Distribution Agreement

In presenting this thesis or dissertation as a partial fulfillment of the requirements for an advanced degree from Emory University, I hereby grant to Emory University and its agents the non-exclusive license to archive, make accessible, and display my thesis or dissertation in whole or in part in all forms of media, now or hereafter known, including display on the world wide web. I understand that I may select some access restrictions as part of the online submission of this thesis or dissertation. I retain all ownership rights to the copyright of the thesis or dissertation. I also retain the right to use in future works (such as articles or books) all or part of this thesis or dissertation.

Signature:

_____ _____

Eric Choi Date

Rotationally Symmetric Planes in Comparison Geometry

By

Eric Choi
Doctor of Philosophy

Mathematics

Igor Belegradek, Ph.D.
Co-advisor

Shanshuang Yang, Ph.D.
Co-advisor

Aaron Abrams, Ph.D.
Committee Member

David Borthwick, Ph.D.
Committee Member

Accepted:

Lisa A. Tedesco, Ph.D.
Dean of the Graduate School

Date

Rotationally Symmetric Planes in Comparison Geometry

By

Eric Choi
Ph.D., Emory University, 2012

Co-advisors: Igor Belegradek, Ph.D., Shanshuang Yang, Ph.D.

An abstract of
A dissertation submitted to the Faculty of the Graduate School
of Emory University in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
in Mathematics
2012

Abstract

Rotationally Symmetric Planes in Comparison Geometry
By Eric Choi

Kondo-Tanaka generalized the Toponogov Comparison Theorem so that an arbitrary noncompact manifold M can be compared with a rotationally symmetric plane M_m (defined by the metric $dr^2 + m^2(r)d\theta^2$), and they used this to show that if M_m satisfies certain conditions, then M must be topologically finite. We substitute one of the conditions for M_m with a weaker condition and show that our method using this weaker condition enables us to draw further conclusions on the topology of M. We also completely remove one of the conditions required for the Sector Theorem, another important result by Kondo-Tanaka. Cheeger-Gromoll showed that if M has nonnegative sectional curvature, then M contains a boundaryless, totally convex, compact submanifold S, called a *soul*, such that M is homeomorphic to the normal bundle over S. We show that in the case of a rotationally symmetric plane M_m, the set of souls is a closed geometric ball centered at the origin, and if furthermore M_m is a von Mangoldt plane, then the radius of this ball can be explicitly determined. We prove that the set of critical points of infinity in M_m is equal to this set of souls, and we make observations on the set of critical points of infinity when M_m is von Mangoldt with negative sectional curvature near infinity. Finally, we set out conditions under which M_m can be guaranteed an annulus free of critical points of infinity and show that we can construct a von Mangoldt plane M_m that is a cone near infinity and for which $m'(r)$ near infinity is prescribed to be any number in $(0, 1]$.

Rotationally Symmetric Planes in Comparison Geometry

By

Eric Choi
Ph.D., Emory University, 2012

Co-advisors: Igor Belegradek, Ph.D., Shanshuang Yang, Ph.D.

A dissertation submitted to the Faculty of the Graduate School
of Emory University in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
in Mathematics
2012

Acknowledgments

Before proceeding with my expressions of gratitude, I would like to bring to the reader's attention that all of the results in chapter 4 (from section 4.3 to the end of the chapter) and chapters 5 to 7 (in their entirety) are, unless otherwise stated, due to joint work with Dr. Igor Belegradek and Dr. Nobuhiro Innami.

I am deeply grateful to Dr. Igor Belegradek. Though he is not on Emory faculty, he kindly agreed to take me on as thesis student simply and only for the noble cause of helping a young mathematician grow. Ever since, he gave of himself tirelessly, and I have learned so much from his knowledge and creativity. It gave me great pleasure to hear him tell me that advising me was a rewarding experience for him.

I am thankful to Drs. Jim Lu and Shanshuang Yang for making it possible for me to work with Dr. Belegradek; I know this took considerable time and trouble.

Dr. Marty Lewinter opened my eyes to the beauty of mathematics. In his heart, he could never do enough to teach and inspire his students, and without his friendship, encouragement, and unflagging faith in me, I would not have made it through graduate school.

I can never forget all that Dr. Virginia Shadron, past assistant dean of Laney Graduate School, did for me during my most difficult moments. I have the deepest respect for her compassion and wise counsel.

Sangjune Lee gave me the idea of approaching Dr. Belegradek and asking him to be my advisor. Ever since we started at Emory together, he has been an indispensable mentor and source of moral support.

I could not have passed my preliminary exams without all the help from Steve La Fleur. Steve is very kind and very patient.

And finally, my wonderful family: Mom and Dad were always there for me and wanted nothing but the best for me; Chris and Scarlet, my second parents, showed me so much undeserved kindness and love; my sister Lynn and her husband Brandon were exceptionally understanding, tolerant, and compassionate—even for family; my wife's sister Alexis and her husband Young always seemed to think that when it came to helping and feeding us, they literally had endless time, energy, and money; and the rest of my wife's family has been just as lovely.

And of course there is my wife herself, Evelyn. Supporting me through my studies over the last five years has been a real challenge, but Evelyn did so as a true soulmate would—with complete and unconditional love. She deserves this Ph.D. as much as I do.

to Evelyn