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Haoran Wang March 29, 2017

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An abstract of
a thesis submitted to the Faculty of Emory College of Arts and Sciences
of Emory University in partial fulfillment
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Bachelor of Sciences with Honors

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Abstract

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We study the flow of soft hydrogel particles out of a quasi-two-dimensional hopper. The hopper chamber is set thin enough for all the particles to stay in one plane without overlapping with each other. Since hydrogels have a significantly low friction coefficient, the system is considered as a frictionless system. We examine the probability of a clog forming as the particles flow out, as a function of the ratio of the size of hopper exit vs. the average diameter of the particles. We find that clogging of these soft particles requires the hopper exit to be quite small, only about twice larger than the particle diameter. Also, we observe the clogging probability increases as we reduce the influence of gravity (by tilting the hopper chamber away from vertical). The clogging happens more often when the exit is slightly larger than the size of the particles, which is much different from the observation of hard particles.

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Acknowledgements

Writing this thesis is a huge challenge for me. I have never written a formal thesis before and I fully appreciate all the other students who take the same honors program. I must say a huge thank you to Dr. Eric Weeks, who are my advisor and a great teacher that led through the whole project. I worked in his research group for almost two years, from my junior year. Starting from a seemingly impossible project of emulsion to a much concrete project I am doing right now, he taught a lot that I had never learned from the other teacher through my life. Besides asking a lot of stupid questions, it seems like the only thing I did is bombing him with the emails and questions for a lot of basic questions.

I also will acknowledge Carlos, the post-doc in the group. He helped a lot when I did my first project and guided me onto a right road for doing research. Although he is a very demanding person, without his encouragement and supervision, I will not become the person I am right now.

Now, I will graduate soon. Looking back the whole four years, although it is not related to the project I did, I must thank every faculty who helped me and as an international student, I appreciate every single suggestion and criticism on me. I did a lot of mistake but I am here now, with all your helps.

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Chapter 1: Introduction

Physics is always focusing on explaining real world phenomena. For example, the very classical homework question in introduction physics class that an object falls from a three-story building is a model to study the free-body fall. In the physics context, we sometimes simplify the real-life question, pursuing a relatively accurate approximation rather than considering every potential attribute. Overall, physicists care more on the underlying principles so that we often categorize bunch of similar concrete systems and abstract them into one model. The system I studied in my research is called 2D Granular System. As the name implies, the system is comprised of granular materials which are a collection of discrete solid, macroscopic particles that when they interact with the surroundings, they lose energy. And the system is defined as two-dimensional since the particles will be placed on a surface, only having one layer. This system is a good model to analyze the practical problems such as traffic, objects on a conveyor belt, etc. Every product on the conveyor can be regarded as a macroscopic particle and since they cannot be on the top of another one, the whole system can be defined as a 2D system. The problem every engineer face in this case is how to arrange the conveyors to achieve the maximum efficiency, at the same time, saving the budgets. This involves several big questions: at what speed are the conveyors running, how long and how wide should a certain conveyor be, how to make two conveyors confluence, etc. The minimum requirement of the solutions for all the problems is that the system cannot clog. Thus, there are a lot of study focusing on the clogging of the system in different environments. All the study try to decompose the underline reason for clogging and to dig out how some factors, friction for example, influence the flow of the particles. Although the setups of the system can be various, the particles can be roughly grouped into two categories, hard particles and soft particles. Hard particles are the particles that can hardly be deformed, such as hard metal. On the other hand, the soft particles can be deformed to a certain degree, based on the characteristics of the material it composed of. A typical soft material is polymers, such as hydrogels, the material I used in my experiment.

A good example that physicists use the 2D granular system to solve the real life question is the study of how people exit a room under panic. Every person can be defined as a hard granular particle, although humans are relatively soft, and all the particles are rushing to the only exit of the room. How to make the casualty reduce to its potential minimum is the goal of the research. In order to do that, physicists did different simulations to find out the optimal size of the door and also the number of the doors in a room of a certain size. This becomes the guideline for the fire inspection.

What I did in my experiment was to find the probability of clogging of soft particles in a two-dimensional hopper when the size of the exit changed. We also investigate how the external force, gravitation force in this case, affects the clogging. The result shows that the clogging of soft particles is far different from the clogging of hard particles. Besides unveiling the change of the clogging probability respect to the size of the exit, we also concluded that the soft particles would be much unlikely to be clogged under a higher driven force whereas prior work shows hard particles would be much likely to be clogged.

Chapter 2: Literature Review

Over ten years ago, Kiwing To (To *et al*, 2001) and his colleagues did a research on clogging of hard disks in a 2D hopper. They made a quasi-2D enclosure containing two angular blocks and one of them could be moved back and forth in order to control the size of the exit. A rotor was connected to the device so that the whole hopper could be flipped, a design aiming for quick reset and repeat the experiment. 200 hard metal disks with 5mm diameter were put into the hopper. The schematic setup is shown in Fig. 1.

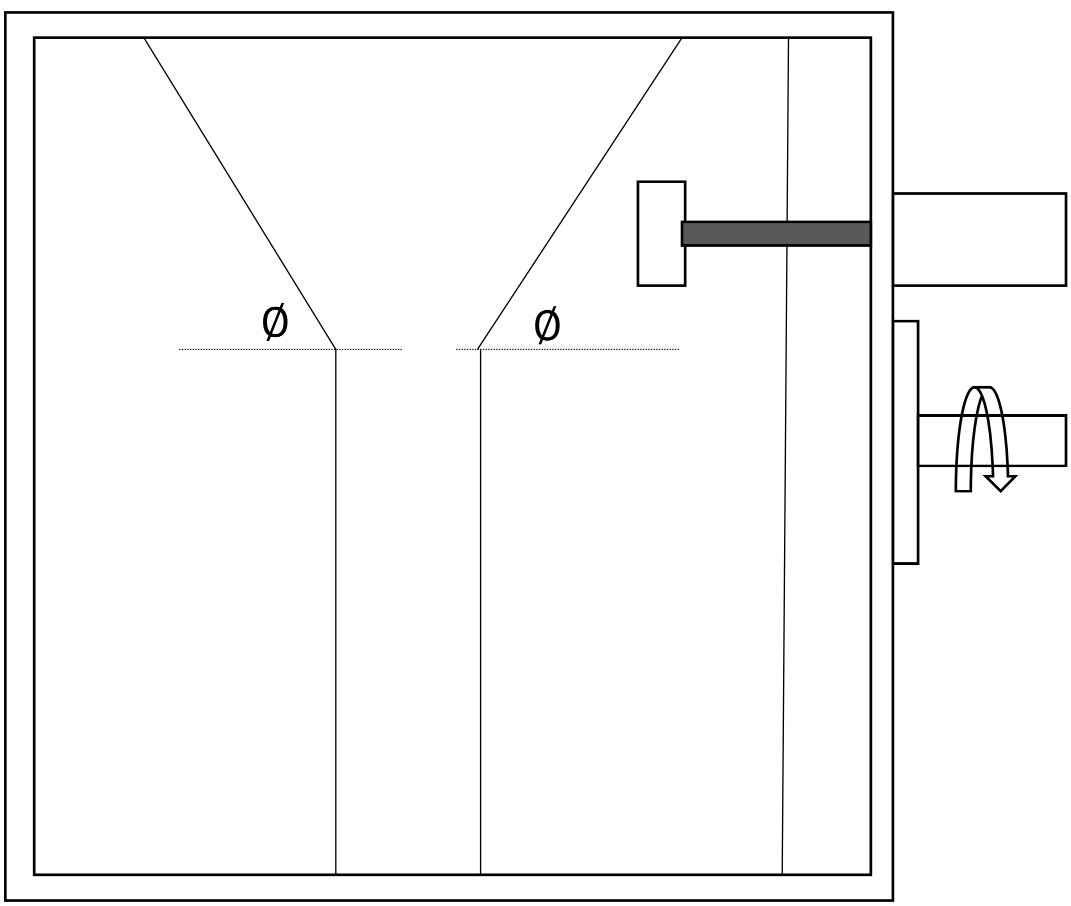

Figure 1, the diagram of the experiment setup. The plate was attached to a bar and the right block could be moved by the outside motor, ∅ is the angle of the angular block.

They did the experiment using three different angular blocks: 34°, 60° and 75°, and counted the times when the system clogged, calculated the probability of clogging respect to the ratio of the exit vs the diameter of the disk. What they found was the clogging probability started to decrease when the ratio reached 3.3 and became zero when the ratio was over 5.5. In the experiment, the plate was set upright which means the gravitational force was fully exerted on each of the disks. In this setup, the main forces on the disks were the gravitational force, the forces exerted by the other particles when they were squeezed together and the force given by the wall when particles touched the wall. The effect of friction between the particles was minimized by polishing the metal disks. The clog was caused by the formation of arch at the hopper exit and they found the horizontal span of the arch was always greater than the exit, which means the arch was always set on the angular blocks rather than the exit. Another important feature of the arch was that it was always convex, shown in Fig. 2, which was the necessary condition for equilibrium if we neglected the friction.

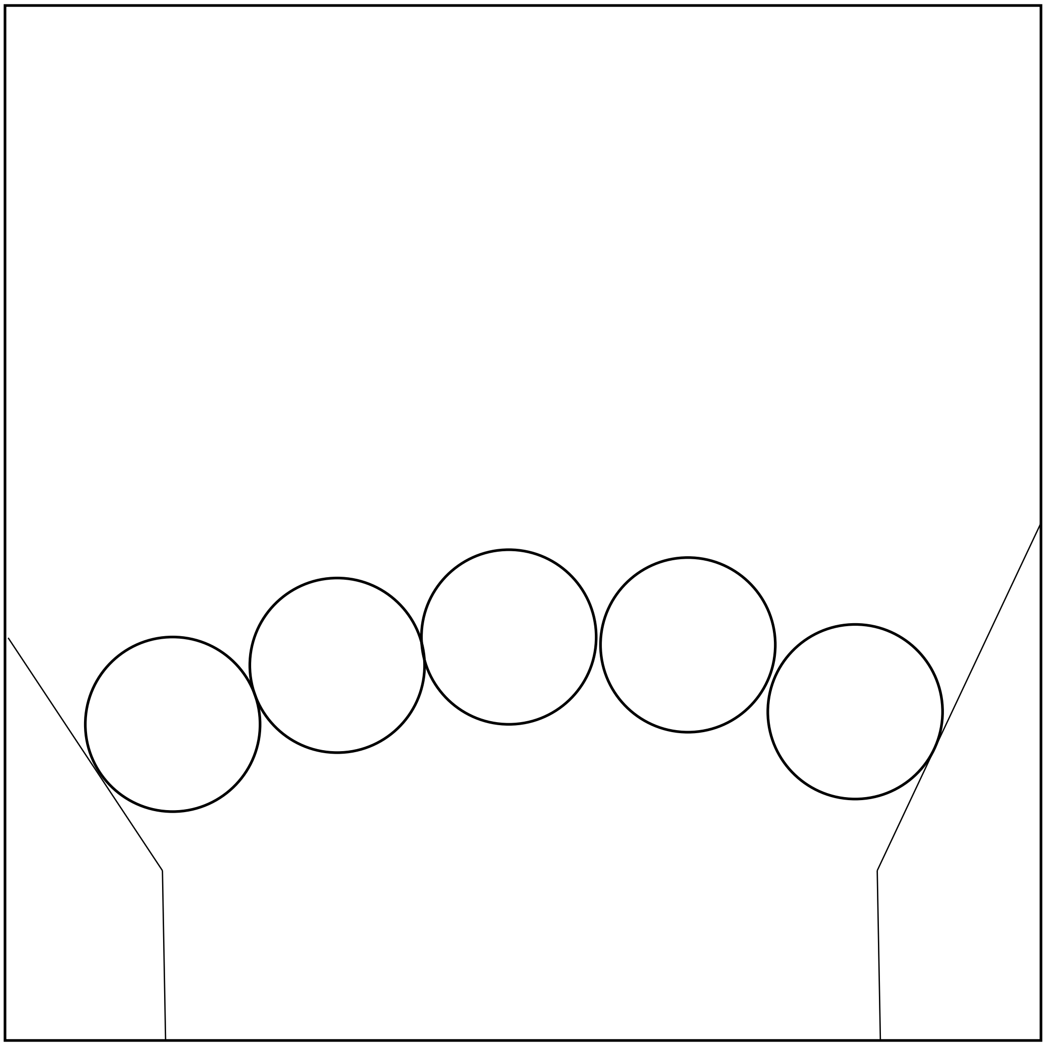

Figure 2, sketch of a typical arch formed near the opening.

Besides this experiment, they also found out the angle of the angular block, if below certain critical degree, did not matter the probability of clogging. They also did an experiment where they magnified the influence of friction between disks. However, the focus of To's experiment is at the first experiment where they neglect the inter-particle friction. His research inspires me and my advisor Dr. Eric Weeks to expand the experiment to soft particles.

Another group also did a very interesting research on the flow and clogging of hard particles. Dr. Zuriguel (Zuriguel *et al*, 2014) and his colleagues did a series of experiments on different hard particles and they introduced different "candidates" that they thought might affect the clogging. They designed the experiments from big objects such as a crowd of sheep, to a relatively small colloid system. They also did a simulation of pedestrians passing through a small door. In this simulation, they controlled the size of the door, the random forces exerted on the pedestrians at a direction perpendicular to the door and the desired velocity. The simulation was constructed based on the Social Force Model (Genovese *et al*, 2011). They observed consecutive exiting of pedestrians, which they defined as bursts, separated by a period of time that no pedestrians could exit, defined as clogs. In this case, they set the minimum duration of flow for a burst to be 5 seconds. They plotted the histogram of the burst size, which was how many people going through the door, rescaled by the average burst size and found out the histogram followed the exponential distribution, which means a small flow happened much likely than a big flow. They also plotted the complementary cumulative distribution function (CDF) of the time lapse t between flow of pedestrians. They used rigorous method (Clauset *et al.*, 2009) to fit the power-law tail,

$$t^{-a} \tag{1}$$

The minimum duration of a burst was set during the process in order to ensure the fit is valid. While they changed the parameters, the desired velocity, the random force and the size of the door, the power a varies. The power a is an important parameter to determine whether the system was finally clogged or not. A bigger a showed a faster probability decay, which indicates a longer rest time between burst was very unlikely. On the other hand, when a was below 2, the average time lapse, the average clog time, diverged. They concluded that within this region, the average flow rate was not well-defined since the probability of a much longer clog was not diminishing. Thus, they interpreted a system with an a below 2 as a clogging system. While they decreased the desired velocity, they discovered a transition from a clogged system to an unclogged system. The desired velocity of a pedestrian was the velocity of the pedestrian moving towards the door. When the velocity was below the desired velocity, the pedestrian would accelerate; in other word, a force would be exerted in the direction towards the door. The lower the velocity was, comparing to the desired velocity, the higher the force would be. On the other hand, if the velocity of the pedestrian was higher than the desired velocity, a force would be exerted in the direction away from the door. The lower the desired velocity was, the smaller the force would be to push the pedestrian to that velocity. The force was maximized when the system was clogged. At static state, the velocities of every pedestrian were zero. Since the simulation was designed to make the pedestrians move at the desired velocity, the additional forces would be applied to every pedestrian towards the door and forces would be huge.

Besides the simulation of pedestrians, they also did an experiment that was similar to To's experiment (To *et al*, 2001). They used a quasi-2D inclined silo and concluded that the system would experience a transition from clogged to unclogged while decreasing the gravitational force. Therefore, combining all the experiments, they concluded that the higher the driven forces was, the more likely the system would be clogged eventually.

My advisor Dr. Eric Weeks (Hong *et al*, 2015), based on other researches, designed a simulation to basically reproduce To's experiment (To *et al*, 2001) but using soft particles rather than metal disks. He set up the simulation by Durian "bubble model" (Durian, 1995), and according to this model, the motion of the particle was controlled by these forces: the viscous force, Eq. 2, the resistance when a particle tended to slip by another particle, the repulsive force, Eq. 3, when two particles overlapping each other, the force exerted by the wall, which is similar to Eq. 3 with the radius of the wall equal to zero, the dragging force exerted by the plates, Eq. 4, and the gravitational force, Eq. 5.

$$\overrightarrow{F_{ij}^{vis}} = b(\overrightarrow{v_i} - \overrightarrow{v_j}) \quad (2)$$

b is the coefficient of the viscous force, and $\overrightarrow{v_i}$, $\overrightarrow{v_j}$ denotes the velocity alongside the radius for a random particles i and a random particle j which contacts with i.

$$\overrightarrow{F_{ij}^{repulsive}} = F_0 \left[\frac{1}{|\overrightarrow{r_i} - \overrightarrow{r_j}|} - \frac{1}{R_i - R_j}\right] \overrightarrow{r_{ij}} \quad (3)$$

F_0 is defined by the surface tension; $\overrightarrow{r_i}$ and $\overrightarrow{r_j}$ are the position of the two particles; R_i and R_j are the radii of the two particles and $\overrightarrow{r_{ij}} = \overrightarrow{r_j} - \overrightarrow{r_i}$.

$$\overrightarrow{F_i^{plates}} = -cR_i^2 \overrightarrow{v_i} \quad (4)$$

c is the coefficient of the force.

$$\overrightarrow{F_i^{gravity}} = -gR_i^2 \vec{y} \quad (5)$$

g is the gravitational constant and $\vec{y}$ is the unit vector alongside the vertical direction.

In the simulation, since the velocity of the particles were not very high, the viscous force and the force exerted by the plates were very small, which became unimportant comparing to other forces. Therefore, the motion of the particles was mainly affected by the repulsive force, the force by the wall and the gravitational force. There were totally 800 soft particles with 10% poly-dispersity in the simulation and the parameters were the ratio of the width of the exit vs the average diameter of the particles and the ratio of the gravitational constant vs F_0. What she found was that the critical region where the probability decreased dramatically was much narrower than the size found by To. The region from the simulation started roughly from 1.5 and ended at around 2.3. In To's experiment, the size of the region was 2.2 and the simulation data was 0.8, which is about 0.36 the size of To's data. While varying the ratio of gravitational constant vs F_0, the simulation showed a very different result comparing to Zuriguel's experiments (Zuriguel *et al,* 2014). When the gravitational force increased, the ratio of gravitational constant vs F_0

becoming bigger, the ratio of the exit vs the average diameter for absolute flow system became smaller. The whole clogging probability curve shifted to the left, which means the system was less likely to be clogged. The ratio for original setup to get the 100% flow was about 2.3 whereas the ratio for the higher gravity setup was about 1.1, meaning the system did not need a large opening for flow, indicating the requirement for getting a flow state decreased. Also, when the gravitational force became weaker, the probability curve shifted to the right, with a slight wider critical region. The shifts of the probability curve demonstrated the influence of the driven force on soft particle system might be totally different from that on hard particle system. The reaction of the soft particle system on the driven force was opposite to that of the hard particle system. This interesting conclusion led to my experiment, where I partially reproduced the simulation in the real life.

Chapter 3: Methodology

The hopper I used in my experiment was made of acrylics. The thicknesses of the top and bottom plates are 1.27 cm. The width of the experimental space, which can hold the hydrogels inside, is 73.48 cm. The height of that space is 55 cm. There is a rectangular reservoir at the top of the device with 45 cm wide and 9 cm high. The total space of the reservoir is 405 cm^2. The reservoir is designed to contain all 200 hydrogels, supposed the diameters of all the particles are 1.6 cm. The specification of the device is shown in Fig. 3:

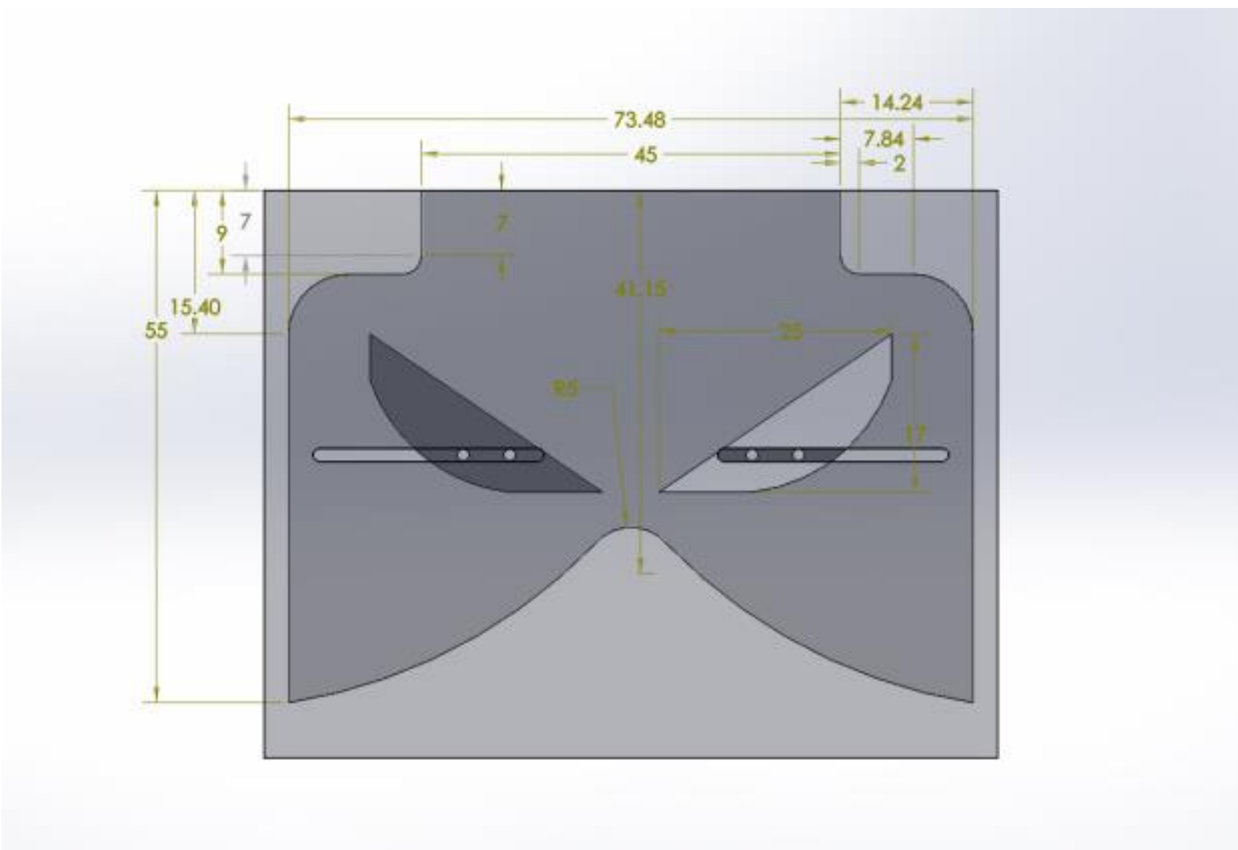

Figure 3, specification of the hopper

The two angular blocks are at an angle of 34° and are fixed by two bolts. There are two slots on the plates so that the two blocks are both movable. There is a metal bar attached to the plate so that I can flip the plate to repeat the experiment. The side channels are wide enough for most cases that while the particles are moving back to the reservoir, they will not be clogged at either side. The bottom is made like a mountain in order to catch the hydrogels safely since hydrogels, although quite soft, are fragile and not good at resisting impact. Therefore, reducing the falling distance from the hopper open is an effective way to protect them. The two curve alongside the "mountain" are made to let the particles easily roll down to the side so that when I flip the plate, all particles can go back to the reservoir smoothly. That is also the reason why the bottoms of those two angular plates are cut to be rounder. The picture of the device is shown below:

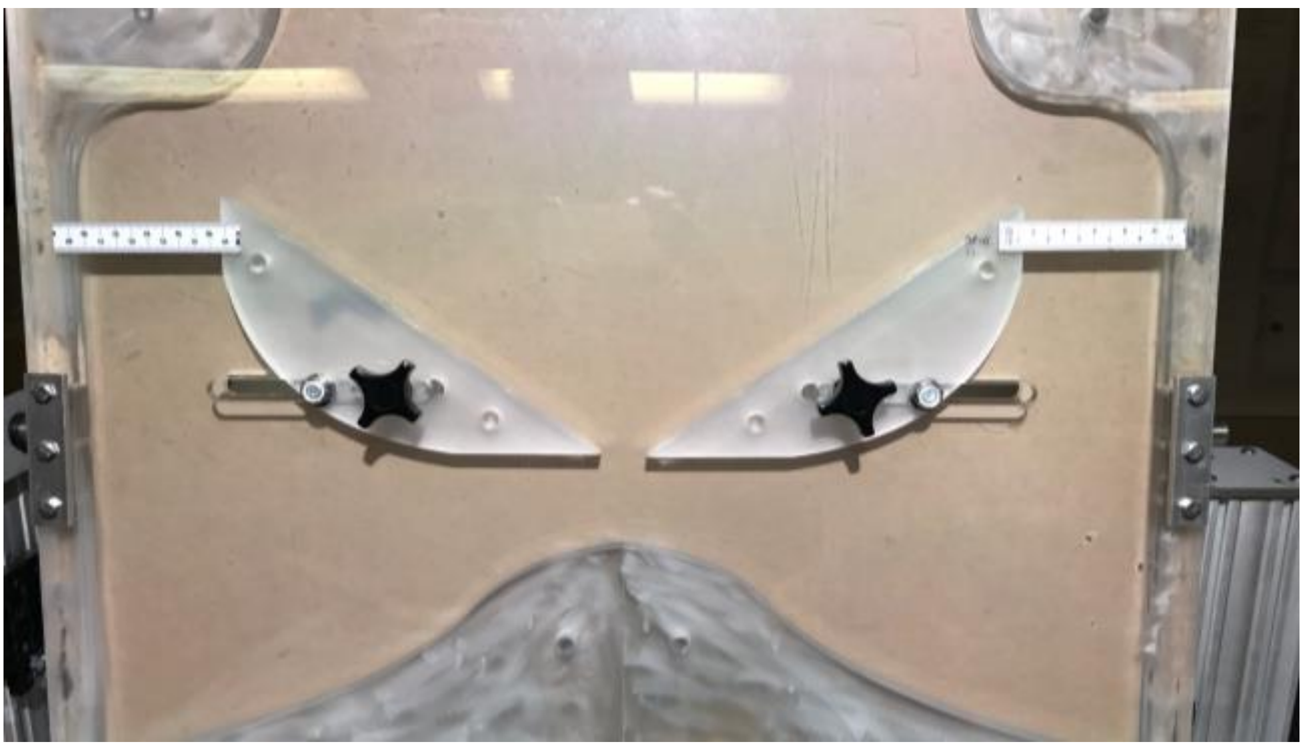

Figure 4, the hopper after setup. The two scales on both sides indicates the size of the exit.

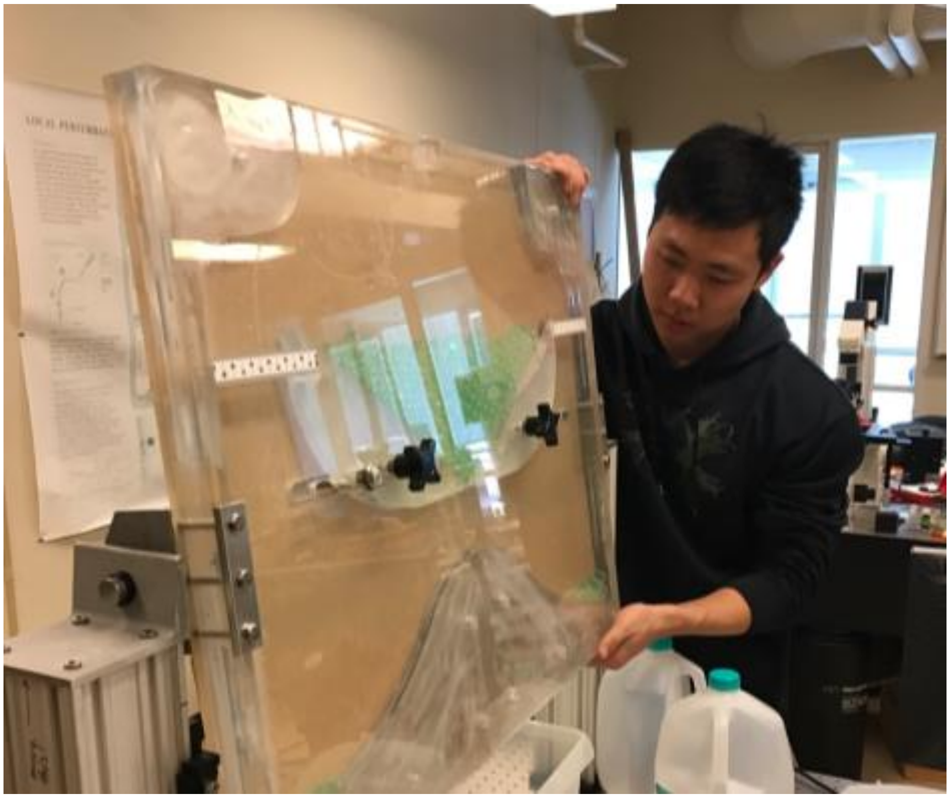

Figure 5, the metal bar behind the plate. The whole device can be flipped.

The green particles in Fig. 5 are the hydrogels. We buy the hydrogels from Amazon and soak them into distilled water for days until they fully swollen up. The size of the hydrogels will be several times bigger after they bloated, as shown in Fig. 6.

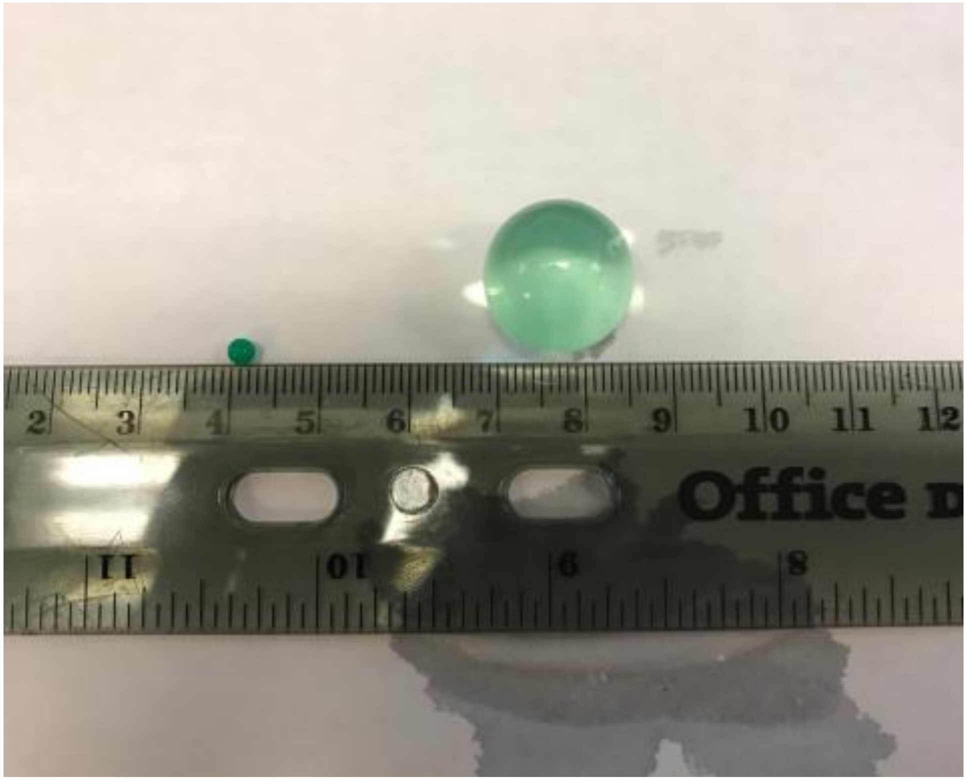

Figure 6, the size comparison between a dry hydrogel and a bloated one. The diameter of the dry one is about 0.25 cm and the diameter of the bloated one is about 1.6 cm, which is 6.4 times bigger.

The hydrogels are selected by a set of sieves. The apertures of the sieves are 1.3 cm, 1.45 cm and 1.6 cm. After the selection, the average size of the hydrogels is 1.57 cm. The process of selection was done by the two steps. First, all the hydrogels would go through the combination of 1.3 cm sieve and 1.6 cm sieve, which was the coarse selection. Then, the gels left would go through the 1.45 cm sieve in order to narrow the size range, which was the fine selection. The figure below shows the size distribution of 161 random hydrogels after the selection:

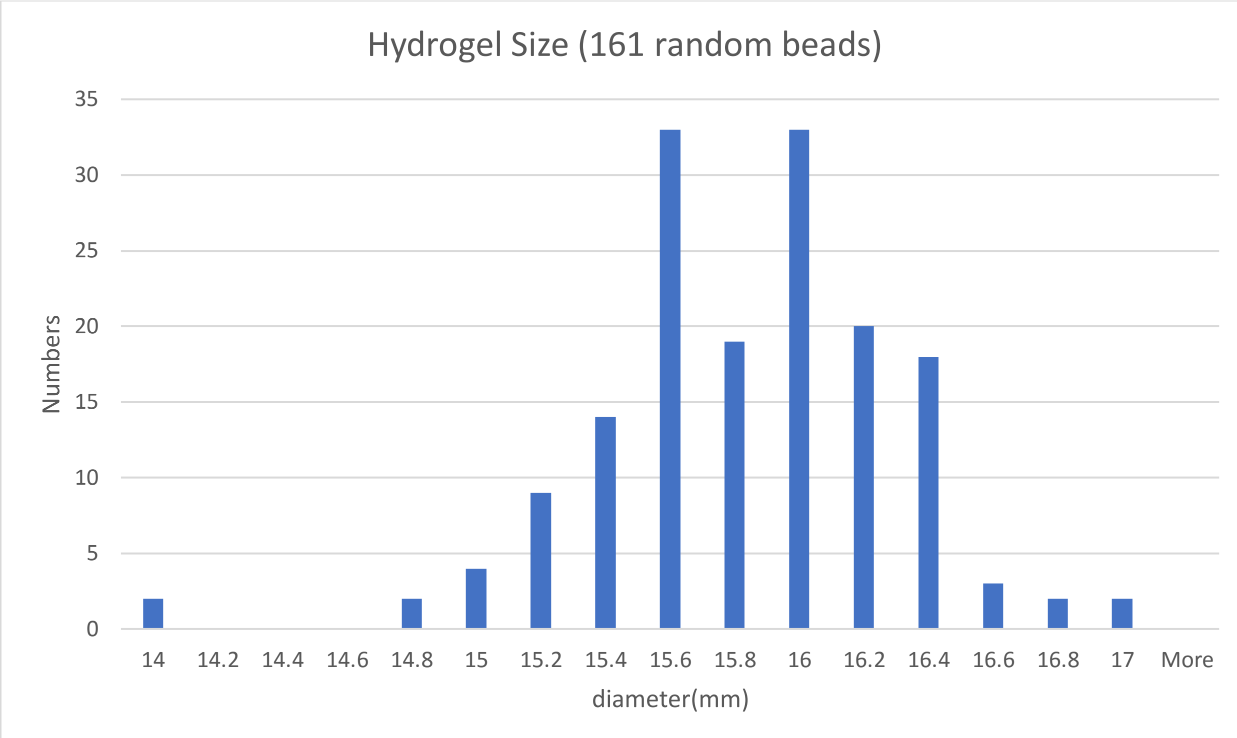

Figure 7, the size distribution of 161 random hydrogels. The distribution is approximate to a normal distribution, except the center, 15.8 mm, has a huge dent whereas the neighbors, 15.6 mm and 16 mm, have the highest frequency.

After the sieve system, there was still some hydrogels that were smaller than 1.45 cm or bigger than 16 cm. The reason why this happened, besides the error while measuring, is the hydrogels are very slippery that they could slip through the sieve if the they were not too big comparing to the aperture of the sieve. The procedure is similar to To's experiment (To *et al*, 2001). First, the size of the exit is set and all 200 selected hydrogels are put into the hopper. Then, the top open is sealed by tape so that the system is almost fully closed, except the two slots for moving the blocks. The device will be flipped to repeat the experiment 50 times for each size of the exit. Every 10 times, the total number of particles inside the hopper will be counted and new hydrogels will be added to the hopper if the total number is less than 200. Since the hydrogels were relatively fragile, some of them would be broken when repeatedly hit the wall of the device. The broken particles would be excluded from the hydrogel pool and would be replaced. And the arch formed by broken particles would not be counted. The number of clogging will be recorded and the probability of clogging for a specific opening is calculated by

$$P_{clogging} = \frac{N_{clogging}}{N_{trial}} \tag{6}$$

$N_{clogging}$ is the number of clogging and N_{trial} is the number of trial per opening.

The device was first set at 15° inclination, which was the angle between the plate and the horizontal surface. There is a mechanism at the post of the device to stop the motion of the plate.

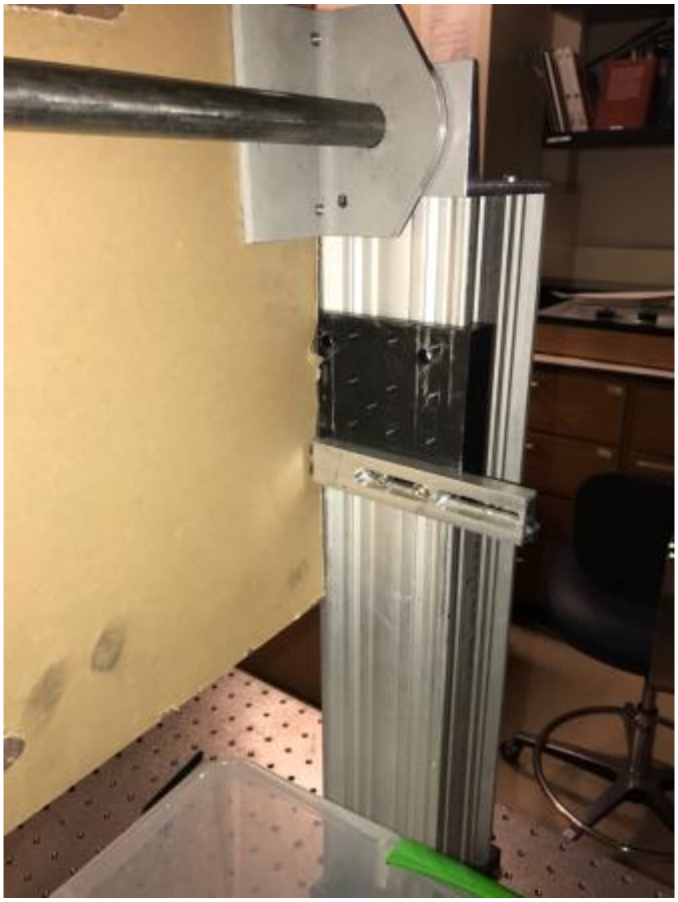

Figure 8, the stop mechanism. Here it was set at 90°. The black plastic plate can be move up and down and the metal bar can be screwed at different position in order to get the right angle.

I tried 7 different opening sizes and the data are shown in Table 1.

For second experiment, the inclination angle was set at 90°. The whole procedure is the same as the that of the 15° experiment. However, in this case, I tried 5 different opening sizes, shown in Table 2. The reason why the sample size was reduced is that at 90°, the gravitational force experienced by the hydrogels are 4 times bigger than that of the 15°, which means that it is easier for hydrogels to be broken. Although a bigger sample size would be better, it was not very practical during the process.

Chapter 4: Results

The probability curve for clogging were plotted after all data were collected. The results are shown below:

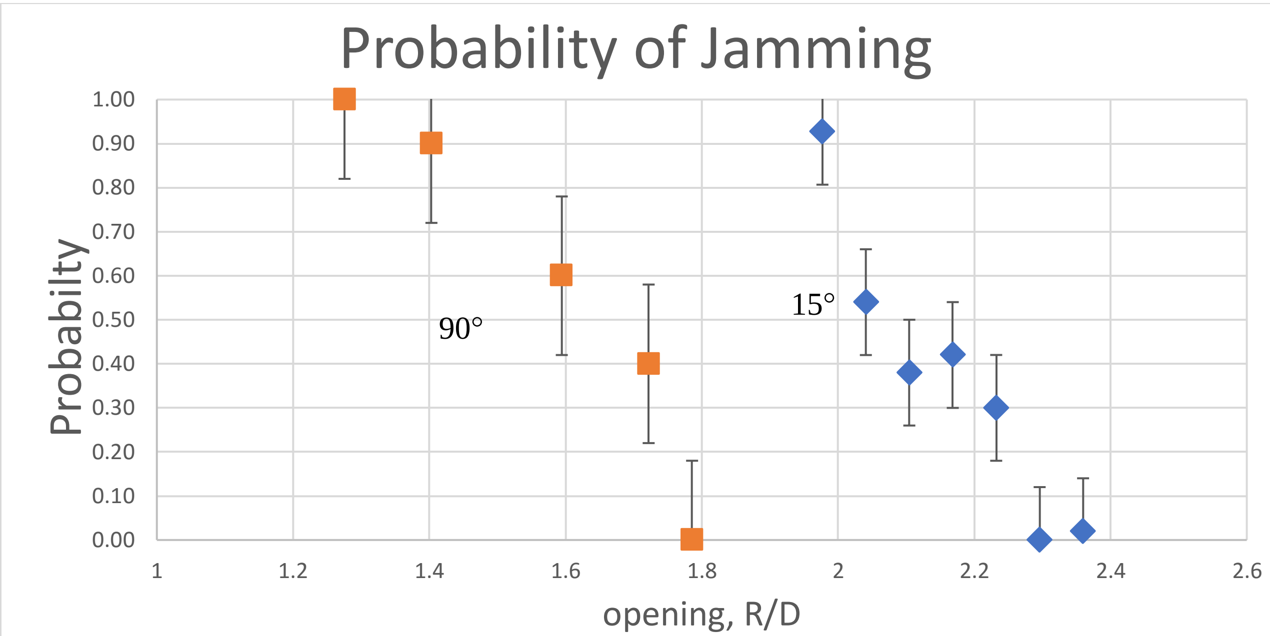

Figure 9, the probability curves for system at 90° and 15°. The blue diamond curve is the probability curve for the data at 15°. The orange square curve is the probability curve for the data at 90°. The curve with higher gravitational force shifts left. The error bar indicates the uncertainty for two sets of data. The error bar for 90° data is larger than that for 15° one since the number of trial per opening for the 90°'s are fewer than that for the 15°'s. There were 50 trials per opening for the 15° and 10 trials per opening for the 90° since the hydrogels broke too often that 50 trials per round was not practical.

width of the exit (cm)	opening, w/d	P_clogging
3.7	2.36	0.02
3.6	2.30	0.00
3.5	2.23	0.30
3.4	2.17	0.42
3.3	2.10	0.38
3.2	2.04	0.54
3.1	1.98	0.93

Table 1, the probability of clogging for different opening at 15°. w is the width of the exit. d is the average diameter.

width of the exit (cm)	opening, w/d	P_clogging
2.8	1.79	0
2.7	1.72	0.4
2.5	1.59	0.6
2.2	1.40	0.9
2	1.28	1

Table 2, the probability of clogging for different opening at 90°. w is the width of the exit and d is the average diameter. Each opening was test 10 times because of the fragility of hydrogels.

Figure 10, a clogged system at 15° inclination angle, with an opening w/d = 2.23

Figure 11, a clogged system at 90° inclination angle, with an opening w/d = 1.59

Fig. 10 and Fig. 11 show two typical clogging for both experiment setups. As shown in the figure, both systems have a three-particle arch near the exit of the hopper. The hydrogels left in Fig. 10 is much more than those in Fig. 11. According to Fig. 9, we can easily see, except the noise around 2.2, both curves share the same trend that while the opening increases, the probability of clogging decreases, which match the experiments did by To (To *et al*, 2001) and by Zuriguel (Zuriguel *et al*, 2014). As mentioned in the previous section, the gravitational force experienced by the particles when the inclination angle was 90° is around four times that when set at 15°. The difference between how particles react to different gravitational force is demonstrated in Fig. 12 below:

Figure 12, the two close-ups at the arches at different inclination angles. The figure on the left is the system at 90° and the figure on the right is the system at 15°. It is clear to see that the particles on the left figure, especially the one at the bottom right, deformed much severely then the one on the right figure at the same position.

And the curve for 90° is at the left side of the plot, which means it is easier for the system to be clogged at 15° rather than 90°. The conclusion, based on the data, is that the driven force exerted on a soft particle 2D system will decrease the clogging probability. The higher the driven force is, the less likely a system will be clogged. It is exact the opposite to the hard particle system, where a higher driven force will cause a higher clogging probability.

Chapter 5: Summary and Discussions

In our experiment, we partially reproduced the experiment To (To *et al*, 2001) did but using a soft hydrogels rather than hard metal disks. Moreover, inspired by Zuriguel's research (Zuriguel *et al*, 2014), we introduced the gravity as a parameter of the driven force exerted on the system. We put 200 hydrogels, with average diameter 1.57 cm, into the hopper and repeated the experiment 50 times for each given opening. The system was set at two different inclination angles, 15° and 90°, where the gravitational force exerted on the particles at 90° is approximately 4 times larger than that at 15°. We found out the probability curve of 90° system shifts to the left comparing to the curve at 15°, which indicates that the higher driven force will result in a system which is less likely to be clogged. On the other hand, the conclusion drawn by Zuriguel's experiments shows that for hard particles, the higher driven force will result in a system which is more likely to be clogged. Therefore, we found out the reaction of a soft particle system to the driven force is the opposite to that of a hard particle system.

There are three problems appeared while doing the experiments. The first and the most severe problem is the collection mechanism of the hopper sometimes blocked the flow of the hydrogels. This problem happened much often when the gravitational force was small. Since the gravitational force was small, while the hydrogels were caught by the "mountain", the particles would not roll down the hill; rather, they formed a relatively stable cluster and supported all the particles up in the hopper. This would have certain influence for the flow of particles and the system might be more probable to be clogged under this circumstance. The solution of this problem will be separate the collection mechanism and the hopper, making an independent collector. The second problem is that the reservoir is not big enough. Although 200 particles could fit in the top part during the experiment, when I flipped the plate, not all particles fell right in the hopper. Some of them fell through the side channels. If the width of the reservoir gets reduced and the height gets increased, the problem could be solved. The third problem is the residue of broken hydrogels. While doing the experiment, some of the particles would be broken due to the impact when hit the collector or hit the top reservoir. The small pieces of broken particles might affect the flow or the formation of potential arch. The solution could be that we can make the system open by separating the collector from the hopper. In that case, we can use sieve to select the good ones and get rid of the broken pieces.

Overall, the experiments show an interesting result that when dealing with clogging problem of a 2D system, the soft particles act much differently from the hard particles. This is a good indication for future study on this system that people should consider designing different procedures for making a more comprehensive investigation. And the most important and argent thing to understand is where the difference comes from. As people found out the critical region of clogging, there might be a critical region of softness that when the softness of a material come cross the region, the transition of behavior happens. By figuring out the details behind this phenomenon, we can have a much better understanding of the 2D granular system.

Bibliography

1: D. J. Durian. Foam Mechanics at the Bubble Scale. *Phys. Rev. Lett.*, **75**, 4780—4783 (1995).

2: D. Genovese and J. Sprakel. Crystallization and intermittent dynamics in constricted microfluidi flows of dense suspensions. *Soft Matter*, **7**, 3889-3896 (2011).

3: Zuriguel, *et al*. Clogging transition of many-particle systems flowing through bottlenecks. *Scientific Reports*, **4**, 7324 (2014).

4: K. To, P. Lai and H. K. Pak. Jamming of Granular Flow in a Two-Dimensional Hopper. *Phys. Rev. Lett.*, **86**, 71--74 (2001).

5: A. Clauset, C. R. Shalizi and M. E. J. Newman. Power-law distributions in empirical data. *SIAM Review*, **51(4)**, 661-703 (2009).

6: X. Hong, M. Kohne, and E. R. Weeks. Clogging of soft particles in 2D hoppers. *ArXiv: 11512.02500* (2015).