" it is in Husserl's language simply *nonsense* (*Unsinn*). However, the phrase "*All men are mortal, and this man is not mortal"* although contradictory, does express a meaningful judgment. There is a content and a graspable horizon of meaning even if the meanings seem to oppose one another. Ontological grammar, like logical grammar, is concerned with well formulated systems of judgment. Cortois suggests that, at this level of logic, the threshold of formalization is already at work, that is to say, the expressive powers of natural language are already tending to the clarification and axiomatic presentation of the latent system of apriorical rules which govern the possible forms of sense[361]. Ontological grammar, even at this fundamental level, is explicitly capable of bracketing intuitive contents

[360] See Cortois "Husserl" 1996, 31.
[361] See Cortois "Husserl" 1996, 34.

of sense in order to highlight the latent principles of organization which subtend the forms of meaning. This is why Husserl states that the first step of formalization is the definition of the grammatical rules which organize the system.

The second level of abstraction extends the theory of meaningful or well-formed judgments into the definition of *the rules of transformation* and derivability, according to which new systems of well-formed judgment can be constructed on the basis of prior systems. It is at this level that Husserl's logic most closely resembles logic in the traditional sense, for here is where the explicit theorization of consistency, non-contradiction, and predication would occur on behalf of apophantics. Likewise, on the ontological plane, this is the level of *mathesis formalis*, at which the procedures of mathematical deduction become thematized in their own right and the domain of natural language as a bearer of latent logical forms is definitively surpassed. Here the organization and distribution of mathematical objects and operations display a tendency to congregate around unexpected centers of complexity - an anticipation of the theory of systems which Husserl will formulate at the third level of logical abstraction. Cortois calls this tendency of mathematical theories to form "relatively isolated clusters" pre-cursors of the possibility of fully axiomatized formal mathematical theories[362]. This is the level at which Husserl locates set theory and number theory and many of the conceptual revolutions of abstract algebra, which were also formative for Cavaillès.

[362] See Cortois "Husserl" 1996, 37.

The third level of logical abstraction is posed by the very possibility of formalization and axiomatic. It constitutes a meta-theoretical reflection on the possibilities of formal deductive systems – it is a theory of theories or a system of systems. Husserl writes,

> Any science whatsoever is a multiplicity of truths – not haphazardly thrown together, but combined and relating in any case to a unitary province. *When* does the whole that comprises the infinite multiplicity of propositions making up a science have a *systematic unity-form that can be constructed a priori,* on the basis of a finite number of pure axiom-forms, *by means of logical-categorical concepts? When* is the group of axiom-forms that defines a theory-form *definite* in the province-form, correlatively, a "*mathematical" or "definite" multiplicity?* If this condition is fulfilled, the unity-form of the whole if the system-form of a "deductive", a "theoretically explanatory", science.[363]

At the apophantic level, the theory of deductive systems is a theory of theories. Procedurally, apophantics at the level of second order abstraction resembles thematization, for it takes as its object a set of operations which have already abstracted a more primordial object. It is simply a matter of constructing a rational sequence which is always capable of taking one set of results as objects for another series of investigations. The significance of this power of indefinite abstraction is obvious to Husserl, for it allows the different branches of already existing science to be located within a geography of science (Husserl speaks of local and global "province-forms") in such a way that transcendental categories of belonging emerge capable of holistically integrating the differentially developed levels of unevenly mature sciences. Phenomenology thus reveals the transcendental unity of science by clarifying the constitutive acts of its epistemic orientation.

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[363] Husserl 1978, 102-103.

At the formal ontological level, the theory of deductive systems is what Husserl calls a theory of manifolds or multiplicities. Husserl is here in immediate contact with the nineteenth century use of the term in the work of Bernard Riemann, for a manifold "designates a purely formally defined domain of 'objects' ruled by abstract operations obeying certain general laws[364]." The theory of abstract objects is a purified field of structural possibilities of objects. Even after the contents of intuition have been bracketed and reduced to their constitutive prejudice, relegated to their appropriate levels and coordinated with their guiding forms of evidence, the problem of the *structure* of the domain of objects in general remains. The global elaboration of the object plane is the task of the theory of manifolds. Here set theory itself would take its place as a member of a broader class or phylum of possible organizations of mathematical theories. Husserl states:

> *Mathesis universalis* (from now on the name is always equivalent to logical analytics) is, *for apriori reasons, a realm of universal construction;* aside from the operative elements, it is entirely a realm of operational formations, which, despite their infinity, can be governed a priori. In it occur, as the highest level, the *deductive* system-forms and *no others.* Precisely this is the answer to the question of when a science or a scientifically closed group of propositions has, according to purely analytic (mathematical) principles, a unitary, mathematically constructible, system structure.[365]

Mathesis universalis, as opposed to *mathesis formalis,* is totalizing, that is to say, it articulates a complete deductive phylum. It is the plane upon which the *styles of deduction* are inscribed, or, as Husserl prefers to say, it is the plane upon which the categories of objectivity come into focus as pure regions of possible epistemic

[364] Cortois "Husserl" 1996, 41.
[365] Husserl 1978, 103.

intention. Every epistemic intention, regardless of its degree of self-consciousness, tends toward a category of objectification which is, in turn, a system of deduction as a category of abstract objectivity.[366] Here again the nineteenth century seems to be at the center of Husserl's reflections, for the theory of manifolds borrows explicitly from Riemann (as the name indicates) and from Hankel and Grassmann, who also figured prominently in Cavaillès' theory of rational sequences. Indeed, Husserl's theory of manifolds and Cavaillès' conceptual dialectic both attempt to take seriously the notion that objects and operations, on the plane of mathematical deduction, are capable of an open ended progress of abstraction which is both creative and constrained. For Husserl, however, the convergence of apophantics and ontology is determined in the final analysis by a phenomenology of lived experience within which all formal ontology becomes the grid of a logic of truth – the objective categories of progressively realized epistemic horizons which are discovered through the agency of an intentional development.

To summarize, Husserl's FTL, as a theory of science, presents the epistemologist with a clearly defined set of tasks. According to Suzanne Bachelard, Husserl's project passes through three successive stages. The first stage corresponds with the basic division of apophantics and ontology. For Bachelard this distinction must be understood as the necessity of observing that it is always possible to distinguish "the categories of signification and the correlative categories

[366] See Husserl 1978, 106.

of the object."[367] The second stage elucidates the apriorical rules which are latent within experience and which determine the enabling conditions of objective knowledge. Here again there are two sets of regulative principles "depending on whether they [the regulative principles] are grounded in the categories of signification or in the categories of the object."[368] Syllogistics defines the rules of signification, whereas the apriori theory of the object (*mathesis formalis)* determines the possible forms of the object. Finally, as we have seen, the third task requires the construction of a theory of theories. Here the theory of the possible forms of theories is also a description of the terrain of objectivity at the level of meta-logic, for the general description of the province of the object discloses a holism of objective domains correlative with epistemic intention. Formal apophantics and formal ontology find themselves united in a logic of truth which locates all judgments in relation to a determinate horizon of the realization of the possible forms of objectivity. This then is the logical architecture Husserl proposes as a theory of science and which Cavaillès encounters in a spirit of lively contestation.

For Cavaillès, Husserl's phenomenology "represents the deepening synthesis" of the philosophy of consciousness, on the one hand, and of logicism, on the other. Its purported advantage over these doctrines stems from its refusal to grant priority to either the "acts" of consciousness or the "objects" of logic and mathematics. As Cavaillès reconstructs the plan of Husserl's FTL, he will isolate, on

[367] Bachelard 1968, 38.
[368] Bachelard 1968, 38.

the one hand, the movement by which Husserl eliminates the diversity and complexity of mathematical experience in order to arrive at a homogenous unity of reason and experience, and on the other, the inability of Husserl's phenomenology to grasp the rational sequence of mathematical intelligibility in its genuine becoming. Additionally, Cavaillès will invoke Husserl's concept of "historicity" in order to demonstrate that the sedimentation of meaning-forms which Husserl takes as constitutive of formal ontology cannot in fact explain either the origin of genuinely mathematical thought or the system of transformations internal to mathematical concepts. Cavaillès' critique of phenomenology as a genre of historical epistemology will ultimately be two-fold. Husserl does not offer the historian or the epistemologist a convincing historical methodology; rather, history is invoked as a form of eidetic necessity by which the chain of logical evidences will be reduced to their ultimate origin. For Cavaillès, Husserl is not interested in the actual history of mathematics but only in the rational reconstruction of eidetic variation. Like all philosophies of consciousness, phenomenology is unable to learn anything about mathematics from the actual history and practice of mathematics. Finally, because Husserl does not attend to the historical development of mathematics, he is unable to reconstruct the authentic conceptual genesis of what Cavaillés calls the genuine mathematics of the infinite. In neglecting the patient conceptual analysis required of the historian of mathematics Husserl cannot help but resurrect the most pernicious ideology of both the philosophies of consciousness and logical empiricism, namely, the refusal to grant to mathematics its own proper content.

For Husserl, as we have seen, all judgments imply a correlative ontology. The thematic separation of formal apophantics and formal ontology should not be taken as definitive; on the contrary, apophantics and ontology enjoy a solidarity within the higher synthesis of a phenomenological logic of truth. The form of judgment as such is motivated from its origin by an orientation toward objectivity. For Cavaillès, the form of judgment must be understood as ultimately expressing a "state of things" (*Sachverhalt*). Judgment then, as such, is destined to disappear as an intelligible sequence in its own right; it will always find itself resolved to an ontological state of affairs and ultimately to the form of an objective reckoning with the world. Cavaillès sees in this unity of judgment and ontology the disappearance of the rational sequence seized as the principle of an autonomous development of the mathematical object. Not only does the basic structural parity of apophantics and ontology forbid the appearance of an autonomous rational sequence; it also imposes a homogeneity on all the levels of judgment proper to the development of any formal system. Cavaillès cites Husserl:

> 'No matter how many intermediate determinations of nominalized substrates there may be at the various stages, what matters ultimately are the lowest and primary substrates, in the sciences the objects of their domain; it is their determination which is intended through all the intermediate levels.' What emerges here is a kind of *principle of reducibility* by which, on the one hand, the scope and the true significance of every judgment are led back to a relation between primary objects and, on the other, homogeneity is established between judgments from various levels.[369]

The principle of reducibility is the same as the phenomenological *epoché*; it is the successive bracketing of the natural attitude or phenomenological prejudice proper

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[369] Cavaillés 1970, 391. The citation is from Husserl 1978, 100-101.

to each level of judgment. The hierarchy of the forms of evidence should be understood as ultimately revealing the disposition of consciousness towards the object. Again, there is no theorization of the rational sequence of mathematics; instead, there is a phenomenological theory of the forms of evidence used in a hierarchical logic of truth.

Cavaillès was perhaps the first of Husserl's Francophone readers to clearly define the intelligible structure of the theory of science in Husserl's FTL. Moreover, he was undoubtedly the first to note the structural demotion of mathematics in Husserl's phenomenology to the a priori outline of a formal ontology. It is this subordination of mathematics to formal ontology which for Cavaillès best characterizes the theory of science in Husserl. This same subordination also causes Husserl to discount the possibility of an intelligible content entirely unique to mathematics. The necessary convergence of formal apophantics and formal ontology allows Husserl to unify the historically extant sciences against the transcendental horizon of the epistemic intentions which found them in their inquiry and unite them in the hierarchy of their eidetic evidences. In short, for Husserl, there can be only one kind of knowledge, the progressive objectification of the world and of experience.[370] Cavaillès writes:

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[370] Cavaillès' critique of Husserl here is exactly the same as his repudiation of Brunschvicg, a fact which has to my knowledge escaped the existing criticism on Cavaillès. Although the implications of this convergence cannot be developed fully here, it is at least possible to suggest that while Brunschvicg represented the highest form of the philosophy of consciousness, for Cavaillès he seems to escape the more forceful criticism leveraged against Husserl because in Brunschvicg there is an entirely different evaluation of the epistemological implications of the history of

> The *mathesis formalis* gives the determination of the possibilities of objects, while the apohantic gives the determination of objects…Moreover, the authority of logic over physics is thereby likewise explained. In fact, it is one and the same movement which, through mathematics, extends itself even to the realities of the world. There is no knowledge which can be halted in its course toward the self enclosed intelligibility of a rational system. To know has only one meaning, and that is to attain the real world.[371]

Logic dictates the necessary structure of judgment in physics, as in all the disciplines, because there is one movement of epistemic intention toward the world extending from mathematics all the way to the empirical sciences. Here Husserl denies any internal differentiation at work between the originary intention of formal ontology and its applications. There may well be an increasing level of complexity and of "sedimentation" between the accumulative layers of formality, but the rational sequence itself has no internal power of self-revision, self-differentiation, or the retroactive consolidation of its own rational contents – the features which define the very essence of the intelligible in Cavaillès.

From the perspective of Cavaillès' epistemology of mathematical experience, Husserl's inability to recognize a fundamental power of self-differentiation internal to the rational sequence necessarily leads to the liquidation of mathematics as an autonomous discipline. As Husserl notes in the FTL, the mathematician need not concern himself with the possible application of mathematics to the applied

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mathematics. In Brunschvicg, the history of science transforms its own significance for the existing state of science according to the dialectic of objectivity and rationalization defined in the previous chapter. In other words, for Brunschvicg and Cavaillès the *present state of science defines the historical significance of science's own history*. The self consciousness of French epistemological historiography in this regard does not seem to have any equivalent in Husserl's theory of science, at least not in Cavaillès' appraisal of Husserl.

[371] Cavaillés 1970, 392.

sciences. This artificial truncation of mathematics as a self-sufficient exercise is, however, an illusion from the perspective of phenomenology. On this point Cavaillés cites Husserl who writes: "It [mathematics] need not bother with the fact that the relation to a possible application, indeterminate and always open, belongs to its proper formal logical-signification[372]." For Husserl mathematics remains phenomenologically vague if it remains only a "subtle mental game," nor can it be conceived apart from its possible applications which, in fact, manifest its ontological significance as the apriori structure of formal ontology. Therefore, Cavaillès can write:

> If applications are excluded, there is no such thing as mathematical knowledge. Mathematics, conscious of its original meaning, of what it truly is, divides itself into two parts: applied mathematics which is physics, and formal mathematics which is logic. It is only because he has forgotten his vocation that the mathematician can claim to be self-sufficient.[373]

In the light of a transcendental logic, there is no mathematics as such, only physics on the one hand (the mathematization of the system of judgment proper to the world of motion) and "formal mathematics" or logic as the formal theory of judgment, on the other. The self-sufficiency of mathematics is an illusion which can only arise in the absence of a phenomenological investigation of the forms of evidence upon which mathematical deductions rely.

The structure of the sciences must ultimately serve the needs of life in Husserl's theory of science, a proposition which Cavaillès would no doubt endorse if only it did not claim to secure thereby the essence of mathematical intelligibility.

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[372] Cavaillés 1970, 392. Cavaillés cites Husserl 1978, 97.
[373] Cavaillés 1970, 393.

Mathematics, for Husserl, is justified by its practical outcomes. It is oriented towards the utility of acts. In physics this occurs primarily through the mathematization of perception, on the one hand, and the formalization of prediction, on the other. Cavaillès notes that these allied movements in Husserl form the basis of all mathematical idealization. Mathematical abstraction arises as a result of the idealization required in order to impose homogeneity on the diversities of sense experience. Husserl locates the phenomenological condition of the possibility of the idea of the infinite in a similar idealization of experience.

> What does it mean to idealize, if not to smooth over the extrinsic, that which for the scientific mind is tied to the facticity (i.e., arising from the fact) of its present actualization and can make sense only in relation to it. Hence the invention of straight lines and perfect planes, the positioning of an infinite and homogenous space. The infinite is elimination of the arbitrary, the arbitrary in the choice of an example, in the reputed obstacle to research and to construction. Measurement, a consequence of the postulate of homogeneity, is the domination of lived space through coordination and iteration of its actual apprehensions.[374]

For Husserl, the "infinite is the elimination of the arbitrary;" it is the purity of a negative space arrived at through elimination, as in Kant's procedure of arriving at the formal by eliminating content. In both cases, abstraction becomes the negative idealization of experience; it is modeled, albeit negatively, on experience. For Cavaillès, by contrast, mathematical abstraction cannot be conceived in this way, for it entertains no necessary relation to lived experience. Husserl's homogenization of the forms of eidetic evidence forces all conceptual entities into the same epistemic horizon of intention. This homogenization or principle of reducibility of all

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[374] Cavaillès 1970, 402-403.

abstraction to the same "elimination of the arbitrary" in experience can in no way capture the conceptual genesis of genuinely mathematical abstraction, which, as we have seen, entails a constantly renegotiated articulation of form and content.

Transcendental logic, on Cavaillès' reading, entails a transcendental subject for whom the progressive forms of the objectivity of the world and lived experience exhaust the possibilities of rational experience. There is only one reason, and it is the reason of the transcendental subject in its orientation toward the object. This amounts to a kind of dogmatic ontology which always discovers the same origin beneath all of its rational activities:

> …throughout this ontology, there remains the one object which is always intended, whose being is irrevocably posed, namely, the individual in the real world. And so we find again the thesis dear to empiricist logicism that mathematics does not have a proper content of knowledge. It is an organization or combination of what is already there, which alone matters, and we can undo and retie their interlacings without augmenting or diminishing the reality known.[375]

Husserl's truth logic results in an equivocation between abstract knowledge and actual knowledge, or between pure mathesis and its applications. Husserl, along with the positivists and the logicists, can only conceive pure mathematical abstraction in the form of a tautology because they fail to arrive at a concept of abstraction proper to mathematics. What disappears in this equivocation is what Cavaillès calls "the progressive rational development" of the rational sequence. Mathematical abstraction as such must be defined as the progression of the rational sequence, which refers to its own material and formal constraints without needing

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[375] Cavaillès 1970, 403.

to make reference either to a prior intuition or to a horizon of epistemic intention. Husserl's FTL results in a "short circuit from the idea of an abstract ontology to actual knowledge." In so doing it passes over the singularity of mathematical abstraction and the historical fact of its progressive nature:

> The mathematical sequence possesses an internal coherence which cannot be treated brusquely. Its progressive character is essential and the decisions which neglect this lose themselves in the void.[376]

Cavaillès argues that any epistemology of mathematics which fails to understand the "internal coherence" of the mathematical sequence loses itself in the void of its own imagining. It is impossible to dictate to mathematics what its own nature must be; rather, an immanent surveillance is required if there is to be any meaningful exploration of the self-differentiating rational sequence.

For Cavaillés, Husserl's theory of *nomological science* best expresses the pervasive ambition of phenomenology to present mathematics as a field of knowledge in which it is possible (and ultimately necessary) to exhaustively define "the system of objects which constitute its domain."[377] The theory of rational sequences had attempted to demonstrate, on the contrary, that it is impossible for even the most elementary mathematical object to be defined exhaustively relative to any single conceptual plane of operations. For Cavaillés, the elaboration of the paradigm and the vertical super-position of thematization present the rational sequence of mathematics as a constantly evolving set of practical effects and explanatory procedures which require a constant deepening and reconciliation of

[376] Cavaillés 1970, 403-404.
[377] Cavaillès 1970, 404.

the formal and material aspects of mathematical experience. By way of contrast, Husserl's regulative ideal of a nomological science is premised on the *total transparency* of the rational sequence to itself, or what amounts to the same, the simultaneous presence of all the objectively necessary formal properties of an axiomatic system existing on the same conceptual plane. As Husserl writes in the FTL:

> …the idea of a *"nomological science",* or correlatively the idea of an *infinite province* (in mathematico-logical parlance, a multiplicity) governable by an explanatory nomology, includes the idea that there is no truth about such a province that is not deducibly included in the "fundamental laws" of the corresponding nomological science – just as, in the *ideal Euclid*, there is no truth about space that is not deducibly included in the "complete" system of space-axioms.[378]

Husserl's nomological science contains all the "fundamental laws" of the objective province it corresponds to. In contrast to Cavaillès' dual injunction that mathematical becoming expresses at once genuine novelty and implacable necessity, Husserl's nomological science of mathematics can only include the apodictic necessity of tautological identity. There is no genuine novelty of mathematical knowledge in Husserl. For Cavaillès, this is a tautological conception of mathematical formalization because it compresses the "entire content of mathematics" into a single moment, thereby once again dispensing with the need to generate an epistemology which would attempt to take mathematics seriously in the moments of its *becoming* and according to the irreconcilable leaps in complexity which are the evidence of its evolution.

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[378] Husserl 1978, 96.

Cavaillès poses two fundamental objections to Husserl's nomological theory of mathematics. The first is historical and involves the results of Gödel's incompletion theorems. The second is epistemological and returns to Husserl's notion of the infinite as merely the abstract negation of the actual. For Cavaillès, mathematical abstraction begins with the infinite. The infinite is not given to the imagination even in negative form. The infinite cannot be imagined; it can only be thought, and this thought is the commencement of genuine mathematical abstraction. These objections are in fact internally related to one another, as this section will show, for Gödel's work demonstrates that the axiomatic formalization of a mathematical theory does not preclude novelties arising within the system, which can in turn only be shown to be consistent by the elaboration of a more powerful theory and so on *ad infinitum.* For Cavaillès, only finite theories can truly be nomological and the emergence of the infinite as a genuine theme of mathematical development definitively precludes the nomological presentation of mathematics as formal ontology.

The significance of Gödel's work for modern mathematics can hardly be overestimated. Cavaillès offers a stark summation of the results of the incompletion theorem:

> …every theory containing the arithmetic of whole numbers, i.e., practically every mathematical theory, is necessarily non-saturated. A proposition can be asserted which is neither the consequence of the axioms nor in contradiction with them: *tertiam datur.*[379]

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[379] Cavaillès 1970, 405.

For Cavaillès, the incompletion theorem is the manifestation of a curious necessity proper to mathematical abstraction: the iteration of that which is novel without being contingent. The unity of novelty and necessity is characteristic of mathematical progress and precisely what a nomological epistemology is incapable of understanding insofar as it recognizes only deduction from first principles. In his reflections on the philosophical implications of foundational projects in modern mathematics, Gödel writes:

> …it turns out that in the systematic establishment of the axioms of mathematics, new axioms, which do not follow by formal logic from those previously established, again and again become evident…it is just this becoming evident of more and more new axioms on the basis of the meaning of the primitive notions that a machine cannot imitate.[380]

The reference to the machine implies the possibility of automating the process of mathematical formalization which would procedurally be the reduction of mathematical formalization to the iterations of an algorithm. In this regard Husserl's nomological theory is also unwittingly a theory of the equivalence of the structure of mathematical theories with their computability. This is of course true; the error consists in the reduction of mathematical intelligibility to the axiomatic presentation of the mathematical theory. Formal logic cannot exhaust the domain of mathematical intelligibility. For Cavaillès and Gödel, the perpetual emergence of new axiom systems forbids the closure of the conceptual field of mathematics on the basis of its formalization. As Cavaillès writes: "…the previous investigations have

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[380] Kurt Gödel, "The Modern Development of the Foundations of Mathematics in the Light of Philosophy", in *Kurt Gödel, Collected Works Volume III, Unpublished Essays and Lectures* , ed. Solomon Feferman. (New York, Oxford University Press, 1995), 385.

indicated the difference between the closure of the field of objects and the closure (or saturation) of its conceptual system."[381]

As the dialectic of the paradigm and the thematic attempted to demonstrate, the elaboration of an object-domain is inseparable from the simultaneous suppression and generalization of the object: to elaborate the field is to transform it rather than to exhaust it. There is no end to the elaborations of mathematics because its conceptual system cannot be saturated. It must be noted that the nature of the relationship between Cavaillès and Husserl, relative to the work of Kurt Gödel, cannot be definitively resolved in Cavaillès' favor based on this citation. In fact, in the essay cited above, Gödel claims that Husserl's phenomenology is the philosophical methodology best suited to the "clarification of meaning" which contemporary mathematical programs of foundation require. Moreover, according to Gödel, Husserl's phenomenology extends and usefully revises "the Kantian conception of mathematics." In place of Kant's "geometrical intuitions," Gödel asserts that Husserl inserts a properly "mathematical" and "set theoretical" order of derivations "from a finite number of axioms." Gödel seems to endorse a philosophical lineage which Cavaillès is keen to repudiate partially on the authority of Gödel's own work. As this section has shown, Cavaillès' substantive disagreement with Husserl intersects with Gödel's work relative to the problem of the *impossibility of exhausting the conceptual field of mathematical abstraction.* Husserl's nomological theory of mathematics as formal ontology would seem to be refuted by Gödel's

[381] Cavaillès 1970, 405.

demonstration of the necessary incompletion of all axiomatic presentations of mathematical theories from the perspective of the problem of the consistency of the theory. The fact that Husserl mistakenly asserts that a mathematical theory can be exhaustively determined in advance, based on a priori assumptions, can in principle be explained historically since the *Formal and Transcendental Logic* was published in 1929 and the first results of the incompletion theorems were published in 1931. Nevertheless, even bracketing the historical circumstances, the epistemological objection Cavaillès raises must still be addressed: if Husserl claims that mathematics is the apriori structure of formal ontology but cannot account for the emergence of mathematical novelty, why should we accept the formal equivalence of mathematics and ontology? On this issue, Cavaillès seems to be a more incisive reader of Gödel relative to Husserl than Gödel himself.[382]

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[382] In order to do justice to the complexity of the problem outlined here an entirely separate study would be necessary. Some promising work has already been done which moves in this direction. The earliest effort to diagram more precisely the relationship between Cavaillès, Husserl and Gödel is probably in Suzanne Bachelard, *A Study of Husserl's Formal and Transcendental Logic*, trans. Lester E. Embree (Evanston: Northwestern University Press, 1968.) On Gödel, 52, 62, 129. On Cavaillès, 12, 52, 62, 206, 220. On the same vexed triangulation but filtered through Derrida's encounter with Cavaillès in his masters thesis on Husserl's genetic phenomenology see Michael Roubach, "Derrida and Cavaillès: Mathematics and the Limits of Phenomenolgy," *International Journal of Philosophical Studies,* Vol. 18(2) (2010.) For an incisive comparison of the objectivity of mathematical objects in Husserl and Gödel see Dagfinn Follesdal, "Gödel and Husserl" in *Naturalizing Phenomenology: Issues in Contemporary Phenomenology and Natural Science,* eds. Jean Petitot, Francisco J. Varela, Bernard Pachoud and Jean-Michel Roy. (Stanford: Stanford University Press, 1999), 385. Finally, for a direct presentation of Husserl and Gödel from the perspective of a scholar intimately familiar with Cavaillès see Pierre Cassou-Nogués, "The Two-Sidedness and the Rationalistic Ideal of Formal Logic: Husserl and Gödel," in Luciano Boi, Pierre Kerzberg and Frédéric Patras, eds.,

From Cavaillès' perspective, armed with the results of Gödel's work, nomological theories would be the exceptions rather than the rule in mathematics: "[o]nly the theories smaller than arithmetic, that is, the theories which may be called quasi-finite, can be nomological."[383] Mathematical abstraction cannot be correctly modeled nomologically, nor can it be reduced to a set of automated procedures (although Husserl notes in passing that formal apophantics is in principle capable of being automated). Instead, if the emergence of novel axioms is characteristic of mathematical abstraction as an essentially progressive movement, then the epistemology of mathematics must be able to distinguish between the established domains of mathematical contents and the emergent domains of mathematical becoming. This would be the underlying necessity of Cavaillès' philosophy of the concept. The results of Gödel's incompletion theorems seem to force the issue:

> …genuine mathematics begins with the infinite. The incorporation of a theory into a larger theory is clearly subject to the sole condition of non-contradiction, but by virtue of the same result of Gödel, the noncontradiction of a theory can be demonstrated only within a more powerful theory. The demonstration retains its interest, which is to concentrate on a single procedure or a canonic system of procedures, the doubts being distributed over an indeterminate polymorphy of intertwined procedures. But there is no longer that apodictic assurance from the start. One must commit oneself to the canonic procedure, to the indefinite iteration of its employment. Thus the deductive sequence is essentially the creator of the contents that it attains. The possibility of assembling some privileged assertions at the outset is a source of illusion if we forget the operational rules which alone give them a meaning.[384]

Rediscovering Phenomenology: Phenomenological Essays on Mathematical Beings, Physical Reality, Perception and Consciousness (Dordrecht: Springer, 2007), 309.
[383] Cavaillès 1970, 405.
[384] Cavaillès 1970, 405-406.

Cavaillès can assert that mathematics begins with the infinite because he identifies mathematics with the progressive transformations of its becoming rather than its consolidation under a nomological theory. The limits of formalization themselves indicate the impossibility of "assembling some privileged assertions at the outset", for this would amount to decoupling what is essential in the dialectical description of progressive mathematical abstraction – the objects of mathematics are fully objective insofar as they are able to extend their effects through precisely defined operations, just as the operations are objective insofar as they are correlated with objective domains whose meaning they determine.

Husserl remains uncharacteristically naïve in his transcendental logic because he assumes that a static genesis of the mathematical object in the consciousness that intends it can capture everything that it is possible to say about the object. This is to deny the strange autonomy and creativity of the rational sequence which must be understood as "the creator of the contents that it attains." The meaning of a mathematical theory cannot be excavated by phenomenological analysis; it adheres in the immanent configuration of the relation between the object and the operation. Moreover, rather than a retroactive orientation towards the origin of the mathematical object in an act of consciousness, the philosophy of the concept is oriented toward the emergence of future novelties. *It identifies the intelligible with the creation of new formal contents and the articulation of a new relation between operation and object.* This is the nature of the intelligibility of the rational sequence in Cavaillès' philosophy of the concept and precisely what Husserl effaces with his nomological theory. Nowhere is this fundamental limitation of Husserl's theory of science more acutely on display than in its characterization of the infinite, as Cavaillés asserts:

> The body of a theory is a certain operative homogeneity – which the axiomatic presentation describes – but when the theory involves the infinite, the iteration and the complications furnish results and an intelligible system of contents which are impossible to dominate, and an internal necessity obliges it to surpass itself through an expansion otherwise unpredictable and which appears as an expansion only after the fact. There is no juxtaposition anymore than there is an initial fixation. It is the entire body of mathematics which develops itself through the steps and in a variety of forms, and it is likewise this which in its entirety, technical devices included, accomplishes or does not accomplish the very function or knowledge.[385]

Axiomatic formalization perfectly captures the mathematical duality of object and operation as *already accomplished acts*; in fact, formalization only becomes possible in the wake of "a certain operative homogeneity" which is itself the result of a stable configuration of formal content. However, theories involving the infinite (or any theory encompassing arithmetic) entail effects which cannot be dominated by axiomatic presentation. Cavaillès' analysis of the history of mathematical concepts of the infinite (especially in his secondary thesis *Remarques sur la formation de la théorie abstraite des ensembles* [1938]) obliges him to clarify how the nature of the rational sequence in turn requires a global redescription of mathematics itself. This is why Cavaillès asserts that "genuine" mathematics begins with the infinite – the outline of mathematical abstraction which emerges with the problem of the infinite refuses to resign itself to a simple axiomatic stability. As Cavaillès writes, if we are attentive to the movement of the rational sequence, we observe at work within

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[385] Cavaillès 1970, 406.

mathematical abstraction "an internal necessity" which is at once unpredictable from the present configuration of mathematical knowledge while also appearing entirely necessary after its accomplishment. This then leads Cavaillès to formulate the necessary consequence of the inherently progressive nature of mathematics for the history of mathematics: the conceptual outline of mathematics at any moment must be grasped as a totality of effects with a stable signification *while at the same time remaining poised on the threshold of a transformation which will "accomplish the very function of knowledge"- the revision of its own significance based on the new articulation of formal content.* Contra Husserl *there is no fixed history of mathematics; the very meaning of this history is indexed to the present of mathematics, which, by being progressive in its nature, requires that the significance of the history of mathematics itself be continuously revised in the light of the contemporary articulation of formal content.*

The concept of the infinite finds itself astride multiple necessities in Cavaillès. It indicates the internal limits of formalization, especially as evidenced by Gödel's work, thus allowing Cavaillès to disqualify Husserl's nomological characterization on the grounds that Husserl overestimates the significance of formalization within mathematics. More seriously, the nomological theory renders what is most crucial about mathematical abstraction invisible to the phenomenologist: the necessary emergence of mathematical novelty. The concept of the infinite is, therefore, the beginning of mathematics for Cavaillès, whereas for Husserl it is a regulative idealization based on the negation of irregularities within experience. Finally, the theory of the rational sequence transforms the epistemological significance of the history of mathematics which must be reconceived each time the intelligible system of the formal contents of mathematics accomplishes "the function of knowledge" by revising its own conceptual organization. For Cavaillès, the dual orientation of Husserl's transcendental logic (with formal apophantics occupying the pole of subjectivity and formal ontology occupying the pole of objectivity) cannot describe or make sense of the actual progress of mathematics because it denies any genuine content to mathematics as such:

> Neither objective logic – the analysis or combination of the formations already produced by spontaneous logic and presented for communication in a systematic aspect which can be extrinsic…nor subjective logic – foundation of the first inasmuch as it relates its products to the activity of absolute consciousness – can account for either the actual progress or the structures which mark it off.[386]

By referring the objectivity of mathematical knowledge to the acts of a transcendental consciousness, Husserl bypasses the real problem of an epistemology of mathematical experience: reconciling the immanent necessity of its established formal contents with the unpredictable necessity of its emergent forms. Husserl, like all philosophers of consciousness, must make logic transcendental because he is incapable of learning anything from the objects of mathematics themselves.

The epistemological status of history within phenomenology is directly related to the transcendental characterization of logic. If logic is transcendental then history has an invariant meaning because everything that it is possible to know

[386] Cavaillès 1970, 407.

about the contents of mathematical science must have already been predetermined by the acts of some antecedent consciousness. When logic becomes transcendental mathematical novelty becomes illegible.

> History reveals authentic meaning to the extent that it permits us to rediscover lost links, first to identify automatisms and sedimentations as such, then to revitalize them by thrusting them back into their conscious actuality…The return to the origin is a return to the original. We have seen that to make comprehensible in the phenomenological sense is not to change a plan or reduce a content into something other than itself but to dissociate the entanglements, to pursue the referential indicators in order to bring out into the open the polished system of acts which "no longer refer to anything." In this sense, says Fink, phenomenology should be considered an archaeology.[387]

The meaning of history is disclosed by the "untangling" of sedimented intentions which must ultimately refer to the objective clarification of consciousness in its relationship to the world. There is a destiny implicit in the relation of consciousness to its objects which requires that the past constantly maintain a stable signification if it is to function as the repository of a hidden "polished system of acts" no longer capable of phenomenological reduction. The stability of the historical sequence is its inertia: its authority is a result of its deductive stasis.

This situation is entirely intolerable for Cavaillès who proposes an antinomy proper to the philosophy of consciousness relative to the history of mathematics as one of perpetual revision and progressive transformation.

> …if there is a consciousness of progress, there is not a progress of the consciousness. Now one of the essential problems of the doctrine of science is that progress itself may not be augmentation by volume by juxtaposition, in which the prior subsists with the new, but a continual revision of contents by deepening and eradication. What comes after is more than what existed

[387] Cavaillès 1970, 408.

> before, not because it contains it or even because it prolongs it but because it departs from it and carries in its content the mark of its superiority, unique every time, with more consciousness in it – and not the same consciousness.[388]

The antinomy is that of consciousness presenting itself with the progression of its acts or consciousness itself progressing and, thereby, revising the very significance of its acts. If consciousness is invariable, then so are the rational sequences of its acts; if the rational sequence is variable, perhaps consciousness will find itself transformed. The philosophy of consciousness is exemplified by Husserl's phenomenology because it expresses this antinomy directly: the phenomenologist can perform an archaeological analysis of his own eidetic evidences and "reverse engineer" the epistemic intention at the origin of all his mathematical activities. He will not thereby have arrived at any transformation of his concepts and he will have abandoned the dynamism of the rational sequence by referring the reality of mathematics to an intentional consciousness. Cavaillès is ultimately supremely pessimistic of the form of consciousness which Husserl seems to endorse, not because it attempts to return to "things themselves", but because it abandons the possibility of its own progress by cultivating a paralyzing and epistemologically inert fetishism of its own originary acts.

The rational sequence is a dialectical philosophy of the mathematical concept. It is a progressive interlacing of objects and operations which constitute an emergent field of objective relations between abstract mathematical entities. It is constantly renegotiating its own historical signification because transformation is

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[388] Cavaillès 1970, 409.

what drives it: by its nature it is an indefinite progression. There is no stable historical referent in the domain of mathematical abstraction. It is therefore illegitimate to institute a theory of science which requires a stable and unitary historical sequence, on the one hand, and a transcendental logic, on the other. There is no transcendence which can dictate or enforce the progress of mathematical novelty. There is only the immanence of the sequence itself which produces and revises its own forms and contents. Cavaillès ends his essay with an affirmation of the "generating necessity" of a philosophy of the concept:

> There is no consciousness which generates its products or is simply immanent to them. In each instance it dwells in the immediacy of the idea, lost in it and losing itself with it, binding itself with other consciousnesses (which one would be tempted to call other moments of consciousness) only through the internal bonds of the ideas to which these belong. The progress is material or between singular essences, and its driving force is the need to surpass each of them. It is not a philosophy of consciousness but a philosophy of the concept which can provide a theory of science. The generating necessity is not the necessity of an activity, but the necessity of a dialectic.[389]

The final lines of Cavaillès' LTS defy easy interpretation but suggest the program by which a philosophy of consciousness must be subordinated to a philosophy of the concept. The repudiation of a consciousness immanently generating its products is a blanket refusal of a theory of science conceived along the lines of Husserl's phenomenology. In the first place there is no consciousness which is capable of being perfectly present to itself in all of its acts, least of all the abstract movements of mathematical experience which manifest themselves over the course of a staggered progression. The philosophy of consciousness is, therefore, disqualified as

[389] Cavaillès 1970, 409.

a viable theory of science in that it assumes it has access to the totality of mathematical abstraction from the perspective of its own illuminated interiority.

The progression of the rational sequence describes the dynamics of conceptual transformation under conditions of enlivening tension: progress is driven by the relation between "singular essences," those mathematical singularities of real but not yet actual effects which must be elaborated in order to render explicit the boundaries of the domain of the object. There is an internal necessity operating between singular essences which requires that they be surpassed: here the vertical dimension of thematization takes up the object domain as a member of a more abstract class and, thereby, defines the morphology of an operative field uniting previously diverse objects which then become relative to an abstract universal. Cavaillés speaks of a material progress between singular essences which would define the generating necessity of mathematical abstraction. Invoking Granger's concept of a science of formal contents, this chapter has argued that such a material progress can be understood as the necessity of constantly reconfiguring the relationship between the material and formal levels of mathematical experience. For Cavaillès, this dialectical transformation is described by the theory of rational sequences and its oscillating modes of concept production: the paradigmatic elaboration of the object domain and the thematic super-position of a field of operations. Cavaillés affirms that no philosophy of consciousness can justify simultaneously the objective necessity of mathematical abstraction and the constant transformation and revolutionary deformation of its own formal limits. The price the philosophy of consciousness pays for its own transcendence is to remain blind to that which is essential in mathematics: the novelty by which the history of mathematics renews itself.

Chapter Four - Gaston Bachelard's Normative Epistemological Program and the History of Science

Gaston Bachelard (1884-1962) assumed the chair of Professor of the History and Philosophy of the Sciences at the Sorbonne in 1940. The prestige of this academic post comes as no surprise given that Bachelard had already published a handful of important works in the epistemology and history of science between 1927 and 1940. The director of Bachelard's primary thesis at the Sorbonne, *Essai sur la connaisance approché* (1927) was Abel Rey (1873-1940), an important philosopher and historian of science, as well as the founder of the Institut d'histoire des sciences et des techniques. Bachelard's secondary thesis, *Etude sur l'évolution d'un problème de physique: la propogation thermique dans les solides* (1927) was directed by Léon Brunschvicg.

The period between 1927 and 1940 is a time of extraordinary productivity for Bachelard. In this span of thirteen years, Bachelard constructs a highly original philosophy of science and a distinctive methodology in the historiography of science.[390] 1940 is also a somewhat paradoxical year for Bachelard insofar as the

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[390] In 1929 Bachelard published an important study on Einstein's theory of relativity, *La valeur inductive de la relativité.* This is followed by a book on the phenomenology of time (partially a refutation of Bergson's philosophy of duration), *L'intuition de l'instant* in 1931. A study of the rational principles underlying the organization of the chemical elements, *La pluralisme cohérente de la chimie moderne* follows in 1933. An epistemological history of the concept of the atom, *Les intuitions atomistiques (essai de classification)* appears in 1933, to be followed a year later by a book on the epistemological novelty of modern science, *La nouvel esprit scientifique* in 1934. Bachelard publishes another book explicitly critiquing the philosophy of Bergson, *La dialectique de la durée* (1936) and an epistemological analysis of the Non-Euclidean structure of space underpinning contemporary physics, *L'expérience de l'espace dans la physique contemporain* in 1937. In 1938 Bachelard publishes his

newly appointed professor of the history and philosophy of science stops publishing works of epistemology at this time and turns his attention to the literary imagination. He does not resume his study of the epistemology of modern science until 1949, when he publishes *La rationalisme appliqué*, perhaps his single most systematic work. Two more important works of epistemology appear in the fifties, *L'activité rationaliste de la physique contemporaine* in 1951, and *Le matérialisme rational* in 1953.

This chapter will primarily address works drawn from the middle period of Bachelard's epistemological research. The chapter has four objectives. 1) To examine the nature of the relationship between epistemology and the history of science in Bachelard's epistemological program. 2) To locate Bachelard relative to his predecessors in the epistemological tradition; especially to the philosophies of mathematics developed by Brunschvicg and Cavaillés, and to Meyerson's interpretation of relativity theory. 3) To determine the nature of scientific objectivity in Bachelard's epistemological analysis of mathematical-physics in the twentieth century. 4) To reconstruct Bachelard's fundamental critique of the inadequacy of traditional philosophies of science.

The chapter is divided into five sections. In the first section I introduce Bachelard's militant defense of the autonomy of epistemological critique against competing programs in the philosophy of science. In the second section I analyze the

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most well known work, *La formation de l'esprit scientifique* and a book of literary criticism on images of fire in poetry and literature, *La psychanalyse de feu.* Finally, in 1940, Bachelard publishes a programmatic work arguing for a new metaphysics of science, *La philosophie du non*.

philosophical significance of Bachelard's work based on the novel conception of the history of science it makes possible. In section three I examine Bachelard's characterization of "epistemological modernity:" the new configuration of rational and experimental procedures which emerges in the wake of the physics of relativity. In the fourth section I turn to Bachelard's concept of the "epistemological obstacle" and his critique of the spontaneous philosophy of lived experience: realism. Finally, in section five, I demonstrate the normative epistemological commitments underpinning Bachelard's fundamental critique of the traditional philosophies of science.

Defending the Autonomy of Epistemological Critique

What is the relationship between the history of science and epistemology in the work of Gaston Bachelard? The determination of this relationship, so I claim, is the very core of Bachelard's thought. Since, in effect, the correct distillation of the profoundly normative claims which Bachelard systematically repeats regarding this relation can yield the entire positive content of Bachelard's system of thought, it is worth quoting him at some length. In *L'Activité rationaliste de la physique contemporaine* (1951) Bachelard writes:

> In an address on the occasion of Max Planck's sixtieth birthday in 1918, Einstein described the real epistemological situation as follows, Hermann Weyl tell us: "The historical development has shown that among the imaginable theoretical constructions there is invariably one that proves to be unquestionably superior to the others." This *superiority* is not merely a question of fact. It is not simply characteristic of a historical moment. This superiority of the explanatory values of a hypothesis shows itself clearly in rational convictions. *It is precisely the sign of a rational value, a cultural value,*

> *an epistemological value* [my italics.] It is science's destiny that rational values impose themselves. They impose themselves historically. *The history of science is led by a kind of autonomous necessity* [my italics]. Philosophy of science should systematically take as its task the determination and classification of the hierarchy of epistemological values. General discussions on the *value of science* are quite futile if one does not see that all scientific thought makes one sensitive to a psychic value of the highest rank.[391]

In the ready-made terms of a classical philosophical opposition, Bachelard here seems to claim that scientific *facts* (the superiority of a given 'theoretical construction') are not all equal, that some display a definitive *superiority* by virtue of their participation in a domain of *rational values.* Before proceeding to the systematic presentation of the ideas underpinning Bachelard's philosophy of rational values, it is instructive to read the text of Hermann Weyl (1885-1955) from which Bachelard, with all due subtlety, has carefully selected the historical materials upon which he bases his claims. In the methodological excursus of his *Philosophy of Mathematics and Natural Science* (1949) Weyl, as we have seen, invokes the authority of his friend and colleague Albert Einstein in order to develop a theory of the relationship between experimentation and theoretical explanation in the physics of relativity. The full citation of Einstein is as follows:

> "The historical development has shown that among the imaginable theoretical constructions there is invariably one that proves to be unquestionably superior to all others. Nobody who really goes into the matter will deny that the world of perceptions determines the theoretical

[391] Gaston Bachelard, *L'Activité rationaliste de la physique contemporaine* (Paris: Presses Universitaires de France, 1951), 47. English translation in Joseph J. Kocklemans and Theodore J. Kisiel, *Phenomenology and the Natural Science* (Evanston: Northwestern University Press, 1970), 345.

> system in a virtually unambiguous manner, although no logical way leads to the principles of the theory."[392]

Bachelard has clearly omitted Einstein's own epistemological position, which flatly affirms the determination of "the theoretical system" by the "world of perceptions," a more or less modifiable empiricism that flies in the face of Bachelard's unapologetic *rationalism of superior scientific values.* No doubt the practicing historian of science would be fully within her rights to question Bachelard's willful appropriation of Einstein in this context. Doesn't Bachelard's editorial ventriloquism transform Einstein into the mouthpiece of an epistemological position he clearly does not endorse when we examine his statement in its full context? Unfortunately for our historian, and for Einstein himself, it does not matter what Einstein's actual epistemological convictions may have been. Bachelard is not concerned with the opinions of philosophers, historians or, indeed, even of scientists when it comes to the matter of clarifying the epistemological structure and function of the system of concepts which define mathematical-physics in the twentieth century. The first principle which must be borne in mind when examining the work of Gaston Bachelard is that his epistemology is *resolutely normative.* Bachelardean epistemology is not a synthesis of historical and philosophical methods; it is a determinate relation between an epistemological program and a very precise concept of the history of science which this epistemology produces and makes legible.

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[392] Hermann Weyl, *Philosophy of Mathematics and Natural Science* (Princeton: Princeton University Press, 1949), 153.

At the same time, however, the primacy of epistemology is what enables Bachelard to formulate a *rigorous history of science* or, to use Bachelard's expression, a *rectified* history of science which is able to discern in the historical development of the natural sciences a *continuous hierarchy of rational epistemological values,* on the one hand, and a *discontinuous series of epistemological obstacles* which have been definitively overcome, on the other. It is also necessary to clarify the nature, scope, and principle of legitimation by which epistemology, as conceived by Bachelard, holds at bay the combined authority of the philosophical tradition, the historical guild, and the practicing research communities of scientists themselves.

Bachelard's entire oeuvre is polemical: he writes *against* the philosophers who would attempt to determine the nature and necessity of scientific development without ever entering into the complexity of its actual thought or practice. For Bachelard, it is not a question of illuminating scientific activity through an act of philosophical reflection, rather, it is a matter of constructing, laboriously, the philosophy *of* (subjective genitive) scientific practice which philosophy, guided by its own imperatives, is too timid to dream of. Bachelard is no less critical of the history of science. Historians of science, lacking epistemological norms to guide their reconstruction of the narratives of scientific progress, all to easily succumb to the illusion that science progresses continuously from one plateau of technical and theoretical expertise to the next. The historian of science (in Bachelard's polemical vision) is prone to look for antecedents and anticipations where there are in fact only ruptures. The historian seeks to explain the present state of science by a laudatory narration of the great men and the great discoveries that make its present state seem both necessary and inevitable. The epistemologically rectified history of science is a narration of genuine disruption and irreducible novelty: anytime a rational value is at stake it will never be explained with reference to the past; every coining of a new rationalist value is an emancipation and reframing of the significance of the history of science itself. It is the present which embraces and consolidates the past, not the past which contains in itself the germinal outline of the future. Finally, the scientists themselves are not reliable witnesses to their own rational conduct, succumbing to what Bachelard calls the “diurnal philosophy” of the scientist: technician by day, amateur philosopher by night. Here again it is the inadequate philosophy of the academy which has too easily seduced the scientist, and which obscures the genuinely novel philosophical procedures of techno-science.

For Bachelard epistemology is not answerable to the philosophers, the historians, or to what the *scientists think they are doing*; the novelty and radicality of Bachelard’s project is that it explicitly seeks to install itself in the *immanent rationality of science.* Epistemology, so conceived, is a theoretical orientation which affirms without reservation the absolute autonomy of science and the singularity of the historical process which thus comes to light *once this epistemological autonomy is adequately conceptualized.* The normative force of epistemology, as Bachelard understands it, is derived from the affirmation that science is able to produce, *and to transform,* its own rational norms and procedures of objectification entirely immanently. Bachelard thus demonstrates, in full solidarity with the epistemological tradition that precedes him, that *a truly normative epistemology is inseparable from the affirmation of the rationality immanent within science.* For Bachelard, science (including mathematics) is the only truly autonomous domain of rationality. On this point Bachelard is in perfect solidarity with his colleague, Jean Cavaillès. For Bachelard, as for Cavaillès, science revolutionizes itself and transforms the epistemological criteria of its own objectivity. The nature of scientific objectivity for both thinkers is to transform itself historically by an evolution of concepts which are only retro-actively unified.

For Bachelard, as for his predecessors in the epistemological tradition, there is no contradiction in claiming that science produces its rational norms internally, and that these rational norms are subject to historical transformation. Bachelard affirms that the normativity of science is instantiated historically and is subject to revision. The lucidity and brevity of this claim belies the formidable efforts which were required in order to formulate it as a coherent research program. Dominique Lecourt has diagrammed with precision the evolution of concepts by which Bachelard was able to arrive at this set of inter-locking claims. Indeed, Lecourt is responsible for introducing the designation "historical epistemology" as a methodological and substantive description of Bachelard's work and of epistemology in twentieth century France more generally. In the following section I will determine the extent to which Lecourt's characterization of Bachelard's project correctly identifies the concepts central to Bachelard's epistemology and the novel approach to the history of science these concepts make possible. At the same time, however, while acknowledging my debt to Lecourt's admirable formulation of Bachelard's project, I will also offer an alternative characterization of what I take to be central to Bachelard's epistemological program: the nature of objectivity in science and the dialectic by which this objectivity transforms itself historically.

The Philosophical Significance of Bachelard's Project

Bachelard's thought proceeds polemically, as noted above, it would, however, be a dangerous simplification of his work to claim that his polemicism is therefore exclusively destructive of other intellectual projects. Bachelard destroys in order to construct: the history of philosophy must be dismantled in order to make room for a philosophy capable of learning from the progress of the natural sciences rather than pretending to dominate them. Lecourt cites Bachelard's address to the International Congress of the Philosophy of Science in 1949 in order to highlight the conflict of *values* motivating Bachelard's repudiation of mainstream philosophy.

> If one were to draw up a general table of contemporary philosophy, one could not but be struck by the tiny space occupied in it by the *philosophy of the sciences*. In a more general way, *philosophies of knowledge* seem in disfavour in our day. The effort to know appears to be sullied with utilitarianism; however unanimously accepted, scientific concepts are held to be merely utility values. The man of science, whose thought is so opinionated and ardent, whose thought is so living, is presented as an abstract man. By degrees all the values of the studious man, of the skillful man fall into discredit. Science is no more than a minor adventure, an adventure in the chimerical countries of theory, in the obscure labyrinths of artificial experiments. By an incredible paradox, to hear the critics of scientific

> activity, the study of nature leads scientists away from natural values, the rational organization of ideas is prejudicial to the acquisition of new ideas.[393]

The "studious man," Bachelard's man of science, is here accused of excessive abstraction. His science is at best an expression of utilitarian values. Science is the incarnation of a dangerous temptation by which the aspiring intellect is led "away from natural values." This is the crux of the matter for Bachelard. This naturalization of value by which the immediate becomes the most desirable contains within itself the tacit repudiation of the whole of scientific culture, as Bachelard understands it. The *life* and *experience* of value is here dangerously one sided. Bachelard's polemical orientation must be understood not as a direct refutation of the values which proceed from the interrogation of lived experience, but as the recognition that science pursues its own system of values, irreducible to those of a strictly utilitarian expansion of technological prowess. Bachelard's systematic repudiation of Bergson is founded on precisely this defense of science as *irreducible to a merely problem solving activity.* Bachelard does not seek to destroy the values of the humanist – his task is more intricate; he must demonstrate that the system of values which modern science "imposes," to use Bachelard's word, is entirely incompatible with the values of common sense and lived experience.

As Lecourt notes, adopting this language of epistemological values means that Bachelard is constantly demoting one set of philosophical commitments while promoting another; the repudiation of what might be called the philosophies of

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[393] Gaston Bachelard, *L'engagement rationaliste* (Paris: Presses Universitaires de France, 1972), 35. Cited in Dominique Lecourt 1975, 11.

lived experience is inseparable from the construction of a new table of values which Bachelard will ultimately call his "applied rationalism" and "rational materialism." For Lecourt, the positive content of Bachelard's new epistemological values can be expressed as a profound "philosophical thesis:"

> Bachelard is stating…that the truth of a scientific truth *'imposes itself' by itself.* In Spinozist terms: 'veritas norma *sui*' (the truth is its own measure). In Leninist terminology: Bachelard is posing the thesis of the objectivity of scientific knowledges. He is posing it, not discussing it. He does not seek to found, to *guarantee* this objectivity. He is not concerned to pose to scientific knowledge the traditional questions of its claims to validity. This point is crucial, for we maintain that this position is a *materialist* position. A position which enables Bachelard to take a step outside the theoretical space of what idealist philosophy called the 'problem of knowledge.'[394]

Lecourt introduces the Spinozist theme of the immanence of scientific truth and its verification. I am entirely in accord with Lecourt's Spinozist characterization of the nature of scientific truth in Bachelard.[395] I cannot endorse the attribution of a *materialist* orientation which immediately follows.[396] For Lecourt, Bachelard's

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[394] Lecourt 1975, 12.

[395] Some beautiful pages on Bachelard's potentially unwitting Spinozism have been written by Didier Gil in *Autour de Bachelard, esprit et matiére, un siècle français de philosophie des sciences (1867-1962)* (Paris: Éditions Les Belles Lettres, 2010), 139-170.

[396] Lecourt also develops a Marxist Leninist materialist reading of Bachelard in *Bachelard, ou le jour et la nuit* (Paris: Éditions Grasset et Fasquelle, 1974), 165-171. Michel Vadée offers a symmetrical but opposing view in his *Bachelard, ou le nouvel idéalisme épistémologique* (Paris: Éditions Sociales, 1975). For Lecourt, Bachelard definitively emancipates himself from the closed circuit of idealist epistemology which, according to Lecourt, interrogates the notional structure of scientific concepts only to discover, without exception, the epistemological features of its own consciousness. This critique of transcendental idealism was already developed in the nineteen forties by Jean Cavaillès, as I demonstrated in Chapter Three. Lecourt claims Bachelard frees himself from the history of philosophy only by assuming a materialist position relative to the epistemological foundations of science. For Vadée, it is just the opposite scenario: the philosophical pre-suppositions allowing

materialism escapes the idealist problematic of "the theory of knowledge" – an inherently closed field of epistemological investigations which purport to unearth the foundational conditions of possibility of scientific knowledge. Lecourt rightly disengages Bachelard from this current in the philosophy of science. His error is then to assert that because Bachelard is unconcerned with foundational accounts of the validity of scientific knowledge (for example, those that would proceed from *a priori* principles as in the Neo-Kantian theories of *wissenschaftslehre*), he therefore simply *poses* the *fact* of scientific objectivity without giving an account of its emergence or the procedures of its self justification.

For Lecourt, insofar as Bachelard is a materialist, it is because the objectivity of science does not require a philosophical mandate derived from the history of idealist philosophy. So be it. This is unfortunately a dangerous compression of the formidable conceptual apparatus Bachelard develops in order to describe, with often breath taking precision, the *technical and theoretical procedures by which the history of mathematical-physics attests to the progressive elaboration of an*

Bachelard to formulate his concepts remain unanalyzed and operate from behind the scenes as it were, dictating an entirely idealist theory of knowledge whose authority is derived solely from the increasing abstraction it is able to generate. This fundamental dead-lock between two of Bachelard's most noted critics has since become a veritable object-lesson in the history of the secondary literature. Jean-François Minko M'Obame offers a masterful surveillance and diagnosis of the *differend* which seems to separate Lecourt and Vadée in *La conception bachelardienne de la connaissance scientifique: le rationalisme de Gaston Bachelard* (Paris: Les editions connaissances et savoirs, 2011.) M'Obame claims that Bachelard must be taken at his word, and cannot be inserted into the field of ready made options available to the history of the philosophy of science without entirely distorting the nature of his work. I am inclined to agree, and I distinguish my own position from the criticisms of Lecourt and Vadée relative to the problem of the dialectic governing the production of objectivity in the history of science.

increasingly rigorous objectification of the phenomena of physics. Contrary to Lecourt's characterization of an objectivity given by fiat, I will pursue a dialectical presentation of the conceptual labors by which Bachelard describes a tendency characteristic of modern science: the progressive "concretion of the abstract" by way of the *phenomeno-technics* Bachelard invokes in the place of a phenomenology of scientific experience in order to underscore the fact that in modern science, even the technical apparatus is the expression of a rational procedure. The concretion of the abstract is always accompanied by a simultaneous "abstraction of the concrete," by which the apparent simplicity of the phenomena of experience are made to yield a mathematical complexity which locates them in the territory of a formal mathematical system in no way answerable to the requirements of a positivist epistemology of scientific knowledge. The system of mutual rectification thus established between the concrete and the abstract becomes subject to perpetual revision and oscillation; it is this space of the *concrete-abstract* (the technical apparatus of scientific activity which in Bachelard are materialized concepts), and the *abstract-concrete* (the mathematical elaboration of the network of relations which obtain, algebraically, between the measurable variables of a given phenomena), which defines the veritable *axis of objectification* by which science progressively *produces* its own field of objectivity. I am not persuaded that Lecourt's concept of Bachelard's materialism adequately describes this process.

Although I distance myself from Lecourt's interpretation of the epistemological status of Bachelard's supposed materialism, I am in complete agreement with his general characterization of Bachelard's epistemology as essentially historical. Lecourt writes:

> …Gaston Bachelard's discovery is precisely to have recognized and then to have theoretically reflected the fact that science has no object outside its own activity; that it is in itself, in its practice, productive of its own norms and of the criterion of its existence…by adopting as its object scientific knowledge in its movement, epistemology is dealing with a *historical process.*[397]

The philosophy of science has failed to understand "that science has no object outside its own activity." It has failed to grasp the immanence of rational norms in scientific activity. Bachelard's key epistemological claim is that science produces its own rational norms and even "the [criteria] of its existence." Science is therefore immanently productive of its own epistemological norms. The philosopher might be willing to concede this first strictly epistemological point, but for Bachelard epistemology is not an abstract or merely theoretical exercise: science also produces the very devices through which it realizes the objective phenomena it studies. Bachelard's vision of science is not that of a mirror held up to nature: it is that of an apparatus which produces the phenomenon. As this section will demonstrate, for Bachelard, there is no room for nature in the movement of scientific objectivity. Nature is unregulated and capricious. Science begins by holding lived experience at a distance. The philosophical novelty of Bachelard is thus to remove science from any relation to experience which would make it a mere discourse about reality. Bachelard is neither an idealist nor a realist: he is an epistemologist – he studies the

[397] Lecourt 1975, 26.

history of the forms of scientific objectivity or, what amounts to the same, the history of the reality which science produces.

Following Lecourt, but bracketing his emphasis on Bachelard's materialism in order to accentuate the creative dialectic of the problem of scientific objectivity, it is possible to isolate three provisionally distinct but ultimately overlapping theses which define Bachelard's epistemological program. 1) Science has no object but itself: it produces its own rational norms and the objective phenomena it studies. Science is immanently creative both of its *concepts* and its *objects.* 2) The nature of science is procedural: nothing is given by fiat. Everything is produced. Everything is equally subject to revision and reconstruction. Science is constantly transforming its objects and ceaselessly deepening the rational syntheses by which it formulates its own objectivity. 3) The concepts and objects of science transform themselves according to a historical process which is inseparable from the epistemological description of science.

Bachelard's epistemology is thus strongly normative in a dual sense. It is *critical* of any philosophy or theory of science (sociological, anthropological, historical etc.) that would seek to undermine the ability of science to strictly regulate the production of its own concepts. In this regard Bachelard is especially critical of philosophical *idealism* and *realism*, the former attributing the objectivity of the phenomenon to the internal mechanics of the mind's cognitive apparatus, the latter reducing the rational and productive power of scientific reason to an always deficient *translation* of the phenomenon, which exists as an unknown quality on the hither-side of the real. Bachelard's great *idealist* antagonists are Descartes and Kant, as the subsequent sections will show. Bachelard's principle *realist* antagonists are Emile Meyerson and Henri Bergson. Bachelard's epistemological project is also profoundly *constructive*; its normative force proceeds from the affirmation that science truly *thinks* the objective phenomenon insofar as it also *produces* the phenomenon. In his final epistemological work, Bachelard will characterize the epistemology of mathematical physics in terms of a genuine "ontogenesis" which cannot be reduced to a singular "flat" ontology assigning the same conditions of possibility to all the objective phenomena of scientific activity. Modern science effectively multiplies the real at the same time that it becomes capable of thinking it.

I characterize Bachelard as a thinker of the immanent rationality of techno-science owing to the dual normative force of his epistemology and his thorough integration of the cognitive, technical, and social aspects of scientific research and practice. To claim, as Bachelard does, that science has no object other than its own progressive realization is to make progress and transformation the very essence of scientific activity. The immanence of the epistemological norm thus requires the perpetual revision and progressive deepening of the rational syntheses which produce the successive forms of scientific objectivity. The historicity of science which emerges from this perspective bears within itself the movement of rational necessity. Bachelard's applied rationalism and technical materialism is thus also a theory of the history of science as the history of the forms of objectivity. The epistemological configuration allows the historical perspective to emerge and returns to this history incessantly in order to keep a dual record of the obstacles that have been overcome and the technical phenomena and conceptual apparatus that remain vital and utterly contemporary for the scientific mind.

In the subsequent sections I will trace the evolution of Bachelard's epistemological thought relative to the three normative commitments which govern his work; the rationality immanent to scientific practice, the necessarily progressive nature of scientific activity, and the novel conception of the history of science which becomes legible through the normative framework of epistemological critique. I will pay particular attention to Bachelard's discussion of the procedures by which modern science fabricates the novel forms of objectivity which populate relativistic physics, and to the relationship between mathematical knowledge and the concepts of the natural sciences.

Epistemological Modernity: Reconstructing the New Scientific Mind

In *Le nouvel esprit scientifique* (1934) Bachelard locates the novelty of the new scientific mind in the relationship it establishes between mathematics and the experimental realization of physical phenomena. Bachelard characterizes the tendency of the modified rationalism which guides the new physics in terms of an "epistemological vector" which "points from the rational to the real." The crisis confronting contemporary philosophy (in 1934), Bachelard claims, is its inability to account for the rational procedures which instruct the physicist in the organization of a new form of objectivity. Instead of following the new pattern of thought which physics itself practices, the philosopher makes use of ready made concepts in a futile effort to locate contemporary scientific thought on the spectrum of an outdated metaphysics. Bachelard intends to interrupt the programmatic repetition of philosophy's failure to actually *think alongside physics.* In order to do so, Bachelard claims, it is necessary to reverse the relationship between science and philosophy. In Bachelard's view, it is science which should instruct philosophy, and philosophy which should learn to adapt its metaphysics to the findings of the most recent physics. Bachelard writes:

> Sooner or later scientific thought will become the central subject of philosophical controversy; science will show philosophers how to replace intuitive, immediate systems of metaphysics with systems whose principles are debatable and subject to experimental validation. What does it mean to say that science can "rectify" metaphysics? As an example…consider how "realism" changes, losing its naïve immediacy, in its encounter with scientific skepticism. Similarly, "rationalism" need not be a closed system; *a priori* assumptions are subject to change (witness the weakening of Euclid's postulates in non-Euclidean geometry, for example). It should therefore be of some interest to take a fresh approach to the philosophy of science, to examine the subject without preconceptions and free from the straitjacket imposed by the traditional vocabulary of philosophy.[398]

The philosopher's vocabulary is a "straitjacket" because traditional philosophy does not understand that in order to comprehend the new physics of relativity it must radically enliven all of its concepts, essentially setting the entire spectrum of metaphysical oppositions into motion. Realism and rationalism are not opposed for the scientific mind. The philosophical mind, however, has yet to understand this. The realism of the philosopher is an experiential immediacy which science can

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[398] Gaston Bachelard, *The New Scientific Spirit,* Trans. Arthur Goldhammer (Boston: Beacon Press, 1984), 2-3.

never integrate into the abstraction of its concepts. In complimentary fashion, the philosopher's rationalism forbids the rectification of the principles of reason according to the historical lessons of experience. Philosophy seems to neutralize the conceptual creativity of physics by capturing it between two epistemological dead-ends: a rational procedure which cannot be enlivened by experience (traditional rationalism, especially Descartes and Kant), and a realist immediacy which cannot be integrated into thought (traditional realism, especially Comte, Bergson and Meyerson).

Bachelard intends to cut this Gordian knot by invoking thought at its most dynamic: mathematics in the upheaval of its conceptual transformations. The equation of the highest form of thought with the transformation of mathematical norms will remain paradigmatic for Bachelard. This defines the point of epistemological solidarity uniting Bachelard with Brunschvicg's progressive philosophy of mathematical intelligence and with Cavaillès' dialectical philosophy of the concept. Non-Euclidean geometry is perhaps Bachelard's favorite case-study of an irreversible epistemological transformation in the history of science. It elegantly expresses the capacity of even the most strident of rational procedures (the postulates of geometry grounded in spatial intuition) to uproot themselves; to transform the universal into the particular by constructing a new field of encompassing abstraction:

> Non-Euclidean geometry was not invented in order to contradict Euclidean Geometry. It is more in the nature of an adjunct, which makes possible an extension of the idea of geometry to its logical conclusion, subsuming Euclidean and non –Euclidean alike in an over-arching "pan-geometry." First

> constructed in the margins of Euclidean geometry, non-Euclidean geometry sheds a revealing light on the limitations of its predecessor. The same may be said of all the new varieties of scientific thought, which have time and again pointed up gaps in earlier forms of knowledge. We shall discover that the new doctrines share many of the same characteristic features, the same methods of extension, inference, induction, generalization, complimentarity, synthesis and integration – all equivalents for the idea of novelty. And the novelty in question is profound: a novelty not of discovery but of method.[399]

Bachelard first dismantles the notion that the evolution of concepts in mathematics proceeds by contradiction. The "non" in Non-Euclidean geometry, as in Bachelard's own "philosophy of no," suppresses the particular in order to affirm a superior multiplicity: a pan-geometry. Following this basic principle, Bachelard's multiple re-conceptualizations of rationalism can be understood as an effort to fulfill the requirements of a "pan-rationalism." This is above all a reflection on the methodology of modern science. For Bachelard, that which is genuinely novel in the epistemological transformations of mathematics and natural science is the intelligible pattern "of extension, inference, induction, generalization, complimentarity, synthesis and integration" which one discovers by studying the history of science. The new scientific mind is defined by "a novelty not of discovery but of method." It is this methodology of novelty which makes modern science incomparably superior to philosophy as a source of conceptual innovation. Accordingly, the implications of non-Euclidean geometry for philosophy are profound: "Science in effect creates philosophy. Philosophy must therefore modify

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[399] Bachelard 1984, 8.

its language if it is to reflect the subtlety and movement of contemporary thought."[400]

The lesson Bachelard wishes to draw from non-Euclidean geometry is that if even the image of universal reason given by the intuitive clarity of Euclid's postulates can be totally revised, then there is no rational procedure which cannot be amended. If mathematics is the image of thought, it is not because it fulfills the ideal of a changeless identity, but because it demonstrates the upheaval and transformation characteristic of thought at its most rigorous and inventive. Mathematics issues a normative command to the epistemological description of science: you must work to transcend the bases of your rationality and to construct a new horizon of intelligibility. The rationalist pole of scientific objectivity, as exemplified by mathematics, is itself an open multiplicity in continuous transformation.

Mathematics is thus the indispensable source of two epistemological norms: it is fully autonomous and, therefore, functions as a model upon which to establish the necessary autonomy of the natural sciences. It is also capable of self revision and, therefore, signals the procedural nature of scientific objectivity as a rational project capable of indefinite self correction or rectification, to use Bachelard's preferred expression. Additionally, against contemporaneous projects in the logicist foundations of mathematics which would seek to reduce mathematics to a branch of logic (i.e., the work of Russell, Carnap, Tarski), or worse, to reduce it to a form of

[400] Bachelard 1984, 3.

language, Bachelard will insist that mathematics is a *thought*, indeed the model of thought itself. In *L'Activité rationaliste de la physique contemporaine* he writes:

> One must break with this commonplace so dear to skeptical philosophers who consider mathematics merely as a *language.* Rather, mathematics is a *thought*, a thought which is certain of its language. The physicist thinks experiments with this mathematical thought.[401]

Mathematics is the thought of modern physics; *it is that which thinks in science*, and it composes its reflection in a language which is sure of itself. If mathematics is not a language, then it is equally impossible to conceive the mathematical notation of the laws of physics as *translations of external phenomena.* In keeping with the model of scientific objectivity which Bachelard will develop over the course of his works in the thirties, mathematics is not merely the description of reality; it is the construction of an experience which is rationally structured and only intelligible by virtue of being constructed. Mathematics is not a translation of the phenomenon into the language of reason. It is reason constructing the intelligible form of the objective phenomenon. Bachelard, Brunschvicg, and Cavaillés all share this programmatic rejection of mathematics as reducible to logic or as an exaggeratedly formal language. They uniformly condemn any attempt to reduce the rational content or structure of mathematical reason to anything other than its own conceptual resources, which must remain strictly *sui generis.* A separate project would be required in order to map the multiple conceptual affiliations of these convergent philosophies of mathematics. This convergence is, however, the very

[401] Bachelard 1970, 326.

core of the normative force authorizing the autonomy of science for the epistemological tradition.

Bachelard, however, recognizes that the problem of determining the nature of objectivity in the natural sciences is not the same as determining the nature of objectivity in mathematics. While explicitly acknowledging the centrality of mathematics as the very model of an immanently self-rectifying rational norm, Bachelard nevertheless does not develop a thorough epistemological description of mathematical objectivity. This is a striking lacuna in a body of work which insists that the epistemological novelty of modern science stems from its effective saturation by mathematics. The reasons for this lacuna are no doubt partially historical. In the preface to *La formation de l'esprit scientifique*, which appeared in 1938, Bachelard announces his plans to write a companion volume which would have addressed the objectivity of mathematics in its own right. Although this volume never appeared, and Bachelard soon developed other interests ( ]his vast mediation on the elements of the "material imagination" in poetry and literature) he does maintain, in that work and elsewhere, that science, unlike mathematics, proceeds by overcoming its own errors. The history of science is inseparable from the history of error. By contrast, the history of mathematics is, strictly speaking, without error; it consists in a radical revision, expansion and simultaneous differentiation and integration of its own contents. The nature of this conceptual revision is incompatible with the dialectic of error and rectification which drives the history of science.

Mathematics and science consequently have very different relationships to their respective histories. Bachelard's entire theory of scientific objectivity is rooted in the necessity of the obstacles which constantly confront the progressive realization of the objective phenomenon. A history of mathematical objectivity would look very different from a history of mathematical-physics, and Bachelard does not provide any indication of how one should pursue such a history.[402] Despite the absence of a sustained reflection on mathematical objectivity, Bachelard does offer a very clear epistemological description of mathematics as a science of pure relations which can be discovered through analysis. In the case of non-Euclidean geometries, the system of rational correspondences underlying the multiplicity of geometric objects is obtained through the algebraic definition of the geometric form. Bachelard writes:

> …the way in which mathematicians prove that "different" geometries are actually equivalent is to show that they "correspond" to the same algebraic objects ("invariant forms"). Once a correspondence of this kind had been established, the fear of eventually finding contradictions in exotic geometries was dispelled: It was no more to be feared that a contradiction would be found in Lobachevski's geometry than in Euclid's, since any geometric contradiction would be reflected in the algebraic form and hence in all the geometries in correspondence. The cornerstone of the whole edifice, then, is the algebraic form. Put simply, algebra contains all relations and nothing but relations. The "equivalence" of different geometries is defined in terms of relations, and it is as relations that geometries have reality, not by reference to any object, experience, or intuition.[403]

The "algebraic form" is the rational kernel of the "pan-geometry" described by Lobachevski – algebra is capable of expressing the pure rational relation which

[402] This is exactly the project of Jean Cavaillès, as I demonstrated in chapter three.
[403] Bachelard 1984, 29.

encompasses the diversity of N dimensional geometries without "reference to any object, experience, or intuition." The exclusion of any reference to experience is crucial: it entirely liquidates realism as a suitable model for mathematical objectivity. In the place of realism or, indeed, of any intuitive foundation of mathematics, Bachelard installs a theory of purely rational transformations.

> Mathematics takes on reality by establishing correspondences. Mathematical objects are known through their transformations. To the mathematical object we may say, Tell me how you are transformed and I will tell you what you are.[404]

The reality of mathematical objects consists of the system of their correspondences and coherent rules of transformation. Bachelard, like his friend and colleague Jean Cavaillès, invokes abstract algebra as the paradigm of a science of pure relations. The rational "object" underlying the apparent diversity of geometrical forms is an algebraic structure, a "pure structure" which gives the rule by which any geometrical form can be constructed. The "realization of the form" is what is real in the domain of mathematics. The objectivity of mathematics is defined by the principle of construction itself, in this case, an algebraic structure which abstractly unifies all possible geometries without contradiction.

Although a complete account of what a Bachelardean treatment of the objectivity of mathematics would look like is beyond the scope of this project, Bachelard's insistence on the "epistemological vector" leading from the rational to the real is common both to his discussion of mathematics and to his presentation of objectivity in the natural sciences. The problem of objectivity for Bachelard is one of

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[404] Bachelard 1984, 29.

realization. This is the feature which unites the mathematical domain and the scientific domain. Scientific objectivity, as discussed previously, is unlike mathematics in that it proceeds through the encounter and rectification of error. Although the rational synthesis accomplished by scientific objectivity is modeled on mathematics, it requires a different set of concepts in order to realize its object. Bachelard writes:

> Any work of science, no matter what its point of departure, cannot become fully convincing until it crosses the boundary between the theoretical and the experimental: *Experimentation must give way to argument, and argument must have recourse to experimentation.* Every application is a form of transcendence…the phenomenology of science divides, according to one set of epistemological polarities, into two realms, that of the picturesque and that of the comprehensible (which is just another way of saying that science may be viewed in either realistic or rationalistic terms)…Yet the orientation of the epistemological "vector" seems clear. It surely points from the rational to the real and not, as all philosophers from Aristotle to Bacon professed, from the real to the general.[405]

Scientific objectivity is not the concretion of a transcendental or experientially derived epistemic conviction, but a reflexivity of mutual verification established between two heterogeneous orders – the theoretical passes into the experimental, just as the experimental gives rise to theory. Experimentation and argument do not stand in relation to one another as evidence would stand to explanation; the experiment is not the material confirmation of an idea. The experiment is itself already the materialization of an idea, just as the argumentative and theoretical structure of physics is already a program which realizes a very precise form of experience. The construction of scientific objectivity does not rely on foundational

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[405] Bachelard 1984, 3-4.

certainty derived from simple elements, be they empirical or rational. One does not start from either a "picturesque" or "comprehensible" world view (although philosophy will always be tempted by this antinomy). As Bachelard states, objectivity begins by a double transcendence of experiment and argument – one does not start with the real, one arrives at it. The epistemological vector of modern science must replace all philosophies of naïve realism and rationalism with a philosophy of *realization*: "To put it another way, the application of scientific thought seems to me to tend essentially toward reality..."[406]

For Bachelard, science only recognizes the reality it is capable of producing. Objectification is verification, realization is comprehension. The problem of scientific methodology is internal to the objectification of the rational phenomena.

> The only way to achieve objectivity is to set forth, in a discursive and detailed manner, a method of objectification… Scientific observation is always polemical; it either confirms or denies a prior thesis, a preexisting model, an observational protocol. It shows as it demonstrates; it establishes a hierarchy of appearances; it transcends the immediate; it reconstructs first its own models and then reality.[407]

Bachelard's polemical strategy now changes its orientation. It is no longer a matter of defending the autonomy of science against inadequate philosophies of science that distort the nature of scientific objectivity. It is objectivity itself that is polemical. Even observation is stripped of the neutrality which is supposed to inform the disinterested observer. Bachelard draws our attention to the truth of science, which is not its passivity or disinterest, but its always active and polemical orientation. To

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[406] Bachelard 1984, 4.
[407] Bachelard 1984, 12-13.

observe is to polemicize. For every observation there is always a theoretical construct at stake and an entire organization of experience which must be transformed into a "hierarchy of appearances," as Bachelard writes.

This hierarchy of appearances is the system of confirmed (verified) observations which must also be understood, in turn, as successive breaks from the immediacy of experience. Every observation engages us in this breaking up and re-organization of experience. Scientific knowledge is polemical in its nature. The progress of objectivity is the cultivation of a polemic against experience in the service of a reality which must be produced. The *will* of the scientist must be this active polemicization against experience. Every passivity must be transformed into the reality of an activity. It is this passage from the immediacy of passive experience to the discrete and regulated affirmation of an epistemological activity that defines the movement from observation to experimentation.

> And once the step is taken from observation to experimentation, the polemical character of knowledge stands out even more sharply. Now phenomena must be selected, filtered, purified, shaped by instruments; indeed, it may well be the instruments that produce the phenomena in the first place. And instruments are nothing but theories materialized. The phenomena they produce bear the stamp of theory throughout.[408]

At the level of cognition the passage from observation to experimentation is no doubt continuous and constantly renegotiated. There is, however, a material difference present in experimentation which establishes a bulwark against sliding back into the disorganized immediacy of appearances. The presence of the scientific instrument makes the technological selection, cultivation, and purification of the

[408] Bachelard 1984, 13.

phenomenon absolutely central to the realization of scientific objectivity. For Bachelard, all scientific objectivity is more accurately described as technological-scientific objectivity. Technology is the realization of the scientific mind and scientific instruments "are nothing but theories materialized."

The abstract and the concrete are not opposed in Bachelard's epistemology of scientific objectivity: they are mutually reinforcing. This is perhaps nowhere more clearly demonstrated than in the notion that scientific instruments are the technical realization of theoretical entities. Theory in fact *requires* technological realization, just as experiment requires theoretical elaboration. The phenomena produced experimentally in the laboratory are *realized theories*, and the apparatus of techno-science is the concretion of thought. Bachelard, in a parodic inversion of Kant, often states that the noumenal and the phenomenal are not epistemologically opposed to one another so much as entangled and subjected to constant dialectical reversal by the procedures of modern science. Experimentation is the vector by which techno-science *realizes the noumenal;* it is the transformation of theory into the technical apparatus of a fully *materialized noumenon*, which becomes an objective phenomenon. Technology is the vector of this realization of the noumenal.

> The dialectical relationship between the scientific phenomenon and the scientific noumenon is not leisurely and remote but rapid and strict; after a few revisions, scientific projects always tend toward effective realization of the noumenon. A truly scientific phenomenology is therefore essentially a phenomeno-technology. Its purpose is to amplify what is revealed beyond appearance. It takes its instruction from construction.[409]

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[409] Bachelard 1984, 13.

The normative force of epistemology again manifests itself in the "rapid and strict" transformation of the theoretical noumenon into the realized phenomenon. Objectivity is quite literally a machine which materializes the noumenal. This is why science cannot be encompassed by phenomenology alone: there are no "primary" phenomena in science. Everything that is objective is technical. A phenomeno-technology or phenomeno-technics is required in order to understand the reality that surpasses, by technical purification, the disordered immediacy of appearances.

Scientific objectivity, when understood as the machinic or technological production of the phenomenon, is the core of what Bachelard will characterize as "epistemological modernity." At the beginning of the twentieth century there is a mutation in the objective order of science which signals a new organization of the objective phenomena and a new epistemological depth of rationality. Between Newton and Einstein there can be no epistemologically identical form of objectivity. Newton is not the precursor of Einstein, he belongs to a *different objective universe*. Just as a pan-geometry can encompass all of Euclidean geometry, the Einsteinean universe encompasses the Newtonian universe. With Einstein we can comprehend Newtonian mechanics down to the subatomic structure of the apple that struck his head, but Newton can know nothing of the curved space-time which explains gravity.

The threshold of epistemological modernity appears to be simultaneous with a technological and conceptual reorganization of epistemological obstacles. The formula of this new epistemological formation is by now familiar, it is the passage from the rational to the real, or the technological realization of the theoretical noumenon. Bachelard does not hesitate to use the language of the miraculous when describing this transformation of the epistemological order.

> Wonderworking reason designs its own miracles. Science conjures up a world, by means not of magic immanent in reality but of rational impulse immanent in mind. The first achievement of the scientific spirit was to create reason in the image of the world; modern science has moved on to the project of constructing a world in the image of reason. Scientific work makes rational entities real, in the full sense of the word.[410]

Reason designs its own miracles: this should be the motto of Bachelard's epistemology. The affirmation of the rational impulse immanent to science is here expressed with programmatic severity. Science is a procedure of world making, even if it proceeds by the most discrete increments. Objectivity is not only a material efficacy; it is a rational consolidation of experience and a principled coherence of relations which is itself suggestive of new technical applications.

Bachelard discerns two stages in the history of the scientific mind. In the first stage, the scientific mind makes reason in the image of the world; it extends the senses by standardizing the measurement of observations and by refining the parallelism of geometry and its algebraic notation.[411] The rational coherence of the algebraic structure underlying geometric measurement remains intuitively related to the observable world. This is a rationalism which remains an image of the world. Epistemological modernity begins when reason no longer imagines itself in mere correspondence to the world of the senses, but when it instead begins to construct

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[410] Bachelard 1984, 13.

[411] This procedure was treated at length in the discussion of Léon Brunschvicg's examination of analytic geometry in Descartes. See pp. 152-170.

"a world in the image of reason." The realization of the rational is a new principle of objectivity. The novelty of techno-scientific objectivity was not anticipated by Newtonian objectivity.

> It is a mistake, in my view, to regard Newtonian physics as a first approximation to Einsteinian physics, because the subtleties of relativity have nothing to do with any sophisticated application of Newtonian principles. It is therefore incorrect to say that the Newtonian world resembles the Einsteinian in rough outlines. It is only after one has adopted the relativistic standpoint that it becomes apparent how, by making various simplifying assumptions, Einstein's formulas can be made to yield numerical results similar to Newton's. One does not proceed from the first to the second by amassing data, perfecting measurements, and making slight adjustments to first principles. What is needed is some totally new ingredient. It is a "transcendental induction" and not an "amplifying induction" that leads the way from classical to relativistic physics.[412]

The history of science, as rectified by epistemological reflection, discloses different procedures of objectivity at the heart of the historical development of scientific rationality. The appearance of a continuity of development, however natural it may seem to the historian of science armed with a mass of technical details describing the gradual consolidation of a new objective world view, can only seem disingenuous to the epistemologist. Epistemological reflection discerns a conceptual reorganization which subtends the accumulation of data and the progressive rectification of measurement. The transition from Newton to Einstein is not a gradual transition along a continuum of uniform rational development. It is a phase transition analogous to the transformation of a state of matter. The Newtonian universe is the universe of rational mechanics and the physics of solids – it is the equivalence of the extended body and the intelligible body. At the very limit of its

[412] Bachelard 1984, 44.

rational powers it integrates its observations through the intelligible forms of the integral and differential calculus. The Einsteinian universe is the universe of non simultaneous reference – it is the four dimensional cosmos of curved spaced time which is only intelligible through the pan-geometries developed in the nineteenth century by Bernard Riemann (1826-1866), Nikolai Lobachevski (1792-1856) and David Hilbert (1862-1943). The physics of relativity is abstract before it is concrete: it fashions its cosmos out of the tensor calculus and its fields of variations.

It must be noted that Bachelard's language concerning the transition from the Newtonian universe to the Einsteinian universe is opposed, term for term, to the language of Emile Meyerson, especially to the vocabulary of Meyerson's 1925 book on the physics of relativity, *The Relativist Deduction.* Meyerson's encounter with relativity was treated in the first chapter. For the moment, all that it is necessary to recall is that Bachelard's "transcendental induction" corresponds with, and intends to refute, Meyerson's "relativistic deduction." Bachelard stages his most direct encounter with Meyerson in *La Valeur inductive de la Relativité* (1929), where he writes, regarding the novelty of relativity:

> Relativity is more than a definitive renewal of the way we think of the physical phenomena, it is a method progressive discovery… Historically speaking, the appearance of the theory of relativity is equally surprising. If there is, in effect, a doctrine which historical antecedents cannot explain, it is that of Relativity.[413]

The induction of relativity signals the operation of a rational and mathematical machinery which cannot have been deduced from the preceding conceptual system

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[413] Gaston Bachelard, *La valeur inductive de la relativité* (Paris: J. Vrin, 1929), 6.

of Newton. The differend which separates Meyerson from Bachelard as epistemological interpreters of the history of science, so I claim, is crystallized in their different encounters with relativity. Meyerson's relativity confirms the fundamental postulate of "the Philosophy of the Intellect:" when the mind wishes to know something it eliminates sensory diversities until it is able to formulate a theory of causality. Relativity, despite its great complexity, is simply the most sophisticated theory of causality the human mind has yet produced. The theory of relativity is not different in *principle* from the Newtonian concept of physical causality because they both strive to reduce the complexity of appearances to a stable causal postulate. For Bachelard this is epistemologically untenable.

Meyerson represents an epistemology of static realism because he insists that the same intellectual principle (the principle of identity and the causal postulate), governs the development of all scientific concepts. For Bachelard, relativity, epistemologically speaking, must be an inversion of realism. It is not, however, an idealism. It manifests a rigorous principle of parity between hypotheses as they are constructed (the constancy of the speed of light), and as they are applied (to use Bachelard's example, in order measure the distances between celestial bodies the speed of light must be a constant, thus its generalization is also the rule of its application.) Here experience is not simply the external reality which informs reason, it is co-constituted through an *experimental rationality* which produces a new form of objective experience. Part of relativity's rigorous objectivity for Bachelard is the "ease and tranquility" with which it resists the objections of experience. The methodological manifestation of this counter-intuitive abstraction in the mathematics of relativity is, following an expression of M. von Laue, the expression of physical principles in the simplest mathematical forms. Bachelard claims that what 'is simple in mathematics is *clearly synthetic* and *evidently fecund."*[414]

The relation to be considered when thinking about the simplicity of mathematical formulations is not that between a purely physical nature and a purely formal mathematics. It is instead a matter of approaching a physics which is already organized mathematically, "geometrized in its very core," as Bachelard writes. Therefore, it is a matter of selecting between two mathematical physics: either one "curbs the mathematical impulse" or one affirms a "pan-mathematicism." Bachelard clearly prefers the latter. One must resist the temptation of realism in order to understand relativity. All predicates have become relations in the relativistic universe: "One starts with relation, all realism is nothing but a mode of the expression of relation… the order must be inverted, one always proves the existence by the relation."[415] In his book on relativity, Bachelard insists that whereas Meyerson favors an epistemology of *explanation* (according to which relativity would be understood as a superior theory insofar as it succeeds in explaining a reality that still exceeds it), what is called for is an epistemology of construction: "*…mais avant d'expliquer, il faut construire."*[416] There is a plane of epistemological

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[414] Bachelard 1929, 186.
[415] Bachelard 1929, 211.
[416] Bachelard 1929, 202.

construction which is anterior to the explanation of theory; *before one can explain, one must construct.* If one interprets science as primarily about *explanation,* as Meyerson does, then the epistemological significance of objectivity is distorted by the bias of realism. This is Meyerson's error. The epistemological question concerning relativity is not whether or not it is grounded in the real. Realism paradoxically assumes the real as a starting point. This is its epistemological fallacy. The real is something one arrives at, not something one departs from. The epistemological question should therefore be: "in which direction and according to what organization of thought can one securely approach the real?[417]"

For Bachelard, realism is a specious belief in essences which exist independently of the procedures of objectification which alone guarantee the intelligibility of the scientific phenomenon. Epistemological modernity begins when one realizes that essences can only be said to be objective when they are treated as functions of relations: "It is the relation which says all, which proves everything, which contains the whole; it is the totality of the phenomenon taken as a mathematical function."[418] The first accomplishment of science was to make reason in the image of the world – the rationalization of the real. The second accomplishment of science, which we are only now beginning to understand, is to make the world in the image of reason - the realization of the rational. Relativity is the event which signals this epistemological transformation. As such, the history of science assumes a new significance for the epistemologist, it is no longer the record

[417] Bachelard 1929, 203.
[418] Bachelard 1929, 210.

of reason assuring itself it has conquered reality. The history of science instead manifests a profound restlessness and a desire to transcend itself. The history of objectivity is not the consolidation of a single unchanging paradigm of reason (as Bachelard finds in Meyerson), it is an essentially *progressive* rectification of the techno-scientific assemblage which allows objectivity to materialize the noumenon – to produce the objective phenomenon.

> If it is granted that scientific thought is, in its essence, a process of objectification, then it follows that its real motive force is rectification and extension. This is where the dynamic history of thought is written. *It is when a concept changes its meaning that it is most meaningful.* For it is then that it becomes, in all truth, an event, a conceptualization.[419]

The dynamic history of thought is the history not only of rectified error (indispensable for any scientific idea), but of the transformation of objectivity itself. Bachelard aligns the scientific concept with the event of its transformation: a conceptualization. The discontinuous history of conceptualization is the dynamic history of thought. This history of conceptualization becomes legible only through epistemological reflection.

The history of science as the history of conceptualization is not only the discovery of the epistemological discontinuity of science, it also allows "the scientific spirit to sit in judgment on its own past."[420] That which was beyond theoretical examination in Newton (the resolute immediacy of space and time), is subjected to experimental construction in Einstein. Space and time are empty categories of thought for Newton and Kant – this is the last peaceful alliance of science and

[419] Bachelard 1984, 54.
[420] Bachelard 1984, 55.

philosophy for Bachelard. Epistemological modernity begins when space and time become mathematical functions, not the conditions of thought, but the results of calculation. The Newtonian universe is the crowning achievement of a one dimensional image of reason. The history of the scientific mind demonstrates how inadequate this initial form of objectivity is to the new "mathematical organization of experimental possibilities" which dominate scientific objectivity after Einstein.

The nature of science is to reform its model of objectivity and the nature of the scientific mind is to rectify itself: "…the scientific spirit is essentially a way of rectifying knowledge, a way of broadening the horizon of what is known...Its structure is its awareness of its historical errors."[421] The epistemological purity of science, its exclusive self reference, does not shelter it from historical variability. On the contrary, it makes the rectification of knowledge necessary. There is an autonomous domain of scientific truth, but it is the history of rectified errors. The stability of the relation between the rational and the sensible in Euclidean geometry must be overthrown by a sur-rationalism, a pan-geometry, and a pan-mathematicism. Descartes' epistemological doctrine of clarity and distinctness tells us nothing about the duality of the particle and the wave; the distinctness of one requiring the obscurity of the other. The new scientific mind is non-Euclidean, non-Cartesian, and non-Kantian. Scientific knowledge proceeds by encountering unstructured experience and subjecting it to incremental rationalization in order to

[421] Bachelard 1984, 171-172.

extract from it the materialization of the noumenal: this is the dialectic of objectification in the wake of relativity.

The problem for Bachelard in the twenties and the thirties is to develop a robust account of epistemological modernity. His work during this period is an intensive investigation of the history of physics and of the novelty represented by the theory of relativity. *La nouvel esprit scientifique* (1934) is the most programmatic formulation of epistemological modernity from this period. The publication of *La formation de l'esprit scientifique* in 1938 clearly demonstrates that Bachelard felt the need to revisit the problem of the scientific mind and to clarify the nature of the *epistemological obstacles* which constantly obstruct the progress of science. If *La nouvel esprit scientifique* is above all concerned with the moments of conceptual transformation in the history of science, then *La formation de l'esprit scientifique* is concerned with the obstacles which impede scientific conceptualization. In order to better grasp the history of science as the history of rectified error, Bachelard proposes to develop an entire "psycho-analysis of objective knowledge" which would have as its task the analysis of the spontaneous philosophy of experience which defines the pre-scientific mind: realism.

In Praise of Abstraction: Overcoming the Realist Epistemological Obstacle

La formation de l'esprit scientifique is a defense of abstract thought against the distorting influence of "experience that is ostensibly concrete and real." *La nouvel esprit scientifique* already posited the epistemological rupture between experience and scientific reason, but it did not present a systematic theory of the nature of lived experience or the structure of the obstacles which impede the development of scientific thought. Bachelard asks how it is that scientific thought emerges from the spontaneous realism of lived experience when the system of rational values informing the construction of scientific objectivity diametrically oppose the intuitive values of lived experience. Bachelard thus requires a mediating notion between the realms of scientific experience and lived experience: he proposes a theory of the *epistemological obstacle* as this mediating notion. The first such obstacle to the formation of the scientific mind is the nature of experience itself:

> …we shall not hesitate to introduce a polemical note at times and argue that experience that is ostensibly concrete and real, natural and immediate presents us with an *obstacle.*[422]

That which is obstructed by the "concrete and real" nature of lived experience is the necessary abstraction of the scientific mind which allows experience to become *dynamic* and *self-critical,* rather than remaining *inert* and *opaque.*

Given that experience itself is an epistemological obstacle for Bachelard, how is it that the scientific mind is able to organize itself in resistance to the nature of experience? Bachelard's answer hinges on the nature of *geometrical representation,* which is simultaneously intuitive and idealizing.

> The scientific mind is first seen clearly and incontrovertibly when it makes representation geometrical, that is to say when it delineates phenomena and

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[422] Gaston Bachelard, *The Formation of the Scientific Mind, A Contribution to a Psychoanalysis of Objective Knowledge* Trans. Mary Mcallester Jones (Manchester: Clinamen Press, 2002), 18.

> puts an experience's decisive events into an ordered series. This is indeed how we arrive at *figured quantity*, halfway between the concrete and the abstract in an intermediate zone where the mind aspires to reconcile mathematics and experience, laws and facts.[423]

Geometrical representation is a powerful method for selecting the "decisive events" of experience and arranging them in "an ordered series." Geometrical representation is not yet a working method of scientific objectivity, but it is a principle of organization which is capable of transforming lived experience into a system of abstract representation. What is most important for Bachelard is the dynamic relationship between the abstract and the concrete which becomes possible thanks to the intuitive appeal of geometric forms. The domain of *figured quantities* is a dialectical threshold between "mathematics and experience," and "laws and facts."

Truly compelling models of geometrical representation often correspond with well known philosophical and scientific systems. Bachelard cites Cartesian and Newtonian systems of mechanics and Fresnel's optics as examples. However, even in such celebrated case studies from the history of science and philosophy, geometrical representation still fails to move beyond "a *naïve realism of spatial properties"* which depends on an all too eagerly established conformity between the rational "order of reasons" (after Descartes), and the diversities of sense experience. The progressive emancipation of the rational from the sensible works to break this intuitive correspondence between the order of reasons and the domain of experience. Bachelard writes:

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[423] Bachelard 2002, 17.

> We gradually feel the need to work *beneath* space, so to speak, at the level of those essential relations upholding both space and phenomena. Scientific thought is thus drawn toward "constructions" that are more metaphorical than real, towards 'configuration spaces' of which perceptible space is, after all, only one poor example. The role of mathematics in contemporary physics therefore goes far beyond simple geometrical description. Mathematicism is not descriptive but formative. The science of reality is no longer content with the phenomenological *how*: the mathematical *why* is what it seeks.[424]

As Bachelard had already argued in *La valeur inductive de la relativité,* the construction of the real is epistemologically superior to the explanation of the real. Here it is the intervention of mathematics that allows the physicist to transcend the naïve realism of space in favor of a purely abstract construction of space.

Bachelard emphasizes the priority of mathematical construction over phenomenological description as the characteristic feature of the new scientific mind. According to Bachelard, the second great age of scientific reason begins in 1905, when Einstein first publishes the quartet of articles which would give rise to the special theory of relativity. The next twenty five years are witness to an historically unprecedented acceleration of the rate of scientific discovery as the methodology of a new form of scientific objectivity rapidly reorganizes the research programs of mathematical physics. Relativity "deformed primordial concepts that we thought were fixed forever."[425] The first decades of the twentieth century are characterized by a new physics which displays an "amazing intellectual maturity," and which definitively breaks away from any conceptual overlap with the history of philosophy. Bachelard finds in this mutation of the scientific mind not only an

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[424] Bachelard 2002, 17.
[425] Bachelard 2002, 19.

entirely unprecedented system of epistemological concepts, but a corresponding need to invent a philosophy capable of productively engaging with the new physics. *La formation de l'esprit scientifique* and Bachelard's following book, *La philosophie du non* (1940), are his attempts to outline this new philosophy.

Philosophy, reconceived as an ally of the new scientific mind ,rather than as an obstacle to be overcome, becomes the vehicle of new judgments of value. What is required is a critique of the values of the scientific mind and the pre-scientific mind. Curiously, as soon as Bachelard has defined the new task of philosophy in terms of the critique of values (explicitly citing Nietzsche's trans-valuation of all values), he promptly renames it a "psychoanalysis of objective knowledge." Much has been written about Bachelard's infelicity to the concepts of psychoanalysis and his haphazard appropriation of Freud and Jung, it would seem to be more accurate and more faithful to Bachelard's project, however, to interpret the use of "psychoanalysis" as a figural instantiation of a well defined philosophical operation: the critique of realism and the affirmation of the techno-rationalism which defines the new scientific mind.[426] Here Bachelard explicitly links the two notions:

> …the task of the philosophy of science is very clear: it is to psychoanalyze interest, to destroy all utilitarianism, however disguised its form and lofty the status it claims, and to turn the mind from the real to the artificial, from the natural to the human, from representation to abstraction.[427]

The philosophy of science now becomes a polemical instrument which serves the interests of science: it is a weapon which diagnoses and destroys the self evident

[426] See in particular Marie-Louise Gouhier, "Bachelard et la psychoanalyse," in *Bachelard, colloque de Cerisy* (Paris: Union Général D'Éditions, 1974), 138.
[427] Bachelard 2002, 21.

values of realism. It seeks out the pre-critical utilitarianism of the apologist of experience and refers the mind to the domain of abstraction. For Bachelard, the critique of realist values is also the *defense* and, indeed, the *celebration* of rational values.

> Now more perhaps than it has ever done, the scientific mind needs to be defended and *illustrated* just as Du Bellay's *Défense et illustration de la langue française* strove both to defend the French language and to illustrate it in the sense of making it illustrious, that is to say conferring honour upon it and celebrating it. This celebration…must be normative and coherent. It must make the pleasure of mental stimulation in the discovery of truth a very conscious and active one. And out of truth it must make our brains. The love of science must be an autogenous psychic dynamism. In the purity that a psychoanalysis of objective knowledge gives it, *science is the aesthetic of the intellect.*[428]

Here Bachelard draws together a hierarchy of normative values which span the epistemological, the psychological, and the social dimensions of science. Epistemologically, science must be defended from naïve philosophical realism and the equally uninformed paralysis of dogmatic rationalism. Psychologically, "the pleasure of mental stimulation" must be encouraged and cultivated in order to stimulate the study of science and the re-organization of mental habits which modern science requires of the student and the scientist. Pedagogically and socially, the "love of science" must be promoted against the merely utilitarian pursuit of industrial efficiency or technical superiority in the market-place. Finally, in what is perhaps the coda of Bachelard's entire philosophy, the psychoanalysis of objective knowledge reveals that "*science is the aesthetic of the intellect.*"

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[428] Bachelard 2002, 21.

Before turning to Bachelard's account of the psychological and developmental sequences by which the intellect liberates itself from the spontaneous realism of experience and installs itself within the normative order of scientific objectivity, it is necessary to understand why experience itself constitutes such a formidable epistemological obstacle to the formation of the scientific mind. Scientific experience is above all experience that is capable of being *verified*. Verification in Bachelard can only be the result of a laborious process of the re-organization of experience. The realism of our daily lives, despite its profound intuitive coherence, is entirely lacking in that rational coherence which is the condition of possibility of experimental verification.

> Since everyday experience is not organized and *composed,* we are of the opinion that it cannot actually be *verified.* It remains a fact. It cannot give us a law. If we are to confirm what is true scientifically, we have to verify it from several points of view. Thinking an experience means therefore giving coherence to an initial pluralism.[429]

The domain of facts is entirely congruent with the domain of lived experience. Every fact announces itself by the solidity of its appearance. A fact does not need to be analyzed; it offers itself as the spontaneous phenomenality of experience. It is therefore appropriate to say that experience itself constitutes a fact. For the epistemologist, however, the unitary fact of experience is like a sea of qualitative transformation. Every fact is a plurality of qualities. In order to *think experience,* the phenomenal pluralism of the fact must become the coherent rationalism of the *experiment.*

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[429] Bachelard 2002, 22.

> A *scientific* experiment is … one that *contradicts ordinary, everyday* experience. Moreover, immediate everyday experience always has a kind of tautological character, developing in the realm of words and definitions; what it lacks in fact is the perspective of *rectified errors* that in our view characterizes scientific thought. Ordinary, everyday experience is not organized and *composed*; it is, at the very most, made up of juxtaposed observations and we are struck by the fact that the old epistemology established continuity between observation and experimentation whereas experimentation ought instead to distance itself from the ordinary conditions of observation.[430]

Experience is an immediacy of facts: an immersion in qualities. The association of any number facts will indeed be arbitrary in the absence of an independent epistemological norm capable of governing the organization of qualitative relationships. Bachelard pushes the skepticism of the empiricist even further: experience is not only arbitrary, that is to say, without transcendental principles of organization, it can only give rise to *tautologies*; everything that appears is self enclosed, a fact sufficient unto itself but incapable of rational integration into a coherent whole. *The experiment does not seek to confirm the nature of experience, but to contradict it.* Experience, as continuous transformation, is incapable of recognizing error. The condition of possibility of verification is the recognition of error. The experiment makes error possible. To err is to establish the possibility of verification.

The possibility of error emerges from a re-organization of experience which experimentation artificially imposes on experience. The history of truth is the history of rectified errors. Bachelard cannot help but compare epistemological modernity to classical epistemology from the perspective of the foundational

[430] Bachelard 2002, 22.

necessity of error. For Newton and Kant, one imagines Bachelard saying, experimentation would have been the disciplined confirmation of appearances, and therefore the illumination of an intelligible relation which, although hidden beneath experience, was itself perfectly consistent with experience. Such a coordination of reason and experience can only yield an image of reason which faithfully reflects the world. Bachelard insists on a contrary image of reason *which affirms itself* and so remakes the world in its own image. The experiment is the disruption of experience and the isolation of error, rather than the rational integration of observations in order to eliminate errors. For Bachelard, scientific knowledge always has this fragmentary, disruptive nature. Knowledge is not part of an intuitive, rational whole. It is not a fragment of immediacy, but a fragmenting of the immediate:

> Knowledge of reality is a light that always casts a shadow in some nook or cranny. It is never immediate, never complete. Revelations of reality are always recurrent. Reality is never 'what we might believe it to be': it is always what we ought to have thought. Empirical thought is clear in *retrospect*, when the apparatus of reason has been developed. Whenever we look back and see the errors of our past, we discover truth through a real intellectual repentance. Indeed, we know *against* previous knowledge, when we destroy knowledge that was badly made and surmount all those obstacles to spiritualization that lie in the mind itself.[431]

Scientific knowledge is knowledge of reality. The nature of the real however does not precede the experimental process which isolates it by trial and error. Scientific truth does not correspond to the intuitive convictions of the realist, it is a polemical re-organization of experience and a history of rectified error. Reality is as normative as epistemology: it is not what we think it is but "what we ought to have thought." It

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[431] Bachelard 2002, 24.

only appears retro-actively once we have destroyed badly made concepts and unverifiable assumptions. Empiricism only becomes possible after a rational apparatus has been built up. It is not the starting point, but the result of an experimental investigation.

Scientific reality is constructed out of this tissue of rectified errors. This is the productive dialectic of that which remains empirical for the new scientific mind. This empiricism is above all the residue of a rational procedure: it is embodied and stabilized by the apparatus of scientific experimentation,; an apparatus which is itself the "materialized theory" of scientific concepts. The epistemologist must carefully trace the network of relations by which modern science constructs the real out of this distribution of the destructive and creative powers of reason. For Bachelard, it is the *constructive* aspect of scientific reason which has been effaced by the traditional historiography of science. Science must remain polemical because it must constantly go beyond the boundaries not only of experience, but of its own rational limits. Science must systematically endeavor not only to rationalize the facts of experience, but to produce the reality of its own reason.

> Reason alone can dynamise research for it is reason alone that goes beyond ordinary experience (immediate and specious) and suggests scientific experiment (indirect and fruitful). It is therefore this striving towards rationality and towards construction that must engage the attention of epistemologists. We can see here what distinguishes the epistemologist's calling from that of the historian of science. Historians of science have to take ideas as facts. Epistemologists have to take facts as ideas and place them within a system of thought. A fact that a whole era has misunderstood remains a *fact* in historians' eyes. For epistemologists however, it is an *obstacle*, a counter-thought.[432]

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[432] Bachelard 2002, 27.

Ordinary experience, the immanence of the fact, is specious because it is immediate. Scientific experimentation tends toward the rational because it does not take facts for granted, but wishes to denature them by locating them in a system of rational relations. Modern science is this tendency which goes from the rational to the real, and from the fact to the rectified error. The epistemologist is distinguished from the historian by this relationship to facts. Every fact, subjected to epistemological verification, becomes meaningful insofar as it becomes an idea, that is to say, an element in a system of thought. The fact is the endlessly renewed primary obstacle for scientific thought. Experience itself ceaselessly renews this "counter-thought."

An epistemologically rectified history of science would thus become a history of obstacles overcome. The epistemologist differs from the historian by filtering the unreflective reality of the fact through the normative lens of rational values. Here the determinate role of error is again crucial: the error is what disappears from the history of science. The history of the epistemologist is the intellectual destiny of ideas.

> It seems to us…that epistemologists – and this is what distinguishes them from historians –should, when dealing with all the knowledge of a particular period, draw attention to the productive ideas. For epistemologists, ideas must not just have had proven existence, they must also have had an intellectual destiny. We shall not hesitate therefore to ascribe to error – or to intellectual futility, which is pretty much the same thing – any truth which is not part of a general system, any experiment, however accurate, which remains unconnected with a general method of experimentation , and any observation which, however real and positive it may be, is made known in a false perspective of verification.[433]

[433] Bachelard 2002, 22.

The method of experimentation makes error possible in the first place, but the rectification of error is the beginning of verification, and thus the beginning of objectivity. The history of science as filtered through epistemological norms is a rectified history of science: it is not the chronicle of facts but the rational coherence of rectified errors. Bachelard thus distinguishes the history of science from all other subjects of historical investigation. The history of science must be written with epistemological criteria in place: historiography is incomplete without epistemology.

> History is…intrinsically hostile to all normative judgments. We are obliged however to take a normative view if we wish to evaluate the efficacy of thought. Not everything we find in the history of scientific thought contributes to the development of that thought, far from it. There are some kinds of knowledge which, even though they are accurate, bring useful research to a premature end. Epistemologists must be selective then in their use of the material historians provide. They have to evaluate these documents from the standpoint of reason and indeed from the standpoint of developed reason for it is only now that we can really judge the errors of the mind's past.[434]

Bachelard constructs a new concept of the history of science as a history which sits in judgment on its own past. The methodology of the historian is usually designed to correct precisely this retroactive judgment of the past by the present. History as written by the victorious is an obstacle to historical objectivity. The objectivity of science, however, cannot exist without the rectification of error: the history of science is in part the construction of a system of values which stands against the history of nations and men, the world of everyday experience, and the irrational domination of facts. The history of science, like epistemology, must evaluate the

[434] Bachelard 2002, 27.

errors of history and of the mind; it must judge that which has a future and that which has proven to be intellectually barren. For Bachelard, historical objectivity and epistemological objectivity do not coincide: in order to write the history of science a new methodological synthesis is required, an historical epistemology.

Epistemology discloses the rupture which science introduces into experience in order to approach the progressive synthesis of objectivity. This rupture is magnified and repeated at every level of experience: the abstract triumphs over the concrete again and again, but not without ceaselessly altering the borders of abstraction and lending precision to the concrete. The scientific instrument is paradigmatic of this "instructed materialism" of the concrete: the spectrometer of the industrial chemist isolates the presence of the atomic elements only after learning how to eliminate everything that is impure in nature. The spectrometer is in fact the materialized theory of atomic structure. Nevertheless, its effects could not be more immediate : it does not survey the real "as it is," but as it "ought it to be" according to the theory of atomic weight and the measurement, by optical diffraction, of the variable density of atomic nuclei. The abstract and the concrete are rigorously correlated in the manufacturing of the apparatus of science. In the same way, scientific objectivity is not the statement of the facts of experience, but the realization of the rational by the production of the phenomena of science.

Science does not distinguish discovery from invention: it rectifies the errors of primary experience in order to successively approach an always approximate knowledge of the real, an always rationalized materialism of experimental verification, and an always applied rationalism of mathematical construction. The nature of the intellect for Bachelard thus reveals itself in the progressive development of scientific objectivity. This is why Bachelard can claim that "science is the aesthetic of the intellect." Before the aesthetic of the intellect can be revealed as essentially scientific, the mind must pass through an evolutionary process which, through a displacement and repetition of the developmental schemes of nineteenth century positivism, corresponds with a "*a law of the three stages* for the scientific mind."[435]

The three stages of the scientific mind are the *concrete,* the *concrete-abstract* and the *abstract.* The concrete stage is fully immersed in the lived reality of phenomena and all the imaginative richness of the images of nature. The epistemological themes which traverse concrete images of the natural world encompass all the diversities of experience: they are at once profoundly heterogeneous and strangely unified. There is no stable system by which the diverse becomes the uniform – any number of analogies, secret correspondences, or motivated hierarchies of being may suddenly co-ordinate the complexity of appearances in the service of an underlying principle of order. The epistemological profile of the concrete image of nature is therefore profoundly capricious. At this level of development the difference between an ordered universe and an unruly universe might hinge on an unlimited array of qualitative features. The concrete image of nature is also an investment in the immediacy of values: being is directly

[435] Bachelard 2002, 20.

animated by value; it pulses with a significance which might manifest itself suddenly in any number of ways. For the concrete mind, the universe appears saturated by values of all types. The concrete gives way to the concrete-abstract when "the mind adds geometrical schemata to physical experience and draws on a philosophy of simplicity."[436] Geometrical representation overlays physical experience but does not change its nature: the philosophy of simplicity is that of a shared reality common to the abstract and the concrete. This is the first achievement of science; the creation of reason in the image of the world. Finally, the abstract mind

> sets to work on information deliberately abstracted from the intuition of real space, deliberately detached from immediate experience and even engaged in an open polemic with primary reality, which is always impure and formless.[437]

The scientific mind is here recognizable in its modern form: it is polemical, actively engaged in the construction of forms of intelligibility, and systematically negating "primary reality."

The psychoanalysis of objective knowledge must not only distinguish among the three developmental stages of the scientific mind, it must learn to recognize the types of interest or fascinated attention which correspond to each stage of development. Bachelard thus invokes a corresponding "law of the three stages of the soul." The "child like soul," characterized by undifferentiated curiosity, remains "passive even in the joy of thinking." The "teacherly soul" celebrates the dogmatism

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[436] Bachelard 2002, 20.
[437] Bachelard 2002, 20.

of its “first abstraction” and exists in a permanent state of gratification. Bachelard is especially critical of the teacherly soul which is content to rest “throughout its life on the laurels of its schooldays…wholly devoted to that deduction which so conveniently bolsters authority.”[438] It is only the soul of the “suffering scientific consciousness” which has learned to value “the dangerous game of thought that has no stable experimental support” that proves capable of embracing genuine abstraction, in so doing “refining and possessing the world’s thought.”[439]

The psychology of the scientific mind requires this relentless classification of its developmental schemes because, for Bachelard, it is a relatively recent phenomenon and nothing guarantees its historical survival. The psychoanalysis of objective knowledge is not only a diagnostic program, it is also designed to function as a selective mechanism, identifying and magnifying the characteristics of the new scientific mind in order to engineer a robust and elastic scientific culture. Bachelard’s epistemology is also normative in this partisan, constructive sense: he wishes to defend the new scientific mind on the cultural, pedagogical and social terrain which it needs in order to sustain its further development. Bachelard is convinced that the nature of experience itself works against the rational values of scientific objectivity, he must therefore take extraordinary steps in order to defend the small but vital community of rational inter-subjectivity which has crystallized in the wake of relativistic physics.

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[438] Bachelard 2002, 21.
[439] Bachelard 2002, 21.

Bachelard evolves a system of concepts which not only seek to re-describe the epistemological contours of modern science, but which also serve a pedagogical and social agenda. *La formation de l'esprit scientifique* is unique among Bachelard's books insofar as it attempts to demonstrate the inter-dependence of epistemological problems, pedagogical methods, and social norms. The epistemological problem at stake is the determination of the nature of scientific objectivity. The pedagogical method, by way of the psychoanalysis of objective knowledge, is to delimit the nature of the scientific mind and to design a method of instruction which cultivates the cognitive habits required to master the phenomeno-technics of a new scientific culture. The social norm, which as we shall see calls for nothing less than the re-organization of society, is derived from the system of rational values that inform epistemological modernity. This is perhaps the most radical claim Bachelard makes in *La formation de l'esprit scientifique.* In order to provide a schematic outline of the relationship between epistemology, pedagogy, and the social organization of scientific labor in Bachelard, it is necessary to return to what I claim is the central affirmation of *La formation de l'esprit scientifique*: *Science is the aesthetic of the intellect.*

Aesthetic concerns are not excluded from the psychoanalysis of objective knowledge, on the contrary, they occupy the most exalted place in Bachelard's table of rational values for the aesthetic of the intellect manifests itself as science. This startling affirmation nevertheless remains a relatively unexplored aspect of Bachelard's philosophy and no doubt gestures toward the possibility of a radical integration of the two sides of Bachelard's oeuvre: the aesthetic and the epistemological. Bachelard himself does not develop this claim any further in *The Formation of the Scientific Mind* other than to note, at the very end of the book, that the recognition of the nature of the intellect as essentially scientific requires not only a reform of philosophy, but of society. The mediating institution between science and society in Bachelard's estimation is school, therefore Bachelard proposed a *permanent rectification of the intellect by an indefinite pedagogy of scientific literacy.*

> The principle of *continued culture* is…the root of modern scientific culture…Only In the work of science can you love what you destroy; only here can you continue the past by repudiating it, and honor your teachers by contradicting them. When that is the case schooling does indeed go in throughout your whole life…There is science only if schooling is permanent. It is this schooling – and this school – that science must found. Social interests will then be reversed once and for all: society will be made for school, not school for society.[440]

A very strong pedagogical program accompanies the psychoanalysis of objective knowledge. Its aim is nothing less than to combat the sclerosis of all customary ways of thinking. The history of science offers a very clear pedagogical model: there can be no stable position of intellectual mastery, the vitality of the mind only extends as far as the re-structuring of the intellect. For Bachelard then, the ultimate lesson of the psychoanalysis of objective knowledge is a pedagogical imperative: the scientific mind must be in a state of perpetual reform, that is to say, it must never stop learning to denature itself and to pursue a more advanced synthesis of the rational and the empirical in order to further advance the realization of the rational.

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[440] Bachelard 2002, 249.

In order to understand the full extent of the normative force of Bachelard's epistemology, it is necessary to dwell a moment longer on the pedagogical and social implications of Bachelard's epistemology, even if he devotes very few pages directly to these subjects.[441] Bachelard does not hesitate to pursue the implications of his pedagogical imperatives all the way to the restructuring of society. Bachelard formulates nothing less than a vision of social order directly inspired by the structure of the scientific mind. The rational development of the scientific mind is not one element of culture among others: *it is the supremely normative element of culture, the permanent rectification of rational values.* As such, pedagogical institutions have a truly indispensable mission as the custodians of the highest values. From this perspective educational institutions would not be made in order to serve the interests of society; society would be engineered in order to support the requirements of the educational institutions. To speak of the *normative force* of Bachelard's epistemology is, in my view, to invoke this broader range of outcomes. Epistemology thus conceals within its profound asceticism the mechanism of a global transformation: the reshaping of society in the interests of the rational values of science.

La formation de l'esprit scientifique introduces the notion of a psychoanalysis of objective knowledge in order to clarify the system of rational values underlying the various developmental stages of the scientific mind. Bachelard's motivation as a

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[441] For an excellent treatment of Bachelard's pedagogical and social views see Didier Gil, *Bachelard et la culture scientifique* (Paris: Presses Universitaires de France, 1993.)

psychoanalyst of objective knowledge is not exclusively epistemological, he also intends to contribute to the rectification and continued development of scientific culture. In order for science to flourish it must be pedagogically and socially sound: the mind must be directed away from the pluralism of experience and towards the coherent rectification of error. The social organization of scientific labor also requires careful management. In the final analysis, Bachelard laments the subordination of pedagogy to the interests of society: it is society which should serve the needs of the most rigorous pedagogical practice; the cultivation of scientific minds capable of extending the work of the scientific objectification of the rational. The normative force of epistemology thus encompasses pedagogical and social interests within the same polemical re-orientation of values. Science cannot be subordinated to the spontaneous philosophies of experience, to the habits and inclinations of the mind, or to the historical variables of a more or less capricious social order.

To claim that science is autonomous and that it is the repository of rational values is then also to argue *against* the autonomy of other systems of value. Let there be no doubt: scientific activity is the central occupation of human civilization for Bachelard. Indeed, it is hard to see how such a conclusion could be avoided if scientific objectivity is not the discovery of the real, but the realization of the rational, *the creation of a new world, a new physics, and a new cosmology: in short, the construction of an entirely rational universe.* There is a teleological aspect to Bachelard's thought which is rarely appreciated; it is the ambition of science to remake the world in its own image. Bachelard does not doubt that this ambition will be thwarted, even forced into retreat. His goal therefore is to carve out, as clearly as possible, the intellectual profile of epistemological modernity in the hope that it may serve as a model for the continued rectification of the real. *La philosophie du non* (1940) is the epistemological profile of scientific objectivity in the wake of relativity and a reclamation of philosophical possibilities authorized by science but ignored by philosophy.

A Program for the Rationalization of the Real: Non-Euclidean Geometry for the Mathematician, Non-Philosophy for the Epistemologist.

It is impossible to over-emphasize Bachelard's experience of the extraordinary acceleration of scientific development during his own life-time. In 1938 he writes:

> Over a period of twenty five years [1905-1930], ideas appear that signal an amazing intellectual maturity, any one of which would suffice to shed luster on the century. Among these are quantum mechanics, Louis de Broglie's wave mechanics, abstract mechanics, and doubtless there will soon be abstract physics which will order all the possibilities of experience.[442]

The new epistemological configuration of objectivity is marked by this acceleration of theoretical and technological revolutions. The image of the world is recurrently dismantled and reconstructed on the basis of progressively more radical rational syntheses. There does not seem to be any rational principle limiting this development. On the contrary, the nature of reason, illustrated by the accelerated intervals of scientific revolution, suggests that the reformation of reality is itself the

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[442] Bachelard 2002, 19.

defining feature of objectivity. Here historical and epistemological observations converge: the nature of the scientific intellect is to reform itself, thereby reforming the nature of objectivity. The historical acceleration of these reforms indicates that science is now in possession of a fruitful methodology for the progressive reinvention of its own material and rational bases. Is there any evidence that philosophy has been able to stay abreast of these changes? For Bachelard, the answer is no, and the philosophers are not alone in their intellectual paralysis. The scientists themselves are often to blame.

> …the philosophy of science remains corralled in the two extremes of knowledge: in the study by philosophers of principles which are too general and in the study by scientists of results which are too particular. It exhausts itself against these two epistemological obstacles which restrict all thought: the general and the immediate. It stresses first the *a priori* then the *a posteriori,* and fails to recognize the transmutation of epistemological values which contemporary scientific thought constantly executes between *a priori* and *a posteriori*, between experimental values and rational values.[443]

Bachelard here suggests the operation of a new antinomy of reason (necessarily astride the mathematical and the dynamic) which would be that of the *general* and the *immediate*; contradictions which proceed from a mistaken attribution of the nature and limits of the boundaries of scientific reason. The philosopher errs by succumbing to the temptation of excessive generality : philosophy reduces science to deductions which proceed from systems of evidence. What matters is that the rational procedures of deduction and the materiality of evidence are given at the outset. It does not particularly matter if the evidence is

[443] Gaston Bachelard, *The Philosophy of No, A Philosophy of the New Scientific Mind* Trans. G.C. Waterston (New York: The Orion Press, 1968), 5.

rational (as in Descartes), and therefore characterizes science as a system of rational mechanics (Ampere, Laplace, Cournot, Carnot etc.), or if the evidence is strictly empirical (as in Comte), and therefore characterizes science as a system of Positive laws (Newton, Comte, Bergson, Meyerson etc.) So much for the philosophers. Do the scientists fare any better? They err on the side of the particular, which does not escape the antinomy of immediacy. The results of a particular experiment do not establish the boundaries of scientific reason: they are themselves experimental corrections of reason and reality at the same time, and in the heart of the same objective mechanism.

For Bachelard, despite the apparent philosophical controversies animating the debate between *a priori* and *a posteriori* accounts of scientific knowledge, the result is always the same. Philosophy may transform science into the manifestation of the structure of the mind or into the confirmation of the structure of reality, in either case it fails to understand that science has no object other than itself. The philosophical debates concerning the nature of science are *epistemologically identical* to Bachelard: it makes no difference to the epistemologist if it is Descartes, Hegel, Bergson or Einstein discoursing on the nature of science. They may each seem to produce incompatible theories of knowledge, but it is possible to recognize a deeper identity forcing them towards a convergence of errors: what is passed over by the philosopher and the scientist alike is the "transmutation of epistemological values;" the actual *work* accomplished by science in the production of objectivity. The *efficacy* of science must be referred to the mutation of epistemological values it accomplishes: by forcing the experimental and rational determinations of objectivity to enter into new relations of the abstract-concrete (i.e. the pan-mathematicsm underlying experimental physics), and the concrete-abstract (i.e. the materialized theory of the scientific apparatus), science forces the evolution of objectivity: it produces the real out of the rational. The epistemological vector of this transformation of values cannot be modeled by *a priori* or *a posteriori* theories of knowledge, which assume, by definition, the stability of the limits of knowledge on the one hand, and the non-decomposable positivity of the empirical fact, on the other.

Bachelard discovers a “double movement” from the rational to the experimental and vice versa which makes science capable of reflexive transformations which neither philosophy nor the scientists themselves have been able to articulate. Although Bachelard does not spare the scientists in his critique of the discourses surrounding science, there is reason to suspect that an inflexible philosophical tradition is the real target of his ire. The well know diagram of the distribution of the philosophies of science from *La rationalisme appliqué* (1949) allows us to condense Bachelard’s principal objections to the traditional philosophies of science into a single image.

Figure 2. <u>Diagram of the Distribution of the Philosophies of Science.</u>

Idealism
|
Conventionalism
|
Formalism
|
Applied Rationalism and Technical Materialism
|
Positivism
|
Empiricism
|
Realism[444]

Bachelard's epistemological project occupies the central axis of the diagram. Applied rationalism and technical materialism represent the modifiable extremes of a concretizing abstraction, on the one hand (i.e., the furnishing of experimental possibilities through purely mathematical resources), and an abstracting materiality, on the other (i.e. the phenomeno-technics of scientific instruments.) It is important to note that even the extreme poles of Bachelard's diagrammatic representation of scientific objectivity are already hybrid, reflexive composites of the abstract and the concrete. Indeed, there are no pure substances in Bachelard. Science can conceive only relations and modifications. This is entirely in conformity with the de-realization of substance and the valorization of relations as depicted most forcefully by Bachelard in his book on relativity, *La valeur inductive de la relativité.*

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[444] This is a modified reproduction of the diagram that appears in Bachelard's *Le rationalism appliqué.* See Gaston Bachelard, *Le rationalisme appliqué* (Paris: Presses Universitaires de France, 1949), 5.

Bachelard locates his project on the central line of the diagram in order to express his distance from the philosophical extremes of idealism and realism. More profoundly, however, the symmetrical distribution of philosophical orientations originate from the central configuration of Bachelard's own project. Lecourt is the most perceptive reader of Bachelard's epistemological diagram, noting that, owing to its symmetry

> by folding the schema about its center, they [the philosophical extremes of the diagram] can be made to coincide…each of the doctrines can be inverted into the apparently contradictory doctrine without its nature being changed thereby…the *nature* of each of the doctrines resides not in itself but in the *fold,* the fixed point in philosophical space.[445]

This should draw our attention to the significance of Bachelard's own position within this philosophical space, which is not only central but *determinate.* Bachelard's diagram allows us to see that he locates himself directly and self consciously at the locus of an event which philosophy itself only responds to passively. Lecourt speaks of a "dispersion of philosophical possibilities" which correspond to an event philosophy finds itself unable to articulate directly. Here we must set aside Lecourt's analysis (which will ultimately concern philosophy's inability to theorize itself), and substitute again the *immanence of rational values* within science as the philosophical event which organizes the philosophical space of the diagram. Bachelard is the thinker for whom science has no object but itself. Its rational values are entirely internal – it transforms itself, paradoxically, by remaining faithful to its own immanence; by not compromising itself by importing

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[445] Lecourt 1975, 62.

values from any other domain. The dispersion of the philosophies of science as expressed by Bachelard's diagram illustrate the importation of values foreign to science. Realism and Idealism import the most drastically opposed values, accordingly, they are at the extremes of the diagram. By the same logic, their opposition only makes sense given the already fully constituted position of the antagonist: the realist needs the idealist, just as the conventionalist needs the empiricist, just as the formalist needs the positivist. None of these philosophical orientations are capable of seeing past the values they project onto science. By distorting the value which science accords to itself (the immanent normativity of rational values- exemplified, in the extreme, by the autonomy of mathematics), they fail to perceive the conceptual genesis of scientific objectivity, and they fail to understand the nature of scientific evolution.

The philosophy of science is a domain of *forced choices* founded on a common error: the failure to acknowledge the normative configuration of science as an epistemological construct. You are free to be an idealist *or* a realist, a formalist *or* a positivist. In any case you will fail to grasp the nature of scientific objectivity if you compromise the normativity of science by failing to recognize the immanence of its rational values. I have characterized the nature of the epistemological norm in terms of immanence. *The normativity of science for Bachelard, and for the tradition of historical epistemology, is characterized by immanence.* The ontological structure of science is normative because it is an ontology of immanence: science has no object but itself. The epistemological structure of science is normative because it is an epistemology of immanence: science furnishes the criteria of its intelligibility through self-reference, as exemplified by the immanent intelligibility of mathematical objects. The system of values informing science are normative because they are immanent: science takes itself as its own end. The historiographic methodology of science is only complete when the epistemological critique has revealed the immanence of the rational norm: the progressive transformation of the conditions of objectivity and the successive realizations of the rational.

All of these moments are unified and inseparable: there is a rigorously immanent rational procedure at work in the heart of scientific objectivity according to which the extremes of the philosophical spectrum must be shown, *in fact,* to be in a constant state of reciprocal transformation; the empirical becoming the rational and the rational illuminating itself through the generation of experimental applications.

> …if one could translate into philosophical terms the double movement which at present animates scientific thought one would perceive that there has to be alternation between *a priori* and *a posteriori,* that empiricism and rationalism in scientific thought are bound together by a strange bond, as strong as the bond which joins pleasure and pain. Indeed the *one triumphs by assenting to the other*: empiricism needs to be understood; rationalism needs to be applied. An empiricism without clear, coordinated, deductive laws can be neither thought nor taught; a rationalism without palpable proofs and without application to immediate reality cannot fully convince. The value of an empirical law is proved by making it the basis for a chain of reasoning. A chain of reasoning becomes legitimate by becoming the basis of an experiment.[446]

The sign of immanence at work in the constitution of scientific objectivity is discernible in the effects of the transformations it makes possible: empiricism only

[446] Bachelard 1968, 6.

succeeds as that which is genuinely empirical, that is to say, as the experimental arm of objectivity, by assenting to rational explanation. Rationalism only succeeds as genuinely rational, that is to say, as the rational coherence of objective experience, by assenting to the technological embodiments of application. Here the coherent extremes of Bachelard's table of philosophical oppositions are deployed in relations of resonance rather than antagonism. The value of the empirical law is derived from the argumentative chain it makes possible, just as the order of reasons gives rise to experimental possibilities. Scientific objectivity proliferates by multiplying this matrix of concrete abstractions and abstract concretions, or by *concretizing* that which is abstract and by *abstracting* that which is concrete.

If, as Bachelard claims in *La valeur inductive de la relativité*, only relations are determinate for epistemological modernity, then it is easy identify the minimum difference which scientific objectivity requires in order to put its relations into effect: the passage from the concrete to the abstract and from the abstract to the concrete. Given that such transformations will happen simultaneously at different locations in the scientific apparatus, there will be two complimentary but opposed differential pairs of transformative relations, at minimum, determining the boundaries of scientific objectivity at any given moment. Following the work of Deleuze and Guattari, we can characterize this boundary of objectivity as a threshold of deterritorialization and reterritorialization: a boundary at which material substances are transformed into intelligible forms (the abstraction of the concrete), and intelligible forms are transformed into techno-scientifically produced phenomena (the concretion of the abstract).

Scientific objectivity is thus the constitution of a novel form of reality, or as Bachelard prefers to say, it is the realization of the rational. It is not surprising, then, that despite the symmetrical but opposing doctrines of idealism and realism as depicted in Bachelard's diagram, it is the realist who bears the brunt of Bachelard's criticism. The nature of scientific objectivity is opposed, term for term, to the always already constituted objectivity of the realist.

> …*realization* takes precedence over reality. By doing so it demotes reality. A physicist only really knows reality when has made it come real, when he is thus master of the eternal rebeginning of things and when he constitutes within himself an eternal return to reason. The ideal of realization is, moreover, very demanding: any theory which realizes partially must realize *totally*. It cannot be right in a fragmentary manner. Theory is mathematical truth which has not yet found its complete realization. The scientist must seek out this complete realization. He must force nature to go as far as our mind goes.[447]

Realization, as active construction of the real, must triumph over reality, the inert and unverifiable multiplicity of facts. The nature of scientific knowledge is not speculative or theoretical, it is only complete when it has realized itself in the fabrication of the objective phenomenon. As Bachelard writes, the ideal of realization cannot brook half measures, it is always a totalizing achievement: the rational and the empirical must totally draw out the intelligible and material boundaries of the objective phenomenon. The nature of science and the nature of scientific objectivity are thus mutually penetrating: by reforming science one

[447] Bachelard 1968, 30.

reforms the real. The internal drive which transforms the scientific concept also redraws the boundaries of scientific objectivity.

> There is only one way to bring about the advance of science, and that is to give the lie to science as already constituted, to change its constitution, in other words. The realist is not in a good position to do this because, apparently, realism is a philosophy whose adherents are always right. It is a philosophy which assimilates everything or at least which absorbs everything. It does not *constitute itself* because it believes itself to be already constituted, *a fortiori* it never changes its constitution. Realism is a philosophy which never becomes involved, whereas rationalism always becomes involved and stakes its entire being on every experiment.[448]

The realist stands accused of a gross misrecognition both of the nature of science (which he assumes is constituted once and for all by the agreement of theory and observation), and of the nature objectivity (which he assumes is the correspondence of thought and things.) The realist has yet to enter the world of epistemological modernity. He does not understand that science is never more incisive than when it is transforming its own structure; that the scientific concept is never more acutely drawn toward the lucid articulation of the real than when it transforms the very structure of reality. By contrast, the realist is never called upon to constitute his philosophy or to fashion the concepts which would enable him to engage the world he takes for granted. For Bachelard, realism is a philosophy of profound passivity, it is a kind of waking dream from which it is impossible to be roused. The most grotesque epistemological error of the realist is his assumption that reason and reality alike are monolithic and unchangeable. If reality is obscure, then the mind must be patient, it must make itself the obscure or veiled mirror of a

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[448] Bachelard 1968, 27.

world that is reluctant to reveal its secrets. There can be no greater calamity from the perspective of the rational values of science than to impose this dual neutrality on thought and things. For Bachelard, a rational value is above all a rectified image or a corrected error: it is the result both of concrete labor and abstract reflection. Reason is only rational to the extent that it can be educated by the "most richly structured experience:"

> …science informs reason. Reason must obey science, the most highly evolved science, science in the process of evolution. Reason has no right to put a premium upon an immediate experience; on the contrary it must put itself in balance with the most richly structured experience. In all circumstances the *immediate* must yield to the *constructed*.[449]

Bachelard's normative imperative is very clear: reason must obey science because science is "the most richly structured experience." The immediacy of the real must be replaced with the ornate technicality of the objective. Only the construction of the real can educate reason. If you would become rational, you must create a world which is itself rational. You must realize the rational, and thereby express the nature of the intellect.

It is science which guides reason and objectivity which shapes experience. The place of reason in Bachelard's epistemology is therefore quite strictly analogous to that of the student in the physics laboratory, or even better, to the physicist in the laboratory. It is reason which must be instructed by a coherent experience, and not experience which is informed by a coherent reason. *Science plays the curious role of that which gives form to reason, even as it constructs itself.* The shape of Bachelard's

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[449] Bachelard 1968, 122-123.

gnoseological universe only appears once it is has been clearly determined that science guides and informs reason in all its aspects. Just as philosophy must learn from science rather than seeking to illuminate its foundations, reason must construct its own rationality in the image of scientific objectivity. Science is not the manifestation of human reason: human reason is the artifact and effect of scientific objectivity. Mathematics is characterized by the same autonomy in relation to reason.

> …Arithmetic is not founded upon reason. It is the doctrine of reason which is founded upon arithmetic. Before knowing how to count I could hardly know what reason was. In general the mind must adapt itself to the conditions of knowing. It must create in itself a structure which corresponds to the structure of knowing. It must mobilize itself around articulate expressions which correspond to the dialectics of knowledge…Reason, once again be it said, must obey science. Geometry, physics, and arithmetic are sciences; the traditional doctrine of an absolute, unchanging reason is only one philosophy, and it is an obsolete philosophy.[450]

Arithmetic is not a manifestation of reason, reason is a reflection of arithmetic. The "structure of knowing" is given by mathematical objectivity quite independently of the mind. Bachelard's epistemological system thus has three parts which can be imagined as concentric spheres. The core of the sphere is the pure and immanent normativity of mathematical objectivity. In my view, it is possible to locate the work of Jean Cavaillès entirely within the boundaries of this resolutely normative core without compromising any element of Bachelard's system. Mathematics, as we have seen, does not however constitute its objectivity through the rectification of errors. Bachelard writes:

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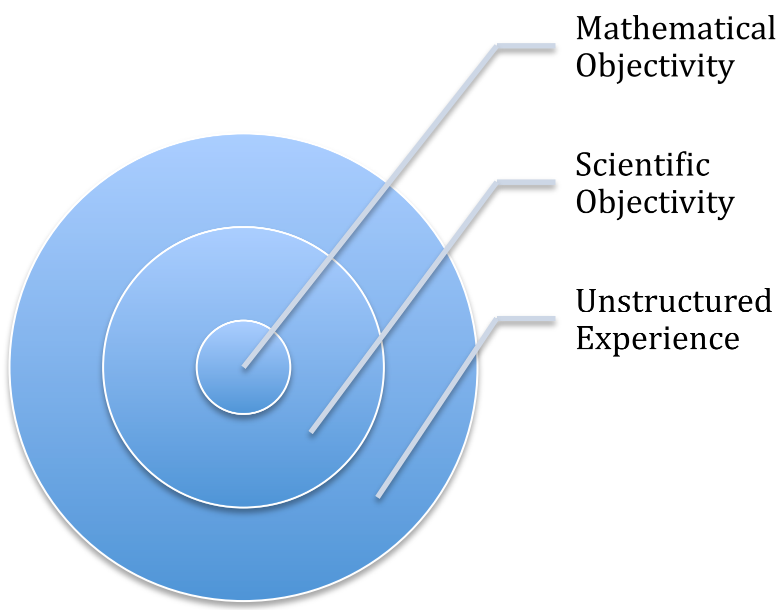
[450] Bachelard 1968, 123.

> …the growth of the mathematical mind is very different from that of the scientific mind as it strives to understand physical phenomena. Indeed, the history of mathematics is wonderfully regular. There are periods when it comes to a halt. There are though no periods of error. None of the arguments we are putting forward in this book [*The Formation of the Scientific Mind*] has any bearing on mathematical knowledge. Our arguments here deal only with knowledge of the objective world.[451]

Bachelard must set aside the problem of mathematical objectivity and simply appropriate the immanent validation of its epistemological norms. Scientific objectivity occupies the second layer of the sphere and has two valences: it faces the objectivity of mathematics, with which it shares a rich system of concepts and writing technologies, but it also engages in the construction and rectification of highly structured experiences. It thus constructs its objectivity out of a dialectic of the concrete and the abstract. Finally, at the outermost layer of the spheres of normativity, there is the unstructured immediacy of experience.

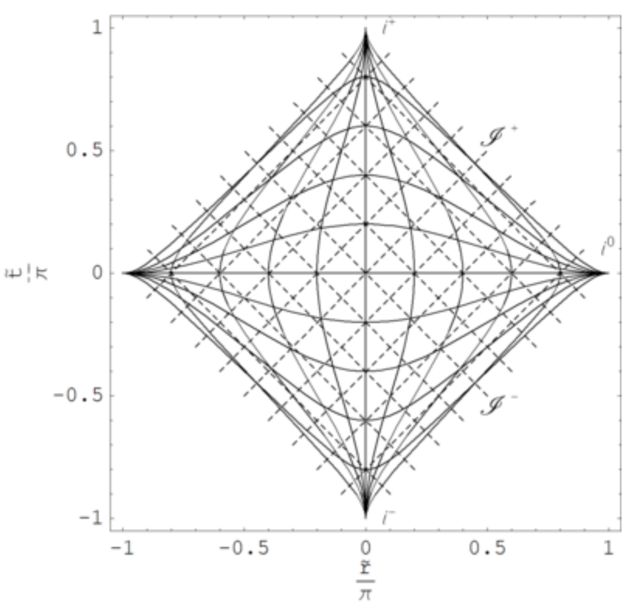
[451] Bachelard 2002, 32.

Figure 3. <u>The Structure of Epistemological Norms in Bachelard</u>



Mathematical objectivity is located at the center of Bachelard's epistemological project. The structure of mathematical objectivity is self-positing. Here the rational and the real are identical. Scientific objectivity mediates between the pure immanence of mathematical objectivity and the qualitative diversity of sense experience. For scientific objectivity, the real and the rational do not coincide: the rational must be constructed and made an object of experience. Finally, on the surface of the sphere, unstructured experience consisting only of the facts of sensation renders objectivity unthinkable. The relation of the real to the rational thus spans three possible configurations: the real *is* the rational for mathematical objectivity. The real *becomes* the rational through the construction of scientific objectivity. The real *negates* the rational as unstructured experience. Bachelard's entire epistemological project operates between the extremes of a radically unstructured experience, on the one hand, and an absolute immanence of rational normativity, on the other. The nature of scientific objectivity requires the constant deformation of these boundaries. In the final analysis, a static representation of Bachelard's epistemology is insufficient. It would be necessary to invoke, in the spirit of Bergsonian duration, a moving image of the machinery of scientific objectivity. Bachelard ceaselessly invokes the *materialization of the rational* and the *rationalization of the material*; the diagram of this transformation would require, perhaps, a torsion in more than three dimensions.

Figure 4. The Carter-Penrose Diagram of Minkowski Space[452]



[452] The Cater-Penrose Diagram of Minkowksi space. From http://www.physics.thetangentbundle.net/wiki/General_relativity/Carter-Penrose_diagram (accessed March 18th, 2014).

Minkowski space, unlike the Euclidean space of our intuition and daily experience, is defined by a topology of four dimensions: three space-like dimensions and one time-like dimension. The representation of any co-ordinate of space-time can be located in this four-dimensional space-time. The fully generalized theory of relativity, however, challenges even the abstraction of Minkowski's topology of space-time by introducing a variable curvature into this four dimensional space defined by the effect of gravity. *The curvature of space-time* generalizes the abstraction of Minkowski space and requires the development of an encompassing geometry of N dimensions and a corresponding calculus of variations. The epistemological gulf separating the spontaneous realism of experience from the structured abstraction of scientific objectivity is literally the exponential differentiation of the geometric manifold of space-time. The epistemological break between the progressive orders of scientific abstraction repeats itself with programmatic necessity, compounding abstractions and consolidating theoretical and experimental possibilities in a rational synthesis that is both inexorable and unpredictable.

Philosophy, Bachelard maintains, is incapable of thinking this progressive or algorithmic compounding of the conceptual procedures internal to scientific objectivity. Just as thought must take its measure from science ("…out of truth it [science] must make our brains…"), so philosophy must be remade in the image of science. The normativity of science cannot be compromised. As Cavaillès writes regarding the conceptual mechanism driving the transformations of mathematical objectivity, the rational sequence cannot be treated brusquely. It can only be affirmed.

Conclusion: The Uses of the History of Science for Philosophy

> Which is more pretentious: to claim memory or judgment? Errors of judgment are accidental, but alteration is of the essence of memory. About reconstructions in the history of science one must make a point that has repeatedly been made about reconstructions in other fields of history – political, diplomatic, military, and so on: namely, that contrary to Leopold von Ranke's dictum, the historian can never claim to represent things as they really were (*wie es eigentlich gewesen*).[453]
>
> -Georges Canguilhem, "The Role of Epistemology in Contemporary History of Science"

Epistemology, as I have argued throughout this work, unambiguously determines the legibility of conceptual transformations in the history of science. The history of science, I claim, is itself the effect of epistemological criteria of judgment. I have emphasized the *normative force* of epistemology in Meyerson, Brunschvicg, Cavaillès and Bachelard in order to reconstruct the *normative theory of science* which subtends the epistemological tradition. It is this underlying theory of science, as I have argued, which gives theoretical consistency to the tradition in the face of otherwise diverse philosophical orientations and objects of inquiry. I characterize this normative theory of science in terms of the uncompromising affirmation of the immanence of rational norms to the concepts of mathematics and the natural sciences. As Bachelard makes clear, science has no object other than itself, a formulation which can be generalized and applied to the tradition as a whole.

The autonomy of science is the first principle of the normative theory of science. The philosophical achievement of the tradition of historical epistemology is

[453] Georges Canguilhem, *Ideology and Rationality in the History of the Life Science* Trans. Arthur Goldhammer (Cambridge: MIT Press, 1988), 2.

to have demonstrated that the immanence of rational norms does not exclude but in fact necessitates the differentiation of the rational contents of science. There is no contradiction between the epistemological autonomy of science, the historical instantiation of the rational procedures of science, and the progressive transformation of the rational norms of science. Accordingly, the normative theory of science at the heart of the epistemological tradition is defined by three philosophical theses which express the principle of the immanent self-differentiation of the rational contents of science.

The first thesis upholds the autonomy of science. The objectivity of mathematics is the foundation of the autonomy of science and the paradigmatic instantiation of an epistemological norm which is defined immanently through recursive self-reference. The second thesis affirms that science is procedural rather than the result of a fixed methodology. Scientific concepts are defined not by their crystallization into the accomplished forms of well established theoretical and experimental systems, but by the elaboration of new rational procedures and experimental possibilities. The scientific concept, perceived in the light of the necessity of the rational norm that drives its transformations, is fundamentally oriented towards a futurity which cannot be anticipated by the existing state of science. The scientific concept discloses the *becoming* of science. The third thesis asserts that the historiography of scientific concepts can only emerge from the procedures of epistemological analysis. Science always bears a determinate relationship to its own history. The *epistemological history of a science* is not necessarily the history of science described by the historian. The history of science, absent epistemological rectification, is not constrained by the normative force of the genesis and development of scientific concepts. The history of the scientific concept, as revealed by epistemological analysis, is itself always immanent to the contemporary state of science. As the epistemological organization of science changes, so too does the history of the concept. *The epistemological history of science is therefore just as mutable as the scientific concept.* These three philosophical theses can be expressed by the epistemological formula IMMANENCE=SELF-DIFFERENTIATION.

The normative force of the theory of science thus conjoins the autonomy of the scientific concept to the necessarily historical instantiation of scientific rationality and to the progressive historical transformation of rational norms. Insofar as science is truly the *becoming* of the rational norm, the historicity of science, to reprise Brunschvicg's formulation, expresses nothing other than the immanent self-differentiation of the rational contents of science. The historical epistemologists, though unified by their adherence to this normative theory of science, do not uniformly discover the same history of science. In fact, as the dissertation demonstrates, the research programs of historical epistemology end up proliferating many potentially *differing* histories of science. It is therefore necessary to conclude that a resolutely normative theory of science, while constituting the epistemological condition of possibility of a rigorous history of scientific concepts, does *not* authorize a monolithic or unified history of science. On the contrary, the normative force of the theory of science requires *the perpetual revision of the history of science.* As science is remade, history is reborn. The affirmation of the immanent self differentiation of the rational norm is inseparable from the active construction of the history of science as a perpetually renewed operation. It is an epistemological contradiction to speak of *the* history of science as if it were a singular entity. In the light of epistemological critique there can only be *multiple* histories of science. The autonomy of science and the immanence of the rational norm manifest themselves as a proliferating multiplicity of epistemological histories.

I have reconstructed a constellation of such histories in the previous chapters. Meyerson's Philosophy of the Intellect is an epistemological history of the forms of causality and of the perpetual transformation of the procedures by which time becomes intelligible to the explanatory concepts of the natural sciences. This history of the forms of time and causality is open ended and must be perpetually re-described in accord with the changing forms of the irrational. Einsteinian relativity is at once the continuity of a deductive sequence and the birth of a new form of the intelligibility of time. Brunschvicg's Philosophy of the Intelligence is an epistemological history of the co-constitution of the forms of rationality and the forms of objectivity. The Philosophy of Judgment is also a history of the progressive rectification of a rational norm that first appeared among the Pythagoreans, but that did not achieve the equipoise of a truly rational objectivity and a truly objective rationality until Einstein's theory of relativity. The history of mathematics as constructed by Jean Cavaillès is a proscenium stage upon which the drama of the scientific concept unfolds. Cavaillès demonstrates the absolute solidarity of the rational sequence of mathematics and the historical transformation of mathematical concepts. It is no surprise then, that Cavaillès is the first to explicitly formulate the normative theory of science as a dialectical philosophy of the scientific concept. Bachelard's epistemological analysis of scientific objectivity as the techno-scientific production of the phenomenon is also an epistemological history of the rectified errors of mathematical physics. *In each case the normative theory of science is inseparable from a singular history of the transformations of the scientific concept.* There is no history more contentious or restless than the history of objectivity. By the same logic, there is no epistemological program more dynamic than the epistemology of immanence.

The history of philosophy in twentieth century France is marked by successive appropriations of the epistemological tradition. In the light of the normative theory of science which I defend in this dissertation, the relationship between epistemology and the history of science is, to quote Bachelard, necessary and strict. Epistemology precedes history. By way of conclusion, I will examine two well known historical appropriations of the epistemological tradition. The first is Louis Althusser's appropriation of Bachelard's concept of the "epistemological rupture" in order to demonstrate the historical emergence, in the political economy of Marx, of a science of history: historical materialism. The second is Michel Foucault's presentation of the work of his friend and teacher Georges Canguilhem.

In each case epistemology is invoked in order to define the relationship between philosophy and the history of science. For Althusser and Foucault, the determination of this relationship serves wildly different objectives. My analysis will determine the extent to which Althusser and Foucault recognize the normative force animating the epistemological tradition.

The strict identification of the theory of knowledge with the philosophy of science has led some commentators to declare that the philosophy of science itself is not one philosophical specialty among others, but the bedrock of philosophical activity. Louis Althusser summarized this view for the participants of his seminar at the *École normal supérieure* in 1964.

> Regarding the philosophy of science, it arises at the origins of philosophy: from Plato to Husserl and Lenin (in *Materialism and Empirio-Criticism*), by way of Cartesian philosophy, eighteenth-century rationalist philosophy,Kant, Hegel and Marx, the philosophy of science is much more than one part of philosophy among others: it is philosophy's *essential* part, to the extent that, at least since Descartes, science, the existing sciences (mathematics with Descartes, then physics in the eighteenth century, then biology and history in the nineteenth, and since then mathematics, physics, mathematical-logic and history) serve as a *guide* and a *model* for every philosophical reflection.[454]

Althusser proceeds to develop a theory of the necessary unity of epistemology and historical reflection in order to serve the requirements of a Marxist science of history, *historical materialism*. For Althusser, the significance of epistemology as the "*guide* and *model* for every philosophical reflection" is its diagnostic capacity:

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[454] Althusser's text serves as the introduction to Pierre Macherey's presentation of the philosophy of science in Georges Canguilhem. See Pierre Macherey, "Georges Canguilhem's Philosophy of Science: Epistemology and History of Science," in *In a Materialist Way: Selected Essays,* Ed. Warren Montag, Trans. Ted Stolze. (New York: Verso, 1998,) 161-162.

epistemological critique allows the philosopher to distinguish *science* from *ideology*. It should not come as a surprise, he informs his audience, that the two are often conflated, even in the history of science. The resources of epistemology allowed Althusser to characterize the theoretical contribution of Marx as having achieved an “epistemological rupture” in the writing of *Capital* which allowed historical materialism to transform itself into a genuine science of political economy and of economic history.

Althusser’s engagement with epistemology is not limited to the rehabilitation of Marxist political economy as the science of history. Epistemology is not only a useful historical tool, capable of identifying crucial structural transformations in the history of discourses; it obliges us to revise and complicate the notion of history itself.

> For some years, under the effect of a specific theoretical conjecture… the old conception as much of the history of science as of the philosophy of science (Epistemology) has been called into question. Some *new* paths have been opened, in epistemology by Cavaillès, Bachelard and Vuillemin, and in the History of Science by Canguilhem and Foucault.[455]

Althusser here presents the epistemology and history of science as two quite distinct enterprises. They represent two different “theoretical novelties.” The new epistemologists (although Cavaillès and Bachelard were active as early as the nineteen twenties) are almost ethnographers of science in action, they are recognizable by their “scrupulous respect for the reality of real science.”[456] The new historians of science are equally engaged by the “reality of real history.” They have

[455] Montag 1998, 162-163.
[456] Montag 1998, 163.

discovered "that in history *things no longer happen as one used to believe.*"[457] It is certainly striking, Althusser informs his audience, that the "reality" of scientific labor and of historical processes have been effaced until now. Therefore, one of the most valuable results of the epistemological and allied histories of science is to have disclosed the ideological mechanisms that accompany science at every moment of its conceptual production.

A number of results follow from this. First, the necessity of distinguishing the reality of science from the spontaneous interpretation of society and of scientists themselves. Here it must be noted that the philosophy of science is no help except as a treasury of ideological obfuscations. Science is certainly not the spontaneous manifestation of the truth, nor is the labor of the scientist a simple excavation of reality or a series of revelations. Here the ideology of science participates fully in the revelatory power of religion. Instead, beneath this harmony of appearances, the epistemologist uncovers a complex system of self-regulating concepts, the correction of experience, the construction of norms, and the elimination of extraneous variables. There is an entire *life of the concept* quite distinct from the personality of the researcher or from the biography of the great scientist. Second, this life of the scientific concept has its own history. "The real becoming" of science is the history of an organic but invisible "totality of theory-concepts-methods" which runs beneath the visible history of science as an accomplished system of facts entirely congruent with reality. The "genuine history of science" is the mutation of

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[457] Montag 1998, 163.

the concepts which run beneath it. This history is rife with rupture, paradox, and philosophical novelty. Above all, it forces the historian to renounce "the old idealist schema of a *continuous* mechanistic…or dialectical…progress, without breaks, paradoxes, set backs or leaps forward."[458]

Althusser wishes to caution the reader of the new epistemologists and historians of science from mistaking the wild tumult of conceptual reorganization at play beneath the stable configuration of the established sciences as endorsing any kind of irrationalism. He writes: "In truth, this new epistemology and the new history of science that is its basis are the scientific form of a truly *rational* conception of their object."[459] The revitalization of epistemology and history does not lead to a distortion or caricature of the sciences in question; quite to the contrary, perhaps for the first time in the history of philosophy, science appears illuminated by its own rationality. The history of science is the foundation of the new epistemology because it is the re-examination of the historical record which discloses *the real history of the conceptual transformation*. Althusser's version of the epistemology and history of science is therefore attuned to this discovery of the real nature of the history of science: this history is marked by discontinuity, rupture and revolution. It only presents a unified narrative of progress through more or less self-conscious acts of retroactive consolidation. Althusser thus introduces a high degree of historiographic self consciousness into the history of science which distinguishes it from the ideological obfuscations of history unaided by epistemological critique.

[458] Montag 1998, 164.
[459] Montag 1998, 164.

Althusser's institutional location and spectacular visibility at the *École normal* allowed an extremely productive network of epistemological research to coalesce around his vision of epistemology and the history of science. He is unquestionably one of the sources, alongside his colleague Georges Canguilhem at the *École normal*, of a brief but influential fluorescence of epistemology in the nineteen sixties, at the very heart of academic philosophy no less.[460] Althusser persuasively synthesizes a complicated history of epistemological research in France which extends well into the nineteenth century, beginning with Comte, and which achieves a kind of apotheosis in the distinguished career of Georges Canguilhem. That is to say, Althusser's narration of the epistemological tradition is itself an act of intellectual appropriation designed to serve the interests of a philosophical project very much of its time: the "theoretical anti-humanism" of the mid sixties as exemplified by the trio of Althusser as the representative of a Marxist philosophy of history, Jacques Lacan as the representative of a psychoanalysis of mathematically structured relations in the unconscious, and Michel Foucault as the archaeologist of the human sciences.[461] This deliberately simplified diagram of the

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[460] A number of publications have recently appeared which attempt to chronicle some of the fervor surrounding what might characterized as an "epistemological renaissance" at the ENS in the mid sixties. The *Cahiers pour l'Analyse,* founded by students of Althusser and Jacques Lacan in 1966, was at the center of a lively integration of epistemology, philosophy, literature, psycho-analysis, linguistics and the history of science. A selection of texts from the *Cahiers* and a series of interviews with contributors can be found in Peter Hallward and Knox Peden, *Concept and Form, Volume One, Key Texts from the Cahiers pour L'Analayse* (New York: Verso, 2012.)

[461] For a discussion of the relationship between the epistemological tradition and the Marxist and Psychoanalytic traditions in France during the sixties and early

philosophical "moment" of the sixties in France is here deployed strictly as a diagnostic and heuristic device designed to assess the strategic significance of Althusser's appropriation of "epistemology" as a critical historical methodology.[462]

In short, Althusser unambiguously subordinates epistemology to the history of science: "In truth, this new epistemology and the new history of science that is its basis are the scientific form of a truly *rational* conception of their object." At the heart of Althusser's engagement with the epistemological tradition, so I claim, one will invariably discover a very precise *philosophy of history.*[463] This philosophy of history is above all one which seeks to establish the theoretical possibility of Marxism as the *science of history.* Althusser finds in the epistemological tradition,

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seventies see Tom Eyers, *Post-Rationalism, Psychoanalysis, Epistemology, and Marxism in Post-War France* (New York: Bloomsbury, 2013.) Eyers provides an insightful account of the theoretical re-organization of psychoanalysis in the wake of Althusser's appropriation of the categories of epistemological critique.

[462] Frédéric Worms develops the concept of the philosophical "moment" as a device which captures the emergence of coherent philosophical problems in the history of philosophy. See Frédéric Worms, *La philosophie en France au xxe siècle. Moments.* (Saint-Amand: Éditions Gallimard, 2009.) On the epistemological "moment" as defined by the rupture between Bachelard and Bergson see pp. 339-355. On the constellation of philosophical "moments" which define the conceptual upheavals of the sixties in France see Patrice Maniglier, Ed., *Le moment philosophique des années 1960 en France* (Paris: Presses Universitaires de France, 2011.) On the epistemological moment see David Rabouin, "Structuralisme et comparatisme en sciences humaines et en mathématiques: un malentendu?" pp. 37- 57, and Alberto Gualandi, "La renaissance des philosophies de la nature et le question de l'humain," pp. 59 – 72. On Althusser see Andrea Cavazzini, "La pratique d'Althusser: d'un marxisme à l'autre," pp. 237-253.

[463] On the relationship of Althusser's epistemologically inflected reading of Marx to other Marxist philosophies of history see Alfred Schmidt, *History and Structure: An Essay on Hegelian-Marxist and Structuralist Theories of History,* Trans. Jeffrey Herf, (Cambridge: MIT Press, 1981.) For a critique of historicism in twentieth century French philosophy and of Althusser's incorporation of Bachelard's model of the history of science see Robert Young, *White Mythologies: Writing History and the West* (New York: Routledge, 1990), 22-27 and 48-68.

especially in Gaston Bachelard's conception of the "epistemological rupture," a methodological postulate which functions as a criterion of demarcation between the "pre-scientific" work of the young Marx, and the "scientific analysis" of the mature Marx, especially in *Capital.*[464] The science at stake for Althusser is Historical Materialism.[465] This concept of a *science* of history, so I claim, profoundly distorts the *normative content governing the structure of the natural sciences* according to the epistemological tradition. *History,* as such, is not and cannot be a science for the epistemological tradition. Althusser's deployment of the epistemological tradition thus willfully contradicts the *concept of science* developed by this very tradition. To paraphrase Cavaillès, the rational sequence of the sciences cannot be treated brusquely; to do so is to undermine that which is resolutely objective in science: the autonomy of the concept and the genetic process by which the scientific concept transforms itself historically.

Although Althusser situates his own project at the very heart of this historical recuperation of a genuinely rational history of science, he cannot help but fatally compromise his own ambition by destroying the normative force at the core of the epistemological tradition: *science is normative because it is an epistemology of immanence; the only object of science is science itself.* What is missing from Althusser's appropriation of epistemology as a critical history of science is,

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[464] On Althusser's appropriation of the concept of the "epistemological rupture" from Bachelard see Étienne Balibar, "From Bachelard to Althusser: the concept of 'epistemological break,'" *Economy and Society,* vol. 7 no. 3 (1978): 207-237.
[465] See Louis Althusser and Étienne Balibar, *Reading Capital*, Trans. Ben Brewster, (New York: Verso, 1970.)

precisely, the normative formulation of the epistemological structure of science. This is not only a technical oversight; it is the liquidation of the conceptual integrity of the tradition. Althusser invokes epistemology in order to dissolve its normative force. He does not pay heed to the exceedingly strict methodological protocols which regulate the relationship of epistemology to the history of science. These protocols can be summarized in the three philosophical theses which, I have argued, express the constitutive normative claims of the epistemological tradition. *Thesis One: The Immanent Rationality of Science*. Science is governed by a strictly immanent epistemological norm – all philosophies of consciousness, experience, or social and historical determination distort this immanent normativity and cannot serve as the foundation of a truly rigorous theory of science. *Thesis Two: The Procedural Nature of Science.* Science is defined by the process of its internal transformations rather than its given state at any historical moment. Its characteristic epistemological features only become legible through the analysis of conceptual transformations. *Thesis Three: The Primacy of Epistemology over History.* The epistemological critique of science is the condition of possibility of a new concept of the history of science. There is undoubtedly a complex relationship between the history of science and the epistemology of science, but epistemological analysis precedes historical reconstruction. The history of science is an effect of epistemological reflection.

Michel Foucault, in very different circumstances and according to very different theoretical motives, also attempts to isolate the use value of the history of science for philosophy in the work of his friend and teacher, Georges Canguilhem. In the introduction to the English translation of Canguilhem's *The Normal and the Pathological* (1991), Foucault proposes an alternative intellectual genealogy to the dominant phenomenological tradition in post war France. Foucault writes:

> Without ignoring the cleavages which, during these last years after the end of the war, were able to oppose Marxists and non-Marxists, Freudians and non-Freudians, specialists in a single discipline and philosophers, academics and non-academics, theorists and politicians, it does seem to me that one could find another dividing line which cuts through all these oppositions. It is the line that separates a philosophy of experience, of sense and of subject and a philosophy of knowledge, of rationality and of concept. On the one hand, one network is that of Sartre and Merleau-Ponty; and then another is that of Cavaillès, Bachelard and Canguilhem. In other words, we are dealing with two modalities according to which phenomenology was taken up in France, when quite late – around 1930 – it finally began to be, if not known, at least recognized.[466]

Foucault's diagnosis of a profound ambivalence at the heart of French philosophy in the twentieth century is here identified with the forces determining the reception history of phenomenology. The opposition of the "philosophers of the concept" and the "philosophers of experience" paraphrases the polemical orientation of Jean Cavaillès in his *Logic and Theory of Science.* The historical origins of this opposition, as Foucault notes, can easily be extended well into the nineteenth century, as witnessed by the ever shifting boundaries of the debate between positivist and "intellectualist" philosophers (Brunschvicg being the paradigmatic intellectualist philosopher of the late nineteenth and early twentieth century), on the one hand, and spiritualist and intuitive philosophers, on the other.

[466] Georges Canguilhem, *The Normal and the Pathological* Trans. Carolyn R. Fawcett with Robert S. Cohen (New York: Zone Books, 1991), 8.

The debate between Bachelard and Henri Bergson in the twenties and thirties (of which the Meyerson/Bachelard encounter is a variant) is occasioned by the same epistemological controversies which opposed the readings of Husserl given by Sartre and Cavaillès at about the same time. At the heart of the matter is the relation of the subject to science. For Bachelard and Cavaillès, subjectivity is a remainder which must be reduced in order to clearly perceive the autonomy of science. For Sartre and Husserl, the subject is a transcendental formation which hovers over the objective formulations of reason and colors the history of the mind with its implacable immanence. Sartre's *Transcendence of the Ego* (1935) and Cavaillès' two doctoral theses (1938), as Foucault reminds us, interrogate two very different aspects of Husserl's phenomenology. The former seeks to generalize the transcendental foundations of the phenomenological reduction, while the latter seeks to demonstrate the illegitimacy of Husserl's invocation of logic as the foundation of mathematics. Sartre will develop a philosophy of abyssal freedom out of the resources of phenomenology, while Cavaillès will reduce phenomenology to the narcissism of a tautological subjectivity.

For Foucault, in the theoretical and social turbulence of the sixties, as the disciplinary regimes of philosophy and the human sciences seem on the verge of fracturing into a thousand formalisms, it is not the phenomenological philosophies of experience which seem capable of diagnosing the fate of reason, but the epistemological philosophers of the history of science. Foucault connects the appeal of the epistemological tradition for the various structuralisms of the sixties to an older problematic represented by the philosophical significance of the history of science. The historicity of science, as revealed by the epistemologists, must be related to the history of the crisis of reason which the philosophers of the eighteenth century posed as the question of enlightenment. For Foucault, then, the epistemological tradition is a variation on enlightenment themes: the determination of "the autonomy and sovereignty" of reason in the western philosophical tradition and the discovery of a "historical and critical dimension" of reason opened up by the historical challenges and catastrophic upheavals of this "sovereign reason."

In France and Germany, Foucault informs us, the philosophical problem of the enlightenment (here understood as the traumatizing encounter with the historicity of reason) is expressed very differently. In German philosophy it is the reformation which acts as a center of philosophical gravity, organizing a constellation" of social and political reflections on society" which give a certain continuity to the critical projects of Hegel, Marx and the Frankfurt School. In France, Foucault claims, the history of science is the vehicle through which the problems of the philosophy of the enlightenment manifest themselves. Therefore, in a strange conjugation, Foucault links the epistemological tradition in France to the Frankfurt school in Germany. Despite the differences in style and content, both traditions encounter the problem of

> a rationality which makes universal claims while developing in contingency; which asserts its unity and yet proceeds only by means of partial modification when not by general recastings; which authenticates itself through its own sovereignty but which in its history is perhaps not dissociated from inertias, weights which coerce it, subjugate it. In the history of science in France as in German critical theory, what we are to examine

> essentially is a reason whose autonomy of structures carries with itself the history of dogmatisms and despotisms – a reason which, consequently, has the effect of emancipation only on the condition that it succeeds in freeing itself of itself.[467]

There is much to be admired in Foucault's thematization of the epistemological tradition in this seemingly strange alliance with the Frankfurt school. Foucault's identification of the dissolution of the contradiction of a unitary reason and a contingent historical sequence seems particularly apt. Rather than simply opposing one another, the historical and the rational dimensions of reason disclose a hidden mechanism of reason itself – the emancipation of the rational norm *from itself* across the successive acts of its transformations.

Georges Canguilhem, as an especially prominent and celebrated historian of science in France, is thereby a privileged inheritor of this philosophical problematic by which reason is recurrently emancipated from the despotic formations of its own history. Canguilhem distinguishes himself in an already distinguished tradition by a surprising tactic. Unlike Bachelard and Cavaillès, his colleagues in this "emancipation of the rational norm," he does not concern himself with the aggressive formalization of knowledge which lends such prestige and authority to the history of mathematics and physics. Canguilhem is a historian and philosopher of the life sciences and introduces a change of perspective into the history of science which focuses on "the middle regions where knowledge is much less deductive" than the celestial intricacies of relativity theory. Nevertheless, the sciences of life are no less complex than the theory of relativity. Part of Canguilhem's significance is to

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[467] Canguilhem 1991, 12.

have so clearly demonstrated "that the theoretical importance of the problems raised by the development of a science are not perforce in direct proportion to the degree of formalization reached by it."[468]

Canguilhem reorganizes the procedures of the history of science using the tools and methods developed by colleagues like Bachelard and Alexandre Koyré (1892-1964). Foucault claims that the recognition of discontinuities and ruptures in the history of science do not have the same revelatory force for Canguilhem as they might have had for Bachelard. This is because Canguilhem has fully internalized the reality of the history of science as discontinuous, whereas Bachelard had to arrive at this conclusion. Therefore, for Canguilhem

> marking discontinuities is neither a postulate nor a result, but rather a "way of doing," a process which is an integral part of the history of science because it is summoned by the very object which must be treated by it. In fact, this history of science is not a history of the true, of its slow epiphany; it would not be able to claim that it recounts the progressive discovery of a truth "inscribed forever in things or in the intellect," except to imagine that contemporary knowledge finally possesses it so completely and definitively that it can start from it to measure the past.[469]

The history of science is not the history of truth gradually revealing itself. If that were the case, history would be a purely quantitative variable: at given moments a greater or lesser amount of the secret order of existence would be available to the scientist, the only difference between historical moments being that of a sum which changes in size without altering its nature. On the contrary, the relationship which differently constituted moments in the history of science bear towards one another

[468] Canguilhem 1991, 13.
[469] Canguilhem 1991, 14.

is what Foucault, paraphrasing Canguilhem, calls a differential effect of "speaking truth," an always reconstituted, re-articulated and self correcting way of establishing the possibility of truth-hood and false-hood in the history of science. In order for science to "speak the truth," it must work through its own previous articulations, not in order to extend them, but to suspend them and to articulate, in turn, an entirely unprecedented elaboration of the possible forms of reality.

This relationship to truth as an emancipation from previous despotic formulations is, in turn, a new despotism. The emancipatory and despotic forms of knowledge are not intrinsic to that which is "spoken of as true" at a given historical moment; they are purely structural dispositions: the emancipatory is that which displaces the despotic; the despotic is that which follows the emancipatory. The epistemological significance of error comes to light in the alternating illumination of this strange but necessary modification of epistemological norms. Foucault writes:

> Error is not eliminated by the muffled force of a truth which gradually emerges from the shadow but by the formation of a new way of "speaking true." One of the conditions of possibility because of which a history of science was formed in at the beginning of the eighteenth century was, as Canguilhem notes, the awareness that there had been recent scientific "revolutions:" that of algebraic geometry and the infinitesimal calculus, or Copernican and Newtonian cosmology.[470]

Error is not displaced by that which contradicts it; it is taken up by a movement of rectification which does not meet it in the symmetry of an inverted reflection, but which locates it in a system of rational relations by which it becomes, as Bachelard might say, not "that which I thought in error," but "that which I ought to have

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[470] Canguilhem 1991, 15.

thought." The epistemological complement of error is not the truth symmetrical to it in its own time, but the truth which recasts it in the light of a newly formulated obligation of the truth: a new epistemological norm. This is the retro-active consolidation of the history of science by the provisional oversight of a contemporary epistemological norm. If something like the history of science first becomes possible at the beginning of the eighteenth century, this is for reasons which are themselves of interest to the history of science: antecedent reformulations of the epistemological structure of mathematics and cosmology in the seventeenth century.

For Canguilhem, the history of science emerges as a conceptually distinct enterprise with its own unique theoretical content by opposing the normative constraints of the scientist to the normative constraints of the historian. The history of science *is not* the normativity of the historian applied to science. The history of science comes into being by taking into account:

> the epistemological point of view between the pure historian and the scientist himself. This point of view is that which causes a "hidden, ordered progression" to appear through different episodes of scientific knowledge: this means that the processes of elimination and selection of statements, theories, objects are made at each instant in terms of a certain norm; and this norm cannot be identified with a theoretical structure or an actual paradigm because today's scientific truth is itself only an episode of it – let us say provisional at most.[471]

The hidden progression of the concepts and objects of science is ordered by the recurrent structure of an epistemological norm which does not cease, in turn, to modify itself. As Foucault notes, the epistemological structure of the norm at any

[471] Canguilhem 1991, 16.

given moment cannot be identified with Thomas Kuhn's "paradigmatic science" because the differential pair of historical epistemology is not that of "paradigmatic science" and "revolutionary science," but of that of "error" and "rectified error" or better, "that which I thought" and "that which I ought to have thought." The normative force of epistemology does not disclose the simultaneous incommensurability of worlds, as in Kuhn's paradigm shifts; it discloses a temporality uniquely inflected with a uniform obligatory tone– one which displaces the errors of the past into the possible articulations of a new valence of the true, or that which *it will have been necessary to think.*

> This history of science, says Canguilhem quoting Suzanne Bachelard, can construct its object only "in an ideal space-time." And this space-time is given to the history of science neither by the "realist" time accumulated by the historian's erudition nor by the idealized space authoritatively cut out by today's science, but by the point of view of epistemology. The latter is not the general theory of all science or of every possible scientific statement; it is the search for normativity within different scientific activities, such that they have effectively been brought into play. Hence we are dealing with an indispensable theoretical reflection which a history of science can form for itself in a way different from history in general; and conversely, the history of science opens up the area for analysis which is indispensable in order for epistemology to be something other than the simple reproduction of schemes within a science at a given moment. In the method used by Canguilhem, the elaboration of "discontinuist" analyses and the elucidation of the history of science/epistemology relation go hand in hand.[472]

The "discontinuist" analyses of Georges Canguilhem follow the movements of an elusive normativity which is not simply a model of science in the form of a confirmed and stable relation to the truth. What is most normative in science is the *emergence of the norm*, not the constituted relation between variables in a system

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[472] Canguilhem 1991, 16-17.

which fabricates the truth, but the reformulation of the structure by which it is possible to elaborate the true. The relation of epistemology to the history of science is not that of the model and the copy; one does not scour the past in search of analogues or variations on the contemporary themes of scientific rationality. One searches for that which is always normative in the history of science: the configuration by which the scientific concept is able to resist all that would speak in its place. This is by no means a simple task; simply insisting on the immanence or the autonomy of the theory of science only discloses the formidable problem which Meyerson, Brunschvicg, Bachelard, Cavaillès and Canguilhem all face, that of distinguishing *within the very normativity of science* another thematization of the norm which must escape it and stand against it, not as a contradictory affirmation of the truth, but as the emancipation from error and the beginning of what "one ought to have thought."

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