Distribution Agreement

In presenting this thesis as a partial fulfillment of the requirements for a degree from Emory University, I hereby grant to Emory University and its agents the non-exclusive license to archive, make accessible, and display my thesis in whole or in part in all forms of media, now or hereafter now, including display on the World Wide Web. I understand that I may select some access restrictions as part of the online submission of this thesis. I retain all ownership rights to the copyright of the thesis. I also retain the right to use in future works (such as articles or books) all or part of this thesis.

Leah Dodell April 17, 2013

Revisiting Biblical Games in a Bayesian Framework

by

Leah Dodell

Sue Mialon
Adviser

Department of Economics

Sue Mialon

Adviser

Shomu Banerjee

Committee Member

Karen Stolley

Committee Member

2013

Revisiting Biblical Games in a Bayesian Framework

By

Leah Dodell

Sue Mialon

Adviser

An abstract of
a thesis submitted to the Faculty of Emory College of Arts and Sciences
of Emory University in partial fulfillment
of the requirements of the degree of
Bachelor of Arts with Honors

Department of Economics

2013

Abstract

Revisiting Biblical Games in a Bayesian Framework
By Leah Dodell

In this paper, I revisit a few of the most debated tales from the Old Testament and model them in the framework of Bayesian games. I model three situations- Jacob's deception of Isaac, G-d's ten plagues, and Abraham's sacrifice- as dynamic games with private information. By solving for the Perfect Bayesian Equilibria that occur in the Torah, I find conditions that must hold for characters to be willing to take the actions that they do. I also examine how characters' actions would have changed if they had held different values. My results shed light on which interpretations of biblical stories hold the most weight when characters maintain consistent beliefs and act upon them in a sequentially rational manner.

Revisiting Biblical Games in a Bayesian Framework

By

Leah Dodell

Sue Mialon

Adviser

A thesis submitted to the Faculty of Emory College of Arts and Sciences
of Emory University in partial fulfillment
of the requirements of the degree of
Bachelor of Arts with Honors

Department of Economics

2013

Acknowledgements

Special thanks to my adviser, Sue Mialon, for her guidance throughout this process and to my committee members, Shomu Banerjee and Karen Stolley, for their support and participation. They have all served as incredible role models during my time at Emory and without their encouragement, this thesis would not have been possible. I would also like to thank my family- Mom, Dad, and Sylvie- for always being there for me with patience and love.

Table of Contents

I. Introduction

In his book *Biblical Games: A Strategic Analysis of Stories in the Old Testament*, Steven J. Brams applies game theory to stories from the Torah in order to justify the rationality behind G-d's and humanity's decisions. He models biblical scenarios as normal form games, creates payoff structures for every character involved, and then explains the outcomes that occur in the Bible by solving for pure strategy Nash equilibria. Because games are defined as "interdependent decision situations whose outcomes depend on the choices of all players", Brams does not treat G-d as omniscient in his models. Although G-d creates human beings and therefore may know their general nature, Brams argues, He has also given people free will. He therefore interacts with biblical characters in a game theoretic fashion in which He cannot always get "His way" (Brams, 2011).

Although Brams offers a new, mathematical approach to biblical exegesis, he simplifies his analyses by limiting his models to those with complete information: the timing, feasible moves, and payoffs of games are common knowledge to all players. In many cases, however, a more accurate way of modeling biblical scenarios involves incomplete information with G-d and biblical characters possessing private information about their "types". Just as biblical characters cannot be sure as to whether, for example, G-d is of a more merciful or adamant type, G-d cannot be sure about humans' true personalities because He has given them free will. Characters' decisions therefore depend on their beliefs about other characters, and their best responses in different scenarios can be derived using Bayesian probabilities.

In the analyses that follow, I revisit a few of the most debated tales from the Old Testament and model them in the framework of Bayesian games. I model three situations- Jacob's deception of Isaac, G-d's ten plagues, and Abraham's sacrifice- as dynamic games with

private information. By solving for the Perfect Bayesian Equilibria that occur in the Torah, I find conditions that must hold for characters to be willing to take the actions that they do. I also examine how characters' actions would have changed if they had held different values. My results shed light on which interpretations of biblical stories hold the most weight when characters maintain consistent beliefs and act upon them in a sequentially rational manner.

II. Jacob's "Deception": *Toledot*

Jacob's "Deception": The Text

[1] When Isaac was old and his eyes were too dim to see, he called his older son Esau and said to him, "My son." He answered, "Here I am." [2] And he said, "I am old now, and I do not know how soon I may die. [3] Take your gear, your quiver and bow, and go out into the open and hunt me some game. [4] Then prepare a dish for me such as I like, and bring it to me to eat, so that I may give you my innermost blessing before I die."

[5] Rebekah had been listening as Isaac spoke to his son Esau. When Esau had gone out into the open to hunt game to bring home, [6] Rebekah said to her son Jacob, "I overheard your father speaking to your brother Esau, saying, [7] 'Bring me some game and prepare a dish for me to eat, that I may bless you, with the Lord's approval, before I die.' [8] Now, my son, listen carefully as I instruct you. [9] Go to the flock and fetch me two choice kids, and I will make of them a dish for your father, such as he likes. [10] Then take it to your father to eat, in order that he may bless you before he dies." [11] Jacob answered his mother Rebekah, "But my brother Esau is a hairy man and I am smooth-skinned. [12] If my father touches me, I shall appear to him as a trickster and bring upon myself a curse, not a blessing." [13] But his mother said to him, "Your curse, my son, be upon me! Just do as I say and go fetch them for me."

[14] He got them and brought them to his mother, and his mother prepared a dish such as his father liked. [15] Rebekah then took the best clothes of her older son Esau, which were there in the house, and had her younger son Jacob put them on; [16] and she covered his hands and the hairless part of his neck with the skins of the kids. [17] Then she put in the hands of her son Jacob the dish and the bread that she had prepared.

[18] He went to his father and said, "Father." And he said, "Yes, which of my sons are you?" [19] Jacob said to his father, "I am Esau, your first-born; I have done as you told me. Pray sit up and eat of my game, that you may give me your innermost blessing." [20] Isaac said to his son, "How did you succeed so quickly, my son?" And he said, "Because the Lord your God granted me good fortune." [21] Isaac said to Jacob, "Come closer that I may feel you, my son-whether you are really my son Esau or not." [22] So Jacob drew close to his father Isaac, who felt him and wondered. "The voice is the voice of Jacob, yet the hands are the hands of Esau." [23] He did not recognize him, because his hands were hairy like those of his brother Esau; and so he blessed him.

[24] He asked, "Are you really my son Esau?" And when he said, "I am," [25] he said, "Serve me and let me eat of my son's game that I may give you my innermost blessing." So he served him and he ate, and he brought him wine and he drank. [26] Then his father Isaac said to him, "Come close and kiss me, my son"; [27] and he went up and kissed him. And he smelled his clothes and he blessed him, saying, "Ah, the smell of my son is like the smell of the fields that the Lord has blessed.

[28] "May God give you Of the dew of heaven and the fat of the earth, Abundance of new grain and wine. [29] Let peoples serve you, And nations bow to you; Be master over your brothers, And let your mother's sons bow to you. Cursed be they who curse you, Blessed they who bless you."

[30] No sooner had Jacob left the presence of his father Isaac-after Isaac had finished blessing Jacob-than his brother Esau came back from his hunt. [31] He too prepared a dish and brought it to his father. And he said to his father, "Let my father sit up and eat of his son's game, so that you may give me your innermost blessing." [32] His father Isaac said to him, "Who are you?" And he said, "I am your son, Esau, your first-born!" [33] Isaac was seized with very violent trembling. "Who was it then," he demanded, "that hunted game and brought it to me? Moreover, I ate of it before you came, and I blessed him; now he must remain blessed!" [34] When Esau heard his father's words, he burst into wild and bitter sobbing, and said to his father, "Bless me too, Father!" [35] But he answered, "Your brother came with guile and took away your blessing." [36] [Esau] said, "Was he, then, named Jacob that he might supplant me these two times? First he took away my birthright and now he has taken away my blessing!" And he added, "Have you not reserved a blessing for me?" [37] Isaac answered, saying to Esau, "But I have made him master over you: I have given him all his brothers for servants, and sustained him with grain and wine. What, then, can I still do for you, my son?" [38] And Esau said to his father, "Have you but one blessing, Father? Bless me too, Father!" And Esau wept aloud. [39] And his father Isaac answered, saying to him,

"See, your abode shall enjoy the fat of the earth
And the dew of heaven above.
[40] Yet by your sword you shall live,
And you shall serve your brother;
But when you grow restive,
You shall break his yoke from your neck."

[41] Now Esau harbored a grudge against Jacob because of the blessing which his father had given him, and Esau said to himself, "Let but the mourning period of my father come, and I will kill my brother Jacob." [42] When the words of her older son Esau were reported to Rebekah, she sent for her younger son Jacob and said to him, "Your brother Esau is consoling himself by planning to kill you. [43] Now, my son, listen to me. Flee at once to Haran, to my brother Laban. [44] Stay with him a while, until your brother's fury subsides- [45] until your brother's anger against you subsides-and he forgets what you have done to him. Then I will fetch you from there. Let me not lose you both in one day!"

[46] Rebekah said to Isaac, "I am disgusted with my life because of the Hittite women. If Jacob marries a Hittite woman like these, from among the native women, what good will life be to me?"

Jacob's "Deception": Background

In the Torah portion *Toledot*, or "Descendants", the Jewish patriarch Isaac faces a difficult choice. He knows that he must impart a paternal blessing on his elder son, Esau, who he favors over his younger son, Jacob. However, Esau has displayed "unworthiness to serve as the next figure in the patriarchal line" by marrying outside of the Hebrew clan (Berlin, 55). In addition, G-d told Isaac's wife, Rebekah, when she gave birth that "two separate peoples [would] issue from [her] body; one people shall be mightier than the other, and the older shall serve the younger", indicating Jacob's rightful place as the father of the Israelites (Berlin, 53). When Isaac calls Esau forth for his blessing, he is "old and his eyes [are] too dim to see" (Berlin, 55). And so a game of deception begins: Rebekah, upon hearing Isaac call for his son, runs to Jacob and convinces him to dress up as his brother and even cover himself in goat's hair to imitate Esau's hairiness. She then instructs him to go and receive the paternal blessing from Isaac.

The events that ensue can be modeled as a dynamic game in which Isaac's son first decides whether to tell the truth about his identity or to lie when he approaches his father. Upon hearing that either Jacob or Esau has arrived, Isaac then decides whether to verify the identity of his son (with "verify" meaning getting the affirmation of another person rather than simply asking for himself) and subsequently whether or not to give his son the blessing. What are the conditions that make the biblical outcome (Jacob lies and says that he is Esau, Isaac does not verify his son's identity, and Isaac blesses Jacob) possible? Why did Jacob choose to deceive his father and why did his deception succeed? The answer can be found by using a signaling game (see *Figure 1: Jacob-Isaac Game*) and solving for the game's Perfect Bayesian Equilibria. Doing so gives this tale of "deception" a twist: Isaac must have wanted to bless Jacob over Esau in order to do so without verification. He was happy to be duped.

Jacob's "Deception": Literature Review

The Jacob-Isaac "Deception" Game relates to economic literature on signaling games and strategic communication. In a basic signaling game as laid out by Cho and Kreps (1987), an informed sender who privately observes a random variable (or has the probability, "*p*", of being of a certain "type") chooses a message that an uninformed receiver observes. The receiver then decides what action to take based on what the message conveys about the sender's type. Both players' payoffs depend on the sender's information as well as the action chosen by the receiver. The signaling game structure applies to strategic communication games in which the sender's, or speaker's, message is a verbal message that the receiver, or listener, must interpret.

Current literature on strategic communication distinguishes between "cheap-talk" games in which lying is costless and communication games in which the sender bears some cost for lying. Navin Kartik (2008) explains that such lying costs can arise for various reasons including a) ex post state verification that results in penalties if misreporting is detected, b) costs of manipulating information, and c) humans' intrinsic aversion to lying. Each of these factors plays a role in the Jacob-Isaac game and is reflected in characters' payoff rankings: a) if Isaac verifies Jacob's identity, Jacob will lose the paternal blessing that he seeks, b) it takes more effort for Jacob to pretend to be Esau or Esau to pretend to be Jacob than to simply state the truth, and c) although he desires the blessing, Jacob also values honesty to his father. Using his approach, Kartik finds pooling Perfect Bayesian Equilibria that feature language inflation: all senders claim to be of a higher type than they truly are, or they tend to lie. However, receivers recognize that senders are lying and adjust their expectations accordingly.

Separating Perfect Bayesian Equilibria can still arise when receivers are naïve. Kartik, Ottaviani and Squintani (2007) use the costly signaling game model in scenarios with receivers

that will believe whatever they hear as true. The Jacob-Isaac game presents a twist to their model as the receiver in the game, Isaac, only pretends to be naïve. He blesses the son who claims to be Esau preferring that he actually be Jacob.

Isaac also has the opportunity to verify the truth of his son's message before taking his final action. Grossman and Milgrom (1981) present the idea of "verifiable disclosure" that explains that senders actually cannot lie but can merely withhold information because their information is verifiable. In the case of the signaling game between Isaac and his son, Isaac makes the active choice not to verify the truth of the message that he receives.

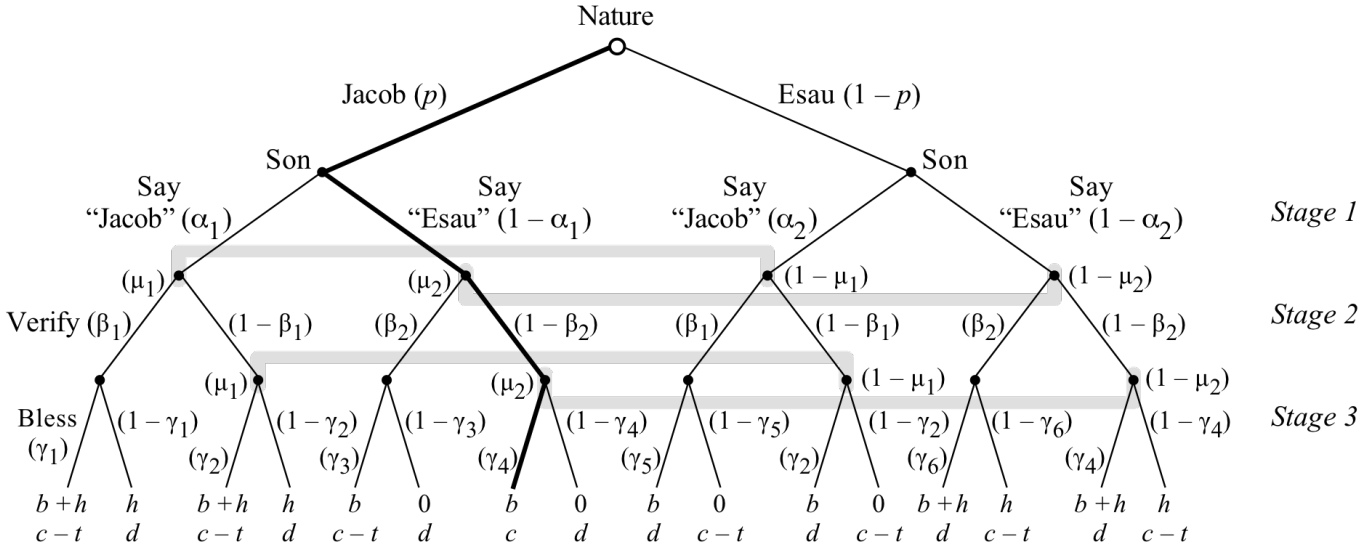
My presentation of the Jacob-Isaac game provides a new economic communication model in which the receiver *prefers* being lied to because verification of the truth is costly for him. Whereas current communication games focus on scenarios in which receivers are better off when they believe the truth, this model introduces a cost, (*t*), to the receiver for recognizing the truth when he does what he prefers. As in standard costly communication models, the sender of the more undesirable type in this model would receive a higher payoff if he could induce the receiver to do what he wanted him to do by telling the truth, but he is more likely to get the receiver to do what he wants him to do by lying. However, the receiver prefers to remain "in the dark" about the lie. Such scenarios can arise when public awareness of receivers' knowledge could hurt their reputation.

The structure for this game is based on a simplified version of the model of figurative or indirect speech from Mialon and Mialon (2012). In their game, a speaker of type θ1 can choose to speak literally ("θ1") or figuratively ("**θ**2") and a speaker of type θ2 can choose to speak literally ("**θ**2") or very literally ("θ2"). Both speakers would like the listener to interpret the literal meaning of their message from the technique that they choose. Using the model, a pooling

Perfect Bayesian Equilibrium can be derived in which the type θ1 chooses to speak figuratively ("**θ2**"), type 2 chooses to speak literally ("**θ2**"), and the listener interprets the literal meaning of the message ("θ1"). A similar pooling equilibrium can be found for the model below of what occurs in *Toledot* in which Jacob faces a similar choice as speaker θ1, Esau as speaker θ2, and Isaac as the listener. Both Jacob and Esau prefer to say that they are Esau: Jacob chooses to lie (corresponding with "speak figuratively", "**θ2**") and Esau chooses to tell the truth (corresponding with "speak literally", "**θ2**").

Jacob's "Deception": The Model

Figure 1: Jacob-Isaac Game



Note: Any unmarked node is assigned to Isaac

Parameters:
h= value of honesty
b= value of receiving a blessing
c= value of blessing Jacob/G-d's preference
d= value of blessing Esau/Isaac's personal preference
t= cost of blessing Jacob publicly

In this game, Isaac's son holds private information (whether he is Jacob or Esau) and his possible payoffs are defined using (*h*) as the value of honesty and (*b*) as the value of receiving a blessing. Nature moves first by determining whether the son Jacob comes forward (with probability "*p*") or the son Esau comes forward (with probability "1– *p*"). The son then decides whether to say that he is Jacob (α_1/α_2) or say that he is Esau ($1 - \alpha_1/1 - \alpha_2$). Next, Isaac faces two information sets: one from the scenario in which he hears that Jacob is before him (μ_1) and one from the scenario in which he hears Esau is before him (μ_2). Because he is blind, he decides whether to verify that the son before him is who he claims to be by calling for someone else (β_1/β_2) or to not verify his son's identity and just go along with his word ($1 - \beta_1/1 - \beta_2$). If he does not verify his son's identity, he again faces the two information sets (μ_1) and (μ_2). Next, he decides whether to bless his son (γ_i) or not ($1 - \gamma_i$), i=1,…,6. His payoffs are defined using (*c*) as the value of blessing Jacob/fulfilling G-d's preference, (*d*) as the value of blessing Esau/fulfilling his own preference, and (*t*) as the cost of blessing Jacob publicly, which encompasses both the cost of breaking with tradition as well as hurting the son he loves.

In the Torah, Jacob comes forth and claims to be Esau, Isaac does not verify his identity, and Isaac blesses him. A Perfect Bayesian Equilibrium for this scenario would have the characters' possible actions as follows (with the actions they take in the Bible bolded):

Jacob: $\boldsymbol{\alpha_1 = 0}$
Esau: $\alpha_2 = 0$
Isaac Hearing "Jacob": $\beta_1 = 1$
and Verifying Jacob: $\gamma_1 = 0$
and Not Verifying Jacob: $\gamma_2 = 0$
and Verifying Esau: $\gamma_5 = 1$
Isaac Hearing "Esau": $\boldsymbol{\beta_2 = 0}$
and Verifying Jacob: $\gamma_3 = 0$
and Not Verifying Jacob: $\boldsymbol{\gamma_4 = 1}$
and Verifying Esau: $\gamma_6 = 1$

In other words, if Jacob comes forward, he must prefer saying that he is Esau and if Esau comes forward, he must also prefer saying that he is Esau (note that although one son lies and one tells the truth, this is still a pooling equilibrium as both sons would say "Esau"). If Isaac hears that Jacob is before him, he must prefer to verify his son's identity. If Isaac were to verify which son was before him, he would need to prefer not blessing Jacob and blessing Esau. If Isaac hears that Esau is before him, he must prefer to not verify his son's identity and bless him.

Below is formal explanation of the Perfect Bayesian Equilibrium described above as well as other possible Perfect Bayesian Equilibria for the model. The first scenario derives conditions for what makes what occurs in the Bible occur (Jacob lies, Isaac does not verify his identity, and Isaac blesses him). The second scenario changes these conditions to see the impact it would have on characters' choices.

Solutions for Perfect Bayesian Equilibria

Assumptions
$\gamma_1 = 0$, $\gamma_3 = 0$, $\gamma_5 = 1$, $\gamma_6 = 1$

If Isaac chooses to "verify", he will not bless when Jacob is before him and will bless when Esau is before him. It is therefore common knowledge that Isaac prefers blessing Esau to blessing Jacob publicly: ($d > c - t$).

Scenario 1
What conditions must hold for the scenario that occurs in the Bible to work out? The answer can be found by using backward induction:

Stage 3

- If Isaac reaches his final information set at μ_2, he faces the following expected payoffs:

E(Bless)/(γ_4) = $\mu_2 (c) + (1 - \mu_2) (d)$
E(Don't Bless)(1-γ_4) = μ_2 (d) + $(1 - \mu_2) (c - t)$

To prefer "Bless", or $\gamma_4 = 1$, the following must hold: $\mu_2 (c - d) + (1 - \mu_2) (d - c + t) > 0$

Therefore, if $c > d$, Isaac will choose "Bless" at this stage regardless of μ_2.

- If Isaac reaches his final information set at μ_1, he faces the following expected payoffs:

E(Bless)/(γ_2) = μ_1 (c – t) + (1 – μ_1) (d)
E(Don't Bless)(1 – γ_2) = μ_1 (d) + (1 – μ_1) (c – t)

To prefer "Bless", or $\gamma_2 = 1$, the following must hold: μ_1 (c – t – d) + (1 – μ_1) (d – c + t) > 0

Therefore, Isaac will "Bless", $\gamma_2 = 1$, only if $\mu_1 < \frac{1}{2}$. This condition makes sense when $c > d > c - t$: Isaac is only willing to bless when he hears "Jacob" if he thinks it is more likely for the son before him to actually be Esau. Otherwise, he risks publicly giving Jacob the blessing, resulting in (c – t).

Stage 2

- Now let's move up to Isaac's decision of whether or not to verify that the son before him is Esau when he hears that Esau is before him (again at μ_2). Now, given $\gamma_4 = 1$, his expected payoffs are as follows:

E (Verify)/(β_2) = d
E (Don't Verify)/(1 – β_2) = μ_2 (c) + (1 – μ_2) (d)

To prefer "Don't Verify", or $\beta_2 = 0$, the following condition applies: $c > d$. This condition is consistent with the condition for Isaac to bless when he hears Esau's name.

To summarize so far: for Isaac to prefer "Bless" when he hears Esau, $\gamma_4 = 1$, and to "Not Verify" when he hears Esau, $\beta_2 = 0$, $c > d$ must hold

- What happens when Isaac hears that Jacob is before him- will he verify or not? Let us continue with the condition that $c > d$. Let us also start with the condition that $\mu_1 > \frac{1}{2}$ so that $\gamma_2 = 0$.

E (Verify)/(β_1) = d
E (Don't Verify)/(1-β_1) = μ_1 (d) + (1 – μ_1) (c – t)

In this case, Isaac will always choose "Verify", or β_1=1.

When $\mu_1 < \frac{1}{2}$ and $\gamma_2 = 1$, Isaac faces the following expected payoffs:

E (Verify)/(β_1) = d
E (Don't Verify)/(1-β_1) = μ_1 (c – t) + (1 – μ_1) (d)

Again, Isaac will always choose to "Verify", or $\beta_1 = 1$.

To summarize so far: when $\gamma_2 = 0$, $\beta_1 = 1$, $\mu_1 > \frac{1}{2}$ and when $\gamma_2 = 1$, $\beta_1 = 1$, $\mu_1 < \frac{1}{2}$

Stage 1

- Finally, let's turn to Jacob's initial decision of whether to say he is Jacob or to say he is Esau. He faces the following expected payoffs:

E(Say "Jacob")/(α_1) = β_1 [(γ_1)(*b* + *h*) + (1 − γ_1) (*h*)] + (1 − β_1) [(γ_2)(*b* + *h*) + (1 − γ_2)(*h*)]
E(Say "Esau")/(1 − α_1) = β_2 [(γ_3)(*b*) + (1 − γ_3) (0)] + (1 – β_2) [(γ_4)(*b*) + (1 − γ_4)(0)]

Under the assumptions $\gamma_1 = 0$, $\gamma_3 = 0$, $\gamma_5 = 1$, and $\gamma_6 = 1$, these payoffs simplify to:

E(Say "Jacob")/(α_1) = β_1 (*h*) + (1 – β_1) [(γ_2)(*b* + *h*) + (1 – γ_2)(*h*)]
E(Say "Esau")/(1-α_1) = (1 – β_2) [(γ_4)(*b*) + (1 − γ_4)(0)]

As shown above, Jacob's decision depends on the values of β_1, β_2, γ_2, and γ_4.

Under the scenario in which $c > d > c - t$, with $\mu_1 > ½$, we have found that $\beta_1 = 1$, $\beta_2 = 0$, $\gamma_2 = 0$, and $\gamma_4 = 1$

Plugging these values into Jacob's expected payoffs, we find that he prefers to say that he is Esau, or $\alpha_1 = 0$, when $b > h$.

- Does Jacob's choice change when $c > d$ still holds but $\mu_1 < ½$? In this case, $\beta_1 = 1$, $\beta_2 = 0$, $\gamma_2 = 1$, and $\gamma_4 = 1$, and Jacob's choice still looks as follows:

E(Say "Jacob")/(α_1) = β_1 (*h*) + (1 – β_1) [(γ_2)(*b* + *h*) + (1 − γ_2)(*h*)]
E(Say "Esau")/(1 – α_1) = (1 – β_2) [(γ_4)(*b*) + (1 – γ_4)(0)]

Again, Jacob will prefer to say that he is Esau when $b > h$. Given Isaac will verify the son before him when he says "Jacob", Jacob is better off saying that he is Esau as long as he does not mind lying to obtain the blessing.

- What about Esau? His preference of whether to lie or tell the truth affects the value of μ_1, which equals (p)(α_1) / [(p)(α_1) + (1-p)(α_2)].

E(Say "Jacob")/(α_2) = β_1 [(γ_5)(*b*) + (1 − γ_5) (0)] + (1 − β_1) [(γ_2)(*b*) + (1 − γ_2)(0)]
E(Say "Esau")/(1-α_2) = β_2 [(γ_6)(*b* + *h*) + (1 − γ_6) (*h*)] + (1 − β_2) [(γ_4)(*b* + *h*) + (1 − γ_4)(*h*)]

Under the assumptions $\gamma_1 = 0$, $\gamma_3 = 0$, $\gamma_5 = 1$, and $\gamma_6 = 1$, these payoffs simplify to:

E(Say "Jacob")/(α_2) = β_1 (*b*) + (1 − β_1) [(γ_2)(*b*) + (1 − γ_2)(0)]
E(Say "Esau")/(1-α_2) = β_2 (*b* + *h*) + (1 − β_2) [(γ_4)(*b* + *h*) + (1 – γ_4)(*h*)]

Regardless of the values of β_1, β_2, γ_2, and γ_4, Esau prefers saying that he is Esau ($\alpha_2 = 0$).

Therefore, μ_1 is undefined and there are two possible Perfect Bayesian Equilibria that would work for the outcome that occurs in the Torah under the conditions $\boldsymbol{c > d > c - t}$ **and** $\boldsymbol{b > h}$:

1) $\boldsymbol{\alpha_1 = 0;}$ $\alpha_2 = 0$**;** $\beta_1 = 1$; $\boldsymbol{\beta_2 = 0;}$ $\gamma_1 = 0$; $\gamma_2 = 0$; $\gamma_3 = 0$; $\boldsymbol{\gamma_4 = 1;}$ $\gamma_5 = 1$; $\gamma_6 = 1$; $\mu_1 > ½$; $\mu_2 = p$
2) $\boldsymbol{\alpha_1 = 0;}$ $\alpha_2 = 0$**;** $\beta_1 = 1$; $\boldsymbol{\beta_2 = 0;}$ $\gamma_1 = 0$; $\gamma_2 = 1$; $\gamma_3 = 0$; $\boldsymbol{\gamma_4 = 1;}$ $\gamma_5 = 1$; $\gamma_6 = 1$; $\mu_1 < ½$; $\mu_2 = p$

Scenario 2
How would the situation change if Isaac preferred blessing Esau to blessing Jacob so that $d > c > c - t$?

Stage 3

- If Isaac reaches his final information set at μ_2, he faces the following expected payoffs:

E(Bless)/(γ_4) = $\mu_2 (c) + (1 - \mu_2) (d)$
E(Don't Bless)$(1 - \gamma_4) = \mu_2 (d) + (1 - \mu_2) (c - t)$

To prefer "Bless", or $\gamma_4 = 1$, the following must hold: $\mu_2 (c - d) + (1 - \mu_2) (d - c - t) > 0$

Now, under the situation when $d > c$, this condition depends on the values of (t) and μ_2. Specifically, Isaac will choose "Bless", $\gamma_4 = 1$, when $\mu_2 < c - d - t/ 2c - 2d - t$. Isaac will prefer "blessing" only when he believes that the son before him is more likely to be Esau, and how much more likely depends on how much he cares about (t). If (t) is small, Isaac will just need to believe with a slightly higher probability that Esau is before him. If (t) is large, only a high probability that Esau is before him will make him bless.

- If Isaac reaches his final information set at μ_1, he faces the following expected payoffs:

E(Bless)/(γ_2) = $\mu_1 (c - t) + (1 - \mu_1) (d)$
E(Don't Bless)$(1 - \gamma_2) = \mu_1 (d) + (1 - \mu_1) (c - t)$

To prefer "Bless", or $\gamma_2 = 1$, the following must hold: $\mu_1 (c - t - d) + (1 - \mu_1) (d - c + t) > 0$

Again, Isaac will "Bless", $\gamma_2 = 1$, only if it is more likely that Esau is before him ($\mu_1 < ½$). The value of (t) is less influential in this case because there is no avoiding it in either decision of whether to bless or not: if Jacob comes forward, Isaac hears "Jacob", and Isaac decides to bless, he receives ($c - t$). If Esau comes forward, Isaac hears "Jacob", and he decides not to bless, he also receives ($c - t$) because he will end up blessing the "wrong" son. Without hearing that "Esau" has come before him, Isaac has no opportunity to be tricked into receiving just (c).

Stage 2

- Now let's move up to Isaac's decision of whether or not to verify that the son before him is Esau when he hears that Esau is before him. Now, if $\gamma_4 = 1$, his expected payoffs are as follows:

E (Verify)/(β_2) = d
E (Don't Verify)/$(1 - \beta_2) = \mu_2 (c) + (1 - \mu_2) (d)$

To prefer "Don't Verify", or $\beta_2 = 0$, the following condition applies: ($c > d$), contradicting ($d > c$). Therefore, Isaac must prefer "Verify", or $\beta_2 = 1$, when he cares more about blessing Esau than blessing Jacob.

What if $\gamma_4 = 0$? His expected payoffs are as follows:

E (Verify)/(β_2) = d
E (Don't Verify)/($1 - \beta_2$) = $\mu_2 (d) + (1 - \mu_2) (c - t)$

Isaac will again prefer to "Verify", or $\beta_2 = 1$.

To summarize so far: when $\gamma_4 = 1$, $\beta_2 = 1$, $\mu_2 < c - d - t / 2c - 2d - t$
when $\gamma_4 = 0$, $\beta_2 = 1$, $\mu_2 > c - d - t / 2c - 2d - t$

- What about when Isaac hears that Jacob is before him? If $d > c$ and $\gamma_2 = 1$,

E (Verify)/(β_1) = d
E (Don't Verify)/($1 - \beta_1$)= $\mu_1 (c - t) + (1 - \mu_1) (d)$

Isaac will always prefer to "Verify", $\beta_1 = 1$.

If $\gamma_2 = 0$,

E (Verify)/(β_1) = d
E (Don't Verify)/($1 - \beta_1$) = $\mu_1 (d) + (1 - \mu_1) (c - t)$

Again, Isaac will always prefer to "Verify", $\beta_1 = 1$.

To summarize so far: when $\gamma_2 = 1$, $\beta_1 = 1$, $\mu_1 < ½$
when $\gamma_2 = 0$, $\beta_1 = 1$, $\mu_1 > ½$

Stage 1

- Finally, let's turn to Jacob's initial decision of whether to say he is Jacob or to say he is Esau. He faces the following expected payoffs:

E(Say "Jacob")/(α_1) = $\beta_1 [(\gamma_1)(b + h) + (1 \gamma_1) (h)] + (1 - \beta_1) [(\gamma_2)(b + h) + (1 - \gamma_2)(h)]$
E(Say "Esau")/($1 - \alpha_1$) = $\beta_2 [(\gamma_3)(b) + (1 - \gamma_3) (0)] + (1 - \beta_2) [(\gamma_4)(b) + (1 - \gamma_4)(0)]$

Under the assumptions $\gamma_1 = 0$, $\gamma_3 = 0$, $\gamma_5 = 1$, and $\gamma_6 = 1$, these payoffs simplify to:

E(Say "Jacob")/(α_1) = $\beta_1 (h) + (1 - \beta_1) [(\gamma_2)(b + h) + (1 - \gamma_2)(h)]$
E(Say "Esau")/($1 - \alpha_1$) = $(1 - \beta_2) [(\gamma_4)(b) + (1 - \gamma_4)(0)]$

As shown above, Jacob's decision depends on the values of β_1, β_2, γ_2, and γ_4.

Under the scenario in which $d > c > c - t$, we have found the following possible situations:

1) $\beta_1 = 1$, $\beta_2 = 1$, $\gamma_2 = 1$, and $\gamma_4 = 1$; $\mu_1 < ½$; $\mu_2 < c - d - t / 2c - 2d - t$
2) $\beta_1 = 1$, $\beta_2 = 1$, $\gamma_2 = 0$, and $\gamma_4 = 1$; $\mu_1 > ½$; $\mu_2 < c - d - t / 2c - 2d - t$
3) $\beta_1 = 1$, $\beta_2 = 1$, $\gamma_2 = 1$, and $\gamma_4 = 0$; $\mu_1 < ½$; $\mu_2 > c - d - t / 2c - 2d - t$
4) $\beta_1 = 1$, $\beta_2 = 1$, $\gamma_2 = 0$, and $\gamma_4 = 0$; $\mu_1 > ½$; $\mu_2 > c - d - t / 2c - 2d - t$

Plugging these values into Jacob's expected payoffs, we find that he always prefers to say that he is Jacob, (α_1=1).

- Which situations above line up with Esau's preferences? His choice again affects the values of μ_1 and μ_2.

$\mu_1 = (p)(\alpha_1) / [(p)(\alpha_1) + (1 - p)(\alpha_2)$
$\mu_2 = (p)(1 - \alpha_1) / [(p)(1 - \alpha_1) + (1 - p)(1 - \alpha_2)$

As shown in Scenario 1, Esau's expected payoffs look as follows:

E(Say "Jacob")/(α_2) = β_1 (b) + (1 – β_1) [(γ_2)(b) + (1 – γ_2)(0)]
E(Say "Esau")/(1-α_2) = β_2 ($b + h$) + (1 – β_2) [(γ_4)($b + h$) + (1 – γ_4)(h)]

Regardless of the values of β_1, β_2, γ_2, and γ_4, Esau prefers saying that he is Esau ($\alpha_2 = 0$).

Therefore, $\mu_1 = 1$ and $\mu_2 = 0$ so that only 2) can work as a PBE: $\alpha_1 = 1$, $\alpha_2 = 0$, β_1=1, $\beta_2 = 1$, $\gamma_2 = 0$, and $\gamma_4 = 1$; $\mu_1 > ½$; $\mu_2 < c - d - t / 2c - 2d - t$

If Isaac valued blessing Esau over blessing Jacob no matter what, ($d > c > c - t$), he would verify and not bless when he hears that Jacob is before him under the belief that he is more likely to be Jacob. He would verify and bless when he hears that Esau is before him under the belief that he is more likely to be Esau. Both Jacob and Esau would prefer to tell the truth about their identities.

When $d > c$, Jacob will prefer saying that he is Jacob to saying that he is Esau: he will tell the truth because he knows that his father will prefer blessing his brother no matter what ($d > c > c - t$). Isaac will verify which son is before him whether he hears "Jacob" or "Esau" to ensure himself the opportunity of receiving (d), his highest payoff. Jacob is therefore better off telling the truth and receiving (h) than lying and receiving nothing.

Jacob's "Deception": Implications of the Model

In the Torah, Jacob comes forth and claims that he is Esau. Upon hearing Jacob's voice, Isaac has the opportunity to verify that the son before him is truly his elder one. He can call for another pair of eyes if he wants to be completely certain that the son before him is Esau.

However, he does not. He asks to "feel" his son and "smell[s]" him to make sure that he is "really [his] son Esau", but he never gets absolute confirmation. The Torah makes this clear by having him wonder, "the voice is the voice of Jacob, yet the hands are the hands of Esau" (Berlin, 56). He decides to go ahead and impart his blessing without receiving any public verification. Doing so gives him a payoff of (*c*) rather than (*d*). We can therefore conclude that Isaac values blessing his more deserving son to blessing his favorite son. However, a cost (*t*) must exist that prevents Isaac from blessing his deserving son publicly and makes him happy to be duped.

Does the Torah give any supporting evidence that such a cost exists? Isaac questions his son's identity as soon as he arrives, asking him "how did you succeed so quickly, my son?" to which Jacob replies, "the Lord your G-d granted me good fortune" (Berlin, 56). According to Midrash, "the moment Isaac heard his son mention G-d's name, he knew it was Jacob, and not Esau"(Plaut, 189). At this point, Isaac's "own blindness and the ruse of Rebekah come literally as a godsend" for he can now bless Jacob without having to face Esau with the truth" (Plaut, 186). One can therefore view the tale of Jacob's deception as one of deception within deception: "Rebekah and Jacob lay elaborate plans for deceiving Isaac, while unbeknown to them Isaac looks for ways to deceive himself" (Plaut, 186). Isaac's continual questions to ensure that the son before him is Esau may therefore actually be tests to ensure that the ruse being pulled on him is sophisticated enough to convince Esau that he was able to be tricked. This is why Isaac's method of "verification" cannot be considered true verification. The signaling game above presents Isaac as a more devious biblical character. He was not an old man easily manipulated; rather, he was wily enough to find a way to bless his deserving son without having to directly hurt the son he loved.

Jacob's "Deception": Comparison With Other Commentaries

The Perfect Bayesian Equilbria found in *Scenario 1* rely on the conditions that Isaac would have preferred blessing Jacob to blessing Esau if he could do so secretly, $c>d>c-t$, and that Jacob valued taking the blessing over remaining honest to his father, $b>h$. Do other Torah commentaries agree with these conditions?

Examination of $c > d > c - t$

Although the Torah explicitly states that "Isaac favors Esau", it also states that G-d foresees that "the older [son] shall serve the younger [son]", making it clear that Jacob should receive the paternal blessing (Berlin, 53). The Torah reveals that Isaac has some awareness of G-d's preference due to Esau's transgressions: when Esau took two Hittite wives, the text states "they were a source of bitterness to Isaac and Rebekah" (Berlin, 55). Isaac therefore may have preferred giving the blessing to Jacob to continue his familial line. The famous medieval commentator Rashi goes so far as to claim that "Isaac grew dim-eyed so that Jacob might get the blessing" to support the condition $c > d$. He wanted Jacob to have the blessing all along and thus "turned a blind eye to the charade" (Plaut, 186, 189). Berlin and Brettler in their *Jewish Study Bible* detect "a note of broad humor" in the situation that further suggests Isaac's awareness of the ruse: when someone comes before a blind man and declares his identity, they explain, "should not [the] blind man…put more trust in his hearing?" (Berlin, 56).

Current commentaries also shed light on the existence of (t), a cost preventing Isaac from blessing Jacob openly. The importance of the tradition of blessing the first-born son, part of (t), certainly exists in the Torah. According to commentator E.A. Speiser, the custom of leaving a paternal legacy "had been established by divine covenant", making "the emphasis on transfer of the birthright in a deathbed blessing with Yahweh's approval…readily appreciated" (Speiser,

213). However, Speiser also explains that "birthright…was often a matter of the father's discretion rather than chronological priority" (213). Therefore, Isaac may not have caused a public scandal by choosing his younger son over his older one. What, then, might prevent him from blessing Jacob publicly if (*c*) outweighs (*d*)? The emotional tone of the scene following Isaac's deliverance of the blessing provides a possible answer.

There is no denying that when Esau approaches his father, both father and son act "shaken and helpless" (Speiser). Isaac's distressed reaction may come from "the stark fact that the deed [that he has performed] cannot be undone", an interpretation based on the condition $d > c$. From this perspective, Isaac is "seized with violent trembling" due to the realization that he has been tricked. However, Isaac's "violent trembling" may come from another source: his guilt for hurting his favorite son. Isaac cares for Esau so much that, although he no longer has a blessing left to give, he still succumbs to Esau's desire to receive one. Using the structure of a paternal blessing, he guarantees Esau a prosperous life despite the future dominance of his brother. He focuses his attention on trying to console Esau, revealing a love for him, part of (*t*), which would have deterred him from blessing Jacob openly.

Examination of $b > h$

Do other commentators agree that Jacob valued receiving the blessing more than remaining honest to his father? Based on his actions, Jacob certainly seems to care more about obtaining the blessing than fulfilling his father's wishes. On the outset, he "practices outrageous deceit on a helpless father and a guileless brother, and he is rewarded for his deed" (Plaut, 185). However, this interpretation omits a number of outside factors. As commentators like Rashi note, Isaac may not be as "helpless" as he seems due to his awareness of the trick. In addition, although Jacob gets away with his ruse, his "immorality does find its implicit judgment in the

unfolding of his life, which turns into a prolonged tragedy" (Plaut, 185). Jacob goes on to be duped by his father-in-law, Laban, who tricks him into marrying his elder daughter, Leah, when Jacob wants to marry his younger daughter, Rachel. With this scenario, the Torah restores order by giving immoral Jacob a taste of his own medicine- ironically, using another conflict based on sibling confusion.

Commentators who argue that, despite his actions, Jacob is still an admirable character point to Rebekah and G-d's role in *Toledot*. According to medieval rabbi David Kimhi, Rebekah, not Jacob, may deserve more of the blame for acting immorally (Matalon, 247). Rebekah herself proclaims in the Torah that if Jacob is caught, "his curse [should] be upon [her]". Jacob initially responds with hesitation to Rebekah's plan, saying "if my father touches me, I shall appear to him as a trickster and bring upon myself a curse, not a blessing" (Berlin, 55). His reaction reveals that he does possess (*h*), the value of remaining honest. He eventually concedes to the ruse only with Rebecca's urging. Therefore, his completion of the trick may reflect more a desire to obey his mother than a desire to deceive his father (Matalon, 247). Other commentators explain that even if Jacob does value the blessing more than honesty, he only feels that way because he needs to fulfill G-d's will. Saint Jerome excuses Jacob's deed as a "laudable lie" and Theodoret claims that "Jacob, having acquired the birthright, was entitled to the blessing" (Plaut, 185).

Under any of the interpretations above, the problem of Jacob's morality seems to remain the same because "Jacob believes that he is deceiving his father and he acts on this belief" (Plaut, 186). However, the signaling model gives this analysis a twist: Jacob chooses to act the way that he does because he knows that Isaac values (*c*) over (*d*). He actually *does Isaac a favor* by pretending to be Esau. He acts as morally as he can in the given situation. As shown in *Scenario 2*, when Jacob knows that Isaac prefers blessing Esau, $d > c$, he tells the truth about his identity.

He therefore lies in the Torah not only because he knows that he deserves the blessing but also because he knows that his father knows that he deserves it.

III. G-d's Plagues: *Va-era and Bo*

G-d's Plagues: The Text

14 And the Lord said to Moses, "Pharaoh is stubborn; he refuses to let the people go. 15 Go to Pharaoh in the morning, as he is coming out to the water, and station yourself before him at the edge of the Nile, taking with you the rod that turned into a snake. 16 And say to him, 'The Lord, the God of the Hebrews, sent me to you to say, "Let My people go that they may worship Me in the wilderness." But you have paid no heed until now. 17 Thus says the Lord, "By this you shall know that I am the Lord." See, I shall strike the water in the Nile with the rod that is in my hand, and it will be turned into blood; 18 and the fish in the Nile will die. The Nile will stink so that the Egyptians will find it impossible to drink the water of the Nile.'"

19 And the Lord said to Moses, "Say to Aaron: Take your rod and hold out your arm over the waters of Egypt — its rivers, its canals, its ponds, all its bodies of water — that they may turn to blood; there shall be blood throughout the land of Egypt, even in vessels of wood and stone." 20 Moses and Aaron did just as the Lord commanded: he lifted up the rod and struck the water in the Nile in the sight of Pharaoh and his courtiers, and all the water in the Nile was turned into blood 21 and the fish in the Nile died. The Nile stank so that the Egyptians could not drink water from the Nile; and there was blood throughout the land of Egypt. 22 But when the Egyptian magicians did the same with their spells, Pharaoh's heart stiffened and he did not heed them — as the Lord had spoken. 23 Pharaoh turned and went into his palace, paying no regard even to this. 24 And all the Egyptians had to dig round about the Nile for drinking water, because they could not drink the water of the Nile.

25 When seven days had passed after the Lord struck the Nile, 26 the Lord said to Moses, "Go to Pharaoh and say to him, 'Thus says the Lord: Let My people go that they may worship Me. 27 If you refuse to let them go, then I will plague your whole country with frogs. 28 The Nile shall swarm with frogs, and they shall come up and enter your palace, your bedchamber and your bed, the houses of your courtiers and your people, and your ovens and your kneading bowls. 29 The frogs shall come up on you and on your people and on all your courtiers.'"

G-d's Plagues: Background

In the book of Exodus, a new Egyptian pharaoh rises to power who enslaves the Israelite people and demands that all Hebrew males be thrown into the Nile river after birth. G-d calls upon Moses, an Israelite raised within Pharaoh's court, to command Pharaoh to free the Hebrew

people from bondage. The Torah portion *Va-era,* or "*and I appeared*", begins the story of the ten plagues that G-d inflicts upon the Egyptians in an attempt to convince Pharaoh to "let [His] people go" (Berlin, 114). In *Va-era* and the portion that follows, *Bo*, or "*come*", G-d administers the following plagues in order of increasing intensity: blood, frogs, lice, wild animals, pestilence, boils, hail, locusts, darkness, and slaying of the firstborn. It is not until after the tenth plague, the slaying of every firstborn Egyptian son, that Pharaoh finally yields to G-d's command and frees the Israelites.

What would compel G-d to impose ten plagues on the Egyptians and Pharaoh to wait until the tenth plague to surrender? Modeling the situation as a sequential game in which both G-d and Pharaoh hold private information helps to provide an answer. In such a game, Pharaoh possesses some breaking point, P*, at which he will be willing to give up his slaves that is unknown to G-d and G-d possesses some maximum level of power, P**, to inflict plagues that is unknown to Pharaoh. G-d will inflict plagues of increasing intensity until Pharaoh believes that G-d's power exceeds his own and is willing to yield to prevent future suffering. The model employs hazard functions for both Pharaoh and G-d that depend on the relationship between the level of plague that G-d inflicts, P, with P* and P**. The outcome of the model shows that in order for a repeated two-player game to work, G-d must endure some cost for imposing harsher and harsher plagues. He and Pharaoh therefore face one another in a fair fight, both unaware of each other's limits and suffering from the infliction of more plagues. When Pharaoh finally yields, G-d proves Himself the more powerful contender.

G-d's Plagues: Literature Review

In *Biblical Games*, Brams presents the ten plagues as a static game that is not a "genuine two player game" because Pharaoh serves as a "tool" for G-d. He bases this game on the Torah's

statement that G-d "hardened" Pharaoh's heart and therefore had control of Pharaoh's actions (Berlin, 122). In his model, G-d both chooses the state of nature (whether Pharaoh remains "submissive" or "defiant up to a point") and a strategy (to "stop" the plagues or "continue" the plagues). Brams argues that along with wanting to save the Israelites, G-d "wants to teach Pharaoh and the Egyptians a lesson" about his power. He therefore "controls [Pharaoh's] behavior" so that Pharaoh will remain defiant up until a certain point and continues his plagues so that His power is revealed (Brams, 88).

Brams' model relies on G-d stripping Pharaoh of his free will- a choice inconsistent with G-d's initial decision to give all humans, even those as cruel as Pharaoh, the option to make their own decisions. Commentators such as Rabbi Eliyahu Dessler and Dr. I.H. Hertz therefore interpret the phrase "G-d hardened Pharaoh's heart" as an expression of Pharaoh's reaction to G-d's continuation of the plagues: Pharaoh still has the ability to make a choice after experiencing each plague, but "every time Pharaoh refuses to allow the Jews to leave, his personality changes and it becomes psychologically easier for him to refuse to free the Jewish people" (Hertz). Therefore, "when the Torah states that God hardened Pharaoh's heart, it means that God allowed the natural psychological consequences of saying "no" to influence Pharaoh's personality" rather than that G-d transformed Pharaoh into His puppet (Dessler).

To resolve the inconsistency between G-d giving humanity free will and G-d controlling Pharaoh to make the plagues continue, I present a model in which Pharaoh retains his free will yet still remains defiant up until a certain point. The model accurately reflects the ten-round game only when Pharaoh is unaware of the extent of G-d's power, G-d is unaware of Pharaoh's breaking point, and G-d endures some cost for inflicting harsher plagues. If Pharaoh is aware that G-d has a maximum level of intensity for His plagues that is lower than his breaking point, his

dominant strategy is to continually refuse to yield within the finitely repeated game. He would rather just endure the cost of the plagues rather than the cost of the plagues plus the cost of losing his slaves. If Pharaoh is aware that G-d's power is higher than his own, his dominant strategy is to yield in the first round so that he will endure just the cost of the first plague and losing his slaves rather than multiple plagues and losing his slaves. A model in which Pharaoh yields only eventually must therefore involve G-d holding private information about the extent of his power: Pharaoh cannot be sure whether the game will end or not. Similarly, if G-d is aware of Pharaoh's breaking point, the prediction of the game does not involve ten plagues. In this case, G-d will either a) start out by inflicting the level of plague that would make Pharaoh yield immediately or b) start out with a low level plague with the intent of making subsequent plagues harsher to punish Pharaoh until he reached his breaking point. If Pharaoh were to observe G-d inflicting a low level plague in the first round of the game, he would know that G-d was of a type who wanted to continue making his plagues harsher until he yielded. Pharaoh would therefore yield after the first round because he would rather lose his slaves immediately rather than lose his slaves plus endure the pain of all future plagues. Because the game lasts for ten rounds rather than just one, G-d must remain unaware of the extent of Pharaoh's willingness to endure more plagues. Finally, in order for a G-d unaware of Pharaoh's breaking point to be willing to start out with a low level plague, He must suffer from inflicting harsher and harsher plagues. If He is indifferent between inflicting moderate and harsh plagues, He is better off inflicting a harsh plague immediately to better ensure that Pharaoh will yield.

G-d's Plagues: The Model

Parameters

P = harshness of plague
P* = maximum amount of harshness Pharaoh can endure (*Pharaoh's breaking point*)
P** = maximum amount of harshness G-d can inflict (*extent of G-d's power*)

Unknown to Pharaoh: $P^{**} \in [0, \infty]$

Suppose F is the cumulative density function, cdf, of P** and f is the probability density function, pdf, of P**. Then $F(P_1) < F(P_2)$ if and only if $P_1 < P_2$, $F(0)= 0$, and $F(\infty) = 1$. Upon observing P_t in each round t, the probability that G-d's maximum power $P^{**} = P_t$ is $q_t = f(P_t) / 1 - F(P_t)$, the hazard rate. Suppose that as P increases, the hazard rate decreases.

q_t= probability that the P** is $P_t = f(P_t) / 1 - F(P_t)$
$1-q_t$= probability that the P that Pharaoh observes is $P_t < P^{**}$

Unknown to G-d: P*

Suppose G is the cdf of P* and g if the pdf of P*. Then, $G(P_1) < G(P_2)$ if and only if $P_1 < P_2$, $G(0) = 0$ and $G(\infty) =1$. Upon observing Pharaoh's refusal after P_t, the probability that his breaking point is $P^* = P_t$ is $g(P_t)/ 1 - G(P_t)$, the hazard rate. Suppose that as P increases, the hazard rate increases.

r_t = probability that P* is $P_t = g(P_t)/ 1 - G(P_t)$
$1 - r_t$ = probability that $P_t < P^*$

<u>*Structure*</u>

Stage 1
Nature moves first and determines G-d's level of P** and Pharaoh's level of P*
Stage 2
G-d, unaware of Pharaoh's level of P*, decides what level of P to inflict
Stage 3
Pharaoh, unaware of G-d's level of P** but having observed the level of intensity of P, decides whether to yield or refuse

Payoffs:
S= value of freeing the Israelites
A= value of keeping slaves
P= cost of inflicting/enduring a plague

- At each round t of the repeated game,

G-d:
G-d inflicts a plague of intensity P_t,
-if $P_t > P^*$, Pharaoh yields: G-d receives $S-P_t$
-if $P_t < P^*$, Pharaoh refuses, and thus the expected payoff to G-d from inflicting P_t is:
$r_t(S - P_t) + (1 - r_t)(-P_t + U_{t+1}) = -P_t + r_tS + (1 - r_t) U_{t+1}$

It is in G-d's best interest to use the lowest level of P that He can. His hazard rate must be increasing for Him to continue to raise P_t. He will raise P_t until it reaches P* and the game can end.

Pharaoh:
If Pharaoh yields, he receives $-P_t - A$
If Pharaoh refuses, he receives $-P_t + (1 - q_t)(V_{t+1})$

Pharaoh will refuse as long as $A > (1 - q_t)(V_{t+1})$.
$(1 - q_t)$ must be significantly low, which requires q_t being high. A is time-invariant while q_t and V_{t+1} are updated at each period. Thus, there is a time t* at which the two are equal if $(1 - q_t)$ is an increasing function over time. That is, if the hazard rate, q_t, is decreasing.

- At the final round t = k of the repeated game,

G-d inflicts a plague of intensity P_k,
-if Pharaoh yields, he receives $-P_k - A$
-if Pharaoh refuses, he receives $-P_k + (1 - q_k)(V_{k+1})$
Pharaoh will yield because $A < (1 - q_k)(V_{k+1})$

Pharaoh has become certain enough that the game will keep continuing, or $(1 - q_t)$ has become high enough, so that he would rather yield now than keep enduring future plagues.

G-d's Plagues: Implications of the Model

The repeated bilateral private information game proves that G-d's infliction of the ten plagues does not require that He possess harsh intentions. If G-d is unaware of P*, as in the model, He cannot begin with a low level plague with the goal of making subsequent plagues harsher and harsher because for all He knows, Pharaoh will give in after the first round and there will be no more plagues. He starts with a moderate plague and increases P only gradually because higher levels of P hurt Him as well. G-d claims that He will prove Himself as the Eternal with the belief that He can withstand administering the plagues for longer than Pharaoh can withstand experiencing them. When Pharaoh yields after the slaying of the firstborn, G-d's theory proves correct.

G-d's Plagues: Comparison With Other Commentaries

Most rabbinic sources agree that "the primary goal of the plagues…is the demonstration of G-d's unparalleled power" (Miller, 39). However, commentators disagree over whether G-d knows that He will inflict ten plagues in advance or whether He adjusts His strategy as Pharaoh proves more and more stubborn. Commentators such as Rashi and Ramban argue that G-d has planned His plagues: He removes Pharaoh's free will not only so that He can display enough marvels for Israel to "recognize [His] power" but also so that Pharaoh can receive "a fitting punishment which he would have otherwise escaped" by surrendering after the first round (Miller, 39). Meanwhile, Rabbis Plaut and Stein explain that Pharaoh chooses to keep refusing G-d on his own accord. In the course of the plagues, "Pharaoh…begins to understand a hitherto never encountered divine force" and, in the final round of the game, "Pharaoh's and G-d's authorities are shown to be unequal" (Plaut, 418-19). G-d therefore accomplishes His stated goal of proving Himself more powerful than Pharaoh without needing to control the leader. The repeated game model in which G-d remains unaware of Pharaoh's breaking point supports this view as G-d alters His strategy in each round of the game by making His plagues more and more severe and Pharaoh eventually yields due to his own changing beliefs about G-d's power.

Commentators also struggle with the concept of a G-d willing to inflict such cruel plagues on the Egyptian people- especially the slaughter of the firstborn. According to Avivah Gottlieb Zornberg in her book *The Particulars of Rapture: Reflections on Exodus*, "the master narrative of G-d as loving and benevolent redeemer of the Israelites is challenged by the killing of the Egyptians' firstborn". She poses the question, "what sin could the babies and the captives possibly have committed to deserve this punishment?" and explains "the answer posits an evil God" (Zornberg, 143). The repeated game above helps to reconcile the issue of G-d's plagues

revealing His "dark" nature. The game shows that G-d does not enjoy inflicting harsher and harsher plagues: he suffers from raising His level of P. He only makes His plagues as terrible as He does out of necessity. In order to defeat Pharaoh and free the Jews, He must go to His own extremes.

IV. Abraham's Sacrifice: *Akedat Yitzhak*

Abraham's Sacrifice: The Text

[1] After these things God tested Abraham and said to him, "Abraham!" And he said, "Here I am." [2] He said, "Take your son, your only son Isaac, whom you love, and go to the land of Moriah, and offer him there as a burnt offering on one of the mountains of which I shall tell you." [3] So Abraham rose early in the morning, saddled his donkey, and took two of his young men with him, and his son Isaac. And he cut the wood for the burnt offering and arose and went to the place of which God had told him. [4] On the third day Abraham lifted up his eyes and saw the place from afar. [5] Then Abraham said to his young men, "Stay here with the donkey; I and the boy will go over there and worship and come again to you." [6] And Abraham took the wood of the burnt offering and laid it on Isaac his son. And he took in his hand the fire and the knife. So they went both of them together. [7] And Isaac said to his father Abraham, "My father!" And he said, "Here I am, my son." He said, "Behold, the fire and the wood, but where is the lamb for a burnt offering?" [8] Abraham said, "God will provide for himself the lamb for a burnt offering, my son." So they went both of them together.

[9] When they came to the place of which God had told him, Abraham built the altar there and laid the wood in order and bound Isaac his son and laid him on the altar, on top of the wood. [10] Then Abraham reached out his hand and took the knife to slaughter his son. [11] But the angel of the LORD called to him from heaven and said, "Abraham, Abraham!" And he said, "Here I am." [12] He said, "Do not lay your hand on the boy or do anything to him, for now I know that you fear God, seeing you have not withheld your son, your only son, from me." [13] And Abraham lifted up his eyes and looked, and behold, behind him was a ram, caught in a thicket by his horns. And Abraham went and took the ram and offered it up as a burnt offering instead of his son. [14] So Abraham called the name of that place, "The LORD will provide"; as it is said to this day, "On the mount of the LORD it shall be provided."

[15] And the angel of the LORD called to Abraham a second time from heaven [16] and said, "By myself I have sworn, declares the LORD, because you have done this and have not withheld your son, your only son, [17] I will surely bless you, and I will surely multiply your offspring as the stars of heaven and as the sand that is on the seashore. And your offspring shall possess the gate of his enemies, [18] and in your offspring shall all the nations of the earth be blessed, because you have obeyed my voice." [19] So Abraham returned to his young men, and they arose and went together to Beersheba. And Abraham lived at Beersheba.

Abraham's Sacrifice: Background

In the Torah portion *Akedat Yitzhak,* or "The Binding of Isaac", G-d decides to "test" Abraham, the first Jewish patriarch, by asking him to "take [his] son, [his] only one, the one [he] loves, Isaac" and "offer him…as a burnt-offering" on a mountain in the land of Moriah. Abraham brings Isaac to the mountain, but just before he can slay his son, an angel calls out to stop him and praises him for his "fear" of G-d.

A simple interpretation of the story is that Abraham reveals himself as a "knight of faith" through his willingness to kill his only son at G-d's command (Kierkegaard, 87). He values obedience to the Lord above all else, and because of his obedience, G-d rewards him and his descendants with an eternal blessing. However, does Abraham's act prove that he is completely faithful? Even an Abraham who wavers in faith may still choose to offer his son as a sacrifice given his beliefs about G-d. If Abraham believes that it is more likely for G-d to be merciful and save Isaac, he will offer his son regardless of how much he values obedience.

How, then, can G-d's test be a true test of faith? Is there a way to prove there exists a "Faithful Abraham" who will place obedience to G-d above all else, even the expected death of his son? A dynamic game theory model (see *Figure 2: Abraham-G-d Game*) in which both Abraham and G-d hold private information about their types helps to provide an answer. The outcome that occurs in the Torah (Abraham obeys G-d and G-d saves Isaac) could occur under a variety of scenarios in the model, making G-d's test an imperfect test of faith.

Abraham's Sacrifice: Literature Review

In *Biblical Games*, Brams presents *Akedat Yitzhak* as a test of Abraham's devotion to G-d and assigns three types for Abraham: "Faithful", "Wavers Somewhat", and "Wavers Seriously".

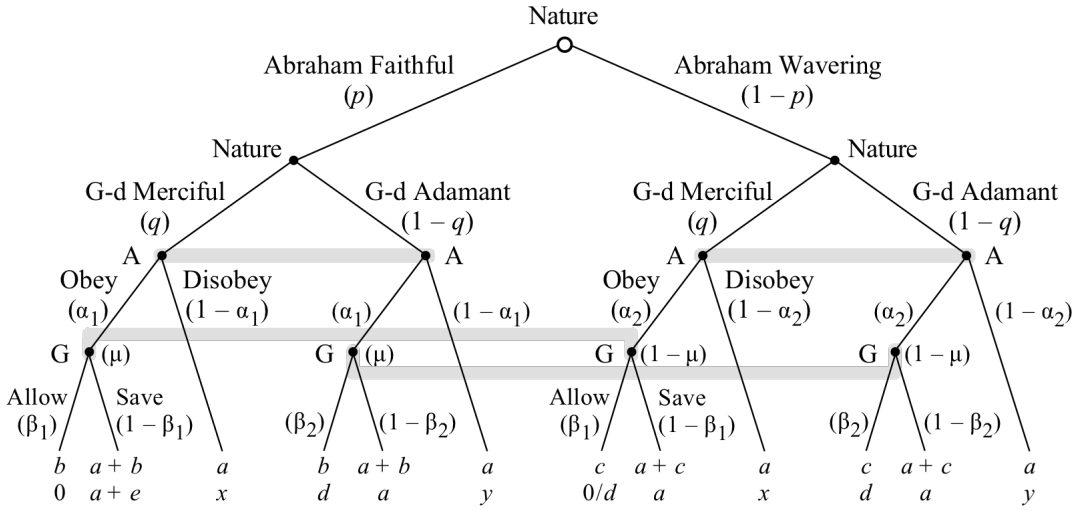
He shows that regardless of Abraham's type, his best response is to offer Isaac given that G-d prefers to save Isaac when he is offered. Because Brams' explanation relies on Abraham anticipating G-d's preferences before making his own decision, his model can be made more complete by adding in Abraham's beliefs about G-d. Just like G-d is unsure about whether Abraham tends to be faithful, a type that obeys no matter what, or wavering, a type that obeys only when he thinks G-d will save his son, Abraham is unsure about whether G-d tends to be merciful, a type that prefers to save Isaac, or adamant, a type that prefers to stick to His word. Consequently, Abraham's choice depends on the probabilities that he assigns to G-d being of one or another type.

Another game theorist, Pawlowitsch, builds off of this idea with her paper "Meaning, Free Will, and the Certification of Types in a Biblical Game" in which she argues that *Akedat Yitzhak* is not a story about G-d trying to determine Abraham's type but rather one in which G-d attempts to reveal His own type to Abraham. She explains that according to the medieval Jewish scholar Maimonides, G-d only tests a righteous man "when He knows about a righteous person that he will fulfill His will, and He wants to grant him merit". G-d would not test "the wicked, who would not listen to Him…thus, all tests mentioned in the Torah are for the benefit of the tested party" (Pawlowitsch, 5). She therefore models the game between G-d and Abraham as a dynamic game in which only G-d holds private information about His type: He will either always stop the sacrifice from occurring (p, G-d_o) or always allow it (1-p, G-d+o). In her game, G-d moves first by deciding whether or not to ask Abraham to offer Isaac. She calculates a Bayesian-Nash Equilibrium in which G-d decides to ask Abraham, Abraham offers Isaac, and G-d saves Isaac under the belief that G-d is more likely of the type that would stop the sacrifice.

Just as Brams' model fails to incorporate Abraham's beliefs about G-d, Pawlowitsch's model fails to incorporate G-d's beliefs about Abraham. Even if aware that Abraham is righteous, G-d cannot be certain about whether Abraham is willing to obey Him no matter what. In my model, I introduce both characters' sets of beliefs into a bilateral private information game. This game reveals the conditions under which G-d's commandment serves as a true test of Abraham's type (rather than one in which a wavering Abraham would still be compelled to obey). I explore the possibility of both a G-d who intends to save Isaac all along (as Pawlowitsch displays) and a G-d who wants His test to work as a true test and therefore only prefers saving Isaac when Abraham is faithful. This model proves that while G-d's test can work to prove Abraham's faith under certain conditions, it is more likely that His command fails to distinguish between a completely faithful and a wavering Abraham. This makes *Akedat Yitzhak* more a story about the ability to reconcile morality with obedience to G-d than a story that extols complete obedience to a higher power.

Abraham's Sacrifice: The Model

Figure 2: Abraham-G-d Game



Note: Nodes marked "A" are assigned to Abraham; nodes marked "G" are assigned to G-d

Parameters:
a= value of Isaac's life
b= value of obeying G-d for Faithful Abraham
c= value of obeying G-d for Wavering Abraham
d= value to G-d of sticking to His word
e= value to G-d of rewarding Faithful Abraham
x= Merciful G-d's payoff when Abraham disobeys
y= Adamant G-d's payoff when Abraham disobeys

In this game, Nature makes the first two moves by determining whether Abraham is "Faithful" (p) or "Wavering" ($1 - p$) and whether G-d is "Merciful" (q) or "Adamant" ($1 - q$). Next, Abraham decides whether to obey (α_1/α_2) or disobey ($1 - \alpha_1/1 - \alpha_2$) G-d's command to offer Isaac. If Abraham obeys, G-d faces two information sets, both defined as μ, which equals $p(\alpha1)/[(p)(\alpha1) + (1 - p)(\alpha2)]$. He decides based on μ whether to allow Abraham to go through with the sacrifice (β_1/β_2) or to save Isaac ($1 - \beta_1/1 - \beta_2$).

<u>*Solutions for Perfect Bayesian Equilibria*</u>

Scenario 1
If the G-d who saves Isaac in the Torah plans to save Isaac no matter what, can His test work as a true test of Abraham's faith? In this scenario, Merciful G-d prefers saving Isaac (receiving either $a + e$ or a) to allowing the sacrifice (receiving 0 or 0) while Adamant G-d prefers allowing the sacrifice (receiving d) to saving Isaac (receiving a). Therefore, ($d > a$): only Adamant G-d possesses the value of sticking to His word and values it more than Isaac's life, so that $\beta_1 = 0$, and $\beta_2 = 1$.

What conditions must hold for a separating equilibrium to occur in which Faithful Abraham prefers obeying ($\alpha_1 = 1$) and Wavering Abraham prefers disobeying ($\alpha_2 = 0$)?

<u>Faithful Abraham</u>
$E(\text{Obey}) = q\,[(\beta_1)(b) + (1 - \beta_1)(a + b)] + (1 - q)\,[(\beta_2)(b) + (1 - \beta_2)(a + b)]$
$E(\text{Disobey}) = q(a) + (1 - q)\,a$

With $\beta_1 = 0$ and $\beta_2 = 1$,

$E(\text{Obey}) = q(a + b) + (1 - q)\,(b)$
$E(\text{Disobey}) = a$

When $b > a\,(1 - q)$, $\alpha_1 = 1$

<u>Wavering Abraham</u>
$E(\text{Obey}) = q\,[(\beta_1)(c) + (1 - \beta_1)(a + c)] + (1 - q)\,[(\beta_2)(c) + (1 - \beta_2)(a + c)]$

E(Disobey) = $q(a) + (1 - q)\ a$

With $\beta_1 = 0$ and $\beta_2 = 1$,

E(Obey) = $q(a + c) + (1 - q)\ (c)$
E(Disobey) = a

When $c < a\ (1 - q)$, $\alpha_2 = 0$

The separating equilibrium only holds when Faithful Abraham values obedience more than Isaac's life so that he will be willing to obey even when he thinks it more likely for G-d to go ahead with the sacrifice (q is close to 0). Meanwhile, Wavering Abraham must value obedience much less than Isaac's life so that even if he believes G-d is more likely to be merciful (q is close to 1) he is still willing to disobey. This is an unlikely case, for if Wavering Abraham believes that G-d is likely to save Isaac, he would get a higher payoff from obeying than disobeying.

A more likely case is a pooling equilibrium in which both Faithful and Wavering Abraham obey G-d, as it does not rely on such a small value for (c). In this scenario, G-d can not be sure as to whether the Abraham that obeyed him is truly a "knight of faith" or not.

To summarize, two possible Perfect Bayesian Equilibria that follow the Torah in this case are:

1) **$\alpha_1 = 1$**, $\alpha_2 = 0$, **$\beta_1 = 0$**, $\beta_2 = 1$; $b > a > c$ and $c < a\ (1 - q)$ (*Abraham must have been Faithful*)
2) **$\alpha_1 = 1$, $\alpha_2 = 1$**, **$\beta_1 = 0$**, $\beta_2 = 1$; $b > a > c$ and $c > a\ (1 - q)$ (*Abraham could have been Faithful or Wavering*)

Scenario 2

How does the situation change when Merciful G-d does not strictly prefer saving Isaac? Merciful G-d's preferences could be logically altered so that he possesses an additional benefit, (d), when allowing Wavering Abraham to sacrifice Isaac with the condition that ($a + e > d > a$). Now, Merciful G-d prefers to save Isaac only when Abraham is of the faithful type. If Abraham wavers, He would rather go through with the sacrifice to punish Abraham for disloyalty. This type of G-d is therefore a G-d who plans to test Abraham rather than show mercy no matter what. Will His test now succeed?

Under this scenario, Merciful G-d's strategy depends on μ, which = $p\ (\alpha 1) / [(p)\ (\alpha 1) + (1 - p)\ (\alpha 2)]$

E(Allow) = $\mu\ (0) + (1 - \mu)\ (d)$
E(Save)= $\mu\ (a + e) + (1 - \mu)\ (a)$

His strategy is:
If $\mu > d - a/d + e$, Save ($\beta_1 = 0$)
If $\mu < d - a/d + e$, Allow ($\beta_1 = 1$)
If $\mu = d - a/d + e$, Mix

If Merciful G-d chooses to save, $\beta_1 = 0$ and $\mu > d - a/d + e$. This leaves us with the same outcome as in the prior scenario for Wavering Abraham: he still has the incentive to mimic Faithful Abraham if (c) is not sufficiently small. If (c) is small enough so that Faithful and Wavering Abraham perform different actions, $\alpha_1 = 1$, $\alpha_2 = 0$, and $\mu = 1$. This is consistent with Merciful G-d choosing to save, as $\mu > d - a/d + e$, and therefore provides us with another separating equilibrium. However, the possibility of a pooling equilibrium remains. If Wavering Abraham chooses to mimic Faithful Abraham so that $\alpha_1 = 1$ and $\alpha_2 = 1$, then $\mu = p$. Merciful G-d will still choose to save if $p > d - a/d + e$.

If Merciful G-d plays a mixed strategy, μ, or $p\,(\alpha_1) / [(p)\,(\alpha_1) + (1 - p)\,(\alpha_2)]$, must equal some value ($d - a/d + e$), meaning a separating equilibrium cannot hold as it would result in $\mu = 1$ or $\mu = 0$.

To summarize, three possible Perfect Bayesian Equilibria that follow the Torah in this case are:

1) $\boldsymbol{\alpha_1 = 1}$, $\alpha_2 = 0$, $\boldsymbol{\beta_1 = 0}$, $\beta_2 = 1$; $\mu > d - a/d + e$; $b > a > c$ and $c < a(1-q)$ (*Abraham must have been Faithful*)
2) $\boldsymbol{\alpha_1 = 1, \alpha_2 = 1}$, $\boldsymbol{\beta_1 = 0}$, $\beta_2 = 1$; $\mu > d - a/d + e$; $p > d - a/d + e$; $b > a > c$ and $c > a(1-q)$ (*Abraham could have been Faithful or Wavering)*
3) $\boldsymbol{\alpha_1 = 1}$ **or mix,** $\boldsymbol{\alpha_2 = 1}$ **or mix,** $\boldsymbol{\beta_1 =}$ **mix**, $\beta_2 = 0$; $\mu = d - a/d + e$ (*Abraham could have been Faithful or Wavering)*

We could derive similar scenarios in which Adamant G-d mixes or both Merciful and Adamant G-d mix. Both will result in similar outcomes as above with possible PBE that make G-d's "test" ineffective.

Abraham's Sacrifice: Implications of the Models

Based on the analysis above, when G-d saves Isaac as He does in the Torah, His test does not prove Abraham's total obedience to Him nor His completely merciful nature. The outcome that occurs can arise under a variety of scenarios, including some in which G-d himself wavers over whether or not to save Isaac. In addition, although G-d's angel claims that Abraham's actions prove that he "fear[s]" G-d, they may prove just the opposite: Abraham may decide to offer Isaac because he does *not* fear that G-d will actually kill the boy. An interesting extension of the model would involve deriving conditions under which G-d's test could work to signal Abraham's type: would it require that G-d plan to allow the sacrifice?

Abraham's Sacrifice: Comparison With Other Commentaries

According to religious thinker Ernst Simon, two different types of interpretations of *Akedat Yitzhak* exist: 1) "existentialist" explanations that say that Abraham shows "absolute submission of his will to G-d's command" by offering Isaac and 2) "rationalist" explanations that claim that Abraham does not need to renounce his moral obligations as a father because G-d never intends to kill Isaac (Simon, 16).

Supporters of the "existentialist" side, including the Hellenistic Jewish philosopher Philo and the nineteenth century Danish philosopher Soren Kierkegaard, argue that Abraham proves his "greatness" to G-d through his willingness to offer his son (corresponding to solutions in which only Faithful Abraham obeys, $\alpha_1 = 1$ and $\alpha_2 = 0$, and Merciful G-d does not always prefer to save Isaac). According to Philo, while "others in the history of mankind have offered themselves and their children for a cause in which they believed…Abraham's sacrifice was unprecedented in that he was not governed by motives of custom, honor, or fear, but solely by the love of G-d" (Jacobs). Kierkegaard supports this view by explaining in his famous work *Fear and Trembling* that Abraham, by offering his only son at G-d's command, is willing to "suspend the ethical" in order to fulfill his "individual relationship to his G-d" (Jacobs).

Meanwhile, "rationalist" commentators, including medieval thinkers Maimonides and Obadja Sforno along with twentieth century Rabbi W. Gunter Plaut, are more "disturbed at the idea of G-d's testing Abraham, as if the purpose of the *Akedat* were to provide G-d with information He did not previously possess" (Jacobs). They agree with the model laid out by Pawlowitsch in which G-d knows how Abraham will behave and tests the patriarch merely to reveal His own mercy (corresponding to situations in which Abraham may be Faithful or Wavering, $\alpha_1 = 1$ and $\alpha_2 = 1$, and Merciful G-d will always save Isaac, $\beta_1 = 0$). Sforno explains

that in bringing Isaac forth, "Abraham has to convert…his love of G-d…from the potential to the actual, in order to resemble G-d whose goodness is always actual, the aim of creation being that man imitates his Creator" (Jacobs). As a supremely "good" being, G-d would never actually allow Isaac to die. However, G-d must "make it difficult, yea, impossible, to understand His actions, so as to give man the opportunity to believe, that is, to ground his faith in trust and freedom" (Plaut, 45-7).

The bilateral private information model supports both the "existentialist" and "rationalist" interpretations of *Akedat Yitzhak*. Perfect Bayesian Equilibria can be derived for scenarios in which G-d is willing to kill Isaac as well as those in which G-d always prefers to save the future patriarch. In addition, Abraham could be a "knight of faith" or a wavering father for the game to occur as it does in the Torah. According to Simon, the Torah may leave the characters' natures open to interpretation for a reason: to "warn against too facile an identification of religion with naturalistic ethics" (Simon, 16). G-d makes an unethical demand of Abraham, but, as the incomplete information model shows, Abraham need not abandon his moral standards to obey this demand because of the belief (but not the certainty) that G-d will save Isaac. Viewing *Akedat Yitzhak* as a game that includes such private information supports Simon's analysis of the primary purpose of the story: to show that "Judaism is neither a secular system of morals nor a blind devotion to a supernatural power" (Simon, 16).

V. Conclusion

The three models above transform interactive biblical scenarios into dynamic games with private information. They help to prove rationales for characters that are not stated explicitly in the Torah: why Jacob deceives his father, why G-d imposes ten plagues, and why Abraham offers Isaac as a sacrifice. The implications of their outcomes- Isaac knowing that Jacob is

tricking him, G-d and Pharaoh both suffering from the infliction of plagues, and the possibility of Abraham not being completely obedient nor G-d completely merciful- provide insight into the Torah's view on justice versus morality. The biblical characters in each story face situations where justice and morality seem to conflict: Jacob deserves a blessing but can only receive it through deception, Pharaoh deserves punishment but can only receive it by G-d inflicting horrors on Egypt, and Abraham must obey G-d but can only do so by betraying his son. However, the conditions derived in the incomplete information games reveal that the characters need not abandon their morals to do what is right. Jacob does not truly deceive Isaac because Isaac wants to be deceived. G-d begins his plagues at the lowest level possible because it pains Him to continue imposing terror. Finally, Abraham does not renounce his moral obligations as a father in order to obey G-d when he believes that G-d will show mercy. Examining these scenarios using Bayesian game theoretic techniques shows that biblical characters are able to reconcile justice with morality. The Old Testament therefore teaches that humanity can find ways to balance doing what is necessary with doing what is ethical.

VI. References

Berlin, Adele, Marc Zvi. Brettler, and Michael A. Fishbane. *The Jewish Study Bible: Jewish Publication Society Tanakh Translation*. Oxford: Oxford UP, 2004. Print.

Blinder, Yaakov. *The Torah: With Ramban's Commentary Translated, Annotated, and Elucidated*. *Bereishis/Genesis Volume I*. Brooklyn, NY: Mesorah Publications. 2004

Brams, Steven J. *Biblical Games: A Strategic Analysis of Stories in the Old Testament.* Cambridge, MA: MIT, 1980. Print.

Brams, Steven J. *Game Theory and the Humanities: Bridging Two Worlds*. Cambridge, MA: MIT, 2011. Print.

Cho, I-K. and Kreps, D. M. "Signaling games and stable equilibria." *Quarterly Journal of Economics* 102 (1987): 179-221.

Gorman, Michael J. *Elements of Biblical Exegesis: A Basic Guide for Students and Ministers.* Grand Rapids, MI: Baker Academic, 2009.

Harrington, Joseph Emmett. *Games, Strategies, and Decision Making*. New York: Worth, 2009. Print.

Jacobs, Louis. "The Problem of the Akedah in Jewish Thought." *Kierkegaard's Fear and Trembling: Critical Appraisals*. Ed. Robert L. Perkins. University: University of Alabama, 1981. 1-9.

Kartik, Navin. "Strategic Communication with Lying Costs." *Review of Economic Studies* 76.4 (2009): 1359-395.

Kartik, N., M. Ottaviani, and F. Squintani. "Credulity, Lies, and Costly Talk." *Journal of Economic Theory* 134.1 (2007): 93-116.

Kierkegaard, Soren. Fear and Trembling. Radford, VA: Wilder Publications, 2008.

Matalon, Guy. "Rebekah's Hoax." *Jewish Bible Quarterly* 36.4 (2008): 243-51.

Miller, Chaim. *The Gutnick Edition Chumash- Book of Exodus: With Rashi's Commentary, Targum Onkelos, Haftaros and Commentary Anthologized from Classic Rabbinic Texts and the Works of the Lubavitcher Rebbe*. Brooklyn, NY: Kol Menachem, 2002.

Pawlowitsch, Christina. "Meaning, Free Will, and the Certification of Types in a Biblical Game." Proc. of International Game Theory Conference, Stony Brook, NY. N.p., 23 Apr. 2012. Web.<http://www.gtcenter.org/Archive/2012/Conf/Pawlowitsch1511.pdf>

Plaut, W. Gunther. “Notes on the Akedat.” *Central Conference of American Rabbis Journal* 17 (January 1969): 45–7.

Plaut, W. Gunter and David E.S. Stein. *The Torah: A Modern Commentary*. URJ Press, 2005. Print.

Simon, Ernst. “Torat Hayyim”. *Conservative Judaism* 12 (1958): 16-19

Speiser, E. A. *Genesis*. Garden City, NY: Doubleday, 1964. Print.

Steinberg, Milton. *Anatomy of Faith*. New York: Harcourt, Brace, 1960.

Zornberg, Avivah Gottlieb. *The Particulars of Rapture: Reflections on Exodus*. New York: Doubleday, 2001