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Essays in Macroeconomics

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Essays in Macroeconomics

By

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M.A., Emory University, 2015

Advisor: Tao Zha, Ph.D.

An abstract of
A dissertation submitted to the Faculty of the
James T. Laney School of Graduate Studies of Emory University
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Abstract

Essays in Macroeconomics
By Tong Xu

This dissertation contains three chapters. The first chapter estimates three widely-used DSGE models for the Chinese economy. The structural estimation methodology allows us to analyze the business cycle behaviors for the Chinese economy. We find that (1) the standard Taylor rule as the monetary policy does not fit well with the Chinese data; (2) the spillovers between housing market and the real economy are more likely to work through the collateral constraints on the entrepreneur side rather than on the household side. The second chapter studies the optimal monetary policy in China in a unified theoretical and empirical framework. We build a simple New Keynesian model with asymmetric loss function for Central Bank, adaptive learning expectation, and targeted growth rate to derive the optimal monetary policy for China consistent with the empirical findings. Different from traditional stabilizing Taylor rule, the derived optimal monetary policy for China has a pro-growth feature and is endogenously regime-switching. We estimate the structural model with Chinese data and make inference on the preference of Central Bank from the estimation results. Also, we conduct several forecast exercises and find that the money growth rate has to grow in a relatively low speed in the long run to achieve a reasonable forecast for the output growth rate. The third chapter develops a model of financial intermediation in which the dynamic interaction between regulator supervision and banks' loophole innovation generates credit cycles. Our model generates pro-cyclical bank leverage and asymmetric credit cycles. We show that a crisis is more likely to occur and the consequences are more severe after a longer boom. In addition, we investigate the welfare implications of a maximum leverage ratio in the environment of loophole innovation.

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Contents

Tables

Figures

Chapter 1

Introduction

1.1 Monetary Policy and Housing Market in China

The recent rapid and stable growth in the Chinese economy for the last several decades has been impressive around the world. How to explain this phenomenon is important not only for understanding the success in the past for China but also for forecasting the future economy development. During the last few years, there emerge more and more literature using quantitive model to explain some facts in the Chinese economy [14, 72]. However, most papers focus on the problem of resources allocation during the economy transition process. They provide valuable insights on the low-frequency changes or the growth trend for China, but they omit the high-frequency fluctuations or the cycles. On the one hand, the cycles contain information about the mechanisms in the economy. On the other hand, the cycle parts will become more and more important when the economy approaches its long-run steady state. In this chapter, we will focus on explaining the high-frequency fluctuations in the Chinese economy, and the new available quarterly data for the Chinese economy makes this task feasible.

A new generation of medium-scale dynamic stochastic general equilibrium (DSGE) models have been widely used in the macroeconomic analysis. Those models contain many frictions and shocks and try to explain the dynamics in the economy. From the methodology perspective, many of those models adopt the Bayesian estimation methodology to identify key parameters and shock processes. Different from Vector Autoregression (VAR) analysis, which mainly captures the dynamics and interactions among the data series, the micro-founded structural models provide more information on the mechanisms and are more suitable for policy analysis. Different from structural models with parameters calibrated or estimated through method of moments, this methodology does not only rely on certain moments of data but takes advantage of the whole distribution information. Also, as the computation technology advances, the computation burden for Bayesian estimation has become less a issue.

This chapter focuses on two issues of the Chinese economy. The first one is the monetary policy rule in China. Monetary policy has been widely used across the world as a tool to affect the economy by Central Banks. How the Central Bank in China controls money supply and the effects of monetary policy on real economy are important not only for research but also for policy making. The second one is the housing market. The recent booming in the housing market in China has attracted a lot of attention from academia and policy makers. There are many debates on the effects of roaring housing price on real economy and the potential systemic risk associated with housing market. In this chapter, we combine structural models with newly available data to address those two important questions in China.

In this chapter, we apply three mainstreams DSGE models to the Chinese economy. First, we estimate Smets and Wouters [71]'s model using quarterly data over the period 1996Q1–2016Q4. This model is the backbone New Keynesian model with widely accepted real frictions and nominal stickiness. Since this model does not contain specific complicate mechanism, it serves very well as the starting point for investigating the main relationship among key variables such as consumption, investment, and inflation. In the estimation process, we find that original model provides very strange results related to the capital share and consumption and investment have different trends over the sample period, so we use variants from the original Smets and Wouters [71]'s model to address those issues in the estimation process. The nominal stickiness provides a role for the monetary policy and allows us to study the monetary policy in China. From the estimation results, we find the estimated monetary policy in China has a small coefficient on inflation, and its coefficients related to output are very close to zero. This is different from the widely-used Taylor rule in U.S. with large coefficients on both inflation and output, which captures the role of the Federal Bank System as a stabilization force in the economy. This difference raises the question whether the standard Taylor rule works well as a simple way to characterize the monetary policy rule in China, because the Central Bank in China may have a different preference [18].

The recent boom in the housing market in China has drawn a lot of attention for research. To investigate the housing market, we adopt two DSGE models with housing in the economy. We estimate Iacoviello and Neri [42]'s model using quarterly data over the period 2004Q2–2016Q2. This model includes both real sector and housing sector and studies the spillovers between the two sectors. The main mechanism to generate the spillover from the housing sector to the real economy comes from the borrowing constraint on the part of some households. Both housing production technology and housing demand preference generate the endogenous dynamics in residential investment and housing prices. Fluctuations in house prices affect the borrowing constraint of a fraction of households and the relative profitability of producing new homes, which generates feedback effects for the expenditure of households and firms. The estimation results show that both housing production technology and housing demand preference shocks can explain almost all the fluctuations in the residential investment and housing prices. However, those two shocks cannot generate enough fluctuations for other variables in the model. This finding is very difficult to concile with the fact that both investment and housing market have experienced rapid growth during the last decade in China, so it implies that the spillovers between housing market and real economy may not work through the borrowing constraint on the households' side.

The other DSGE model with housing is Liu et al. [49]. We estimate their model using quarterly data over the period 2004Q2–2016Q2. This model builds the link between housing price and investment through the borrowing constraint on the entrepreneurs' side. An increase in the housing demand from the households triggers the competing demand for the land/housing between households and entrepreneurs and starts a financial spiral that drives the co-movement between land/housing price and investment. The estimation results show that investment technology shocks can explain land price fluctuations in the long run, and housing demand shocks have sizable effects on investment. So this model can generate sufficient spillovers between the housing market and the real economy. This result provides some evidence that it is more likely to explain the rapid growth both in investment and housing market through the mechanism that entrepreneurs are borrowing constrained by their land value. However, this model builds on real business cycle models and does not have nominal rigidities, so we cannot use it to study inflation and monetary policy.

In the chapter 2, we discuss the related literature in Section 2.1. In Section 2.2, we discuss data and methodology in this chapter. In Section 2.3, we estimate Smets and Wouters [71]'s model and discuss the results. In Section 2.4, we estimate Iacoviello and Neri [42]'s model and discuss the results. In Section 2.5, we estimate Liu et al. [49]'s model and discuss the results. In Section 2.6, we summarize the estimation results and generally discuss the issues related to applying these models to the Chinese economy.

1.2 Optimal Monetary Policy in China

Monetary policy plays an important role in the economy. Taylor rule has been widely used in the literature to model the monetary policy rule in advanced countries since Taylor [76]. The micro-foundation for Taylor rule comes from the stabilization role of monetary policy. However, for an emerging economy like China, monetary policy may serve other roles than only stabilizing one. Especially considering the huge political and economic institutional differences between China and advanced economy such as U.S., we should not expect the monetary policy in China plays the same role and follows similar rules. One recent empirical finding provides evidence on the pro-growth role for the monetary policy in China [18]. This motivates this chapter to investigate the monetary policy in a unified theoretical and empirical framework.[1]

The first contribution of this chapter is that we derive the optimal monetary policy from Central Bank's asymmetric loss function. To obtain a consistent form with [18]'s empirical finding, we assume a specific loss function for the Central Bank, adaptive learning expectations in the economy, and revenue tax for firms. The specific loss function for the Central Bank help us capture the asymmetric trade-off in the monetary policy. Adaptive learning expectations help us avoid complicated endogenous regime-switching model and are crucial in the identification strategy for the estimation. Revenue tax for firms can introduce the targeted GDP growth rate into the monetary policy. Targeted GDP growth rate is one feature for China. Chinese government announces annual GDP growth rate target each year, and this target plays an important role in the monetary policy.

Second, we estimate the whole structure model including the monetary policy. Even though the monetary policy is endogenously regime-switching, we are able to estimate the whole model. With the estimation results, we can identify the weight coefficients in the loss function for the Central Bank. Our inference for preference of the Central Bank in China is a novel approach, and it comes from the unified framework between theoretical model and empirical estimation in this chapter. Even though [18] provide solid empirical results for the monetary policy, lack of structural model prevents them from conducting inference on the preference of the Central Bank.

Third, we conduct several conditional forecast analyses based on our estimated structural model. Due to the endogenous regime-switching property for the monetary policy, we cannot use normal forecast method for linear dynamic models. Instead, we adopt simulation method and update regime each period to conduct the forecasts. The recent decline in the GDP growth rate in China has received many discussions, and it is widely believed that this will become the "new normal" state for the Chinese economy. In the forecast exercises, we find that in order to achieve a reasonable forecast for the output growth rate we need the money growth rate to keep a relatively low speed in the long run. Otherwise, the output growth rate reverts back to a high level.

[1] This chapter is adapted from my joint work with Kaiji Chen and Tao Zha, "Optimal Monetary Policy in China".

This chapter is linked to a strand of literature about Taylor rule and optimal monetary policy. Taylor [76] proposes the Taylor rule and is the first to link the monetary policy with inflation and output. A series of works including Rotemberg and Woodford [66], Woodford [80, 81] derive the Taylor rule from the stabilizing role for the Central Bank. However, monetary policy rule in their papers focuses on the stabilization, while the monetary policy in our chapter includes more roles. Rudebusch and Svensson [67] use estimated model and loss function to compare among different monetary policy rules. Different from their paper, we use the estimated monetary policy rule to infer the preference for the Central Bank rather than compare the performances of different monetary policy rules.

Our chapter is also linked to another strand of literature on the monetary policy in China. This chapter is closely related to Chen et al. [18]. On the one hand, they adopt VAR framework, but our chapter adopts the structural DSGE approach. On the other hand, they estimate the monetary policy rule and study the effect of monetary policy on investment through the bank lending channel, while our chapter investigates the micro-foundation for the monetary policy in China and makes inference on the Central Bank's preference. Chang et al. [15] have monetary policy in their structural DSGE model. However, their focus is on the interaction between capital control and monetary policy, and their monetary policy is in the Ramsey sense rather than a policy rule.

Another literature related to this chapter is conditional forecast. Waggoner and Zha [79] propose a method to conduct conditional forecast for Bayesian VAR. Maih [50] discusses how to conduct conditional forecast for Bayesian estimated DSGE model. Higgins et al. [40] apply conditional forecast to the Chinese economy. Our chapter also conducts several conditional forecast analyses. However, due to the non-linearity of the monetary policy, we cannot adopt traditional conditional forecast suitable for linear models, so we rely on the simulation method to provide the forecast results.

The structure for Chapter 3 is as follows. In Section 3.1, we derive the optimal monetary policy with asymmetric Central Bank's loss function and the structural model. In Section 3.2, we estimate the monetary policy and the rest of the structural model and infer the Central Bank's preference with the estimated parameters. In Section 3.3, we conduct several conditional forecasts to study the future economy performance under different scenarios.

1.3 Loophole Innovation and Supervision

Banks and other financial intermediaries play a prominent role in the economy by channeling funds from savers to borrowers. In the wake of the recent financial crisis, there is a surge in research aimed at understanding the relationship between financial intermediaries, financial instability and macroeconomic fluctuations. In this chapter, we build a dynamic model of financial intermediation that emphasizes the interaction between the regulator and banks. We show that banks' moral hazard can endogenously lead to financial instability, and generate boom-bust credit cycles. In particular, the longer the boom, the more likely there will be a crisis and the more severe the consequences will be, which corresponds to Minsky [56]'s hypothesis that good times sow the seeds for the next financial crisis. Moreover, the model's predictions reconcile well with some empirical facts related to credit cycles. For instance, our model predicts that banks' leverage is pro-cyclical, consistent with the findings in Adrian and Shin [3, 4]. The model also generates asymmetric credit cycles, i.e., long periods of credit booms followed by sudden and sharp busts, while recovery is slow and gradual, as documented in Reinhart and Reinhart [64].[2]

The key element of our chapter is banks' risk-shifting problem. It is now widely accepted that excessive risk-taking by banks contributed to the financial crisis of 2007-2009. While the causes of excessive risk-taking remain subject to debate, many observers and policymakers believe that bank supervision failure is one of the key contributing factors (Acharya and Richardson [1], Acharya et al. [2], Freixas et al. [33]).[3] Indeed, several countries have made great efforts to improve their supervision of banks in the aftermath of the crisis.

What could explain the failure of bank supervision? Among various factors, financial innovation is mentioned as one key factor that can undermine the effectiveness of the regulator's supervision (see, e.g., Silber [70], Miller [55], Kane [44], Tufano [77]). Undoubtedly, good financial innovations provide numerous benefits to the economy.[4] However, there are also bad financial innovations that create new ways for financial institutions to get around current supervision and take excessive risks, which we refer to as loophole innovations in this chapter. For instance, Stein [73] argues that second-generation securitization, like subprime CDOs, is a

[2] This chapter is adapted from my joint work with Jianxing Wei, "A Model of Bank Credit Cycles".

[3] Other mentioned factors include, for instance, shortcomings in financial institutions' incentive structures and risk management practices, misplaced reliance on credit rating agencies, etc.

[4] For instance, financial innovations help improve risk sharing, complete the market, reduce trade costs, see Beck et al. [7] for an excellent survey of the debate on the "bright" and "dark" sides of financial innovation.

bad financial innovation that evolved in response to flaws in prevailing models and incentive schemes. Another related example is Credit Default Swap (CDS). CDS was widely used to free up regulatory capital in banks' balance sheets prior to the crisis. However, when the risky assets of banks and the insurer are correlated, banks can use CDS to engage in regulatory arbitrage and take excessive risks under the Basel regulatory framework (Yorulmazer [84]). As is illustrated in the recent financial crisis, the regulator was slow to understand the danger of loophole innovations in some instances, facilitating banks' excessive risk-taking. In this chapter, we put a central emphasis on how banks' loophole innovation affects the effectiveness of regulator supervision, and investigate its macroeconomic implications over the credit cycles. To the best of our knowledge, this chapter is the first to explicitly model the dynamic interaction between regulator supervision and banks' loophole innovation.

In the model, banks borrow from depositors in the form of debt to finance their investment opportunities. The investment opportunities could be safe or risky projects. Due to limited liability, banks are subject to a risk-shifting problem. Therefore, banks have incentives to take on inefficient risky projects, in which they enjoy the upside of payoff if projects succeed but depositors bear the loss if projects fail. One solution to this moral hazard problem is market discipline: depositors impose a leverage constraint on banks. If banks have enough "skin in the game", they will behave properly. However, market discipline is costly in the sense that it limits a bank's investment capacity.

Another complementary solution to the moral hazard problem is supervision by the regulator. In this chapter, we formally distinguish regulation from supervision, in terms of the verifiability of bank information and actions, following Eisenbach et al. [26].[5] Through actively monitoring banks' activities, the regulator can promote banks' safety and soundness. The leverage constraint and supervision from the regulator work together to address banks' risk-shifting problem. The better the regulator's supervision ability, the more banks can relax their borrowing constraint. As a result, the size of the banks depends on depositors' beliefs regarding the regulator's supervision ability. When depositors' confidence in the regulator's competence is high, they will permit banks to take high leverage without worrying about the risk-shifting problem. If the regulator's ability is perceived to be low, depositors have to tighten the leverage

[5]Regulation is written into law and enforced through the courts, so it can only be contingent on verifiable information. In contrast, supervision involves the assessment of the safety and soundness of banks through monitoring by the regulator, and corrective actions in response to the assessment. Supervision can be contingent on non-verifiable information.

constraint to make sure banks behave properly.

Even though regulator supervision helps banks to increase their leverage from an ex ante perspective, banks always have incentives to find loopholes to circumvent regulator supervision ex post. In our chapter, we model loophole innovation as discovering a new type of risky project which is not currently supervised by the regulator, thereby providing banks with opportunities to take on risky projects without being monitored by the regulator. This acts as an endogenous opposing force diminishing the regulator's expertise in supervision. When the loophole innovation eventually succeeds, the regulator's supervision becomes less effective. However, the regulator and depositors are not aware of the new loopholes immediately, so banks take on inefficient risky projects, thereby leading to massive defaults and a severe decline in output. After a bust, depositors realize that the regulator's expertise has become obsolete and they lose confidence in the financial system. In response, they constrain banks from taking high leverage to prevent their risk-taking activities, which implies a sharp contraction of the banking sector.

We incorporate regulator supervision and banks' loophole innovation into a dynamic model. We assume that the regulator's expertise in supervising banks regarding previous loopholes gradually improves through a learning-by-doing process. This assumption is supported by some recent studies on how prudential supervision works in practice (see, e.g., Dahlgren [20], Dudley [24], Eisenbach et al. [26]).[6] As the regulator's expertise grows, it has two effects on banks' moral hazard problem. On the one hand, it eases banks' risk-shifting problem related to previous loopholes, which allows banks to take a higher leverage. Therefore, banks have a larger investment size, and the total output goes up. In this way, the economy experiences a boom accompanied with rising leverage in the banking sector. On the other hand, however, banks will also engage in loophole innovation more actively. When supervision is more effective and banks' leverage is higher, the gain from finding a new loophole is larger. Banks' efforts to conduct loophole innovation increase, and thus banks are more likely to discover a new loophole. If the loophole innovation is successful, it provides banks with a new type of risky project which is not supervised by the regulator. There is a crisis in the economy.

Our main result of the dynamic model is that the interaction between regulator supervision

[6] For instance, according to Eisenbach et al. [26], "The current structure and organization of FRBNY FISG supervisory staff dates from a significant reorganization that took place in 2011. That reorganization drew on lessons learned during the financial crisis to reshape the internal structure of the group and the way that staff interacts with one another to enhance communication and facilitate identification of emerging risks through a greater emphasis on cross-firm perspectives. The reorganization was designed to foster enhanced and more frequent engagement between senior supervisory staff and senior managers and members of the board of directors at supervised firms".

and banks' attempts to circumvent supervision can lead to regime changes in banks' moral hazard problem, and generates macroeconomic fluctuations. In our economy, the sources of the economic downturns endogenously come from banks' loophole innovations. Moreover, the longer the boom, the more likely there will be a crisis and the more severe the consequences will be. This is because as banks' leverage rises in boom periods, they have stronger incentives to find new loopholes. Furthermore, the business cycles are asymmetric in our economy: periods of gradual expansions in banks' leverage, investment, and aggregate output are followed by sudden and sharp contractions, and then the economy starts the gradual growth again. This result arises from the asymmetric nature of loophole innovation. Although the regulator takes time to gradually improve its supervision ability through a learning-by-doing process, its expertise can be severely undermined the moment that new loopholes are discovered.

The 2007-2009 financial crisis is a good example to illustrate our mechanism. Before the crisis, banks discovered vulnerabilities in the rules of regulation and supervision, and by exploiting these loopholes, they took excessive risks. When the massive failures occurred and the crisis unfolded, regulators and investors realized that there had been so many cracks in the financial system. As Timothy F. Geithner[34] recognized, "Our regulatory framework was built in a different era for a long extinct form of finance. It long ago fell behind the curve of market developments. Parts of the system were crawling with regulators but parts of the system were without any meaningful oversight. This permitted and even encouraged arbitrage and evasion on an appalling scale." In response to the vulnerabilities in the financial system, investors cut their lending to the banks and there was a sharp deleveraging process in the financial sector.

We also investigate the regulation implications of this model. We consider the regulation with a maximum leverage ratio. The regulator's supervision ability can be seen as the state of the economy. Banks' loophole innovation effort determines the evolution rules for the regulator's supervision ability, which characterizes the stationary distribution of the economy in the long run. We find that under certain conditions the regulator would set a maximum leverage ratio to restrict the upper-bound leverage for the banks. This regulation has two effects. First, it reduces banks' leverage and can potentially decrease output in boom periods. Second, it decreases success probability of loophole innovation. A lower loophole innovation success probability shifts the stationary distribution of the economy towards more favorable states, which improves the average output in the long run. The regulator will trade off these two effects to set the optimal maximum leverage ratio.

The model's empirical implications are broadly consistent with the stylized facts found in many empirical studies. First, Schularick and Taylor [69] study 14 developed countries over 140 years, concluding that a long period of credit growth is the best single predictor of financial crises. Second, Reinhart and Reinhart [64] find that credit cycles are asymmetric: long periods of credit expansion are followed by sudden stops, and then gradual recovery. Third, Adrian and Shin [3, 4] find that financial intermediaries' leverage is pro-cyclical over the business cycles. Fourth, Dell'Ariccia et al. [22] find that during a boom, financial intermediaries' lending standards decrease and loan default rates increase, which is accompanied by massive failures in the financial sector. Our model's results reflect these facts within a unified framework.

This chapter contributes to the existing literature in several ways. First, unlike most regulation and supervision literature which focuses on static models, this chapter studies regulator supervision in a dynamic framework. Second, complementary to a small but growing literature on endogenous business cycles, which focus on non-financial firms, this chapter provides a novel mechanism to generate endogenous credit cycles originated from the financial sector. By analyzing the dynamics of banks' moral hazard problem, this chapter is able to rationalize some of the key features of the credit cycles that are not explained by the existing literature. Third, this chapter provides a new rationale for the maximum leverage ratio when there is an interaction between regulator supervision and banks' loophole innovation. We show that tightening banks' leverage ratio involves a systemic risk and output trade-off, and the regulator can lower the likelihood of systemic crises at the cost of decreasing output in boom periods.

Chapter 4's structure is as follows. Section 4.1 discusses the related literature. Section 4.2 presents the static model for bank risk-shifting, supervision, and loophole innovation. Section 4.3 nests the static model in a dynamic model, analyzing the macroeconomic implications of the interaction between banks' loophole innovation and regulator supervision. Sector 4.4 investigates the welfare implications of the maximum leverage ratio. Section 4.5 adds learning about unknown loopholes and the regulator's investigation choice in the model. Section 4.6 discusses several setups in the model.

Chapter 2

DSGE Models for Chinese Economy

2.1 Literature

This chapter fits into several strands of literature such as quantitive modeling for China, data for Chinese economy, monetary policy, and housing in China. In this section, we will discuss related papers.

Our chapter follows the literature about studying Chinese economy with quantitive models [18, 72]. Song et al. [72] study the China's economic transition process focusing on the resources reallocation between State-Owned Enterprise and private firms. Calibrated to match several data moments in China, their model can explain high economic growth, long-term high returns on capital, and a large trade surplus in China between 1992 and 2007. Chen et al. [18] present empirical facts for Chinese economy and build a theoretical model to explain the increase in investment-to-output ratio and changes in the loan structure for banks. In their model, collateral constraint on capital plays an important role, and they focus on the resources allocation between light and heavy sectors. Both papers use quantitive methods to study the Chinese economy, but our chapter is different from them in two ways. First, different from their calibration methodology, our chapter adopts the Bayesian estimation approach. Rather than matching several moments of Chinese data, we can take advantage of more information from the distribution of the data series, which is feasible recently mainly due to the availability of constructed quarterly Chinese data series by Higgins and Zha [39]. Second, their papers focus more on explaining the trend part for the Chinese economy, while our chapter mainly studies the cycle or fluctuation part. To study the cycle part, we have to include various shocks processes in the model, and our chapter is more suitable for this task.

Different from U.S., data availability has always been an issue for studying the Chinese economy. Usually there are only officially published annual data, which provides very few observations. This limited sample can explain partly why most quantitive research on China employ the calibration approach. However, very recently Higgins and Zha [39] construct and update quarterly and monthly Chinese data seires, which greatly enlarges the sample size and makes estimation approach available for the Chinese economy. Higgins et al. [40] adopt the Vector-autoregression (VAR) approach to study the Chinese economy and provide reasonable unconditional and conditional forecasts for the future Chinese economy. Similar to Higgins et al. [40], our chapter takes advantage of Higgins and Zha [39] to increase the sample size for the data series, but we use the structural DSGE model for estimation so we can understand more structural details for the micro-foundation for the Chinese economy. We will also provide some unconditional and conditional forecasts with our estimated DSGE models.

Our chapter is related to a huge literature on monetary policy. One key element in the New Keynesian framework is the monetary policy rule. Most literature studying U.S. economy follow Taylor [76] to specify the monetary policy rule as the interest rate rule. However, there is still a question whether the Taylor rule works in China since China has a very different political and economic institution from U.S.. China has a officially determined loan rate and deposit rate for a long time, so the interest rate mechanism from the demand channel may not be applied in China. Chen et al. [18] provide empirical evidence that the monetary policy in China mainly works through bank credit channel and there exists regime-switching for Chineses monetary policy. In Chen et al. [17]'s paper, they find the monetary policy can affect the real economy through shadow banks in China. Different from them, our chapter tries to identify the problems of monetary policy rule through investigating the performance of estimated models. Zhang [85] compares the two monetary policy rules in China. Chang et al. [15] studies the linkage between the monetary policy and capital control in China. Chen et al. [19] studies the effects of credit quota for Chinese monetary policy through GMM approach. Different from their research, our chapter uses Bayesian estimation methodology for DSGE model. Dai et al. [21] also apply Bayesian estimation methodology for the Chinese economy and does not favor the New Keynesian framework. Different from their focus on the model comparison, we try to diagnose which part in the model may lead to the estimation problems, and also we study the recent housing boom in China.

Our chapter is also related to the literature on Chinese housing market. Several papers document the rapid development in the housing market for China [29, 82, 83]. They provide very important evidence that the high-speed development in the housing market accompanies the impressive advance in the growth of real economy in China in the last decade. Although it has been an important feature for China in the last decade, there exist very few papers explaining this phenomenon. Chen and Wen [16] builds a quantitive model to explain this great housing boom from the perspective of rational bubble. Recently, there are two paper [60, 62] using Bayesian estimated DSGE model to study the housing market and real economy in China. They build their models on Iacoviello and Neri [42] and Iacoviello [41]. This chapter uses more models and focuses on the difficulty for current main stream DSGE models to explain the Chinese data series.

2.2 Data and Methodology

2.2.1 Data

Most data used in this chapter comes from Higgins and Zha [39]. They constructed quarterly Chinese data, which has been in other studies such as Chang et al. [14]. The quarterly data greatly improves the size and quality of the sample for China and makes the structural estimation for the models feasible. In Iacoviello and Neri [42] and Liu et al. [49], we use additional data about land prices from Wu et al. [82].

All the data used in this chapter is listed in the Table 2.1. All variables except LandPriceGyourko_Q_SA comes from Higgins and Zha [39], and LandPriceGyourko_Q_SA comes from Wu et al. [82]. Later we will show how we construct Chinese series of variables corresponding to their U.S. counterparts based on these raw data.

2.2.2 Methodology

Our estimation method follows Bayesian approach. We linearize the equations describing the equilibrium around the balanced growth path steady state. The solution takes the form of a state-space model for given parameters, which can be used to calculate the likelihood function. A system of measurement equations links the observed data to the state variables in the equations. A standard Kalman-filter algorithm can be applied to the system of measurement and state equations in the form of likelihood function. Multiplying the likelihood by the prior distribution leads to a posterior kernel, which is proportional to the posterior density function. We follow a Bayesian approach to estimate the model. We transform the data for computing the likelihood function, choose prior distributions for the parameters, and estimate the posterior distribution.

In practice, we use both Dynare and Dr Tao Zha's C/C++ code to search for the posterior mode. As is well-known, one difficulty in the numerical optimization is to find the global peak instead of a local one. All the algorithm can only guarantee a local peak, so we need to start searching from many initial guesses of the parameter values and pick the best one out of all the local peaks. For complicated model such as Liu et al. [49][1], we may need to use advanced

[1]The complexity of the model, on the one hand, depends on the number of equations in the model. On the other hand, it depends on the shape of the posterior kernel. As is discussed in its online appendix, Liu et al. [49] finds its posterior kernel is full of ridges and local peaks, where Dynare has a hard time to find the mode of the posterior distribution. Even though in theory, given sufficient random draws, Dynare could be able to find the same mode, but the limitation of time makes it not feasible in reality. For the Chinese data, Dr Tao Zha's C/C++ code performs better for this model, which is consistent with the finding in Liu et al. [49]'s paper.

Table 2.1: Data Period: China

Variable	**Label**	**Period**
NGDP_va_Q_SA	Nominal GDP	1992Q1 - 2017Q2
DGDP_va_Q_SA	GDP Implicit Deflator	1992Q1 - 2017Q2
Pop_Q	Population	1950Q1 - 2016Q4
NC_Q_SA	Nominal Consumption	1990Q1 - 2016Q4
NGFCF_Q_SA	Nominal GFCF	1990Q1 - 2016Q4
AverageWageSplice_Q_SA	Wage	1992Q1 - 2016Q4
Emp_Q_SAAvg	Employment	1992Q1 - 2016Q4
SpliceRepo7Day_Q	7-Day Repo Rate	1996Q1 - 2017Q2
GovtGFCF_Q_SA	Nominal GFCF(Government)	1995Q1 - 2016Q4
HHNGFCF_Q_SA	Nominal GFCF(Household)	1992Q1 - 2016Q4
PriceGFCFAltPad_Q	GFCF Price	1993Q1 - 2016Q4
LandPriceGyourko_Q_SA	House Price	2004Q1 - 2016Q2
NonConstrSecEmp_Q_SAAvg	Non-construction Sector Employment	1992Q1 - 2016Q4
ConstrSecEmp_Q_SAAvg	Construction Sector Employment	1992Q1 - 2016Q4
AvgUrbWageNonConsAlt_Q	Non-construction Sector Avg Urban Wage	1992Q1 - 2016Q4
AvgUrbWageConsAlt_Q	Construction Sector Avg Urban Wage	1992Q1 - 2016Q4
LandPriceGyourko_Q_SA	Land Price	2004Q1 - 2016Q2
PriceCexHousing_Q	CPI price of nondurables and services excluding residence	1993Q1 - 2016Q4
LevNFESTLoanFOF_Q	Non-Financial Enterprise ST Loan	1994Q1 - 2016Q4
LevNFEMLTLoanFOF_Q	Non-Financial Enterprise M&LT Loan	1995Q1 - 2016Q4

algorithm with Dr Tao Zha's C/C++ code, which demands large extra amount of computation and the fast speed in C/C++ makes it feasible.

2.3 Smets-Wouters Model

2.3.1 Model Overview

Smets and Wouters [71] provides a canonical New Keynesian model. This model features both nominal and real rigidities in the economy. It has imperfect competition in intermediary good and labor market to capture the sticky price and wage, which allows monetary policy to play a role in the business cycles. More specifically, the two markets follow Calvo [13]'s assumption that in each period firms or labor union face fixed probabilities to change their price or wage. Also, it consists of habit formation, investment adjustment cost, and variable capital utilization to capture the real rigidities in the economy. To generate fluctuations in the business cycles, in addition to the standard total factor productivity shocks, the model includes two intertemporal-margin shocks (risk premium shocks and investment-specific technology shocks), two intratemporal-margin shocks (price and wage mark-up shocks), and two policy shocks (exogenous spending shocks and monetary policy shocks). It also contains labor-augmenting technological progress to capture the balanced steady-state growth path.

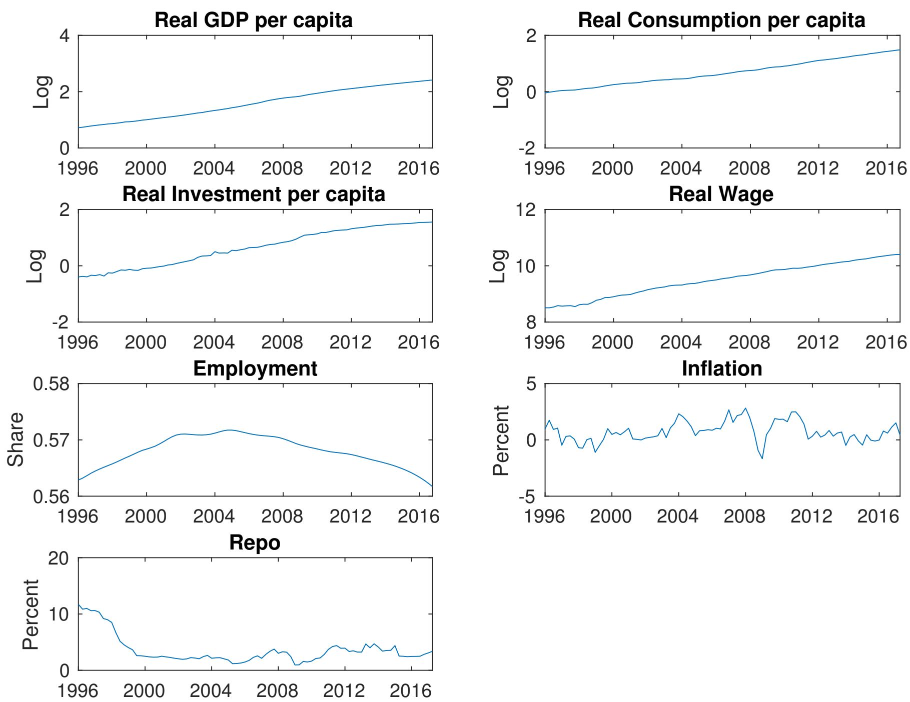
In this model, the representative household maximizes an infinite-horizon utility with consumption goods and labor efforts. The utility contains a time-varying external consumption habit part. Labor is differentiated by a union, which has monopoly power to allow for a sticky nominal wage. Household rents capital service to intermediary-goods firms and decides capital accumulation facing investment adjustment costs. Intermediary-goods firms rent labor and capital service to produce intermediary goods and set their prices following Calvo model to allow for a sticky price. Final-goods firms buy intermediary goods and use them to produce and sell final goods in the competitive markets. Details about the model setup and derivation of log-linearization equations are in the Appendix A.

2.3.2 Data Construction

Smets and Wouters [71] estimated their model with seven macroeconomic quarterly U.S. time series as observable variables: the log difference of real GDP, real consumption, real investment and the real wage, log hours worked, the log difference of the GDP deflator, and the federal funds rate. To get the correspondent Chinese time series, we construct them with data in the Table 2.1 as follows: (1) construct "real GDP per capita" with "NGDP_va_Q_SA / DGDP_va_Q_SA / Pop_Q"; (2) construct "real consumption per capita" with "NC_va_Q_SA / DGDP_va_Q_SA / Pop_Q"; (3) construct "real investment per capita" with "NGFCF_va_Q_SA / DGDP_va_Q_SA / Pop_Q"; (4) construct "real wage" with "AverageWageSplice_Q_SA / DGDP_va_Q_SA"; (5) since there is no hours data for China, we use quarterly employment share out of total population as a proxy. So construct "hours" with "Emp_Q_SA / Pop_Q"; (6) construct "GDP deflator" with "DGDP_va_Q_SA"; (7) construct "federal funds rate" or "interest rate" with the 7-day repo in China "SpliceRepo7Day_Q"[2].

From the Table 2.1, we can see that different data time series cover different time span, and we will use the intersection of them to estimate the model. Thus, the model is estimated over the sample period from 1996Q1 to 2016Q4. The time series are plotted in the Figure 2.1. We can see that real GDP per capita, real consumption per capita, real investment per capita, and real wage have been growing since 1996Q1. Employment share has a hump-shape, which peaks around 2005. Inflation has been fluctuating, and there was a big drop around 2009. The 7-day repo started high from 1996Q1 and became relatively stable after 2000.

Figure 2.1: Chinese Data Time Series



[2]We use 7-day repo instead of 1-day repo because 7-day repo is more commonly used in practice.

Table 2.2 shows the statistical summary for the data series. First, we can see the quarterly growth rate for real GDP per capita is around 2% for the last twenty years. Second, the growth rates for real consumption per capita and real investment per capita are around 1.8% and 2.3% respectively. This means that investment has grown faster than consumption for China. Third, compared to GDP and consumption, investment in China has been much more volatile with a standard deviation of around 3% for the growth rate.

Table 2.2: SW Data Summary: 1996Q1 to 2016Q4

Variable	**mean**	**std**	**min**	**p25**	**median**	**p75**	**max**
log diff of real GDP	2.044	0.558	0.605	1.636	1.960	2.310	3.923
log diff of real consumption	1.853	0.746	0.258	1.440	1.883	2.396	3.300
log diff of real investment	2.334	2.992	-5.103	0.703	1.962	3.383	13.714
log diff of real wage	2.222	1.774	-3.313	1.493	2.160	3.006	8.298
employment share	0.568	0.003	0.562	0.566	0.568	0.571	0.572
inflation	0.733	0.902	-1.666	0.150	0.678	1.152	2.825
interest rate	0.896	0.633	0.238	0.545	0.644	0.981	2.929

2.3.3 Parameter Estimates

Our calibrated parameter values and prior distributions mainly follow the setup in Smets and Wouters [71]. Like their model, we fix five parameters. We set the depreciation rate δ at 0.025, the steady-state mark-up in the labor market ϕ_w at 1.5, and the curvature parameters of the Kimball aggregators in the goods and labor market ε_p and ε_w both at 10. Besides, we set the exogenous spending-GDP ratio g_y at 20 percent match the Chinese data.

We follow Smets and Wouters [71] exactly to set the prior distributions for the parameters as in the Table 2.3, with one exception that balanced growth rate $\bar{\gamma}$ is set to have a mean of 2 and standard deviation of 0.5 to fit the Chinese long-run growth rate.

Column (1) in the Table 2.3 gives the mode of the posterior distribution of the parameters. The trend growth rate is around 2.14 percent quarterly, which is consistent with the average growth rate for the Chinese economy. However, the capital share α is very close to zero, which highly contrasts with the high capital share of income in China. To deal with this issue, we fix the capital share at 0.5 and re-estimate the model, and the results are in the column (2). We can see that the trend growth rate now deviates from the average growth rate of Chinese economy. Also, from Table 2.2, we can see that real consumption and investment grow with very different pace during the sample period, where investment grows about 0.5% faster than consumption. During the last two decades, the consumption share out of GDP has dropped from 46% to below 40%, while the investment share has increased around 10% from 33%. Since we are more interested in the high frequency dynamics rather than the low frequency trend changes, we detrend real GDP, real consumption, and real investment separately with linear trend to capture the high-frequency fluctuations in the data series. In estimation, we set the trend growth rate at 2, the capital share at 0.5, and fit the detrended time series in the model to search for the posterior. The results are in the column (3) in the Table 2.3.

First, we can see that the parameter σ_c is smaller than one for all the models, which implies the risk aversion is not large in China. Second, the persistence parameters for both productivity and investment shocks are very large. Third, the government spending shocks are persistent. Last, let us look at parameters related to the monetary policy. The persistence coefficient for the interest rate ρ is around 0.85 to 0.9 from the estimation, so interest rate is relatively persistent. The coefficient for the inflation r_π is close to one compared to the value of two in U.S., which implies the monetary policy in China does not respond much to the inflation. The coefficients related to output r_y and $r_{\Delta y}$ are not large, and they are very close to zero in the model (2) and (3). From the results related to the monetary policy, it seems that different from the traditional understanding of Taylor rule in the U.S. economy where interest rate responds sufficiently to changes in inflation and output, those factors do not feed into the monetary policy a lot in the estimation results for China. This is consistent with the findings of Chen et al. [18], who identify the monetary policy rule in China and find it is different from the standard Taylor rule.

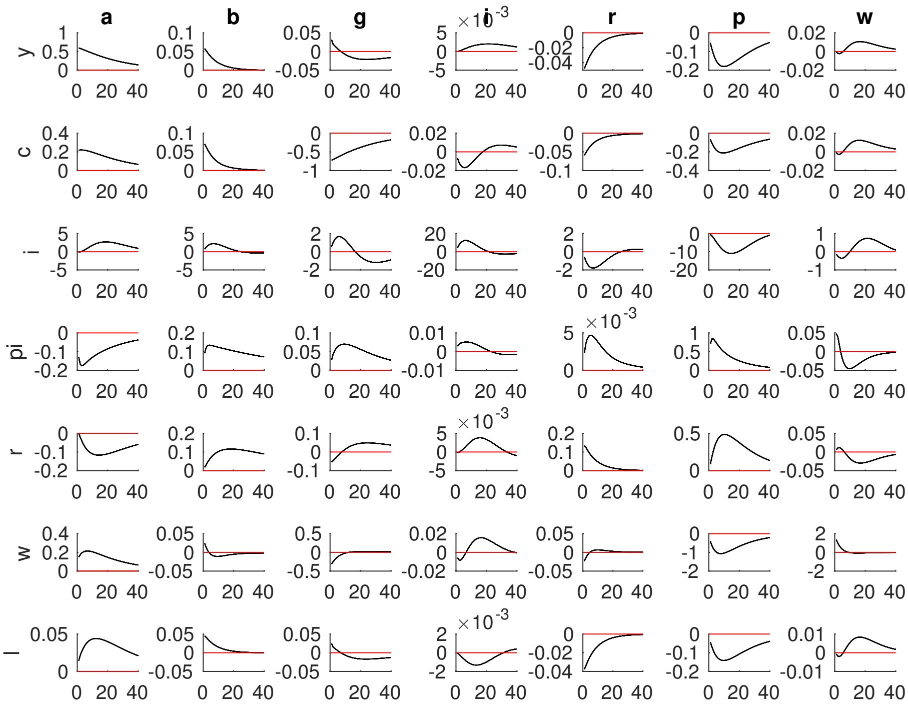
Table 2.3: SW Prior and Posterior Distributions

	Prior			Model Posterior		
	Distribution	Mean	Std	(1)	(2)	(3)
φ	NORMAL	4	1.5	5.5241	7.2773	5.1991
σ_c	NORMAL	1.50	0.375	0.0071	0.0006	0.6060
λ	BETA	0.7	0.1	0.9936	0.9680	0.9135
ξ_w	BETA	0.5	0.1	0.7404	0.8668	0.7091
σ_l	NORMAL	2	0.75	2.7834	3.2862	2.9593
ξ_p	BETA	0.5	0.10	0.7631	0.9608	0.6691
ι_w	BETA	0.5	0.15	0.3988	0.2214	0.5485
ι_p	BETA	0.5	0.15	0.3690	0.4547	0.3847
ψ	BETA	0.5	0.15	0.6200	0.9018	0.9236
ϕ_p	NORMAL	1.25	0.125	1.2680	1.5846	1.7953
r_π	NORMAL	1.5	0.25	1.0912	1.0108	1.1345
ρ	BETA	0.75	0.10	0.8789	0.8553	0.9094
r_y	NORMAL	0.125	0.05	0.1220	-0.000	-0.075
$r_{\Delta y}$	NORMAL	0.125	0.05	0.0877	0.0001	0.0107
$\bar{\pi}$	GAMMA	0.625	0.1	0.6113	0.6017	0.5679
$100(\beta^{-1}-1)$	GAMMA	0.25	0.1	0.1575	0.4030	0.3048
$\bar{l}$	NORMAL	0.0	2.0	-0.261	6.9849	-3.782
$\bar{\gamma}$	NORMAL	2.0	0.5	2.1396	0.0919	
α	NORMAL	0.3	0.05	0.0007		
μ_p	BETA	0.5	0.2	0.4628	0.1504	0.2469
μ_w	BETA	0.5	0.2	0.8738	0.2496	0.8691
ρ_{ga}	BETA	0.5	0.2	0.9263	0.4521	0.3445
ρ_a	BETA	0.5	0.2	0.9593	0.9736	0.9787
ρ_b	BETA	0.5	0.2	0.9811	0.9997	0.1574
ρ_g	BETA	0.5	0.2	0.9607	0.9909	0.8019
ρ_i	BETA	0.5	0.2	0.8771	0.9521	0.9762
ρ_r	BETA	0.5	0.2	0.0578	0.1279	0.2080
ρ_p	BETA	0.5	0.2	0.9415	0.4064	0.7205
ρ_w	BETA	0.5	0.2	0.8475	0.3035	0.9057
σ_a	InvGAMMA	0.1	2.0	0.4545	0.4273	0.3283
σ_b	InvGAMMA	0.1	2.0	0.0829	0.0334	0.2721
σ_g	InvGAMMA	0.1	2.0	0.6076	1.2852	1.5310
σ_i	InvGAMMA	0.1	2.0	0.6803	0.1675	0.3911
σ_r	InvGAMMA	0.1	2.0	0.1374	0.1511	0.1623
σ_p	InvGAMMA	0.1	2.0	0.0847	0.3278	0.2177
σ_w	InvGAMMA	0.1	2.0	0.8778	0.6934	0.7207

2.3.4 Properties of the Estimated Model

In this part, we will study the dynamic properties from the perspectives of impulse response functions and variance decompositions.

Figure 2.2: SW Model 1: Impulse Response



In the Figure 2.2 to 2.4, we plot the impulse response functions for the Model 1 to 3. In the figures, we include seven important variables (output ‘y’, consumption ‘c’, investment ‘i’, inflation ‘pi’, interest rate ‘r’, wage ‘w’, and labor ‘l’) and all the shocks (TFP technology ‘a’, risk premium ‘b’, exogenous spending ‘g’, investment technology ‘i’, monetary policy ‘r’, price mark-up ‘p’, and wage mark-up ‘w’). For Model 1, several things stand out. First, the responses of investment to all shocks are relative large. Second, the investment technology shock only has significant impacts on investment but very small effects on other variables. For Model 2, there are strong non-stationarity in the estimated model because all the impulse respones do not come back to zero even after long periods. For Model 3, first we can see that all variables have relatively strong responses to the investment shocks. Second, we can see that there exist many fluctuations in the impulse response functions. These fluctuations may mainly come from the respones of monetary policy, i.e. interest rate (row ‘r’). In the Table 2.3, we can see that the coefficient for output in the monetary policy is negative, which is different from the standard Taylor rule with a positive coefficient on output. Thus, this also provides some evidence that Taylor rule or interest rate mechanism may not apply to the Chinese economy.

Figure 2.3: SW Model 2: Impulse Response

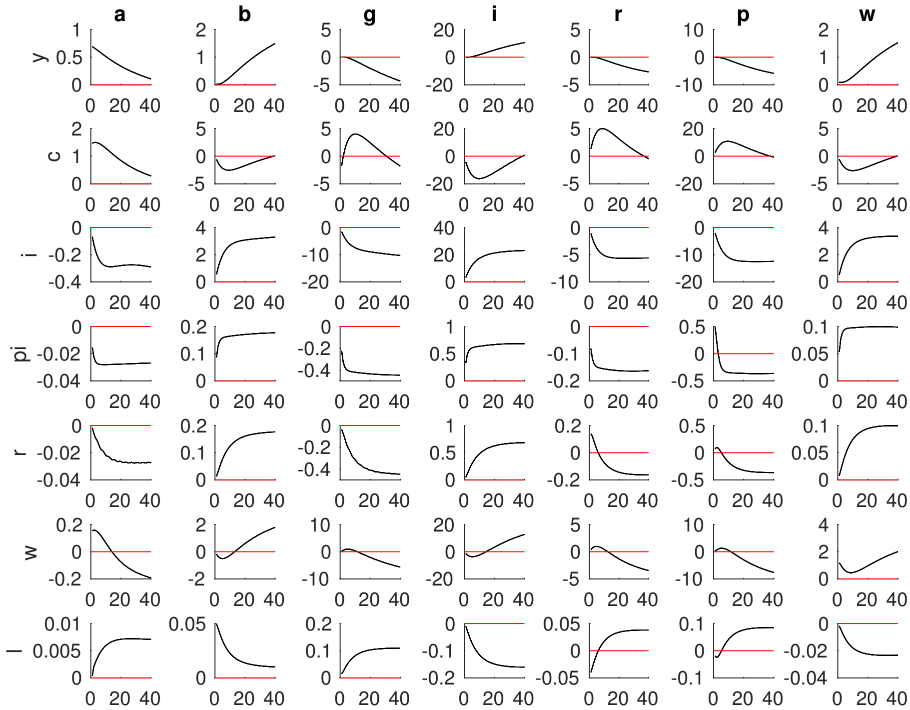
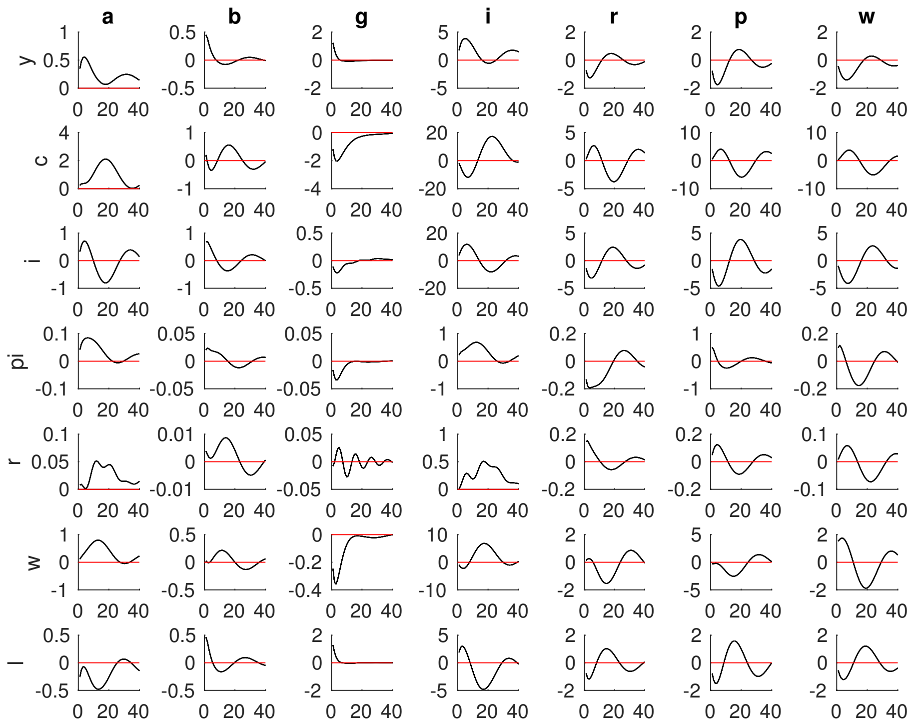


Figure 2.4: SW Model 3: Impulse Response



The variance decomposition results are in the Tables 2.4 and 2.5.[3] For Model 1, We can see that TFP shocks explain most fluctuations of output. Exogenous spending shocks explain most fluctuations of consumption. Investment shocks are responsible for large portions of both short-

[3]Since Model 2 is not very stationary, here we only present results for Model 1 and 3.

and long-run variance decomposition of investment. Price mark-up shocks are the dominant factors for the changes in inflation. Monetary policy shocks play an important role in the short-run effects on interest rate, but in the long run price mark-up shocks dominate. In the short run, wage mark-up shocks explain large part of changes in wage, but in the long-run price mark-up shocks have largest effects. For labor, risk premium shocks, monetary policy shocks, and price mark-up shocks are important in the short run. However, in the long run, only price mark-up shocks stand out. For Model 3, we can see that investment shocks are dominant in the long run for all the variables, which is consistent with previous findings in the impulse response functions. But in the short run, price mark-up shocks, monetary policy shocks, wage mark-up shocks are important for inflation, interest rate, and wage respectively.

Table 2.4: SW Model 1: Variance Decomposition

	a	b	g	i	r	p	w
				Output			
Unconditional	88.24	0.26	0.26	0.00	0.18	11.01	0.03
1Q	97.38	0.91	0.26	0.00	0.62	0.81	0.00
4Q	95.63	0.71	0.12	0.00	0.48	3.03	0.00
8Q	92.97	0.53	0.06	0.00	0.36	6.04	0.00
40Q	88.16	0.27	0.21	0.00	0.18	11.12	0.03
				Consumption			
Unconditional	9.91	0.22	80.20	0.03	0.15	9.44	0.02
1Q	8.12	0.86	89.63	0.00	0.59	0.77	0.00
4Q	8.69	0.65	87.39	0.02	0.44	2.78	0.00
8Q	9.15	0.47	84.58	0.04	0.32	5.41	0.00
40Q	9.88	0.23	80.12	0.03	0.16	9.52	0.02
				Investment			
Unconditional	3.83	1.38	1.29	40.21	0.90	52.13	0.22
1Q	0.02	2.57	1.63	92.96	1.68	1.02	0.08
4Q	0.03	2.58	1.61	88.25	1.68	5.73	0.08
8Q	0.33	2.43	1.40	77.78	1.58	16.39	0.05
40Q	3.76	1.38	1.16	40.04	0.90	52.52	0.21
				Inflation			
Unconditional	5.65	7.48	1.55	0.00	0.00	84.95	0.34
1Q	3.20	1.60	0.14	0.00	0.00	94.60	0.44
4Q	3.92	2.19	0.32	0.00	0.00	93.39	0.15
8Q	4.39	2.78	0.58	0.00	0.00	92.03	0.19
40Q	5.59	5.88	1.47	0.00	0.00	86.68	0.35
				Interest Rate			
Unconditional	6.36	10.34	1.38	0.00	1.52	80.12	0.24
1Q	0.00	1.13	10.07	0.00	61.98	26.66	0.13
4Q	1.83	2.29	2.44	0.00	18.30	74.98	0.12
8Q	3.19	2.96	0.70	0.00	6.72	86.37	0.04
40Q	6.07	7.27	1.14	0.00	1.64	83.59	0.26
				Wage			
Unconditional	3.90	0.00	1.09	0.01	0.00	79.62	15.34
1Q	1.06	0.02	4.55	0.00	0.02	7.58	86.74
4Q	2.23	0.01	3.73	0.00	0.01	34.34	59.65
8Q	2.96	0.01	2.35	0.00	0.00	59.78	34.87
40Q	3.76	0.00	1.09	0.01	0.00	79.51	15.58
				Labor			
Unconditional	11.12	2.01	2.08	0.00	1.39	83.14	0.23
1Q	3.41	33.64	9.88	0.00	23.13	29.91	0.00
4Q	6.02	15.53	2.67	0.00	10.56	65.17	0.02
8Q	7.03	7.15	0.93	0.00	4.87	79.97	0.02
40Q	10.42	2.09	1.68	0.00	1.45	84.08	0.24

Table 2.5: SW Model 3: Variance Decomposition

	a	b	g	i	r	p	w
				Output			
Unconditional	1.48	0.25	1.02	71.51	5.25	11.65	8.8051
1Q	1.94	3.24	24.00	49.97	8.34	9.48	3.0031
4Q	1.66	0.81	3.79	65.20	8.58	13.81	6.1200
8Q	1.39	0.37	1.67	68.63	6.40	12.98	8.5364
40Q	1.45	0.27	1.10	70.42	5.48	12.08	9.1736
				Consumption			
Unconditional	1.11	0.06	0.50	79.79	3.47	8.61	6.4290
1Q	1.10	0.73	25.83	64.94	1.39	4.45	1.5300
4Q	0.22	0.11	6.36	77.17	4.01	8.12	3.9841
8Q	0.19	0.05	2.76	78.06	3.83	8.83	6.2458
40Q	1.09	0.07	0.59	78.54	3.69	9.08	6.9149
				Investment			
Unconditional	0.41	0.13	0.01	72.56	5.28	12.65	8.9429
1Q	0.50	2.23	0.05	72.89	8.91	10.95	4.4442
4Q	0.38	0.43	0.03	73.85	7.14	12.06	6.0684
8Q	0.23	0.16	0.02	75.22	5.21	11.05	8.0748
40Q	0.40	0.13	0.01	72.64	5.29	12.57	8.9373
				Inflation			
Unconditional	1.00	0.06	0.06	76.06	5.36	13.04	4.3905
1Q	0.49	0.12	0.07	15.54	5.20	75.64	2.9068
4Q	1.58	0.16	0.30	39.58	11.06	43.90	3.3875
8Q	1.90	0.13	0.22	60.98	10.73	24.19	1.8422
40Q	1.03	0.06	0.06	74.80	5.71	13.80	4.4986
				Interest Rate			
Unconditional	0.86	0.02	0.11	90.92	2.68	3.71	1.6726
1Q	0.25	0.06	0.25	0.21	88.91	9.99	0.3026
4Q	0.09	0.01	0.39	37.58	39.37	20.59	1.9382
8Q	0.17	0.01	0.35	65.42	16.59	14.49	2.9399
40Q	0.82	0.02	0.13	89.95	3.09	4.14	1.8189
				Wage			
Unconditional	1.17	0.07	0.08	76.38	3.85	10.74	7.6871
1Q	0.35	0.01	1.72	28.72	0.51	2.34	66.3144
4Q	0.90	0.00	1.59	52.89	0.77	0.92	42.9073
8Q	3.01	0.12	1.35	46.56	1.32	4.82	42.7935
40Q	1.17	0.07	0.09	74.94	4.01	11.31	8.3799
				Labor			
Unconditional	0.73	0.17	0.61	77.37	4.51	10.20	6.3746
1Q	0.94	3.06	23.48	51.37	8.14	8.56	4.4144
4Q	0.20	0.92	5.30	61.50	9.41	14.34	8.3017
8Q	0.43	0.68	3.75	59.06	8.42	15.25	12.3667
40Q	0.70	0.19	0.70	76.37	4.71	10.52	6.7662

2.4 Iacoviello-Neri Model

2.4.1 Model Overview

Iacoviello and Neri [42] builds on Smets and Wouters [71] and includes several new elements to investigate the interaction between housing market and real economy in the business cycles. First, similar to Smets and Wouters [71], it has nominal and real rigidities in the economy. Second, it has a multi-sector structure with housing and non-housing goods. By introducing the housing goods, it allows us to study whether the developments of housing sector (residential investment and housing prices) are just a reflection of macroeconomic activities in the real economy or it might be an important driving force of business cycles. Third, it contains financial frictions in the household sector. In particular, some (impatient) households suffer from borrowing constraint where housing serves as collateral. This allows housing sector to have spillover effects for the real economy through changing the borrowing constraint for the impatient households. Fourth, it introduces housing demand shocks from the household side and housing technology shock from the housing sector to capture the demand and supply sides for the housing market, and it further helps explain how much fluctuations in the real economy can be explained by the housing sector activities.

This model features two sectors, heterogeneity in households, and collateral constraints tied to housing values. On the demand side, there are two types of households: patient and impatient. Both patient and impatient households work, consume, and owning housing. However, patient ones own capital and supply funds to firms and impatient households. Impatient households borrow from patient ones up against their housing collateral. On the supply side, the non-housing sector uses capital and labor to produce consumption goods and business capital for both sectors. The housing sector produces new homes combining business capital with labor and land. Details about the model setup and derivation of log-linearization equations are in the Appendix B.

2.4.2 Data Construction

Iacoviello and Neri [42] estimated their model with ten observables: real consumption, real residential investment, real business investment, real house prices, nominal interest rates, inflation, hours and wage inflation in the consumption sector, hours and wage inflation in the housing

sector. We construct the correspondent Chinese observables as follows: (1) we construct "real consumption" with "NC_va_Q_SA / DGDP_va_Q_SA / Pop_Q"; (2) we construct "real residential investment" with "HHNGFCF_Q_SA / PriceGFCFAltPad_Q / Pop_Q"[4]; (3) we construct "real business investment" with
"(NGFCF_Q_SA - GovtGFCF_Q_SA - HHNGFCF_Q_SA) / PriceGFCFAltPad_Q / Pop_Q"; (4) we construct "real house prices" with "LandPriceGyourko_Q_SA / DGDP_va_Q_SA". There exist office house price indices calculated and reported by the National Bureau of Statistics of China. However, those house price indices are widely criticized for under-estimating the housing prices in China [29, 83]. In the estimation process, it is the growth rate mattering rather than the level. We believe that the growth rate of land price is a reasonable proxy for that of housing price. (5) we construct "nominal interest rates" with "SpliceRepo7Day_Q"; (6) we construct "inflation" as the log difference of "DGDP_va_Q_SA"; (7) Since hours data is not available in China, we use employment share as a proxy for hours as previously. We construct "hours in the consumption sector" with "NonConstrSecEmp_Q_SAAvg / Pop_Q"; (8) we construct "wage inflation in the consumption sector" as the log difference of "AvgUrbWageNonConsAlt_Q"; (9) we construct "hours in the housing sector" with "ConstrSecEmp_Q_SAAvg / Pop_Q"; (10) we construct "wage inflation in the housing sector" as the log difference of "AvgUrbWageConsAlt_Q". We take the common time span for all the data, and the sample period is from 2004Q2 to 2016Q2.

We plot the data series in the Figure 2.5. From the figure, we can see that both residential investment and the land price have been increasing around the last decade, which are consistent with what we observe for the Chinese economy. However, the growth moment for business investment has slowed down a little since around 2012. The employment share for consumption sector has been declining, which largely comes from the population aging problem in China. But the employment in the housing sector has been increasing, which is consistent with the booming housing market. Table 2.6 shows the statistical summary for those time series. First, the growth rates of real residential investment and real land price are both higher than that of real business investment in the last decade and much higher than the growth rate of real consumption. Second, the growth rate of residential investment is more volatile than that of business investment, but the standard deviation of growth rate of real land price is much higher.

[4]Since there is no price deflator for housing investment in China, we use fixed asset investment price deflator as proxy.

Figure 2.5: Chinese Data Time Series

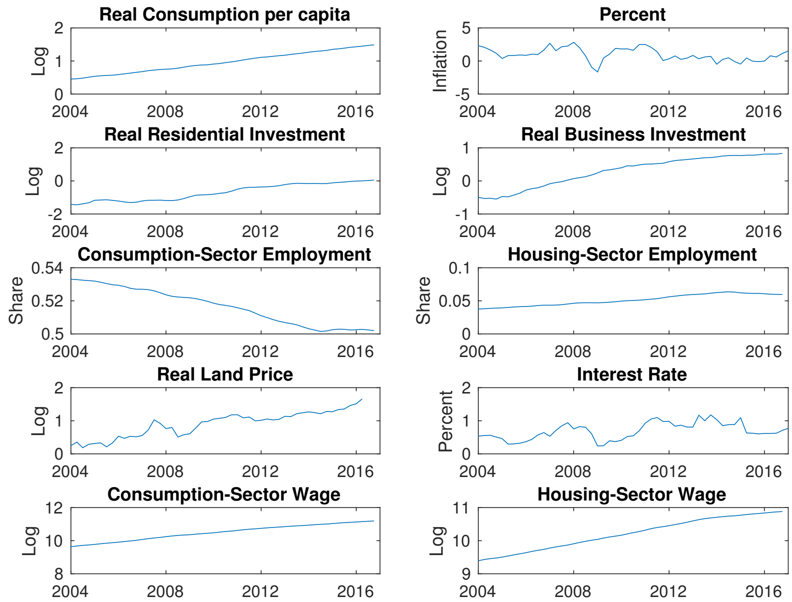


Table 2.6: IN Data Summary: 2004Q2 to 2016Q2

Variable	**mean**	**std**	**min**	**p25**	**median**	**p75**	**max**
log diff of real consumption	2.026	0.669	0.639	1.554	2.051	2.506	3.300
inflation	0.942	0.970	-1.666	0.331	0.834	1.707	2.825
log diff of real residential investment	2.910	4.225	-5.490	0.301	2.336	5.010	15.165
log diff of real business investment	2.666	2.599	-3.625	1.108	2.400	4.563	9.217
cons-sector employment share	0.517	0.011	0.501	0.506	0.518	0.527	0.533
hous-sector employment share	0.051	0.008	0.038	0.044	0.050	0.060	0.064
log diff of real land price	2.888	10.300	-28.785	-1.159	2.699	8.104	31.004
interest rate	0.694	0.258	0.238	0.520	0.642	0.883	1.177
log diff of cons-sector wage inflation	3.102	0.828	1.762	2.454	2.964	3.667	5.131
log diff of hous-sector wage inflation	2.982	0.866	1.047	2.350	3.184	3.640	4.578

2.4.3 Parameter Estimates

Our calibrated parameters and prior distributions follow exactly the same setup as Iacoviello and Neri [42]. We set the discount factors of the patient and impatient households at 0.9925 and 0.97 respectively. The low discount factor for impatient households guarantees that the borrowing constraint is binding for them around steady state. We set $X = 1.15$, which means that the mark-up in the consumption-good sector is 15 percent in the steady state. Similarly, we set the mark-up in the labor market $X_{w,c} = X_{w,h} = 1.15$. Since the persistent shock in the monetary policy is difficult to identify in the estimation, we set $\rho_s = 0.975$. We fix the depreciation rates for housing, consumption-sector capital, and housing-sector capital equal to $\delta_h = 0.01$, $\delta_{kc} = 0.025$, and $\delta_{kh} = 0.03$ respectively. For goods production function, we set the capital share $\mu_c = 0.35$. For housing production function, we set the capital share $\mu_h = 0.1$, land share $\mu_l = 0.1$, and intermediate goods share $\mu_b = 0.1$. Regarding the borrowing constraint parameter, we set the LTV ratio $m = 0.85$, which means that the total debt has to be less that 85% of the expected value of the housing for the impatient households. We set our prior distributions for the parameters exactly following Iacoviello and Neri [42] in the Table 2.7.

The results are in the Table 2.7. Several things stand out. First, the habit parameter for the patient households is much smaller that that for the impatient households. Second, the parameters related to monetary policy seem to be consistent with the traditional Taylor rule. The persistence coefficient for interest rate r_R is about 0.82. The coefficient for inflation r_π is much higher than one. The coefficient for output r_Y is positive, which means higher output results in a increase in the interest rate for the monetary policy. Third, all shocks related productivity, ρ_{AC}, ρ_{AH}, and ρ_{AK}, are persistent with persistence coefficients higher than 0.95. Regarding the standard deviations for the shocks, housing productivity and housing demand shocks have relatively large ones. This is consistent with the data summary in the Table 2.6, where growth rate of land prices has the largest standard deviation and growth rate of residential investment has the second largest ones. To match with the data characteristics, the model needs to have very large shock standard deviations for housing demand and housing productivity. Fourth, there is one strange result, which is a negative trend for the investment technology, i.e. $\gamma_{AK} = -0.016$. This result comes from the fundamental conflict between the balance path relationship in the model and the observed data in the Chinese economy. In the model, there exists a relationship for the growth rates in the balanced growth path, which is that the growth rate for the consumption is equal to the sum of growth rates for housing prices and residential investment. However, from the Table 2.6, we can see that both the growth rate for land price (2.8%) and the growth rate for residential investment (2.9%) are higher than the growth rate for consumption (2.0%) in the last decade. In this sense, Iacoviello and Neri [42]'s model has a hard time to explain the boom in the housing market.

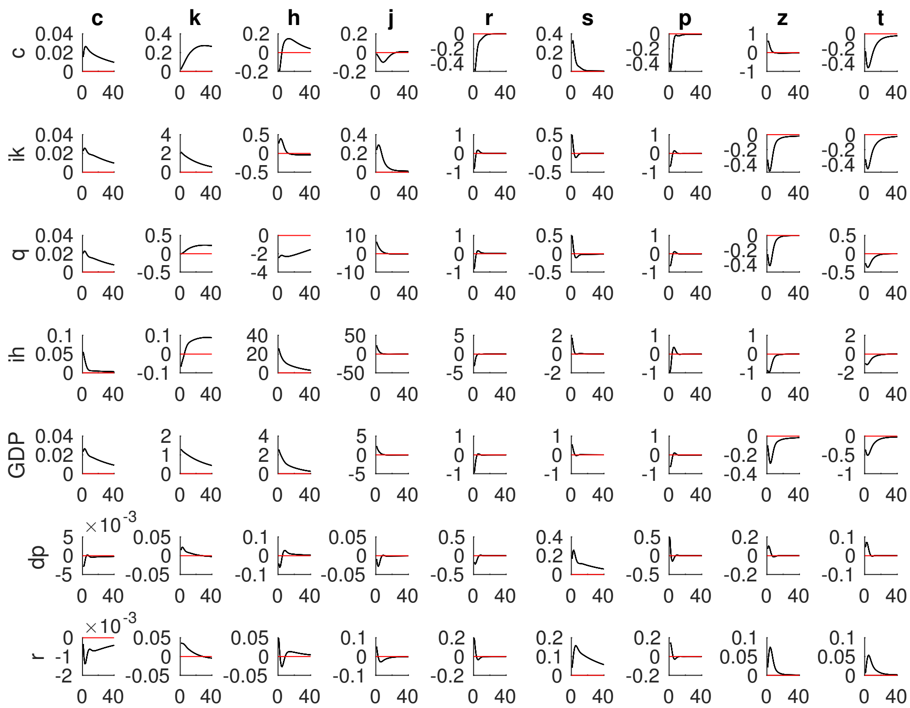
Table 2.7: IN Prior and Posterior Distributions

	Prior			Model Posterior
	Distribution	Mean	Std	Mode
ε	BETA	0.5	0.075	0.329
ε'	BETA	0.5	0.075	0.718
η	GAMMA	0.5	0.1	0.448
η'	GAMMA	0.5	0.1	0.456
ξ	NORMAL	1	0.1	0.928
ξ'	NORMAL	1	0.1	0.756
$\phi_{k,c}$	GAMMA	10	2.5	28.69
$\phi_{k,h}$	GAMMA	10	2.5	6.524
α	BETA	0.65	0.05	0.793
r_R	BETA	0.75	0.1	0.819
r_π	NORMAL	1.5	0.1	1.749
r_Y	NORMAL	0	0.1	0.131
θ_π	BETA	0.125	0.05	0.652
ι_π	BETA	0.125	0.05	0.964
$\theta_{w,c}$	BETA	0.625	0.1	0.664
$\iota_{w,c}$	BETA	0.25	0.1	0.054
$\theta_{w,h}$	BETA	0.0	2.0	0.863
$\iota_{w,h}$	BETA	2.0	0.5	0.129
ζ	BETA	0.3	0.05	0.987
γ_{AC}	NORMAL	0.5	0.2	0.034
γ_{AH}	NORMAL	0.5	0.2	0.016
γ_{AK}	NORMAL	0.5	0.2	-0.016
ρ_{AC}	BETA	0.5	0.2	0.964
ρ_{AH}	BETA	0.5	0.2	0.966
ρ_{AK}	BETA	0.5	0.2	0.952
ρ_j	BETA	0.5	0.2	0.798
ρ_z	BETA	0.5	0.2	0.789
ρ_τ	BETA	0.5	0.2	0.842
σ_{AC}	InvGAMMA	0.5	0.2	0.000
σ_{AH}	InvGAMMA	0.1	2.0	0.113
σ_{AK}	InvGAMMA	0.1	2.0	0.021
σ_j	InvGAMMA	0.1	2.0	1.486
σ_z	InvGAMMA	0.1	2.0	0.019
σ_τ	InvGAMMA	0.1	2.0	0.019
σ_R	InvGAMMA	0.1	2.0	0.002
σ_p	InvGAMMA	0.1	2.0	0.010
σ_s	InvGAMMA	0.5	0.2	0.000
$\sigma_{n,h}$	InvGAMMA	0.1	2.0	0.246
$\sigma_{w,h}$	InvGAMMA	0.1	2.0	0.005

2.4.4 Properties of the Estimated Model

In the Figure 2.6, we plot the impulse response functions for the estimated model. In the figure, we include seven variables (consumption 'c', business investment 'ik', housing price 'q', residential investment 'ih', GDP 'GDP', inflation 'dp', interest rate 'r') and all the shocks (consumption-sector technology 'c', investment technology 'k', housing-sector technology 'h', housing preference 'j', temporary monetary policy 'r', persistent monetary policy 's', price mark-up 'p', discount factor 'z', labor supply 't'). First, we can see discount factor shocks affect consumption mostly in the short run, but investment technology shocks have large long-run effects on consumption. Second, investment technology shocks are the most dominant forces in determining business investment. Third, housing technology shocks have negative effects on housing price, while housing preference shocks have positive ones. Housing preference shocks have short-term effects, but it takes long periods before the effects of housing technology shocks to fade. Fourth, both housing technology shocks and housing preference shocks have large and positive effects on residential investment. Similar to housing prices, housing technology shocks have larger long-term effects. Fifth, investment technology shocks, housing technology shocks, and housing preference shocks have large effects on GDP, because they are dominant forces for important components of GDP such as consumption, business investment, and resident investment. Sixth, two monetary policy shocks and price mark-up shocks have relatively large effects on inflation and interest rate. But only persistent monetary policy shocks have long-run effects.

Figure 2.6: IN Model: Impulse Response



Next let us look at the variance decomposition results in the Table 2.8. Most results are consistent with the implications from the impulse response functions. First, investment technology shocks explain most of the variance for consumption and business investment. Second, housing technology and housing preference shocks account for most of the fluctuations for housing price and residential investment. Third, persistent monetary policy shocks are important to explain the variance for inflation and interest rate. Since housing sector is an important feature in the Iacoviello and Neri [42]'s model compared to Smets and Wouters [71]'s model, we will focus on two shocks related to housing, housing technology shock and housing preference shock. Even though these two shocks explain more than 95% variance for housing price and residential investment, they are not important for other variables. Especially they can only explain less than 2% unconditional variance for business investment and around 6% unconditional variance for consumption. In this sense, it is very hard for this model to explain the high-speed development in both business investment and housing sector for China in the last decade. In the Table 2.6, we can see that business investment, residential investment, and land price all have experienced a growth rate higher than 10 percent annually, and this is an important characteristics for Chinese economy. However, these facts are very hard to concile with Iacoviello and Neri [42]'s model. To obtain high growth for residential investment and housing price, we need persistent positive housing preference shocks. But in the model, housing preference shocks do not have large effects on business investment.

Table 2.8: IN Model: Variance Decomposition

	c	k	h	j	r	s	p	z	t
				Consumption					
Unc	0.14	46.68	5.14	1.00	7.99	3.71	7.09	11.28	16.92
1Q	0.02	0.05	3.84	0.02	24.14	9.67	16.16	40.35	5.71
4Q	0.06	0.33	2.71	0.16	20.42	9.30	19.29	30.56	17.13
8Q	0.10	1.58	2.80	0.85	17.55	8.09	15.80	24.71	28.48
40Q	0.16	29.37	6.69	1.30	10.68	4.95	9.46	14.95	22.41
				Business Investment					
Unc	0.01	91.14	1.12	0.77	1.22	0.46	1.46	1.77	2.01
1Q	0.00	72.27	1.15	0.88	10.71	3.75	7.77	1.75	1.68
4Q	0.01	78.69	2.40	1.43	4.01	1.51	5.03	3.66	3.22
8Q	0.01	82.41	2.19	1.46	2.65	1.00	3.18	3.53	3.53
40Q	0.01	90.48	1.15	0.83	1.32	0.50	1.59	1.91	2.16
				Housing Price					
Unc	0.00	0.89	64.66	33.22	0.27	0.10	0.30	0.26	0.25
1Q	0.00	0.00	12.11	84.85	1.37	0.47	0.91	0.13	0.13
4Q	0.00	0.00	17.39	80.01	0.69	0.26	0.82	0.43	0.36
8Q	0.00	0.01	26.16	71.28	0.59	0.22	0.66	0.55	0.49
40Q	0.00	0.50	58.61	39.43	0.32	0.12	0.36	0.31	0.30
				Residential Investment					
Unc	0.00	0.00	79.43	19.88	0.28	0.08	0.04	0.09	0.15
1Q	0.00	0.00	55.58	43.10	0.84	0.24	0.08	0.06	0.08
4Q	0.00	0.00	65.67	33.33	0.49	0.14	0.05	0.12	0.16
8Q	0.00	0.00	71.97	27.14	0.38	0.11	0.05	0.12	0.19
40Q	0.00	0.00	79.12	20.19	0.28	0.08	0.04	0.09	0.16
				GDP					
Unc	0.01	34.43	47.26	12.58	1.40	0.51	1.08	0.52	2.16
1Q	0.00	10.86	43.10	34.77	5.78	1.94	2.60	0.06	0.84
4Q	0.00	15.18	49.65	26.00	3.08	1.13	2.31	0.47	2.13
8Q	0.00	19.57	51.97	20.10	2.25	0.82	1.72	0.72	2.82
40Q	0.01	31.66	49.08	13.24	1.48	0.54	1.13	0.54	2.27
				Inflation					
Unc	0.00	0.29	1.29	0.21	12.37	55.45	26.18	2.71	1.46
1Q	0.00	0.07	0.56	0.02	9.89	9.27	77.66	1.80	0.69
4Q	0.00	0.23	1.44	0.30	19.86	30.91	40.70	4.30	2.22
8Q	0.00	0.28	1.41	0.29	17.38	37.99	36.77	3.78	2.05
40Q	0.00	0.26	1.36	0.22	13.13	52.77	27.79	2.87	1.55
				Interest Rate					
Unc	0.00	2.04	1.19	1.32	8.04	72.05	8.26	4.32	2.72
1Q	0.00	1.95	3.57	4.19	57.21	2.60	29.71	0.68	0.05
4Q	0.00	2.82	1.90	2.03	27.59	26.13	29.03	7.54	2.93
8Q	0.00	2.74	1.58	2.16	17.00	45.88	17.43	8.56	4.61
40Q	0.00	1.93	1.29	1.46	8.95	69.29	9.20	4.80	3.03

2.5 Liu-Wang-Zha Model

2.5.1 Model Overview

Liu et al. [49] develop a DSGE model to investigate the co-movements between land prices and business investment. Different from Smets and Wouters [71] and Iacoviello and Neri [42], this model only contains real rigidities such as consumption habit formation and investment adjustment costs and does not include nominal rigidities. In this sense, this model fits more into the real business cycle model rather than New Keynesian one. However, this model features borrowing constrained entrepreneurs with land as collateral, which has the potential to link the fluctuations in land prices to those in business investment. This mechanism is different from Iacoviello and Neri [42], where the borrowing constraints lie on impatient households. Thus, this model could deliver positive co-movements between land prices and business investment while Iacoviello and Neri [42] could not.

The setup for the model is as follows: the economy is populated by two types of agents, households and entrepreneurs. Households consume goods, provide labor supply, and enjoy housing (land services). Entrepreneurs consume goods and conduct capital investment. Entrepreneurs use labor, capital, and land to produce output, which can be used fro consumption goods and capital investment. Details about the model setup and derivation of log-linearization equations are in the Appendix C.

2.5.2 Data Construction

Liu et al. [49] fitted their model to six U.S. time series: the real price of land, the inverse of relative price of investment, real per capita consumption, real per capita investment (in consumption units), real per capita nonfarm non financial business debt, and per capita hours worked. We construct the correspondent Chinese time series as follows: (1) we construct "real price of land" with "LandPriceGyourko_Q_SA / PriceCexHousing_Q"; (2) we construct "inverse of relative price of investment" with "PriceCexHousing_Q / PriceGFCFAltPad_Q"; (3) we construct "real per capita consumption" with "NC_Q_SA / PriceCexHousing_Q / Pop_Q"; (4) we construct "real per capita investment" with "NGFCF_Q_SA / PriceCexHousing_Q / Pop_Q"; (5) we construct "real per capita nonfarm non financial business debt" with "(LevNFEST-LoanFOF_Q+LevNFEMLTLoanFOF_Q) / PriceCexHousing_Q / Pop_Q"; (6) we construct "per capita hours worked" with "Emp_Q_SAAvg / Pop_Q". And the sample period is from 2004Q2 to 2016Q2 due to the data availability for land price.

Figure 2.7: Chinese Data Time Series

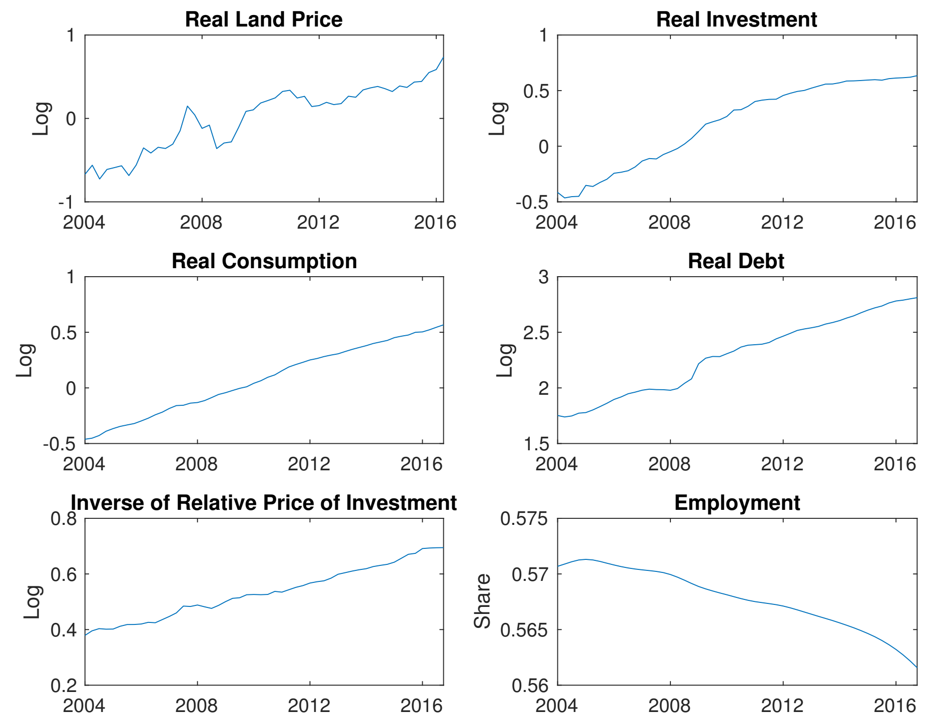


Table 2.9: LWZ Data Summary: 2004Q2 to 2016Q2

Variable	**mean**	**std**	**min**	**p25**	**median**	**p75**	**max**
log diff of real land price	2.870	10.239	-28.133	-1.517	2.424	8.318	29.564
log diff of inverse of rel. price of inv	0.641	0.625	-0.639	0.186	0.603	1.086	2.450
log diff of real consumption	2.008	0.808	0.308	1.381	1.970	2.449	3.897
log diff of real investment	2.104	2.399	-4.956	0.392	1.788	3.384	9.875
log diff of real debt	2.117	2.100	-1.305	1.195	1.976	2.649	13.553
employment share	0.568	0.002	0.563	0.566	0.568	0.570	0.571

We plot the Chinese data series in the Figure 2.7. First, we can see that the inverse of relative price of investment is increasing, which reflects that capital goods technology grows faster than consumption goods technology. Second, the real debt has been increasing in the last decade. Table 2.9 shows the statistical summary for these time series. First, we can see that real land price grows faster than real consumption, real investment, and real debt. Second, we can see that the inverse of relative price of investment grows at the average rate of 0.64% each quarter, which demonstrates the relative faster advance in the capital goods technology. Third, the growth rates for real investment and real debt show the similar magnitude in the mean and standard deviation. Their average growth rates are slightly higher than that of real consumption but much more volatile.

2.5.3 Parameter Estimates

We follow exactly the same setup as Liu et al. [49] to estimate our model. In their model, they not only directly fix the values for some parameters but also target some moments for the economy. First, we fix the capital share α at 0.3 and LTV ratio in the steady state $\bar{\theta}$ at 0.75. Second, we target the following moments in the steady state: (1) the real interest rate is 1.01; (2) the quarterly capital-output ratio is 4.6194; (3) the quarterly investment-capital ratio is 0.0523; (4) the quarterly land-output ratio is 2.6; (5) the quarterly housing-output ratio is 5.8011. From those five moments, we can back out five parameter values (β, $\bar{\lambda}_a$, $\bar{\varphi}$, ϕ, and δ) after obtaining the estimated parameter values. We set our prior distributions for the parameters exactly following Liu et al. [49] in the Table 2.10.[5]

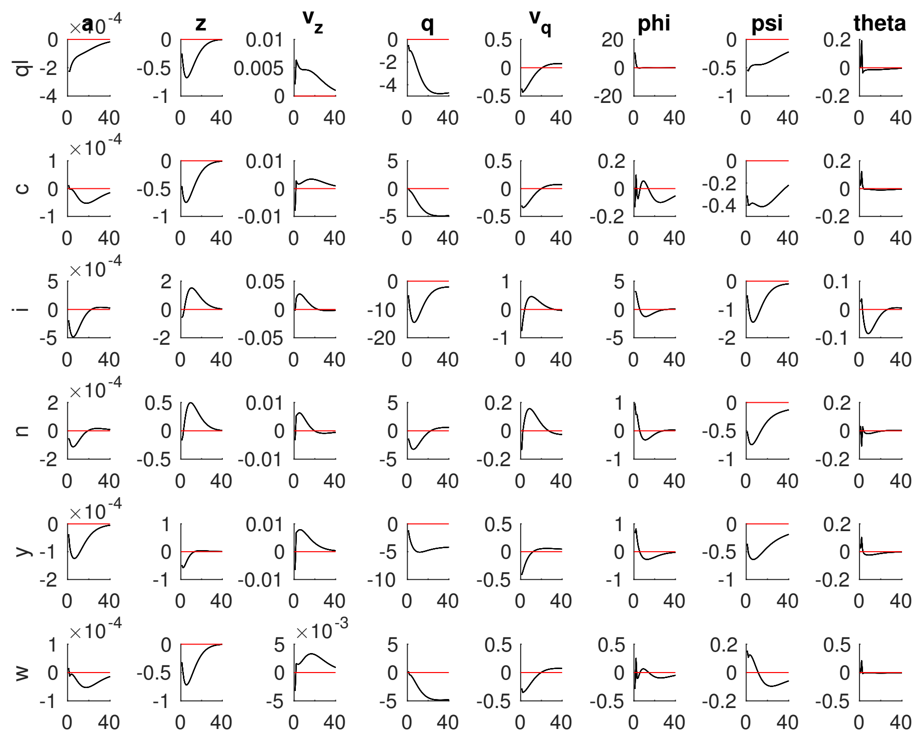
The estimation results are in the Table 2.10. First, we can see that households have small consumption habit coefficient while entrepreneurs have a large one. Second, the persistence coefficients for discount factor shocks, transitory and permanent investment technology shocks, labor supply shocks are relatively large, which means that those shocks are persistent. Third, housing demand shocks have large standard deviation.

[5]The prior distributions for standard deviations follow Gamma distribution with hyper-parameter values of 0.3261 and 1.45e-4. Their priors do not have finite mean and standard deviation, but their 90% probability interval are between 0.0001 and 2.

Table 2.10: LWZ Prior and Posterior Distributions

	Prior			Model Posterior
	Distribution	Mean	Std	Mode
γ_h	BETA	0.333	0.235	0.236
γ_e	BETA	0.333	0.235	0.934
Ω	GAMMA	2	2	3.132
$100(g_\gamma - 1)$	GAMMA	0.618	0.453	0.128
$100(\bar{\lambda}_q - 1)$	GAMMA	0.618	0.453	0.127
ρ_a	BETA	0.333	0.453	0.886
ρ_z	BETA	0.333	0.453	0.537
ρ_{v_z}	BETA	0.333	0.453	0.019
ρ_q	BETA	0.333	0.453	0.998
ρ_{v_q}	BETA	0.333	0.453	0.964
ρ_φ	BETA	0.333	0.453	0.271
ρ_ψ	BETA	0.333	0.453	0.967
ρ_θ	BETA	0.333	0.453	0.000
σ_a	InvGAMMA	Inf	Inf	0.000
σ_z	InvGAMMA	Inf	Inf	0.009
σ_{v_z}	InvGAMMA	Inf	Inf	0.000
σ_q	InvGAMMA	Inf	Inf	0.005
σ_{v_q}	InvGAMMA	Inf	Inf	0.008
σ_φ	InvGAMMA	Inf	Inf	9.548
σ_ψ	InvGAMMA	Inf	Inf	0.005
σ_θ	InvGAMMA	Inf	Inf	0.010

Figure 2.8: LWZ Model: Impulse Response



2.5.4 Properties of the Estimated Model

In the figure 2.8, we plot the impulse response functions for the estimated model. In the figure, we focus on six variables (land price 'ql', consumption 'c', investment 'i', labor 'n', output 'y', wage 'w') and all the shocks (discount factor 'a', permanent component of technology 'z', transitory component of technology 'v_z', permanent component of investment technology 'q', transitory component of investment technology 'v_q', housing demand 'phi', labor disutility 'psi', and collateral constraint 'theta'). First, we can see that the permanent component of investment technology shocks has large effects on all the variables. And its effects on land price, consumption and wage are long-lasting. Second, housing demand shocks have large effects on land price in the short-run but not in the long run, while effects of permanent component of investment technology shocks on land price are small at the beginning but become large in the long run. Also, housing demand shocks have sizable effects on investment. Third, the directions of permanent component of investment technology shocks and housing demand shocks are the same for land price and investment. This provides some potential to concile with the fact that there have been booms both in investment and in housing/land price in China in the last decade.

The variance decomposition results are in the Table 2.11. First, in the long run, the permanent components for investment technology shocks are dominant for all the variables. This is consistent with a large persistent coefficient for this shock. Second, in the short run, other than land price, housing demand shocks have sizable effects on the variances of investment, labor, and wage. Third, technology shocks explain almost half of the variance for consumption in the short run.

Table 2.11: LWZ Model: Variance Decomposition

	a	z	v_z	q	v_q	φ	ψ	θ
				Land Price				
Unc	0.00	0.06	0.00	98.12	0.02	1.69	0.09	0.00
1Q	0.00	0.05	0.00	0.21	0.11	99.36	0.25	0.00
4Q	0.00	0.73	0.00	2.63	0.49	95.26	0.83	0.02
8Q	0.00	1.85	0.00	10.26	0.73	85.80	1.31	0.02
40Q	0.00	0.62	0.00	81.69	0.18	16.67	0.82	0.00
				Consumption				
Unc	0.00	0.07	0.00	99.83	0.01	0.00	0.07	0.00
1Q	0.00	49.56	0.01	0.23	22.34	4.24	23.46	0.11
4Q	0.00	46.31	0.00	23.48	12.37	1.01	16.34	0.46
8Q	0.00	25.08	0.00	61.68	4.98	0.27	7.85	0.10
40Q	0.00	0.92	0.00	98.11	0.14	0.03	0.77	0.00
				Investment				
Unc	0.00	0.66	0.00	97.63	0.07	1.00	0.60	0.00
1Q	0.00	0.90	0.00	68.57	1.59	28.23	0.70	0.00
4Q	0.00	0.15	0.00	92.79	0.18	5.93	0.94	0.00
8Q	0.00	0.44	0.00	96.36	0.09	2.13	0.96	0.00
40Q	0.00	0.99	0.00	96.47	0.11	1.50	0.90	0.00
				Labor				
Unc	0.00	1.27	0.00	93.76	0.13	1.19	3.62	0.00
1Q	0.00	0.88	0.00	60.78	0.57	29.31	8.39	0.03
4Q	0.00	0.32	0.00	86.40	0.10	6.55	6.55	0.05
8Q	0.00	1.12	0.00	90.60	0.15	2.59	5.49	0.02
40Q	0.00	2.42	0.00	88.37	0.23	2.26	6.67	0.01
				Output				
Unc	0.00	0.02	0.00	99.79	0.01	0.03	0.13	0.00
1Q	0.00	9.82	0.00	58.53	7.24	18.95	5.42	0.02
4Q	0.00	3.22	0.00	88.11	1.31	4.08	3.22	0.03
8Q	0.00	1.23	0.00	94.78	0.44	1.23	2.29	0.01
40Q	0.00	0.18	0.00	98.47	0.07	0.29	0.96	0.00
				Wage				
Unc	0.00	0.07	0.00	99.90	0.01	0.00	0.00	0.00
1Q	0.00	30.71	0.00	12.41	24.14	25.41	7.27	0.03
4Q	0.00	44.53	0.00	29.26	15.90	6.16	2.44	1.67
8Q	0.00	24.70	0.00	67.06	5.73	1.36	0.78	0.35
40Q	0.00	0.84	0.00	98.89	0.15	0.05	0.04	0.00

2.6 Discussion

This section summarizes the estimation results in the above sections and discuss issues of applying DSGE estimation models to the Chinese economy.

For Smets and Wouters [71]'s model, we get very strange estimation results about the capital share in the goods production function if we estimate the capital share and the growth trend for the model. This issue may come from the different growth rate for consumption and investment during the last two decades. Another issue is related to the monetary policy rule. The estimated results for different variants of Smets and Wouters [71]'s model seem to be different from the traditional Taylor rule in U.S.. The Federal Reserve Bank has long built a reputation for targeting the inflation rate, so the inflation coefficient in the monetary policy for U.S. is larger than that in China. In the Model 2 and 3, we can see that the coefficients related to output is negative for current output and close to zero for output growth. However, those coefficients for U.S. are positive to reflect the stabilization objective of the central bank in U.S. [81]. However, Chen et al. [18] find that monetary policy has a stimulus role rather than a stabilization role in China. This raises the question about validation of Taylor rule in China. To understand more about the monetary policy in China, we also need to characterize the objective of the Central Bank in China other than the standard stabilization objective, which may be consistent with the empirical evidence.

Both Iacoviello and Neri [42] and Liu et al. [49] have housing in their models and can provide some insights on the recent housing boom in China. Iacoviello and Neri [42]'s model is hard to concile with the fact that both housing price and residential investment have been growing very rapidly in the last decade. Also, in their model, there are not many spillovers between housing market and investment, which may be hard to explain the impressive joint growth of investment and housing market observed in China. This may come from the main mechanism of Iacoviello and Neri [42]'s model, which relies mainly on the effects of housing price on consumption for the constrained households. Thus, the effects of housing market on investment are only indirect. Liu et al. [49] has a different mechanism than Iacoviello and Neri [42], which is based on the effects of housing market on borrowing constraint for the entrepreneurs, so housing price can affect the investment decisions for the entrepreneurs in a more direct channel. The estimated results show that Liu et al. [49]'s model can generate substantial spillovers between the housing market and investment, which is more consistent with the observed facts in China. In this sense, Liu et al. [49]'s model has more potential to explain the recent housing booms in China. However, this model is based on real business cycles model, so we cannot study the role of monetary policy here, which may be an interesting topic for future research.

Chapter 3

Optimal Monetary Policy in China

3.1 Model

In this section, we will provide a micro-foundation for the whole structural model and derive the optimal monetary policy for the Central Bank. First, we derive the three New Keynesian equations with targeted output from households' problem, firms' problem, and aggregate price dynamics. Second, we introduce the Central Bank's asymmetric loss function and discuss how it is related to the specific institutional background in China. Third, we derive the optimal monetary policy with the Central Bank's loss function and New Keynesian equations with targeted output and discuss the conditions under which we can get the monetary policy rule consistent with Chen et al. [18].

There are three elements in this model different from the standard simple New Keynesian model. First, we assume that agents in the model use adaptive learning expectations, so we can avoid the difficulty of solving an endogenous regime-switching model. Second, we let the government collect revenue tax on the firms to introduce the targeted output growth rate in the model. Third, the Central Bank's loss function deviates from the standard quadratic from and has some asymmetric features. In the model below, we will discuss more about those elements in detail.

3.1.1 New Keynesian Equations with Targeted Output

3.1.1.1 Household's Problem

There is a representative household that consumes goods, supplies labor, accumulates bonds, holds shares in firms, receives lump-sum transfer payments from government, and accumulates money. It's problem is

$$\max \; E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \varphi \frac{N_t^{1+\psi}}{1+\psi} + \nu \log(M_t/P_t) \right]$$

where C_t is a consumption index given by

$$C_t = \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

with $C_t(i)$ representing the quantity of good i consumed by the representative household in the period t. We assume that there exist a continuum of goods denoted by the interval $[0, 1]$. The budget constraint within each period is

$$\int_0^1 P_t(i)C_t(i)di + B_t + M_t \leq (1+R_{t-1})B_{t-1} + M_{t-1} + W_tN_t + T_t$$

where T_t includes both dividends from firms and lump-sum transfer payments from government in the period t.

The household's decision on allocating its consumption expenditures among the differentiated goods can be separately solved from other decisions. This problem requires that the consumption index C_t be maximized for any given level of consumption expenditures $\int_0^1 P_t(i)C_t(i)di$. The solution to this problem gives us the optimal demand for differential goods

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t, \ \forall\ i \in [0,1]$$

where $P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon}di\right)^{\frac{1}{1-\varepsilon}}$. And we can also get the following equation

$$P_tC_t = \int_0^1 P_t(i)C_t(i)di$$

Taking advantage of the above results, we can re-write household's problem

$$\max_{C_t,N_t,M_t,B_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \varphi\frac{N_t^{1+\psi}}{1+\psi} + \nu\log(M_t/P_t)\right]$$

subject to the intra-period budget constraint

$$P_tC_t + B_t + M_t \leq (1+R_{t-1})B_{t-1} + M_{t-1} + W_tN_t + T_t$$

The optimal decisions can be described by the following first-order conditions

$$\begin{aligned}
\partial N_t: &\quad \frac{\varphi N_t^{\psi}}{C_t^{-\sigma}} = \frac{W_t}{P_t} \\
\partial B_t: &\quad 1 = \beta E_t\left(\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{P_t}{P_{t+1}}(1+R_t)\right) \\
\partial M_t: &\quad \nu\left(\frac{M_t}{P_t}\right)^{-1} = C_t^{-\sigma}\frac{R_t}{1+R_t}
\end{aligned}$$

We can log-linearize the F.O.C. around the steady state, and we get

$$\partial N_t: \quad \psi \hat{n}_t + \sigma \hat{c}_t = \hat{w}_t - \hat{p}_t \tag{3.1}$$

$$\partial B_t: \quad \hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma}(\tilde{r}_t - E_t \tilde{\pi}_{t+1}) \tag{3.2}$$

$$\partial M_t: \quad \hat{m}_t - \hat{p}_t = \sigma \hat{c}_t - \frac{1}{R(1+R)}\tilde{r}_t \tag{3.3}$$

where we use 'hat' to denote percentage deviation from the steady state for variables and 'tilde' to denote linear deviation from the steady state for variables[1].

3.1.1.2 Aggregate Price Dynamics

Next we will derive the aggregate price dynamics. Here we follow Calvo [13]'s setup by assuming that in each period only a measure of $1-\theta$ firms can reset their prices while the other θ measure of firms cannot adjust their prices. Since each firm faces the same price-setting problem, they choose the same price P_t^* if they can reset their price in the period t and the other firms continue their last-period price $P_{t-1}(i)$. From $P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$, we can get

$$P_t^{1-\varepsilon} = \int_0^1 P_t(i)^{1-\varepsilon} di = (1-\theta)(P_t^*)^{1-\varepsilon} + \theta \int_0^1 P_{t-1}(i)^{1-\varepsilon} di = (1-\theta)(P_t^*)^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon}$$

Then we can get the inflation

$$\Pi_t^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}$$

where $\Pi_t \equiv P_t/P_{t-1}$. We log-linearize it around the steady state, and we get

$$\tilde{\pi}_t = (1-\theta)(\hat{p}_t^* - \hat{p}_{t-1}) \tag{3.4}$$

3.1.1.3 Firm's Problem

The firms produce goods with labor and have a linear production technology

$$Y_t(i) = A_t N_t(i)$$

[1]In simplicity, for variable x with a steady state value $\bar{x}$, $\hat{x} = dx/\bar{x}$, and $\tilde{x} = dx$.

A firm with price-resetting opportunity in period t will choose the price P_t^* which maximizes all its discounted profits generated while that price remains effective. Formally, it solves the following problem

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t Q_{t,t+k} \left[(1-\tau_{t+k})P_t^* Y_{t+k}(i) - MC_{t+k}P_{t+k}Y_{t+k}(i)\right]$$

subject to its demand function

$$Y_{t+k}(i) = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} C_{t+k}$$

where $Q_{t,t+k} \equiv \beta^k (C_{t+k}/C_t)^{-\sigma}(P_t/P_{t+k})$ is the stochastic discount factor for nominal profits, MC_{t+k} is the real marginal cost in period $t+k$, and τ_{t+k} is the revenue tax. The revenue tax is set by the government and transfered to the household in lump-sum transfer payments. To introduce the targeted GDP growth rate in the model, we assume that the government sets the revenue tax τ_t to achieve the targeted output growth in the flexible price environment. In China, the targeted growth rate is an important part in the monetary policy [18]. However, in the standard New Keynesian model, it only contains the potential output, which is described as the output under flexible-price environment. Our model introduces the revenue tax in this novel way to include the targeted output growth rate in the New Keynesian framework.

We can get the F.O.C. for the price setting problem that

$$\sum_{k=0}^{\infty} \theta^k E_t Q_{t,t+k} Y_{t+k}(i) \left[(1-\tau_{t+k})P_t^* - \frac{\varepsilon}{\varepsilon-1} MC_{t+k}P_{t+k}\right] = 0$$

By log-linearizing it around the steady state, we can get

$$\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[-\frac{\tau}{1-\tau}\hat{\tau}_{t+k} + \hat{p}_t^* - \hat{mc}_{t+k} - \hat{p}_{t+k}\right] = 0$$

We can re-arrange the equation and get

$$\hat{p}_t^* - \hat{p}_{t-1} = (1-\beta\theta)\left(\hat{mc}_t + \hat{p}_t - \hat{p}_{t-1} + \frac{\tau}{1-\tau}\hat{\tau}_t\right) + \beta\theta(E_t\hat{p}_{t+1}^* - \hat{p}_{t-1})$$

Using equation (3.4), we can express the above equation with inflation

$$\tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \frac{(1-\beta\theta)(1-\theta)}{\theta}\left(\hat{mc}_t + \frac{\tau}{1-\tau}\hat{\tau}_t\right) \tag{3.5}$$

From this equation, we can see that current inflation is positively related to the expected inflation in the next period. Also, current inflation increases as real marginal cost and revenue tax increase.

3.1.1.4 Equilibrium

In the equilibrium, market clearing in the goods markets require

$$Y_t(i) = C_t(i), \forall\ i \in [0,1]$$

Define aggregate output $Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$, and we can get

$$Y_t = C_t.$$

From the labor market clearing, we can get

$$N_t = \int_0^1 N_t(i)di = \frac{Y_t}{A_t}\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di$$

From firms' production function, we can get the real marginal cost

$$MC_t = \frac{W_t}{P_t}\frac{1}{A_t}$$

By log-linearizing the above equations, we can get

$$\hat{y}_t = \hat{c}_t \tag{3.6}$$

$$\hat{n}_t = \hat{y}_t - \hat{a}_t \tag{3.7}$$

$$\hat{mc}_t = \hat{w}_t - \hat{p}_t - \hat{a}_t \tag{3.8}$$

3.1.1.5 Flexible-price Economy

Under flexible-price economy, all firms can reset their prices in each period. We use superscript f to denote the variables under flexible-price economy. Then we can get the following equations.

$$\hat{mc}_t^f = -\frac{\tau}{1-\tau}\hat{\tau}_t$$

$$\hat{mc}_t^f = \hat{w}_t^f - \hat{p}_t^f - \hat{a}_t$$

$$\hat{y}_t^f = \hat{a}_t + \hat{n}_t^f$$

$$\psi\hat{n}_t^f + \sigma\hat{c}_t^f = \hat{w}_t^f - \hat{p}_t^f$$

Substitute with above equations and we can get

$$\hat{y}_t^f = \frac{1+\psi}{\sigma+\psi}\hat{a}_t - \frac{1}{\sigma+\psi}\frac{\tau}{1-\tau}\hat{\tau}_t \tag{3.9}$$

3.1.1.6 Sticky-price Economy

From equations (3.1), (3.7), and (3.8)

$$\begin{aligned}\hat{mc}_t + \frac{\tau}{1-\tau}\hat{\tau}_t &= (\sigma+\psi)\hat{y}_t - (1+\psi)\hat{a}_t + \frac{\tau}{1-\tau}\hat{\tau}_t \\ &= (\sigma+\psi)(\hat{y}_t - \hat{y}_t^f)\end{aligned}$$

Substitute the above equation into equation (3.4) and substitute $\hat{y}_t^f$ with $\hat{y}_t^*$, we get the Phillips curve

$$\tilde{\pi}_t = \beta E_t\tilde{\pi}_{t+1} + \frac{(1-\beta\theta)(1-\theta)(\sigma+\psi)}{\theta}(\hat{y}_t - \hat{y}_t^*) \tag{3.10}$$

Since the government set τ_t to achieve the targeted output under the flexible-price economy, i.e. $\hat{y}_t^*$ is the output associated with the targeted output growth rate.

Substitute equation (3.6) into (3.2) and (3.3), we get the demand equation and money demand equation

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}(\tilde{r}_t - E_t\tilde{\pi}_{t+1}) \tag{3.11}$$

$$\hat{m}_t - \hat{p}_t = \sigma\hat{y}_t - \frac{1}{R(1+R)}\tilde{r}_t \tag{3.12}$$

Equations (3.10), (3.11), and (3.12) describe the optimal decisions for the private agents (household and firms). To close the economy, we need the monetary policy on the part of Central Bank, which will be derived later.

3.1.1.7 Adaptive Learning

In this part, we deviate from the rational expectation and assume that agents' beliefs follows adaptive learning process. Specifically, the adaptive learning expectation about future output in the next period is weighted average of current period output and targeted output., i.e.

$$E_t^a \hat{y}_{t+1} = \alpha_y \hat{y}_t + (1 - \alpha_y) \hat{y}_t^* \tag{3.13}$$

Similarly, the expectation of inflation also follows the adaptive form

$$E_t^a \tilde{\pi}_{t+1} = \alpha_\pi \tilde{\pi}_t \tag{3.14}$$

We adopts adaptive learning belief for several reasons. First, more and more papers in the business cycle literature try to deviate from rational expectation equilibrium to explain the business cycle properties [27, 28, 68]. Second, the adoption of adaptive learning is more from the practical perspective in this chapter. Since the monetary policy follows the endogenous regime-switching process, it is very difficult to solve the rational expectation equilibrium. Furthermore, the Central Bank's problem becomes even more difficult if the Central Bank take the rational expectation of the private agents into consideration when it sets the optimal monetary policy. This is because the monetary policy itself would also alternate the private agents' expectation formation. Third, there has been a discussion about the commitment problem for the Central Bank [81], and the optimal monetary policy may take Ramsey form or discretion form. However, in the adaptive learning framework, commitment problem is not a issue since the private agents' decision is backward looking rather than forward looking. Thus, there is only one optimal monetary policy for the Central Bank. Fourth, the adaptive learning expectation assumption allows us to exploit an identification strategy in Section 3.2, which makes the estimation of the whole model feasible.

3.1.1.8 New Keynesian Equation in the Growth Form

In this part, we will transform output, targeted output, and money supply into the growth rate form, but keep inflation and interest rate in their level form. After substituting adaptive learning equations (3.13) and (3.14) into equations (3.11), (3.12) and (3.10), and transforming them into growth rates, we get the demand equation

$$(1 - \alpha_y)(g_{y,t} - g_{y,t}^*) + \tau(r_t - \alpha_\pi \pi_t) + C_y + u_{d,t} = 0 \tag{3.15}$$

where $\tau = 1/\sigma$. Following Chen et al. [18], we define the growth rate for the targeted output $g^*_{y,t}$ as the logarithm difference between targeted output and last period output, i.e. $y^*_t - y_{t-1}$. Thus, the gap between actual output and targeted output $y_t - y^*_t$ equals to the gap between actual output growth rate and targeted output growth rate $g_{y,t} - g^*_{y,t}$.

The money demand equation is

$$g_{m,t} - \pi_t = \eta_1 g_{y,t} - \eta_r(r_t - r_{t-1}) + C_m + u_{b,t} \tag{3.16}$$

where $\eta_1 = \sigma$, and $\eta_r = 1/(R(1+R))$. Since $\tau = 1/\eta_1$, we can re-write the demand equation (3.15) as

$$(1-\alpha_y)\eta_1(g_{y,t} - g^*_{y,t}) + r_t - \alpha_\pi \pi_t + C_y + u_{d,t} = 0 \tag{3.17}$$

And the inflation equation is

$$\pi_t = \frac{(1-\beta\theta)(1-\theta)(\sigma+\psi)}{(1-\alpha_\pi\beta)\theta}(g_{y,t} - g^*_{y,t}) + C_\pi + u_{s,t}$$

We can simplify the above equation as

$$\pi_t = \kappa(g_{y,t} - g^*_{y,t}) + C_\pi + u_{s,t} \tag{3.18}$$

where $\kappa = \frac{(1-\beta\theta)(1-\theta)(\sigma+\psi)}{(1-\alpha_\pi\beta)\theta}$.

3.1.2 Central Bank's Loss function

To understand the role of the government's asymmetric preference for output growth, we begin with the conventional loss function for output growth

$$L^y_t = \delta\left(g_{y,t} - \bar{g}_{y,t}\right)^2, \tag{3.19}$$

where $\bar{g}_{y,t}$ represents the potential GDP growth. The overall loss function is

$$\begin{aligned} L_t &= L^y_t + \lambda\left(\pi_t - \pi^*\right)^2 + \left(g_{m,t} - g_{m,t-1}\right)^2 \\ &= \delta\left(g_{y,t} - \bar{g}_{y,t}\right)^2 + \lambda\left(\pi_t - \pi^*\right)^2 + \left(g_{m,t} - g_{m,t-1}\right)^2 \end{aligned} \tag{3.20}$$

This loss function suffers from two problems. First, for transitional economies like China, the concept of potential GDP is not well defined. The transition path for the Chinese economy is characterized by steady increases of the share of investment in GDP, the share of medium- and long-term loans in total loans, and the share of revenues in heavy industries in total output since the late 1990s [14]. China is still in the transition process. In such a policy environment, it is practically difficult to define what constitutes potential or trend output growth like those in advanced countries. Second, even if we avoid the issue related to the concept of potential GDP by setting $\bar{g}_{y,t} = g^*_{y,t}$, the quadratic output loss function (3.19) fails to represent the Chinese government's preference for growth to be above the target (that is, more growth is preferred by the Chinese government).

To approximate the government's practical preference in a tractable form, we apply the prospect theory proposed by Kahneman and Tversky [43]. In their decision theory under risk, the value function is (1) concave for gains and convex for losses and (2) steeper for losses than for gains. For our case, the reference point for decision choice is the GDP target $g^*_{y,t}$. Using the prospect theory, we generalize the loss function (3.19) to be[2]

$$L^y_t = \delta_t \left(g_{y,t} - g^*_{y,t}\right)^2 \tag{3.21}$$

with

$$\delta_t = \begin{cases} \delta_b & \text{if } g_{y,t} - g^*_{y,t} < 0 \\ -\delta_a & \text{if } g_{y,t} - g^*_{y,t} \geq 0 \end{cases},$$

where $\delta_b > \delta_a > 0$, this loss function becomes asymmetric. The parametric constraint, $\delta_b > \delta_a > 0$, implies that the marginal loss for the government when actual GDP growth misses the target is larger than the marginal gain when actual GDP growth is already above the target. The negative sign for the weight on $(g_{y,t} - g^*_{y,t})^2$ is necessary to ensure the concave gain required by the prospect theory of Kahneman and Tversky [43]; that is, the loss declines as GDP growth continues to rise in the growth region $g_{y,t} - g^*_{y,t} \geq 0$. This captures the pro-growth preference for the Chinese government. The piecewise quadratic form of loss function (3.21) enables us to obtain a closed-form solution to optimal monetary policy.

The purpose of this chapter is not to justify the generalized loss function (3.21) can apply all the economy around the world. The Chinese government's objective function is unlikely

[2]Potentially the coefficient for inflate λ could also be time-varying, but here we keep it constant to be consistent with Chen et al. [18].

to be the same as the objective function of the representative agent for an economy with a 1.38 billion population. Among others, political and social stability is a top priority of the government; maintaining control of the communist party over economic and social activities is another. Building a micro foundation of the Chinese government's loss function is a complex issue and a challenging task and thus merits a separate research paper.[3] We instead use a straightforward application of the widely used prospect theory of Kahneman and Tversky [43] for the Chinese government's decision making.

3.1.3 Optimal Monetary Policy

In this part, we will derive the Central Bank's optimal monetary policy given the private agents' decision equations (3.16), (3.17), and (3.18). The Central Bank's problem is

$$\min_{g_{m,t}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2}[\delta_t(g_{y,t} - g^*_{y,t})^2 + \lambda(\pi_t - \pi^*)^2 + (g_{m,t} - g_{m,t-1})^2] \right\}$$

$$\begin{aligned} s.t.\ & g_{m,t} - \pi_t = \eta_1 g_{y,t} - \eta_r(r_t - r_{t-1}) + C_m + u_{b,t} \\ & (1-\alpha_y)\eta_1(g_{y,t} - g^*_{y,t}) + r_t - \alpha_\pi \pi_t + C_y + u_{d,t} = 0 \\ & \pi_t = \kappa(g_{y,t} - g^*_{y,t}) + C_\pi + u_{s,t} \end{aligned}$$

We can write down the Lagrangian equation with Lagrangian multiplier $\mu_{1,t}$ for equation (3.16), Lagrangian multiplier $\mu_{2,t}$ for equation (3.17), and Lagrangian multiplier $\mu_{3,t}$ for equation (3.18)

$$\begin{aligned} L = E_0 \textstyle\sum_{t=0}^{\infty} \beta^t \{ \quad & \tfrac{1}{2}[\delta_t(g_{y,t} - g^*_{y,t})^2 + \lambda(\pi_t - \pi^*)^2 + (g_{m,t} - g_{m,t-1})^2] \\ & +\mu_{1,t}[g_{m,t} - \pi_t - \eta_1 g_{y,t} + \eta_r(r_t - r_{t-1}) - C_m - u_{b,t}] \\ & +\mu_{2,t}[(1-\alpha_y)\eta_1(g_{y,t} - g^*_{y,t}) + r_t - \alpha_\pi \pi_t + C_y + u_{d,t}] \\ & +\mu_{3,t}[\pi_t - \kappa(g_{y,t} - g^*_{y,t}) - C_\pi - u_{s,t}]\} \end{aligned}$$

The first-order conditions with respect to $g_{m,t}$, r_t, π_t, and $g_{y,t}$ are as follows

$$\partial g_{m,t} : g_{m,t} - g_{m,t-1} - \beta(E_t g_{m,t+1} - g_{m,t}) + \mu_{1,t} = 0 \tag{3.22}$$

$$\partial r_t : \eta_r \mu_{1,t} + \mu_{2,t} - \beta \eta_r E_t \mu_{1,t+1} = 0 \tag{3.23}$$

$$\partial \pi_t : \lambda(\pi_t - \pi^*) - \mu_{1,t} - \alpha_\pi \mu_{2,t} + \mu_{3,t} = 0 \tag{3.24}$$

$$\partial g_{y,t} : \delta_t(g_{y,t} - g^*_{y,t}) - \eta_1 \mu_{1,t} + (1-\alpha_y)\eta_1 \mu_{2,t} - \kappa \mu_{3,t} = 0 \tag{3.25}$$

[3] See Li and Zhou [48] for a description of Chinese leaders' own preferences and incentives.

Canceling the term $\mu_{3,t}$ from equation (3.24) and (3.25), we get

$$\kappa\lambda(\pi_t - \pi^*) + \delta_t(g_{y,t} - g^*_{y,t}) - (\kappa + \eta_1)\mu_{1,t} + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)\mu_{2,t} = 0$$

Substitute $\mu_{2,t}$ from the above equation with equation (3.23),

$$\kappa\lambda(\pi_t - \pi^*) + \delta_t(g_{y,t} - g^*_{y,t}) - [(\kappa + \eta_1) + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)\eta_r]\mu_{1,t} + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)\beta\eta_r E_t\mu_{1,t+1} = 0 \tag{3.26}$$

Substitute $\mu_{1,t}$ and $E\mu_{1,t+1}$ with equation (3.22), and use adaptive learning for money growth

$$E^a_t g_{m,t+1} = (1-\alpha_m)\bar{g}_m + \alpha_m g_{m,t}$$

$$E^a_t g_{m,t+2} = (1-\alpha_m^2)\bar{g}_m + \alpha_m^2 g_{m,t}$$

Here this adaptive learning expectation is on the part of Central Bank.

we get

$$\begin{aligned} g_{m,t} = & \frac{\kappa\lambda\pi^* + (1-\alpha_m)\beta[(\kappa+\eta_1) + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)(2-\alpha_m\beta)\eta_r]\bar{g}_m}{(\kappa+\eta_1)(1-\alpha_m\beta+\beta) + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)(1+(1-\alpha_m)(2-\alpha_m\beta)\beta)\eta_r} \\ & + \frac{(\kappa+\eta_1) + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)\eta_r}{(\kappa+\eta_1)(1-\alpha_m\beta+\beta) + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)(1+(1-\alpha_m)(2-\alpha_m\beta)\beta)\eta_r} g_{m,t-1} \\ & - \frac{\kappa\lambda}{(\kappa+\eta_1)(1-\alpha_m\beta+\beta) + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)(1+(1-\alpha_m)(2-\alpha_m\beta)\beta)\eta_r} \pi_t \\ & - \frac{\delta_t}{(\kappa+\eta_1)(1-\alpha_m\beta+\beta) + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)(1+(1-\alpha_m)(2-\alpha_m\beta)\beta)\eta_r} (g_{y,t} - g^*_{y,t}) \end{aligned} \tag{3.27}$$

Define

$$\begin{aligned} \gamma_0 \equiv & \frac{\kappa\lambda\pi^* + (1-\alpha_m)\beta[(\kappa+\eta_1) + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)(2-\alpha_m\beta)\eta_r]\bar{g}_m}{(\kappa+\eta_1)(1-\alpha_m\beta+\beta) + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)(1+(1-\alpha_m)(2-\alpha_m\beta)\beta)\eta_r} \\ \gamma_m \equiv & \frac{(\kappa+\eta_1) + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)\eta_r}{(\kappa+\eta_1)(1-\alpha_m\beta+\beta) + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)(1+(1-\alpha_m)(2-\alpha_m\beta)\beta)\eta_r} \\ \gamma_\pi \equiv & - \frac{\kappa\lambda}{(\kappa+\eta_1)(1-\alpha_m\beta+\beta) + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)(1+(1-\alpha_m)(2-\alpha_m\beta)\beta)\eta_r} \\ \gamma_{y,t} \equiv & \begin{cases} \gamma_{y,a} = \frac{\delta_a}{(\kappa+\eta_1)(1-\alpha_m\beta+\beta) + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)(1+(1-\alpha_m)(2-\alpha_m\beta)\beta)\eta_r}, & \text{if } g_{y,t} - g^*_{y,t} \geq 0 \\ \gamma_{y,b} = -\frac{\delta_b}{(\kappa+\eta_1)(1-\alpha_m\beta+\beta) + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)(1+(1-\alpha_m)(2-\alpha_m\beta)\beta)\eta_r}, & \text{if } g_{y,t} - g^*_{y,t} < 0 \end{cases} \end{aligned}$$

Given that all the adaptive learning parameters (α_y, α_π, and α_m) are smaller than one, the sufficient condition to get the signs that $\gamma_m < 1$, $\gamma_\pi < 0$, $\gamma_a > 0$, and $\gamma_b < 0$ are

$$(1-\alpha_y)\eta_1 - \alpha_\pi \kappa \geq 0 \tag{3.28}$$

Thus, we can get the following proposition

Proposition 1. *Under the adaptive learning expectation and the parameter configuration that* $(1-\alpha_y)\eta_1 - \alpha_\pi\kappa \geq 0$, *optimal monetary policy for the Central Bank's problem follows the form*

$$g_{m,t} = \gamma_0 + \gamma_m g_{m,t-1} + \gamma_\pi \pi_t + \gamma_{y,t}(g_{y,t} - g^*_{y,t}), \tag{3.29}$$

where $\gamma_m < 1$, $\gamma_\pi < 0$, *and the output coefficient is time-varying as*

$$\gamma_{y,t} = \begin{cases} \gamma_{y,a} > 0, & \textit{if } g_{y,t} - g^*_{y,t} \geq 0 \\ \gamma_{y,b} < 0, & \textit{if } g_{y,t} - g^*_{y,t} < 0 \end{cases}.$$

Proposition 1 provides the optimal monetary policy form in the model. This monetary policy rule is similar to the empirical finding in Chen et al. [18], but the difference is the timing in the right hand of the equation. Their empirical rule depends on the lagged inflation rate and lagged gap between actual and targeted output growth rate. Our derived monetary policy depends on current inflation rate and current gap between actual and targeted output growth rate, which shares the same feature with the standard Taylor rule. Even though there is difference in the timing, both monetary policies share the same asymmetric feature.

3.2 Estimation

In this section, we will estimate the structural models with Chinese data. First, we discuss the construction for the data series. Second, we estimate the monetary policy rule following Chen et al. [18]. Third, we estimate the rest of the structural model. Fourth, we make inference on the preference of Central Bank based on the estimated model.

3.2.1 Data

In this part, we will discuss the construction of relevant Chinese data series. We will estimate the model following Section 3.1, so the output, targeted output, and money are all in the growth rate, while inflation and interest rate are in their level.

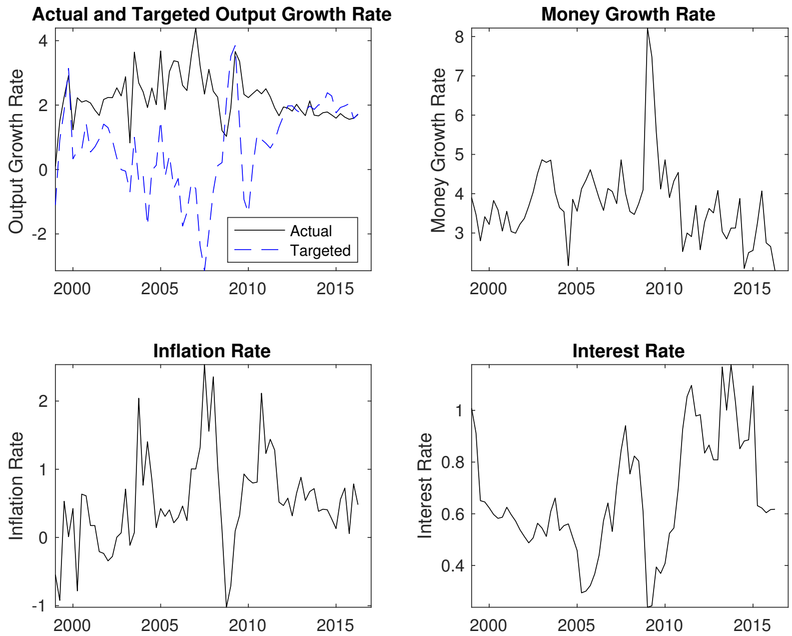
Table 3.1: Statistical Summary (Quarterly)

	mean	std	min	25%	median	75%	max
Output Growth Rate	2.22	0.73	0.10	1.74	2.16	2.51	4.39
Targeted Output Growth Rate	0.71	1.38	-3.14	-0.16	0.90	1.83	3.85
Money Growth Rate	3.74	1.01	2.04	3.13	3.63	4.10	8.22
Inflation Rate	0.52	0.69	-1.02	0.13	0.46	0.81	2.53
Interest Rate	0.67	0.23	0.24	0.53	0.62	0.85	1.18

We construct the Chinese data series following Chen et al. [18]. We construct several data series: (1) real GDP growth rate; (2) targeted GDP growth rate; (3) money (M2) growth rate; (4) inflation rate; (5) interest rate. First, real GDP growth rate is constructed from the nominal GDP level and GDP deflator. We can get the real GDP level first by dividing nominal GDP by the GDP deflator and then calculate real GDP growth rate. Second, for the targeted GDP growth rate, we can calculate the targeted GDP level with the real GDP level four quarters ago and the announced annual GDP growth rate. Next we calculated the targeted GDP growth rate with the targeted GDP level and the real GDP level in the last period. Third, we construct the money growth rate with the levels of M2. Fourth, we use consumer price index to calculate the inflation rate. Fifth, we use 7-day repo rate to construct the interest rate.

The statistical summary for the constructed data is in the Table 3.1. First, we can see that the real GDP growth rate is around 2.2 percent quarterly, which reflects high-speed growth in China. Second, the targeted output growth rate is lower than the actual one on average. This is consistent with Chen et al. [18], who argue that targeted output growth rate often reflects the conservative prospect of the government for the GDP growth. Third, the money growth rate is higher than GDP growth rate. In the same time the money growth is more volatile. Fourth, the inflation rate is around 0.52 percent quarterly on average, but there are periods with negative inflation rate within the sample. Fifth, the interest rate is about 0.67 percent quarterly, and it is relatively stable across the sample period.

Figure 3.1: Quarterly Data



We plot the data series in the Figure 3.1. First, we can see from the figure that the volatility for the real GDP growth rate has decreased after 2010. Also the average GDP growth rate seems to decrease during the same period. Second, there is a peak in the money growth rate around 2009, which corresponds to the stimulus period in China to cope with the financial crisis. Third, the inflation dropped a lot during the financial crisis period due to the weak demand. Fourth, for the 7-day repo, the average repo rate after 2010 is higher than that before 2010.

3.2.2 Estimation for Monetary Policy Rule

Following Chen et al. [18], we characterize the monetary policy rule in China as

$$g_{m,t} = \gamma_0 + \gamma_m g_{m,t-1} + \gamma_\pi \pi_{t-1} + \gamma_{y,t}(g_{y,t-1} - g^*_{y,t-1}) + \sigma_{m,t}\varepsilon_{m,t} \tag{3.30}$$

where the output coefficient is time-varying with the form

$$\gamma_{y,t} = \begin{cases} \gamma_{y,a} > 0, & \text{if } g_{y,t-1} - g^*_{y,t-1} \geq 0 \\ \gamma_{y,b} < 0, & \text{if } g_{y,t-1} - g^*_{y,t-1} < 0 \end{cases}$$

and the subscript "a" stands for "above target" and "b" for "below target". Also, we allow for heteroskedasticity between the two states in the estimation

$$\sigma_{m,t} = \begin{cases} \sigma_{m,a} > 0, & \text{if } g_{y,t-1} - g^*_{y,t-1} \geq 0 \\ \sigma_{m,b} < 0, & \text{if } g_{y,t-1} - g^*_{y,t-1} < 0 \end{cases} .$$

The timing of the monetary policy is different from the derived model, but it is the same as Chen et al. [18]'s specification. In their VAR framework, timing is very important for their identification strategy of the monetary policy shocks. In our model, timing is not crucial, because all variables on the right hand of equation (3.30) ($g_{m,t-1}$, π_{t-1}, and $g_{y,t-1} - g^*_{y,t-1}$) are observable. Whether they are lagged values or current values does not affect the identification of monetary policy rule. However, the adaptive learning expectations in the rest of the model are important, because we are able to avoid solving endogenous regime-switch rational expectation equilibrium. To be consistent with Chen et al. [18], we follow the same timing setup for the monetary policy rule. Thus, the estimation results for the monetary policy rule are the same as Chen et al. [18] in the Table 3.2.

Table 3.2: Estimated monetary policy

Coefficient	Estimate	SE	p-value
γ_m	0.391^{***}	0.101	0.000
γ_π	-0.397^{***}	0.121	0.001
$\gamma_{y,a}$	0.183^{***}	0.060	0.002
$\gamma_{y,b}$	-1.299^{***}	0.499	0.009
$\sigma_{m,a}$	0.005^{***}	0.001	0.000
$\sigma_{m,b}$	0.010^{***}	0.002	0.000

Note. "SE" stands for standard error. The three-star superscript indicates a 1% significance level.

From Table 3.2, we can see first that the coefficient for lagged money growth γ_m is positive and smaller than one, which means that there exists some stickiness in the money supply and the money growth rate is stable in the long run. Second, the coefficient for the inflation γ_π is negative, which means that money growth rate decreases as the lagged inflation increases. This is consistent with the traditional anti-inflation role of Central Bank. Third, the coefficient of output growth rate above target $\gamma_{y,a}$ is positive, and that of output growth rate below target $\gamma_{y,b}$ is negative. This is the main empirical evidence in Chen et al. [18] to argue that the monetary policy is pro-growth in China. The positive coefficient of output growth rate above target means that money supply goes up as the actual output growth rate increases in the last period when it is higher than the targeted one, which represent the pro-growth inclination of the Central Bank. But the negative coefficient of output growth rate below target means that money supply increases as the lagged output gap enlarged. Also, we can see that the absolute value of $\gamma_{y,b}$ is much larger than $\gamma_{y,a}$, which implies the reaction of money supply is more sensitive to the output growth rate gap in the below-target state. Fourth, the shock standard deviation in the below-target state is about twice the size of the one in the above-target state. This implies that the money supply shocks are more volatile in the below-target state.

We can see that the Chinese monetary policy is different from the traditional Taylor rule, which is widely used in advanced countries. One underlying rationale for the traditional Taylor rule is the stabilization role of the Central Bank regarding inflation and output. However, the Central Bank or government in China may have very different preference compared to those advanced countries. Chen et al. [18] provides a detailed introduction and discussion on the institutional background for the role and preference of Central Bank in China. That is why we should expect the monetary policy rule has a very different form in China.

3.2.3 Estimation for the Structural Model

The rest of the model are the structural models related to the private agents. Those equations include (3.16), (3.17), and (3.18), i.e.

$$g_{m,t} - \pi_t = \eta_1 g_{y,t} - \eta_r (r_t - r_{t-1}) + C_m + u_{b,t}$$

$$(1 - \alpha_y)\eta_1 (g_{y,t} - g^*_{y,t}) + r_t - \alpha_\pi \pi_t + C_y + u_{d,t} = 0$$

$$\pi_t = \kappa (g_{y,t} - g^*_{y,t}) + C_\pi + u_{s,t}$$

We assume that all shock processes follow AR(1) process, so the shock processes are

$$u_{d,t} = \rho_d u_{d,t-1} + \varepsilon_{d,t} \tag{3.31}$$

$$u_{b,t} = \rho_b u_{b,t-1} + \varepsilon_{b,t} \tag{3.32}$$

$$u_{s,t} = \rho_s u_{s,t-1} + \varepsilon_{s,t} \tag{3.33}$$

For the measurement equations, we use data series including output growth rate, money growth rate, interest rate, inflation rate, and targeted output growth rate.

$$g_{x,t}^{Data} = g_{x,t} \tag{3.34}$$

$$g_{m,t}^{Data} = g_{m,t} \tag{3.35}$$

$$r_t^{Data} = r_t \tag{3.36}$$

$$\pi_t^{Data} = \pi_t \tag{3.37}$$

$$g_{x,t}^{*Data} = g_{x,t}^{*} \tag{3.38}$$

Here there is no expectation terms in the private agents' part of the model. This is very important for the identification strategy for a model with endogenous regime-switching monetary policy. There has been a literature discussing the Markov-Regime Switching DSGE models, but most of them focus on the exogenous regime switching case [30, 32]. In this model, we have an endogenous regime-switching monetary policy. It would be very difficult to solve this model in rational expectation case, and let alone to estimate this model.

The prior distributions are in the Table 3.3. We assume that the prior distributions of η_1, η_r, and κ all follow the Gamma distribution. Since η_1 is the risk aversion parameter for the households, we set its prior with a mean of 2 and a standard deviation of 1.4. The parameter η_r captures the effects of interest rate on the money demand. We set its prior covering a large range because we do not have much prior information about its value. The parameter κ governs the Phillips curve, which measures the effects of output gap on the inflation. We set this parameter with both mean and standard deviation of 0.9. The parameters, α_y and α_π, determine the adaptive learning process. According to the adaptive learning literature, they should be between zero and one, which both captures the belief stickiness and guarantees the stabilization of the system. We set their prior following Beta distribution with a mean of 0.5 and a standard deviation of 0.22. We set the constant terms in the equations (3.16), (3.17), and (3.18) following Normal distribution with a zero mean and a standard deviation of 5, because we do not have much prior information on those constant terms. For the shock processes, we assume that all the persistence coefficients follow Beta distribution with a mean of 0.33 and a standard deviation of 0.23. For the priors of standard deviation parameters, we follow Liu et al. [49] to set them with inverse Gamma distribution and the 90 percentile interval covering from

1e-4 to 2.

Table 3.3: Parameters Priors and Posterior Mode

	Prior Distribution	Mean	Std	90% interval	Mode	90% interval
η_1	Gamma(2,1)	2	1.4142	[0.35,4.74]	2.9715	[2.12, 4.38]
η_r	Gamma(1,0.02)	50	50	[2.56,149.78]	11.989	[5.52, 19.17]
κ	Gamma(1,1/0.9)	0.9	0.9	[0.04,2.69]	0.2332	[0.09, 0.32]
α_y	Beta(2,2)	0.5	0.2236	[0.13,0.86]	0.9664	[0.91, 0.99]
α_π	Beta(2,2)	0.5	0.2236	[0.13,0.86]	0.0461	[0.00, 0.11]
C_y	Normal(0,5)	0	5	[-8.22,8.22]	-0.790	[-1.00, -0.64]
C_m	Normal(0,5)	0	5	[-8.22,8.22]	-3.475	[-6.82, -1.62]
C_π	Normal(0,5)	0	5	[-8.22,8.22]	0.2034	[-0.00, 0.44]
ρ_b	Beta(1,2)	0.3333	0.2357	[0.02,0.77]	0.0276	[0.00, 0.26]
ρ_d	Beta(1,2)	0.3333	0.2357	[0.02,0.77]	0.7230	[0.62, 0.88]
ρ_s	Beta(1,2)	0.3333	0.2357	[0.02,0.77]	0.4848	[0.30, 0.61]
σ_b	InvGamma(0.3261,1.452e-4)	Inf	Inf	[1e-4,2]	2.1896	[1.65, 3.14]
σ_d	InvGamma(0.3261,1.452e-4)	Inf	Inf	[1e-4,2]	0.1467	[0.11, 0.21]
σ_s	InvGamma(0.3261,1.452e-4)	Inf	Inf	[1e-4,2]	0.5039	[0.45, 0.61]

From the estimation results, we can see that the parameter η_1 has a mode around 3. The mode for the parameter η_r is around 12, which means that the real money demand increases about 12 percent if the real interest rate increase 1 percent. The mode for the parameter κ is 0.23, and this means that 1 percent increase in the output growth rate gap leads to 0.23 percent increase in the inflation in the Phillips curve. Regarding the adaptive learning parameters, α_y is close to one and α_π is close to zero. This implies that when agents form their belief about future output, they put a lot of weight on current output and very little weight on the targeted output. But when agents form their belief about inflation in the next period, they do not put much weight on current inflation and believe inflation in the next period is more close to the average inflation in the long run. We can see that all the shock processes are not very persistent from the persistence coefficients of the shocks. The shock in the money demand equation has a relatively large standard deviation mode, which captures the high volatility of money growth rate in the Table 3.1.

From this estimation, we can get the parameter values related to the equations (3.16), (3.17), and (3.18). However, due to the endogenous regime-switching monetary policy rule, there is non-linearity in the model, so we cannot conduct standard analysis such as impulse response functions and variance decomposition. Alternatively, we will conduct several forecast analyses in the next section.

3.2.4 Inference for Central Bank's Preference

In the Proposition 1, we establish the optimal monetary policy under the Central Bank's preference. The coefficients in the optimal monetary policy rule link with the preference parameters in the Central Bank's loss function and the structure parameters in the model. One condition for the parameters in the Proposition 1 is $(1-\alpha_y)\eta_1 - \alpha_\pi\kappa \geq 0$. From the Table 3.3, we can see that this condition is satisfied, so the derived optimal monetary policy has the same signs as the estimated monetary policy.

Given the estimated monetary policy and the structural model, we are able to identify some the preference parameters, specifically the three relative weight coefficients. We cannot separately identify the discount factor and adaptive learning parameter for Central Bank. From equation (3.27) and the estimation results, we can get

$$\begin{aligned}
\lambda =& -\frac{\gamma_\pi}{\gamma_m\kappa}((\kappa+\eta_1) + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)\eta_r) = 18.6 \\
\delta_a =& \frac{\gamma_a}{\gamma_m}((\kappa+\eta_1) + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)\eta_r) = 2.0 \\
\delta_b =& -\frac{\gamma_b}{\gamma_m}((\kappa+\eta_1) + ((1-\alpha_y)\eta_1 - \alpha_\pi\kappa)\eta_r) = 14.2
\end{aligned}$$

The Central Bank's loss function in each period is in the form $\delta_t(g_{x,t} - g^*_{x,t})^2 + \lambda(\pi_t - \pi^*)^2 + (g_{m,t} - g_{m,t-1})^2$. Note that all the weights are relative one compared to the money growth, which has a normalized weight of one. From the above results, we can see that the weight on the inflation λ is much larger than that on the money growth, and to compensate one percent of inflation fluctuation, the Central Bank would like to tolerate about 4.3 percent change in the money growth rate. The weight on output in the above-target regime is twice the weight to the money growth, which means that one percent increase in the money growth rate can be compensated by 0.7 percent increase in the output growth rate for the Central Bank. However, the weight on output in the below-target regime is much larger, which means that in this regime Central Bank would like to sacrifice one percent increase in the money growth for at least 0.23 percent increase in the output. This means that in the below-target regime Central Bank are more willing to increase money growth to promote the output growth.

3.3 Forecast

In this section, we will conduct several forecast exercises to help us understand the structural DSGE model for the Chinese economy. Due to the non-linearity property of the monetary policy, it is meaningless to conduct unconditional forecasts, because the future shock realizations are able to affect the monetary policy rule. This issue leads us to the conditional forecasts. There is a literature about the methodology of conditional forecasts in the Bayesian VAR and DSGE framework [50, 79]. Higgins et al. [40] also explore the role of monetary policy within the conditional forecast framework. However, most papers in the conditional forecast literature use linear models, while this chapter has a monetary policy rule with endogenous regime-switching property. This non-linearity in the monetary policy prevents us from adopting those conditional forecast methods in the literature, so we turn to the simulation method to conduct the conditional forecast. Since the annual targeted GDP growth rate is set by the government, we treat it as the conditional variable in the forecast exercises. The algorithm is as follows:

Algorithm: given the estimation results for the sample period between 1 and T, we conduct conditional forecast for the period between $T+1$ and $T+h$.

(I) For simulation $i = 1, 2, \ldots, N$,

   (1) calculate the targeted growth rate in the period $t \in [T+1, T+h]$ with annual targeted GDP growth rate, output level one year ago, and lagged output level;

   (2) random draw money supply shock $u_{m,t}$, demand shock $u_{d,t}$, money demand shock $u_{b,t}$, and supply shock $u_{s,t}$ independently for the forecast period $t \in [T+1, T+h]$ (the standard deviation of money supply shocks is varying depend on lagged gap between actual output growth rate and targeted output growth rate);

   (3) calculate the money supply using equation (3.30);

   (4) solve the model in the period t for output growth $g_{y,t}$, inflation rate π_t, and interest rate r_t;

   (5) repeat the above steps (i) to (iii) for period $t+1$ until covering all the periods from $T+1$ to $T+h$, and we get one simulation for the forecast period.

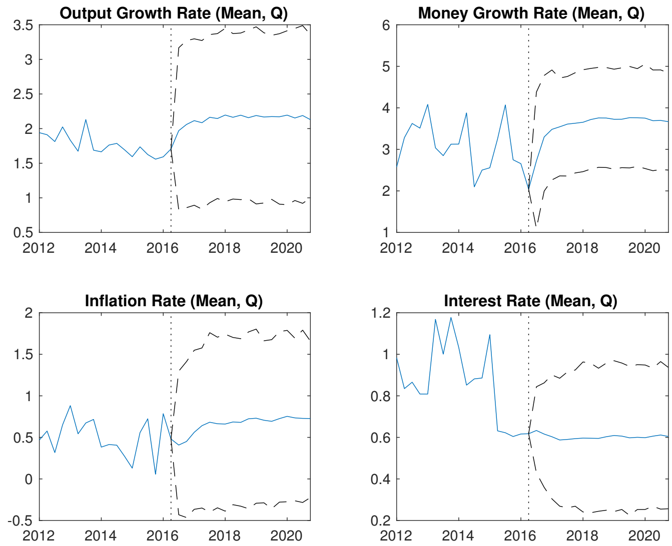
(II) repeat the above steps until we get N simulations, and calculate the mean and confidence interval for the conditional forecast.

One important variable in the model the targeted GDP growth rate, and we assume that the annual targeted GDP growth rate is 6.5% in all the forecasting periods. But we need to transform the annual targeted GDP growth rate into the targeted output growth rate, which depends on the output growth rate path in the past. We will consider three scenarios: (I) the annual targeted GDP growth rate is 6.5%; (II) the annual targeted GDP growth rate is 6.5% and the monetary policy shocks are two standard deviation lower than zero for the first four quarters in the forecast period; (III) the annual targeted GDP growth rate is 6.5% and the money growth rate is two standard deviation lower than its average. Scenario (I) is conditional on the annual targeted GDP growth rate but allows totally random shocks in the model. Scenario (II) is conditional on both the annual targeted GDP growth rate and the money supply shocks in the first four quarters in the forecast period, and all other shocks and money supply shocks in later periods are still random. Scenario (III) is conditional on the annual targeted GDP growth rate and the money growth rate, so monetary policy rule and money supply shocks are abstract from the model. All other shocks are still random. Note that scenarios (I) and (II) still adopt the endogenous monetary policy rule in the model for the forecast period, but scenario (III) adopts a different monetary policy which sticks to a constant money growth rate.

3.3.1 Forecast Scenario I

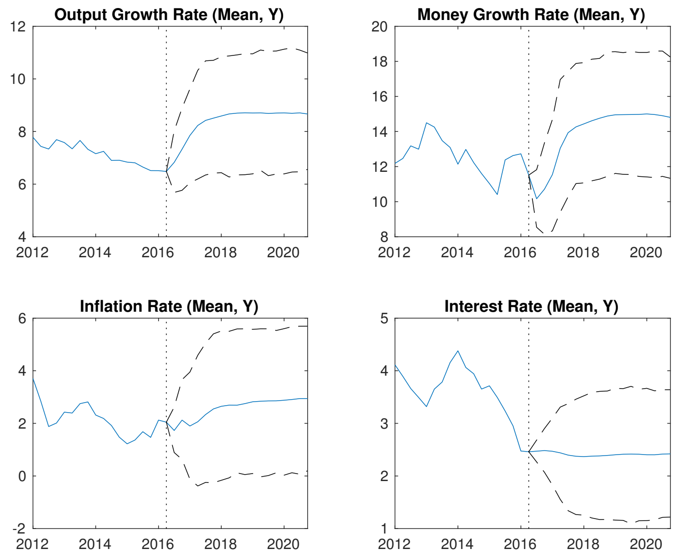
In this part, we conduct conditional forecasts given the annual targeted GDP growth rate is 6.5%. We will focus on four variables: output growth rate, money growth rate, inflation rate, and interest rate.

Figure 3.2: Scenario I Forecast: Quarterly



Note: blue solid line is mean, and black dashed line is 90% confidence interval.

Figure 3.3: Scenario I Forecast: Year-to-Year



Note: blue solid line is mean, and black dashed line is 90% confidence interval.

The forecast results are in the Figure 3.2 and 3.3. Figure 3.2 plots the forecasts on the quarterly basis, while Figure 3.3 transform the forecasts into year-to-year data, which is more closer to the data we discuss generally. Even though they are in different basis, the main message is still the same. We can see that the output growth rate goes back to the relatively high level in the forecast period. This is an issue for all the estimated models, which has a mean-reversing property. All the variables will go to their sample mean in the long run. Since China has kept growing at a high speed for a long time, it is very difficult to obtain a reasonable forecast which captures the trend consistent with recent decline in the GDP growth rate. It would be very hard to believe that China can continue to keep the annual growth rate higher than 8% in the long run. Similarly, after the temporary decline at the beginning of the forecast period, money growth rate reverts to a high level about 14% annually. This is consistent with the high growth rate in the money supply for China in the sample period, but it is not realistic to predict such high money growth rate can sustain in the future. The forecasts for inflation rate and interest rate are still within the reasonable range.

Figure 3.4: Scenario I Forecast: Regime

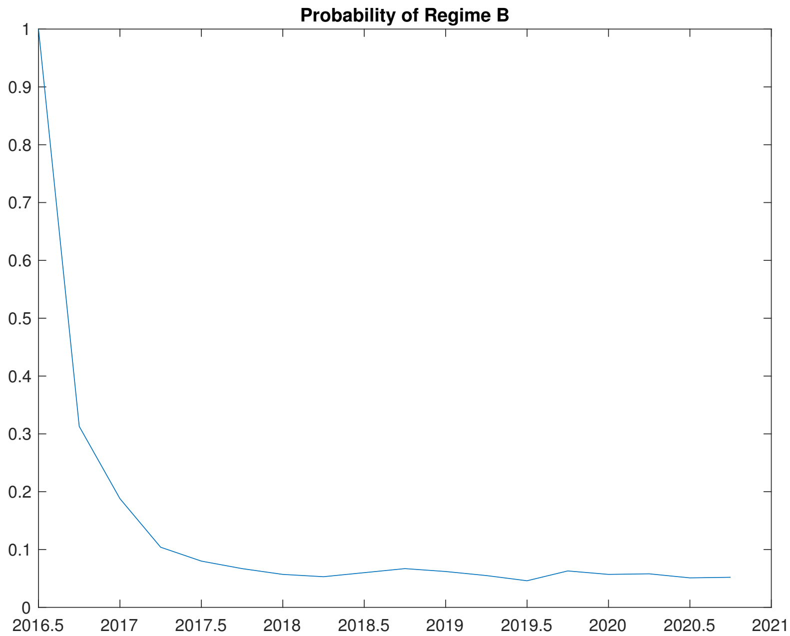
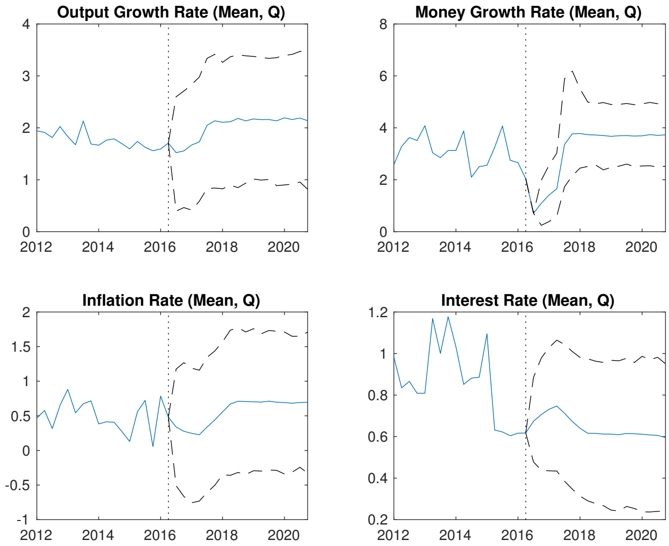


Figure 3.4 plots the probability of the economy staying in the below-target regime, and the probability is based on the simulation frequencies. We can see that at the beginning of the forecast period, the economy has a very high probability in the below-target regime due to the low GDP growth rate at the end of the sample. But after about four quarters, the probability of falling in the below-target regime is stably about 10%. When the output growth rate is stable, the targeted growth rate is also stable, so the probability of falling in the below-target regime is relatively stable.

3.3.2 Forecast Scenario II

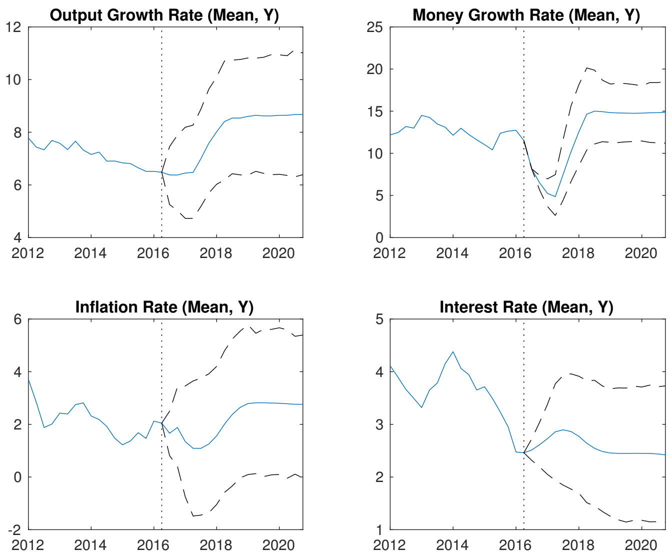
In this part, we conduct conditional forecasts given the annual targeted GDP growth rate is 6.5% and the monetary policy shocks are two standard deviation lower than zero for the first four quarters in the forecast period. After the first four quarters, money supply shocks follow random distribution. This scenario studies the case where Central Bank still follows the monetary policy rule but it would like to lower the money supply temporarily.

Figure 3.5: Scenario II Forecast: Quarterly



Note: blue solid line is mean, and black dashed line is 90% confidence interval.

Figure 3.6: Scenario II Forecast: Year-to-Year

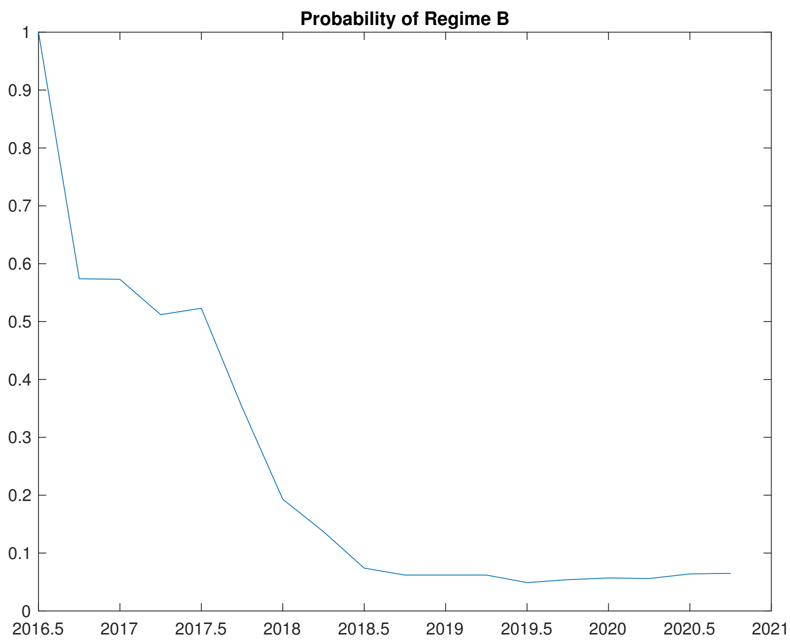


Note: blue solid line is mean, and black dashed line is 90% confidence interval.

The forecast results are in the Figure 3.5 and 3.6. Figure 3.5 plots the forecasts on the quarterly basis, while Figure 3.6 transform the forecasts into year-to-year data. We can see that the output growth rate in the first several quarters in the forecast period follows the declining trend at end the of the sample period, and this is because the negative monetary policy shocks. After those periods, the output growth rate jumps back to very high level just like the scenario I. It seems that it would be very hard to obtain a reasonable prospect for the output growth in China without the help of slowing money growth in the long run. The money growth rate plummets at the beginning and reverts back to its sample average later. The temporary negative shocks for the money supply also decrease inflation rate and increase interest rate temporarily, but in the long run they come back to their stable level.

Figure 3.7 plots the probability of the economy staying in the below-target regime. We can see that the probability of falling in the below-target regime is relatively high in the first four quarters in the forecast period due to the negative money supply shocks. After that, the probability reverts back to stable low probability.

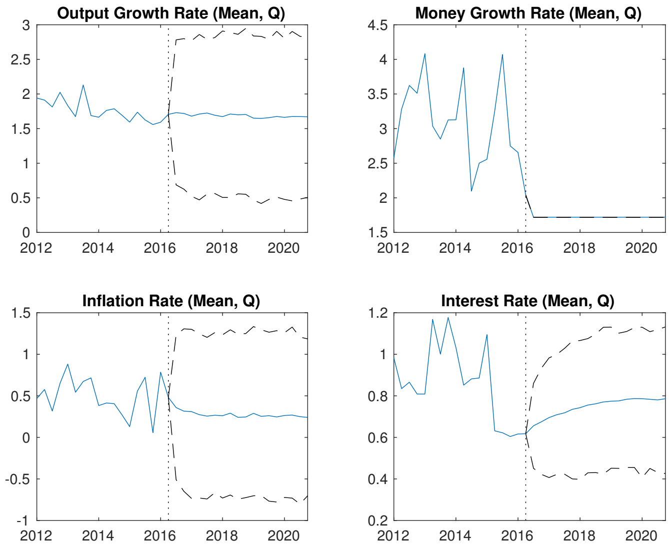
Figure 3.7: Scenario II Forecast: Regime



3.3.3 Forecast Scenario III

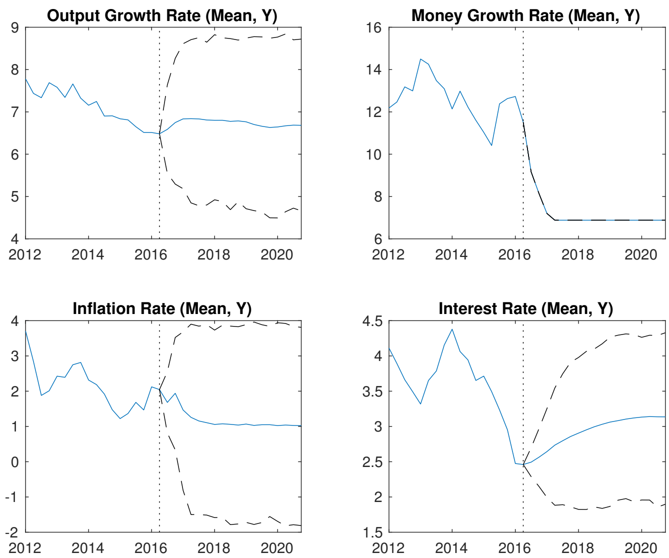
In this part, we conduct conditional forecasts given the annual targeted GDP growth rate is 6.5% and the money growth rate is two standard deviation lower than its average. Thus, in this part, the monetary policy in the forecast period is not following the equation (3.30), but the money growth rate stays a certain level (about annual growth rate 6.88%). This scenario describe the case that the Central Bank suddenly gives up its former monetary policy rule and commits to increasing the money supply in a stable pace. We will study the forecast performance in this scenario.

Figure 3.8: Scenario III Forecast: Quarterly



Note: blue solid line is mean, and black dashed line is 90% confidence interval.

Figure 3.9: Scenario III Forecast: Year-to-Year



Note: blue solid line is mean, and black dashed line is 90% confidence interval.

The forecast results are in the Figure 3.8 and 3.9. Figure 3.8 plots the forecasts on the quarterly basis, while Figure 3.9 transforms the forecasts into year-to-year data. We can see that in this scenario the year-to-year output growth rate is between 6% and 7% in the long run, which seems to be a reasonable forecast. The inflation rate is around 1% in the long run. There is a rising trend for interest rate, but in the long run it stabilizes around 3%. Compared with scenario I and II, we can see that we need a relatively low money growth rate to concile with the fact of declining GDP growth rate recently in China.

Chapter 4

A Model of Bank Credit Cycles

4.1 Literature

This chapter is linked to different strands of the literature on banks' risk-taking, financial innovation, financial crises, and credit cycles.

Our chapter follows the literature on banks' risk-taking and financial stability (see, e.g., Keeley [45], Suarez [74], Matutes and Vives [54], Boyd and Nicoló [11] and Martinez-Miera and Repullo [53]). Unlike most literature, which assumes an exogenous capital structure, in our chapter, bank's leverage is endogenously chosen by the bank as a commitment device to reduce moral hazard. In this respect, our chapter is mostly related to a recent paper by Dell'Ariccia et al. [22], in which it is shown how interest rate affects a bank's risk-taking when the bank can choose its leverage optimally. However, none of these papers consider the role of regulator supervision in alleviating banks' moral hazard.

Our work is related to the literature on regulator supervision (see, e.g., Dewatripont et al. [23], Bhattacharya et al. [8], Prescott [63], Marshall and Prescott [51], Rochet [65]). More recently, Eisenbach et al. [25, 26] formally distinguish bank supervision and regulation and develop a static framework to explain the relationship between supervisory efforts and bank characteristics observed in the data. We depart from this literature by focusing on the connection between the regulator's competence and credit cycles. In this respect, our chapter is closely related to Morrison and White [57, 58]. They show that crises will only occur when public confidence in the regulator's ability to detect bad banks through screening is low. While the regulator's ability is constant in the static model in Morrison and White [57, 58], we study the dynamic interaction between regulator supervision and banks' loophole innovation. In this regard, we consider our model a first attempt to formalize Kane [44]'s influential idea of "regulatory dialectic".

Our interest in endogenous business cycle relates to Suarez and Sussman [75], Martin [52], Favara [31], Myerson [59], and Gu et al. [38]. Among these papers, our chapter is mostly related to Myerson [59], who shows how boom-bust credit cycles can be sustained in economies with moral hazard in financial intermediation. Unlike Myerson [59], our model focuses on the role of regulator supervision in curbing moral hazard in financial intermediation, and more importantly, our chapter generates richer macroeconomic implications consistent with stylized facts found in the empirical literature.

Our work is also linked to the literature on asymmetric business cycles. Some papers, including Veldkamp [78], Ordoñez [61], and Kurlat [46], study the asymmetric nature of the credit cycles from the perspective of the asymmetric information flow over the cycles. A recent paper by Asriyan and Vanasco [6] studies the role of financial intermediaries' learning in generating and amplifying the informational cycles. Our chapter also features a regulator whose expertise grows through learning-by-doing. The key difference is that, in our chapter, the shock to the fundamental is endogenously generated by the banking sector itself rather than exogenously. And our chapter also stresses the role of banks' leverage over the cycle, which is absent in their paper.

There is an emerging literature studying the close relationship between boom and bust in the business cycles. In Gorton and Ordoñez [36, 37], booms are associated with loss of information while crises happen when the economy transits from information-insensitive states to information-sensitive states. Boz and Mendoza [12] and Biais et al. [9] emphasize the role of investors' belief regarding the strength of a financial innovation in generating boom and bust. Good belief builds up in boom periods, but adverse realization of the fundamental decreases belief dramatically and leads to a bust. Boissay et al. [10] build a model featuring an interbank market with moral hazard and adverse selection problems. Increased savings during expansions drive down the return on loans, and when the fundamental becomes weak, the interbank market freezes due to an agency problem, which leads to a bank crisis. Unlike these papers, we build a model focusing on the interaction between regulator supervision and banks' loophole innovation.

4.2 Static Model

Consider an economy with a mass-one continuum of banks, a large mass of households, and a regulator. All parties are risk-neutral. Each bank is endowed with w. Banks can use their own money ω and raise deposit (or more generally issue debt liabilities) from households to make investments. A household can invest in a storage technology with a fixed return of r_0, or invest in the banks as a depositor. We assume that the deposit market is competitive, and there is no deposit insurance, so households are willing to invest in the banks as long as they break even relative to the return on storage technology. If a bank borrows x from depositors, the bank's investment size would be $\omega + x$.[1] We denote a bank's leverage as $L \equiv \frac{\omega+x}{\omega}$. Banks are protected by limited liability and repay depositors only in case of success.

[1] In this chapter, a bank's capital structure is endogenously determined, rather than exogenously given. This treatment is supported by two observations under existing bank regulations. First, a bank's true leverage may be higher than the regulatory limit because banks can overstate capital by not recognizing losses. Second, banks can save on capital by engaging in regulatory arbitrage of capital requirements.

Banks can invest in a safe project or in a risky project. The safe project's payoff is R/η^s with probability η^s and zero with probability $1-\eta^s$, so the expected return of the safe project is R. The risky project is more likely to fail than the safe project but will pay more if it succeeds. More specifically, the success probability of the risky project is $\eta < \eta^s$, and the payoff conditional on project success is $\bar{\lambda}R/\eta$, so the expected payoff of the risky project is $\bar{\lambda}R$. Banks can choose the success probability of the risky project, η, within the interval $[\underline{\eta}, \bar{\eta}]$, with $\bar{\eta} < \eta^s$ and $\bar{\lambda}/\bar{\eta} > 1/\eta^s$. As η is lower, the risky project is less likely to succeed, but, conditional on success, the payoff is higher. Therefore, η is also a measure of the riskiness of the risky project. The lower is η, the more risky is the project. We assume that $R > r_0 > \bar{\lambda}R$. Thus, the safe project has the highest expected return, and the risky project has a lower expected return than the storage technology. Banks' project choices are not observed by depositors and are not contractable.

There is a benevolent regulator who can supervise the banks. To model regulator supervision, we assume that the regulator can prevent banks from choosing high riskiness when taking the risky project.[2] More specifically, when the regulator's supervision ability is η^*, banks can only choose the risky project success probability within the interval $[\eta^*, \bar{\eta}]$. The setup regarding supervision is similar to Eisenbach et al. [26], where the regulator can take corrective actions to reduce the variance of bank' return.

However, the regulator's supervision is not perfect. Sometimes banks may discover a new type of risky project, which is not immediately known by the regulator and households. We call this discovery a successful loophole innovation. If a loophole innovation succeeds, banks are able to take the new risky project with any riskiness levels, since the regulator does not realize that it exists. Borrowing the setup from technology innovation literature, such as Aghion and Howitt [5] and Laeven et al. [47], we assume that only one bank is capable to conduct loophole innovation.[3] We call this bank the capable bank. It is costly for the capable bank to conduct loophole innovation. When the capable bank's effort is $e \in [0, 1]$, the loophole innovation succeeds with probability e. The cost for the capable bank is $\frac{1}{2}ce^2 \cdot (\omega + x)$, where c is the coefficient governing the cost of innovation. If a loophole innovation succeeds, all banks learn about the new loophole, and a risky project immune from supervision is available to them.

The timing of the static model is as follows: at the beginning of the period, each bank

[2] High riskiness would correspond to low success probability in our model.

[3] We can generalize this assumption for N banks, as long as N is finite. Otherwise, innovations succeed every period.

offers a deposit menu to households, which specifies the leverage of the bank and deposit rate. Households decide whether or not to make deposits in the banks. After that, one of the banks knows it is the capable one and exerts loophole innovation effort. If the loophole innovation is successful, a new type of risky project emerges, and all other banks learn about it. Banks make project choices and choose riskiness levels under the regulator's supervision if they invest in the risky project. At the end of the period, banks' projects pay off. Banks pay back the depositors if their projects succeed. Otherwise, they default and go bankrupt.

To characterize the equilibrium, we take the following steps. First, we describe banks' menu choice problem, in which banks choose the deposit menus to maximize their expected profits. Second, we solve the capable bank's loophole innovation problem, in which the capable bank chooses the effort to conduct loophole innovation, given its leverage and interest rate. Third, we impose the equilibrium condition that the expected loophole innovation success probability is consistent with the capable bank's innovation effort, and solve the equilibrium.

First, we describe banks' menu choice problem. A bank's expected profit given leverage and interest rate is[4]

$$(1-p)\max\{RL-\eta^s r(L-1), \bar{\lambda}RL-\eta^* r(L-1)\}+p\max\{RL-\eta^s r(L-1), \bar{\lambda}RL-\underline{\eta} r(L-1)\} \quad (4.1)$$

where p is the probability that a loophole innovation succeeds in equilibrium, which both banks and households take as given. The first term is bank's expected profit if the loophole innovation fails. When the loophole innovation fails, banks are monitored by the regulator, thus the highest riskiness available to them is η^*. Due to limited liability, banks would like to choose the highest riskiness η^* if they invest in the risky project. Banks optimally decide between the safe project and the risky project with riskiness η^*. The second term is bank's expected profit if the loophole innovation is successful. If the loophole innovation succeeds, it provides banks with a new type of risky project to circumvent the regulator's supervision. In this case, if banks invest in the new risky project, they can choose the riskiness $\underline{\eta}$. Banks decide between the safe project and the risky project with riskiness $\underline{\eta}$ to maximize their expected profits.

In this chapter, we focus on the case that banks' leverage is always constrained by the risk-shifting problem, so that their leverage is finite. The following assumption is the sufficient

[4]Since there is a continuum of banks and only one of them is capable, each bank expects itself to be the capable one with a probability of measure zero. Thus, banks do not consider the cost of loophole innovation when they choose the leverage at the beginning of the period.

condition that guarantees it.

Assumption 1. $\frac{(1-\bar{\lambda})R}{(\eta^s-\bar{\eta}^*)r_0} < 1$.

This assumption implies two things. First, banks with sufficiently high leverage will choose the risky project. Second, the maximum supervision ability is not high enough to fully eliminate banks' risk-shifting problem.

To analyze banks' menu choice problem, we can divide the possible menus into three areas according to equation (4.1). First, if the deposit menu $\{L, r\}$ satisfies $RL - \eta^s r(L-1) \geq \bar{\lambda}RL - \underline{\eta}r(L-1)$, banks will never choose the risky project since the safe project yields a higher expected profit. Second, if $\{L, r\}$ satisfies $\bar{\lambda}RL - \underline{\eta}r(L-1) > RL - \eta^s r(L-1) \geq \bar{\lambda}RL - \eta^* r(L-1)$, which project banks will choose depends on whether there is a successful loophole innovation or not. If the loophole innovation fails, banks will be under the monitoring of the regulator with supervision ability η^*. Therefore, they choose the safe project. Otherwise, banks will take advantage of the loophole to circumvent the supervision, and choose the new risky project with the highest riskiness. Third, if $\{L, r\}$ is in the area such that $RL - \eta^s r(L-1) < \bar{\lambda}RL - \eta^* r(L-1)$, banks will always choose the risky project, even if the loophole innovation fails. Banks choosing menus in this area have to offer households very high interest rates to attract deposits, which yields a negative expected profit for banks. Thus, menus in this area are never optimal for banks. In other words, the feasible menus have to provide banks with incentives to invest in the safe project if there is no successful loophole innovation. We can write this incentive compatibility constraint as

$$RL - \eta^s r(L-1) \geq \bar{\lambda}RL - \eta^* r(L-1) \tag{4.2}$$

The left side of the constraint is a bank's expected profit from taking the safe project. The right side is a bank's expected profit from taking the risky project if the loophole innovation fails, in which case the regulator's supervision ability is η^*.

Given the leverage, banks would like to offer the lowest possible interest rate to attract deposit from households. Since households are rational, they would conjecture banks' project choices given the leverage, and demand a deposit rate that leaves them indifferent between depositing in the bank and investing in the storage technology. Therefore, the interest rate in the deposit menu is related to the leverage. We define $L_0 \equiv 1/(1-(1-\bar{\lambda})R\eta^s/((\eta^s - \underline{\eta})r_0)))$ and $L^* \equiv 1/(1-(1-\bar{\lambda})R((1-p)\eta^s + p\underline{\eta})/((\eta^s - \eta^*)r_0)))$. It is easy to see that for a small loophole innovation success probability p, L^* is larger than L_0.

To raise money from depositors, the interest rate needs to be sufficiently high to compensate for the bank's risk. The interest rate that leaves depositors indifferent between depositing in the bank and investing in the storage technology is

$$r = \begin{cases} \frac{r_0}{\eta^s}, & \text{if } L \leq L_0 \\ \frac{r_0}{(1-p)\eta^s + p\underline{\eta}}, & \text{if } L_0 < L \leq L^* \\ \frac{r_0}{\eta^*}, & \text{if } L > L^* \end{cases} \tag{4.3}$$

First, if a menu has a leverage lower than or equal to L_0 and an interest rate r_0/η^s, the bank's expected profit from taking the safe project is always higher than the risky project. Therefore, the bank will never invest in the risky project, even if there is a successful loophole innovation. Since the safe project succeeds with probability η^s, the interest rate for depositors to break even is r_0/η^s. Second, for a menu with a leverage between L_0 and L^* and an interest rate $r_0/((1-p)\eta^s + p\underline{\eta})$, the bank's expected profit from taking the safe project is higher than the risky project when the loophole innovation fails and lower than the risky project when the loophole innovation succeeds. With probability p, the loophole innovation is successful, and banks will choose the risky project with the highest riskiness $\underline{\eta}$. With probability $1-p$, the loophole innovation fails, and banks will choose the safe project. From an ex-ante perspective, the bank succeeds with probability $(1-p)\eta^s + p\underline{\eta}$, thus depositors demand an interest rate of $r_0/((1-p)\eta^s + p\underline{\eta})$. Third, if a bank's leverage is higher than L^*, it will always take the risky project even if the loophole innovation fails, so the interest rate needs to be as high as r_0/η^* to compensate for the risk. It is easy to see that this leads to a negative profit for banks. Therefore, banks will never choose a leverage higher than L^*.

In the case that p is small, we can show that a bank's expected profit with menu $\{L^*, r_0/((1-p)\eta^s + p\underline{\eta})\}$ is higher than that with menu $\{L_0, r_0\}$, so all banks will choose a leverage of L^*. From now on, we will focus on this case.

Next, let us solve the loophole innovation effort problem of the capable bank. After all banks raise deposits, one bank knows that it is the capable one, and it can exert effort to conduct loophole innovation. Given the leverage level and deposit rate, the innovation effort problem of the capable bank is

$$\max_e \; (1-e)[RL - \eta^s r(L-1)] + e[\bar{\lambda}RL - \underline{\eta}r(L-1)] - \frac{1}{2}ce^2L \tag{4.4}$$

With probability $1-e$, the loophole innovation fails, so the capable bank chooses the safe project. With probability e, the loophole innovation is successful, so the capable bank chooses the risky project with riskiness $\underline{\eta}$.

The first-order condition can be written as

$$-\left[R-\eta^{s}r(1-1/L)\right]+\left[\bar{\lambda}R-\underline{\eta}r(1-1/L)\right]=ce \tag{4.5}$$

First, we can see that given the leverage and interest rate, a higher loophole innovation cost coefficient c reduces the capable bank's innovation effort. Second, other things equal, a higher leverage L induces the capable bank to choose a higher loophole innovation effort. This is because when the bank's leverage is higher, the gain from finding a new loophole is larger. Third, a higher interest rate r results in a higher loophole innovation effort, since a new loophole provides the capable bank with an opportunity to avoid paying interest.

The definition of equilibrium in the static model is as follows.

Definition 1. *An equilibrium in the static model consists of the success probability of loophole innovation and decision rules* $\{L(\eta), r(\eta), e(\eta)\}$ *such that (i) the deposit menu* $\{L(\eta), r(\eta)\}$ *solves the banks' problem* (4.1) *given* (4.3)*; (ii)* $e(\eta)$ *solves the capable bank's problem* (4.4)*; (iii) the success probability of loophole innovation is consistent with the capable bank's innovation effort, i.e.,* $p=e$.

In equilibrium, the ex ante probability that the loophole innovation succeeds must be equal to the innovation effort chosen by the capable bank, i.e., $p=e$. The break-even condition for depositors implies that the interest rate is

$$r_0=\left[(1-e)\eta^{s}+e\underline{\eta}\right]r \tag{4.6}$$

Here with probability $1-e$, the loophole innovation fails. Banks take on the safe project, which has a success probability η^{s}. With probability e, the loophole innovation succeeds,and banks take on the risky project with success probability $\underline{\eta}$. From an ex-ante view, the bank succeeds with probability $(1-e)\eta^{s}+e\underline{\eta}$. The interest rate r_0 compensates for the bank's default risk.

As we mentioned before, we need the loophole innovation success probability p to be small, so that banks will choose a leverage of L^{*}. The following lemma shows that a large innovation cost c will guarantee this.

Lemma 1. *Under Assumption 1 and a large innovation cost coefficient c, the incentive compatibility constraint equation* (4.2) *is always binding for each bank.*

With Lemma (1), we can solve the bank's problem in an explicit form. From equations (4.2), (4.6), and (4.5), we can solve the innovation effort, deposit rate, and leverage in equilibrium given the supervision ability η^*,

$$e = \frac{\eta^* - \underline{\eta}}{\eta^s - \eta^*} \frac{(1-\bar{\lambda})R}{c} \tag{4.7}$$

$$r = \frac{r_0}{(1-e)\eta^s + e\underline{\eta}} \tag{4.8}$$

$$L = \frac{1}{1 - \frac{(1-\bar{\lambda})R}{(\eta^s - \eta^*)r}} \tag{4.9}$$

From the above equations, we have the following proposition.

Proposition 2. *Under Assumption 1 and a large innovation cost coefficient c, as the regulator's supervision ability increases,*

(I) the capable bank's loophole innovation effort increases;

(II) the banks' deposit rate increases;

(III) the banks' leverage increases.

The total output depends on whether the loophole innovation succeeds or not. If the loophole innovation fails, all banks take the safe project, and the total output is $(R - r_0)\omega L$. If the loophole innovation is successful, all banks take the risky project, and the total output is $(\bar{\lambda}R - r_0)\omega L$. Thus, the expected output at the beginning of the period is $\omega \cdot [(1-e)(R-r_0) + e(\bar{\lambda}R - r_0)]L$. We plot these results in Figure 4.1.

Next we study the comparative statics. We focus on how the cost coefficient of loophole innovation (c), the expected payoff of the safe project (R), and the relative payoff of the risky project ($\bar{\lambda}$) will affect banks' deposit rate (r), leverage (L), and capable bank's innovation effort (e).

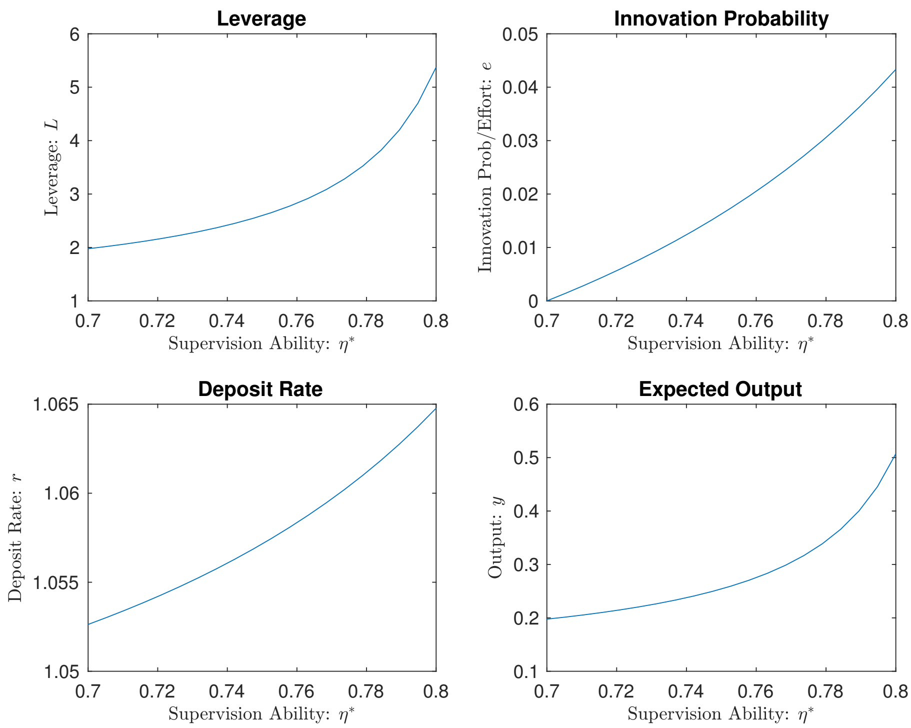
Lemma 2. *Under Assumption 1 and a large innovation cost coefficient c,*

(I) the capable bank's loophole innovation effort e decreases in c, increases in R, and decreases in $\bar{\lambda}$;

(II) the banks' deposit rate r decreases in c, increases in R, and decreases in $\bar{\lambda}$;

(III) the banks' leverage L is increases in c, increases in R, and decreases in $\bar{\lambda}$;

Figure 4.1: Relationship with Supervision Ability



It is easy to see that with a larger innovation cost coefficient c, the capable bank will exert less effort to conduct loophole innovation, thus the success probability of the loophole innovation decreases. A lower loophole innovation success probability will reduce the deposit rate demanded by households, since banks are less likely to take the risky project. And a lower deposit rate relaxes banks' incentive compatibility constraint, so banks can have a higher leverage.

The effects of increasing the expected payoff of the safe project, R, are more complicated. On the one hand, a larger R makes the safe project more attractive, which directly dampens the incentive of loophole innovation. On the other hand, a larger R also increases the leverage of banks, which indirectly gives the capable bank stronger incentive to innovate. The latter effect dominates the former one, so the capable bank's innovation effort increases. Following a similar logic, a larger $\bar{\lambda}$, increases the attractiveness of the risky project, but the low leverage associated with it decreases the capable bank's incentive to innovate. Overall, the capable bank's innovation effort is lower with a larger $\bar{\lambda}$.

4.3 Dynamic Model

4.3.1 Setup

In this section, we extend the static model into a dynamic model. Each bank lives for one period. Each bank is endowed with ω at the beginning of each period. To raise deposit, banks offer deposit menus to households. In each period, only one bank has a capable idea, and it chooses the effort it will make to conduct loophole innovation. The loophole innovation, once successful in one period, spreads in two dimensions. First, all banks in that period learn about it and are able to exploit the loophole, as in the static model. Second, all banks in the periods following the successful loophole innovation will learn about it.

One key element in the dynamic model is the evolution of the regulator's supervision ability. There are two countervailing forces that affect the regulator's supervision ability. On the one hand, a successful loophole innovation discovers a new type of risky project that is off the radar of the regulator's accumulated monitoring skills, which undermines the regulator's expertise. On the other hand, after a successful loophole innovation, the regulator recognizes the existence of a new loophole and starts to investigate it.[5] Over time, the regulator learns more and more about the new loophole, and improves its monitoring skills each period through learning-by-doing. As mentioned in the introduction, the assumption that the regulator engages in learning-by-doing is supported by some recent empirical papers (see, e.g., Dahlgren [20], Dudley [24], Eisenbach et al. [26]). These papers find that, in reality, regulators have drawn on lessons learned during the financial crisis and make effort to improve their supervision abilities.

In this chapter, we capture the regulator's learning-by-doing in a reduced form. More specifically, the evolution law for the regulator's supervision ability for a new loophole, i.e., the regulator's supervision ability η_t in the period t since the last loophole innovation that succeeded in the period $\hat{t}$ is

$$\eta_t = \begin{cases} \eta_k^*, & \text{if } t - \hat{t} = k < K \\ \eta_K^*, & \text{if } t - \hat{t} = k \geq K \end{cases} \tag{4.10}$$

Here k is the period following the last successful loophole innovation. If a loophole innovation is successful in one period, regulator supervision becomes ineffective for this new loophole. From the next period on, the regulator's supervision ability starts to evolve gradually according

[5] Since there is an infinite number of banks in the economy, the public can infer the occurrence of a successful loophole innovation from the share of bank failures at the end of the period.

to the evolution law. We assume that η_k^* increases with k, so regulator's supervision ability regarding the new loophole increases for each period. After K periods, it will stay constant unless another new loophole innovation succeeds. This guarantees that there is an upper-bound for the regulator's supervision ability, so banks' risk-shifting problem always exists. For each loophole, we denote the regulator's supervision ability space as $\{\eta_1^*, \eta_2^*, \ldots, \eta_k^*, \ldots \eta_K^*\}$.

Regarding banks' project choices in each period, we need to consider two possible cases. First, if a loophole innovation is successful, all banks can take the new risky project without being detected by the regulator. Second, if a loophole innovation fails, it is easy to see that if banks want to take risky projects, they would only take the risky project discovered in the latest loophole innovation. This is because the regulator's supervision ability is lowest for the risky project discovered in the latest loophole innovation, so banks can choose the highest riskiness when taking the new risky project.

When banks borrow from depositors at the beginning of each period, whether the loophole innovation will succeed or fail is not yet known. Thus the regulator's supervision ability for the latest discovered loophole determines the deposit contracts between banks and depositors, and the capable bank's loophole innovation effort. Therefore, the regulator's supervision ability related to the latest loophole is sufficient to describe the state for the economy, which implies that regulator's supervision ability space for the latest discovered loophole, $\{\eta_1^*, \eta_2^*, \ldots, \eta_k^*, \ldots \eta_K^*\}$, is also the state space for the economy.

The timing of the dynamic model is as follows: at the beginning of each period, the regulator's supervision ability is updated according to the evolution law, which is common knowledge. Banks offer deposit menus to households. Households decide whether or not to make deposits in the banks. After banks raise deposits, one of the banks knows it is the capable bank, and it chooses to make loophole innovation effort. If the loophole innovation is successful, all other banks can learn from it. Banks make project choices and choose riskiness if they invest in risky projects. At the end of each period, projects pay off. Banks pay back the depositors if their projects succeed. Otherwise, they default. In the next period, the regulator improves its supervision ability on the loopholes according to the evolution law.

4.3.2 Dynamics

Within each period, the problem is the same as the static model. As shown in the static model, in normal times without successful loophole innovation, all banks choose the safe project. However, if loophole innovation is successful, all banks choose the risky project. Given the regulator's supervision ability η_t in the period t, we have the following results

$$e_t = \frac{\eta_t - \underline{\eta}}{\eta^s - \eta_t} \frac{(1-\bar{\lambda})R}{c} \tag{4.11}$$

$$r_t = \frac{r_0}{(1-e_t)\eta^s + e_t\underline{\eta}} \tag{4.12}$$

$$L_t = \frac{1}{1 - \frac{(1-\bar{\lambda})R}{(\eta^s - \eta_t)r_t}} \tag{4.13}$$

As the regulator's supervision ability improves, banks have a higher leverage ratio. If the loophole innovation fails in the period t, all banks choose the safe project. A fraction $1-\eta^s$ of banks fail at the end of the period, and the output is $y_t^n = \omega \cdot (R - r_0)L_t$. Thus the output in the economy increases as banks' leverage rises. We say that the economy is in boom. However, if the loophole innovation succeeds in the period t, all banks choose the risky project. A fraction $1-\underline{\eta}$ of banks default, and the output is $y_t^i = \omega \cdot (\bar{\lambda}R - r_0)L_t$. Due to the widespread defaults and declining output, we say that there is a crisis in the economy when a loophole innovation is successful.

Proposition 3. *Under Assumption 1 and a large innovation cost coefficient c, the longer the boom,*

(I) the higher the bank's leverage;

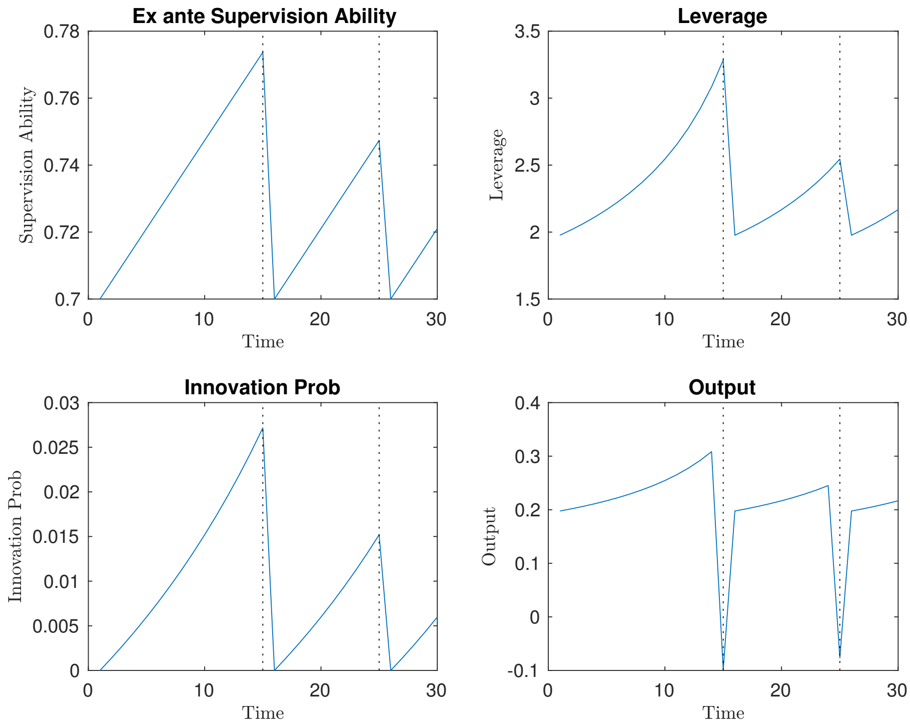
(II) the more likely a crisis is to occur;

(III) conditional on a crisis occurring, the larger the decline in output.

Since the regulator improves its supervision ability each period through learning-by-doing, the regulator's supervision ability is higher when the boom is longer. From Proposition 2, we know that banks' leverage and the capable bank's loophole innovation effort increase with supervision ability. Therefore, banks' leverage is higher for a longer boom, and at the same time capable banks' innovation effort is higher, which implies that crises are more likely to happen. Conditional on loophole innovation being carried out, the output is $y_t^i = \omega \cdot (\bar{\lambda}R - r_0)L_t$. Since $\bar{\lambda}R < r_0$, the greater the leverage, the larger the drop in output.

To illustrate Proposition 3, we simulate a certain path of loophole innovation in the economy. The results are in Figure 4.2. Two successful loophole innovations take place in the period 15 and 25, so there are crises in these two periods. The boom period before the first crisis is longer than the one before the second. As we can see, both leverage and output increase in boom periods. The longer the boom, the higher the leverage and output. At the same time, the capable bank's innovation effort also increases, which means there is a higher probability that a crisis is to occur. When the loophole innovation eventually succeeds, banks choose the risky project. As is shown in Figure 4.2, conditional on a crisis occurring, the drop in output is larger in the first crisis.

Figure 4.2: Dynamics



Note: The dotted vertical lines indicate the periods when loophole innovation occurs.

4.3.3 Long-run Distribution Properties

Next, we investigate the long-run distribution for the economy. As we have shown before, the regulator's supervision ability regarding the latest discovered loophole characterizes the states of the dynamic economy. Given the regulator's supervision ability η_i^*, all banks offer the same contracts to households, which determines the leverage, deposit contract, and capable bank's loophole innovation effort. At the same time, the evolution of the supervision ability state depends on whether loophole innovation succeeds or not. To make a more general case, we let the regulator's supervision ability for known risky projects grow with probability q, and stay at the same level with probability $1 - q$. Whether or not the regulator's supervision ability grows is public knowledge. Note that when $q = 1$, we go back to the previous case where supervision ability grows in each period with certainty. If the current supervision ability is η_i^*, i.e., $\eta_t = \eta_i^*$, we can write down the general rule for supervision ability evolution. For the case $i < K$,

$$\eta_{t+1} = \begin{cases} \eta_{i+1}^*, & \text{with prob. } q \text{ in case of no successful loophole innovation;} \\ \eta_i^*, & \text{with prob. } 1-q \text{ in case of no successful loophole innovation;} \\ \eta_1^*, & \text{in case of successful loophole innovation.} \end{cases} \tag{4.14}$$

For the case $i = K$,

$$\eta_{t+1} = \begin{cases} \eta_K^*, & \text{in case of no successful loophole innovation;} \\ \eta_1^*, & \text{in case of successful loophole innovation.} \end{cases} \tag{4.15}$$

For supervision ability η_i^*, with probability $1 - e_i$, loophole innovation fails in the current period. In this case, with probability q, the regulator's supervision ability will evolve to η_{i+1}^* if $i < K$, or stay at η_K^* if $i = K$ in the next period. With probability $1-q$, the regulator will stay at the same level of supervision ability η_i^*. With probability e_i, loophole innovation succeeds in the current period. In this case, the regulator's supervision ability resets to η_1^* in the next period. Thus, the regulator's supervision ability follows a Markov process. We can write the transition matrix for the Markov process as

$$P = \begin{bmatrix} e_1 + (1-q)(1-e_1) & q(1-e_1) & 0 & \dots & 0 \\ e_2 & (1-q)(1-e_2) & q(1-e_2) & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ e_{K-1} & 0 & 0 & \dots & q(1-e_{K-1}) \\ e_K & 0 & 0 & \dots & 1-e_K \end{bmatrix} \tag{4.16}$$

The element P_{ij} denotes the probability that the economy evolves from state i in the current period, to state j in the next period. If the current state is i, the regulator's supervision ability is η_i^*, and the capable bank's innovation effort is e_i. For states $1 \leq i < K$, with probability e_i, the loophole innovation succeeds, and the economy will evolve to state 1 in the next period. With probability $1 - e_i$, the loophole innovation fails, and the economy will evolve to the next state $i+1$ with probability q and stay at the same state i with probability $1-q$ in the next period. For state K, the difference is that the economy will stay the same state in the next period if there is no successful loophole innovation in the current period.

Since there is only a finite number of recurrent states which follow a Markov process, we can deduce the following lemma.

Lemma 3. *Under Assumption 1 and a large innovation cost coefficient c, there is a stationary distribution π for the supervision ability Markov process, i.e., $\pi = \pi P$.*

The stationary distribution π is a $1 \times K$ row vector, where the ith element π_i is the probability of the economy with supervision ability η_i^*. Since the first state occurs only after a successful loophole innovation, the first element π_1 equals the probability of crises in the long run.

As is shown in Lemma 2, when the values of parameters such as c, R, and $\bar{\lambda}$ change, the success probability of loophole innovation changes. This leads to changes in the transition matrix and the stationary distribution. We can deduce the following lemma.

Lemma 4. *Under Assumption 1 and a large innovation cost coefficient c, if c increases, R decreases, or $\bar{\lambda}$ increases, the probability of the lowest supervision ability state decreases, and the probability of the highest supervision state increases.*

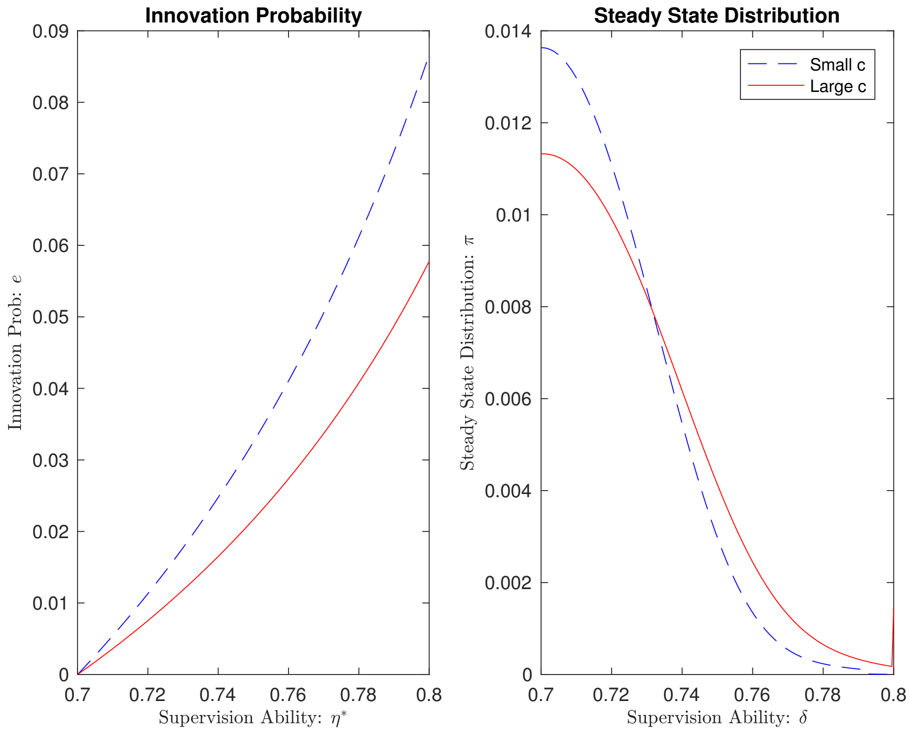
The intuition is that if c increases, R decreases, or $\bar{\lambda}$ increases, the success probability of loophole innovation in each state decreases. On the one hand, this implies that there are fewer crises, and the economy is less likely to return to the lowest supervision ability state. On the other hand, the economy is more likely to evolve into the state with higher supervision ability, thus the probability of the highest supervision ability state increases.

We can further characterize the property for the whole distribution in the following proposition.

Proposition 4. *Under Assumption 1 and a large innovation cost coefficient c, if c increases, R decreases, or $\bar{\lambda}$ increases, the new stationary distribution will first-order stochastic dominate the original one.*

First-order stochastic dominance means that the cumulative density function of the new stationary distribution is lower than that of the original one, so the whole distribution shifts to the higher supervision states on average. In other words, the probability that the regulator has a high supervision ability is higher in the long run. In Figure 4.3, we plot the innovation probability and stationary distribution with different innovation cost coefficients. As shown in the figure, we can see that with a small c, the stationary distribution has a higher probability for low supervision ability, i.e., the economy is more likely to stay in the low supervision ability states in the long run.

Figure 4.3: Long-run Stationary Distribution



4.4 Regulation: Maximum Leverage Ratio

In this section, we discuss the policy implications of banks' loophole innovation. When a capable bank engages in loophole innovation, it will not internalize the negative externalities for other banks. The negative externalities of loophole innovation have two dimensions. First, successful loophole innovation will reduce the output in the current period by allowing all banks to invest in inefficient risky projects. Second, after a new loophole innovation, the regulator has to learn about it and improve its supervision ability gradually from the start. This leads to a low leverage for the banks in the following periods. These externalities provide the regulator with justification for setting the maximum leverage ratio to curb loophole innovation probability. As shown before, when the regulator has a high supervision ability, the market allows the banks to have a high leverage. But at the same time, the market-determined leverage results in a high probability of innovation. To curb the high probability of innovation, the regulator can set a maximum leverage ratio for the banks.

Under the regulator's supervision ability η_i^*, let us denote the market-determined leverage as L_i^m. Here L_i^m is the bank's privately optimal leverage where there is no regulation, as in the benchmark model. Now suppose that the regulator sets the maximum leverage ratio as $\bar{L}$. If $L_i^m \leq \bar{L}$, banks can choose the market-determined leverage without violating the regulation. In this case, the maximum leverage ratio will not affect the bank's decision. However, if $L_i^m > \bar{L}$, regulation constrains banks' leverage choices. Banks cannot choose the privately optimal leverage L_i^m due to the regulation, instead they can only take a leverage of $\bar{L}$. From Proposition 2, we know that L^m increases with the regulator's supervision ability, so regulation is more likely to be effective when supervision ability is high. We refer to the states that the maximum leverage ratio constrains market-determined leverage as the affected states.

When the leverage regulation is effective, banks optimally choose the regulated maximum leverage, and the incentive compatibility constraint becomes slack. The first order condition for the capable bank's innovation effort is

$$-\left[R - \eta^s r(1 - 1/\bar{L})\right] + \left[\bar{\lambda} R - \underline{\eta} r(1 - 1/\bar{L})\right] = ce \tag{4.17}$$

and the interest rate in the equilibrium is

$$r = \frac{r_0}{(1-e)\eta^s + e\underline{\eta}} \tag{4.18}$$

By solving the above two equations, we can get the innovation probability $\bar{e}$ when banks' leverages are restricted by the regulation. Thus, the innovation probability under regulation is

$$e_i^r = \begin{cases} e_i^m, & \text{if } L_i^m \leq \bar{L} \\ \bar{e}, & \text{if } L_i^m > \bar{L} \end{cases} \tag{4.19}$$

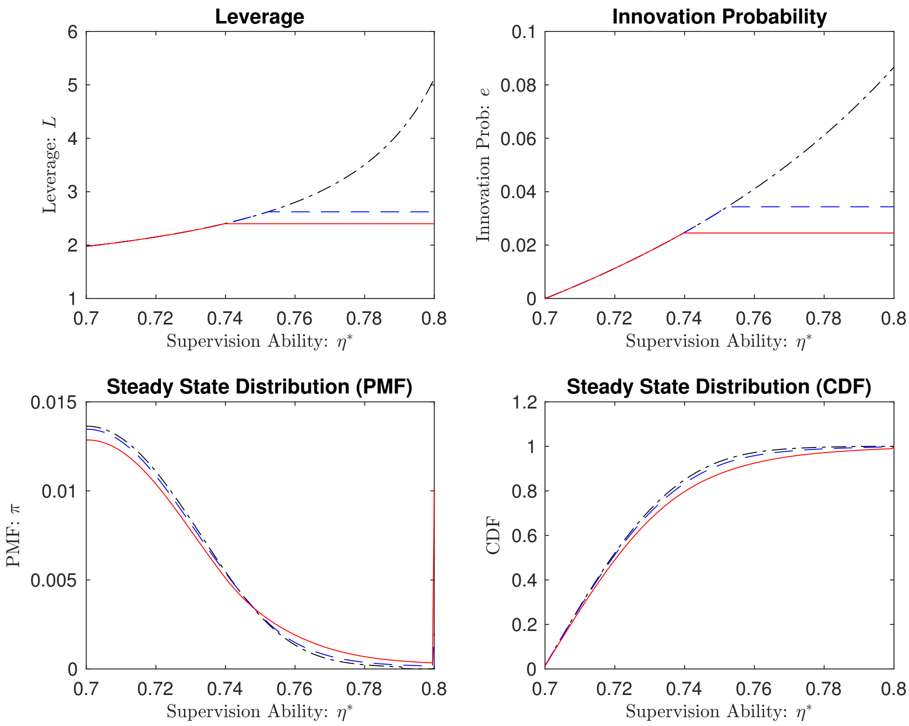
Since banks' leverage is constrained with a maximum leverage ratio, the innovation probability under regulation is always smaller than or equal to that without regulation, i.e., $e_i^r \leq e_i^m$. With the above innovation probability, we can write the transition matrix under regulation as

$$P^r = \begin{bmatrix} e_1^r + (1-q)(1-e_1^r) & q(1-e_1^r) & 0 & \dots & 0 \\ e_2^r & (1-q)(1-e_2^r) & q(1-e_2^r) & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ e_{K-1}^r & 0 & 0 & \dots & q(1-e_{K-1}^r) \\ e_K^r & 0 & 0 & \dots & 1-e_K^r \end{bmatrix} \tag{4.20}$$

where the element $P^r(i,j)$ is the probability of moving from state i to state j in the next period. With this transition matrix, we can get the stationary distribution under regulation, π^r. Since we know that certain states' innovation probabilities are smaller with regulation if there are some $L_i^m > \bar{L}$, we can compare the two stationary distributions with and without regulation in the following proposition.

Proposition 5. *Under Assumption 1 and a large innovation cost coefficient c, if the regulator sets a maximum leverage lower than the highest one determined by the market, the stationary distribution under regulation will first-order stochastic dominate the one without regulation.*

Figure 4.4: Maximum Leverage Ratio



Note: Black dot-dashed line: without regulation; Blue dashed line: lenient regulation; Red solid line: strict regulation.

The results are shown in Figure 4.4. The figure includes three cases: without regulation, lenient regulation (high maximum leverage), and strict regulation (low maximum leverage). In fact, we can consider the case without regulation as a special case of regulation, when the maximum leverage ratio is sufficiently high for banks' leverage choice to never be restricted. As we can see in the figure, as regulation becomes stricter, leverage and innovation probability under more states deviate from the case with only market discipline. Also, the leverage and innovation probability in those affected states are lower under stricter regulation. The changes in innovation probability affect the transition matrix and also stationary distribution. As we see in the graph, the stationary distribution shifts more to the high states under stricter regulation.

We assume that the regulator sets the maximum leverage ratio to maximize average output in the long run. The expected output in state i is

$$y_i^r = \omega \cdot [(1 - e_i^r)(R - r_0) + e_i^r(\bar{\lambda} R - r_0)]L_i^r \tag{4.21}$$

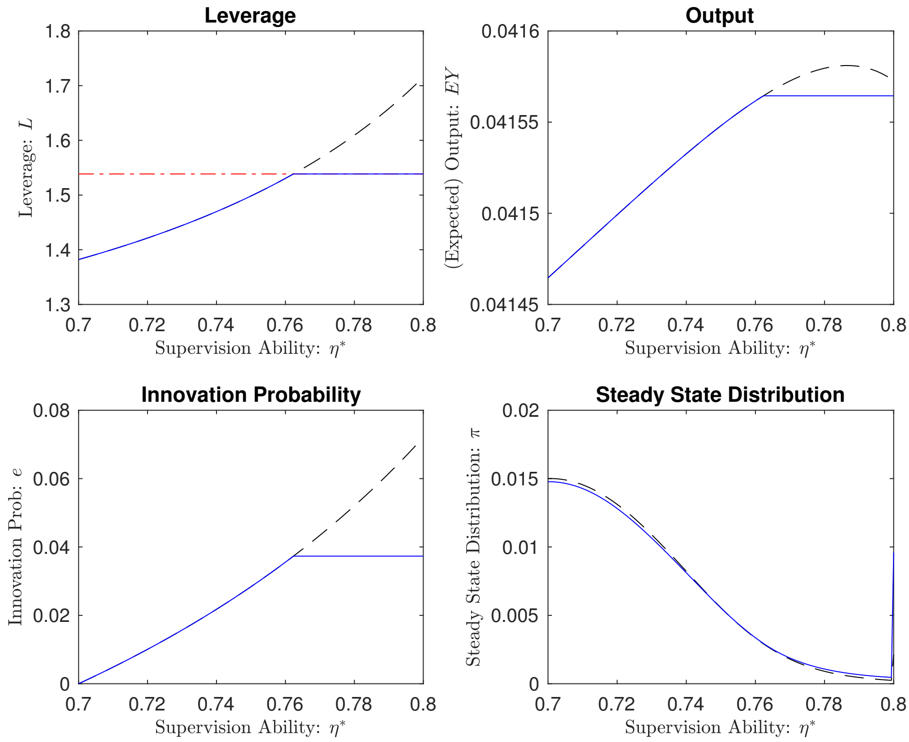
and the average output in the stationary distribution is

$$EY = \sum_{i=1}^{K} \pi_i^r y_i^r \tag{4.22}$$

The maximum leverage ratio can affect average output in two ways. First, it can directly affect expected output y_i^r in certain states through its effects on leverage and loophole innovation success probability. Effective regulation decreases banks' leverage in the affected states, which has a negative effect on output in the affected states given the expected output per unit investment. But at the same time, regulation reduces loophole innovation success probability, which increases the expected output per unit investment. The overall effect of regulation on expected output in the affected states depends on which effect dominates. Usually when supervision ability is high, the former effect dominates, so expected output in the affected states will decrease with strict regulation. Second, it can affect the stationary distribution π_i^r through its effect on loophole innovation success probability. Strict regulation will shift the distribution towards high states, which usually have a higher output. If regulation decreases expected output in the affected states, there is a trade-off for the regulator between expected output in affected states and the probability of staying in high states in the stationary distribution.

In certain parameter space, the regulator optimally chooses a maximum leverage level at which the incentive compatibility constraint is not binding when the regulator has a high supervision ability. The results are shown in Figure 4.5. As we can see, the optimal regulation sets a maximum leverage ratio which is effective in some states. Expected output in those affected states becomes lower under regulation. However, the loophole innovation success probability is also reduced in those high supervision ability states, because the capable bank has less incentive to innovate under regulation. The change of loophole innovation success probability shifts the stationary distribution. Compared to the case of no regulation, the economy has a higher probability of staying in high supervision ability states, as shown in the fourth graph.

Figure 4.5: Optimal Regulation



Note: Black dashed line: without regulation; Blue solid line: optimal regulation.

4.5 Learning about Loophole Innovation

In the previous sections, if a loophole innovation succeeds, the regulator and investors have full knowledge about the existence of the new loophole by the end of the period. Next we study the dynamics when there is some uncertainty concerning whether or not there has been an unknown loophole in the economy.

We assume that there are N banks in the economy. Each bank lives for one period. A finite number of banks can prevent the revelation of the existence of a new loophole through the fraction of failed banks. In each period, one of the N banks is a capable one and can choose to make effort to conduct loophole innovation.

Regarding the uncertainty surrounding an unknown loophole, we assume that at the end of each period, the public only observes the number of bank failures. Therefore, the public needs to infer whether bank failures come from the safe projects or risky ones, and updates its belief about an unknown loophole using this information.[6] Let us denote the public's belief about the probability of there being an unknown loophole as θ.

[6] On the contrary, if the public can observe banks' payoff, they know whether banks have invested in risky projects, and can clearly infer the existence of a loophole.

Unlike in previous sections, we give the regulator the additional role of investigator. Since there is uncertainty about the existence of loophole innovation, the regulator can pay a fixed cost χ/r_0 to investigate the banking sector at the beginning of each period, and the investigation result are publicly observed. We assume that the investigation cost comes from a lump-sum tax from households. If there is a loophole, the public knows about it, and the regulator's supervision ability for it starts to grow gradually from the lowest level. If there is no unknown loophole, it is revealed to the public, and the supervision ability evolves. Thus, investigation plays two roles in the model. First, it eliminates the uncertainty regarding an unknown loophole. Second, it is the starting point for the gradual growth of supervision ability for a certain type of risky project. Eisenbach et al. [25] discusses that one of the supervisory jobs for the central bank is "discovery examination", which focuses on understanding a specific business activity and filling the knowledge gap. In our model, investigation from the regulator serves a similar role.

The timing is as follows: at the beginning of each period, the regulator decides whether or not to investigate. If it investigates and finds a loophole, its supervision ability resets to the lowest level. The investigation result is publicly observed, and the public updates its belief regarding an unknown loophole. Banks offer menus of leverage and deposit rate to households. Households decide whether or not to make deposits in the banks. After banks raise deposits, banks know whether there is a loophole that is unknown to the regulator, and they learn about the loophole if there is one. One of the banks knows it is the capable bank, and it makes loophole innovation effort. If the innovation innovation is successful, all other banks can learn about it. Then banks make project choices under the regulator's supervision. At the end of each period, projects pay off. Banks pay back the depositors if their projects succeed. Otherwise, they default. The public updates its belief about the existence of an unknown loophole in the economy. In the next period, the regulator's supervision ability on known risky projects evolves.

Consider the deposit menus banks offer to depositors. As in previous sections, we focus on the case where the innovation cost coefficient is large so that the success probability of loophole innovation is small. It is easy to see that banks will offer at most two types of contracts, one with low leverage and the other with high leverage. The first one is that banks offer a leverage and deposit rate menu $\{L_0, r_0/\eta^s\}$, where $L_0 \equiv 1/(1-(1-\bar{\lambda})R\eta^s/((\eta^s-\underline{\eta})r_0)))$. Under this menu, the bank will never invest in any risky project even if there is an unknown loophole, so they only need to pay a low interest rate r_0/η^s to allow the depositors to break even. Also, if the capable bank offers this menu, it will have no incentive to conduct loophole innovation. Thus, the expected profit of the banks choosing this menu is

$$\pi_0 = RL_0 - r_0(L_0 - 1)$$

The second one is a menu with a leverage higher than L_0, and a deposit rate that allows depositors to break even. For a large innovation cost coefficient, the incentive compatibility constraint is binding, i.e., $R - \eta^s r(1 - 1/L) = \bar{\lambda}R - \eta^* r(1 - 1/L)$. Since there is uncertainty about an unknown loophole, the belief about the probability that there is an unknown loophole, θ, plays a role in the deposit contract. Note that the capable bank will make loophole innovation efforts only if it chooses the high-leverage menu. Therefore, the number of banks choosing the high-leverage menu affects the success probability of loophole innovation, which also determines the expected profit of banks with high-leverage menus. Let us use n to denote the banks choosing a high-leverage menu. We have the following results related to banks choosing a high-leverage menu

$$e = \frac{\eta^* - \underline{\eta}}{\eta^s - \eta^*}\frac{(1-\bar{\lambda})R}{c} \tag{4.23}$$

$$r = \frac{r_0}{(1-\theta)(1-\frac{n}{N}e)\eta^s + [\theta + (1-\theta)\frac{n}{N}e]\underline{\eta}} \tag{4.24}$$

$$L = \frac{1}{1 - \frac{(1-\bar{\lambda})R}{(\eta^s - \eta^*)r}} \tag{4.25}$$

This menu is only feasible if the interest rate r is not higher than R/η^s. Since r depends on θ, this implies that the belief that there is an unknown loophole cannot be too large.

The expected profit for banks choosing a high-leverage menu is

$$\pi(n, \theta, \eta^*) = (1-\theta)\left(1 - \tfrac{n}{N}e\right)[RL - \eta^s r(L-1)] + \left[\theta + (1-\theta)\tfrac{n}{N}e\right][\bar{\lambda}RL - \underline{\eta}r(L-1)] - \tfrac{1-\theta}{N}\tfrac{1}{2}ce^2L$$

With probability $(1-\theta)\left(1 - \frac{n}{N}e\right)$, no unknown loophole existed before this period, and no new loophole innovation is carried out in this period, so banks with a high-leverage menu will choose the safe project. With probability $\theta + (1-\theta)\frac{n}{N}e$, either there is unknown loophole, or a new loophole is discovered in this period, so banks with a high-leverage menu invest in the risky project evading the regulator's supervision. The probability that one high-leverage bank is a capable one is $1/N$, and it will exert innovation effort when there is no unknown loophole. We can see that $\pi^*(n, \theta, \eta^*)$ decreases in n, decreases in θ, and increases in η^* for a large innovation cost coefficient. The number of banks choosing high leverage, n, is endogenously determined in the equilibrium, where no bank has the incentive to switch to the low leverage menu. Let n^* denote the number of banks choosing a high-leverage menu in equilibrium, then

$$n^* = \begin{cases} 0, & \text{if } \pi(1, \theta, \eta^*) < \pi_0 \\ n, & \text{if } \pi(n, \theta, \eta^*) \geq \pi_0 > \pi(n+1, \theta, \eta^*) \\ N, & \text{if } \pi(N, \theta, \eta^*) \geq \pi_0 \end{cases} \tag{4.26}$$

Firstly, if banks' expected profit with the low-leverage menu is higher than with the high-leverage menu, even if only one bank chooses the high-leverage menu, all banks will offer the low-leverage one. This case occurs when the belief is very pessimistic, i.e., θ is large. Secondly, if banks' expected profit with the high-leverage menu is higher than with the low-leverage menu, even if all banks choose low-leverage menu, all banks will offer the high-leverage one. This case occurs when the belief is very optimistic, i.e., θ is small. Thirdly, when θ is in the medium range, some banks may choose the high-leverage menu while others choose the low-leverage one. The number of banks choosing the high-leverage menu is determined in such a way that banks' expected profit with high leverage is higher than or equal to the expected profit with low leverage, with one extra bank switching to the high-leverage menu making banks prefer the low-leverage menu. For a bank that chooses the high-leverage menu, let r^*, L^*, and π^* denote respectively the bank's interest rate, leverage, and expected profit in equilibrium. For a capable bank, let us use e^* to denote its loophole innovation effort in equilibrium.

Next we consider the belief updating problem. At the end of each period, the public can update its belief about the existence of an unknown loophole from the performance of banks in that period. For the banks choosing the low-leverage menu, there is no information about the existence of an unknown loophole since they never choose risky projects. Thus, all the information related to belief updates comes from those banks choosing the high-leverage menu. If the public observes m banks failing out of n^* banks choosing the high-leverage menu, the updated belief is

$$\tilde{\theta}(m, \theta, \eta^*) = \frac{\left[\theta + (1-\theta)\frac{n^*}{N}e^*\right] \underline{\eta}^m (1-\underline{\eta})^{n^*-m}}{(1-\theta)\left(1 - \frac{n^*}{N}e^*\right)(\eta^s)^m (1-\eta^s)^{n^*-m} + \left[\theta + (1-\theta)\frac{n^*}{N}e^*\right] \underline{\eta}^m (1-\underline{\eta})^{n^*-m}} \tag{4.27}$$

For a certain belief θ and supervision ability η^*, the updated belief after observing banks' performance can only have $n^* + 1$ possible values. Let us denote by $\mathcal{M}(\theta, \eta^*)$ the set for all possible updated belief,

$$\mathcal{M}(\theta, \eta^*) = \{\tilde{\theta}(m, \theta, \eta^*) | m = 0, 1, \ldots, n^*\}$$

For a belief $\tilde{\theta}(m)$ in the set $\mathcal{M}(\theta, \eta^*)$, the probability that the public will have that belief after observing banks' performance is

$$\Gamma(\tilde{\theta}(m)|\theta, \eta^*) = \binom{n^*}{m}(1-\theta)\left(1 - \frac{n^*}{N}e^*\right)(\eta^s)^m(1-\eta^s)^{n^*-m} + \binom{n^*}{m}\left[\theta + (1-\theta)\frac{n^*}{N}e^*\right]\underline{\eta}^m(1-\underline{\eta})^{n^*-m}$$

At the beginning of each period, if the public belief is too pessimistic, all banks will offer low-leverage menu, and there is no belief updating. Let $\bar{\theta}(\eta^*)$ denote the threshold belief at which at least one bank will choose the high-leverage menu given the supervision ability η^*. It satisfies the following condition

$$\pi(1, \bar{\theta}(\eta^*), \eta^*) = \pi_0$$

Since $\pi(n, \theta, \eta^*)$ decreases in θ and increases in η^* for a large c, we get the following lemma

Lemma 5. *Under Assumption 1 and a large innovation cost coefficient c, there is an unique belief threshold, above which no bank will choose the high-leverage menu, and thus the belief about an unknown loophole is not updated in the period. The belief threshold increases in the regulator's supervision ability.*

When the belief θ is higher than $\bar{\theta}(\eta^*)$, no bank chooses the high-leverage menu, so there is no update about an unknown loophole from the banks' performance. The belief at the end of the period will be the same as θ. We call $\bar{\theta}(\eta^*)$ the belief-update threshold because there is updating of belief only if the belief is lower than $\bar{\theta}(\eta^*)$ for supervision ability η^*.

Next we discuss the effects of changes in belief and supervision ability on the economy. The analysis is complicated by the fact that the number of banks choosing the high-leverage menu also changes with these factors. We use n^*e^*/N, $[(N - n^*)r_0/\eta^s + n^*r^*]/N$, and $[(N - n^*)L_0 + n^*L^*]/N$ to denote expected innovation effort, average interest rate, and average leverage respectively. We have the following proposition

Proposition 6. *Under Assumption 1 and a large innovation cost coefficient c,*

(I) if θ increases, n^* stays the same or decreases.

(i) if n^* stays the same, expected innovation effort stays the same, average interest rate increases, average leverage decreases;

(ii) if n^* decreases, expected innovation effort decreases, interest rate may increase, decrease or stay the same, average leverage decreases.

(II) if supervision ability η^* increases, n^* stays the same or increases. Expected innovation effort increases, average interest rate increases, average leverage increases.

For the first part of Proposition 6, the effects of belief θ mainly come from its effect on interest rate. When it is large, depositors worry about the unknown loophole, so banks choosing the high-leverage menu have to pay a high interest rate. A higher interest rate lowers the leverage through incentive compatibility constraint. Its effect on capable bank's loophole innovation effort comes from the extensive margin, i.e., banks switch to low-leverage menu. For the second part of Proposition 6, the effects of supervision ability could come from both the intensive and extensive margin. For the intensive margin, banks choosing the high-leverage menu can offer a higher leverage. This also leads to a higher loophole innovation effort if the capable bank chooses the high-leverage menu. If more banks choose the high-leverage menu with increasing supervision ability, this increases the average leverage and expected innovation probability from the extensive margin. This shows that the results in Proposition 2 are robust even if we include learning in the model.

Unlike in previous sections, the regulator faces an investigation problem now, i.e., when to pay a fixed cost to investigate whether there is an unknown loophole. The regulator uses a lump-sum tax from households to fund the investigation cost. The regulator has a discount factor β, and its aim is to maximize the discounted expected output including the loss from the investigation cost. The expected output, given belief θ and supervision ability η^*, is

$$y(\theta,\eta^*) = n^*\left[(1-\theta)\left(1-\tfrac{n^*}{N}e^*\right)(R-r_0) + \left(\theta+(1-\theta)\tfrac{n^*}{N}e^*\right)(\bar{\lambda}R-r_0)\right]L^* + (N-n^*)(R-r_0)L_0 \quad (4.28)$$

The regulator makes a decision concerning investigation based on the belief at the beginning of each period, which is the same as updated belief based on banks' performance in the last period. If the regulator does not investigate, the belief stays the same, and banks offer menus based on this belief. Otherwise, the belief will reset to zero after investigation, since the investigation eliminates the uncertainty about an unknown loophole in the economy. If the regulator finds a loophole through investigation, the regulator has to accumulate supervision ability from the beginning for the new type of risky project. If the investigation does not find a loophole, the regulator's supervision ability continues to evolve from the last period. Let $\tilde{\theta}$ be the belief before investigation in the current period, and $\tilde{\theta}'$ be the belief before investigation in the next period. We can write down the regulator's problem in the recursive form

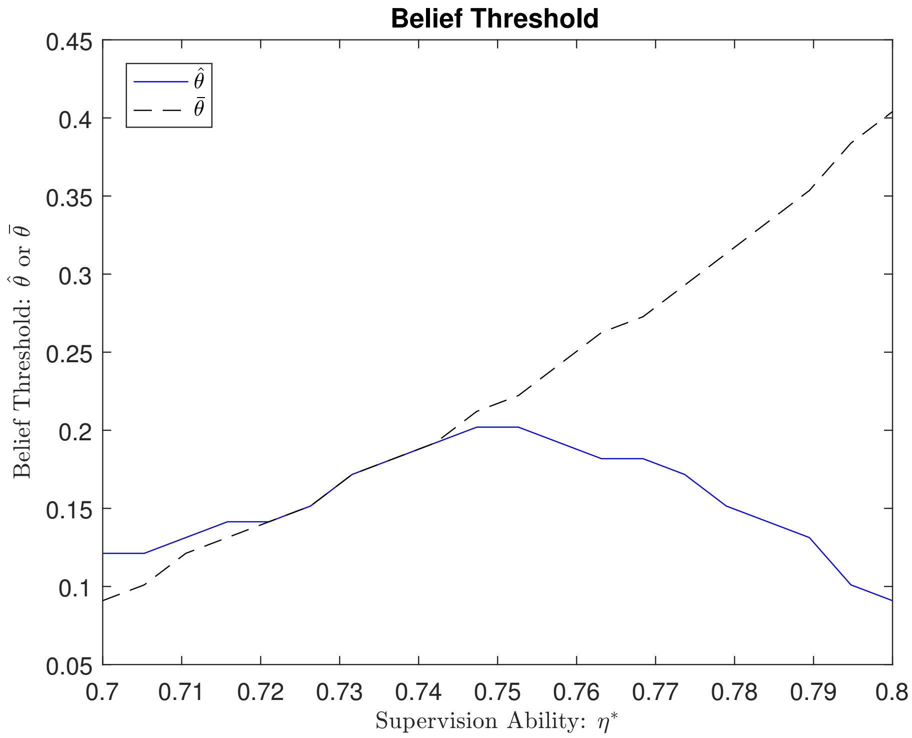
$$\begin{aligned} V(\tilde{\theta}, \eta_i^*) = \max_{d \in \{0,1\}} \quad & (1-d) \quad \left[y(\tilde{\theta}, \eta_i^*) + \beta \textstyle\sum_{\tilde{\theta}' \in \mathcal{M}(\tilde{\theta}, \eta_i^*)} \Gamma(\tilde{\theta}' | \tilde{\theta}, \eta_i^*)(q * V(\tilde{\theta}', \eta_{i+1}^*) + (1-q) * V(\tilde{\theta}', \eta_i^*)) \right] \\ & + d \quad \left\{ -\chi + \tilde{\theta} \left[y(0, \eta_1^*) + \beta \textstyle\sum_{\tilde{\theta}' \in \mathcal{M}(0, \eta_1^*)} \Gamma(\tilde{\theta}' | 0, \eta_1^*)(q * V(\tilde{\theta}', \eta_2^*) + (1-q) * V(\tilde{\theta}', \eta_1^*)) \right] \right. \\ & \left. \quad + (1-\tilde{\theta}) \left[y(0, \eta_i^*) + \beta \textstyle\sum_{\tilde{\theta}' \in \mathcal{M}(0, \eta_i^*)} \Gamma(\tilde{\theta}' | 0, \eta_i^*)(q * V(\tilde{\theta}', \eta_{i+1}^*) + (1-q) * V(\tilde{\theta}', \eta_i^*)) \right] \right\} \end{aligned} \tag{4.29}$$

If the regulator chooses not to investigate, i.e., $d = 0$, the expected output is $y(\tilde{\theta}, \eta_i^*)$, the belief in the next period $\tilde{\theta}'$ is updated from $\tilde{\theta}$ through the banks' performance, and the supervision ability evolves to η_{i+1}^* with probability q and stays the same with probability $1-q$ in the next period. If the regulator chooses to investigate, i.e., $d = 1$, it needs to collect the tax from the household and pay the fixed cost at the beginning of the period, and the related loss in the output is χ. If the regulator finds a loophole through investigation, the regulator has to accumulate its expertise for this new type of risky project from the beginning, and its supervision ability resets to the lowest level. The expected output is $y(0, \eta_1^*)$ in the current period, and the supervision ability and belief evolve following the rules. If the regulator does not find a loophole, the expected output is $y(0, \eta_i^*)$, and the supervision ability and belief evolve. In this economy, the belief $\tilde{\theta}$ and the supervision ability for a known loophole η_i^* are important states characterizing the evolution of the economy.

We can see that if the investigation cost is zero, the regulator will choose to investigate each period, because eliminating uncertainty can increase banks' leverage and reduce the risk related to an unknown loophole. Thus, there is no uncertainty about an unknown loophole when banks offer menus to households. The results will be the same to those in Section **??**. If the investigation cost is too large, there are some absorbing states with positive probability, where the economy will stay forever once it enters. Since the supervision ability for an known loophole increases with positive probability, the absorbing states can only include the highest supervision ability η_K^*. If the belief at the beginning of the period is higher than the belief-update threshold $\bar{\theta}(\eta_K^*)$, banks will always choose the low-leverage menu if the regulator does not investigate. Given a sufficiently large investigation cost, the regulator would not choose to investigate. In this case, there is no belief update and no evolution for the supervision ability, and the economy stays where it is.

The case with a medium investigation cost is more interesting. We plot the belief thresholds in Figure 4.6 for a certain parameter space where the investigation cost is not too large or too small. The black dashed line denotes the belief-update threshold. If the belief θ is higher than this threshold, all banks will choose the low-leverage menu, and there will be no update regarding an unknown loophole. As is shown in Lemma 5, the threshold is higher for high supervision ability, and the black dashed line is higher on the right side. The blue solid line denotes the belief threshold for investigation. If the belief is higher than the threshold, the regulator will investigate whether there is an unknown loophole. We can see that the relationship between the investigation threshold and the supervision ability is not monotonic. Within the belief-update region, on the one hand, higher supervision ability leads to higher leverage, and the drop in the expected output will be larger if there is an unknown loophole. This force leads to the belief threshold decreasing with the supervision ability. In the extreme case where β is zero, it is easy to show that the belief threshold for investigation is a decreasing function for supervision ability. On the other hand, the investigation cost is irreversible, so investigation is an option for the regulator. There could be a wait-and-see effect. The regulator may need more information before paying the fixed investigation cost. This force makes the regulator willing to delay investigation. Within the no-belief-update region, the regulator has an incentive to investigate to eliminate the uncertainty so that banks can have higher leverage. Also, since the belief-update threshold increases with supervision ability, the regulator may withhold investigation to allow the belief to fall below the threshold in the higher supervision ability state. Thus, the relationship between the belief threshold for investigation and supervision ability may not be monotonic.

Figure 4.6: Belief Threshold



Note: Black dashed line: belief threshold for no belief update; Blue solid line: belief threshold for investigation.

4.6 Discussion

In this section, we discuss several setups in the model in relation to the loophole innovation and regulator's learning-by-doing.

Regarding the loophole innovation, we assume that there is only one capable bank in each period. We can easily extend the model to the case that there is a finite number of capable banks. In this case, we can show that the choice of innovation effort of one capable bank depends on the choices of other banks. This extension makes the model more complicated, without adding any new insights. Second, we assume that all current and future banks can learn about the new loophole if the capable bank succeeds. There are two reasons that we make this assumption. Firstly, since a successful loophole innovation provides banks with the opportunities to take the risky projects, which the regulator tries to forbid, the capable bank cannot rely on any legal system to protect the successful loophole innovation. Secondly, there is no competition among banks in the model, so the capable bank has no incentive to prevent other banks from taking advantage of the new loophole.

In this chapter, we model the regulator's learning-by-doing process in a reduced form. We do this for two reasons. First, we treat the regulator's supervision as passive in the model, so we can focus on the decisions of banks, especially the loophole innovation. Second, the passive evolution of the regulator's supervision ability makes our model much more tractable. However, in Section **??**, we add an investigation role for the regulator, which can be considered as a form of active learning-by-doing. In a companion paper on shadow banking that is still working in progress, we provide a complete micro-foundation for investors' learning-by-doing, which we expect to incorporate into this model in the future.

Chapter 5

Conclusion

5.1 Summary for Monetary Policy and Housing Market in China

In Chapter 2, we estimate three mainstream DSGE models with newly available Chinese quarterly data. The results for Smets and Wouters [71]'s model show that the estimated monetary policy is not consistent with the widely used Taylor rule for U.S. data. The results for Iacoviello and Neri [42]'s model show that there is not much spillovers between the housing market and the real economy, which implies that borrowing constraint on the households' side is not important to explain the rapid growth of both business investment and housing market. Liu et al. [49]'s model can generate sufficient co-movement between housing market and investment through borrowing constraint on the entrepreneurs' side, but this model is abstract from nominal rigidities. Thus, there are two main findings in this chapter. First, the standard Taylor rule as monetary policy does not fit well with the Chinese data. Second, housing price plays a role in the real economy mainly through relaxing the borrowing constraint of entrepreneurs.

There are still several issues to solve in the future for us to apply newly-developed Bayesian-estimated DSGE models for the Chinese economy. First, as discussed by Song et al. [72] and Chang et al. [14], the Chinese economy in the past few decades has been experiencing the economic transition. It would be very helpful if we can differentiate between the low-frequency transition part and the high-frequency fluctuation part. In this way, we may understand better the interaction between long-term trend and short-term cycles for the Chinese economy. Second, for New Keynesian models, we need to find a reasonable way to describe the monetary policy for China. The Taylor rule has been widely used in the DSGE models for the U.S. economy, and literature tries to justify it from the stabilization perspective. However, Chen et al. [18] provides empirical evidence that the standard Taylor rule does not apply in China. In this chapter, we also raise the question whether the standard Taylor rule applies for China. We need more research on how to model the monetary policy in China so we can incorporate it into the DSGE model. Third, the recent boom in the housing market has been phenomenal in China. Both Iacoviello and Neri [42] and Liu et al. [49] attribute it to the demand shocks for housing, which are not explained in the model. It could be an important topic for future research to dig into the structural determinants of these shocks. In the end, as time goes, we believe that the Bayesian-estimated DSGE models have even larger potential to understand the Chinese economy due to the improvements in the data availability and accuracy.

5.2 Summary for Optimal Monetary Policy in China

In Chapter 3, we conduct several investigations related to the monetary policy in China. First, we derive the endogenous regime-switching monetary policy, which is consistent with the empirical findings from [18]. To derive this optimal monetary policy, we need asymmetric loss function for the Central Bank, adaptive learning expectation, and revenue tax. Second, we estimate the monetary policy rule and the rest equations of the structural model. Based on the estimation results, we are able to make inference about the preference of Central Bank. Third, we conduct several conditional forecast exercises based on the estimated model. We find that we need a relatively low money growth rate to achieve a reasonable forecast for the output growth.

The preference of the Central Bank is a very important issue for research and policy making. Our chapter proposes a simple loss function to characterize the Central Bank's preference in China. But we believe that in reality its preference could be very complex. It would be very interesting to investigate the motivation and preferences of the Central Bank with detailed information about institutional background and decision processes. With those knowledge, we are able to refine our model and characterize the monetary policy better in the model. Also, during the derivation of the optimal monetary policy, we assume that the agents use adaptive learning expectation. One reason underlying this assumption is the difficulty related to solving an endogenous regime-switching model. It would be an interesting direction to study whether the adaptive learning belief will lead to the same equilibrium as the rational expectation. If it is the case, this provides more evidence to support the adoption of adaptive learning in our model.

5.3 Summary for A Model of Bank Credit Cycles

In Chapter 4 we develop a model on the dynamic interaction between regulator supervision and banks' loophole innovation, and study its implications on banks' credit cycles. In the model, banks' leverages are constrained due to a risk-shifting problem. The regulator supervises the banks to ease this moral hazard problem, and its expertise in supervision improves gradually through learning-by-doing. At the same time, banks can engage in loophole innovation to circumvent supervision, which acts as an endogenous opposing force diminishing the value of the regulator's accumulated expertise. In equilibrium, banks' leverage and loophole innovation move together with the regulator's supervision ability. The model shows that long periods of gradual expansion in banks' leverage, investment, and aggregate output, are followed by sudden and sharp recessions. In our model, even in the absence of exogenous perturbations, banks themselves can become the sources of adverse shocks to the real economy. We show that the longer the boom, the more likely there is a crisis and the more severe the consequences, which corresponds to Minsky's hypothesis that good times sow the seeds for the next financial crisis. The model's empirical implications are broadly consistent with the stylized facts from empirical studies related to credit cycles.

Based on this model, we also discuss the welfare implications of a maximum leverage ratio in the environment of loophole innovation. We show that the regulator faces a trade-off between financial stability and output in boom periods. A higher maximum leverage ratio is associated with higher output in good times but more frequent crises, while a lower maximum leverage ratio is associated with lower output in good times but less frequent crises. Also, we extend the benchmark model by allowing households to have uncertainty regarding the regulator's supervision ability, and study how the economy evolves with both the regulator's supervision ability and households' beliefs regarding the regulator's supervision ability.

In the chapter, the sources of credit cycles come from the interaction between regulator supervision and banks' loophole innovation. Without a doubt, there are other important sources for the credit cycles, which have been widely discussed in the literature. We consider our mechanism as a novel and complementary one to those in the previous literature. To highlight our mechanism, we have omitted other sources for the business cycles from this chapter. We can potentially incorporate some common shocks in the business cycle literature into our model.

Although the present model is stylized, it would be interesting to test the implications of this model with data in future work. First, our model shows that longer boom periods predict higher probability of crises and more severe consequences. We can test the relationship between conditional frequency, as well as consequences of crises and the length of boom periods with cross-country data. Second, as more data on regulation and supervision emerges, we can study the linkage between bank regulation and business cycle patterns across countries.

Appendices

A Smets-Wouters Model

A.1 Overview

This model provides a canonical New Keynesian model. This model features both nominal and real rigidities in the economy. It has imperfect competition in intermediary good and labor market to capture the sticky price and wage, which allows monetary policy to play a role in the business cycles. More specifically, the two markets follow Calvo [13]'s assumption that each period firms or labor union face a fixed probability to change their price or wage. Also, it consists of habit formation, cost of adjusting investment, and variable capital utilization to capture the real rigidity in the economy. To generate fluctuations in the business cycles, in addition to the standard total factor productivity shocks, the model also includes two intertemporal-margin shocks (risk premium shocks and investment-specific technology shocks), two intratemporal-margin shocks (price and wage mark-up shocks), and two policy shocks (exogenous spending shocks and monetary policy shocks). It also contains labor-augmenting technological progress to capture the balanced steady-state growth path.

A.2 Environment

A.2.1 Final Goods Producers

$$\max_{Y_t, Y_{i,t}} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di$$

$$s.t. \int_0^1 G\left(\frac{Y_{i,t}}{Y_t}; \varepsilon_t^p\right) di = 1 \tag{1}$$

F.O.C.

$$Y_{i,t} = Y_t G'^{-1}\left[\frac{P_{i,t}}{P_t}\int_0^1 G'\left(\frac{Y_{j,t}}{Y_t}; \varepsilon_t^p\right)\frac{Y_{j,t}}{Y_t} dj\right]$$

Zero-profit condition

$$P_t = \int_0^1 P_{i,t} G'^{-1}\left[\frac{P_{i,t}}{P_t}\int_0^1 G'\left(\frac{Y_{j,t}}{Y_t}; \varepsilon_t^p\right)\frac{Y_{j,t}}{Y_t} dj\right] di \tag{2}$$

A.2.2 Intermediate Goods Producers

Cost minimization problem

$$\min_{K^s_{i,t}, L_{i,t}} W_t L_{i,t} + R^k_t K^s_{i,t}$$

$$s.t.\ Y_{i,t} = \mathrm{e}^{\varepsilon^a_t} (K^s_{i,t})^\alpha (\gamma^t L_{i,t})^{1-\alpha} - \gamma^t \Phi \tag{3}$$

F.O.C. (same for all firms)

$$K^s_t = \frac{\alpha}{1-\alpha} \frac{W_t}{R^k_t} L_t \tag{4}$$

and marginal cost

$$MC_t = \frac{W_t}{(1-\alpha)\gamma^{(1-\alpha)t} \mathrm{e}^{\varepsilon^a_t} (K^s_t / L_t)^\alpha} = \frac{W_t^{1-\alpha} (R^k_t)^\alpha}{\alpha^\alpha (1-\alpha)^{1-\alpha} \gamma^{(1-\alpha)t} \mathrm{e}^{\varepsilon^a_t}} \tag{5}$$

Price setting problem

$$\max_{\tilde{P}_{i,t}} E_t \sum_{s=0}^{\infty} \xi_p^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} [\tilde{P}_{i,t} X^p_{t,s} - MC_{t+s}] Y_{i,t+s}$$

$$s.t.\ Y_{i,t+s} = Y_{t+s} G'^{-1} \left(\frac{\tilde{P}_{i,t} X^p_{t,s}}{P_{t+s}} \tau^p_{t+s} \right)$$

where

$$X^p_{t,s} = \begin{cases} 1 & \text{for } s = 0 \\ \prod_{l=1}^{s} \pi_{t+l-1}^{\iota_p} \pi^{1-\iota_p} & \text{for } s = 1, \ldots, \infty \end{cases}$$

and

$$\tau^p_{t+s} = \int_0^1 G' \left(\frac{Y_{i,t+s}}{Y_{t+s}}; \varepsilon^p_{t+s} \right) \frac{Y_{i,t+s}}{Y_{t+s}} di$$

F.O.C.

$$E_t \sum_{s=0}^{\infty} \xi_p^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} Y_{i,t+s} \left[\tilde{P}_{i,t} X^p_{t,s} + (\tilde{P}_{i,t} X^p_{t,s} - MC_{t+s}) \frac{1}{G'^{-1}(\cdot)} \frac{G'(\cdot)}{G''(\cdot)} \right] = 0$$

let

$$\eta^p_{t+s} \left(\frac{\tilde{P}_{i,t} X^p_{t,s}}{P_{t+s}}; \varepsilon^p_{t+s} \right) \equiv - \frac{1}{G'^{-1}(\cdot)} \frac{G'(\cdot)}{G''(\cdot)}$$

then

$$E_t \sum_{s=0}^{\infty} \xi_p^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \frac{Y_{i,t+s}}{\eta^p_{t+s}(\cdot) - 1} \left[\tilde{P}_{i,t} X^p_{t,s} - \frac{\eta^p_{t+s}(\cdot)}{\eta^p_{t+s}(\cdot) - 1} MC_{t+s} \right] = 0 \tag{6}$$

since $\tilde{P}_{i,t}$ is the same, from equation (2) (approximately),

$$P_t = (1-\xi_p)\tilde{P}_t G'^{-1}\left[\frac{\tilde{P}_t}{P_t}\tau_t^p\right] + \xi_p \pi_{t-1}^{\iota_p}\pi^{1-\iota_p}P_{t-1}G'^{-1}\left[\frac{\pi_{t-1}^{\iota_p}\pi^{1-\iota_p}P_{t-1}}{P_t}\tau_t^p\right] \tag{7}$$

A.2.3 Household

$$\max_{C_t,I_t,B_t,Z_t,\bar{L}_t} E_t\sum_{s=0}^{\infty}\beta^s\left[\frac{1}{1-\sigma_c}(C_{t+s}-\lambda\bar{C}_{t+s-1})^{1-\sigma_c}\right]\mathrm{e}^{\frac{\sigma_c-1}{1+\sigma_l}\bar{L}_{t+s}^{1+\sigma_l}}$$

$$s.t.\ C_{t+s}+I_{t+s}+\frac{B_{t+s}}{\mathrm{e}^{\varepsilon_{t+s}^b}R_{t+s}P_{t+s}}-T_{t+s}\leq\frac{B_{t+s-1}}{P_{t+s}}+\frac{W_{t+s}^h\bar{L}_{t+s}}{P_{t+s}}+\frac{R_{t+s}^k Z_{t+s}K_{t+s-1}}{P_{t+s}} \tag{8}$$
$$-a(Z_{t+s})K_{t+s-1}+\frac{Div_{t+s}}{P_{t+s}}$$

$$K_{t+s}=(1-\delta)K_{t+s-1}+\mathrm{e}^{\varepsilon_{t+s}^i}\left[1-S\left(\frac{I_{t+s}}{I_{t+s-1}}\right)\right]I_{t+s} \tag{9}$$

where

$$K_{t+s}^s=Z_{t+s}K_{t+s-1} \tag{10}$$

and

$$\bar{L}_{t+s}=\int_0^1 L_{l,t+s}dl$$

F.O.C.

$$(\partial C_t)\ \Xi_t=(C_t-\lambda C_{t-1})^{-\sigma_c}\mathrm{e}^{\frac{\sigma_c-1}{1+\sigma_l}\bar{L}_t^{1+\sigma_l}} \tag{11}$$

$$(\partial\bar{L}_t)\ \frac{W_t^h}{P_t}=(C_t-\lambda C_{t-1})\bar{L}_t^{\sigma_l} \tag{12}$$

$$(\partial B_t)\ \Xi_t=\beta\mathrm{e}^{\varepsilon_t^b}R_t E_t\left[\frac{\Xi_{t+1}}{\pi_{t+1}}\right] \tag{13}$$

$$(\partial I_t)\ \Xi_t=\Xi_t^k\mathrm{e}^{\varepsilon_t^i}\left[1-S\left(\tfrac{I_t}{I_{t-1}}\right)-S'\left(\tfrac{I_t}{I_{t-1}}\right)\tfrac{I_t}{I_{t-1}}\right]+\beta E_t\left[\Xi_{t+1}^k\mathrm{e}^{\varepsilon_{t+1}^i}S'\left(\tfrac{I_{t+1}}{I_t}\right)\left(\tfrac{I_{t+1}}{I_t}\right)^2\right]$$

let

$$Q_t\equiv\frac{\Xi_t^k}{\Xi_t}$$

then

$$(\partial I_t)\ 1=Q_t\mathrm{e}^{\varepsilon_t^i}\left[1-S\left(\tfrac{I_t}{I_{t-1}}\right)-S'\left(\tfrac{I_t}{I_{t-1}}\right)\tfrac{I_t}{I_{t-1}}\right]+\beta E_t\tfrac{\Xi_{t+1}}{\Xi_t}\left[Q_{t+1}\mathrm{e}^{\varepsilon_{t+1}^i}S'\left(\tfrac{I_{t+1}}{I_t}\right)\left(\tfrac{I_{t+1}}{I_t}\right)^2\right] \tag{14}$$

$$(\partial K_t)\ \Xi_t^k = \beta E_t \left[\Xi_{t+1} \left(\frac{R_{t+1}^k Z_{t+1}}{P_{t+1}} - a(Z_{t+1}) \right) + \Xi_{t+1}^k (1-\delta) \right]$$

substitute with Q_t, we get

$$(\partial K_t)\ Q_t = \beta E_t \frac{\Xi_{t+1}}{\Xi_t} \left[\left(\frac{R_{t+1}^k Z_{t+1}}{P_{t+1}} - a(Z_{t+1}) \right) + Q_{t+1}(1-\delta) \right] \tag{15}$$

$$(\partial Z_t)\ \frac{R_t^k}{P_t} = a'(Z_t) \tag{16}$$

A.2.4 Intermediate Labor Union Sector

Labor packers (final labor provider)

$$\max_{L_t, L_{l,t}} W_t L_t - \int_0^1 W_{l,t} L_{l,t} dl$$

$$s.t.\ \int_0^1 H\left(\frac{L_{l,t}}{L_t}; \varepsilon_t^w\right) dl = 1$$

F.O.C.

$$L_{l,t} = L_t H'^{-1} \left[\frac{W_{l,t}}{W_t} \int_0^1 H'\left(\frac{L_{l,t}}{L_t}; \varepsilon_t^w\right) \frac{L_{l,t}}{L_t} dl \right]$$

zero-profit condition

$$W_t = \int_0^1 W_{l,t} H'^{-1} \left[\frac{W_{l,t}}{W_t} \int_0^1 H'\left(\frac{L_{j,t}}{L_t}; \varepsilon_t^w\right) \frac{L_{j,t}}{L_t} dj \right] dl \tag{17}$$

Labor unions (intermediate labor provider)

$$\max_{\tilde{W}_{l,t}} E_t \sum_{s=0}^{\infty} \xi_w^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} [\tilde{W}_{l,t} X_{t,s}^w - W_{t+s}^h] L_{l,t+s}$$

$$s.t.\ L_{l,t+s} = L_{t+s} H'^{-1} \left(\frac{\tilde{W}_{l,t} X_{t,s}^w}{W_{t+s}} \int_0^1 H'\left(\frac{L_{l,t+s}}{L_{t+s}}; \varepsilon_{t+s}^w\right) \frac{L_{l,t+s}}{L_{t+s}} dl \right)$$

where

$$X_{t,s}^w = \begin{cases} 1 & \text{for } s = 0 \\ \prod_{l=1}^{s} \gamma \pi_{t+l-1}^{\iota_w} \pi^{1-\iota_w} & \text{for } s = 1, \ldots, \infty \end{cases}$$

F.O.C.

$$E_t \sum_{s=0}^{\infty} \xi_w^s \frac{\beta^s \Xi_{t+s} P_t}{\Xi_t P_{t+s}} \frac{L_{l,t+s}}{\eta_{t+s}^w(\cdot) - 1} \left[\tilde{W}_{l,t} X_{t,s}^w - \frac{\eta_{t+s}^w(\cdot)}{\eta_{t+s}^w(\cdot) - 1} W_{t+s}^h \right] = 0 \tag{18}$$

where

$$\eta^w_{t+s}\left(\frac{\tilde{W}_{l,t}X^w_{t,s}}{W_{t+s}};\varepsilon^w_{t+s}\right)\equiv-\frac{1}{H'^{-1}(\cdot)}\frac{H'(\cdot)}{H''(\cdot)}$$

since $\tilde{W}_{l,t}$ is the same, from equation (17) (approximately),

$$W_t=(1-\xi_w)\tilde{W}_tH'^{-1}\left[\frac{\tilde{W}_t}{W_t}\tau^w_t\right]+\xi_w\gamma\pi^{\iota_w}_{t-1}\pi^{1-\iota_w}W_{t-1}H'^{-1}\left[\frac{\gamma\pi^{\iota_w}_{t-1}\pi^{1-\iota_w}W_{t-1}}{W_t}\tau^w_t\right] \tag{19}$$

where

$$\tau^w_t\equiv\int_0^1H'\left(\frac{L_{l,t}}{L_t};\varepsilon^w_t\right)\frac{L_{l,t}}{L_t}dl$$

A.2.5 Government Policies

Monetary policy

$$\frac{R_t}{R}=\left(\frac{R_{t-1}}{R}\right)^{\rho}\left[\left(\frac{\pi_t}{\pi}\right)^{r_\pi}\left(\frac{Y_t}{Y^p_t}\right)^{r_y}\right]^{1-\rho}\left(\frac{Y_t/Y_{t-1}}{Y^p_t/Y^p_{t-1}}\right)^{r_{\Delta y}}\mathrm{e}^{\varepsilon^r_t} \tag{20}$$

where Y^p_t is potential output (output under flexible prices and wages in the absence of the two "mark-up" shocks). Government budget

$$P_tG_t+B_{t-1}=T_t+\frac{B_t}{R_t}$$

where

$$G_t=\mathrm{e}^{\varepsilon^g_t}g_yy\gamma^t$$

A.2.6 Resource Constraint

$$C_t+I_t+G_t+a(Z_t)K_{t-1}=Y_t \tag{21}$$

A.3 Detrending

Notation

$$
\begin{aligned}
c_t &= \tfrac{C_t}{\gamma^t} \\
i_t &= \tfrac{I_t}{\gamma^t} \\
y_t &= \tfrac{Y_t}{\gamma^t} \\
y_{i,t} &= \tfrac{Y_{i,t}}{\gamma^t} \\
y_t^p &= \tfrac{Y_t^p}{\gamma^t} \\
k_t^s &= \tfrac{K_t^s}{\gamma^t} \\
k_{i,t}^s &= \tfrac{K_{i,t}^s}{\gamma^t} \\
k_t &= \tfrac{K_t}{\gamma^t} \\
\tilde{p}_t &= \tfrac{\tilde{P}_t}{P_t} \\
r_t^k &= \tfrac{R_t^k}{P_t} \\
mc_t &= \tfrac{MC_t}{P_t} \\
w_t &= \tfrac{W_t}{P_t \gamma^t} \\
w_t^h &= \tfrac{W_t^h}{P_t \gamma^t} \\
\zeta_t &= \Xi_t \gamma^{\sigma_c t}
\end{aligned}
$$

From equation (1), we get

$$\int_0^1 G\left(\frac{y_{i,t}}{y_t}; \varepsilon_t^p\right) di = 1 \tag{22}$$

From equation (7), we get

$$1 = (1-\xi_p)\tilde{p}_t G'^{-1}(\tilde{p}_t \tau_t^p) + \xi_p \pi_{t-1}^{\iota_p} \pi^{1-\iota_p} \pi_t^{-1} G'^{-1}(\pi_{t-1}^{\iota_p} \pi^{1-\iota_p} \pi_t^{-1} \tau_t^p) \tag{23}$$

From equation (3), we get

$$y_{i,t} = \mathrm{e}^{\varepsilon_t^a} (k_{i,t}^s)^{\alpha} (L_{i,t})^{1-\alpha} - \Phi \tag{24}$$

From equation (4), we get

$$k_t^s = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t^k} L_t \tag{25}$$

From equation (5), we get

$$mc_t = \frac{w_t}{(1-\alpha)e^{\varepsilon_t^a}(k_t^s/L_t)^\alpha} = \frac{w_t^{1-\alpha}(r_t^k)^\alpha}{\alpha^\alpha(1-\alpha)^{1-\alpha}e^{\varepsilon_t^a}} \tag{26}$$

From equation (6) (divided by P_t), we get

$$E_t \sum_{s=0}^{\infty} \xi_p^s \beta^s \gamma^{(1-\sigma_c)s} \frac{\zeta_{t+s}}{\zeta_t} \frac{y_{i,t+s}}{\eta_{t+s}^p(\cdot)-1} \left[\tilde{p}_{i,t} \frac{\prod_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi^{1-\iota_p}}{\prod_{l=1}^s \pi_{t+l}} - \frac{\eta_{t+s}^p(\tilde{p}_{i,t} \frac{\prod_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi^{1-\iota_p}}{\prod_{l=1}^s \pi_{t+l}}; \varepsilon_{t+s}^p)}{\eta_{t+s}^p(\cdot)-1} mc_{t+s} \right] = 0 \tag{27}$$

From equation (9), we get

$$k_t = \frac{(1-\delta)}{\gamma} k_{t-1} + e^{\varepsilon_t^i} \left[1 - S(\frac{i_t \gamma}{i_{t-1}}) \right] i_t \tag{28}$$

From equation (10), we get

$$k_t^s = \frac{Z_t k_{t-1}}{\gamma} \tag{29}$$

From equation (11), we get

$$\zeta_t = (c_t - \frac{\lambda}{\gamma} c_{t-1})^{-\sigma_c} e^{\frac{\sigma_c - 1}{1+\sigma_l} \bar{L}_t^{1+\sigma_l}} \tag{30}$$

From equation (12), we get

$$w_t^h = (c_t - \frac{\lambda}{\gamma} c_{t-1}) \bar{L}_t^{\sigma_l} \tag{31}$$

From equation (13), we get

$$\zeta_t = \beta \gamma^{-\sigma_c} e^{\varepsilon_t^b} R_t E_t \left[\frac{\zeta_{t+1}}{\pi_{t+1}} \right] \tag{32}$$

From equation (14), we get

$$1 = Q_t e^{\varepsilon_t^i} \left[1 - S\left(\frac{i_t \gamma}{i_{t-1}}\right) - S'\left(\frac{i_t \gamma}{i_{t-1}}\right) \frac{i_t \gamma}{i_{t-1}} \right] + \beta \gamma^{-\sigma_c} E_t \frac{\zeta_{t+1}}{\zeta_t} \left[Q_{t+1} e^{\varepsilon_{t+1}^i} S'\left(\frac{i_{t+1}\gamma}{i_t}\right) \left(\frac{i_{t+1}\gamma}{i_t}\right)^2 \right] \tag{33}$$

From equation (15), we get

$$Q_t = \beta \gamma^{-\sigma_c} E_t \frac{\zeta_{t+1}}{\zeta_t} \left[r_{t+1}^k Z_{t+1} - a(Z_{t+1}) + Q_{t+1}(1-\delta) \right] \tag{34}$$

From equation (16), we get

$$r_t^k = a'(Z_t) \tag{35}$$

From equation (18) (divided by P_t), we get

$$E_t \sum_{s=0}^{\infty} \xi_w^s \beta^s \gamma^{(1-\sigma_c)s} \frac{\zeta_{t+s}}{\zeta_t} \frac{L_{l,t+s}}{\eta_{t+s}^w(\cdot)-1} \left[\tilde{w}_{l,t} \frac{\prod_{l=1}^s \pi_{t+l-1}^{\iota_w} \pi^{1-\iota_w}}{\prod_{l=1}^s \pi_{t+l}} - \frac{\eta_{t+s}^w(\frac{\tilde{w}_{l,t}}{w_{t+s}} \frac{\prod_{l=1}^s \pi_{t+l-1}^{\iota_w} \pi^{1-\iota_w}}{\prod_{l=1}^s \pi_{t+l}}; \varepsilon_{t+s}^p)}{\eta_{t+s}^w(\cdot)-1} w_{t+s}^h \right] = 0 \quad (36)$$

From equation (19), we get

$$w_t = (1-\xi_w)\tilde{w}_t H'^{-1}\left[\frac{\tilde{w}_t}{w_t}\tau_t^w\right] + \xi_w \pi_{t-1}^{\iota_w} \pi^{1-\iota_w} \pi_t^{-1} w_{t-1} H'^{-1}\left[\frac{\pi_{t-1}^{\iota_w} \pi^{1-\iota_w} \pi_t^{-1} w_{t-1}}{w_t}\tau_t^w\right] \quad (37)$$

From equation (20), we get

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho} \left[\left(\frac{\pi_t}{\pi}\right)^{r_\pi} \left(\frac{y_t}{y_t^p}\right)^{r_y}\right]^{1-\rho} \left(\frac{y_t/y_{t-1}}{y_t^p/y_{t-1}^p}\right)^{r_{\Delta y}} \mathrm{e}^{\varepsilon_t^r} \quad (38)$$

From equation (21), we get

$$c_t + i_t + \mathrm{e}^{\varepsilon_t^g} g_y y + \frac{a(Z_t)}{\gamma} k_{t-1} = y_t \quad (39)$$

A.4 Steady State

Steady state

$$\tilde{p} = 1$$

$$z = 1$$

$$a(1) = 0$$

$$q = 1$$

$$S(\gamma) = S'(\gamma) = 0$$

$$S''(\gamma) = \varphi$$

$$\frac{a'(1)}{a''(1)} = \frac{1-\psi}{\psi}$$

Steady state relationship (find $r, r^k, c/y, i/y, k/y, wL/c$)

$$w = \tilde{w}$$

$$y = y_i = (k^s)^{\alpha} L^{1-\alpha} - \Phi$$

$$L = \bar{L} = L_l$$

r. From equation (32), we get

$$r = \frac{\pi}{\beta\gamma^{-\sigma_c}}$$

r^k. From equation (34), we get

$$r^k = \frac{1}{\beta\gamma^{-\sigma_c}} - (1-\delta)$$

k/y. From zero-profit conditions for intermediate goods producers, we get

$$\phi_p \equiv \frac{\Phi + y}{y} = \frac{1}{mc} = \frac{\eta^p}{\eta^p - 1} = \frac{(k^s)^\alpha L^{1-\alpha}}{y}$$

From equation (26), we get

$$w = \left[\frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{\phi_p (r^k)^\alpha}\right]^{\frac{1}{1-\alpha}}$$

From equation (25), we get

$$\frac{k^s}{L} = \frac{\alpha}{1-\alpha}\frac{w}{r^k}$$

From equation (24), we get

$$\frac{k^s}{y} = \phi_p \left(\frac{k^s}{L}\right)^{1-\alpha}$$

From equation (29), we get

$$\frac{k}{y} = \gamma \frac{k^s}{y}$$

i/y. From equation (28), we get

$$\frac{i}{y} = \frac{i}{k}\frac{k}{y} = (\gamma - 1 + \delta)\frac{k^s}{y}$$

c/y. From equation (39), we get

$$\frac{c}{y} = 1 - \frac{i}{y} - g_y$$

wL/c. From equation (36), we get

$$w^h = \frac{\eta_w - 1}{\eta_w} w = \frac{w}{\phi_w}$$

Then from equation (25)

$$\frac{w^h L}{c} = \frac{w}{\phi_w}\left(\frac{L}{k^s}\right)\left(\frac{k^s}{y}\right)\left(\frac{y}{c}\right) = \frac{1}{\phi_w}\frac{1-\alpha}{\alpha}\frac{r^k (k^s/y)}{c/y}$$

A.5 Log-Linearization

SW(1). From equation (39), we get

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{i}_t + \frac{r^k}{\gamma}\frac{k}{y}\hat{z}_t + \underbrace{g_y \varepsilon_t^g}_{\varepsilon_t^g}$$

SW(2). From equation (30), we get

$$\hat{\zeta}_t = (\sigma_c - 1)L^{1+\sigma_l}\hat{l}_t - \frac{\sigma_c}{1-\frac{\lambda}{\gamma}}\left(\hat{c}_t - \frac{\lambda}{\gamma}\hat{c}_{t-1}\right) = \frac{\sigma_c - 1}{1-\frac{\lambda}{\gamma}}\frac{w^h L}{c}\hat{l}_t - \frac{\sigma_c}{1-\frac{\lambda}{\gamma}}\left(\hat{c}_t - \frac{\lambda}{\gamma}\hat{c}_{t-1}\right)$$

From equation (32), we get

$$\hat{\zeta}_t = \varepsilon_t^b + \hat{r}_t - E\hat{\pi}_{t+1} + E\hat{\zeta}_{t+1}$$

Then

$$\hat{c}_t = \frac{\frac{\lambda}{\gamma}}{1+\frac{\lambda}{\gamma}}\hat{c}_{t-1} + \frac{1}{1+\frac{\lambda}{\gamma}}E\hat{c}_{t+1} + \frac{\sigma_c - 1}{\sigma_c\left(1+\frac{\lambda}{\gamma}\right)}\frac{w^h L}{c}(\hat{l}_t - E\hat{l}_{t+1}) - \frac{1-\frac{\lambda}{\gamma}}{\sigma_c\left(1+\frac{\lambda}{\gamma}\right)}(\hat{r}_t - E\hat{\pi}_{t+1} + \varepsilon_t^b)$$

SW(3). From equation (33), we get

$$\hat{i}_t = \frac{1}{1+\beta\gamma^{1-\sigma_c}}\hat{i}_{t-1} + \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}}E\hat{i}_{t+1} + \frac{1}{(1+\beta\gamma^{1-\sigma_c})\gamma^2\varphi}\hat{q}_t + \underbrace{\frac{1}{(1+\beta\gamma^{1-\sigma_c})\gamma^2\varphi}\varepsilon_t^i}_{\varepsilon_t^i}$$

SW(4). From equations (34) and (32), we get

$$\hat{q}_t = \beta\gamma^{-\sigma_c}(1-\delta)E\hat{q}_{t+1} + \beta\gamma^{-\sigma_c}r^k E\hat{r}_{t+1}^k - (\hat{r}_t - E\hat{\pi}_{t+1} + \varepsilon_t^b)$$

SW(5). From equation (22), we get

$$\hat{y}_t = \int_0^1 \hat{y}_{i,t}di$$

With equation (24), we get

$$\hat{y}_t = \phi_p(\alpha\hat{k}_t^s + (1-\alpha)\hat{l}_t + \varepsilon_t^a)$$

SW(6). From equation (29), we get

$$\hat{k}_t^s = \hat{k}_{t-1} + \hat{z}_t$$

SW(7). From equation (35), we get

$$\hat{z}_t = \frac{a'(1)}{a''(1)}\hat{r}_t^k = \frac{1-\psi}{\psi}\hat{r}_t^k$$

SW(8). From equation (28), we get

$$\hat{k}_t = \frac{1-\delta}{\gamma}\hat{k}_{t-1} + \left(1-\frac{1-\delta}{\gamma}\right)\hat{i}_t + \left(1-\frac{1-\delta}{\gamma}\right)\underbrace{\varepsilon_t^i}_{(1+\beta\gamma^{1-\sigma_c})\gamma^2\varphi\varepsilon_t^i}$$

SW(9). From equation (26), we get price mark-up

$$\mu_t^p \equiv -\hat{mc}_t = \alpha(\hat{k}_t^s - \hat{l}_t) + \varepsilon_t^a - \hat{w}_t$$

SW(10). From equation (23), we get

$$\hat{\tilde{p}}_t = \frac{\xi_p}{1-\xi_p}(\hat{\pi}_t - \iota_p\hat{\pi}_{t-1})$$

From equation (27), we get

$$\hat{\tilde{p}}_t = -\xi_p\beta\gamma^{1-\sigma_c}(\iota_p\hat{\pi}_t - E\hat{\pi}_{t+1}) - \tfrac{1-\xi_p\beta\gamma^{1-\sigma_c}}{1+(\phi_p-1)\varepsilon_p}\mu_t^p - \frac{(1-\xi_p\beta\gamma^{1-\sigma_c})\frac{1}{\eta^p}\frac{\partial\eta^p}{\partial\varepsilon_p}(\phi_p-1)}{1+(\phi_p-1)\varepsilon_p}\varepsilon_t^p + \xi_p\beta\gamma^{1-\sigma_c}E\hat{\tilde{p}}_{t+1}$$

where $\varepsilon_p \equiv \frac{\eta^{p\prime}}{\eta^p}$.

Then

$$\begin{aligned}\hat{\pi}_t = &\frac{\iota_p}{1+\beta\gamma^{1-\sigma_c}\iota_p}\hat{\pi}_{t-1} + \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}\iota_p}E\hat{\pi}_{t+1}\\ &-\frac{1-\xi_p\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}\iota_p}\frac{1-\xi_p}{\xi_p}\frac{1}{1+(\phi_p-1)\varepsilon_p}\mu_t^p\\ &\underbrace{-\frac{(1-\xi_p)(1-\xi_p\beta\gamma^{1-\sigma_c})\frac{1}{\eta^p}\frac{\partial\eta^p}{\partial\varepsilon_p}(\phi_p-1)}{\xi_p(1+\beta\gamma^{1-\sigma_c}\iota_p)(1+(\phi_p-1)\varepsilon_p)}\varepsilon_t^p}_{\varepsilon_t^p}\end{aligned}$$

SW(11). From equation (25), we get

$$\hat{r}_t^k = -(\hat{k}_t^s - \hat{l}_t) + \hat{w}_t$$

SW(12). From equation (31), we get wage mark-up ($\hat{\hat{l}}_t = \hat{l}_t$)

$$\mu_t^w \equiv \hat{w}_t - \hat{w}_t^h = \hat{w}_t - \left(\sigma_l \hat{l}_t + \frac{1}{1-\frac{\lambda}{\gamma}}(\hat{c}_t - \frac{\lambda}{\gamma}\hat{c}_{t-1})\right)$$

SW(13). From equation (37), we get

$$\hat{\hat{w}}_t = \frac{1}{1-\xi_w}\hat{w}_t + \frac{\xi_w}{1-\xi_w}(-\hat{w}_{t-1} + \hat{\pi}_t - \iota_w \hat{\pi}_{t-1})$$

From equation (36), we get

$$\begin{aligned} \hat{\hat{w}}_t = &-\xi_w \beta\gamma^{1-\sigma_c}(\iota_w \hat{\pi}_t - E\hat{\pi}_{t+1}) + (1-\xi_w\beta\gamma^{1-\sigma_c})\hat{w}_t \\ &-\frac{1-\xi_w\beta\gamma^{1-\sigma_c}}{1+(\phi_w-1)\varepsilon_w}\mu_t^w - \frac{(1-\xi_w\beta\gamma^{1-\sigma_c})\frac{1}{\eta^w}\frac{\partial\eta^w}{\partial\varepsilon_w}(\phi_w-1)}{1+(\phi_w-1)\varepsilon_w}\varepsilon_t^w + \xi_w\beta\gamma^{1-\sigma_c}E\hat{\hat{w}}_{t+1} \end{aligned}$$

where $\phi_w \equiv \frac{\eta^w}{\eta^w - 1}$ and $\varepsilon_w \equiv \frac{\eta^{w\prime}}{\eta^w}$.

Then

$$\begin{aligned} \hat{w}_t = &\frac{1}{1+\beta\gamma^{1-\sigma_c}}\hat{w}_{t-1} + \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}}(E\hat{w}_{t+1} + E\hat{\pi}_{t+1}) \\ &-\frac{1+\beta\gamma^{1-\sigma_c}\iota_w}{1+\beta\gamma^{1-\sigma_c}}\hat{\pi}_t + \frac{\iota_w}{1+\beta\gamma^{1-\sigma_c}}\hat{\pi}_{t-1} - \frac{1-\xi_w\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}}\frac{1-\xi_w}{\xi_w}\frac{1}{1+(\phi_w-1)\varepsilon_w}\mu_t^w \\ &\underbrace{-\frac{(1-\xi_w)(1-\xi_w\beta\gamma^{1-\sigma_c})\frac{1}{\eta^w}\frac{\partial\eta^w}{\partial\varepsilon_w}(\phi_w-1)}{\xi_w(1+\beta\gamma^{1-\sigma_c}\iota_w)(1+(\phi_w-1)\varepsilon_w)}\varepsilon_t^w}_{\varepsilon_t^w} \end{aligned}$$

SW(14). From equation (38), we get

$$\hat{r}_t = \rho\hat{r}_{t-1} + (1-\rho)[r_\pi\hat{\pi}_t + r_y(\hat{y}_t - \hat{y}_t^p)] + r_{\Delta y}[(\hat{y}_t - \hat{y}_t^p) - (\hat{y}_{t-1} - \hat{y}_{t-1}^p)] + \varepsilon_t^r$$

14 variables: $\hat{y}_t$, $\hat{c}_t$, $\hat{i}_t$, $\hat{q}_t$, $\hat{k}_t^s$, $\hat{k}_t$, $\hat{z}_t$, $\hat{r}_t^k$, $\hat{\mu}_t^p$, $\hat{\pi}_t$, $\hat{\mu}_t^w$, $\hat{w}_t$, $\hat{l}_t$, $\hat{r}_t$.

Flexible Economy

$$\hat{y}_t^f = \frac{c}{y}\hat{c}_t^f + \frac{i}{y}\hat{i}_t^f + \frac{r^k}{\gamma}\frac{k}{y}\hat{z}_t^f + \underbrace{g_y\varepsilon_t^g}_{\varepsilon_t^g}$$

$$\hat{c}_t^f = \frac{\frac{\lambda}{\gamma}}{1+\frac{\lambda}{\gamma}}\hat{c}_{t-1}^f + \frac{1}{1+\frac{\lambda}{\gamma}}E\hat{c}_{t+1}^f + \frac{\sigma_c - 1}{\sigma_c\left(1+\frac{\lambda}{\gamma}\right)}\frac{w^h L}{c}(\hat{l}_t^f - E\hat{l}_{t+1}^f) - \frac{1-\frac{\lambda}{\gamma}}{\sigma_c\left(1+\frac{\lambda}{\gamma}\right)}(\hat{r}_t^f + \varepsilon_t^b)$$

$$\hat{i}_t^f = \tfrac{1}{1+\beta\gamma^{1-\sigma_c}}\hat{i}_{t-1}^f + \tfrac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}}E\hat{i}_{t+1}^f + \tfrac{1}{(1+\beta\gamma^{1-\sigma_c})\gamma^2\varphi}\hat{q}_t^f + \underbrace{\frac{1}{(1+\beta\gamma^{1-\sigma_c})\gamma^2\varphi}\varepsilon_t^i}_{\varepsilon_t^i}$$

$$\hat{q}_t^f = \beta\gamma^{-\sigma_c}(1-\delta)E\hat{q}_{t+1}^f + \beta\gamma^{-\sigma_c}r^k E\hat{r}_{t+1}^{k,f} - (\hat{r}_t^f + \varepsilon_t^b)$$

$$\hat{y}_t^f = \phi_p(\alpha\hat{k}_t^{s,f} + (1-\alpha)\hat{l}_t^f + \varepsilon_t^a)$$

$$\hat{k}_t^{s,f} = \hat{k}_{t-1}^f + \hat{z}_t^f$$

$$\hat{z}_t^f = \frac{1-\psi}{\psi}\hat{r}_t^{k,f}$$

$$\hat{k}_t^f = \frac{1-\delta}{\gamma}\hat{k}_{t-1}^f + \left(1 - \frac{1-\delta}{\gamma}\right)\hat{i}_t^f + \left(1 - \frac{1-\delta}{\gamma}\right)\overbrace{(1+\beta\gamma^{1-\sigma_c})\gamma^2\varphi\varepsilon_t^i}^{\varepsilon_t^i}$$

$$0 = \alpha(\hat{k}_t^{s,f} - \hat{l}_t^f) + \varepsilon_t^a - \hat{w}_t^f$$

$$\hat{r}_t^{k,f} = -(\hat{k}_t^{s,f} - \hat{l}_t^f) + \hat{w}_t^f$$

$$0 = \hat{w}_t^f - \left(\sigma_l\hat{l}_t^f + \frac{1}{1-\frac{\lambda}{\gamma}}(\hat{c}_t^f - \frac{\lambda}{\gamma}\hat{c}_{t-1}^f)\right)$$

A.6 Shock Processes

Exogenous spending shock

$$\varepsilon_t^g = \rho_g\varepsilon_{t-1}^g + \eta_t^g + \rho_{ga}\eta_t^a$$

Risk premium shock

$$\varepsilon_t^b = \rho_b\varepsilon_{t-1}^b + \eta_t^b$$

Investment-specific technology shock

$$\varepsilon_t^i = \rho_i\varepsilon_{t-1}^i + \eta_t^i$$

Total factor productivity shock

$$\varepsilon_t^a = \rho_a\varepsilon_{t-1}^a + \eta_t^a$$

Price mark-up shock

$$\varepsilon_t^p = \rho_p\varepsilon_{t-1}^p + \eta_t^p - \mu_p\eta_{t-1}^p$$

Wage mark-up shock

$$\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w$$

Monetary policy shock

$$\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r$$

Measurement Equation

$$Y_t = \begin{bmatrix} dlGDP_t \\ dlCON_t \\ dlINV_t \\ dlWAG_t \\ lHOURS_t \\ dlP_t \\ FEDFUNDS_t \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{l} \\ \bar{\pi} \\ \bar{r} \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{c}_t - \hat{c}_{t-1} \\ \hat{i}_t - \hat{i}_{t-1} \\ \hat{w}_t - \hat{w}_{t-1} \\ \hat{l}_t \\ \hat{\pi}_t \\ \hat{r}_t \end{bmatrix}$$

where l and dl stand for 100 times log and log difference, respectively; $\bar{\gamma} = 100(\gamma - 1)$ is the common quarterly trend growth rate to real GDP, consumption, investment and wages; $\bar{\pi} = 100(\pi - 1)$ is the quarterly steady-state inflation rate; and $\bar{r} = (\beta^{-1}\gamma^{\sigma_c}\pi - 1)$ is the steady-state nominal interest rate; $\bar{l}$ is steady-state hours worked, which is normalized to be equal to zero.

B Iacoviello-Neri Model

B.1 Overview

This comprehensive DSGE model consists of four main elements: (1) a multi-sector structure with housing and non-housing goods; (2) nominal rigidities (prices and wages); (3) financial frictions in the household sector (collateral constraint); (4) a rich set of shocks (sectoral productivity shock, housing preference shock, preference shock, ISTC shock, CB inflation target shock, labor supply shock,...). Thus, this model can be viewed as SW model plus a housing sector (with price and quantity), and it tries to answer the following two questions: (1) What are the main driving forces of fluctuations in the housing market? (2) How large are the spillovers from the housing market to the wider economy?

B.2 Environment

B.2.1 Households

Patient households work, consume, and accumulate housing. They own the productive capital of the economy, and supply funds to firms on the one hand, and to impatient households on the other hand. They also own land and rent land to the final good production firm. And they supply intermediate goods to the firm.

Patient household

$$\max E_0 \sum_{t=0}^{\infty} (\beta G_C)^t z_t \left(\Gamma_c \ln(c_t - \varepsilon c_{t-1}) + j_t \ln h_t - \frac{\tau_t}{1+\eta} (n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi})^{\frac{1+\eta}{1+\xi}} \right) \tag{40}$$

$$\begin{aligned} s.t.\ c_t + \frac{k_{c,t}}{A_{k,t}} + k_{h,t} + k_{b,t} + q_t h_t + p_{l,t} l_t - b_t = \frac{w_{c,t} n_{c,t}}{X_{wc,t}} + \frac{w_{h,t} n_{h,t}}{X_{wh,t}} \\ + \left(R_{c,t} z_{c,t} + \frac{1-\delta_{kc}}{A_{k,t}} \right) k_{c,t-1} + (R_{h,t} z_{h,t} + 1 - \delta_{kh}) k_{h,t-1} + p_{b,t} k_{b,t} - \frac{R_{t-1} b_{t-1}}{\pi_t} \\ + (p_{l,t} + R_{l,t}) l_{t-1} + q_t (1-\delta_h) h_{t-1} + Div_t - \phi_t - \frac{a(z_{c,t}) k_{c,t-1}}{A_{k,t}} - a(z_{h,t}) k_{h,t-1} \end{aligned} \tag{41}$$

where $\Gamma_c = (G_C - \varepsilon)/(G_C - \beta \varepsilon G_C)$, $\Gamma_c' = (G_C - \varepsilon')/(G_C - \beta' \varepsilon' G_C)$, and $Div_t = (1 - 1/X_{wc,t}) w_{c,t} n_{c,t} + (1 - 1/X_{wh,t}) w_{h,t} n_{h,t} + (1 - 1/X_t) Y_t$. j_t represents housing preference shock (demand shock). z_t denotes intertemporal preference shock. And τ_t is labor supply shock.

adjustment cost on capital

$$\phi_t = \frac{\phi_{kc}}{2}\left(\frac{k_{c,t}}{k_{c,t-1}} - G_K\right)^2 \frac{k_{c,t-1}}{A_{k,t}} + \frac{\phi_{kh}}{2}\left(\frac{k_{h,t}}{k_{h,t-1}} - G_C\right)^2 k_{h,t-1} \tag{42}$$

adjustment cost on capacity utilization

$$a(z_{c,t}) = R_c\left(\frac{1}{2}\varpi z_{c,t}^2 + (1-\varpi)z_{c,t} + \left(\frac{\varpi}{2} - 1\right)\right) \tag{43}$$

$$a(z_{h,t}) = R_h\left(\frac{1}{2}\varpi z_{h,t}^2 + (1-\varpi)z_{h,t} + \left(\frac{\varpi}{2} - 1\right)\right) \tag{44}$$

where R_c and R_h are steady state for rent rate of capital in goods and housing sector.

Impatient household do not accumulate capital and do not own finished good firms or land (the dividends only come from labor unions). And they accumulate housing and borrow from patient households for the maximum possible amount against it.

Impatient household

$$\max E_0 \sum_{t=0}^{\infty}(\beta' G_C)^t z_t\left(\Gamma_c' \ln(c_t' - \varepsilon' c_{t-1}') + j_t \ln h_t' - \frac{\tau_t}{1+\eta'}((n_{c,t}')^{1+\xi'} + (n_{h,t}')^{1+\xi'})^{\frac{1+\eta'}{1+\xi'}}\right) \tag{45}$$

$$s.t.\ c_t' + q_t h_t' - b_t' = \frac{w_{c,t}' n_{c,t}'}{X_{wc,t}'} + \frac{w_{h,t}' n_{h,t}'}{X_{wh,t}'} + q_t(1-\delta_h)h_{t-1}' - \frac{R_{t-1} b_{t-1}'}{\pi_t} + Div_t' \tag{46}$$

$$b_t' \leq m E_t\left(\frac{q_{t+1} h_t' \pi_{t+1}}{R_t}\right) \tag{47}$$

where $Div_t' = (1 - 1/X_{wc,t}')w_{c,t}' n_{c,t}' + (1 - 1/X_{wh,t}')w_{h,t}' n_{h,t}'$.

B.2.2 Technology

Competitive flexible price/wholesale firms that produce wholesale consumption goods and housing using two technologies, and a final good firm (described below) that operates in the consumption sector under monopolistic competition. Wholesale firms hire labor and capital services, and purchase intermediate goods to produce wholesale goods Y_t and new houses IH_t.

Wholesale firms

$$\max \frac{Y_t}{X_t} + q_t IH_t - \left(\sum_{i=c,h} w_{i,t} n_{i,t} + \sum_{i=c,h} w_{i,t}' n_{i,t}' + \sum_{i=c,h} R_{i,t} z_{i,t} k_{i,t-1} + R_{l,t} l_{t-1} + p_{b,t} k_{b,t}\right) \tag{48}$$

$$s.t.\ Y_t = (A_{c,t}(n_{c,t}^{\alpha} n_{c,t}'^{1-\alpha}))^{1-\mu_c}(z_{c,t}k_{c,t-1})^{\mu_c} \tag{49}$$

$$IH_t = (A_{h,t}(n_{h,t}^{\alpha} n_{h,t}'^{1-\alpha}))^{1-\mu_h-\mu_b-\mu_l}(z_{h,t}k_{h,t-1})^{\mu_h} k_{b,t}^{\mu_b} l_{t-1}^{\mu_l} \tag{50}$$

where X_t is the markup of final goods over wholesale goods.

B.2.3 Nominal Rigidities and Monetary Policy

We allow for price rigidities in the consumption sector and for wage rigidities in both the consumption and housing sectors. We rule out price rigidities in the housing market.

We introduce sticky prices in the consumption sector by assuming monopolistic competition at the retail level and implicit costs of adjusting nominal prices following Calvo-style contracts. Each period, a fraction $1 - \theta_\pi$ of retailers set prices optimally, while a fraction θ_π cannot do so, and index prices to the previous period inflation rate with an elasticity equal to ι_π.

Consumption-sector Phillips curve

$$\ln \pi_t - \iota_\pi \ln \pi_{t-1} = \beta G_C(E_t \ln \pi_{t+1} - \iota_\pi \ln \pi_t) - \varepsilon_\pi \ln(X_t/X) + u_{p,t} \tag{51}$$

where $\varepsilon_\pi = (1-\theta_\pi)(1-\beta G_C \theta_\pi)/\theta_\pi$, $u_{p,t}$ is i.i.d. normal shock with zero mean and variance σ_p^2.

Patient and impatient households supply homogeneous labor services to unions. The unions differentiate labor services as in Smets and Wouters (2007), set wages subject to a Calvo scheme and offer labor services to wholesale labor packers who reassemble these services into the homogeneous labor composites $n_{i,t}$ and $n'_{i,t}$ for $i = c, h$.

Wage Phillips curve

$$\ln \omega_{c,t} - \iota_{wc} \ln \pi_{t-1} = \beta G_C(E_t \ln \omega_{c,t+1} - \iota_{wc} \ln \pi_t) - \varepsilon_{wc} \ln(X_{wc,t}/X_{wc}) \tag{52}$$

$$\ln \omega'_{c,t} - \iota_{wc} \ln \pi_{t-1} = \beta' G_C(E_t \ln \omega'_{c,t+1} - \iota_{wc} \ln \pi_t) - \varepsilon'_{wc} \ln(X'_{wc,t}/X_{wc}) \tag{53}$$

$$\ln \omega_{h,t} - \iota_{wh} \ln \pi_{t-1} = \beta G_C(E_t \ln \omega_{h,t+1} - \iota_{wh} \ln \pi_t) - \varepsilon_{wh} \ln(X_{wh,t}/X_{wh}) \tag{54}$$

$$\ln \omega'_{h,t} - \iota_{wh} \ln \pi_{t-1} = \beta' G_C(E_t \ln \omega'_{h,t+1} - \iota_{wh} \ln \pi_t) - \varepsilon'_{wh} \ln(X'_{wh,t}/X_{wh}) \tag{55}$$

where $\omega_{i,t} = \frac{w_{i,t}\pi_t}{w_{i,t-1}}$ is nominal wage inflation, $\varepsilon_{wi} = (1-\theta_{wi})(1-\beta G_C \theta_{wi})/\theta_{wi}$, and $\varepsilon'_{wi} = (1-\theta_{wi})(1-\beta' G_C \theta_{wi})/\theta_{wi}$.

Taylor rule

$$R_t = R_{t-1}^{r_R} \pi_t^{(1-r_R)r_\pi} \left(\frac{GDP_t}{G_C GDP_{t-1}} \right)^{(1-r_R)r_Y} \bar{rr}^{1-r_R} \frac{u_{R,t}}{s_t} \tag{56}$$

where $\bar{rr}$ is the steady-state real interest rate, $u_{R,t}$ i.i.d. normal shock with zero mean and variance σ_R^2, s_t is a stochastic process with high persistence capturing long-lasting deviations of inflation from its steady-state level, due,e.g.,to shifts in the central bank's inflation target, and $GDP_t = C_t + \bar{q}_t IH_t + IK_t$.

B.2.4 Equilibrium

Final goods market

$$C_t + IK_{c,t}/A_{k,t} + IK_{h,t} + k_{b,t} = Y_t - \phi_t \tag{57}$$

Housing market

$$H_t - (1 - \delta_h) H_{t-1} = IH_t \tag{58}$$

Debt market

$$b_t + b_t' = 0 \tag{59}$$

Aggregate

$$C_t = c_t + c_t' \tag{60}$$

$$H_t = h_t + h_t' \tag{61}$$

$$IK_{c,t} = k_{c,t} - (1 - \delta_{kc}) k_{c,t-1} \tag{62}$$

$$IK_{h,t} = k_{h,t} - (1 - \delta_{kh}) k_{h,t-1} \tag{63}$$

B.2.5 Trends and Balanced Growth

Productivity

$$\ln A_{c,t} = t \ln(1 + \gamma_{AC}) + \ln Z_{c,t}, \quad \ln Z_{c,t} = \rho_{AC} \ln Z_{c,t-1} + u_{C,t} \tag{64}$$

$$\ln A_{h,t} = t \ln(1 + \gamma_{AH}) + \ln Z_{h,t}, \quad \ln Z_{h,t} = \rho_{AH} \ln Z_{h,t-1} + u_{H,t} \tag{65}$$

$$\ln A_{k,t} = t \ln(1 + \gamma_{AK}) + \ln Z_{k,t}, \quad \ln Z_{k,t} = \rho_{AK} \ln Z_{k,t-1} + u_{K,t} \tag{66}$$

Growth rates

$$G_C = G_{IK_h} = G_{q\times IH} = 1 + \gamma_{AC} + \frac{\mu_c}{1-\mu_c}\gamma_{AK} \tag{67}$$

$$G_{IK_c} = 1 + \gamma_{AC} + \frac{1}{1-\mu_c}\gamma_{AK} \tag{68}$$

$$G_{IH} = 1 + (\mu_h + \mu_b)\gamma_{AC} + \frac{\mu_c(\mu_h + \mu_b)}{1-\mu_c}\gamma_{AK} + (1 - \mu_h - \mu_l - \mu_b)\gamma_{AH} \tag{69}$$

$$G_q = 1 + (1 - \mu_h - \mu_b)\gamma_{AC} + \frac{\mu_c(1 - \mu_h - \mu_b)}{1-\mu_c}\gamma_{AK} - (1 - \mu_h - \mu_l - \mu_b)\gamma_{AH} \tag{70}$$

B.3 Euler Equations

B.3.1 Patient Household

marginal utility of consumption

$$u_{c,t} = z_t \frac{G_C - \varepsilon}{G_C - \beta\varepsilon G_C}\left(\frac{1}{c_t - \varepsilon c_{t-1}} - \frac{\beta G_C \varepsilon}{c_{t+1} - \varepsilon c_t}\right) \tag{71}$$

marginal utility of housing

$$u_{h,t} = \frac{z_t j_t}{h_t} \tag{72}$$

marginal disutility of working in the goods and housing sector

$$u_{nc,t} = z_t \tau_t n_{c,t}^{\xi}\left(n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi}\right)^{\frac{\eta-\xi}{1+\xi}} \tag{73}$$

$$u_{nh,t} = z_t \tau_t n_{h,t}^{\xi}\left(n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi}\right)^{\frac{\eta-\xi}{1+\xi}} \tag{74}$$

F.O.C.

$$\partial h_t: \quad u_{c,t}q_t = u_{h,t} + \beta G_C E_t[u_{c,t+1}q_{t+1}(1-\delta_h)] \tag{75}$$

$$\partial b_t: \quad u_{c,t} = \beta G_C E_t(u_{c,t+1}R_t/\pi_{t+1}) \tag{76}$$

$$\partial k_{c,t}: \quad u_{c,t}\left(\frac{1}{A_{k,t}} + \frac{\partial \phi_t}{\partial k_{c,t}}\right) = \tag{77}$$

$$\beta G_C E_t\left[u_{c,t+1}\left(R_{c,t+1}z_{c,t+1} - \frac{a(z_{c,t+1})}{A_{k,t+1}} + \frac{1-\delta_{kc}}{A_{k,t+1}} - \frac{\partial \phi_{t+1}}{\partial k_{c,t}}\right)\right] \tag{78}$$

$$\partial k_{h,t}: \quad u_{c,t}\left(1 + \frac{\partial \phi_t}{\partial k_{h,t}}\right) = \tag{79}$$

$$\beta G_C E_t\left[u_{c,t+1}\left(R_{h,t+1}z_{h,t+1} - a(z_{h,t+1}) + 1 - \delta_{hc} - \frac{\partial \phi_{t+1}}{\partial k_{h,t}}\right)\right] \tag{80}$$

$$\partial n_{c,t}: \quad u_{nc,t} = u_{c,t}\frac{w_{c,t}}{X_{wc,t}} \tag{81}$$

$$\partial n_{h,t}: \quad u_{nh,t} = u_{c,t}\frac{w_{h,t}}{X_{wh,t}} \tag{82}$$

$$\partial k_{b,t}: \quad u_{c,t}(p_{b,t} - 1) = 0 \tag{83}$$

$$\partial z_{c,t}: \quad R_{c,t} = \frac{a'(z_{c,t})}{A_{k,t}} \tag{84}$$

$$\partial z_{h,t}: \quad R_{h,t} = a'(z_{c,t}) \tag{85}$$

$$\partial l_t: \quad u_{c,t}p_{l,t} = \beta G_C E_t[u_{c,t+1}(p_{l,t+1} + R_{l,t+1})] \tag{86}$$

B.3.2 Impatient Household

marginal utility of consumption

$$u_{c',t} = z_t\frac{G_C - \varepsilon'}{G_C - \beta'\varepsilon' G_C}\left(\frac{1}{c'_t - \varepsilon' c'_{t-1}} - \frac{\beta' G_C \varepsilon'}{c'_{t+1} - \varepsilon' c'_t}\right) \tag{87}$$

marginal utility of housing

$$u_{h',t} = \frac{z_t j_t}{h'_t} \tag{88}$$

marginal disutility of working in the goods and housing sector

$$u_{nc',t} = z_t\tau_t(n'_{c,t})^{\xi}\left((n'_{c,t})^{1+\xi} + (n'_{h,t})^{1+\xi}\right)^{\frac{\eta-\xi}{1+\xi}} \tag{89}$$

$$u_{nh',t} = z_t\tau_t(n'_{h,t})^{\xi}\left((n'_{c,t})^{1+\xi} + (n'_{h,t})^{1+\xi}\right)^{\frac{\eta-\xi}{1+\xi}} \tag{90}$$

F.O.C.

$$\partial h'_t: \quad u_{c',t}q_t = u_{h',t} + \beta' G_C E_t[u_{c',t+1}q_{t+1}(1-\delta_h)] + E_t\left(\lambda_t \frac{mq_{t+1}\pi_{t+1}}{R_t}\right) \tag{91}$$

$$\partial b'_t: \quad u_{c',t} = \beta' G_C E_t(u_{c',t+1}R_t/\pi_{t+1}) + \lambda_t \tag{92}$$

$$\partial n_{c',t}: \quad u_{nc',t} = u_{c',t}\frac{w'_{c,t}}{X'_{wc,t}} \tag{93}$$

$$\partial n_{h',t}: \quad u_{nh',t} = u_{c',t}\frac{w'_{h,t}}{X'_{wh,t}} \tag{94}$$

$$\tag{95}$$

B.3.3 Intermediate Goods Firms

F.O.C.

$$\partial n_{c,t}: \quad (1-\mu_c)\alpha\frac{Y_t}{X_t n_{c,t}} = w_{c,t} \tag{96}$$

$$\partial n'_{c,t}: \quad (1-\mu_c)(1-\alpha)\frac{Y_t}{X_t n'_{c,t}} = w'_{c,t} \tag{97}$$

$$\partial n_{h,t}: \quad (1-\mu_h-\mu_b-\mu_l)\alpha\frac{q_t IH_t}{n_{h,t}} = w_{h,t} \tag{98}$$

$$\partial n'_{h,t}: \quad (1-\mu_h-\mu_b-\mu_l)(1-\alpha)\frac{q_t IH_t}{n'_{h,t}} = w'_{h,t} \tag{99}$$

$$\partial k_{c,t-1}: \quad \mu_c\frac{Y_t}{X_t k_{c,t-1}} = R_{c,t}z_{c,t} \tag{100}$$

$$\partial k_{h,t-1}: \quad \mu_h\frac{q_t IH_t}{k_{h,t-1}} = R_{h,t}z_{h,t} \tag{101}$$

$$\partial l_t: \quad \mu_l q_t IH_t = R_{l,t} \tag{102}$$

$$\partial k_{b,t}: \quad \mu_b\frac{q_t IH_t}{k_{b,t}} = p_{b,t} \tag{103}$$

where set $l_t = 1$.

B.4 Detrend

B.4.1 Detrend Rules

growth rate G_C:

c_t, c'_t, $k_{h,t}$, $k_{b,t}$, $p_{l,t}$, $R_{l,t}$, b_t, b'_t, $w_{c,t}$, $w'_{c,t}$, $w_{h,t}$, $w'_{h,t}$, Y_t

detrend method: $\cdot/G_C^t$

growth rate G_H:

h_t, h'_t, IH_t

detrend method: $\cdot/G_H^t$

growth rate G_Q:

q_t

detrend method: $\cdot/G_Q^t$

growth rate G_K:

$k_{c,t}$

detrend method: $\cdot/G_K^t$

growth rate G_C^{-1}:

$u_{c,t}$, $u'_{c,t}$

detrend method: $\cdot \times G_C^t$

growth rate G_H^{-1}:

$u_{h,t}$, $u'_{h,t}$

detrend method: $\cdot \times G_H^t$

growth rate Γ_K:

$R_{c,t}$

detrend method: $\cdot \times \Gamma_K^t$

no trend: $n_{i,t}$, $n'_{i,t}$, R_t, $R_{h,t}$

B.4.2 Patient Household

Define the detrended variable $\tilde{x}_t \equiv x_t/G_x^t$. Then the preference becomes

$$\max E_0 \sum_{t=0}^{\infty} (\beta G_C)^t z_t \left(\Gamma_c \ln(\tilde{c}_t - \frac{\varepsilon \tilde{c}_{t-1}}{G_C}) + j_t \ln \tilde{h}_t - \frac{\tau_t}{1+\eta} (\tilde{n}_{c,t}^{1+\xi} + \tilde{n}_{h,t}^{1+\xi})^{\frac{1+\eta}{1+\xi}} + t\Gamma_c \ln G_C + t j_t \ln G_H \right)$$

marginal utility of consumption

$$\tilde{u}_{c,t} = z_t \frac{G_C - \varepsilon}{G_C - \beta\varepsilon G_C} \left(\frac{1}{\tilde{c}_t - \frac{\varepsilon}{G_C}\tilde{c}_{t-1}} - \frac{\beta\varepsilon}{\tilde{c}_{t+1} - \frac{\varepsilon}{G_C}\tilde{c}_t} \right) \tag{104}$$

marginal utility of housing

$$\tilde{u}_{h,t} = \frac{z_t j_t}{\tilde{h}_t} \tag{105}$$

From (41), budget constraint

$$\begin{aligned} \tilde{c}_t + \frac{\tilde{k}_{c,t}}{a_{k,t}} + \tilde{k}_{h,t} + \tilde{k}_{b,t} + \tilde{q}_t\tilde{h}_t + \tilde{p}_{l,t}l_t - \tilde{b}_t = \tilde{w}_{c,t}n_{c,t} + \tilde{w}_{h,t}n_{h,t} \\ + \left(\tilde{R}_{c,t}z_{c,t} + \frac{1-\delta_{kc}}{a_{k,t}}\right)\frac{\tilde{k}_{c,t-1}}{G_K} + (R_{h,t}z_{h,t} + 1 - \delta_{kh})\frac{\tilde{k}_{h,t-1}}{G_C} + p_{b,t}\tilde{k}_{b,t} - \frac{R_{t-1}\tilde{b}_{t-1}}{\pi_t G_C} \\ + (\tilde{p}_{l,t} + \tilde{R}_{l,t})l_{t-1} + \tilde{q}_t(1-\delta_h)\frac{\tilde{h}_{t-1}}{G_H} + \left(1 - \frac{1}{X_t}\right)\tilde{Y}_t - \frac{\phi_{kc}G_K}{2}\left(\frac{\tilde{k}_{c,t}}{\tilde{k}_{c,t-1}} - 1\right)^2 \tilde{k}_{c,t-1} \\ - \frac{\phi_{kh}G_C}{2}\left(\frac{\tilde{k}_{h,t}}{\tilde{k}_{h,t-1}} - 1\right)^2 \tilde{k}_{h,t-1} - \frac{a(z_{c,t})\tilde{k}_{c,t-1}}{a_{k,t}G_K} - a(z_{h,t})\frac{\tilde{k}_{h,t-1}}{G_C} \end{aligned} \tag{106}$$

F.O.C.

$$\partial h_t: \quad \tilde{u}_{c,t}\tilde{q}_t = \tilde{u}_{h,t} + \beta G_Q E_t[\tilde{u}_{c,t+1}\tilde{q}_{t+1}(1-\delta_h)] \tag{107}$$

$$\partial b_t: \quad \tilde{u}_{c,t} = \beta E_t(\tilde{u}_{c,t+1}R_t/\pi_{t+1}) \tag{108}$$

$$\begin{aligned} \partial k_{c,t}: \quad & \tilde{u}_{c,t}\left[\frac{1}{a_{k,t}} + \phi_{kc}\left(\frac{\tilde{k}_{c,t}}{\tilde{k}_{c,t-1}} - 1\right)\right] = \\ & \beta G_C E_t\left[\frac{\tilde{u}_{c,t+1}}{G_K}\left(\tilde{R}_{c,t+1}z_{c,t+1} - \frac{a(z_{c,t})}{a_{k,t}} + \frac{1-\delta_{kc}}{a_{k,t+1}} + \frac{G_K\phi_{kc}}{2}\left(\frac{\tilde{k}^2_{c,t+1}}{\tilde{k}^2_{c,t}} - 1\right)\right)\right] \end{aligned} \tag{109}$$

$$\begin{aligned} \partial k_{h,t}: \quad & \tilde{u}_{c,t}\left[1 + \phi_{kh}\left(\frac{\tilde{k}_{h,t}}{\tilde{k}_{h,t-1}} - 1\right)\right] = \\ & \beta G_C E_t\left[\frac{\tilde{u}_{c,t+1}}{G_C}\left(\tilde{R}_{h,t+1}z_{h,t+1} - a(z_{h,t}) + 1 - \delta_{kh} + \frac{G_C\phi_{kh}}{2}\left(\frac{\tilde{k}^2_{h,t+1}}{\tilde{k}^2_{h,t}} - 1\right)\right)\right] \end{aligned} \tag{110}$$

$$\tag{111}$$

$$\partial n_{c,t}: \quad z_t \tau_t n_{c,t}^{\xi} \left(n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi} \right)^{\frac{\eta-\xi}{1+\xi}} = \tilde{u}_{c,t} \frac{\tilde{w}_{c,t}}{X_{wc,t}} \tag{112}$$

$$\partial n_{h,t}: \quad z_t \tau_t n_{h,t}^{\xi} \left(n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi} \right)^{\frac{\eta-\xi}{1+\xi}} = \tilde{u}_{c,t} \frac{\tilde{w}_{h,t}}{X_{wh,t}} \tag{113}$$

$$\partial k_{b,t}: \quad \tilde{u}_{c,t}(p_{b,t} - 1) = 0 \tag{114}$$

$$\partial z_{c,t}: \quad \tilde{R}_{c,t} = \frac{a'(z_{c,t})}{a_{k,t}} \tag{115}$$

$$\partial z_{h,t}: \quad R_{h,t} = a'(z_{c,t}) \tag{116}$$

$$\partial l_t: \quad \tilde{u}_{c,t} \tilde{p}_{l,t} = \beta G_C E_t[\tilde{u}_{c,t+1}(\tilde{p}_{l,t+1} + \tilde{R}_{l,t+1})] \tag{117}$$

B.4.3 Impatient Household

Do the similar transformation to impatient households' problem. The marginal utility of consumption

marginal utility of consumption

$$\tilde{u}_{c',t} = z_t \frac{G_C - \varepsilon'}{G_C - \beta' \varepsilon' G_C} \left(\frac{1}{\tilde{c}'_t - \frac{\varepsilon'}{G_C} \tilde{c}'_{t-1}} - \frac{\beta' \varepsilon'}{\tilde{c}'_{t+1} - \frac{\varepsilon'}{G_C} \tilde{c}'_t} \right) \tag{118}$$

marginal utility of housing

$$\tilde{u}_{h',t} = \frac{z_t j_t}{\tilde{h}'_t} \tag{119}$$

F.O.C.

$$\partial h'_t: \quad \tilde{u}_{c',t} \tilde{q}_t = \tilde{u}_{h',t} + \beta' G_Q E_t[\tilde{u}_{c',t+1} \tilde{q}_{t+1}(1-\delta_h)] + E_t \left(\tilde{\lambda}_t \frac{m \tilde{q}_{t+1} \pi_{t+1}}{R_t} G_Q \right) \tag{120}$$

$$\partial b'_t: \quad \tilde{u}_{c',t} = \beta' E_t(\tilde{u}_{c',t+1} R_t / \pi_{t+1}) + \tilde{\lambda}_t \tag{121}$$

$$\partial n_{c',t}: \quad z_t \tau_t (n'_{c,t})^{\xi} \left((n'_{c,t})^{1+\xi} + (n'_{h,t})^{1+\xi} \right)^{\frac{\eta-\xi}{1+\xi}} = \tilde{u}_{c',t} \frac{\tilde{w}'_{c,t}}{X'_{wc,t}} \tag{122}$$

$$\partial n_{h',t}: \quad z_t \tau_t (n'_{h,t})^{\xi} \left((n'_{c,t})^{1+\xi} + (n'_{h,t})^{1+\xi} \right)^{\frac{\eta-\xi}{1+\xi}} = \tilde{u}_{c',t} \frac{\tilde{w}'_{h,t}}{X'_{wh,t}} \tag{123}$$

$$\tag{124}$$

From (46)

$$\tilde{c}'_t + \tilde{q}_t \tilde{h}'_t - \tilde{b}'_t = \tilde{w}'_{c,t} n'_{c,t} + \tilde{w}'_{h,t} n'_{h,t} + \tilde{q}_t (1-\delta_h) \frac{\tilde{h}'_{t-1}}{G_H} - \frac{R_{t-1} \tilde{b}'_{t-1}}{\pi_t G_C} \tag{125}$$

From (47)

$$\tilde{b}'_t = mE_t\left(G_Q\frac{\tilde{q}_{t+1}\tilde{h}'_t\pi_{t+1}}{R_t}\right) \tag{126}$$

B.4.4 Intermediate Goods Firms

From (49)

$$\tilde{Y}_t = (a_{c,t}(n^{\alpha}_{c,t}n'^{1-\alpha}_{c,t}))^{1-\mu_c}(z_{c,t}\tilde{k}_{c,t-1}/G_K)^{\mu_c} \tag{127}$$

$$I\tilde{H}_t = (a_{h,t}(n^{\alpha}_{h,t}n'^{1-\alpha}_{h,t}))^{1-\mu_h-\mu_b-\mu_l}(z_{h,t}\tilde{k}_{h,t-1}/G_C)^{\mu_h}\tilde{k}^{\mu_b}_{b,t}l^{\mu_l}_{t-1} \tag{128}$$

F.O.C.

$$\partial n_{c,t}: \quad (1-\mu_c)\alpha\frac{\tilde{Y}_t}{X_t n_{c,t}} = \tilde{w}_{c,t} \tag{129}$$

$$\partial n'_{c,t}: \quad (1-\mu_c)(1-\alpha)\frac{\tilde{Y}_t}{X_t n'_{c,t}} = \tilde{w}'_{c,t} \tag{130}$$

$$\partial n_{h,t}: \quad (1-\mu_h-\mu_b-\mu_l)\alpha\frac{\tilde{q}_t I\tilde{H}_t}{n_{h,t}} = \tilde{w}_{h,t} \tag{131}$$

$$\partial n'_{h,t}: \quad (1-\mu_h-\mu_b-\mu_l)(1-\alpha)\frac{\tilde{q}_t I\tilde{H}_t}{n'_{h,t}} = \tilde{w}'_{h,t} \tag{132}$$

$$\partial k_{c,t-1}: \quad \mu_c\frac{\tilde{Y}_t G_K}{X_t\tilde{k}_{c,t-1}} = \tilde{R}_{c,t}z_{c,t} \tag{133}$$

$$\partial k_{h,t-1}: \quad \mu_h\frac{\tilde{q}_t I\tilde{H}_t G_C}{\tilde{k}_{h,t-1}} = R_{h,t}z_{h,t} \tag{134}$$

$$\partial l_t: \quad \mu_l\tilde{q}_t I\tilde{H}_t = \tilde{R}_{l,t} \tag{135}$$

$$\partial k_{b,t}: \quad \mu_b\frac{\tilde{q}_t I\tilde{H}_t}{\tilde{k}_{b,t}} = \tilde{p}_{b,t} \tag{136}$$

B.4.5 Equilibrium

From (56)

$$R_t = R^{r_R}_{t-1}\pi_t^{(1-r_R)r_\pi}\left(\frac{G\tilde{D}P_t}{G\tilde{D}P_{t-1}}\right)^{(1-r_R)r_Y}\bar{rr}^{1-r_R}\frac{u_{R,t}}{s_t} \tag{137}$$

From (57)

$$\tilde{C}_t + I\tilde{K}_{c,t}/a_{k,t} + I\tilde{K}_{h,t} + \tilde{k}_{b,t} = \tilde{Y}_t - \tfrac{\phi_{kc}G_K}{2}\left(\tfrac{\tilde{k}_{c,t}}{\tilde{k}_{c,t-1}}-1\right)^2\tilde{k}_{c,t-1} - \tfrac{\phi_{kh}G_C}{2}\left(\tfrac{\tilde{k}_{h,t}}{\tilde{k}_{h,t-1}}-1\right)^2\tilde{k}_{h,t-1} \tag{138}$$

From (58)

$$\tilde{h}_t + \tilde{h}'_t - \frac{1-\delta_h}{G_H}(\tilde{h}_{t-1} + \tilde{h}'_{t-1}) = I\tilde{H}_t \tag{139}$$

From (59)

$$\tilde{b}_t + \tilde{b}'_t = 0 \tag{140}$$

B.5 Steady State

From (108)

$$R = \frac{1}{\beta} \tag{141}$$

From (109)

$$R_c = \frac{\Gamma_K}{\beta} - (1 - \delta_{kc}) \tag{142}$$

From (110)

$$R_h = \frac{1}{\beta} - (1 - \delta_{kh}) \tag{143}$$

define

$$r \equiv \frac{R}{G_C} - 1 \tag{144}$$

$$\delta'_h \equiv 1 - \frac{1-\delta_h}{G_H} \tag{145}$$

$$\delta'_{kc} \equiv 1 - \frac{1-\delta_{kc}}{G_K} \tag{146}$$

$$\delta'_{kh} \equiv 1 - \frac{1-\delta_{kh}}{G_C} \tag{147}$$

From (133) and (142)

$$\zeta_0 = \frac{k_c}{Y} = \frac{\beta G_K \mu_c}{\Gamma_K - \beta(1-\delta_{kc})}\frac{1}{X} \tag{148}$$

From (134) and (143)

$$\zeta_1 = \frac{k_h}{qIH} = \frac{\beta G_C \mu_h}{1 - \beta(1-\delta_{kh})} \tag{149}$$

From (107)

$$\zeta_2 = \frac{qh}{c} = \frac{j}{1 - \beta G_Q(1-\delta_h)} \tag{150}$$

From (107)

$$\zeta_3 = \frac{qh'}{c'} = \frac{j}{1 - \beta' G_Q(1-\delta_h) - G_Q(\beta - \beta')m} \tag{151}$$

From (126)

$$\zeta_4 = \left(\frac{R}{G_C} - 1\right)\frac{mG_Q}{R} \tag{152}$$

define

$$\chi_1 \equiv 1 + \delta_h' \zeta_2 (1 - r\zeta_1 - \mu_l - \alpha(1 - \mu_h - \mu_l - \mu_b)) \tag{153}$$

$$\chi_2 \equiv (r\zeta_1 + \mu_l + \alpha(1 - \mu_h - \mu_l - \mu_b))\delta_h' \zeta_3 + \zeta_3\zeta_4 \tag{154}$$

$$\chi_3 \equiv \frac{X - 1 + r\zeta_0 X + \alpha(1 - \mu_c)}{X} \tag{155}$$

$$\chi_4 \equiv 1 + \delta_h' \zeta_3 (1 - (1 - \alpha)(1 - \mu_h - \mu_l - \mu_b)) + \zeta_3\zeta_4 \tag{156}$$

$$\chi_5 \equiv (1 - \alpha)(1 - \mu_h - \mu_l - \mu_b)\delta_h' \zeta_2 \tag{157}$$

$$\chi_6 \equiv \frac{(1 - \alpha)(1 - \mu_c)}{X} \tag{158}$$

then

$$\frac{c}{Y} = \frac{\chi_3\chi_4 + \chi_2\chi_6}{\chi_1\chi_4 - \chi_2\chi_5} \tag{159}$$

$$\frac{c'}{Y} = \frac{\chi_1\chi_6 + \chi_3\chi_5}{\chi_1\chi_4 - \chi_2\chi_5} \tag{160}$$

$$\frac{qIH}{Y} = \delta_h' \left(\zeta_2 \frac{c}{Y} + \zeta_3 \frac{c'}{Y}\right) \tag{161}$$

normalize $\tau = 1$ and from (112) (113) (129), and (131)

$$n_c = \left(\frac{\frac{(1-\mu_c)\alpha}{XX_{wc}}\frac{Y}{c}}{\left(1 + \frac{(1-\mu_h-\mu_b-\mu_l)X}{1-\mu_c}\frac{qIH}{Y}\right)^{\frac{\eta-\xi}{1+\xi}}}\right)^{\frac{1}{1+\eta}} \tag{162}$$

$$n_h = n_c \left(\frac{(1 - \mu_h - \mu_b - \mu_l)X}{1 - \mu_c}\frac{qIH}{Y}\right)^{\frac{1}{1+\xi}} \tag{163}$$

from (122) (123) (130), and (132)

$$n_c' = \left(\frac{\frac{(1-\mu_c)\alpha}{XX_{wc}}\frac{Y}{c'}}{\left(1 + \frac{(1-\mu_h-\mu_b-\mu_l)X}{1-\mu_c}\frac{qIH}{Y}\right)^{\frac{\eta-\xi}{1+\xi}}}\right)^{\frac{1}{1+\eta}} \tag{164}$$

$$n_h' = n_c' \left(\frac{(1 - \mu_h - \mu_b - \mu_l)X}{1 - \mu_c}\frac{qIH}{Y}\right)^{\frac{1}{1+\xi}} \tag{165}$$

from (127) and (148)

$$Y = n_c^{\alpha}(n_c')^{1-\alpha}\left(\frac{\zeta_0}{G_K}\right)^{\frac{\mu_c}{1-\mu_c}} \tag{166}$$

from (128) and (149)

$$IH = n_h^{\alpha(1-\mu_h-\mu_b-\mu_l)}(n_h')^{(1-\alpha)(1-\mu_h-\mu_b-\mu_l)}\left(\zeta_1\frac{Y}{G_C}\frac{qIH}{Y}\right)^{\mu_h}\left(\mu_b Y\frac{qIH}{Y}\right)^{\mu_b} \tag{167}$$

$$c = Y\frac{c}{Y} \tag{168}$$

$$c' = Y\frac{c'}{Y} \tag{169}$$

$$q = \frac{Y}{IH}\frac{qIH}{Y} \tag{170}$$

$$qIH = Y\frac{qIH}{Y} \tag{171}$$

from (148)

$$k_c = \zeta_0 Y \tag{172}$$

from (149)

$$k_h = \zeta_1 qIH \tag{173}$$

from (150)

$$h = \zeta_2\frac{c}{q} \tag{174}$$

from (151)

$$h' = \zeta_3\frac{c'}{q} \tag{175}$$

from (126)

$$b = mqG_Q\frac{h'}{R} \tag{176}$$

$$C = c + c' \tag{177}$$

$$IK = \delta_{kc}'k_c + \delta_{kh}'k_h \tag{178}$$

$$w_c = \alpha(1-\mu_c)\frac{Y}{Xn_c} \tag{179}$$

$$w_h = \alpha(1 - \mu_h - \mu_l - \mu_b)\frac{qIH}{n_h} \tag{180}$$

$$w'_c = (1 - \alpha)(1 - \mu_c)\frac{Y}{Xn'_c} \tag{181}$$

$$w'_h = (1 - \alpha)(1 - \mu_h - \mu_l - \mu_b)\frac{qIH}{n'_h} \tag{182}$$

$$u_c = \frac{1}{c} \tag{183}$$

$$u_{c'} = \frac{1}{c'} \tag{184}$$

$$\lambda = \frac{1 - \beta'/\beta}{c'} \tag{185}$$

B.6 Log-linearization

we now log-linearize the model around the steady state. Define $\hat{x}_t \equiv \log x_t - \log x_{SS}$ $= \log(\frac{x_t}{x_{SS}}) = \log(1 + \frac{x_t - x_{ss}}{x_{ss}}) \approx \frac{x_t - x_{ss}}{x_{ss}}$. From (106)

$$\begin{aligned} c\hat{c}_t + k_c(\hat{k}_{c,t} - \hat{a}_{k,t}) + k_h\hat{k}_{h,t} + qh(\hat{q}_t + \hat{h}_t) - b\hat{b}_t = \frac{1 - \delta_h}{G_H}qh(\hat{q}_t + \hat{h}_{t-1}) \\ + w_c n_c(\hat{w}_{c,t} + \hat{n}_{c,t}) + w_h n_h(\hat{w}_{h,t} + \hat{n}_{h,t}) + (1 - \frac{1}{X})Y\hat{Y}_t + \frac{Y}{X}\hat{X}_t \\ + \frac{Rb}{G_C}(\hat{R}_{t-1} + \hat{b}_{t-1} - \hat{\pi}_t) + \frac{R_c + 1 - \delta_{kc}}{G_K}k_c\hat{k}_{c,t-1} + \frac{k_c}{G_K}(R_c(\hat{R}_{c,t} + \hat{z}_{c,t}) - (1 - \delta_{kc})\hat{a}_{k,t}) \\ + \frac{R_h + 1 - \delta_{kh}}{G_C}k_h\hat{k}_{h,t-1} + \frac{k_h}{G_C}R_h(\hat{R}_{h,t} + \hat{z}_{h,t}) + \mu_l qIH(\hat{q}_t + I\hat{H}_t) \end{aligned} \tag{186}$$

From (107)

$$qu_c(\hat{q}_t + \hat{u}_{c,t}) = \frac{j}{h}(\hat{z}_t + \hat{j}_t - \hat{h}_t) + \beta G_Q(1 - \delta_h)qu_c(E\hat{q}_{t+1} + E\hat{u}_{c,t+1}) \tag{187}$$

From (108)

$$\hat{u}_{c,t} = E\hat{u}_{c,t+1} + \hat{R}_t - E\hat{\pi}_{t+1} \tag{188}$$

From (109)

$$\hat{u}_{c,t} - \hat{a}_{k,t} + \phi_{kc}(\hat{k}_{c,t} - \hat{k}_{c,t-1}) = E\hat{u}_{c,t+1} + \tfrac{\beta}{\Gamma_K}(R_c(E\hat{R}_{c,t+1} + E\hat{z}_{c,t+1}) - (1 - \delta_{kc})E\hat{a}_{k,t+1} + G_K\phi_{kc}(E\hat{k}_{c,t+1} - \hat{k}_{c,t})) \tag{189}$$

From (110)

$$\hat{u}_{c,t} + \phi_{kh}(\hat{k}_{h,t} - \hat{k}_{h,t-1}) = E\hat{u}_{c,t+1} + \beta(R_h(E\hat{R}_{h,t+1} + E\hat{z}_{h,t+1}) + G_C\phi_{kh}(E\hat{k}_{h,t+1} - \hat{k}_{h,t})) \quad (190)$$

From (112)

$$\hat{\tau}_t + \hat{z}_t + \xi\hat{n}_{c,t} + \frac{\eta - \xi}{n_c^{1+\xi} + n_h^{1+\xi}}(n_c^{1+\xi}\hat{n}_{c,t} + n_h^{1+\xi}\hat{n}_{h,t}) = \hat{u}_{c,t} + \hat{w}_{c,t} - \hat{X}_{wc,t} \quad (191)$$

From (113)

$$\hat{\tau}_t + \hat{z}_t + \xi\hat{n}_{h,t} + \frac{\eta - \xi}{n_c^{1+\xi} + n_h^{1+\xi}}(n_c^{1+\xi}\hat{n}_{c,t} + n_h^{1+\xi}\hat{n}_{h,t}) = \hat{u}_{c,t} + \hat{w}_{h,t} - \hat{X}_{wh,t} \quad (192)$$

From (125)

$$c'\hat{c}'_t + qh'(\hat{q}_t + \hat{h}'_t) - \tfrac{1-\delta_h}{G_H}qh'(\hat{q}_t + \hat{h}'_{t-1}) = w'_c n'_c(\hat{w}'_{c,t} + \hat{n}'_{c,t}) + w'_h n'_h(\hat{w}'_{h,t} + \hat{n}'_{h,t}) + b\hat{b}_t - \tfrac{Rb}{G_C}(\hat{R}_{t-1} + \hat{b}_{t-1} - \hat{\pi}_t) \quad (193)$$

From (120)

$$qu_{c'}(\hat{q}_t + \hat{u}_{c',t}) = \tfrac{j}{h'}(\hat{z}_t + \hat{j}_t - \hat{h}'_t) + \beta' G_Q(1-\delta_h)qu_{c'}(E\hat{q}_{t+1} + E\hat{u}_{c',t+1}) + m\tfrac{\lambda q G_Q}{R}(\hat{\lambda}_t + E\hat{q}_{t+1} + E\hat{\pi}_{t+1} - \hat{R}_t) \quad (194)$$

From (126)

$$\hat{b}_t = E\hat{q}_{t+1} + \hat{h}'_t + E\hat{\pi}_{t+1} - \hat{R}_t \quad (195)$$

From (121)

$$u_{c'}\hat{u}_{c',t} = \frac{\beta'}{\beta}u_{c'}(E\hat{u}_{c',t+1} + \hat{R}_t - E\hat{\pi}_{t+1}) + \lambda\hat{\lambda}_t \quad (196)$$

From (122)

$$\hat{\tau}_t + \hat{z}_t + \xi'\hat{n}'_{c,t} + \frac{\eta' - \xi'}{(n'_c)^{1+\xi'} + (n'_h)^{1+\xi'}}((n'_c)^{1+\xi'}\hat{n}'_{c,t} + (n'_h)^{1+\xi'}\hat{n}'_{h,t}) = \hat{u}_{c',t} + \hat{w}'_{c,t} - \hat{X}'_{wc,t} \quad (197)$$

From (123)

$$\hat{\tau}_t + \hat{z}_t + \xi'\hat{n}'_{h,t} + \frac{\eta' - \xi'}{(n'_c)^{1+\xi'} + (n'_h)^{1+\xi'}}((n'_c)^{1+\xi'}\hat{n}'_{c,t} + (n'_h)^{1+\xi'}\hat{n}'_{h,t}) = \hat{u}_{c',t} + \hat{w}'_{h,t} - \hat{X}'_{wh,t} \quad (198)$$

From (127)

$$\hat{Y}_t = (1-\mu_c)\hat{a}_{c,t} + \alpha(1-\mu_c)\hat{n}_{c,t} + (1-\alpha)(1-\mu_c)\hat{n}'_{c,t} + \mu_c(\hat{k}_{c,t-1} + \hat{z}_{c,t}) \tag{199}$$

From (128)

$$I\hat{H}_t = (1-\mu_h-\mu_b-\mu_l)\hat{a}_{h,t} + \mu_b(\hat{q}_t + I\hat{H}_t) + \alpha(1-\mu_h-\mu_b-\mu_l)\hat{n}_{h,t} + (1-\alpha)(1-\mu_h-\mu_b-\mu_l)\hat{n}'_{h,t} + \mu_h(\hat{k}_{h,t-1} + \hat{z}_{h,t}) \tag{200}$$

From (129)

$$\hat{Y}_t - \hat{X}_t - \hat{n}_{c,t} = \hat{w}_{c,t} \tag{201}$$

From (130)

$$\hat{Y}_t - \hat{X}_t - \hat{n}'_{c,t} = \hat{w}'_{c,t} \tag{202}$$

From (131)

$$\hat{q}_t + I\hat{H}_t - \hat{n}_{h,t} = \hat{w}_{h,t} \tag{203}$$

From (132)

$$\hat{q}_t + I\hat{H}_t - \hat{n}'_{h,t} = \hat{w}'_{h,t} \tag{204}$$

From (133)

$$\hat{Y}_t - \hat{X}_t - \hat{k}_{c,t-1} = \hat{R}_{c,t} + \hat{z}_{c,t} \tag{205}$$

From (134)

$$\hat{q}_t + I\hat{H}_t - \hat{k}_{h,t-1} = \hat{R}_{h,t} + \hat{z}_{h,t} \tag{206}$$

From (51)

$$\hat{\pi}_t - \iota_\pi \hat{\pi}_{t-1} = \beta G_C(E_t\hat{\pi}_{t+1} - \iota_\pi\hat{\pi}_t) - \frac{(1-\theta_\pi)(1-\beta G_C\theta_\pi)}{\theta_\pi}\hat{X}_t + u_{p,t} \tag{207}$$

From (137)

$$\hat{R}_t = r_R\hat{R}_{t-1} + (1-r_R)r_\pi\hat{\pi}_t + (1-r_R)r_Y\left(G\hat{D}P_t - G\hat{D}P_{t-1}\right) + u_{r,t} - \hat{s}_t \tag{208}$$

From (139)

$$h\hat{h}_t + h'\hat{h}'_t - \frac{1-\delta_h}{G_H}(h\hat{h}_{t-1} + h'\hat{h}'_{t-1}) = IHI\hat{H}_t \tag{209}$$

From (104)

$$\hat{u}_{c,t} = \hat{z}_t - \frac{G_C}{(1-\beta\varepsilon)(G_C-\varepsilon)}\left(\hat{c}_t - \frac{\varepsilon}{G_C}\hat{c}_{t-1} - \beta\varepsilon\left(E\hat{c}_{t+1} - \frac{\varepsilon}{G_C}\hat{c}_t\right)\right) \tag{210}$$

From (118)

$$\hat{u}_{c',t} = \hat{z}_t - \frac{G_C}{(1-\beta'\varepsilon')(G_C-\varepsilon')}\left(\hat{c}'_t - \frac{\varepsilon'}{G_C}\hat{c}'_{t-1} - \beta'\varepsilon'\left(E\hat{c}'_{t+1} - \frac{\varepsilon'}{G_C}\hat{c}'_t\right)\right) \tag{211}$$

From (52)

$$\hat{w}_{c,t} = \frac{1}{1+\beta G_C}\hat{w}_{c,t-1} + \left(1 - \frac{1}{1+\beta G_C}\right)(E\hat{w}_{c,t+1} + E\hat{\pi}_{t+1}) - \frac{1+\beta G_C \iota_{wc}}{1+\beta G_C}\hat{\pi}_t + \frac{\iota_{wc}}{1+\beta G_C}\hat{\pi}_{t-1} - \frac{(1-\theta_{wc})(1-\beta G_C\theta_{wc})}{\theta_{wc}(1+\beta G_C)}\hat{X}_{wc,t} \tag{212}$$

From (53)

$$\hat{w}'_{c,t} = \frac{1}{1+\beta' G_C}\hat{w}'_{c,t-1} + \left(1 - \frac{1}{1+\beta' G_C}\right)(E\hat{w}'_{c,t+1} + E\hat{\pi}_{t+1}) - \frac{1+\beta' G_C \iota_{wc}}{1+\beta' G_C}\hat{\pi}_t + \frac{\iota_{wc}}{1+\beta' G_C}\hat{\pi}_{t-1} - \frac{(1-\theta_{wc})(1-\beta' G_C\theta_{wc})}{\theta_{wc}(1+\beta' G_C)}\hat{X}'_{wc,t} \tag{213}$$

From (54)

$$\hat{w}_{h,t} = \frac{1}{1+\beta G_C}\hat{w}_{h,t-1} + \left(1 - \frac{1}{1+\beta G_C}\right)(E\hat{w}_{h,t+1} + E\hat{\pi}_{t+1}) - \frac{1+\beta G_C \iota_{wh}}{1+\beta G_C}\hat{\pi}_t + \frac{\iota_{wh}}{1+\beta G_C}\hat{\pi}_{t-1} - \frac{(1-\theta_{wh})(1-\beta G_C\theta_{wh})}{\theta_{wh}(1+\beta G_C)}\hat{X}_{wh,t} \tag{214}$$

From (55)

$$\hat{w}'_{h,t} = \frac{1}{1+\beta' G_C}\hat{w}'_{h,t-1} + \left(1 - \frac{1}{1+\beta' G_C}\right)(E\hat{w}'_{h,t+1} + E\hat{\pi}_{t+1}) - \frac{1+\beta' G_C \iota_{wh}}{1+\beta' G_C}\hat{\pi}_t + \frac{\iota_{wh}}{1+\beta' G_C}\hat{\pi}_{t-1} - \frac{(1-\theta_{wh})(1-\beta' G_C\theta_{wh})}{\theta_{wh}(1+\beta' G_C)}\hat{X}'_{wh,t} \tag{215}$$

From (115)

$$\hat{R}_{c,t} + \hat{a}_{k,t} = \frac{\zeta}{1-\zeta}\hat{z}_{c,t} \tag{216}$$

where $\zeta = \varpi/(1+\varpi)$. From (116)

$$\hat{R}_{h,t} = \frac{\zeta}{1-\zeta}\hat{z}_{h,t} \tag{217}$$

aggregate

$$C\hat{C}_t = c\hat{c}_t + c'\hat{c}'_t \tag{218}$$

$$IKI\hat{K}_t = k_c\left(\hat{k}_{c,t} - \frac{1-\delta_{kc}}{G_K}\hat{k}_{c,t-1}\right) + k_h\left(\hat{k}_{h,t} - \frac{1-\delta_{kh}}{G_C}\hat{k}_{h,t-1}\right) \tag{219}$$

$$\hat{W}_{c,t} = \frac{w_c}{w_c + w'_c}\hat{w}_{c,t} + \frac{w'_c}{w_c + w'_c}\hat{w}'_{c,t} \tag{220}$$

$$\hat{W}_{h,t} = \frac{w_h}{w_h + w'_h}\hat{w}_{h,t} + \frac{w'_h}{w_h + w'_h}\hat{w}'_{h,t} \tag{221}$$

$$\hat{N}_{c,t} = \alpha\hat{n}_{c,t} + (1-\alpha)\hat{n}'_{c,t} \tag{222}$$

$$\hat{N}_{h,t} = \alpha\hat{n}_{h,t} + (1-\alpha)\hat{n}'_{h,t} \tag{223}$$

shock process

$$\hat{a}_{c,t} = \rho_{AC}\hat{a}_{c,t-1} + u_{c,t} \tag{224}$$

$$\hat{a}_{h,t} = \rho_{AH}\hat{a}_{h,t-1} + u_{h,t} \tag{225}$$

$$\hat{a}_{k,t} = \rho_{AK}\hat{a}_{k,t-1} + u_{k,t} \tag{226}$$

$$\hat{j}_t = \rho_j\hat{j}_{t-1} + u_{j,t} \tag{227}$$

$$\hat{z}_t = \rho_z\hat{z}_{t-1} + u_{z,t} \tag{228}$$

$$\hat{\tau}_t = \rho_\tau\hat{\tau}_{t-1} + u_{\tau,t} \tag{229}$$

$$\hat{s}_t = \rho_s\hat{s}_{t-1} + u_{s,t} \tag{230}$$

Equations (186) to (230) determine the following 45 variables: $\hat{c}_t$, $\hat{u}_{c,t}$, $\hat{h}_t$, $\hat{k}_{c,t}$, $\hat{k}_{h,t}$, $\hat{n}_{c,t}$, $\hat{n}_{h,t}$, $\hat{b}_t$, $\hat{z}_{c,t}$, $\hat{z}_{h,t}$, $\hat{c}'_t$, $\hat{u}_{c',t}$, $\hat{h}'_t$, $\hat{n}'_{c,t}$, $\hat{n}'_{h,t}$, $I\hat{H}_t$, $\hat{Y}_t$, $\hat{q}_t$, $\hat{R}_t$, $\hat{\pi}_t$, $\hat{\lambda}_t$, $\hat{X}_t$, $\hat{w}_{c,t}$, $\hat{w}_{h,t}$, $\hat{w}'_{c,t}$, $\hat{w}'_{h,t}$, $\hat{X}_{wc,t}$, $\hat{X}_{wh,t}$, $\hat{X}'_{wc,t}$, $\hat{X}'_{wh,t}$, $\hat{R}_{c,t}$, $\hat{R}_{h,t}$, $\hat{C}_t$, $I\hat{K}_t$, $\hat{W}_{c,t}$, $\hat{W}_{h,t}$, $\hat{N}_{c,t}$, $\hat{N}_{h,t}$, $\hat{a}_{c,t}$, $\hat{a}_{h,t}$, $\hat{a}_{k,t}$, $\hat{j}_t$, $\hat{z}_t$, $\hat{\tau}_t$, $\hat{s}_t$.

Measurement equations (total 10)

Measurement equations link the data with the model variables. Along the BGP, we have (pick C_t as an example) model-generated consumption at time t is $C_t = G_C^t G_0 = G_C^t$ (if normalize $G_0 = 1$), while the data of consumption at time t is C_t^{data}. Therefore $\hat{C}_t \equiv \log C_t^{data} - \log C_t =$

$\log C_t^{data} - t \log G_C = \log C_t^{data} - t \log(1 + G_C - 1) \approx \log C_t^{data} - t(G_C - 1)$. Similarly, $\hat{C}_{t-1} \approx \log C_{t-1}^{data} - (t-1)(G_C - 1)$. Therefore $d\log(C_t^{data}) = \log C_t^{data} - \log C_{t-1}^{data} = \hat{C}_t - \hat{C}_{t-1} + (G_C - 1)$.

aggregate consumption (log difference of real personal consumption per capita)

$$C_t^{data} = \hat{C}_t - \hat{C}_{t-1} + (G_C - 1) \tag{231}$$

business fixed investment (log difference of real private nonresidentialinvestment per capita)

$$IK_t^{data} = I\hat{K}_t - I\hat{K}_{t-1} + (G_K - 1) \tag{232}$$

residential investment (log difference of real private residential investment per capita)

$$IH_t^{data} = I\hat{H}_t - I\hat{H}_{t-1} + (G_H - 1) \tag{233}$$

inflation (log difference of implicit price deflator, demeaned)

$$\pi_t^{data} = \hat{\pi}_t \tag{234}$$

nominal short-term interest rate (demeaned)

$$R_t^{data} = \hat{R}_t \tag{235}$$

real house price (log difference of real house price index deflated with implicit price deflator)

$$q_t^{data} = \hat{q}_t - \hat{q}_{t-1} + (G_Q - 1) \tag{236}$$

hours in consumption sector

$$N_{c,t}^{data} = \hat{N}_{c,t} \tag{237}$$

hours in housing sector (with measurement error)

$$N_{h,t}^{data} = \hat{N}_{h,t} + u_{nh,t} \tag{238}$$

wage inflation in consumption-good sector (demeaned)

$$W_{c,t}^{data} = \hat{W}_{c,t} - \hat{W}_{c,t-1} + \hat{\pi}_t \tag{239}$$

wage inflation in consumption-good sector (demeaned, with measurement error)

$$W_{h,t}^{data} = \hat{W}_{h,t} - \hat{W}_{h,t-1} + \hat{\pi}_t + u_{wh,t} \tag{240}$$

9 shocks and 2 measurement errors: $u_{c,t}$, $u_{h,t}$, $u_{k,t}$, $u_{j,t}$, $u_{z,t}$, $u_{\tau,t}$, $u_{s,t}$, $u_{p,t}$, $u_{r,t}$, $u_{nh,t}$, $u_{wh,t}$

C Liu-Wang-Zha Model

The economy is populated by two types of agents—households and entrepreneurs—with a continuum and unit measure of each type. There are four types of commodities: labor, goods, land, and loanable bonds. Goods production requires labor, capital, and land as inputs. The output can be used for consumption (by both types of agents) and for capital investment (by the entrepreneurs). The representative household's utility depends on consumption goods, land services (housing), and leisure; the representative entrepreneur's utility depends on consumption goods only.

C.1 Environment

C.1.1 The representative household

The household has the utility function

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t A_t\left\{\log(C_{ht}-\gamma_h C_{h,t-1})+\varphi_t\log L_{ht}-\psi_t N_{ht}\right\}, \tag{241}$$

where C_{ht} denotes consumption, L_{ht} denotes land holdings, and N_{ht} denotes labor hours. The parameter $\beta \in (0,1)$ is a subjective discount factor, the parameter γ_h measures the degree of habit persistence. The terms A_t, φ_t, and ψ_t are intertemporal preference, housing preference, and labor supply shocks respectively. They follow the stochastic process

$$A_t = A_{t-1}(1+\lambda_{at}), \quad \ln\lambda_{at} = (1-\rho_a)\ln\bar{\lambda}_a + \rho_a\ln\lambda_{a,t-1} + \varepsilon_{at}, \tag{242}$$

$$\ln\varphi_t = (1-\rho_\varphi)\ln\bar{\varphi} + \rho_\varphi\ln\varphi_{t-1} + \varepsilon_{\varphi t}, \tag{243}$$

$$\ln\psi_t = (1-\rho_\psi)\ln\bar{\psi} + \rho_\psi\ln\psi_{t-1} + \varepsilon_{\psi t}, \tag{244}$$

The budget constraint for the household is given by

$$C_{ht} + q_{lt}(L_{ht}-L_{h,t-1}) + \frac{S_t}{R_t} \le w_t N_{ht} + S_{t-1}. \tag{245}$$

The household chooses C_{ht}, $L_{h,t}$, N_{ht}, and S_t to maximize (241) subject to (242)-(245) and the borrowing constraint $S_t \ge -\bar{S}$ for some large number $\bar{S}$.

C.1.2 The representative entrepreneur

The entrepreneur has the utility function

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t\left[\log(C_{et}-\gamma_e C_{e,t-1})\right], \tag{246}$$

where C_{et} denotes the entrepreneur's consumption and γ_e is the habit persistence parameter.

The production function is given by

$$Y_t = Z_t[L_{e,t-1}^{\phi}K_{t-1}^{1-\phi}]^{\alpha}N_{et}^{1-\alpha}, \tag{247}$$

We assume that the total factor productivity Z_t is composed of a permanent component Z_t^p and a transitory component ν_t such that $Z_t = Z_t^p\nu_{zt}$, where the permanent component Z_t^p follows the stochastic process

$$Z_t^p = Z_{t-1}^p\lambda_{zt}, \quad \ln\lambda_{zt} = (1-\rho_z)\ln\bar{\lambda}_z + \rho_z\ln\lambda_{z,t-1} + \varepsilon_{zt}, \tag{248}$$

and the transitory component follows the stochastic process

$$\ln\nu_{zt} = \rho_{\nu_z}\ln\nu_{z,t-1} + \varepsilon_{\nu_z t}. \tag{249}$$

Capital accumulation follows the law of motion

$$K_t = (1-\delta)K_{t-1} + \left[1-\frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}}-\bar{\lambda}_I\right)^2\right]I_t, \tag{250}$$

where I_t denotes investment, $\bar{\lambda}_I$ denotes the steady-state growth rate of investment, and $\Omega > 0$ is the adjustment cost parameter.

The entrepreneur faces the budget constraint

$$C_{et} + q_{lt}(L_{et}-L_{e,t-1}) + B_{t-1} = Z_t[L_{e,t-1}^{\phi}K_{t-1}^{1-\phi}]^{\alpha}N_{et}^{1-\alpha} - \frac{I_t}{Q_t} - w_tN_{et} + \frac{B_t}{R_t}, \tag{251}$$

where B_{t-1} is the amount of matured debt and B_t/R_t is the value of new debt. We interpret Q_t as the investment-specific technological change. Specifically, we assume that $Q_t = Q_t^p\nu_{qt}$, where the permanent component Q_t^p follows the stochastic process

$$Q_t^p = Q_{t-1}^p \lambda_{qt}, \quad \ln \lambda_{qt} = (1-\rho_q) \ln \bar{\lambda}_q + \rho_q \ln \lambda_{q,t-1} + \varepsilon_{qt}, \tag{252}$$

and the transitory component follows the stochastic process

$$\ln \nu_{qt} = \rho_{\nu_q} \ln \nu_{q,t-1} + \varepsilon_{\nu_q t}. \tag{253}$$

The entrepreneur faces the credit constraint

$$B_t \leq \theta_t E_t[q_{l,t+1} L_{et} + q_{k,t+1} K_t], \tag{254}$$

where $q_{k,t+1}$ is the shadow price of capital in consumption units. θ_t is "collateral shock" that reflects the uncertainty in the tightness of the credit market. We assume that θ_t follows the stochastic process

$$\ln \theta_t = (1-\rho_\theta) \ln \bar{\theta} + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta t}, \tag{255}$$

The entrepreneur chooses C_{et}, N_{et}, I_t, $L_{e,t}$, K_t, and B_t to maximize (246) subject to (247) through (255).

C.1.3 Market clearing conditions and equilibrium

In a competitive equilibrium, the markets for goods, labor, land, and loanable bonds all clear. The goods market clearing condition implies that

$$C_t + \frac{I_t}{Q_t} = Y_t, \tag{256}$$

where $C_t = C_{ht} + C_{et}$ denotes aggregate consumption. The labor market clearing condition implies that labor demand equals labor supply:

$$N_{et} = N_{ht} \equiv N_t. \tag{257}$$

The land market clearing condition implies that

$$L_{ht} + L_{et} = \bar{L}, \tag{258}$$

where $\bar{L}$ is the fixed aggregate land endowment. Finally, the bond market clearing condition implies that

$$S_t = B_t. \tag{259}$$

A competitive equilibrium consists of sequences of prices $\{w_t, q_{lt}, R_t\}_{t=0}^{\infty}$ and allocations $\{C_{ht}, C_{et}, I_t, N_{ht}, N_{et}, L_{ht}, L_{et}, S_t, B_t, K_t, Y_t\}_{t=0}^{\infty}$ such that (i) taking the prices as given, the allocations solve the optimizing problems for the household and the entrepreneur and (ii) all markets clear.

C.2 Derivations of equilibrium conditions

C.3 Euler equations

Denote by μ_{ht} the Lagrangian multiplier for the budget constraint (245). The first-order conditions for the household's optimizing problem are given by

$$\mu_{ht} = A_t \left[\frac{1}{C_{ht} - \gamma_h C_{h,t-1}} - \mathrm{E}_t \frac{\beta \gamma_h}{C_{h,t+1} - \gamma_h C_{ht}} (1 + \lambda_{a,t+1}) \right], \tag{260}$$

$$w_t = \frac{A_t}{\mu_{ht}} \psi_t, \tag{261}$$

$$q_{lt} = \beta \mathrm{E}_t \frac{\mu_{h,t+1}}{\mu_{ht}} q_{l,t+1} + \frac{A_t \varphi_t}{\mu_{ht} L_{ht}}, \tag{262}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\mu_{h,t+1}}{\mu_{ht}}. \tag{263}$$

Denote by μ_{et} the Lagrangian multiplier for the budget constraint (251), μ_{kt} the multiplier for the capital accumulation equation (250), and μ_{bt} the multiplier for the borrowing constraint (254). With these notations, the shadow price of capital in consumption units is given by

$$q_{kt} = \frac{\mu_{kt}}{\mu_{et}}. \tag{264}$$

The first-order conditions for the entrepreneur's optimizing problem are given by

$$\begin{aligned}
\mu_{et} &= \frac{1}{C_{et} - \gamma_e C_{e,t-1}} - \mathrm{E}_t \frac{\beta\gamma_e}{C_{e,t+1} - \gamma_e C_{et}}, && (265)\\
w_t &= (1-\alpha) Y_t / N_{et}, && (266)\\
\frac{1}{Q_t} &= q_{kt}\left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I\right)^2 - \Omega\left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I\right)\frac{I_t}{I_{t-1}}\right] \\
&\quad + \beta\Omega \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} q_{k,t+1}\left(\frac{I_{t+1}}{I_t} - \bar{\lambda}_I\right)\left(\frac{I_{t+1}}{I_t}\right)^2, && (267)\\
q_{kt} &= \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}}\left[\alpha(1-\phi)\frac{Y_{t+1}}{K_t} + q_{k,t+1}(1-\delta)\right] + \frac{\mu_{bt}}{\mu_{et}}\theta_t \mathrm{E}_t q_{k,t+1}, && (268)\\
q_{lt} &= \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}}\left[\alpha\phi\frac{Y_{t+1}}{L_{et}} + q_{l,t+1}\right] + \frac{\mu_{bt}}{\mu_{et}}\theta_t \mathrm{E}_t q_{l,t+1}, && (269)\\
\frac{1}{R_t} &= \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} + \frac{\mu_{bt}}{\mu_{et}}. && (270)
\end{aligned}$$

C.4 Stationary equilibrium

We make the following transformations of the variables along the stationary equilibrium

$$\begin{aligned}
&\tilde{Y}_t \equiv \frac{Y_t}{\Gamma_t}, \quad \tilde{C}_{ht} \equiv \frac{C_{ht}}{\Gamma_t}, \quad \tilde{C}_{et} \equiv \frac{C_{et}}{\Gamma_t}, \quad \tilde{I}_t \equiv \frac{I_t}{Q_t \Gamma_t}, \quad \tilde{K}_t \equiv \frac{K_t}{Q_t\Gamma_t}, \quad \tilde{B}_t \equiv \frac{B_t}{\Gamma_t},\\
&\tilde{w}_t \equiv \frac{w_t}{\Gamma_t}, \quad \tilde{\mu}_{ht} \equiv \frac{\mu_{ht}\Gamma_t}{A_t}, \quad \tilde{\mu}_{et} \equiv \mu_{et}\Gamma_t, \quad \tilde{\mu}_{bt} \equiv \mu_{bt}\Gamma_t, \quad \tilde{q}_{lt} \equiv \frac{q_{lt}}{\Gamma_t}, \quad \tilde{q}_{kt} \equiv q_{kt} Q_t, \qquad (271)
\end{aligned}$$

where $\Gamma_t \equiv [Z_t Q_t^{(1-\phi)\alpha}]^{\frac{1}{1-(1-\phi)\alpha}}$.

Denote by $g_{\gamma t} \equiv \frac{\Gamma_t}{\Gamma_{t-1}}$ and $g_{qt} \equiv \frac{Q_t}{Q_{t-1}}$ the growth rates for the exogenous variables Γ_t and Q_t. Denote by g_γ the steady-state value of $g_{\gamma t}$ and $\lambda_k \equiv g_\gamma \bar{\lambda}_q$ the steady-state growth rate of capital stock. On the balanced growth path, investment grows at the same rate as does capital, so we have $\bar{\lambda}_I = \lambda_k$.

The stationary equilibrium is the solution to the following system of equations:

$$\tilde{\mu}_{ht} = \frac{1}{\tilde{C}_{ht} - \gamma_h \tilde{C}_{h,t-1}\Gamma_{t-1}/\Gamma_t} - \mathrm{E}_t \frac{\beta\gamma_h}{\tilde{C}_{h,t+1}\Gamma_{t+1}/\Gamma_t - \gamma_h \tilde{C}_{ht}}(1+\lambda_{a,t+1}), \tag{272}$$

$$\tilde{w}_t = \frac{\psi_t}{\tilde{\mu}_{ht}}, \tag{273}$$

$$\tilde{q}_{lt} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{h,t+1}}{\tilde{\mu}_{ht}}(1+\lambda_{a,t+1})\tilde{q}_{l,t+1} + \frac{\varphi_t}{\tilde{\mu}_{ht} L_{ht}}, \tag{274}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{h,t+1}}{\tilde{\mu}_{ht}} \frac{\Gamma_t}{\Gamma_{t+1}}(1+\lambda_{a,t+1}). \tag{275}$$

$$\tilde{\mu}_{et} = \frac{1}{\tilde{C}_{et} - \gamma_e \tilde{C}_{e,t-1}\Gamma_{t-1}/\Gamma_t} - \mathrm{E}_t \frac{\beta\gamma_e}{\tilde{C}_{e,t+1}\Gamma_{t+1}/\Gamma_t - \gamma_e \tilde{C}_{et}}, \tag{276}$$

$$\tilde{w}_t = (1-\alpha)\tilde{Y}_t/N_t, \tag{277}$$

$$\begin{aligned} 1 = {} & \tilde{q}_{kt}\left[1 - \frac{\Omega}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}} - \bar{\lambda}_I\right)^2 - \Omega\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}} - \bar{\lambda}_I\right)\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}}\right] \\ & + \beta\Omega \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\frac{Q_t\Gamma_t}{Q_{t+1}\Gamma_{t+1}}\tilde{q}_{k,t+1}\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\frac{Q_{t+1}\Gamma_{t+1}}{Q_t\Gamma_t} - \bar{\lambda}_I\right)\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\frac{Q_{t+1}\Gamma_{t+1}}{Q_t\Gamma_t}\right)^2, \end{aligned} \tag{278}$$

$$\tilde{q}_{kt} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\left[\alpha(1-\phi)\frac{\tilde{Y}_{t+1}}{\tilde{K}_t} + \tilde{q}_{k,t+1}\frac{Q_t\Gamma_t}{Q_{t+1}\Gamma_{t+1}}(1-\delta)\right] + \frac{\tilde{\mu}_{bt}}{\tilde{\mu}_{et}}\theta_t \mathrm{E}_t \tilde{q}_{k,t+1}\frac{Q_t}{Q_{t+1}}, \tag{279}$$

$$\tilde{q}_{lt} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\left[\alpha\phi\frac{\tilde{Y}_{t+1}}{L_{et}} + \tilde{q}_{l,t+1}\right] + \frac{\tilde{\mu}_{bt}}{\tilde{\mu}_{et}}\theta_t \mathrm{E}_t \tilde{q}_{l,t+1}\frac{\Gamma_{t+1}}{\Gamma_t}, \tag{280}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\frac{\Gamma_t}{\Gamma_{t+1}} + \frac{\tilde{\mu}_{bt}}{\tilde{\mu}_{et}}. \tag{281}$$

$$\tilde{Y}_t = \left(\frac{Z_t Q_t}{Z_{t-1}Q_{t-1}}\right)^{-\frac{(1-\phi)\alpha}{1-(1-\phi)\alpha}}[L_{e,t-1}^{\phi}\tilde{K}_{t-1}^{1-\phi}]^{\alpha}N_t^{1-\alpha}, \tag{282}$$

$$\tilde{K}_t = (1-\delta)\tilde{K}_{t-1}\frac{Q_{t-1}\Gamma_{t-1}}{Q_t\Gamma_t} + \left[1 - \frac{\Omega}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}} - \bar{\lambda}_I\right)^2\right]\tilde{I}_t, \tag{283}$$

$$\tilde{Y}_t = \tilde{C}_{ht} + \tilde{C}_{et} + \tilde{I}_t, \tag{284}$$

$$\bar{L} = L_{ht} + L_{et}, \tag{285}$$

$$\alpha\tilde{Y}_t = \tilde{C}_{et} + \tilde{I}_t + \tilde{q}_{lt}(L_{et} - L_{e,t-1}) + \tilde{B}_{t-1}\frac{\Gamma_{t-1}}{\Gamma_t} - \frac{\tilde{B}_t}{R_t}, \tag{286}$$

$$\tilde{B}_t = \theta_t \mathrm{E}_t\left[\tilde{q}_{l,t+1}\frac{\Gamma_{t+1}}{\Gamma_t}L_{et} + \tilde{q}_{k,t+1}\tilde{K}_t\frac{Q_t}{Q_{t+1}}\right]. \tag{287}$$

We solve these 16 equations for 16 variables summarized in the vector

$$[\tilde{\mu}_{ht}, \tilde{w}_t, \tilde{q}_{lt}, R_t, \tilde{\mu}_{et}, N_t, \tilde{I}_t, \tilde{Y}_t, \tilde{C}_{ht}, \tilde{C}_{et}, \tilde{q}_{kt}, L_{et}, L_{ht}, \tilde{K}_t, \tilde{B}_t, \tilde{\mu}_{bt}]'.$$

C.5 Steady state

To get the steady-state value for $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$, we use the stationary bond Euler equations (275) for the household and (281) (described in the Appendix) to obtain

$$\frac{1}{R} = \frac{\beta(1+\bar{\lambda}_a)}{g_\gamma}, \quad \frac{\tilde{\mu}_b}{\tilde{\mu}_e} = \frac{\beta\bar{\lambda}_a}{g_\gamma}. \tag{288}$$

Since $\bar{\lambda}_a > 0$, we have $\tilde{\mu}_b > 0$ and the borrowing constraint is binding in the steady-state equilibrium.

To get the ratio of commercial real estate to output, we use the land Euler equation (280) for the entrepreneur, the definition of $\tilde{\mu}_e$ in (276), and the solution for $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$ in (288). In particular, we have

$$\frac{\tilde{q}_l L_e}{\tilde{Y}} = \frac{\beta\alpha\phi}{1-\beta-\beta\bar{\lambda}_a\bar{\theta}}. \tag{289}$$

To get the investment-output ratio, we first solve for the investment-capital ratio by using the law of motion for capital stock in (283) and then solve for the capital-output ratio using the capital Euler equation (279). Specifically, we have

$$\frac{\tilde{I}}{\tilde{K}} = 1 - \frac{1-\delta}{\lambda_k}, \tag{290}$$

$$\frac{\tilde{K}}{\tilde{Y}} = \left[1 - \frac{\beta}{\lambda_k}(\bar{\lambda}_a\bar{\theta} + 1 - \delta)\right]^{-1} \beta\alpha(1-\phi), \tag{291}$$

where we have used the steady-state condition that $\tilde{q}_k = 1$, as implied by the investment Euler equation (278). The investment-output ratio is then given by

$$\frac{\tilde{I}}{\tilde{Y}} = \frac{\tilde{I}}{\tilde{K}}\frac{\tilde{K}}{\tilde{Y}} = \frac{\beta\alpha(1-\phi)[\lambda_k - (1-\delta)]}{\lambda_k - \beta(\bar{\lambda}_a\bar{\theta} + 1 - \delta)}. \tag{292}$$

Given the solution for the ratios $\frac{\tilde{q}_l L_e}{\tilde{Y}}$ and $\frac{\tilde{K}}{\tilde{Y}}$ in (289) and (291), the binding borrowing constraint (287) implies that

$$\frac{\tilde{B}}{\tilde{Y}} = \bar{\theta} g_\gamma \frac{\tilde{q}_l L_e}{\tilde{Y}} + \frac{\bar{\theta}}{\lambda_q}\frac{\tilde{K}}{\tilde{Y}}. \tag{293}$$

The entrepreneur's flow of funds constraint (286) implies that

$$\frac{\tilde{C}_e}{\tilde{Y}} = \alpha - \frac{\tilde{I}}{\tilde{Y}} - \frac{1-\beta(1+\bar{\lambda}_a)}{g_\gamma}\frac{\tilde{B}}{\tilde{Y}}. \tag{294}$$

The aggregate resource constraint (284) then implies that

$$\frac{\tilde{C}_h}{\tilde{Y}} = 1 - \frac{\tilde{C}_e}{\tilde{Y}} - \frac{\tilde{I}}{\tilde{Y}}. \tag{295}$$

To solve for $\frac{L_h}{L_e}$, we first use the household's land Euler equation (i.e., the housing demand equation) (274) and the definition for the marginal utility (272) to obtain

$$\frac{\tilde{q}_l L_h}{\tilde{C}_h} = \frac{\bar{\varphi}(g_\gamma - \gamma_h)}{g_\gamma(1 - g_\gamma/R)(1 - \gamma_h/R)}, \tag{296}$$

where the steady-state loan rate is given by (288).

Taking the ratio between (296) and (289) results in the solution

$$\frac{L_h}{L_e} = \frac{\bar{\varphi}(g_\gamma - \gamma_h)(1 - \beta - \beta\bar{\lambda}_a\bar{\theta})}{\beta\alpha\phi g_\gamma(1 - g_\gamma/R)(1 - \gamma_h/R)} \frac{\tilde{C}_h}{\tilde{Y}}. \tag{297}$$

Finally, we can solve for the steady-state hours by combining the labor supply equation (273) and the labor demand equation (277) to get

$$N = \frac{(1-\alpha)g_\gamma(1 - \gamma_h/R)}{\bar{\psi}(g_\gamma - \gamma_h)} \frac{\tilde{Y}}{\tilde{C}_h}. \tag{298}$$

C.6 Log-linearized equilibrium system

Upon obtaining the steady-state equilibrium, we log-linearize the equilibrium conditions (272) through (287) around the steady state. We define the constants $\Omega_h \equiv (g_\gamma - \beta(1+\bar{\lambda}_a)\gamma_h)(g_\gamma - \gamma_h)$ and $\Omega_e \equiv (g_\gamma - \beta\gamma_e)(g_\gamma - \gamma_e)$. The log-linearized equilibrium conditions are given by

$$
\begin{aligned}
\Omega_h \hat{\mu}_{ht} &= -[g_\gamma^2 + \gamma_h^2 \beta(1+\bar{\lambda}_a)]\hat{C}_{ht} + g_\gamma \gamma_h(\hat{C}_{h,t-1} - \hat{g}_{\gamma t}) \\
&\quad - \beta\bar{\lambda}_a \gamma_h (g_\gamma - \gamma_h)\mathrm{E}_t\hat{\lambda}_{a,t+1} + \beta(1+\bar{\lambda}_a) g_\gamma \gamma_h \mathrm{E}_t(\hat{C}_{h,t+1} + \hat{g}_{\gamma,t+1}), && (299) \\
\hat{w}_t + \hat{\mu}_{ht} &= \hat{\psi}_t, && (300) \\
\hat{q}_{lt} + \hat{\mu}_{ht} &= \beta(1+\bar{\lambda}_a)\mathrm{E}_t\left[\hat{\mu}_{h,t+1} + \hat{q}_{l,t+1}\right] \\
&\quad + [1-\beta(1+\bar{\lambda}_a)](\hat{\varphi}_t - \hat{L}_{ht}) + \beta\bar{\lambda}_a \mathrm{E}_t \hat{\lambda}_{a,t+1}, && (301) \\
\hat{\mu}_{ht} - \hat{R}_t &= \mathrm{E}_t\left[\hat{\mu}_{h,t+1} + \frac{\bar{\lambda}_a}{1+\bar{\lambda}_a}\hat{\lambda}_{a,t+1} - \hat{g}_{\gamma,t+1}\right], && (302) \\
\Omega_e \hat{\mu}_{et} &= -(g_\gamma^2 + \beta\gamma_e^2)\hat{C}_{e,t} + g_\gamma\gamma_e(\hat{C}_{e,t-1} - \hat{g}_{\gamma t}) + \beta g_\gamma \gamma_e \mathrm{E}_t(\hat{C}_{e,t+1} + \hat{g}_{\gamma,t+1}), && (303) \\
\hat{w}_t &= \hat{Y}_t - \hat{N}_t, && (304) \\
\hat{q}_{kt} &= (1+\beta)\Omega\lambda_k^2 \hat{I}_t - \Omega\lambda_k^2 \hat{I}_{t-1} + \Omega\lambda_k^2(\hat{g}_{\gamma t} + \hat{g}_{qt}) \\
&\quad - \beta\Omega\lambda_k^2 \mathrm{E}_t[\hat{I}_{t+1} + \hat{g}_{\gamma,t+1} + \hat{g}_{q,t+1}], && (305) \\
\hat{q}_{kt} + \hat{\mu}_{et} &= \frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}}{\bar{\lambda}_q}(\hat{\mu}_{bt} + \hat{\theta}_t) + \frac{\beta(1-\delta)}{\lambda_k}\mathrm{E}_t(\hat{q}_{k,t+1} - \hat{g}_{q,t+1} - \hat{g}_{\gamma,t+1}) \\
&\quad + \left(1 - \frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}}{\bar{\lambda}_q}\right)\mathrm{E}_t\hat{\mu}_{e,t+1} + \frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}}{\bar{\lambda}_q}\mathrm{E}_t(\hat{q}_{k,t+1} - \hat{g}_{q,t+1}) \\
&\quad + \beta\alpha(1-\phi)\frac{\tilde{Y}}{\tilde{K}}\mathrm{E}_t(\hat{Y}_{t+1} - \hat{K}_t), && (306) \\
\hat{q}_{lt} + \hat{\mu}_{et} &= \frac{\tilde{\mu}_b}{\tilde{\mu}_e} g_\gamma \bar{\theta}(\hat{\theta}_t + \hat{\mu}_{bt}) + \left(1 - \frac{\tilde{\mu}_b}{\tilde{\mu}_e} g_\gamma \theta\right)\mathrm{E}_t\hat{\mu}_{e,t+1} + \frac{\tilde{\mu}_b}{\tilde{\mu}_e} g_\gamma \bar{\theta}\mathrm{E}_t(\hat{q}_{l,t+1} + \hat{g}_{\gamma,t+1}) \\
&\quad + \beta \mathrm{E}_t \hat{q}_{l,t+1} + (1 - \beta - \beta\bar{\lambda}_a\bar{\theta})\mathrm{E}_t[\hat{Y}_{t+1} - \hat{L}_{et}], && (307) \\
\hat{\mu}_{et} - \hat{R}_t &= \frac{1}{1+\bar{\lambda}_a}\left[\mathrm{E}_t(\hat{\mu}_{e,t+1} - \hat{g}_{\gamma,t+1}) + \bar{\lambda}_a \hat{\mu}_{bt}\right], && (308) \\
\hat{Y}_t &= \alpha\phi\hat{L}_{e,t-1} + \alpha(1-\phi)\hat{K}_{t-1} + (1-\alpha)\hat{N}_t - \frac{(1-\phi)\alpha}{1-(1-\phi)\alpha}[\hat{g}_{zt} + \hat{g}_{qt}], && (309) \\
\hat{K}_t &= \frac{1-\delta}{\lambda_k}[\hat{K}_{t-1} - \hat{g}_{\gamma t} - \hat{g}_{qt}] + \left(1 - \frac{1-\delta}{\lambda_k}\right)\hat{I}_t, && (310) \\
\hat{Y}_t &= \frac{\tilde{C}_h}{\tilde{Y}}\hat{C}_{ht} + \frac{C_e}{\tilde{Y}}\hat{C}_{e,t} + \frac{\tilde{I}}{\tilde{Y}}\hat{I}_t, && (311) \\
0 &= \frac{L_h}{\bar{L}}\hat{L}_{ht} + \frac{L_e}{\bar{L}}\hat{L}_{et}, && (312) \\
\alpha\hat{Y}_t &= \frac{\tilde{C}_e}{\tilde{Y}}\hat{C}_{e,t} + \frac{\tilde{I}}{\tilde{Y}}\hat{I}_t + \frac{\tilde{q}_l L_e}{\tilde{Y}}(\hat{L}_{et} - \hat{L}_{e,t-1}) \\
&\quad + \frac{1}{g_\gamma}\frac{\tilde{B}}{\tilde{Y}}(\hat{B}_{t-1} - \hat{g}_{\gamma t}) - \frac{1}{R}\frac{\tilde{B}}{\tilde{Y}}(\hat{B}_t - \hat{R}_t), && (313) \\
\hat{B}_t &= \hat{\theta}_t + g_\gamma\bar{\theta}\frac{\tilde{q}_l L_e}{\tilde{B}}\mathrm{E}_t(\hat{q}_{l,t+1} + \hat{L}_{et} + \hat{g}_{\gamma,t+1}) \\
&\quad + \left(1 - g_\gamma\bar{\theta}\frac{\tilde{q}_l L_e}{\tilde{B}}\right)\mathrm{E}_t(\hat{q}_{k,t+1} + \hat{K}_t - \hat{g}_{q,t+1}). && (314)
\end{aligned}
$$

The terms $\hat{g}_{zt}$, $\hat{g}_{qt}$, and $\hat{g}_{\gamma t}$ are given by

$$\hat{g}_{zt} = \hat{\lambda}_{zt} + \hat{\nu}_{zt} - \hat{v}_{z,t-1}, \tag{315}$$

$$\hat{g}_{qt} = \hat{\lambda}_{qt} + \hat{\nu}_{qt} - \hat{v}_{q,t-1}, \tag{316}$$

$$\hat{g}_{\gamma t} = \frac{1}{(1-(1-\phi)\alpha)}\hat{g}_{zt} + \frac{(1-\phi)\alpha}{(1-(1-\phi)\alpha)}\hat{g}_{qt}. \tag{317}$$

The technology shocks follow the processes

$$\begin{aligned}
\hat{\lambda}_{zt} &= \rho_z \hat{\lambda}_{z,t-1} + \hat{\varepsilon}_{zt}, && (318)\\
\hat{\nu}_{zt} &= \rho_{\nu_z} \hat{\nu}_{z,t-1} + \hat{\varepsilon}_{\nu_z t}, && (319)\\
\hat{\lambda}_{qt} &= \rho_q \hat{\lambda}_{q,t-1} + \hat{\varepsilon}_{qt}, && (320)\\
\hat{\nu}_{qt} &= \rho_{\nu_q} \hat{\nu}_{q,t-1} + \hat{\varepsilon}_{\nu_q t}. && (321)\\
& && (322)
\end{aligned}$$

There preference shocks follow the processes

$$\begin{aligned}
\hat{\lambda}_{at} &= \rho_a \hat{\lambda}_{a,t-1} + \hat{\varepsilon}_{at}, && (323)\\
\hat{\varphi}_{t} &= \rho_\varphi \hat{\varphi}_{t-1} + \hat{\varepsilon}_{\varphi t}, && (324)\\
\hat{\psi}_{t} &= \rho_\psi \hat{\psi}_{t-1} + \hat{\varepsilon}_{\psi t}. && (325)
\end{aligned}$$

The liquidity shock follows the process

$$\hat{\theta}_t = \rho_\theta \hat{\theta}_{t-1} + \hat{\varepsilon}_{\theta t}. \tag{326}$$

We solve the 19 rational expectations equations, (299) through (317), for the 19 unknowns summarized in the column vector

$$x_t = [\hat{\mu}_{ht}, \hat{w}_t, \hat{q}_{lt}, \hat{R}_t, \hat{\mu}_{et}, \hat{\mu}_{bt}, \hat{N}_t, \hat{I}_t, \hat{Y}_t, \hat{C}_{ht}, \hat{C}_{et}, \hat{q}_{kt}, \hat{L}_{ht}, \hat{L}_{et}, \hat{K}_t, \hat{B}_t, \hat{g}_{\gamma t}, \hat{g}_{zt}, \hat{g}_{qt}]',$$

where x_t is referred to as a vector of state variables. The system of solved-out equations forms a system of state equations.

D Proofs for A Model of Bank Credit Cycles

Proof for Proposition 2

From equation (4.7), we can get

$$\frac{\partial e}{\partial \eta^*} = \frac{\eta^s - \underline{\eta}}{(\eta^s - \eta^*)^2} \frac{(1-\bar{\lambda})R}{c} > 0 \tag{327}$$

Thus, innovation effort (probability) is increasing with supervision ability.

From equation (4.8), we can get

$$\frac{\partial r}{\partial \delta} = \frac{(\eta^s - \underline{\eta})r^2}{r_0} \cdot \frac{\partial e}{\partial \eta^*} > 0 \tag{328}$$

Thus, deposit rate is increasing with supervision ability.

From equation (4.9), we can get

$$\frac{\partial L}{\partial \eta^*} = \frac{(1-\bar{\lambda})R}{(\eta^s - \eta^*)^3 c L^2 r_0}[\eta^s(\eta^s - \eta^*)c - (1-\bar{\lambda})(\eta^s - \underline{\eta})(\eta^s + \eta^* - 2\underline{\eta})R] \tag{329}$$

From the above equation we can see that, as long as $c \geq \frac{(1-\bar{\lambda})(\eta^s - \underline{\eta})(\eta^s + \bar{\eta}^* - 2\underline{\eta})R}{\eta^s(\eta^s - \bar{\eta}^*)}$ holds, leverage is always increasing with supervision ability. Q.E.D.

Proof for Lemma 2

From equation (4.7), we can get

$$\frac{\partial e}{\partial c} = -\frac{\eta^* - \underline{\eta}}{\eta^s - \eta^*} \frac{(1-\bar{\lambda})R}{c^2} < 0 \tag{330}$$

$$\frac{\partial e}{\partial R} = \frac{\eta^* - \underline{\eta}}{\eta^s - \eta^*} \frac{1-\bar{\lambda}}{c} > 0 \tag{331}$$

$$\frac{\partial e}{\partial \bar{\lambda}} = -\frac{\eta^* - \underline{\eta}}{\eta^s - \eta^*} \frac{R}{c} < 0 \tag{332}$$

From equation (4.8), we can get

$$\frac{\partial r}{\partial c} = \frac{(\eta^s - \underline{\eta})r^2}{r_0} \cdot \frac{\partial e}{\partial c} < 0 \tag{333}$$

$$\frac{\partial r}{\partial R} = \frac{(\eta^s - \underline{\eta})r^2}{r_0} \cdot \frac{\partial e}{\partial R} > 0 \tag{334}$$

$$\frac{\partial r}{\partial \bar{\lambda}} = \frac{(\eta^s - \underline{\eta})r^2}{r_0} \cdot \frac{\partial e}{\partial \bar{\lambda}} < 0 \tag{335}$$

From equation (4.9) and under large innovation cost coefficient c, we can get

$$\frac{\partial L}{\partial c} = -\frac{(1-\bar{\lambda})R}{(\eta^s-\eta^*)L^2r^2} \cdot \frac{\partial r}{\partial c} > 0 \tag{336}$$

$$\frac{\partial L}{\partial R} = \frac{1-\bar{\lambda}}{(\eta^s-\eta^*)^2 cL^2 r_0}[\eta^s(\eta^s-\eta^*)c - 2(1-\bar{\lambda})(\eta^s-\underline{\eta})(\eta^*-\underline{\eta})R] \tag{337}$$

$$\frac{\partial L}{\partial \bar{\lambda}} = -\frac{R}{(\eta^s-\eta^*)^2 cL^2 r_0}[\eta^s(\eta^s-\eta^*)c - 2(1-\bar{\lambda})(\eta^s-\underline{\eta})(\eta^*-\underline{\eta})R] \tag{338}$$

It is easy to see that for sufficiently large c, $\frac{\partial L}{\partial R} > 0$ and $\frac{\partial L}{\partial \bar{\lambda}} < 0$. In fact, as long as $\frac{\partial L}{\partial \eta^*} > 0$, $\frac{\partial L}{\partial R} > 0$ and $\frac{\partial L}{\partial \bar{\lambda}} < 0$. Q.E.D.

Proof for Lemma 4

From Lemma 2, we know that when c increases, R decreases, or $\bar{\lambda}$ increases, the innovation probabilities are lower for each state. If we can show that lower innovation probabilities lead to a lower probability in the lowest state and a higher probability in the highest state, we can prove this lemma. We use superscripts o to denote the old states and n to denote the new states.

From $\pi = \pi P$, we can get $\pi(I-P) = 0$. We can write down the relationship between the probabilities of two nearby states as follows

$$\pi_{j+1} = \begin{cases} \frac{q(1-e_j)}{1-(1-q)(1-e_{j+1})}\pi_j, & \text{if } 1 < j < K-1 \\ \frac{q(1-e_{K-1})}{e_K}\pi_{K-1}, & \text{if } j = K-1 \end{cases} \tag{339}$$

With this relationship, we can write down the relationship between the probability of the lowest state and that of others

$$\pi_j = \begin{cases} \prod_{i=1}^{j-1} \frac{q(1-e_j)}{1-(1-q)(1-e_{j+1})} \cdot \pi_1, & \text{if } 1 < j < K \\ \frac{q(1-e_{K-1})}{e_K} \prod_{i=1}^{K-1} \frac{q(1-e_j)}{1-(1-q)(1-e_{j+1})} \cdot \pi_1, & \text{if } j = K \end{cases} \tag{340}$$

We can define Δ_j as

$$\Delta_j = \begin{cases} \prod_{i=1}^{j-1} \frac{q(1-e_j)}{1-(1-q)(1-e_{j+1})}, & \text{if } 1 < j < K \\ \frac{q(1-e_{K-1})}{e_K} \prod_{i=1}^{K-1} \frac{q(1-e_j)}{1-(1-q)(1-e_{j+1})}, & \text{if } j = K \end{cases} \tag{341}$$

So $\pi_j = \Delta_j \cdot \pi_1$ for any $j \geq 2$. It is easy to see that as c increases, R decreases, or $\bar{\lambda}$ increases, all e_j's decrease, so all Δ_j's increase. Substitute π_j into $\sum_{j=1}^{K} \pi_j = 1$, we can get $\pi_1 = 1/(\sum_{j=1}^{K} \Delta_j)$, so π_1 decreases.

We will prove $\pi_K^n > \pi_K^o$ by contradiction. If $\pi_K^n \leq \pi_K^o$, since e_j becomes smaller, from equation (339), we can get $\pi_j^n < \pi_j^o$ for all $1 < j < K$. And from above, we know $\pi_1^n < \pi_1^o$. So $\sum_{j=1}^K \pi_j^n < \sum_{j=1}^K \pi_j^0 = 1$, and there is contradiction. Thus, $\pi_K^n > \pi_K^o$. Q.E.D.

Proof for Proposition 4

To prove the new stationary distribution first-order stochastic dominates the original one, we just need to show that the cumulative probability $\sum_{j=1}^k \pi_j^n$ is smaller or equal to $\sum_{j=1}^k \pi_j^o$ for all k and with strict inequality for some k following the definition of first-order stochastic dominance.

If c increases, R decreases, or $\bar{\lambda}$ increases, $e_j^n < e_j^o$ for all j. From equation (339), it is easy to see that (1) if $\pi_j^n > \pi_j^o$ for some j, this inequality holds for all k larger than j; (2) if $\pi_j^n < \pi_j^o$ for some j, this inequality holds for all k smaller than j. From Lemma 4, there must exist a $1 < k < K$, where $\pi_k^n \leq \pi_k^o$ and $\pi_{k+1}^n > \pi_{k+1}^o$. For $j < k$, $\pi_j^n < \pi_j^o$, so $\sum_{i=1}^j \pi_i^n < \sum_{i=1}^j \pi_i^o$. For $j > k$, $\pi_j^n > \pi_j^o$, so $\sum_{i=j}^N \pi_i^n > \sum_{i=j}^N \pi_i^o$. For $k < j < N$, $\sum_{i=1}^j \pi_i^n = 1 - \sum_{i=j}^N \pi_i^n < 1 - \sum_{i=j}^N \pi_i^o = \sum_{i=1}^j \pi_i^o$. Thus, we can show that $\sum_{i=1}^j \pi_i^n \leq \sum_{i=1}^j \pi_i^o$ for all j and with strict inequality for $j < K$. Q.E.D.

Proof for Proposition 5

With innovation probability under regulation, we can write down the relationship between the probabilities of two nearby states as follows

$$\pi_{j+1}^r = \begin{cases} \frac{q(1-e_j^r)}{1-(1-q)(1-e_{j+1}^r)}\pi_j^r, & \text{if } 1 < j < K-1 \\ \frac{q(1-e_{K-1}^r)}{e_K^r}\pi_{K-1}^r, & \text{if } j = K-1 \end{cases} \tag{342}$$

With this relationship, we can write down the relationship between the probability of the lowest state and that of others

$$\pi_j^r = \begin{cases} \prod_{i=1}^{j-1} \frac{q(1-e_j^r)}{1-(1-q)(1-e_{j+1}^r)} \cdot \pi_1^r, & \text{if } 1 < j < K \\ \frac{q(1-e_{K-1}^r)}{e_K^r} \prod_{i=1}^{K-1} \frac{q(1-e_j^r)}{1-(1-q)(1-e_{j+1}^r)} \cdot \pi_1^r, & \text{if } j = K \end{cases} \tag{343}$$

We can define Δ_j^r as

$$\Delta_j^r = \begin{cases} \prod_{i=1}^{j-1} \frac{q(1-e_j^r)}{1-(1-q)(1-e_{j+1}^r)}, & \text{if } 1 < j < K \\ \frac{q(1-e_{K-1}^r)}{e_K^r} \prod_{i=1}^{K-1} \frac{q(1-e_j^r)}{1-(1-q)(1-e_{j+1}^r)}, & \text{if } j = K \end{cases} \tag{344}$$

From $\sum_{j=1}^{K} \pi_j^r = 1$, we get $\pi_1^r = 1/(\sum_{j=1}^{K} \Delta_j^r)$. If the regulator sets a maximum leverage lower than the highest one determined by the market, there exists at least one e_j^r which is smaller than that without regulation. We can get that all Δ_j^r's are larger than or equal to the correspondent without regulation, Δ_j^m's, and some are strictly larger. Then, $\sum_{j=1}^{K} \Delta_j^r$ is larger, so π_1^r is lower that that without regulation, π_1^m. It is easy to see that π_K^r is higher than the case without regulation, π_K^m.

Since the regulator sets a maximum leverage lower than the highest one determined by the market, there must exist a $1 \leq \bar{k} \leq K$, where all states lower than or equal to $\bar{k}$ are not affected by the regulation, while all states higher than $\bar{k}$ are affected by the regulation. For $j \leq \bar{k}$, $e_j^r = e_j^m$, and for $j > \bar{k}$, $e_j^r < e_j^m$. For equations (339) and (342), we can see that $\pi_j^r < \pi_j^m$ for $j \leq \bar{k}$. And if $\pi_j^r > \pi_j^m$ for some j, this inequality holds for all k larger than j. Since $\pi_K^r > \pi_K^m$, there must exist one $\bar{k} < \hat{k} \leq K$, where $\pi_j^r \leq \pi_j^m$ for $j < \hat{k}$ and $\pi_j^r > \pi_j^m$ for $j \geq \hat{k}$. For $j < \hat{k}$, $\pi_j^r \leq \pi_j^m$ with some strict inequality, so $\sum_{i=1}^{j} \pi_i^r < \sum_{i=1}^{j} \pi_i^m$. For $j \geq \hat{k}$, $\pi_j^r > \pi_j^m$, so $\sum_{i=j}^{K} \pi_i^r > \sum_{i=j}^{K} \pi_i^m$. For $\hat{k} \leq j < K$, $\sum_{i=1}^{j} \pi_i^r = 1 - \sum_{i=j}^{K} \pi_i^r < 1 - \sum_{i=j}^{K} \pi_i^m = \sum_{i=1}^{j} \pi_i^m$. Thus, we can show that $\sum_{i=1}^{j} \pi_i^r \leq \sum_{i=1}^{j} \pi_i^m$ for all j and with strict inequality for $j < K$. Q.E.D.

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