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Matching is not an Emergent Property of the Operant Reserve

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An Abstract of
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2009

Abstract

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Matching theory has been highly successful in describing behavior at equilibrium across a wide variety of live organisms. Despite this success, the dynamics from which equilibrium behavior emerge are not well understood. Numerous dynamic models of behavior have been proposed and while some have been successful in accounting for particular types of behavior, only recent advances in computational modeling, namely the McDowell (2004) model of selection by consequences, have produced results that account for a wide variety of behavior. Catania (2005) found that a computational model of the operant reserve (Skinner, 1938) produced realistic behavior in initial, exploratory analyses. Although Catania's operant reserve computational model demonstrated potential to simulate varied behavioral phenomena, the model was not systematically tested. The current project replicated the Catania model, clarified its capabilities through systematic testing, and determined the extent to which it produces behavior corresponding to matching theory. Significant departures from both classic and modern matching theory were found in model behavior across all conditions. The results suggest that a simple, dynamic operant model does not simulate realistic steady state behavior.

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Acknowledgements

I would like to thank Jack McDowell for his guidance as my thesis advisor; Marcia Caron and Saule Kulubekova for their support; my wife, Britt Berg, for her loving encouragement and support; and my children, Zoe and Anneka, for their love and understanding of a father in graduate school.

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Matching is not an Emergent Property of the Operant Reserve

A wide range of live organism behavior at equilibrium has been shown to correspond to the mathematical function,

$$R = \frac{kr}{r + r_e}, \quad (1)$$

originally proposed by Herrnstein (1970) (McDowell, 1988). This function describes a hyperbolic shape where R is the response rate, r the reinforcement rate, k the maximum response rate, and r_e the reinforcement rate due to extraneous behavior. Equation 1 is one equation of matching theory, which accounts for upwards of 99 percent of the variance in pooled, single-alternative, live animal data (McDowell, 2005). Although Equation 1 describes behavior on single alternative schedules, related functions account for concurrent schedules as well (Herrnstein, 1970).

Despite the success of matching theory (i.e. Equation 1), recent work by McDowell (2005) has shown that the alternative, but related function,

$$R = \frac{kr^a}{r^a + \frac{r_e^a}{b}}, \quad (2)$$

consistently outperforms Equation 1 in terms of percent variance accounted for (pVAF) and production of randomly distributed residuals when fitting the function to data from live organisms. Equations 1 and 2 differ in terms of the exponent, a , and bias parameter, b . The difference follows from the functions that Equations 1 and 2 were derived from. Equation 1 was derived from the matching equation,

$$\frac{R_1}{R_1 + R_2} = \frac{r_1}{r_1 + r_2}, \quad (3)$$

where the R s refer to the rate of responding and the r s refer to the rate of reinforcement for a two-alternative concurrent schedule (Herrnstein, 1970). Although the matching equation originally provided a good fit with live-organism data (Herrnstein, 1961) it could not account for certain types of behavioral phenomena, namely undermatching and bias. (For a full description of undermatching and bias in relation to matching theory see Baum, 1979). Thus, the revised matching equation,

$$\frac{R_1}{R_2} = b \left(\frac{r_1}{r_2} \right)^a, \quad (4)$$

with additional parameters, a and b , was proposed. Equation 4 and derivative functions (including Equation 2) have become known as “modern matching theory” as opposed to “classic matching theory” described by Equation 2 and its derivative functions (including Equation 1) (McDowell, 2005). Although classic matching theory describes live organism data well, in general, some equation forms have been problematic. In contrast, modern matching theory and related functions are consistent with all known data. Thus, the modern matching functions may represent a more general account of equilibrium behavior (McDowell, 2005).

Although matching theory has been useful in understanding behavior at equilibrium, it only provides a description of behavior in terms of a mathematical function, albeit a well supported one. It does not provide an account of the dynamic processes from which steady-state behavior emerges. To this end, a dynamic model of behavior is needed.

A variety of dynamic models have been proposed but few have been widely supported. Models such as optimization (Baum, 1981) and incentive theory (Killeen, 1982) have shown efficacy in modeling certain types of behavior such as behavior on

single-alternative schedules and chained behavior, but do not generalize to fit other types of behavior. Other models based on concepts such as switching principles (Myerson & Hale, 1988) and regulatory principles (Hanson & Timberlake, 1983) predict the response-reinforcement relationship to follow a non-hyperbolic function form. However, a large body of evidence from live-organisms suggests that the relationship between response and reinforcement rates is hyperbolic in form (McDowell, 1988). Overall, many models have fit a particular type of behavior well but lacked generalization to a wide variety of behaviors and environments. This limited success does not provide solid evidence for a fundamental dynamic process.

Several criteria must be met for a dynamic model to demonstrate success in representing a fundamental behavioral process. 1) The model must produce or predict behavior that corresponds to matching theory, which has so far been most successful in describing a wide variety of live-organism behavior. Thus, behavior at equilibrium must be described by a matching function form such as Equations 1 or 2. 2) Dynamic model data at equilibrium must be *best* described by a matching theory function form.

Alternative function forms have not, in general, been observed to describe live-organism behavior at equilibrium successfully (McDowell, 1988). 3) In conjunction with Criterion 2, the residuals resulting from the data-function fit must be random. Regardless of the pVAF, non-random residuals indicate that a function does not accurately describe the data and hence, that an alternative function may be more appropriate (McDowell, 2005).

4) The model must not be so flexible that any behavioral phenomena can be modeled by simply changing enough parameters. 5) The model must account for a wide variety of behavioral phenomena including behavior on schedules representative of common

behaviors and common molecular (momentary or discrete response level) phenomena. At a minimum, both single-alternative and concurrent random-interval schedules must be accounted for given their ubiquitous presence in the research literature.

The McDowell Computational Model of Selection by Consequences

To date, the computational model of selection by consequences proposed by McDowell (2004) has been the only dynamic model of behavior to meet all criteria mentioned previously. Single-alternative schedules (McDowell, 2004; McDowell and Caron, 2007), concurrent schedules (McDowell, Caron, Kulubekova, and Berg, in press), and some molecular behavioral phenomenon (Kulubekova and McDowell, 2008) have been successfully simulated and found consistent with matching theory and live-organism data. In addition to the wide variety of behavior accounted for, the model also has been shown to be robust in its ability to simulate behavior: even after gross changes to some parameters, the model still produces realistic behavior (McDowell, 2004).

The McDowell computational model implemented B. F. Skinner's (1981) idea of behavioral selection by consequences by means of a genetic algorithm. These computational devices have been used to develop novel solutions to a wide variety of problems including animal feeding patterns (Barta, Flynn, Giraldeau, 1997), predicting consumer buying patterns (Hurley, Moutinho, & Stephens, 1995), predicting S&P 500 performance (Allen & Karjalainen, 1999), and optimizing satellite communications (Salcedo-Sanz & Bousoño-Calzon, 2005). Essentially, genetic algorithms work by evaluating a set of solutions to a problem by means of a fitness function, which tests the solution's adequacy for solving the problem. Solutions selected by the fitness function

are then “mated” and “offspring” created by combining parts from two or more potential solutions into a new amalgamated solution, which may then be subjected to random mutation depending on the particular genetic algorithm. Typically, the process is repeated until an end-state criterion is met (Rowland and Weisstein, 2006). Many variations of this basic approach are used in the implementation of genetic algorithms.

The computational model of selection by consequences proposed by McDowell (2004) created a virtual organism with behaviors represented by 100 integers with values between 0 and 1023. The 100 integers made up the virtual organism’s behavior repertoire. The range of 0 to 1023 integer values was split into several sub-ranges. Each sub-range represented a behavior class. During each iteration of the genetic algorithm, one behavior was randomly picked from the 100-behavior repertoire to be emitted. If the behavior’s integer value fell within a specific behavior class, that class was considered activated. This would be analogous in the real-world to a rat pressing a lever, for example. Because emitted behaviors were randomly selected at each iteration of the model, the chance of a behavior class being activated at each iteration was dependent on the number of behaviors currently in that class. For example, if 40 out of the 100 behaviors in the repertoire fell within a particular class during an iteration of the model, the probability of the class being activated during that iteration would be $40/100$, which is .4.

One or more behavior classes were selected as target classes. Behaviors emitted from target classes were reinforced on random interval (RI) schedules by means of a genetic algorithm selection procedure, which had the effect of shifting the distribution of behaviors towards or into the target class. The selection procedure employed a fitness

function to increase the probability that a behavior near or in the target behavior class would be selected as a parent to “mate” and produce a related “offspring” behavior. During iterations where behaviors outside a target class were emitted, or when a target-class behavior was emitted but reinforcement was not available as determined by the RI schedule, parents were chosen randomly.

Mating was accomplished by first choosing two parent behaviors to mate either by means of the selection function or randomly. The parent behavior integers were converted to ten-position binary form, which is a method of representing numbers with only 1’s and 0’s. For example, the integer 25 in ten-position binary form is 0000011001 while 500 is 0111110100 and 724 is 1011010100. An “offspring” behavior was created by randomly selecting which parent’s bits (i.e. 1’s and 0’s) would be passed onto the offspring for each position in a ten-position binary number representation. The mating process was repeated until 100 new “offspring” behaviors, representing the new virtual organism behavior repertoire, were created. In a final step, the new behaviors in the repertoire were subjected to mutation.

One iteration of the McDowell computational model consisted of behavior emission, possible reinforcement, mating, and mutation. In contrast to other applications of genetic algorithms, the McDowell model did not specify an end state or attempt to maximize any quantity. The model produced behavior based only on the simple rules inherent in the model.

The initial results from the McDowell computational model are promising. The response-reinforcement relationship follows a hyperbolic shape and fits the matching functions well for both single-alternative and concurrent schedules (McDowell, 2004;

McDowell and Caron, 2007; McDowell et al., in press). Behavioral phenomena such as undermatching and realistic inter-response time distributions have been shown to be emergent properties of the model as well (McDowell and Caron, 2007; Kulubekova and McDowell, in press).

The Catania Operant Reserve Computational Model

Catania (2005) presented a computational model of behavior based on Skinner's (1938) theory of the operant reserve. In brief, the principle of the operant reserve is that the instantaneous probability of behavior emission is related to the current level of the reserve. With a full reserve, there is a 100 percent probability of that behavior being emitted. Thus a reserve that is 50 percent full would correspond to a 50 percent probability of a behavior being emitted and so on. The reserve is depleted by a finite amount with each behavior emission that is not reinforced. Conversely, if an emitted behavior is reinforced, the operant reserve level increases. In general, those behaviors that are reinforced increase the operant reserve level while those that are not deplete it. When behaviors are reinforced, the operant reserve level increases resulting in an increased probability that the behavior will be emitted in the future.

Catania (2005) implemented the operant reserve in a computational model that combined Skinner's original theory of the operant reserve and a decay function. Like Skinner's theory, the Catania computational model added to the reserve when behaviors were reinforced and subtracted from it when they were not. However, the model relied on several parameters key to the model's functionality: the decay function, the maximum size of the reserve, the increment and decrement size, and the definition of time.

Catania's greatest change to the original theory of the operant reserve was the decay function that, in essence, gave the digital organism short term memory. Each increment to the reserve, resulting from a reinforcement event, depended on the number of responses preceding reinforcement where each response contributed to the increment by an amount determined by the decay function. Thus, responses closest in time to the reinforcement added more to the operant reserve than did responses that occurred further in time. At some point, determined by the decay function, a previous response would not add anything to the operant reserve because it was too distant from the point in time that reinforcement occurred. To avoid overlapping decay functions, Catania did not extend the decay function past a previous reinforcement. This way if reinforcement occurred at Time 1 followed shortly by reinforcement at Time 2, the behaviors previous to the Time 1 reinforcement would contribute to the operant reserve by an amount determined only by the decay function at Time 1 without any addition from the Time 2 function. Figure 1 graphically illustrates this concept.

Catania (2005) used the reciprocal decay function,

$$\rho = \frac{c}{d_0}, \quad (5)$$

where ρ was the incremental contribution of a response, c was the maximum possible incremental contribution of a response, and d_0 was the time from the reinforcement event to the response. Reinforcement events occurred during the time-step immediately after a response making the minimum value of d_0 equal to one. The total increment to the reserve at each reinforcement event was the summation of the incremental contributions of all previous responses provided a previous reinforcement event did not truncate the

decay function. Catania considered alternative function forms such as an exponential or hyperbolic but only used the reciprocal form for simplicity.

The maximum size of the reserve and the increment and decrement sizes are related parameters. The increment and decrement sizes can be defined in terms of percentages of the maximum reserve. For example, a 1,000-unit increment to a reserve with a size of 100,000 units is one percent while the same increment to a reserve of 200,000 units would be .5 percent. Catania (2005) varied the size of all three parameters, but not in a systematic manner despite noting that changing these parameters relative to each other could create a model with coarser or finer behavior. Thus, in a model with a reserve small in comparison to the increment or decrement size, each response would have a greater impact on the output of the model. The opposite would be true for a large reserve.

The final parameter, time, was arbitrarily set by Catania (2005). Each step in a simulation was made equivalent to 0.2s where each step consisted of one opportunity for the model to emit a response. However, the time in seconds was not used consistently throughout the model and the analyses of the model's behavior. Catania (2005) did not specify why the 0.2s number was used but future implementations of the model may benefit from a consistent use of time.

Catania (2005) simulated behavior on random interval (RI), random ratio (RR), fixed interval (FI), concurrent, and several other schedules of reinforcement. However, no systematic method was used to run simulations, report data, or evaluate the effectiveness of the model. Catania supported the viability of the model by mainly presenting qualitative observations of extinction curves and response patterns for

behavior on various schedules. These analyses were also not representative of all model behavior. Catania admittedly only published selected results that provided evidence the simulated contingencies could be implemented, and that correspondence to live-organism behavior could be obtained. Few analyses were quantitative in nature and those primarily consisted of illustrations that some but not all results were similar to live-organism behavior. Catania (2005) did report some consistency with matching theory but again the published analyses were not representative, merely demonstrating that successful outcomes were possible. Based on Catania (2005), the viability of the model remains to be demonstrated by a systematic and rigorous evaluation.

Comparing the McDowell and Catania Computational Models

McDowell's genetic algorithm model and Catania's operant reserve model differ in structure. While the genetic algorithm deals with a repertoire of behaviors, the operant reserve model primarily simulates a single behavior. However, the two models have three fundamental similarities: 1) the output from each model is based on the probability of emission of target behavior(s), 2) the probability of emission is increased after reinforcement, and 3) unreinforced responses decrease the probability of emission. The operant reserve model calculates the emission probability by comparing the current to the total level of the reserve. Additions to the reserve due to a reinforcement event increase the future probability of emission. The genetic algorithm model does not specifically calculate a probability but the output of the model is tied to an indirectly calculated probability. The number of behaviors falling within a class divided by the total number of behaviors, which was 100 in the simulations reported by McDowell (2004), corresponds to the probability of emission. The fitness function employed by McDowell serves the

purpose of stochastically increasing the number of behaviors that fall into the target behavior class. The addition of behaviors within the target class increases the probability of emission. Unreinforced responses implement random mating, in comparison to mating determined by a fitness function. Random mating stochastically decreases the number of behaviors in a target class, which corresponds to a decrease in the probability of future emission.

It is possible that the fundamental principles listed above underlie the basic functionality of the two models albeit from distinct approaches. While the McDowell model has been shown to meet criteria for a viable dynamic model, the Catania model has not. In the case that the Catania model can meet those criteria, evidence that the fundamental properties underlying both models are true dynamic behavioral processes will be strengthened.

Purpose of the Current Project

The current project attempted to replicate the operant reserve model proposed by Catania (2005), clarify the capabilities of the model as it currently exists, and determine the extent to which it produces behavior corresponding to matching theory. All simulations focused on random interval (RI) schedules of reinforcement. This was also the schedule implemented by McDowell (2004) to initially test the evolutionary model for correspondence with matching theory.

Replication of the model was achieved by adapting model functions reported by Catania (2005) for use in the current study. Model behavior was compared to results presented by Catania to ensure successful replication. Primarily, cumulative response

records from the replicated model and results published in Catania (2005) were qualitatively compared for similarities.

Although Catania (2005) defined time as 0.2s per computational cycle, this definition was abandoned in the current project given its arbitrary nature. Instead, all time was referenced in computational “ticks” where a tick is one opportunity for the model to emit a response. This definition of time is similar to one used by McDowell (2004).

Multiple forms of the decay function were implemented and evaluated for their impact on model behavior. The single parameter reciprocal function seen in Equation 5 served as a primary function that other functions was compared against. A single parameter exponential function of the form,

$$\rho = ce^{-d_0}, \quad (6)$$

was be evaluated. In this equation, ρ is the incremental contribution of a response, c affects the maximum incremental contribution, and d_0 is the time from the response to the reinforcement event. The parameter c was manipulated such that the maximum contribution of a response is similar to that of the reciprocal function. A single parameter hyperbola of the form,

$$\rho = \frac{c}{1+d_0}, \quad (7)$$

with parameters, ρ , c , and d_0 similar to the Equation 6 was evaluated. Again the parameter c was manipulated to make the maximum contribution of a response equivalent to the same value in the reciprocal function seen in Equation 5. Finally, a single parameter linear function of the form,

$$\rho = -d_0 + c, \quad (8)$$

was implemented. In this equation, ρ is the incremental contribution of a response, c is the maximum increment contribution, and d_0 is the time from the response to the reinforcement event. The parameter c was manipulated to make the maximum increment contribution similar to the reciprocal function.

The maximum reserve size was set at 100,000 units while the increment and decrement sizes were varied. This allowed the effects of varying the relative sizes of the increment and decrement to the maximum reserve size to be evaluated. Increments and decrements were measured in percentages of the maximum reserve size. Changes in the increment and decrement size were expected to alter the stability of behavior where larger values correspond to greater instability.

Given that the model's correspondence with matching theory was of primary interest, equilibrium states of responding were produced by running the model until examination of time-series of response rates revealed steady-state behavior. A range of possible RI schedules were arranged to determine the maximum time for steady-state behavior to be reached. Data collected before this maximum time point was not considered in matching analyses.

Following precedence set by McDowell (2004), 11 RI schedules with mean intervals of 1, 2, 3, 5, 8, 10, 18, 25, 68, 112, and 200 time ticks were arranged and Equations 1 and 2 fit to the response rate versus reinforcement rate data. To reduce measurement error, 20,000 model cycles were run for each schedule beyond the maximum time taken to achieve steady-state behavior.

Methods

Subject

The participant was a digital organism with one behavior governed by the principles of an operant reserve. Probability of emission at each computational tick was directly proportional to the current level of the reserve. Responses decreased the reserve level while contingent reinforcement increased the reserve level.

Apparatus and Materials

Software to implement the digital organism and conduct the experiments was written in VB.net and run on computers using Windows XP operating system. Computers had at least 498-Mhz Pentium III processors with 500 MB of RAM and 5 GB of hard disk space.

Procedure

For all experiments, time was defined in terms of computational cycles or “ticks.” Although this definition differs from that used by Catania (2005), it is consistent with experiments conducted by McDowell and colleagues and unencumbered by any conversion factors.

Experiment parameters and data from each time tick was stored in comma separated text files for later analysis. Total experiment time, maximum reserve size, increment size, decrement size, decay function, and schedule information were recorded in an experiment parameters reference file. Current time, current reserve level, responses, reinforcements, and reserve increments for each time tick were recorded in a separate experiment details table in the database.

Phase 1 – Programming the model in VB.net

A VB.net program was created to implement the operant reserve model, run experiments using the model, and perform data analysis. The data-analysis program retrieved experiment data saved in the text files and saved processed data to Excel files for further analysis.

The model operated on the following 7-step pseudocode:

1. At the start of an experiment, initial model parameters were collected and the corresponding model variables were set accordingly.
2. For the first and subsequent computational ticks, the probability of response emission was calculated by dividing the current reserve level by the maximum reserve level.
3. A response was emitted if a randomly generated number was larger than the probability calculated in Step 2. If a response was emitted, the reserve was decremented according to the model parameters.
4. The RI schedule was consulted to determine if reinforcement was available. If it was, then the reserve increment was calculated using the decay function specified in the model parameters and added to the reserve.
5. A time counter was incremented one tick.
6. Data from the computational tick was saved.
7. The computation was returned to Step 2 until the specified number of ticks in an experiment has been counted.

Phase 2 – Replication

A single set of model parameters and a single schedule was implemented to replicate the Catania (2005) operant reserve model. The reserve size, maximum increment, decrement sizes, and operant level were set to 100,000 units, 3 percent of the reserve, 1 percent of the reserve size, and 0 percent of the reserve respectively. The reciprocal decay function, namely, Equation 5, where c is the maximum individual increment, was used to calculate the total increment due to reinforcement. The reserve level was initially set to 75,000 units and the model run for 25,000 time ticks on a RI 8 schedule. These specifications are similar to an experiment used to validate the model reported in Figure 4 of Catania (2005). Data from the replication experiment was graphed as a cumulative response record and examined. Replication was considered confirmed if the cumulative record appeared similar to Figure 4 of Catania (2005). Aberrations in the record were examined for possible programming flaws or problems in the model.

Phase 3 – Systematic Model Testing: Schedules and Simple Decay Functions

The model's behavior was systematically tested by varying model parameters independent of each other and observing the effects. Four primary parameters were tested: first, the effect of varying the reserve level decrement (DCR) while keeping the maximum individual increment (MIC) constant; second, varying MIC while keeping DCR constant; third, the effect of the operant level of behavior; and fourth, the effect of different decay functions. To determine the relationship between reinforcement rate and response rate, a set of 11 RI schedule experiments with mean intervals of 1, 2, 3, 5, 8, 10, 18, 25, 68, 112, and 200 time ticks were run for each model parameter being tested. For the purposes of this thesis, these 11 experiments will be referred to as an experiment set.

To determine the effect of DCR on model behavior, a collection of 6 experiment sets was conducted. The MIC was set to 3 percent and the operant level kept at 0 percent for the 6 sets of experiments while the DCR level was varied from 3 percent to .3 percent. One experiment set consisted of the model being run on each of the eleven RI schedules mentioned previously for a single combination of model parameters (i.e. MIC = 3%, operant level = 0%, DCR = 3%).

The effect of MIC and operant level were tested using similar methodology. To evaluate the effect of MIC, DCR was set to 1.5% with a 0 percent operant level for four experiment sets where the MIC was varied from 1.5 percent to 10 percent. The effect of operant level was evaluated by varying the level in three experiment sets of 0, 4, and 8 percent while keeping the MIC and DCR set to 3% and 1% respectively.

For each collection of experiment sets, the model was run on four different decay function forms allowing the differential effect of the particular function form to be evaluated for all model parameter combinations. Equations 5, 6, 7, and 8 (see Table 1) describe simple reciprocal, exponential, hyperbolic, and linear decay functions along with the conversion factors necessary to obtain the maximum increment values. Figure 2 shows a comparison of the four function forms. For all decay functions, the time from the reinforcement event to a particular response, d_0 , was defined to begin at one for the response triggering the reinforcement event. Thus, a response one tick prior to the triggering response had a d_0 value of two and so on.

Phase 4 – Follow-up model testing

Based on the behaviors observed in Phase 3, a series of follow-up experiments was conducted to investigate notable phenomena. These experiments consisted of

running 5 repetitions of experiment sets where initial analyses were inconclusive. The resulting behavioral data was pooled before analysis. The additional 5 repetitions increased the power of statistical analyses and illustrated any consistencies in model behavior across multiple experiment runs.

Data analysis

In all, 52 experiment sets with 572 total experiments were conducted in Phase 3. For Phase 4, a total of 70 experiment sets or 770 experiments were conducted. Each experiment condition was run for 25,000 time ticks. Time series analyses of preliminary model runs revealed that during some experiments with lean schedules, equilibrium states were not reached until 2,000 or 3,000 ticks had occurred. For data analysis, the first 5,000 ticks were discarded to ensure that model behavior had sufficient time to reach an equilibrium state.

Rate and time series data was constructed out of 500 time-tick blocks. This 500 tick unit of analysis is similar to the one used in studies by McDowell and colleagues (McDowell, 2004; McDowell and Caron, 2007; Kulubekova and McDowell, 2008; McDowell, Caron, Kulubekova, and Berg, in press). The total number of responses and reinforcements collected during a 500 tick window was recorded. The number of responses and reinforcements across all 500 tick windows in an experiment run was averaged to obtain the experiment-wise response and reinforcement rates.

The single-alternative classic and modern matching functions seen in Equations 1 and 2, respectively, were fitted to the response and reinforcement rate data using least squares regression. The r_e^a/b expression in Equation 2 was fitted as a single unit because values of r_e and b will be indeterminate (i.e. for any value of r_e chosen, a corresponding b

could be found that would keep the unit, r_e^a/b , constant) if fitted as separate parameters. See McDowell (2005) for a further discussion on fitting the modern matching function. For experiment sets with operant levels higher than 0, an additional, y^+ , parameter was added to Equations 1 and 2 to account for the inability of model behavior to fall below the operant level.

Function parameters were iterated via Microsoft Excel's Solver add-in tool until a minimal sum of squares value was obtained. The classic and modern matching hyperbola parameters were recorded as well as the percent variance accounted for (i.e. pVAF or R^2). For each fit to model data, the standardized residuals were plotted against the predicted response rates and examined for non-random patterns. F-ratios were used to determine if cubic or quartic polynomial trends existed in the residuals. Because the probability of fitting a polynomial to any set of data increases as the polynomial order approaches the number of data points, single experiment set fits, which contained a maximum of 11 data points, were evaluated for 3rd order polynomial residual trends. Pooled data points from Phase 4, which consisted of up to 55 data points, were evaluated for 4th order polynomials. To detect any non-random patterns not accounted for by polynomial curve fitting, residual plots were visually evaluated.

Finally, the overall effectiveness of the model was evaluated by calculating the probability that the number of significant non-random residual trends found was due to random chance (e.g. Type I error) for each decay function across all parameter conditions. The binomial probability function,

$$P = \sum_{k=k_0}^n \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k} , \quad (9)$$

gives P , the binomial probability of finding k_0 or more significant results when n total tests are performed that each have a probability, p , of being due to chance. This method was similar to that used by McDowell (2004) and it allowed all the results from each decay function condition to be evaluated in a single test together. Calculated binomial probabilities less than .05 were considered significant evidence that the model, using a specific decay function, produced behavior inconsistent with matching theory. In other words, a $P < .05$ indicated a probability less than 5% that the number of significant polynomial trends found in a group of behavior data was due to chance alone.

Results

Programming the Model

The model was successfully programmed in Visual Basic .NET 2003. Approximately 2000 lines of new code were written to implement the model computationally. When possible, model functions were programmed using equations and descriptions of functions adapted from Catania (2005). To reduce the possibility of systematic error in the computational model, error checking functions were programmed to check for model behavior inconsistent with expected behavior and to ensure accurate data analysis.

Model Verification

The operant reserve computational model was run for 25,000 ticks on a RI 8 schedule of reinforcement with a 0 percent operant level of behavior, a MIC of 3 percent, and a DCR of 1 percent. A cumulative response record was generated and compared to results reported in Catania (2005). Figure 3 shows cumulative records with lengths of

1000 and 10,000 ticks to illustrate the detailed and overall model behavior for this experiment respectively. An examination of the cumulative record, revealed an overall steady rate of responding with some minor perturbations. There were no patterns in the data that would suggest programming errors or problems in replication of the Catania model. In sum, the model's behavior resembled live organism data and was similar to that reported in Catania (2005).

Systematic Model Testing – Varying the Decrement to the Reserve

The effect of the decrement to the reserve (DCR) after a response was evaluated in a collection of experiments where the DCR was varied while all other model parameters remained constant. The MIC was set to 3 percent of the maximum reserve size and the operant level to 0 percent. Six experiment sets with the DCR set to .3, 1, 1.5, 2.1, 2.7 and 3 percent, respectively, were run for each decay function form. Average response and reinforcement data from each schedule in the experiment set were plotted. Figure 4 shows results for the reciprocal, exponential, hyperbolic, and linear decay function forms. For all decay functions, changing the DCR affected the behavior of the model. At lower DCR values, small increases in reinforcement rates produced large changes in the response rate. Increases in DCR resulted in less sensitivity to reinforcement. As the value of the DCR increased, larger increases in reinforcement rate were needed to produce the same change in response rate. At the highest DCR level (3 percent) where MIC and DCR were equal, the model's performance broke down to the point where the maximum response rate was not achieved. With a linear decay function, the lowest DCR level (DCR = .3%) produced very high rates of responding with very little reinforcement. In fact, the response rate began at approximately 325 responses per

500 ticks under RI 200 conditions as seen in the lower right panel in Figure 4. This pattern of very high responding for very lean schedules was not observed for the other decay function forms.

The overall shape of the model behavior data was asymptotic in form for all decay function types with the exception of experiment sets where the DCR was equal to the MIC (DCR = 3%). Under these conditions, the data formed a straight line for the reciprocal, exponential, and hyperbolic decay function conditions as seen in Figure 4. For the linear decay function, the shape for this condition was curvilinear and possibly asymptotic but this latter assertion could not be confirmed because the model behavior did not achieve the maximal rate of responding.

The type of decay function affected the degree to which the DCR affected model behavior. For reciprocal, exponential, and hyperbolic decay functions, changes in the DCR produced proportional changes in model behavior as illustrated by the consistent spacing of the data across different DCR level experiment sets in Figure 4. The behavior of the model with a linear decay function was markedly different. The difference between model behavior for DCR values .3, 1, 1.5, 2.1 and 2.7 percent was much smaller than model behavior using curvilinear decay functions under the same DCR conditions. Additionally, the model's behavior was greatly affected by a DCR equal to the MIC (DCR = 3%) when compared to a DCR 10 percent lower than the MIC (DCR = 2.7%). The small decrease in DCR relative to the MIC produced behavior much more sensitive to changes in reinforcement rate as seen in the lower right panel in Figure 4. This sensitivity to DCR values close to the MIC level was only observed in experiment sets using linear decay functions.

The classic and modern matching equations were fitted to all experiment set data, the equations' parameters were collected, percent variance accounted for (R^2) calculated, and residuals from each fit plotted and evaluated for non-random patterns. The equation parameters and R^2 values for all fits can be found in Table 2. The R^2 values ranged from .89 to 1.00 (rounded to two decimal places) ($Mdn = .99$) for classic matching fits and between .96 and 1.00 ($Mdn = 1.00$) for modern matching fits. This result taken alone would have indicated a high level of correspondence to matching theory with the modern matching equation describing the data best. However, evaluation of the residuals revealed systematic deviations from both matching equation forms under most conditions.

Figures 5 through 8 show classic and modern matching equations fit to experiment data from reciprocal, exponential, hyperbolic, and linear decay function conditions respectively ($MIC = 3\%$, operant level = 0%, and $DCR = 1\%$). These conditions were chosen because they appeared to produce the most realistic behavior. The bottom plot shows the residuals resulting from the fits. For reciprocal, exponential, and hyperbolic decay functions the best fit classic matching equation clearly deviates from the model data despite high R^2 values. The poorness of fit is reflected in the non-random patterns of residuals and significant polynomial trends for the majority of model conditions. While the modern matching equation described the data better, a close inspection also revealed systematic deviations, which were again reflected in non-random patterns in many of the residuals.

The cause of these systematic departures from the two hyperbolic forms was apparent in the plots of the experiment set data. While the classic matching equation describes a hyperbolic function shape with a single inflection point, the experiment set

data for the reciprocal, exponential, and hyperbolic model conditions had two inflection points as seen in Figures 5 through 7. One of these inflection points was at high response rate similar to location described by the classic matching equation. The second inflection point appeared at lower response rates and was not reflected in the classic matching equation. Although the modern matching equation had additional parameters, a and b , and fit the experiment set data better than the classic equation, the non-random residual patterns indicated that the experiment data was not best described by this function form either. Additionally, the second inflection point was indicative of overmatching.

Although Figures 5 through 7 show results from one DCR value (1%), similar non-random patterns were observed in the residuals from classic and modern equation fits to most experiment sets with curvilinear decay functions as seen in Figures 9, 10, and 11.

In contrast to the reciprocal, exponential, and hyperbolic decay function forms, classic and modern equation fits to model behavior under a linear function form condition produced much better fits. R^2 values for both classic and modern equation fits was 0.99 or above for all experiment sets. Figure 8 shows classic and modern equation fits to model behavior for MIC, operant level, and DCR values of 3, 0 and 1 percent respectively. The second inflection point observed in model behavior under curvilinear decay function forms was not present. Both classic and modern equations with best-fit parameters adhered closely to the data. A qualitative evaluation of the residuals in Figure 8 did not immediately reveal any non-random patterns. However, the model's response rate was highly sensitive to reinforcement rate under these conditions. As shown in Figure 8, there was data for response rates near zero and above 400 but none in between those values. Given this lack of data, the classic and modern equation fits to the data were

effectively for response rates above 400 responses per 500 ticks. An examination of the residuals for all DCR levels under linear decay function conditions, indicated similar gaps in model data for other DCR levels (see Figure 12). Additionally, significant polynomial trends were found for 4 of 6 DCR values tested for both classic and modern matching functions.

Systematic Model Testing – Varying the Increment to the Reserve

The effect of the maximum individual contribution (MIC) was evaluated in a collection of experiments where the MIC was varied but other model parameters held constant. DCR was set to 1.5 percent while operant level remained at 0 percent for all experiments in the collection. This DCR value was chosen because the model appeared to perform best under these conditions and it allowed for a slightly different parameter value than that tested while systematically varying the DCR. Four experiment sets where the MIC was set to 1.5, 3, 5, and 10 percent, respectively, were run for each decay function form. Average response and reinforcement data from each schedule in the experiment were plotted. Figure 13 shows the results for all decay function forms and MIC levels. For reciprocal, exponential, and hyperbolic decay functions, different MIC levels affected model behavior. Higher MIC levels were associated with greater sensitivity to reinforcement rate for all MIC levels except when MIC was equal to DCR (1.5%). In results mirroring those obtained from varying DCR mentioned previously, model performance deteriorated to the point where the maximum rates of reinforcement and responding were not achieved when MIC was equal to DCR. In contrast to the curvilinear decay functions, changes in MIC did not produce differences in model behavior when the decay function was linear as seen in the lower right plot of Figure 13. The only exception

to this was when MIC equaled DCR level, but the model's behavior broke down under this condition did not reach the maximum reinforcement and response rates.

For all decay function conditions, the overall shape of the data was asymptotic in form with the exception of experiment sets where the DCR was equal to the MIC (MIC = 1.5%). Under equal MIC/DCR conditions, the data formed a straight line for the reciprocal, exponential, and hyperbolic decay functions and a curvilinear line for the linear decay function. These results were similar to those obtained when varying DCR. The second inflection point was observed again in results from the reciprocal, exponential, and hyperbolic functions. The severity of this inflection point appeared to vary with MIC level. At lower MIC levels, the second inflection point was more pronounced than at higher MIC levels.

With a linear decay function, the effect of a MIC level equal to the DCR was very pronounced. As mentioned previously, model behavior deteriorated when the MIC equaled the DCR. However, increasing the MIC from 1.5 percent and equal to the DCR to 3 percent, changed the model behavior significantly as seen in the lower right plot in Figure 13.

The classic and modern matching equations were fitted to all experiment set data, the equations' parameters were collected, R^2 values calculated, and the residuals plotted and evaluated for non-random patterns. The equation parameters and R^2 values for all fits can be found in Table 3. For the classic fits, R^2 values ranged from .80 to 1.00 (rounded to two decimal places) ($Mdn = .995$). R^2 values for modern fits were all above .99 ($Mdn = 1.00$). Significant non-random patterns were not found consistently in either classic or modern matching fits (see Table 3). The linear decay function condition appeared to have

the best fits with very high R^2 values and only one condition with evidence of statistically or visually significant non-random residuals.

Figures 14, 15, 16, and 17 show the residual plots for all experiment sets with the DCR set to 1.5 percent, operant level set to 0 percent, and variable MIC.. For the linear decay function condition, a non-random pattern of residuals was evident for the modern fit when the MIC was 3 percent as seen in the top right plot in Figure 17. When the MIC was 5 and 10 percent, no systematic trends were immediately obvious in the residuals. However, there were large gaps in the response rate data due to the model's sensitivity to reinforcement rate for MIC levels of 5 and 10 percent. No data were available below a response rate of approximately 350 responses per 500 ticks. Thus, the classic and modern equation fits were only for response rates above 350.

Systematic Model Testing – Varying the Operant Level

The effect of the operant level of behavior was evaluated in a collection of experiments where operant level was varied while other model parameters were held constant. The MIC and DCR were set to 3 percent and 1 percent, respectively, while the operant level was set to 0, 4, and 8 percent in three experiment sets for each decay function form condition. These MIC and DCR values were chosen because they appeared to produce the most realistic behavior as observed in previous experiments. Average response and reinforcement data from each schedule in the experiment were plotted. Figures 18 to 21 show the results from the reciprocal exponential, hyperbolic, and linear function forms. Changes in the operant level produced changes in model behavior for all decay function forms. The changes in model behavior were similar for reciprocal, exponential, and hyperbolic function forms. Increases in operant level increased the

effect of the inflection point seen at lower response rates. Higher operant levels also changed the y-axis intercept of the data. These effects can be seen in the lower plots of Figures 18, 19, and 20.

The effect of differing operant levels was restricted to low reinforcement rates. As can be seen in the top plots of Figures 18, 19, and 20, the model behavior was similar across all operant levels except for the lowest reinforcement rates. The effect of operant level under curvilinear decay function conditions appeared to be restricted to the leanest schedules of reinforcement.

In contrast, the changes in model behavior due to varying operant levels was markedly different when the decay function was linear. Under zero operant level conditions, the model's behavior was split between very low rates of responding for the lowest rates of reinforcement (RI 200, 112, and 68) and very high rates of responding on richer schedules (RI 25 and above) as seen in Figure 21. Therefore, no reliable data points existed for response rates below 400 responses per 500 ticks. When the operant level was increased to 4 and 8 percent, the response rates for the three leanest schedules increased markedly as seen in the three data point clusters with the lowest response rates in the lower plot of Figure 21. For all other reinforcement rates, different operant levels had no appreciable effect.

The classic and modern matching equations were fitted to all experiment set data. The equation parameters and R^2 values for each fit are listed in Table 4. For classic equation fits, the R^2 values ranged from .90 to 1.00 (rounded to two decimal places). For modern equation fits, R^2 ranged from .99 to 1.00. Figures 22 to 25 show best-fit classic

and modern equations plotted with experiment data for an 8 percent operant level along with the residuals from each fit.

For reciprocal, exponential, and hyperbolic decay functions, the response-reinforcement plots showed significant systematic deviations from the best-fit classic matching function despite large R^2 values (.90 to .96). These deviations were reflected in non-random patterns visible in the residual plots and significant polynomial fits (see Table 4). The modern matching equation fit the experiment data comparably better with all R^2 values greater than .99. Upon visual inspection, the modern matching residuals seen in the lower plots of Figures 22, 23, and 24 did not appear to have non-random patterns. However, when comparing the residuals resulting from fits to data at the three different operant levels (0, 4, and 8 percent), the pattern of residuals was markedly similar across all three datasets (as seen in Figures 26, 27, and 28) even though the experiments were conducted independent of each other. Such consistency across independent experiment runs with different model parameters qualitatively suggested that non-random systematic deviations from the modern matching equation existed but cubic polynomial fits could not detect them.

Figure 25 shows model behavior with a linear decay function and 8 percent operant level along with best-fit classic and modern equation plots. A plot of the residuals revealed systematic deviations from experiment data for the classic and modern matching equations as seen in the lower portion of Figure 25. Polynomial fits to the residuals were significant as well. However, residuals for the 0 and 4 percent operant levels did not contain significant polynomial trends for either classic or modern matching fits (see Table 4). Visual inspection did reveal a likely non-random pattern for the 4 percent

condition but not the 0 percent condition as seen in the top and middle panels of Figure 29. Most notably, the model appeared to be highly sensitive to reinforcement rate under all operant levels. As seen in Figure 25, rates of responding did not drop below 150 responses per 500 time ticks despite being on lean schedules of reinforcement at an 8 percent operant level. This effect was also observed for a 4 percent operant level. For the 0 percent condition, no data response data existed between 50 and 400 responses per 400 ticks.

Follow-up Model Testing - Five Repetition Experiments

After the analyses of the single experiments were completed, an additional series of experiments with 5 repetitions per model condition were conducted in an attempt to clarify if non-random patterns observed qualitatively could be detected with the additional power of pooled datasets. Given the presence of the second inflection point in the majority of data produced under curvilinear decay function conditions, which has not been reported in live organism data to date, only one 5 repetition experiment was conducted using a curvilinear decay function. In contrast, all linear conditions were repeated five times and the pooled data analyzed for non-random patterns.

Reciprocal decay function with 4 percent operant level - five repetitions

The operant model was run 5 times with the MIC, DCR, and operant level set to 3, 1, and 4 percent, respectively, with a reciprocal decay function. This condition was selected for additional repetitions because it produced realistic behavior with no polynomial trends in the residuals from the matching equation fits. However, as noted previously, similar residual patterns were qualitatively observed across model conditions indicating some systematic patterning in the residuals. Although the results were similar

for both hyperbolic and exponential decay function conditions, the reciprocal form was selected for further analysis because it was the decay function used for all data presented in Catania (2005) and therefore most relevant to any comparisons.

Behavior data from the 5 repetitions was pooled and the modern matching function fit to the data. The residuals from this fit were plotted and analyzed for non-random patterns using 1st through 4th order polynomials. Figure 30 shows the residuals from the single experiment set (the top panel), the residuals from the 5 identical, pooled experiment sets (middle panel), and cubic and quartic polynomials fit pooled residuals (bottom panel). A non-random pattern is not clearly detectable by just evaluating the single experiment. In contrast, the close grouping of residuals resulting from the pooled data fit (middle panel), clearly showed some non-random pattern. Although this pattern was easily detectable by qualitative analysis, no significant trend was present ($R^2 = .0091$, $p = .93$) using lower order polynomials (up to 3rd order). However, a quartic polynomial fit was statistically significant ($R^2 = .87$, $p < .00001$). The bottom panel of Figure 33 illustrates the goodness of 4th order polynomial fit in comparison to a 3rd order polynomial.

Five repetition results for all linear decay function conditions

For each linear decay function condition, the computational model was run 5 times with identical parameters. The resulting datasets were pooled within each condition and analyzed together. Classic and modern matching functions were fitted to the pooled data and the resulting residuals plotted and evaluated for 4th order polynomial trends. The matching equation parameters and goodness-of-fit statistics can be seen in Table 5.

Figures 31 and 32 show the residuals from classic and modern matching fits for each DCR value, respectively, with a MIC of 3 percent and an operant level of 0 percent. A qualitative inspection revealed clear non-random patterns for all but one DCR value. While the residuals for this condition (DCR = .3) did not clearly indicate systematic deviations from the matching functions, there was no data for response rates less than 250 responses per 500 ticks. Polynomial fits to the residuals were significant for all conditions and matching functions except for when the DCR was .3 percent (see Table 5) indicating pervasive systematic deviations from both matching function forms.

Results for experiment sets varying MIC were similar. Figures 33 and 34 show residuals from classic and modern matching fits to each MIC value, respectively, with a DCR of 1.5 percent and an operant level of 0 percent. The plots for both classic and modern functions clearly showed non-random patterns in the residuals. Polynomial fits to the residuals were significant for all conditions with the exception of one condition (10 percent MIC and modern matching) which showed a trend towards significance ($p = .08$).

Identical repetitions of experiment sets varying operant level produced results indicating some increased model variability with operant levels above 0 percent. Figures 35 and 36 show residuals from classic and modern matching fits for each operant level, respectively, with a MIC of 3 percent and a DCR of 1 percent. When the operant level was 0 percent, the residuals were more closely clustered together in comparison to the 4 and 8 percent operant levels. However, no data existed for response rates lower than 150 responses per 500 ticks for either 4 or 8 percent despite lean reinforcement schedules. Polynomial fits to the residuals indicated significant non-random patterns in only the 0

percent operant condition. No systematic deviations from classic or modern matching functions were statistically detected for either the 4 or 8 percent operant conditions.

Model Effectiveness within Decay Function Type and Across Conditions

Binomial probabilities were calculated to determine if the significant residual trends, evaluated together, could be due to chance. The number of significant residual trends observed for each decay function condition across all parameter values (i.e. MIC, DCR, and operant level) was counted and a binomial probability calculated using Equation 9. For both classic and modern matching fits to model data, significant binomial probabilities were found for all decay functions indicating that the number of non-random residual patterns was unlikely due to chance (see Table 6). The highest binomial probability was observed for the modern matching fit to exponential decay model data, $P = .025$, where 3 out of 13 experiment sets contained non-random residuals. The binomial probabilities for both classic and modern fits to linear decay model data were lower for the 5 repetition pooled data when compared to data from single repetitions indicating that increased statistical power strengthened the findings from single repetition experiments.

Discussion

Programming and Verification of the Model

A computational model of the operant reserve was successfully programmed and model behavior was qualitatively similar to that reported in Catania (2005). Cumulative response graphs created from model behavior were consistent with those published by Catania (2005). Additionally, response-reinforcement (R-r) plots generated from the model using reciprocal decay functions showed evidence of a two inflection points. A

close inspection of Catania's (2005) Figure 11 suggest that this phenomenon, observed in the current data, is not unique to the current implementation of the model. In sum, there were no observed differences between the current model behavior and that reported in Catania (2005).

Systematic Model Testing

The effect of varying the decrement to the reserve (DCR), maximum individual increment (MIC), and operant level were evaluated for each decay function. Global behavior qualities were consistent for each decay function regardless of DCR, MIC, and operant level values. For all curvilinear decay functions (reciprocal, exponential, and hyperbolic), two inflection points were observed in R-r plots with the exception of when MIC equaled DCR. The pattern of deviations implies overmatching where under-responding occurs on lean schedules and over-responding occurs on rich schedules. While overmatching has been observed in some live organism behavior (Aparicio, 2001), it appears to be an exception to an overall trend towards undermatching. Although under or overmatching has been most commonly measured using concurrent schedules, the additional parameter, a , in the modern matching single-alternative equation accounts for under or overmatching. Values of $a = 1$ correspond to perfect matching while $a < 1$ and $a > 1$ correspond to undermatching and overmatching, respectively. Out of 40 experiment sets in the current study where $MIC \neq DCR$, only one set produced an undermatching exponent, $a < 1$. These results suggest that overmatching is an emergent property of the operant reserve model that uses curvilinear decay functions.

Using a linear decay function, the operant reserve model's behavior was notably different. No second inflection point was observed in any R-r plots. However, the

exponent, a , was larger than 1.0 for all but one set of model parameters tested in the 5 repetition follow-up data. This result suggests that use of a linear decay function reduces the degree of overmatching but does not eliminate it altogether.

The model's sensitivity to changes in the parameter values varied with the type of parameter. No differences in global model behavior were observed between the three curvilinear decay functions. The overall shape of the R-r plots and parameters of the matching equations were very similar in all curvilinear decay function conditions. Model behavior did vary when changing MIC, DCR, and operant level.

For curvilinear functions, changes in DCR affected the model's sensitivity to reinforcement rate. A wide variety of R-r shapes was obtained by varying this parameter. When $DCR = MIC$, the maximum response rate was not reached. The overall shape of the R-r plot under these conditions appeared to be a straight line. A small decrease in the DCR, relative to the MIC, produced greater sensitivity to reinforcement rate and more curvature in the R-r plot. The model became increasingly sensitive to reinforcement rate as DCR was decreased until very small increases in the rate of reinforcement produced very large increases in response rate. For example, under reciprocal decay function conditions with a DCR, MIC, and operant level of .3, 3, and 0 percent, respectively, an increase in the reinforcement rate from 8 to 20 reinforcements per 500 ticks produced an increase in response rate from 125 to 444 reinforcements per 500 ticks. This sensitivity to changes in reinforcement rate resulted in very few data points for the mid-range of response rates for many experiment sets. Overall, the model's response to varying DCR values made it possible to produce a wide variety of R-r plot shapes suggesting that a variety of live organism behaviors could be simulated with the computational model.

The model's sensitivity to DCR changes differed when a linear decay function was used. The greatest difference was observed when DCR was close in value to MIC. Under DCR=MIC conditions, the maximum rate of responding was not achieved despite a maximally rich reinforcement schedule. When the DCR was decreased 10 percent relative to the MIC, model behavior changed notably. In contrast to a 10 percent decrease in DCR using a curvilinear decay function, the linear decay function model behavior immediately became highly sensitive to reinforcement rate. Subsequent decreases in the DCR, produced much smaller changes in the sensitivity to reinforcement when compared to the same decreases but using a curvilinear decay function. At the lowest DCR relative to MIC, the model behavior was so sensitive to reinforcement that even the lowest reinforcement rates produced very high rates of responding. Although using the linear decay function produced less overmatching, the inability to produce a wide range of sensitivity to reinforcement suggests that the model could only simulate a narrow range of live organism behaviors.

Changes in MIC while holding DCR constant produced similar results in all curvilinear decay function conditions. Again, when MIC=DCR, the model was less sensitive to reinforcement rates and the maximum response rate was not reached despite a maximum reinforcement rate. Increasing MIC produced greater sensitivity to reinforcement rates. Additionally, a wide variety of model behavior was produced through varying MIC.

In contrast, variations in MIC produced minimal changes in model behavior when a linear decay function was used. Although the model was insensitive to reinforcement rate when MIC=DCR, as previously observed when varying DCR by itself, changes in

MIC produced no appreciable variation in model behavior when $MIC > DCR$. This result was unexpected given the success in changing DCR to produce a variety of behavior and suggests that DCR has a greater ability to produce change in a model using a linear decay function.

Variations in model behavior produced by changes in the operant level were restricted to the leanest schedules of reinforcement for all decay function types. For all curvilinear decay functions, the second inflection point curvature and the y-axis intercept as seen in an R-r plot increased. Higher operant levels affected both inflection curvature and y-axis intercept. For a linear decay function, an operant level greater than 0 percent produced higher rates of responding for the leanest schedules. These higher rates “filled in” ranges of response rates that were previously missing when the operant level was set to 0 percent. Subsequent and further increases in operant level (e.g. 4 vs. 8 percent) did not produce additionally higher rates of responding for the leanest schedules as observed in curvilinear decay function conditions. This finding suggests that a minimal operant level is necessary to affect the model but further increases have no additional affect when using a linear decay function. It is notable that the minimum response rate was not achieved when the operant level was greater than 0 percent suggesting that even leaner schedules may be possible with a linear decay function and non-zero operant levels.

Fits of the classic and modern matching equations to experiment data suggested that the model behavior was not best accounted for by matching theory. Non-random patterns in the residuals from classic matching equation fits were observed in 34 of 39 single repetition experiment sets using curvilinear decay functions. The modern matching equation fit the curvilinear decay function data better with 20 out of 39 experiment sets

having non-random residual patterns. Despite the incremental success of the modern equation in comparison to the classic equation, potential non-random patterns were still not fully explained. For example, a similar pattern in the residuals was observed when comparing across experiment sets with varying operant reserve levels. However, no polynomial trends (up to cubic) were significant. The pattern appeared to be high-order, systematic pattern where standardized residuals followed a negative-positive-negative-positive pattern. Because a single experiment could only provide up to 11 data points that could be analyzed, higher-order polynomial trends such as quartic could not be evaluated without risking false-positive fits. To obtain the necessary number of datapoints, additional repetitions of the same experiment were conducted. With 5 repetitions of an experiment set, up to 55 datapoints could be fitted and high-order polynomial trends evaluated.

The data resulting from 5 repetitions of the experiment set with 3 percent MIC, 1 percent DCR, and 4 percent operant level under reciprocal decay function conditions provided robust evidence that neither the classic or modern matching equations fit the model's behavior data. A qualitative observation of the pooled residuals from those 5 repetitions was sufficient to confirm the presence of non-random patterns. If the residuals were random one would expect to see a randomly distributed cloud with no observable systematic patterns and no relationship between each repetition's residuals. Instead, the residuals from each repetition of the experiment set are clumped together indicating that similar residual values were obtained from each repetition of the experiment set. Interestingly, when the pooled residuals were evaluated for polynomial trends up to cubic, there were no significant trends. However, quartic polynomial trend

was significant. When observing the quartic trendline overlaid on the pooled residuals, the high-order, non-random pattern is clearly present. Given the similar pattern of residuals from single experiment sets under curvilinear conditions, it is likely that high-order trends exist in that data as well. Additional repetitions of all the curvilinear decay functions experiment sets were not conducted for several reasons. First, the presence of a 2nd inflection point in the all the curvilinear decay function data indicated that the operant model under these conditions did not simulate behavior that was comparable to live-organisms. Second, the indication that overmatching existed also did not support correspondence with live-organism data.

In contrast to curvilinear decay function data, the operant model behavior data under linear decay function conditions from the single repetition experiment sets was inconclusive. Although non-random residual patterns were found in over 65% of the matching function fits, the amount of overmatching was less than with curvilinear decay function conditions and no second inflection point was found. The 5 additional repetitions for each linear decay function condition provided the power necessary to thoroughly evaluate the model for correspondence with matching theory. When the pooled data from the 5 repetition experiment sets was analyzed, non-random residual trends were found for all but two conditions. when the operant level was 4 and 8 percent, a statistically significant polynomial trend in the residuals was not found for either classic or modern fits.

It is notable that the R^2 values for the classic and modern matching equation fits were only moderately informative across all experiments sets. The R^2 values indicated that a high proportion of variance was accounted for by both classic and modern

equations. The lowest R^2 value was .89 indicating that 89 percent of the variance in the experiment set data was accounted for by a matching equation. With live organisms, this would be a respectable R^2 value given the variable nature of such data. The presence of significant polynomial trends in most of the current study data suggest that R^2 values alone are not sufficient to establish that a good fit has been achieved. Robust residual analyses are necessary to establish the existence of a good fit.

Based on the current results, a simple model of the operant reserve is not sufficient to simulate the behavior of live organisms. While the model tested in this study produced behavior that was very similar to that of live organisms, it did not produce behavior that consistently adhered to matching theory. Binomial probability calculations indicated that, when considered together, the number of significant non-random patterns found in the data was unlikely due to chance. It remains possible that some combination of model parameters could reliably produce matching behavior. Knowing which parameters to use is unclear based on the current results although the incremental success of a linear decay function and operant level greater than zero suggest that these areas may warrant further investigation. Given the wide range of values currently tested with minimal success, it is likely that any such “perfect” combination of parameters would be very sensitive to any deviations from the “perfect” combination. It is also possible that a more complex model as summarized by Killeen (1988) could produce realistic behavior. Even Skinner (1938), in his original conceptualization of the reserve, envisioned a more complex model. The added complexity of a complex model can be problematic, however. With each additional component added, the number of possible parameter combinations increases. Thus, the chance of error (i.e. finding a particular set of parameters that

produces realistic behavior when the model itself is not a robust behavior simulator) rises as well.

The current study raised certain methodological implications as well. Problems arose when the residuals from a single experiment set (11 data point maximum) were evaluated. The residual plots clearly showed non-random patterns for many model conditions. Higher order trends were not apparent and polynomial trend analysis was limited to lower order trends because the risk of Type I error. Identical repetitions of experiment sets proved highly informative. First, pooling the additional repetition data added power to the residual trend analysis making higher-order trend analysis possible. Second, residuals could be compared across identical experiment set repetitions. If the residuals were random, then one would expect to see a random cloud of residuals where no relationship existed between residuals from one experiment and another. When the residuals were similar, they appeared clumped together on residual plots indicating the possibility of a systematic pattern. However, the bias inherent in visual inspection of data is a drawback and quantitative techniques are needed to remove any analytical bias. Work with live organisms is unlikely to need such analysis tools because of high random error in such data. As computational models of behavior develop, the opportunity to simulate behavior under truly identical conditions will likely require the development of new analysis tools. These tools should take advantage of the computational environment and be able critically evaluate simulated behavior.

Although the model did not reliably produce realistic behavior, it is notable that the decay function that produced the best behavior was a linear one. The result is surprising because, to date, the decay of memory for previous events has been

conceptualized as curvilinear with reciprocal, exponential, and logarithmic functions being proposed as potential decay functions (Catania and Shimoff, 1996; Killeen, 1994; Mazur, 1987). These curvilinear functions are closely related and therefore it should not be surprising that there were few differences in model behavior across hyperbolic, exponential, and reciprocal decay functions in the current results. Previous research has demonstrated preference for stimuli that reduce the time until a reinforcement event occurs with the degree of preference being proportional to the reduction in time (e.g. delay reduction theory, Fantino, 1969). This relationship between degree of stimuli preference and time until reinforcement has been proposed to be linear in form (A. Reid, personal communication, June 5, 2008). Although this concept of delay reduction is not directly comparable to calculating the increase in response probability by summing incremental values from a series of previous responses (as in the current model), it does provide some indirect evidence that a value of an response is linearly related to time until reinforcement.

Although the current study found evidence that suggested the operant reserve model does not reliably and robustly produce realistic behavior, several results should be followed up in future work and an alternative model evaluated. First, all curvilinear decay function conditions should be evaluated with 5 experiment set repetitions to evaluate whether single experiment sets with non-significant trends in contain higher-order trends. Second, other decay functions should be evaluated and compared to the current results. Specifically, a convex function form, which has not been evaluated previously, could be informative considering the current finding that a less concave function (i.e. linear) produced more realistic behavior. Multi-parameter decay functions could also prove

useful by indicating which decay function properties produce what type of behavior.

Third, the operant levels should be evaluated further because the correspondence with matching theory improved when the operant level was increased. Fourth, the fundamental assumptions of the model such as decay function truncation should be tested to determine if their effect on the model. Finally, it should be noted that studies with concurrent or other more complex schedules are not indicated at this point. It is unlikely that such experiments would provide useful information given that the model does not produce realistic behavior on basic, single alternative schedules.

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Table 1

Simple decay function forms and constant, c , conversion factors.

Function	Equation	Maximum Increment (I_{max}) Conversion Factor
Reciprocal	$\rho = \frac{c}{d_0}, (6)$	$c = I_{max}$
Exponential	$\rho = ce^{-d_0}, (7)$	$c = e \cdot I_{max}$
Hyperbolic	$\rho = \frac{c}{1+d_0}, (8)$	$c = 2 \cdot I_{max}$
Linear	$\rho = -d_0 + c, (9)$	$c = I_{max}$

Table 2

Maximum individual contributions (MIC), maximum individual contribution to decrement ratio (MIC:DCR), operant level, parameters of best-fitting classic and modern hyperbolas, and percent variance accounted for (R^2) by classic and modern hyperbolas.

MIC	DCR	Operant Level	Classic hyperbola Parameters			Modern hyperbola parameters				
			k	r_e	R^2	k	a	$\frac{r_e^a}{b}$	R^2	
Reciprocal decay function										
3	3	0	89847	89538	1.00*	31384	1.01	32968	1.00*	
3	2.7	0	2454	1942	1.00	1202	1.19	140	1.00*	
3	2.1	0	979	427	0.98	7451	0.70	981	0.96*	
3	1.5	0	756	180	0.95*	526	2.35	39537	0.99	
3	1	0	644	77	0.94*	511	2.51	19989	0.99	
3	0.3	0	560	13	0.89*	497	2.95	1095	1.00*	
Exponential decay function										
3	3	0	89853	89532	1.00*	2044	1.10	3029	1.00*	
3	2.7	0	2849	2340	1.00	1901	1.06	2038	1.00	
3	2.1	0	1069	536	0.99	605	1.70	7682	1.00	
3	1.5	0	813	245	0.97*	545	1.98	11541	0.99	
3	1	0	697	124	0.96*	527	2.10	7698	0.99	
3	0.3	0	574	22	0.94*	509	2.18	665	0.99	
Hyperbolic decay function										
3	3	0	89843	89542	1.00*	7925	1.02	8707	1.00*	
3	2.7	0	2041	1523	1.00	1422	1.14	2191	1.00	
3	2.1	0	897	328	0.96	542	2.23	51815	0.99	
3	1.5	0	701	125	0.94	506	3.06	467024	1.00	
3	1	0	605	45	0.92*	496	3.46	238913	1.00*	
3	0.3	0	548	9	0.91*	494	2.79	266	1.00	
Linear decay function										
3	3	0	557	188	1.00	544	1.02	192	1.00	
3	2.7	0	542	25	0.99*	478	1.92	462	1.00*	
3	2.1	0	511	12	1.00*	508	1.05	13	1.00*	
3	1.5	0	515	9	1.00*	498	1.23	16	1.00*	
3	1	0	504	5	1.00	403	1.02	5	1.00	
3	0.3	0	501	2	1.00*	504	0.93	1	1.00*	

* statistically significant ($p < .05$) non-random pattern in the residuals

Table 3
Maximum individual contribution (MIC), decrement (DCR), operant level, parameters of best-fitting classic and modern hyperbolas, and percent variance accounted for (R^2) by classic and modern hyperbolas.

			Classic hyperbola Parameters			Modern hyperbola parameters			
MIC	DCR	Operant Level	k	r_e	R^2	k	a	$\frac{r_e^a}{b}$	R^2
Reciprocal decay function									
1.5	1.5	0	79202	78872	1.00*	122243	0.61	13969	1.00*
3	1.5	0	756	179	0.95*	527	2.34	37550	0.99
5	1.5	0	616	59	0.96*	505	2.27	4920	1.00
10	1.5	0	534	16	0.99*	491	2.06	444	1.00*
Exponential decay function									
1.5	1.5	0	44382	44033	1.00*	40754	0.66	5930	0.94*
3	1.5	0	819	249	0.96*	547	1.95	10293	0.99
5	1.5	0	640	88	0.98	505	2.66	66601	1.00*
10	1.5	0	577	37	0.97*	498	2.19	1908	1.00
Hyperbolic decay function									
1.5	1.5	0	369491	369183	1.00*	2723	1.02	2808	1.00*
3	1.5	0	701	124	0.94	506	3.10	527384	0.99
5	1.5	0	506	8	0.80*	490	3.39	149974	1.00
10	1.5	0	513	10	1.00*	500	1.30	24	1.00
Linear decay function									
1.5	1.5	0	574	190	1.00	471	1.23	381	1.00
3	1.5	0	513	9	1.00*	500	1.17	13	1.00
5	1.5	0	505	8	1.00	508	0.96	7	1.00
10	1.5	0	507	8	1.00	507	0.99	8	1.00

* statistically significant ($p < .05$) non-random pattern in the residuals

Table 4

Maximum individual contributions (MIC), decrement (DCR), operant level, parameters of best-fitting classic and modern hyperbolas, and percent variance accounted for (R^2) by classic and modern hyperbolas.

MIC	DCR	Operant Level	Classic hyperbola Parameters				Modern hyperbola parameters				
			k	r_e	y^+	R^2	k	a	$\frac{r_e^a}{b}$	y^+	R^2
Reciprocal decay function											
3	1	0	644	77	0	0.94*	502	2.62	31474	7	0.99
3	1	4	650	80	0	0.94*	470	3.07	226565	34	1.00
3	1	8	643	76	0	0.94*	451	3.42	962231	50	1.00
Exponential decay function											
3	1	0	697	124	0	0.96*	513	2.23	13805	10	0.99
3	1	4	697	125	0	0.95*	480	2.58	71245	34	1.00
3	1	8	689	119	0	0.95*	461	2.84	235456	49	1.00
Hyperbolic decay function											
3	1	0	605	45	0	0.92*	492	3.46	243398	3	1.00*
3	1	4	612	48	0	0.90*	474	3.53	344643	26	1.00
3	1	8	607	46	0	0.90*	446	4.20	4597042	47	1.00
Linear decay function											
3	1	0	504	5	0	1.00	503	1.02	5	0	1.00
3	1	4	504	5	0	1.00	503	1.00	5	0	1.00
3	1	8	502	5	0	1.00*	508	0.93	5	0	1.00*

* statistically significant ($p < .05$) non-random pattern in the residuals

Table 5

Maximum individual contributions (MIC), decrement (DCR), operant level, parameters of best-fitting classic and modern hyperbolas, and percent variance accounted for (R^2) by classic and modern matching functions resulting from 5 repetition pooled data.

MI C	DC R	Opera nt Level	Classic hyperbola Parameters				Modern hyperbola parameters				
			k	r_e	y_+	R^2	k	a	$\frac{r_e^a}{b}$	y_+	R^2
Varying DCR											
3	3	0	571	188	-	1.00*	533	1.07	233	-	1.00*
3	2.7	0	527	19	-	0.99*	496	1.42	68	-	1.00*
3	2.1	0	513	12	-	1.00*	500	1.20	21	-	1.00*
3	1.5	0	508	8	-	1.00*	503	1.08	10	-	1.00*
3	1	0	507	6	-	1.00*	498	1.15	8	-	1.00*
3	0.3	0	501	1	-	1.00	500	1.02	2	-	1.00
Varying MIC											
1.5	1.5	0	572	188	-	1.00*	468	1.24	390	-	1.00
3	1.5	0	509	9	-	1.00*	503	1.09	11	-	1.00*
5	1.5	0	511	9	-	1.00*	500	1.16	13	-	1.00*
10	1.5	0	510	8	-	1.00*	501	1.14	12	-	1.00*
Varying Operant Level											
			509	6		1.00*	497	1.18	8	0	1.00*
3	1	0			0						
3	1	4	493	5	1	1.00	497	0.99	5	8	1.00
					1						
3	1	8	485	5	1	1.00	483	1.00	5	2	1.00
					9					1	

* statistically significant ($p < .05$) non-random pattern in the residuals

Table 6

Binomial probabilities of finding significant residual trends across 13 model conditions.

Decay Function	Classic		Modern	
	k	P	k	P
Reciprocal	11	3.5×10^{-13}	6	2.0×10^{-5}
Exponential	10	2.4×10^{-11}	3	.025
Hyperbolic	9	1.2×10^{-9}	4	.003
Linear (1 repetition)	6	2.0×10^{-5}	5	2.9×10^{-4}
Linear (5 repetitions)	10	2.4×10^{-11}	9	1.2×10^{-9}

k : number of significant non-random residual patterns observed
 P : binomial probability

Figure Captions

Figure 1. Top graph shows a reciprocal decay function gradient reproduced from Catania (2005). Responses contribute to the operant reserve by an amount determined by the value of the delay gradient at the time coincident with the response. Bottom graph shows a truncated delay gradient utilized to avoid overlapping delay gradients.

Figure 2. Comparison of simple reciprocal, exponential, hyperbolic, and linear decay function forms.

Figure 3. Single alternative cumulative records of an operant reserve computational model on a RI 8 schedule of reinforcement with a reciprocal decay function, 0 percent operant level of behavior, a decrement of 1 percent, and the maximum increment set to 3 percent of the maximum reserve.

Figure 4. Response-reinforcement plots of a computational operant reserve model with MIC = 3 %, 0 % operant level, and varying DCR (.3, 1, 1.5, 2.1, 2.7 and 3 %) for four decay functions (reciprocal, exponential, hyperbolic, and linear).

Figure 5. Response-reinforcement plot with overlaid classic and modern fitted hyperbolas (top graph) for computational operant reserve model with reciprocal decay function, 0 percent operant behavior level, 3 percent MIC, and a 1 percent DCR. Bottom graph shows residuals of classic and modern hyperbolas.

Figure 6. Response-reinforcement plot with overlaid classic and modern fitted hyperbolas (top graph) for computational operant reserve model with exponential decay function, 0 percent operant behavior level, 3 percent MIC, and a 1 percent DCR. Bottom graph shows residuals of classic and modern hyperbolas.

Figure 7. Response-reinforcement plot with overlaid classic and modern fitted hyperbolas (top graph) for computational operant reserve model with hyperbolic decay function, 0 percent operant behavior level, 3 percent MIC, and a 1 percent DCR. Bottom graph shows residuals of classic and modern hyperbolas.

Figure 8. Response-reinforcement plot with overlaid classic and modern fitted hyperbolas (top graph) for computational operant reserve model with linear decay function, 0 percent operant behavior level, 3 percent MIC, and a 1 percent DCR. Bottom graph shows residuals of classic and modern hyperbolas.

Figure 9. Standardized residuals from six classic and modern hyperbola fits to operant reserve computational model behaviors with a reciprocal decay function, 0 percent operant behavior level, and 3 percent MIC for six DCR values.

Figure 10. Standardized residuals from six classic and modern hyperbola fits to operant reserve computational model behaviors with a exponential decay function, 0 percent operant behavior level, and 3 percent MIC for six DCR values.

Figure 11. Standardized residuals from six classic and modern hyperbola fits to operant reserve computational model behaviors with a hyperbolic decay function, 0 percent operant behavior level, and 3 percent MIC for six DCR values.

Figure 12. Standardized residuals from six classic and modern hyperbola fits to operant reserve computational model behaviors with a linear decay function, 0 percent operant behavior level, and 3 percent MIC for six DCR values.

Figure 13. Response-reinforcement plots of a computational operant reserve model with DCR = 1.5 percent, 0 percent operant level, and varying MIC (1.5, 3, 5 and 10 percent) for four decay functions (reciprocal, exponential, hyperbolic, and linear).

Figure 14. Standardized residuals from four classic and modern hyperbola fits to operant reserve computational model behaviors with a reciprocal decay function, 0 percent operant behavior level, and 1.5 percent DCR for four MIC levels.

Figure 15. Standardized residuals from four classic and modern hyperbola fits to operant reserve computational model behaviors with a exponential decay function, 0 percent operant behavior level, and 1.5 percent DCR for four MIC levels.

Figure 16. Standardized residuals from four classic and modern hyperbola fits to operant reserve computational model behaviors with a hyperbolic decay function, 0 percent operant behavior level, and 1.5 percent DCR for four MIC levels.

Figure 17. Standardized residuals from four classic and modern hyperbola fits to operant reserve computational model behaviors with a linear decay function, 0 percent operant behavior level, and 1.5 percent DCR for four MIC levels.

Figure 18. Effect of varying operant behavior level for computational operant reserve model with reciprocal decay function, 3 percent MIC, and a 1 percent DCR. Upper plot shows entire range of results. Lower plot shows lower range of same results.

Figure 19. Effect of varying operant behavior level for computational operant reserve model with exponential decay function, 3 percent MIC, and a 1 percent DCR. Upper plot shows entire range of results. Lower plot shows lower range of same results.

Figure 20. Effect of varying operant behavior level for computational operant reserve model with hyperbolic decay function, 3 percent MIC, and a 1 percent DCR. Upper plot shows entire range of results. Lower plot shows lower range of same results.

Figure 21. Effect of varying operant behavior level for computational operant reserve model with linear decay function, 3 percent MIC, and a 1 percent DCR. Upper plot shows entire range of results. Lower plot shows lower range of same results.

Figure 22. Response-reinforcement plot with overlaid classic and modern fitted hyperbolas (top graph) for computational operant reserve model with reciprocal decay function, 8 percent operant behavior level, 3 percent MIC, and a 1 percent DCR. Bottom graph shows residuals of classic and modern hyperbolas.

Figure 23. Response-reinforcement plot with overlaid classic and modern fitted hyperbolas (top graph) for computational operant reserve model with exponential decay function, 8 percent operant behavior level, 3 percent MIC, and a 1 percent DCR. Bottom graph shows residuals of classic and modern hyperbolas.

Figure 24. Response-reinforcement plot with overlaid classic and modern fitted hyperbolas (top graph) for computational operant reserve model with hyperbolic decay function, 8 percent operant behavior level, 3 percent MIC, and a 1 percent DCR. Bottom graph shows residuals of classic and modern hyperbolas.

Figure 25. Response-reinforcement plot with overlaid classic and modern fitted hyperbolas (top graph) for computational operant reserve model with linear decay function, 8 percent operant behavior level, 3 percent MIC, and a 1 percent DCR. Bottom graph shows residuals of classic and modern hyperbolas.

Figure 26. Standardized residuals from classic and modern hyperbola fits to operant reserve computational model behaviors with a reciprocal decay function, 3 percent MIC, 1 percent DCR, and three operant behavior levels.

Figure 27. Standardized residuals from classic and modern hyperbola fits to operant reserve computational model behaviors with an exponential decay function, 3 percent MIC, 1 percent DCR, and three operant behavior levels.

Figure 28. Standardized residuals from classic and modern hyperbola fits to operant reserve computational model behaviors with a hyperbolic decay function, 3 percent MIC, 1 percent DCR, and three operant behavior levels.

Figure 29. Standardized residuals from classic and modern hyperbola fits to operant reserve computational model behaviors with a linear decay function, 3 percent MIC, 1 percent DCR, and three operant behavior levels.

Figure 30. Standardized residuals from modern hyperbola fits to operant reserve computational model behaviors resulting from single (top panel) and five identical (middle panel) experiments sets with a linear decay function, 3 percent MIC, and 1 percent DCR. Bottom panel shows cubic and quartic polynomial fits to 5 experiment set pooled data.

Figure 31. Standardized residuals from classic hyperbola fits to operant reserve computational model behaviors resulting from five identical experiments with a linear decay function, 0 percent operant behavior level, and 3 percent MIC for six DCR values.

Figure 32. Standardized residuals from modern hyperbola fits to operant reserve computational model behaviors resulting from five identical experiments with a linear decay function, 0 percent operant behavior level, and 3 percent MIC for six DCR values.

Figure 33. Standardized residuals from classic hyperbola fits to operant reserve computational model behaviors resulting from five identical experiments with a linear

decay function, 0 percent operant behavior level, and 1.5 percent DCR for four MIC levels.

Figure 34. Standardized residuals from modern hyperbola fits to operant reserve computational model behaviors resulting from five identical experiments with a linear decay function, 0 percent operant behavior level, and 1.5 percent DCR for four MIC levels.

Figure 35. Standardized residuals from classic hyperbola fits to operant reserve computational model behaviors resulting from five identical experiments sets with a linear decay function, 3 percent MIC, 1 percent DCR, and three operant behavior levels.

Figure 36. Standardized residuals from modern hyperbola fits to operant reserve computational model behaviors resulting from five identical experiments sets with a linear decay function, 3 percent MIC, 1 percent DCR, and three operant behavior levels.







































































