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Multi-factor Loading Uncertainty and Expected Returns

By

Haosi Shen

Jeong Ho (John) Kim

Adviser

Economics

Kaiji Chen

Committee Member

Caroline Fohlin

Committee Member

Juan F. Rubio-Ramirez

Committee Member

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An abstract of
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of Emory University in partial fulfillment
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Bachelor of Arts with Honors

Economics

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Abstract

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β is notoriously difficult to measure, and the direction of the relationship between beta uncertainties and expected returns is *a priori* not obvious. This paper demonstrates that stocks with high factor-loading uncertainty exhibit significant underperformance compared to those with lower factor-loading uncertainty, and systematic risk factors in the canonical multi-factor models cannot account for this negative premium. Our findings show that uncertainty surrounding non-market factor loadings have significantly larger impact than the market beta uncertainty, and additionally, the former subsumes the latter. Investors' uncertainty on the CAPM market beta alone does not have significant explanatory power over expected returns. Additionally, our results indicate that the pricing implications of factor-loading uncertainties are driven by large-cap stocks.

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Contents

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April 2023

1 Introduction

The empirical asset pricing literature suggests a "*zoo*" of risk factors (i.e., sources of non-diversifiable systematic risk), but the examination on investors' learning is comparatively lacking.[1] In turn, expected returns on stocks are modeled as linear functions of these risk factors, and the coefficients on these risk factors are commonly referred to as the "betas". However, most neoclassical pricing frameworks such as the Capital Asset Pricing Model (CAPM) only provide theoretical guidance and do not specify approaches to estimate the proposed factor loadings. Hence, when investors employ the factor pricing models, discrepancies in aspects including the choice of historical data, estimation methods, investment horizons would result in inconsistent estimations of the risk sensitivity measures. Furthermore, β is notoriously difficult to measure. As documented in Vasicek (1973) and Welch (2021), the utilization of historical data to estimate betas may result in an inaccurate assessment of the level of systematic risk bearing in the future. Based on existing studies, the direction of the relationship between beta uncertainties and expected returns is *a priori* not

[1] For example, see the factor zoo in Cochrane (2011).

obvious. Specifically, Armstrong et al. (2013) and Hollstein et al. (2020) examine the asset pricing implications of investor uncertainty on the *CAPM market beta*, and both document a negative premium for the parameter estimation uncertainty. Albeit as the bedrock of factor models of risk-and-return, the CAPM is not sufficient for explaining the cross-section of stock returns. (Barillas et al. (2020), Fama and French (2018)) Therefore, this paper studies the effects of multi-factor loading uncertainty on expected returns. In particular, we are examining whether uncertainties surrounding the multi-factor loading yield an economically and statistically significant premium on cross-sectional stock returns.

To distinguish the effects of non-market factor loading uncertainties, we conduct our analysis using four canonical factor pricing models, out of which three are multi-factor models. Each of these multi-factor models integrates the factors from the prior model with a new factor. This enables us to determine if the non-market betas command significant risk premia themselves, and if they substitute for the risk premium on market loading uncertainty. First, we consider the univariate CAPM to replicate the main findings in Armstrong et al. (2013). Second, we employ the Fama and French (1993) three-factor model incorporating the size and book-to-market factors. Third, we evaluate the Carhart (1997) four-factor model including momentum. Lastly, we utilize a five-factor model incorporating the tradable liquidity factor from Pastor and Stambaugh (2003).

We begin our empirical analysis by Fama-MacBeth two-step regressions, which allows us to directly examine the explanatory power of multi-factor loading uncertainties on the cross sections of stock returns. We dissect the panel data into its time-series and cross-sectional dimensions consecutively. In the first stage, we use firm-specific rolling OLS regressions with 60-month rolling windows at each cross section to obtain the factor-loading estimates and proxies for beta uncertainties. For each stock, we estimate a regression on log excess returns based on the four pricing models respectively. To proxy the uncertainty surrounding each factor beta, we use the squared standard error of the estimated factor-loading. In the second stage, we estimate cross-sectional regressions of log excess returns using the estimated beta uncertainty measure, while explicitly controlling for firm-level characteristics and levels of factor-loadings. Then, we compute the the time-series average of the coefficient estimates.

In contrast to Armstrong et al. (2013), we find that investors' uncertainty on the CAPM market beta *alone* does not have significant explanatory power over expected returns. In the multi-factor models, the market beta uncertainty is generally insignificant as well, suggesting that the risk premium on the market beta uncertainty may be subsumed by multi-factor loading uncertainty. On the other hand, most non-market factors including size, book-to-market, and momentum consistently command significant risk premium in the cross-section of stock returns. Importantly, we observe that directions of the relationships between beta uncertainties and expected returns do not agree among different factors. In particular, uncertainties surround the size and momentum factor loadings are negatively related to expected returns, whereas the book-to-market beta uncertainty yields a positive premium[2]. To the extent that most of the effects were negative, we conjecture that the dominant force is negative. Hence, in order to obtain a coherent asset pricing implication, we propose combining the asset pricing implications across factors by evaluating the performance of Long-Short portfolios that are sorted by multi-factor loading uncertainties. Additionally, the coefficients on liquidity factor loading and its uncertainty are never significant, albeit consistently negative. Therefore, we focus on risk factors in the 4-factor model for capturing multi-factor loading uncertainty in portfolio sorts.

Next, we use portfolio sorting to pool the asset pricing information contained in beta variability across factors in the Carhart (1997) four-factor model. This in turn allows us to construct a trading strategy based on the resulting insights. To further distinguish the effects of market and non-market multi-factors, we are conducting two groups of sorts. First, we perform univariate sorts based on the CAPM beta uncertainty as an alternative approach to replicating the findings in Armstrong et al. (2013). Subsequently, we employ a method

[2]A positive sign is consistent with the assumption of ambiguity-averse investors. Under ambiguity aversion, investors have a preference for lower uncertainty and would require a premium for greater uncertainty implied by parameter uncertainty.

similar to Daniel et al. (1997) and Ang et al. (2006a) to implement triple sorts based on multi-factor loading uncertainty. Our approach uses a dependent sort, by which the stock universe is sorted into quintiles according to their factor-loading uncertainty in each of the three layers — corresponding to the size, book-to-market, and momentum betas respectively. In total, we create 125 portfolios that collectively account for factor-loading uncertainties with respect to non-market factors. To avoid bias introduced by small-cap or large-cap stocks, we compute both the equal-weighted and value-weighted returns for each of the single-sorted and triple-sorted portfolios. For univariate sorts, we construct a long-short portfolio that goes long on the highest market beta uncertainty quintile and short on the lowest quintile. In the case of triple-sorts, we consider the long-short portfolio that goes long the 555 portfolio, consisting of stocks whose factor-loading uncertainties are uniformly maximally uncertain, and goes short with the 111 portfolio, comprising stocks whose factor-loading uncertainties are uniformly minimally uncertain. For each long-short portfolio, we test if its alpha in terms of each benchmark pricing model is significantly different from null.

Our findings indicate that greater uncertainty in factor loadings is indeed associated with lower alpha. Specifically, both the univariate and triple sorted long-short portfolios consistently yield negative alphas. However, the hedge portfolio based on multi-factor loading uncertainty yields larger magnitude alphas, providing valuable insights in terms of investment strategy. In general, the value-weighted portfolios yield statistically significant results, indicating that the pricing implications of factor-loading uncertainties are primarily driven by the risk exposures of large-cap stocks. Institutional investors have a bigger presence in the market for large-cap stocks, as well as greater sophistication and access to expertise and resources. As a result, they are more inclined to take into account the effects of factor-loading uncertainty in making investment decisions. Moreover, we observe that the highest quintile portfolios — i.e. stocks with the highest factor loading uncertainties — largely account for the negative premia of the long-short portfolios.

After establishing the adverse implications of factor loading uncertainties on expected returns, we proceed to examine if the non-market factor loading uncertainties subsume the CAPM beta uncertainty. To investigate this question, we construct portfolios based on quadruple sorts, where the fourth sort represents the market beta uncertainty. This approach allows us to evaluate the pricing effects of the CAPM beta uncertainty while explicitly accounting for the impacts of multi-factor loading uncertainties. Our results suggest that controlling for the size, book-to-market, and momentum factors, the market beta uncertainty is no longer significant, and thus we infer that the non-market factor loading uncertainties incorporate the effects of the CAPM beta uncertainty.

Overall, our study finds that stocks with high factor-loading uncertainty significantly underperform those with lower factor-loading uncertainty, and systematic risk factors in the canonical multi-factor models cannot account for this negative premium. We propose a mispricing effect based on heterogeneous beliefs and short-selling constraints that explains the observed premium. In specific, stocks with higher factor-loading uncertainty are subject to overpricing, in turn delivering lower returns at the following time period. The first potential mechanism is that higher factor-loading uncertainty may proxy for higher belief disagreement across investors. In consistence with Miller (1977), optimistic investors are free to buy the asset at equilibrium, whereas pessimistic investors cannot freely short if short-selling constraints are binding. The pessimistic opinion is thus only partially incorporated into the asset price, thereby creating a bubble. Furthermore, our results are consistent with Hollstein et al. (2020), which demonstrate that the negative premiums resulting from factor-loading uncertainty are primarily influenced by stocks with the highest levels of factor loading uncertainties. Hollstein et al. (2020) indicate that stocks with higher beta uncertainty are typically small, illiquid, and possess higher idiosyncratic volatility, which in turn is associated with limits to arbitrage and high short-selling costs.

The subsequent sections of this paper is structured as follows. In Section 2, we describe the data and estimation methodologies used in our analysis, as well as summary statistics on the key variables. Section 3 presents and discusses our empirical results on the asset pricing implications of multi-factor loading uncertainty. In particular, Subsections 3.1 and 3.2 examine the effect of factor-loading uncertainties on cross-sectional stock returns. Subsections 3.3 and 3.4 analyze the joint effects of multi-factor loading uncertainties through results on portfolio sorts. Section 4 concludes and communicates directions for future work.

2 Data and Methodology

2.1 Data

This study obtains stock return data from the Center for Research in Security Prices (CRSP). Our analysis considers all CRSP firms incorporated in the U.S. and listed on the New York Stock Exchange (NYSE), NYSE American (AMEX, previously known as American Stock Exchange) and National Association of Securities Dealers Automated Quotations (NASDAQ) that are ordinary common shares (with CRSP share codes 10 or 11).

We extract our balance sheet and income statement data from the Standard & Poor's Compustat database. These data are used to estimate stock characteristics that are shown to have explanatory power over cross-sectional stock returns (See Armstrong et al. (2013); Hollstein et al. (2020)), namely the lagged market value of equity (size), lagged turnover, book-to-market ratio, debt-to-equity ratio, return on assets, operating accruals, bid-ask spread, and idiosyncratic volatility. We follow Freyberger et al. (2020) in computing the proxies for firm characteristics, as described in the Appendix (5).

In addition, data on the one-month Treasury Bill rate (risk-free rate), the Fama and French (1993) factors (size and book-to-market), and the Carhart (1997) momentum factor come from Kenneth R. French's data library. Data on the Pastor and Stambaugh (2003) liquidity factor is acquired from Robert Stambaugh's home page. We use the value-weighted return of all NYSE, AMEX, and NASDAQ firms that are classified as ordinary common shares as the proxy for market return.

Our study is based on monthly data from January 1995 to December 2018. To account for the timing when factor pricing became widely utilized among investors, we conduct our empirical analysis on the post-1995 sample period. Fama and French (1993) showed that the size and value factors explain a significant amount of return not captured by the CAPM. Prior returns, or momentum, was initially researched in Jegadeesh and Titman (1993) and was first used along with the Fama-French factors to price mutual fund returns in Carhart (1997). The role of liquidity in asset pricing also dates back to around 1990, when Amihud and Mendelson (1986) relates trading costs to stock returns, and became widely recognized by investors after Pastor and Stambaugh (2003).

2.2 Estimation Methodology

2.2.1 Benchmark Pricing Models

In this study, we investigate the implications of factor-loading uncertainties on expected returns with respect to four benchmark pricing models. First, to replicate the findings of Armstrong et al. (2013), we use the log CAPM as our first benchmark model. In this model, the aggregate factor-loading uncertainty is simply the variance around the beta for excess market return. On top of the univariate model, our analysis is benchmarked against the Fama and French (1993) three-factor model, the Carhart (1997) four-factor model, and a variant five-factor model based on Pastor and Stambaugh (2003).

Through comparing results among various multi-factor pricing models, we aim to (i) test if the negative premium of factor-loading uncertainty applies to risk factors other than the market return, (ii) find the optimal model for generalizing the effect of multi-factor loading uncertainty, and (iii) study the covariance effects of beta uncertainty between different risk factors.

2.2.2 Fama-MacBeth Two-Step Regressions

To estimate the empirical implications factor-loading uncertainty on expected returns, we perform a two-stage regression analysis following the approaches in Fama and Macbeth (1973), where the two stages account for the time-series and cross-section dimensions of the panel data respectively. In particular, we are studying if uncertainties around multi-factor loadings have explanatory power in the cross section of stock returns, after controlling for stock characteristics and the average level of factor loadings.

First, we estimate firm-specific rolling OLS regressions to obtain factor-loading estimates and uncertainties. One advantage of this procedure is its ability to account for time-series variations in the model parameters. The regressions use a 60-months rolling window, corresponding to a 5-year subsample period at each point in time. For each stock i, we are regressing its log excess return on the factors in our four benchmarking pricing models, from now on referred as $CAPM$, $FF3$, $FFC4$, and $PS5$ respectively:

$$\begin{cases} r_{i,t+1} - r_{f,t} = \alpha_{i,t} + \beta_{i,MKT}(r_{m,t+1} - r_{f,t}) + \epsilon_{i,t+1} \\ r_{i,t+1} - r_{f,t} = \alpha_{i,t} + \beta_{i,MKT}(r_{m,t+1} - r_{f,t}) + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \epsilon_{i,t+1} \\ r_{i,t+1} - r_{f,t} = \alpha_{i,t} + \beta_{i,MKT}(r_{m,t+1} - r_{f,t}) + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \\ \qquad\qquad\qquad \beta_{i,UMD}UMD_t + \epsilon_{i,t+1} \\ r_{i,t+1} - r_{f,t} = \alpha_{i,t} + \beta_{i,MKT}(r_{m,t+1} - r_{f,t}) + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \\ \qquad\qquad\qquad \beta_{i,UMD}UMD_t + \beta_{i,LIQ}LIQ_t + \epsilon_{i,t+1} \end{cases} \tag{1}$$

where $r_{i,t+1}$ and $r_{m,t+1}$ are the log return of stock i and the market, $r_{f,t}$ is the log risk-free (one-month Treasury Bill) rate, and SMB_t, HML_t, UMD_t, LIQ_t are the returns of the Size, Book-to-Market, Momentum, and Liquidity portfolios. Our measure for each factor's beta uncertainty is the squared standard error of the estimated factor-loading, denoted by $V_{\beta,i}$.

$$V_{\beta,i} = (\sigma_{\beta,i})^2 \tag{2}$$

For the second stage, we run T cross-sectional regressions of log excess returns using our estimated $\beta_{i,t}$ and $V_{\beta,i}$ for each factor, the convexity adjustment term $\beta_{i,t}^2$ to account for our log transformation, and control variables on stock characteristics as the explanatory variables, where T denotes the number of cross-sections in our monthly sample from January 1995 to December 2018. For each $t \in [t_0, t_T]$, we estimate the following regressions with respect to each benchmark pricing model:

$$r_{i,t+1} - r_{f,t} = \alpha_{i,t} + \lambda_t \cdot \tilde{\beta}_{i,t} + \gamma_t \cdot V_{\tilde{\beta},i,t} + \varphi_t \cdot \tilde{\beta}_{i,t}^2 + controls_{i,t} + \varepsilon_{i,t+1} \tag{3}$$

where $\tilde{\beta}$ for the $CAPM$, $FF3$, $FFC4$, and $PS5$ models respectively are:

$$\begin{cases} \tilde{\beta}_{CAPM} \equiv [\beta_{MKT}]^\top \\ \tilde{\beta}_{FF3} \equiv [\beta_{MKT}, \beta_{SMB}, \beta_{HML}]^\top \\ \tilde{\beta}_{FFC4} \equiv [\beta_{MKT}, \beta_{SMB}, \beta_{HML}, \beta_{UMD}]^\top \\ \tilde{\beta}_{PS5} \equiv [\beta_{MKT}, \beta_{SMB}, \beta_{HML}, \beta_{UMD}, \beta_{LIQ}]^\top \end{cases} \tag{4}$$

The controls consist of proxies for stock characteristics that have been documented to have explanatory power over returns[3], including the lagged market value of equity, lagged turnover, book-to-market ratio, debt-to-equity ratio, return on assets, operating accruals, bid-ask spread, and idiosyncratic volatility (denoted by *lme*, *lturnover*, *BEME*, *Debt2P*, *ROA*, *OA*, *spread*, and *Idio_Vol* respectively). Details on the variable definitions are presented in the Appendix (5).

Then, we compute the time-series average of the coefficient estimates in Equation 3 and evaluate their statistical significance using the Newey-West standard errors with 60 lags. Our model is adjusted to improve statistical robustness in the following aspects. The t-statistics reported using Newey-West standard errors are corrected for heteroskedasticity and autocorrelations in the panel data. To mitigate the effects of abnormal extreme values,

[3]See Armstrong et al. (2013) and Hollstein et al. (2020).

we perform winsorization on all independent variables at the 1% and 99% percentiles[4]. Furthermore, we restrict our analysis to firms that have more than 60 observations to ensure sufficient degrees of freedom.

2.2.3 Portfolio Sorts

To identify the relationship between factor-loading uncertainties and pricing anomalies in stock returns, we employ portfolio sorts to visualize how expected returns vary across different levels of beta variability. At the end of each month t, we sort the stocks based on their beta uncertainties regarding each risk factor in ascending order. For each factor, we sort the stocks into five quintile portfolios, in which the first quintile represents stocks with the lowest factor-loading uncertainty and the fifth quintile captures stocks with the highest factor-loading uncertainty. We are focusing on the behaviors of returns on the long-short portfolios (denoted by $High - Low$) which mimic the investment outcomes of long-short positions at the extreme quintiles. Moreover, Fama and French (2008) mention that equal-weighted hedge portfolios may yield biased pictures as "microcaps" account for more than half of the total number of stocks, yet they make up only a tiny portion of the NYSE, AMEX, and NASDAQ stock universe. Hence, we analyze the performance of both equal-weighted and value-weighted portfolios to avoid either the small or big stocks dominating the results.

We are performing two groups of portfolio sorts. First, we conduct univariate sorts according to the CAPM beta uncertainty as an alternative approach to replicate the findings in Armstrong et al. (2013). Second, we apply a triple-sorts approach based on factor loading uncertainties of non-market factors in the $FFC4$ model, namely size (SMB), book-to-market (HML), and momentum (UMD). We use the $FFC4$ factors instead of $PS5$ since estimates and uncertainties of the liquidity exposure are not statistically significant from our

[4]Winsorization refers to the transformation of statistics by setting values of all extreme outliers to a specified percentile of the data. In our example, we are replacing all observations below the 1st percentile to the value at the 1st percentile, and all data above the 99th percentile to the 99th percentile. Our parameter estimates obtained based on the winsorized data is more robust to the effect of extreme values in comparison to the standard approach.

Fama-MacBeth regressions (See Section 3.2). Our procedure of multi-leveled sorts refers to the construction of characteristic-sorted portfolios in Daniel et al. (1997) and the double-sort methodology of coskewness and downside beta portfolios in Ang et al. (2006a). To reduce the risk of getting false-positive errors, we need to ensure sufficient sample sizes and reasonable diversification in the hedge portfolios. Therefore, we are using dependent sorts described as follow.[5] To jointly evaluate the effects of factor-loading uncertainties in the three cross-sectional factors, we constructed $5 \times 5 \times 5$ quintile portfolios based on a triple-sort on each stock's size, book-to-market, and momentum factors' exposure uncertainty. At each month t, we first sort the stock universe into five quintiles based on their size beta uncertainty (i.e. $V_{\beta_{SMB,i}}$). Then, within each size beta uncertainty quintile, the stocks are further sorted into quintiles based on their book-to-market beta uncertainties (i.e. $V_{\beta_{HML,i}}$). Finally, within each of the 25 SMB/HML beta uncertainty portfolio, we sort the firms into quintiles based on their momentum beta uncertainties (i.e. $V_{\beta_{UMD,i}}$), forming a total of 125 portfolios on multi-factor loading uncertainties.

For each of the 5 single-sorted portfolios and the 125 triple-sorted portfolios, we compute the portfolio returns[6] with respect to each benchmark pricing model[7] by value-weighting and equal-weighting the stocks respectively. In univariate sorts, we consider the long-short portfolio that goes long with the highest $V_{\beta_{MKT}}$ quintile and goes short with the lowest $V_{\beta_{MKT}}$ quintile. Regarding triple sorts, we consider the long-short portfolio that goes long with the 555 portfolio and goes short with the 111 portfolio.[8] For each long-short portfolio, we are

[5] The counterpart of dependent sort is independent sort, by which each firm in the stock universe would be sorted based on their beta uncertainty with respect to each risk factor. Independent sorts would be ideal if we can ensure a sufficient number of stocks that satisfy the sorting criteria of each benchmark portfolio.

[6] When evaluating portfolio performance, "returns" - or alpha - refers to excess returns earned by the portfolio investment relative to the benchmark required return. See Jensen (1968). Here, we are computing the average return from t to $t + 1$.

[7] For each benchmark pricing model (namely $CAPM$, $FF3$, $FFC4$, and $PS5$), the required return is different depending on the risk factors. The benchmark required return is the return expected to be earned based on the amount of (systematic) risk borne by the investor, i.e. captured by the portfolio's β's.

[8] The 555 portfolio denotes stocks whose risk exposures with respect to non-market factors are uniformly maximally uncertain; the 111 portfolio comprises stocks whose risk exposures w.r.t. non-market factors are uniformly minimally uncertain.

testing the hypothesis

$$\begin{cases} H_0 : \alpha_{LS} = 0 \\ H_a : \alpha_{LS} \neq 0 \end{cases} \tag{5}$$

with respect to each benchmark pricing model.

2.2.4 Quadruple Sorts

In the last stage of our analysis, we measure the effect of CAPM beta uncertainty on cross-sectional expected returns while explicitly controlling for beta uncertainties in the multi-factors. The portfolio construction procedure follows our multi-level sorting methodology in Section 2.2.3. Using our 125 benchmark portfolios sorted based on the size, book-to-market, and momentum factors' loading uncertainty, we perform a quadruple-sort. At each month t, within each triple-sorted portfolio, we further sort the stocks into five quintiles based on their $V_{\beta_{MKT}}$, creating a total of $5^4 = 625$ portfolios.

According to results from our triple-sorts, the value-weighted long-short portfolios generally yield statistically significant excess returns, whereas alphas of the equal-weighted hedge portfolios are broadly insignificant. Therefore, in quadruple sorts, we focus on evaluating returns of the value-weighted portfolio sorted based on the CAPM beta uncertainty, controlling for the level of factor-loading uncertainties in the size, book-to-market, and momentum factors.

2.3 Summary Statistics

Table 1 presents the summary statistics for our monthly sample considering all CRSP stocks listed on the NYSE, AMEX, and NASDAQ that are ordinary common shares. Our sample period is from January 1995 to December 2018, but we are testing our analysis for robustness using an extended sample from July 1962 to December 2018. We restrain our analysis to firms with at least 60 monthly observations, and we perform winsorization on all variables at the 1st and 99th percentiles to restrict perturbations introduced by abnormal extreme values. Panel A reports summary statistics on the returns and firm-level characteristics of all stocks in our analysis. Panel B provides statistics for the factor-loading estimates in our four benchmark pricing models. In Panel C, which summarizes the factor-loading uncertainties with respect to each model, we observe a significant degree of cross-sectional variation in the estimated beta uncertainties.

Table 1
Summary Statistics

Panel A: Stock Returns and Characteristics

	# of obs.	Mean	Std. Dev.	Min	Percentiles 5%	Percentiles 50%	Percentiles 95%	Max
$r_{i,t+1} - r_{f,t}$	886,377	0.00855	0.14212	-0.68334	-0.20724	0.00272	0.23954	1.67156
lme	886,377	3.50E+06	1.20E+07	1.64E+03	1.10E+04	3.42E+05	1.61E+07	2.07E+08
lturnover	886,377	0.13741	0.16055	0.00111	0.00832	0.08535	0.45327	1.65253
BEME	886,377	0.73685	0.68019	0.02520	0.12282	0.57190	1.90736	9.15770
Debt2P	886,377	0.64316	1.38256	0.00000	0.00000	0.21363	2.64830	26.80141
ROA	886,377	0.00979	0.15024	-1.06237	-0.27727	0.02800	0.17775	0.36089
OA	886,377	-0.03511	0.10832	-0.75459	-0.19389	-0.03516	0.12423	0.73427
spread	886,377	0.01892	0.03105	-0.00065	0.00027	0.00695	0.07650	0.33147
Idio_Vol	886,377	0.02785	0.02295	0.00311	0.00722	0.02081	0.07346	0.25181

Panel B: Estimated Factor Loadings

	# of obs.	Mean	Std. Dev.	Min	Percentiles 5%	Percentiles 50%	Percentiles 95%	Max
$\beta_{i,CAPM,MKT}$	886,377	1.06004	0.76164	-1.61653	0.05335	0.95175	2.50601	4.94080
$\beta_{i,FF3,MKT}$	886,377	0.99010	0.68656	-1.33119	0.02408	0.91986	2.23845	4.29619
$\beta_{i,FF3,SMB}$	886,377	0.73203	0.91331	-2.20885	-0.50250	0.59705	2.44312	5.51917
$\beta_{i,FF3,HML}$	886,377	0.28272	0.99867	-4.45347	-1.44202	0.31946	1.85529	4.47728
$\beta_{i,FFC4,MKT}$	886,377	0.94499	0.67517	-1.43563	-0.04445	0.89164	2.15075	3.78715
$\beta_{i,FFC4,SMB}$	886,377	0.74956	0.92891	-2.20036	-0.50987	0.61541	2.49814	5.56200
$\beta_{i,FFC4,HML}$	886,377	0.23611	1.00930	-4.18226	-1.53139	0.28613	1.79673	4.59196
$\beta_{i,FFC4,UMD}$	886,377	-0.15337	0.64017	-3.42795	-1.30939	-0.09047	0.76579	2.46012
$\beta_{i,PS5,MKT}$	886,377	0.93905	0.69477	-1.79062	-0.08898	0.88536	2.17505	3.94658
$\beta_{i,PS5,SMB}$	886,377	0.74437	0.93769	-2.18850	-0.52687	0.60855	2.50996	5.48260
$\beta_{i,PS5,HML}$	886,377	0.23759	1.02793	-4.32188	-1.56814	0.28913	1.82037	4.48712
$\beta_{i,PS5,UMD}$	886,377	-0.15710	0.64826	-3.45576	-1.33045	-0.09341	0.76847	2.45681
$\beta_{i,PS5,LIQ}$	886,377	-0.01279	0.62092	-2.31550	-1.06404	-0.01112	1.01980	2.44379

Panel C: Estimated Factor-Loading Uncertainties

	# of obs.	Mean	Std. Dev.	Min	Percentiles 5%	Percentiles 50%	Percentiles 95%	Max
$V_{\beta,CAPM,MKT,\,i}$	886,377	0.23339	0.33420	0.00810	0.02402	0.12053	0.81988	4.40498
$V_{\beta,FF3,MKT,\,i}$	886,377	0.28129	0.39001	0.01165	0.02892	0.14894	0.97549	4.57571
$V_{\beta,FF3,SMB,\,i}$	886,377	0.52635	0.71349	0.01854	0.04865	0.28541	1.81247	6.66395
$V_{\beta,FF3,HML,\,i}$	886,377	0.63911	0.87859	0.02592	0.06373	0.33836	2.23669	9.67263
$V_{\beta,FFC4,MKT,\,i}$	886,377	0.30942	0.42924	0.01206	0.03159	0.16260	1.08057	5.27015
$V_{\beta,FFC4,SMB,\,i}$	886,377	0.53521	0.72345	0.01931	0.04974	0.29127	1.83403	6.76930
$V_{\beta,FFC4,HML,\,i}$	886,377	0.69170	0.95419	0.02856	0.06948	0.36676	2.42185	9.93836
$V_{\beta,FFC4,UMD,\,i}$	886,377	0.30534	0.52181	0.00721	0.02001	0.13251	1.15164	7.15235
$V_{\beta,PS5,MKT,\,i}$	886,377	0.33979	0.46711	0.01245	0.03510	0.18106	1.17526	6.25053
$V_{\beta,PS5,SMB,\,i}$	886,377	0.56696	0.77170	0.02046	0.05304	0.30668	1.94491	7.77895
$V_{\beta,PS5,HML,\,i}$	886,377	0.75283	1.02809	0.03238	0.07602	0.40369	2.60902	10.82512
$V_{\beta,PS5,UMD,\,i}$	886,377	0.31485	0.53335	0.00726	0.02065	0.13764	1.18244	7.27815
$V_{\beta,PS5,LIQ,\,i}$	886,377	0.35603	0.48199	0.01432	0.03796	0.18964	1.22318	5.35774

This table presents the summary statistics for our monthly data over the sample period from January 1995 to December 2018. Panel A reports summary statistics on the returns of all CRSP stocks listed on the NYSE, AMEX, and NASDAQ that are ordinary common shares, as well as the corresponding firm-level characteristics including lagged market value of equity, lagged turnover, book-to-market ratio, debt-to-equity ratio, return on assets, operating accruals, bid-ask spread, and idiosyncratic volatility (denoted by *lme*, *lturnover*, *BEME*, *Debt2P*, *ROA*, *OA*, *spread*, and *Idio_Vol* respectively). Panel B provides summary statistics for estimates of the factor-loadings in the four benchmark pricing models (namely *CAPM*, *FF3*, *FFC4*, and *PS5*). Panel C presents summary statistics for the proxies for factor exposure uncertainties with respect to each benchmark model. All variables are winsorized at the 1st and 99th percentile in each month.

Table 1: Summary Statistics

3 Results

3.1 Revisiting the CAPM Beta Uncertainty Risk Premium

Table 2 presents our results from replicating the empirical findings in Armstrong et al. (2013) using a more recent sample after 1995. For each month t, we estimate the Fama-MacBeth regression in Equation 3 with respect to the univariate CAPM, in which the only risk factor is excess market return. According to Table 2, the coefficient estimates of $V_{\beta_{mkt}}$ in the third, fourth, and fifth columns are neither negative nor statistically significant, regardless if we are controlling for the firm-level characteristics and factor-loadings of the size, book-to-market, and momentum factors.

As addressed in Section 2, we are incorporating a second order term β^2_{MKT} into the cross-sectional regressions for convexity adjustment after log transformation on the excess returns. From the first two columns, it is observed that the convexity adjustment does have consequential effects both in terms of level and statistical significance of the estimated market risk premium. Comparing the last two columns, there is evidence that the firm-level characteristics play an important role in explaining cross-sectional stock returns, since they help improve the average R^2 and yield statistically significant estimates[9].

However, through the third to fifth columns, we observe that both the magnitude and significance of the CAPM factor-loading uncertainty $V_{\beta_{mkt}}$ monotonically decrease as we saturate the regression with more control variables. Moreover, reflecting on the second and third columns, introduction of the market beta uncertainty $V_{\beta_{mkt}}$ does not have material impact on the magnitude and significance of the market risk premium (i.e. β_{mkt}). Given our empirical results in Table 2, we fail to replicate the findings in Armstrong et al. (2013), which shows that firms with high factor-loading uncertainty with regard to the CAPM have lower expected returns cross-sectionally. In the following section, we are extending our

[9] Complete regression results including estimates on all control variables are available upon request.

Table 2

The Cross-Section of Log Excess Returns,
Controlling for CAPM Factor Loadings and Characteristics

Intercept	0.008746	0.008017	0.008071	0.008029	0.005867
	2.74	2.62	2.84	3.00	2.47
$\beta_{i,MKT}$	0.001134	0.002781	0.002319	0.002095	0.003322
	0.63	2.21	2.22	2.04	3.47
$\beta^2_{i,MKT}$		-0.000560	-0.000532	-0.000488	-0.000625
		(0.75)	(1.35)	(1.22)	(1.71)
$V_{\beta\_MKT,\, i}$			0.002473	0.001758	0.000157
			0.67	0.58	0.10
$\beta_{i,SMB}$				-0.000033	-0.000043
				(0.06)	(0.09)
$\beta_{i,HML}$				-0.000149	-0.000131
				(0.27)	(0.29)
$\beta_{i,UMD}$				-0.001010	-0.000266
				(1.15)	(0.44)
Stock Chars.	No	No	No	No	Yes
Mean R-squared	1.86%	2.12%	3.02%	4.01%	6.16%
# of months	288	288	288	288	288
# of firms	9097	9097	9097	9097	9097

This table presents the results from the Fama-MacBeth 2-step regressions with respect to the CAPM, as shown in Equation 2. β and V are the factor-loading estimates and uncertainties obtained from the firm-specific sixty-months rolling window regressions on log excess returns. The regression controls for factor-loadings on the size, book-to-market, and momentum factors (denoted by β^{SMB}, β^{HML}, and β^{UMD} respectively), as well as the firm-level characteristics (reported in *Stock Chars.*). Below the coefficient estimates, the t-statistics are computed using Newey-West standard errors with sixty lags to adjust for heteroskedasticity and autocorrelations. The mean adjusted R-squared's, the number of monthly cross-sections, and the average number of firms at each cross-section in the sample are presented at the bottom.

Table 2: CAPM Beta Uncertainty on the Cross-Section of Log Excess Returns

analysis to a multi-factor pricing scenario, in which we are testing whether there exists a relationship between multi-factor loading uncertainty and expected returns, and whether there is concurrence between the different factors.

3.2 Multi-factor Loading Uncertainty and Expected Returns

Table 3 provides our empirical results on the effects of multi-factor loading uncertainty on the cross-section of log excess returns using the $FF3$, $FFC4$, and $PS5$ benchmark pricing models. At each time t over the 288-month sample period, we estimate the set of Fama-MacBeth regressions in Equation 3 with respect to each multi-factor model. As shown by the statistics in Table 3, the implications of factor-loading uncertainties on expected returns do not appear in uniform directions and degrees of significance when we examine multiple risk factors respectively. Hence, we are generalizing the effect of multi-factor loading uncertainty through modeling layer-sorted portfolios in the ensuing section.

Panel A presents estimation results from the Fama-MacBeth regression using the Fama and French (1993) three-factor model. Our evidence indicates that multi-factor loadings command significant risk premium in the cross-section of stock returns. As in the second and third columns, coefficient estimates for the factor-loading uncertainties of size and book-to-market ($V_{\beta_{smb}}, V_{\beta_{hml}}$) are significant with a 95% confidence interval before and after controlling for stock characteristics. Interestingly, estimates on the CAPM beta uncertainty ($V_{\beta_{mkt}}$) are never significant, suggesting that the risk premium on the market beta uncertainty may be subsumed by multi-factor loading uncertainty. Furthermore, we notice that signs of the coefficient estimates on beta uncertainties are not uniform across factors. In particular, the coefficient on $V_{\beta_{smb}}$ is consistently negative, whereas the coefficient on $V_{\beta_{hml}}$ is consistently positive.The importance of factor-loading uncertainty in pricing returns is confirmed by our results, as we observe an increase in the model's explanatory power according to the coefficient of determination in columns one and two. Comparing the results in columns two and three, there is an increment in the estimates and significance of all three factor-loading uncertainties. Hence, the role of firm-level characteristics as controls is verified as well.

Panel B provides results on the Fama-MacBeth regression estimates benchmarking with

Table 3

The Cross-Section of Log Excess Returns, Controlling for Factor Loadings and Characteristics

This table presents the results from the Fama-MacBeth 2-step regressions in Equation 2 with respect to the *FF3*, *FFC4*, and *PS5* models. β and V are the factor-loading estimates and uncertainties obtained from the firm-specific sixty-months rolling window regressions on log excess returns. The control variables comprise firm-level characteristics (reported in *Stock Chars.*). Below the coefficient estimates, the t-statistics are computed using Newey-West standard errors with sixty lags to adjust for heteroskedasticity and autocorrelations. The mean adjusted R-squared's, the number of monthly cross-sections, and the average number of firms at each cross-section in the sample are presented at the bottom.

Panel A: Benchmarked Against Fama-French 3-Factor Model

Intercept	0.008101	0.008000	0.005882
	2.74	2.71	2.27
$\beta_{i,\,MKT}$	0.002606	0.002361	0.003796
	1.73	1.51	3.00
$\beta_{i,\,SMB}$	0.000669	0.000464	0.000037
	0.67	0.55	0.05
$\beta_{i,\,HML}$	0.000147	0.000358	0.000083
	0.19	0.46	0.12
$V_{\beta\_MKT,\,i}$		-0.021641	-0.044912
		(0.52)	(1.27)
$V_{\beta\_SMB,\,i}$		-0.056580	-0.052813
		(2.27)	(2.61)
$V_{\beta\_HML,\,i}$		0.044568	0.051906
		1.99	3.23
Stock Chars.	No	No	Yes
Convexity Adj.	Yes	Yes	Yes
Mean Adj. R-squared	3.52%	4.22%	6.35%
# of months	288	288	288
# of firms	9097	9097	9097

Panel B: Benchmarked Against Fama-French-Carhart 4-Factor Model

Intercept	0.008059	0.007878	0.005638
	2.77	2.65	2.06
$\beta_{i,\,MKT}$	0.002203	0.002013	0.003567
	1.36	1.20	2.51
$\beta_{i,\,SMB}$	0.000696	0.000532	0.000130
	0.80	0.69	0.19
$\beta_{i,\,HML}$	0.000079	0.000314	0.000099
	0.11	0.42	0.15
$\beta_{i,\,UMD}$	-0.002049	-0.001954	-0.001525
	(1.97)	(1.90)	(1.90)
$V_{\beta\_MKT,\,i}$		0.104316	0.078804
		1.72	1.20
$V_{\beta\_SMB,\,i}$		-0.052362	-0.055925
		(1.16)	(1.30)
$V_{\beta\_HML,\,i}$		0.032189	0.052328
		0.76	1.19
$V_{\beta\_UMD,\,i}$		-0.184766	-0.225396
		(1.90)	(1.98)
Stock Chars.	No	No	Yes
Convexity Adj.	Yes	Yes	Yes
Mean Adj. R-squared	3.88%	4.62%	6.69%
# months	288	288	288
# firms	9097	9097	9097

Panel C: Benchmarked Against Pastor-Stambaugh 5-Factor Model

Intercept	0.007992	0.007920	0.005757
	2.72	2.66	2.13
$\beta_{i,\,MKT}$	0.002179	0.001961	0.003480
	1.42	1.26	2.48
$\beta_{i,\,SMB}$	0.000686	0.000567	0.000184
	0.80	0.76	0.26
$\beta_{i,\,HML}$	0.000153	0.000339	0.000165
	0.21	0.45	0.24
$\beta_{i,\,UMD}$	-0.001840	-0.001749	-0.001460
	(1.87)	(1.84)	(1.91)
$\beta_{i,\,LIQ}$	0.001338	0.001303	0.001280
	0.78	0.78	0.84
$V_{\beta\_MKT,\,i}$		0.260950	0.264896
		1.84	1.63
$V_{\beta\_SMB,\,i}$		-0.109736	-0.125349
		(2.36)	(2.66)
$V_{\beta\_HML,\,i}$		0.125560	0.131377
		2.13	2.19
$V_{\beta\_UMD,\,i}$		-0.130560	-0.190410
		(1.54)	(1.83)
$V_{\beta\_LIQ,\,i}$		-0.112126	-0.078901
		(1.14)	(0.77)
Stock Chars.	No	No	Yes
Convexity Adj.	Yes	Yes	Yes
Mean Adj. R-squared	4.21%	5.02%	7.06%
# months	288	288	288
# firms	9097	9097	9097

Table 3: Multi-Factor Loading Uncertainty on the Cross-Section of Log Excess Returns

the Carhart (1997) four-factor model. After including momentum as the fourth factor, results from the $FF3$ model continue to hold for the SMB, and HML factors in the $FFC4$ model. However, it is important to note that the coefficient on the CAPM beta uncertainty $V_{\beta_{MKT}}$ flips from negative to positive and is insignificant controlling for firm characteristics. This reinforces that the risk premium on the market beta uncertainty may be incorporated by multi-factor loading uncertainty. Another observation is suppression effects of the momentum factor on factor-loading uncertainties of size and book-to-market. In particular, the signs of size and book-to-market factor-loading uncertainties remain negative and positive respectively, yet both estimates lose significance upon the introduction of momentum. In terms of individual risk factor-loadings, excess market return and momentum also have greater marginal effect in comparison to size and book-to-market. One possible interpretation of such phenomena is that the return premia on size and book-to-market are attributable to firm characteristics rather than exposures to pervasive factor risks[10]. Once more, our results show disagreement with Armstrong et al. (2013) on the economic importance of the CAPM factor-loading uncertainty. When controlling for firm characteristics in column three, coefficients on loading uncertainties of all cross-sectional factors (i.e. smb, hml, and umd) increase in terms of magnitude and significance, whereas estimates for the CAPM factor-loading uncertainty becomes insignificant. Lastly, our results indicate that $FFC4$ is a better benchmark pricing model, since $V_{\beta_{umd}}$ consistently yields significantly negative explanatory power on expected returns, and the adjusted R^2 in Panel B is universally improved in comparison to the estimates based on $FF3$.

Panel C presents estimation results from the Fama-MacBeth regression using the five-factor model adapted from Pastor and Stambaugh (2003). Results from the $FFC4$ model mostly hold for the market, size, book-to-market, and momentum factors in the $PS5$ model. As we introduce liquidity as the fifth factor, the model displays improved goodness of fit as shown in

[10]Results in Daniel and Titman (1997) indicate that although stocks with similar sizes and book-to-market ratios covary strongly, it is their tendency to have similar firm-level characteristics rather than systematic factors of distress risk that explains variations in cross-sectional returns. In particular, controlling for stock characteristics, Daniel and Titman (1997) find no discernible relations between expected returns and factor-loadings on the Fama and French (1993) factors.

the mean adjusted R^2s. Moreover, we observe boosted significance in estimates of the factor-loading uncertainties, particularly for size and book-to-market whose effects are suppressed in the context of $FFC4$. Comparing the estimates on $V_{\beta_{mkt}}$ in Panel B and C, coefficients on the CAPM beta uncertainty remain positive under $PS5$, with marginal significance at the 90% confidence interval. Again, our analysis yields contradicting results with evidence presented in Armstrong et al. (2013). Nevertheless, note that both the factor-loading estimate and uncertainty of liquidity exhibit no statistical significance, regardless of inclusion of the control variables on firm characteristics. Therefore, we are not incorporating $V_{\beta_{liq}}$ as a fourth layer when conducting portfolio sorts based on multi-factor loading uncertainties. Instead, the $FFC4$ model is used for capturing multi-factor loading uncertainty hereafter. For the same cause, we are excluding the CAPM beta uncertainty in multi-layered portfolio sorts as $V_{\beta_{mkt}}$ does not display significance under all three models.

3.3 Portfolio Sorts

Table 4 presents our results for portfolio sorts based on the CAPM beta uncertainty and multi-factor loading uncertainty. In general, as indicated by the Newey-West t-statistics of the hedge portfolios, value-weighted portfolios yield statistically significant results, whereas equal-weighted portfolios do not. Additionally, when comparing the value-weighted and equal-weighted returns, we observe that under both univariate and triple-sorts, value-weighted portfolios display more monotonic trends from the lowest to the highest quintile, while equal-weighted portfolios manifest relatively disordered trends across the spectrum of factor-loading uncertainty. Meanwhile, alphas of the value-weighted hedge portfolios monotonically decrease as we append more factors into the benchmark model, whereas the equal-weighted alphas do not reveal noticeable patterns in vertical comparison.

In reference to Fama and French (2008), value-weighted returns are dominated by large-cap stocks, as they account for over 90% of total market capitalization (in the NYSE, AMEX, and NASDAQ stock universe) on average. Thus, we infer that the asset pricing implications

Table 4

Returns of Stocks Sorted By Factor-Loading Uncertainties

Panel A: Portfolio Sorted Based on CAPM Beta Uncertainty

Return (%)	$Low\ V_{CAPM}$	2	3	4	$High\ V_{CAPM}$	$High - Low$	NW t-stat
			Equal-Weighted				
Excess Return	0.8787	0.8799	1.0072	1.2240	1.1086	0.2299	0.47
CAPM Alpha	0.4602	0.3190	0.3192	0.3957	0.1128	-0.3473	-0.85
FF3 Alpha	0.3466	0.1801	0.2015	0.3295	0.1570	-0.1896	-0.67
FF4 Alpha	0.3580	0.2382	0.3353	0.5319	0.4444	0.0864	0.30
PS5 Alpha	0.3498	0.2254	0.3157	0.5143	0.4395	0.0897	0.31
			Value-Weighted				
Excess Return	0.7074	0.7690	0.8560	0.9948	0.5984	-0.1089	-0.24
CAPM Alpha	0.2126	0.0751	0.0007	-0.0304	-0.5683	-0.7809	-2.04
FF3 Alpha	0.1354	0.0185	0.0324	0.0638	-0.4538	-0.5891	-2.42
FF4 Alpha	0.1396	0.0475	0.0844	0.1661	-0.3674	-0.5069	-2.22
PS5 Alpha	0.1360	0.0286	0.0476	0.1615	-0.3631	-0.4991	-2.20

Panel B: Portfolio Triple-Sorted Based on Cross-sectional Factor Loading Uncertainty

Return (%)	$Low\ V_{grp}$	2	3	4	$High\ V_{grp}$	$High - Low$	NW t-stat
			Equal-Weighted				
Excess Return	0.7904	0.7414	0.8686	1.3447	0.3250	-0.4654	-0.75
CAPM Alpha	0.5063	0.1838	0.2069	0.4148	-0.7112	-1.2175	-2.27
FF3 Alpha	0.4225	0.0520	0.1141	0.3394	-0.5962	-1.0187	-2.27
FF4 Alpha	0.3965	0.1256	0.1992	0.5071	-0.1874	-0.5839	-1.10
PS5 Alpha	0.3954	0.1102	0.1893	0.4897	-0.2159	-0.6112	-1.18
			Value-Weighted				
Excess Return	0.7471	0.4475	0.8978	0.2296	-0.4155	-1.1625	-1.83
CAPM Alpha	0.3482	-0.1751	0.0423	-0.8039	-1.5335	-1.8817	-3.02
FF3 Alpha	0.3012	-0.2579	0.0802	-0.7739	-1.4255	-1.7267	-3.05
FF4 Alpha	0.2787	-0.1913	0.0896	-0.6839	-1.1124	-1.3911	-2.01
PS5 Alpha	0.2894	-0.2003	0.0912	-0.7067	-1.0832	-1.3726	-2.04
# of months	288	288	288	288	288	288	288
# of firms	9097	9097	9097	9097	9097	9097	9097

This table presents results of portfolio sorts based on the CAPM beta uncertainty and multi-factor loading uncertainty, as shown in Panel A and B respectively. For each set of portfolio sorts, both equal-weighted and value-weighted returns are provided to avoid either small or big stocks dominating the results. At the end of each month, we sort the stocks based on their beta uncertainties regarding each risk factor in ascending order. For each factor, we sort the stocks into five quintile portfolios, in which the first quintile represents stocks with the lowest factor-loading uncertainty and the fifth quintile captures stocks with the highest factor-loading uncertainty. The hedge portfolios (denoted by "*High - Low*") mimic the investment outcomes of long-short positions at the extreme quintiles. The rightmost column labeled "*NW t-stat*" refers to the t-statistics of excess returns of the hedge portfolios, computed using Newey-West (1987) standard errors with five lags to adjust for heteroskedasticity and autocorrelations. The rows labeled "*Excess Return*" represent the portfolios' realized return less the risk-free rate. "*CAPM Alpha*", "*FF3 Alpha*", "*FFC4 Alpha*", and "*PS5 Alpha*" denote the alphas of the portfolio investment with respect to the CAPM, FF3, FFC4, and PS5 benchmark pricing models.

Table 4: Portfolio Sorts Based on Factor-Loading Uncertainties

of factor-loading uncertainties are driven by risk exposures of the large-cap stocks. Such phenomena may be ascribed to the bigger presence of institutional investors for large-cap stocks[11]. Since institutional investors have access to greater resources and expertise, they tend to exhibit more sophisticated investment behaviors and account for uncertainty from gauging risk exposure.

After controlling for systematic risk factors using the four benchmark models, our results are largely consistent as most portfolios yield negative alphas. Among the value-weighted hedge portfolios, magnitudes of the negative risk premia is substantially larger in portfolios sorted by multi-factor loading uncertainty, which clearly casts insights to investment strategies. Pertinently, similar to the findings of Hollstein et al. (2020) regarding the CAPM beta uncertainty, we observe that the fifth quintile portfolios comprising stocks with the highest factor-loading uncertainties predominantly account for the negative alphas of the long-short portfolios. These results imply that stocks with high factor-loading uncertainty significantly underperform those with lower factor-loading uncertainty, concurring with the arguments in Armstrong et al. (2013). Moreover, the negative correlation between factor-loading uncertainty and expected returns is particularly strong in the multi-factor pricing scenario, and pervasive risk factors in traditional asset pricing models cannot account for this relationship.

To explore the potential mechanisms driving the negative premium resulting from factor-loading uncertainty, we conducted an extensive review of relevant literature on parameter estimation risks. Our findings suggest that a mispricing explanation based on heterogeneous beliefs and short-selling constraints is the most consistent with our results. Specifically, we argue that stocks with higher factor-loading uncertainty tend to be overpriced at the time of factor-loading estimation. As a result, returns in the following period adjust for the mispricing with a negative alpha. In accordance with the opinion model of Miller (1977), we suggest that factor-loading uncertainties can act as proxies for belief disagreement or ambiguity across investors on systematic risk bearing of stocks. At equilibrium, optimistic investors can freely go long with the asset, whereas pessimistic investors are unable to freely

[11] For example, findings of Bushee (2001) suggest that institutional investors that are not subject to strict fiduciary standards exhibit greater focus on the short-term investment horizon, and their portfolio structure tend to overweight near-term earnings and underweight long run values. Such preference may be associated with a preference for more established firms with large market capitalization and higher liquidity.

short if short-selling constraints are binding. By such manner, the pessimistic opinion is only partially incorporated into the asset price, thereby leading to a bubble. However, this hypothesis is in contrast with the risk-return spectrum assuming ambiguity aversion in investors. Moreover, Hollstein et al. (2020) documents that among the stocks sorted by CAPM beta uncertainty, the high quintile generally contains stocks of smaller market-cap, less liquidity, and greater idiosyncratic volatility. These characteristics are likely related to to limits to arbitrage and greater short-selling costs.

3.4 Do Non-Market Factor Loading Uncertainties Subsume the CAPM Beta Uncertainty?

Table 5

Portfolio Sorted Based on CAPM Beta Uncertainty, Controlling for Multi-Factor Loading Uncertainties

Return (%)	$Low\ V_{CAPM}$	2	3	4	$High\ V_{CAPM}$	$High - Low$	NW t-stat
			Value-Weighted				
Excess Return	0.6188	0.8733	0.6946	0.7121	0.7396	0.1207	0.79
CAPM Alpha	0.0139	0.2884	0.0777	0.0446	-0.0136	-0.0275	-0.19
FF3 Alpha	-0.0097	0.2529	0.0513	0.0188	-0.0917	-0.0820	-0.62
FF4 Alpha	0.0263	0.2577	0.0721	0.0262	-0.0602	-0.0865	-0.67
PS5 Alpha	0.0157	0.2526	0.0636	0.0066	-0.0646	-0.0802	-0.61
# of months	288	288	288	288	288	288	288
# of firms	9097	9097	9097	9097	9097	9097	9097

This table presents results of portfolio sorts based on the CAPM beta uncertainty, controlling for multi-factor loading uncertainties. The portfolio returns are computed based on a value-weighted approach, through which stocks with higher market capitalization receive higher weightings. At the end of each month, we triple sort the stocks into 125 portfolios based on their factor-loading uncertainties for size, book-to-market, and momentum in ascending order. Within each portfolio, we then perform a fourth layer sort based on the CAPM beta uncertainties, creating a total of 625 portfolios. For each layer, we sort the stocks into five quintile portfolios, in which the first quintile represents stocks with the lowest factor-loading uncertainty and the fifth quintile captures stocks with the highest factor-loading uncertainty. The hedge portfolios (denoted by "*High - Low*") mimic the investment outcomes of long-short positions at the extreme quintiles. The rightmost column labeled "*NW t-stat*" refers to the t-statistics of excess returns of the hedge portfolios, computed using Newey-West (1987) standard errors with five lags. The rows labeled "*Excess Return*" represent the portfolios' realized return less the risk-free rate. "*CAPM Alpha*", "*FF3 Alpha*", "*FFC4 Alpha*", and "*PS5 Alpha*" denote the alphas of the portfolio investment with respect to the CAPM, FF3, FFC4, and PS5 benchmark pricing models.

Table 5: Portfolio Sorts Based on CAPM Beta Uncertainty, Controlling for Multi-Factors

Table 5 provides the results on our quadruple sorted portfolio, through which we empirically examine the effect of CAPM factor-loading uncertainty on cross-sectional stock returns while explicitly controlling for beta uncertainties in the multi-factors. Since in the previous section (3.3), we observe statistically significant results in the value-weighted portfolios but not the equal-weighted portfolios, we are restricting our analysis to value-weighted portfolios in quadruple sorts. As shown in the last two columns, excluding the effects of multi-factor loading uncertainties, the hedge portfolio with respect to CAPM beta uncertainty no longer yield significant alphas, regardless of the underlying benchmark model. Therefore, we infer that the explanatory power of CAPM beta uncertainty can be accounted for by the multi-factor loading uncertainties, and thus estimation risk of the CAPM beta would not significantly improve our model's predictive power over abnormal stock returns.

4 Conclusion

In conclusion, this paper argues that multi-factor loading uncertainty matters in the cross-section of stock returns. Specifically, we find that stocks with higher factor-loading uncertainty underperform those with lower factor-loading uncertainty, and systematic risk factors in the canonical multi-factor models cannot account for this negative premium. Through Fama-MacBeth regression analysis based on the CAPM and three multi-factor models, we find that directions of the relationships between beta uncertainties and expected returns do not agree among different factors. Moreover, our analysis shows that investors' uncertainty on the CAPM market beta alone does not have significant explanatory power over expected returns.

Using leveled portfolio sorts, we demonstrate that long-short portfolios based on factor-loading uncertainty consistently yield significantly negative alphas. In particular, the pricing implications of factor-loading uncertainties are driven by risk exposures of the large-cap stocks, suggesting a bigger role of institutional investors in accounting for parameter estimation risks. Furthermore, our evidence suggests that uncertainty surrounding non-market factor loading has a significantly larger impact than CAPM beta uncertainty, and that the former subsumes the latter.

For future work, it would be insightful to examine the mechanisms underlying the ambiguity puzzle highlighted by our research. Specifically, it would be interesting to investigate whether factor-loading uncertainty, or more broadly, parameter uncertainty, can serve as a proxy for model uncertainty. If this is the case, higher levels of factor-loading uncertainty would imply greater ambiguity in the amount of systematic risk bearing, which is disliked by ambiguity-averse investors, who would require a positive premium for compensation. However, this stands in contrast to the empirical evidence presented in this paper. In addition, extending our analysis to downside risk could shed further light on the relationship between parameter estimation uncertainty and stock returns. As it has been widely documented that investors are more sensitive to downside risks than to upside gains[12], it would be valuable to investigate whether such asymmetry also exists in the context of factor-loading uncertainty and its associated asset pricing implications. Finally, an additional avenue for future research is to examine the determinants of the negative premium associated with our multi-factor loading uncertainty. For instance, Hollstein et al. (2020) propose that uncertainty surrounding the correlation between asset returns and market returns — rather than uncertainty on stock volatility — is the primary contributing factor to the negative premium associated with their CAPM beta uncertainty.

[12]For example, see Ang et al. (2006a).

5 Appendix: Variable Definitions

The control variables on firm-level characteristics in Section 2.2.2 are computed using monthly CRSP and annual Compustat data. We refer to Freyberger et al. (2020) and Armstrong et al. (2013) in defining the variables. The definitions are as follow:

- **Lagged market value of equity** (Fama and French (1992), "lme"), also known as the size, is the total market capitalization of the previous month. It is defined as price times the shares outstanding ("shrout"). $lme = abs(price) \times shrout$

- **Lagged turnover** ("lturnover") is the ratio of trading volume ("vol") to shares outstanding ("shrout") in the previous month. $lturnover = vol/shrout$

- **Book-to-market ratio** ("BEME") is the ratio of the current book value of equity over market value of equity at the end of the previous fiscal year. The book value of equity is computed as the sum of shareholders' equity, deferred taxes, and investment tax credit less preferred stock.

- **Debt-to-equity ratio** ("Debt2P"), or debt-to-price, is the ratio of long-term debt ("DLTT") and debt in current liabilities ("DLC") to the market capitalization at the end of the previous fiscal year. $Debt2P = \frac{DLTT+DLC}{abs(price)\times shrout}$

- **Return-on-assets** ("ROA") is the ratio of income before extraordinary items to lagged total assets.

- **Operating accruals** ("OA") is the variation in non-cash working capital, excluding depreciation (DP), relative to the previous period's total assets (TA). The difference between non-cash current assets and current liabilities (LCT), debt in current liabilities (DLC), and income taxes payable (TXP) defines non-cash working capital. Non-cash current assets correspond to current assets (ACT) minus cash and short-term investments (CHE). $OA = \frac{(\Delta ACT - \Delta CHE)-(\Delta LCT - \Delta DLC \Delta TXP)-DP}{TA}$

- **Bid-ask spread** ("spread") is the mean daily bid-ask spread for the preceding months.

- **Idiosyncratic volatility** (Ang et al. (2006b), "Idio_Vol") refers to the residual standard deviation obtained from regressing excess returns on the Fama and French (1993) three-factor model. The estimations utilize daily samples covering one month, with a minimum requirement of fifteen observations.

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