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Predicting and Testing a Contemporary Quantitative Model of Punishment

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Bryan Klapes
M.A., Emory University, 2016
B.S., Emory University, 2013

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An abstract of
A dissertation submitted to the Faculty of the
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Abstract

Predicting and Testing a Contemporary Quantitative Model of Punishment

By Bryan Klapes

In this dissertation, five novel quantitative models of punishment based on the generalized matching law (GML; Baum, 1974) were developed. The descriptive accuracies of these models were tested against one another and against the GML in three experiments using information criteria. Experiment 1 entailed a reanalysis of previously collected live organism data. None of the punishment models were supported over the GML. It was hypothesized that this result was likely due to the small number of data points used to fit each model. Thus, two additional experiments were performed using datasets with many more data points per fit. Experiment 2 utilized a well-regarded computational theory of operant behavior known as the Evolutionary Theory of Behavior Dynamics (ETBD; McDowell, 2004). Experiment 3 was a replication of Experiment 2 using human participants who worked on a recently developed rapid-acquisition procedure called a Procedure for Rapidly Establishing Steady-State Behavior (PRESS-B; Klapes et al. 2020). These experiments initially resulted in divergent conclusions: the ETBD predicted that the GML was the superior model, while data generated by PRESS-B showed that a punishment model based on the concatenated GML (cGML; Davison & McCarthy, 1988) was superior to the GML and the other punishment models. Experiment 2 was found to have relatively weak punishing contingencies, however, which was the likely source of the discrepant conclusions. Thus, the cGML-based punishment model is presumed to be the best contemporary quantitative model of punishment.

*Keywords:* matching law; punishment; model development; information criteria

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This dissertation will focus on the effect of punishment on free operant behavior. In the field of quantitative behavior analysis, free operant behavior is often studied in the form of continuous choice. Continuous choice experiments present the participant with two or more alternatives upon which they can perform operant behaviors. They are free to choose between these alternatives, which are always present. Continuous choice can result in many different behavior patterns, depending on the reinforcing contingencies that are in place. For example, there are differences in continuous choice behavior when reinforcer delivery is contingent upon performing the operant behavior after a certain amount of time has elapsed since the last reinforcer delivery on that alternative (i.e., an interval schedule) compared to when reinforcer delivery is contingent upon performing the operant behavior a certain number of times since the last reinforcer delivery on that alternative (i.e., a ratio schedule). When interval schedules of reinforcement deliver reinforcers unpredictably and on two possible alternatives (concurrent variable-interval variable-interval schedules, or conc VI VI), then the participant tends to allocate the same proportion of behavior across the two alternatives as the proportion of acquired reinforcers is allocated across the two alternatives. This pattern of behavior allocation is called matching and has been very effectively modeled by a quantitative statement called the matching law (Herrnstein, 1961). To properly observe the effect of punishment on continuous choice behavior, this dissertation will focus on the situations where conc VI VI schedules of punishment are superimposed on conc VI VI schedules of reinforcement.

## 1.1. The Matching Law

The matching law is a quantitative statement of continuous choice formulated by Herrnstein (1961). In his original experiment, three White Carneuax pigeons served as subjects. He placed them in the standard operant chamber typically used with pigeons (Ferster & Skinner, 1957); these are usually made of metal, large enough for the pigeon to comfortably move about, and contain a response key affixed to one of the sides that detects pecks as operant behaviors. In this case, Herrnstein modified a standard operant chamber to contain two response keys. Thus, he was able to run the pigeons on conc VI VI schedules of reinforcement (where access to grain served as the reinforcer) to assess their continuous choice behavior. The reinforcement schedules were independently scheduled; that is, the reinforcement acquired on one alternative had no effect on the reinforcement schedule on the other alternative.

Herrnstein (1961) exposed the pigeons to four sets of conc VI VI schedules of reinforcement. In each set, he modified the intervals of average reinforcer delivery. For example, during the second of these conc VI VI schedules, one alternative made a reinforcer available approximately every 2.25 minutes after the last reinforcer was acquired while the other alternative made a reinforcer available approximately every 4.5 minutes after the last reinforcer was acquired (conc VI 2.25-min VI 4.5-min). When plotting the proportion of responses and acquired reinforcers on each alternative for each reinforcement schedule, Herrnstein noticed that the proportion of responses allocated across the two alternatives approximated the proportion of reinforcement acquired across the two alternatives (i.e., the pigeons "matched" their response allocation to the reinforcer allocation). Mathematically, Herrnstein's matching law is written as

$$\frac{B_1}{B_1 + B_2} = \frac{R_1}{R_1 + R_2}, \qquad (1a)$$

where $B_1$ is the number of responses emitted on the first alternative, $B_2$ is the number of responses emitted on the second alternative, $R_1$ is the number of acquired reinforcers on the first alternative, and $R_2$ is the number of acquired reinforcers on the second alternative.

Subsequently, Herrnstein (1970) performed algebraic manipulations of his original formulation to develop a quantitative relationship between single-alternative responding and reinforcement. This statement was achieved by treating the first alternative as the sole alternative of interest (i.e., $B_1$ and $R_1$ are now just *B* and *R*) and by treating the second alternative as all other possible behaviors the organism could perform (i.e., $B_2$ and $R_2$ are now $B_e$ and $R_e$; "*e*" subscript for "extraneous"):

$$\frac{B}{B + B_e} = \frac{R}{R + R_e}.$$

It is important to note that because this equation is used in situations when only one alternative's reinforcement rate is experimentally manipulated, $B_e$ and $r_e$ will always be estimated parameters. Given this flexibility of $B_e$, Herrnstein chose to operate under the assumption that there is a finite number of behaviors an organism could perform at any one time; mathematically, this would allow for a constant, *k*, to be substituted for the sum of *B* and $B_e$:

$$\frac{B}{k} = \frac{R}{R + R_e}.$$

By multiplying both sides of the equation by *k*, one achieves

$$B = \frac{k\,R}{R + R_e}.$$

This expression is often referred to as the quantitative law of effect (QLE; de Villiers, 1977), due to its resemblance to the reinforcement tenet of Thorndike's (1911) Law of Effect: "[o]f several responses made to the same situation, those which are accompanied or closely followed by satisfaction to the animal will… be more likely to recur…" (pg. 244).

As the volume of research devoted to continuous choice increased, it became evident that the simple, proportional relation described by Herrnstein's (1961) matching law was insufficient. Lander and Irwin (1968) and Staddon (1968) presented a modified version of Herrnstein's original formulation to account for these insufficiencies as

$$\frac{B_1}{B_2} = b\left(\frac{R_1}{R_2}\right)^a. \qquad (1b)$$

The coefficient, *b*, accounts for asymmetries in response allocation not attributable to the reinforcement rate. For example, two stimuli of differing hedonic magnitude could be used as reinforcers across the two alternatives (e.g., a pigeon exposed to a continuous choice experiment where access to buckwheat acts as the reinforcer on the first alternative and access to hemp acts as the reinforcer on the second alternative; Miller, 1976). This experimental design would result in the subject shifting its response allocation towards the alternative with the "better" reinforcer, even though it may be acquiring the same number of reinforcers on each alternative. The exponent, *a*, accounts for organisms' inherent tendency to allocate slightly more responses than expected to the alternative with the leaner reinforcement rate (i.e., "undermatching"; Myers & Myers, 1977).[1] To fit this equation directly to data, non-linear regression techniques would be necessary. Thus, to permit fitting the model using ordinary least-squares (OLS) regression, Equation 1b is often logarithmically transformed to

$$\log\left(\frac{B_1}{B_2}\right) = a\log\left(\frac{R_1}{R_2}\right) + \log(b).$$

[1] Although uncommon, the opposite effect of "overmatching" (i.e., the organism allocates slightly more responses than expected to the alternative with the richer reinforcement rate) can sometimes occur.

Both Equation 1b and its logarithmic transformation are typically referred to as the generalized matching law (GML; Baum, 1974).

The GML is a better descriptor of concurrent choice behavior than Herrnstein's (1961) original matching law (see review by McDowell, 2013a). The GML's ability to account for much more of the variance in the data (typically assessed by proportion variance accounted for, or PVAF) is due to its ability to effectively model undermatching and systematic bias; undermatching is modeled in the GML by the *a* parameter almost always being estimated as less than 1, whereas bias is modeled by the *b* parameter being estimated as greater than 1 for bias towards the first alternative and less than 1 for bias towards the second alternative. Nonetheless, the GML has a conspicuous omission that makes it unable to account for most naturalistic environments: it only uses reinforcement rates to predict response allocation. Real-life scenarios, in contrast, tend to have both costs and benefits associated with the performance of operant behavior. Hence, the GML, in its current state, cannot handle situations where both reinforcing and punishing contingencies are present. Over the years, two theories of punishment have been used to develop matching-law-based punishment models: the additive and subtractive theories of punishment.

## 1.2. Matching-Law-Based Punishment Models

### *The Additive Theory of Punishment*

The additive theory of punishment is rooted in a phenomenon called behavioral contrast (Reynolds, 1961). Behavioral contrast is an effect that can be seen in a continuous choice experiment; when an organism increases its response rate on one alternative, there will be an observable decrease in the responses allocated to the opposite alternative, even if the reinforcement rate has not changed (Catania, 1963). The implementation of a punisher, by definition, should decrease the rate of responding on the punished alternative. Thus, if one alternative is punished, then there should be an increase in responding on the other alternative (Herman & Azrin, 1964). That is, punishment suppresses behavior on the punished alternative by increasing the value of the other alternative and making the other alternative more attractive.

Noting the dearth of experiments on punishment and continuous choice, Deluty (1976) wished to empirically study the effect of punishment schedules superimposed on concurrent reinforcement schedules. He exposed three albino Charles River CD strain Norway rats to conc random-interval (RI)[2] 1.5-min RI 1.5-min reinforcement (45-mg food pellets) schedules for lever pressing. Next, he re-exposed the rats to the conc RI 1.5-min RI 1.5-min schedule of reinforcement for five conditions and superimposed RI punishment schedules (electric shock to the levers, floor, and sides of the operant chamber) on the two alternatives. During the punishment conditions, the first alternative always had a superimposed RI 6-min punishment schedule, while the RI value of a superimposed punishment schedule on the second alternative varied (viz., RI 0-min, 12-min, 6-min, 4-min, and 3-min). After plotting the response rates on the two alternatives across the different conditions, Deluty noted that the rats' behavior on the first alternative increased as the punishment on the second alternative increased, even though the reinforcement schedule on both alternatives stayed constant across all conditions of the experiment. Based on these results, he proposed an additive matching-law-based model,

[2] RI and VI schedules are functionally equivalent. For VI schedules, the intervals are predetermined and follow a specific sequence (Fleshler & Hoffman, 1962). This sequence leads to a pseudo-random appearance of the intervals. RI schedules are similar but use probability distributions to completely randomize the scheduling of reinforcer delivery. The mean of the distribution represents the RI value for that schedule.

$$\frac{B_1}{B_1 + B_2} = \frac{R_1 + P_2}{R_1 + P_2 + R_2 + P_1}, \qquad (2a)$$

where $P_2$ is the number of punishers acquired on the second alternative and $P_1$ is the number of punishers acquired on the first alternative. This model suggests that, in a two-alternative system, acquired punishers on one alternative add to the reinforcing value of the other alternative. Importantly, this equation was the first matching-law-based model of punishment to be proposed based on empirical data.

A single-alternative human operant study performed by Bradshaw et al. (1978) lends more support for the additive theory of punishment. Bradshaw et al. had four human participants press a lever for monetary reinforcement (acquisition of one pence per reinforcer delivery). Each participant worked in five 10-min conditions, each with its own on VI reinforcement schedule. They then re-exposed these participants to the same set of VI reinforcement schedules but superimposed a VI 170-s punishment (removal of one pence per punisher delivery) schedule on the lever. The QLE (for reference, see p. 3) was fitted to both sessions and the parameters of each fit were compared. For all four participants, there was no difference between the *k* parameter estimated from QLE fits to the baseline condition data and that estimated for the punishment condition data. However, there was a significant increase in the $R_e$ parameter estimates from the QLE fits to the punishment condition data compared to the estimates for the baseline condition data for all four participants. In summary, the results indicated that there was no change in the estimated overall amount of behavior emitted by the organism in punishing conditions; rather there was an increase in the value of reinforcement acquired by performing extraneous behavior compared to the target behavior. The additive theory of punishment would predict this result.

### ***The Subtractive Theory of Punishment***

Estes (1969) categorized punishment theories as unlearning, relearning, and suppression theories. Unlearning theories are predicated on the idea that a punishing contingency will change behavior by eroding an existing reinforcement contingency, or contingencies, in place for that behavior. Relearning theories are based on the conversion of experiences with punishing stimuli into avoidance-based reinforcement behavioral paradigms. One can see that Deluty's (1976) additive model would fall under this category; punishment on one alternative leads to an increase in avoidant behavior away from the punished alternative and onto the other alternative. Estes argued that, although reported in the literature, only a subset of experimental data supported unlearning and relearning theories. Suppressive theories state that punishing contingencies will directly suppress the ongoing behavior, rather than modifying the reinforcing contingency or contingencies also in place for that behavior. Hence, suppressive theories would hypothesize a direct, subtractive relationship between punishment and the targeted behavior.

de Villiers (1977) was the first to suggest a matching-law-based implementation of the subtractive theory using data from de Villiers and Millenson's (1972) "conditioned anxiety" [3] (Estes & Skinner, 1941) experiment. They exposed three rats to a conc RI 2-min RI 2-min schedule of reinforcement (0.1 ccs of sweetened milk). They used a 0.25 milliamp, scrambled electric shock to the grid floor of the chamber as the randomly-delivered, unavoidable punisher, and 200-ms flashes of three lights mounted to the front of the chamber as the CS. de Villiers and

[3] During conditioned anxiety experiments, an aversive stimulus is delivered randomly to the participant. Importantly, this delivery is not contingent on the participant's behavior. When the participants are exposed to conditions where the aversive stimulus is being implemented, another stimulus (auditory, like a tone, or visual, like a light) is delivered. The pairing of the auditory or visual stimulus with the aversive stimulus results in the auditory or visual stimulus becoming a conditioned stimulus (CS) to the aversive stimulus. The participant will then exhibit avoidance or escape behavior in the presence of the CS (hence the name "conditioned anxiety").

Millenson made the reinforcer available for 4.5 s on one alternative and for 1.5 s on the other, which would induce preference for the alternative with a longer availability of sweetened milk. They found that the rats allocated a greater proportion of their behavior to the preferred lever (i.e., the one with 4.5 s of milk availability) when the aversive stimulus was present compared to when it was not.

de Villiers (1977) shows that this response allocation pattern suggests that the aversive stimulus is suppressing behavior equally on both alternatives. Because there was only one punisher delivered across both alternatives, he modeled this idea within Herrnstein's (1961) original matching law (Equation 1) as

$$\frac{B_1}{B_1 + B_2} = \frac{R_1 - P}{R_1 - P + R_2 - P}.$$

When punishers are independently scheduled on each alternative, this model can be more easily compared with Deluty's (1976) additive model (Equation 2a) by modifying it as

$$\frac{B_1}{B_1 + B_2} = \frac{R_1 - P_1}{R_1 - P_1 + R_2 - P_2}. \qquad (3a)$$

This model suggests that acquired punishers directly subtract from the reinforcing value of alternative upon which it is delivered.

***The "Rate-of-Exchange" Parameter***

For non-human animal experiments, reinforcers and punishers are almost always qualitatively inequivalent. For example, Farley and Fantino (1978) exposed pigeons to a set of concurrent schedules of reinforcement (3-second access to mixed grain) with superimposed punishment schedules (electric shock to the pelvic bone). They accounted for the asymmetry between the two qualitatively inequivalent stimuli by incorporating a “rate-of-exchange” parameter that converts shocks to “negative food units” in the subtractive model:

$$\frac{B_1}{B_1 + B_2} = \frac{R_1 - cP_1}{R_1 - cP_1 + R_2 - cP_2}, \qquad (3a')$$

where *c* is a non-negative estimate of the relative subjective magnitude of a reinforcer and a punisher. A negative value for *c* would state that acquired punishment on one alternative is adding to the reinforcing value on that alternative, which is clearly not following the subtractive theory. To eliminate the possibility that the model would result in a negative response allocation prediction, *c* must also be constrained to values less than the smallest ratio of acquired reinforcement to acquired punishment (i.e., *R*/*P*) across all components of the different schedules. Farley (1980) similarly modified Deluty’s (1976) additive model to include a rate-of-exchange parameter as

$$\frac{B_1}{B_1 + B_2} = \frac{R_1 + cP_2}{R_1 + cP_2 + R_2 + cP_1}. \qquad (2a')$$

As with Equation 3a′, *c* must also be positive to retain the model’s additive rationale.

The “rate-of-exchange” parameter has an analog in Prospect Theory (PT; Kahneman & Tversky, 1979), which is a theory of decision making held in high regard by experts in the field of behavioral economics. A major tenet of PT is that choices are context-dependent, particularly when relating gains and losses. This phenomenon is called loss aversion (Kahneman & Tversky, 1984) and can be estimated using a mixed-gamble procedure. This task requires the participant to choose between two or more alternatives that have the same expected value but discrepant probabilities and magnitudes of gains and losses. For example, a mixed-gambles task may require a participant to choose between one alternative with a 75% chance of losing $100 and a

25% chance of winning $300, and another alternative with a 50% chance of losing $100 and a 50% chance of winning $100. Studies using mixed-gambles tasks (e.g., Abdellaoui et al., 2007; Schmidt & Traub, 2002; Tom et al., 2007) have shown that a loss will have approximately twice as much impact on one's behavior as an objectively-equivalent gain (Sokol-Hessner et al., 2009). To incorporate these findings, the inclusion of a conversion factor for the relative subjective magnitudes of punishers and reinforcers appears to be essential for future matching-law-based punishment models to be successful.

### *Comparisons of the Additive and Subtractive Theories*

**de Villiers (1980) study with pigeons.** de Villiers was the first to directly compare the additive and subtractive theories. He focused his analysis on the competing qualitative predictions made by the two theories. Specifically, he used an example scenario where one alternative has a richer reinforcement schedule than the other in a baseline condition. Then, equivalent punishment schedules are superimposed on these same reinforcement schedules in a punished condition. de Villiers showed that the subtractive theory predicts that there will be a greater proportion of responses allocated to the richer alternative (i.e., shift towards overmatching) in the punishment condition compared to the baseline condition, whereas the additive theory predicts that there will be a greater proportion of responses allocated to the leaner alternative (i.e., shift towards undermatching) in the punishment condition compared to the baseline condition. He tested these qualitative predictions in two experiments.

In Experiment 1, de Villiers (1980) exposed three pigeons to an independent conc VI 3-min VI 1-min schedule of reinforcement (3-s access to mixed grain) in a standard two-key operant conditioning chamber. After completing this baseline condition, he replicated the procedure multiple times with a superimposed inter-dependent[4] (Stubbs & Pliskoff, 1969) VI 30-s punishment schedule (electric shock to the bird's pelvic bone) with equal probabilities on the reinforcement schedules. In each punishment condition, he varied the shock intensity. de Villiers calculated the difference in the response allocation proportion from the proportion of acquired reinforcement across the two alternatives (i.e., the predicted response allocation proportion obtained from Equation 1) to determine the shift toward under- or over-matching. Two of the pigeons exhibited an increased preference for the richer alternative (i.e., shift toward overmatching) in all punishment conditions compared to baseline, and this preference increased as the shock intensity increased. The other pigeon exhibited an initial decreased preference for the richer alternative (i.e., shift toward undermatching) in the first punishment condition compared to baseline; however, all the other punishment conditions elicited shifts toward overmatching compared to the baseline condition for this pigeon.

de Villiers (1980) noted that the acquired and scheduled reinforcement rates were quite different for Experiment 1. In Experiment 2, he used inter-dependent VI schedules for both reinforcement and punishment. He exposed another three pigeons to an inter-dependent VI 40-s reinforcement schedule with probabilities of 0.75 and 0.25 for the first and second alternatives, respectively. As in Experiment 1, he superimposed an inter-dependent VI 30-s punishment schedule with equal probabilities on the reinforcement schedules and varied the shock intensity.

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[4] Interdependent schedules have only one interval determining stimulus delivery, with a probability of delivering that stimulus to either alternative. For example, an inter-dependent VI 10-s schedule of reinforcement with probabilities of 0.75 and 0.25 would set-up the delivery of a reinforcer approximately every 10 seconds after the last reinforcer was acquired. Once the interval has elapsed, there is a 75% chance the reinforcer would be available on the first alternative and a 25% chance it would be available on the second alternative. The advantage of this type of schedule is that the acquired and scheduled rates of stimulus delivery are more likely to be similar to each other than when they are delivered according to independently scheduled conc VI VI schedules.

de Villiers showed that one pigeon shifted toward overmatching in all punishment conditions compared to the baseline. The other two pigeons shifted towards overmatching compared to the baseline condition in the two of the three conditions. Altogether, the results of de Villiers' Experiments 1 and 2 support the subtractive theory's prediction over the additive theory's prediction.

**Farley's (1980) study with pigeons.** Farley performed three experiments to bolster de Villiers' (1980) support for the subtractive theory of punishment over the additive theory. Experiment 1 was a comprehensive extension of de Villiers' experiments. Farley exposed three pigeons to a set of five independent conc VI VI schedules of reinforcement (3-sec access to mixed grain) in a standard two-key operant conditioning chamber. After completing this baseline condition, he repeated the set of reinforcement schedules and superimposed a conc VI 30-s VI 30-s punishment schedule (electric shock to the pelvic bone of the bird) on the reinforcement schedules (i.e., a "constant and equal" VI value of superimposed punishment schedules across all five reinforcement schedules). He then plotted the relative rate of responses on the two alternatives against the relative rate of reinforcement acquired on the two alternatives for both conditions. By visual inspection of his plots, he concluded that, like de Villiers' pigeons, the subjects did indeed exhibit a preference shift towards overmatching in the punished condition compared to the baseline condition.

In Experiment 2, Farley (1980) tested another divergence in the qualitative predictions made by the additive and subtractive theories. He showed that when overall reinforcement is held constant and overall punishment is increased, the additive theory predicts that overall response rates will also increase. In this same scenario, the subtractive theory predicts that there will be a suppression of overall response rates. Farley exposed two sets of three pigeons to independent conc VI VI schedules of reinforcement with superimposed independent conc VI VI schedules of punishment. The first set of pigeons was exposed to a conc VI 30-s VI 30-s reinforcement schedule (“nondifferential reinforcement”) in a baseline condition, and three punishment conditions of varying superimposed punishment frequency. The second set of pigeons was exposed to a conc VI 30-s VI 15-s reinforcement schedule (“differential reinforcement”) in a baseline condition, and the same three punishment conditions of superimposed punishment schedules as the other set. The response rates across both alternatives, for all birds (i.e., for both the differential and nondifferential reinforcement groups), were lower in all punishment conditions compared to the baseline condition.

In Experiment 3, Farley (1980) showed the additive theory predicts a shift toward overmatching as the overall reinforcement of a differential reinforcement scenario is increased and punishment is held constant. In this same scenario, the subtractive theory predicts that there will be a shift toward undermatching as the overall reinforcement density is increased. Farley exposed five pigeons (three from Experiment 2, and two with extensive experimental experience) to three independent conc VI VI schedules of reinforcement with an equal reinforcement ratio but increasing reinforcement density (viz., VI 60-s VI 120-s, VI 30-s VI 60-s, and VI 15-s VI 30-s). He then superimposed an independent conc VI 4-min VI 2-min schedule of punishment on all the reinforcement schedules. By examining the relative responses on the richer alternative, Farley concluded that all five birds increasingly shifted toward undermatching as the overall reinforcement density increased. In summary, all three results of Farley’s experiments followed the predictions made by the subtractive theory, rather than the predictions made by the additive theory.

**Critchfield et al.'s (2003) study with humans.** Critchfield et al. extended de Villiers' (1980) and Farley's (1980) comparisons of the additive and subtractive theories to human participants. They conducted three continuous choice experiments on undergraduate students. Using a computerized operant system (Madden & Perone, 1999), participants were exposed to conc VI VI reinforcement schedules (receipt of $0.08 per reinforcer delivery) and superimposed VI punishment (deduction of $0.08 per punisher delivery) schedules where the instrumental response was mouse clicking. Experiment 1 was performed to show that the procedure correctly implemented punishment. They exposed four participants to a conc VI 20-s VI 20-s reinforcement schedule. They then re-exposed the participants to the same reinforcement schedule but superimposed a VI 40-s punishment schedule on the preferred alternative (i.e., the one that generated more responses) from the baseline reinforcement condition. Critchfield et al. showed that the response rate on the punished alternative decreased and the response rate on the unpunished alternative increased from baseline in the punished condition. These results established that the chosen aversive stimulus (i.e., monetary loss) served as an effective punisher.

Critchfield et al.'s (2003) Experiment 2A was a replication of de Villiers' (1980) Experiment 1 with human participants. They exposed five participants to a conc VI 15-s VI 30-s reinforcement schedule. They then re-exposed these participants to the same concurrent reinforcement schedule while superimposing a conc VI 60-s VI 60-s punishment schedule (50% punishment condition) on the reinforcement schedule. They also included another condition where they superimposed a conc VI 30-s VI 30-s punishment schedule (100% punishment condition). Visual inspection of the data indicated a shift towards overmatching in the punishment conditions compared to the baseline condition. Additionally, Critchfield et al. noted that the shift increased as the punishment density increased (i.e., an increased shift towards overmatching in the 100% punishment condition compared to the 50% punishment condition).

Instead of a “constant and equal” superimposition of VI punishment on the reinforcement schedules, Critchfield et al. (2003) used Experiment 2B to test a different punishing contingency. Using the same five participants from Experiment 2A, they implemented a conc VI 12-s VI 60-s reinforcement schedule for the baseline condition. They then re-exposed the participants to this reinforcement schedule under three punishment conditions in which the VI values of superimposed punishment schedules were proportional to that of the VI values of the reinforcement schedules (e.g., a superimposed conc VI 48-s VI 240-s punishment schedule has VI values that are four times the VI values of the concurrent reinforcement schedule upon which it is superimposed). Critchfield et al. explained that, in this scenario, the additive theory predicts that the participants would shift toward greater undermatching in the punishment condition compared to the baseline condition, whereas the subtractive theory would predict no change in response allocation. They concluded, by visual inspection of the plotted data, that three of the five participants showed no change in their response allocation between the baseline and punished conditions.

In Experiment 3, Critchfield et al. (2003) built new additive and subtractive models that incorporated the power-function aspect of the GML as

$$\log\left(\frac{B_1}{B_2}\right) = a\log\left(\frac{R_1 + P_2}{R_2 + P_1}\right) + \log(b) \qquad (2b)$$

and

$$\log\left(\frac{B_1}{B_2}\right) = a\log\left(\frac{R_1 - P_1}{R_2 - P_2}\right) + \log(b) \qquad (3b)$$

for the additive and subtractive theories, respectively. Equations 2b and 3b will be referred to as "generalized" versions of the additive and subtractive models, as they resemble the GML. Note that, as in the original versions of these equations, acquired punishers on one alternative add to the reinforcing value of the other alternative in the generalized additive model, while acquired punishers directly subtract from the reinforcing value of the targeted alternative in the generalized subtractive model.

In Experiment 3A, four participants were exposed to a set of five conc VI VI schedules of reinforcement. They were then re-exposed to that same set of conc VI VI reinforcement schedules, but with a conc VI 85-s VI 85-s punishment schedule superimposed on all schedules (i.e., a "constant and equal" punishment condition). For all four participants,[5] the generalized subtractive model (Equation 3b) accounted for more variance in the data than the generalized additive model (Equation 2b). In Experiment 3B, nine participants were exposed to a set of seven conc VI VI schedules of reinforcement. They were then re-exposed to that same set of conc VI VI reinforcement schedules, but with superimposed punishment schedules with VI values that were always double that of the VI value of the reinforcement schedule upon which it was superimposed (i.e., a "proportional" punishment condition). For seven of the nine participants,[6] the generalized subtractive model accounted for more variance in the data than the generalized additive model.

[5] The data for the VI 11-s VI 80-s schedule were not presented (in Appendix C of the article) for participant 514. This includes the data for this schedule during the punishment condition.

[6] Although the article describes seven conc VI VI reinforcement schedules for this experiment, none of the participants' displayed data (Appendix C) consists of seven schedules. The appendix displays six schedules for participants 209, 252, and 254, five schedules for participants 243 and 253, and four schedules for participants 210, 265, 267, and 268. These schedules also appear to be omitted for the punishment condition.

**Klapes et al. (2018) reanalysis.** In summary, the above studies comparing the additive and subtractive theories (Critchfield et al., 2003; de Villiers, 1980; Farley, 1980) give strong support for the subtractive theory. The most theoretically-advanced of these models is Critchfield et al.'s generalized subtractive model (Equation 3b). However, the best quantitative model of punishment should significantly outperform all models upon which it is based (i.e., "predecessor" models). To appropriately compare models to their predecessors, all models in the comparison must estimate the same predicted variable [the logarithm of the predicted response ratio, $\log(B_1/B_2)$]. Figure 1 shows all previous matching-law-based models in this form. One can see that all models, except Herrnstein's (1961) matching law (Equation 1a), are built from some other model in this figure. For example, the generalized subtractive model (Equation 3b) has two predecessors: de Villiers' (1977) subtractive model (Equation 3a) and the GML (Equation 1b).

The models in Figure 1 are of varying complexity (i.e., they have different numbers of free parameters). Thus, they cannot be compared using just the PVAF. More complex models inherently account for more of the data's variance because the free parameters allow for the model to "wiggle" and fit the data better (i.e., overfitting). When comparing models with varying degrees of complexity, information criteria (IC) can be used. IC consider both a model's complexity and its descriptive accuracy (i.e., its PVAF). Two of the most commonly used ICs in the field of quantitative behavior analysis are the Akaike Information Criterion – corrected (AICc; Akaike, 1998) and the Bayes Information Criterion (BIC; Schwarz, 1978) (e.g., McLean, Grace, & Nevin, 2012; Navakatikyan, 2007). AICc and BIC are calculated as

$$\text{AICc} = n \ln\left(\frac{RSS}{n}\right) + 2K\left(\frac{n}{n-K-1}\right)$$

and

$$BIC = n \ln\left(\frac{RSS}{n}\right) + K \ln(n),$$

respectively, where *n* is the number of data points in the dataset to which the model is fitted, *RSS* is the residual sum of squares (which is related to the PVAF) of the model, and *K* is the number of free parameters in the model.

McArdle et al. (2007) developed a method to calculate a single AICc and BIC value across multiple datasets (i.e., a summary of a model's fit quality across all participants):

$$\text{AICc} = \sum_{i=1}^{N}\left[n_i * \ln\left(\frac{RSS}{n_i}\right)\right] + 2K\sum_{i=1}^{N}\left(\frac{n_i}{n_i - K - 1}\right)$$

and

$$BIC = \sum_{i=1}^{N}\left[n_i * \ln\left(\frac{RSS}{n_i}\right)\right] + KN \ln(n_t),$$

where *N* is the number of datasets used in the analysis, *i* is the index of the dataset $\{i = 1, 2, \ldots N\}$, $n_i$ is the number of data points in the $i^{th}$ dataset, and $n_t$ is the total number of data points in all the datasets combined (i.e. data series). Because IC values are unit dependent, it is best practice to compare models' values against each other. This is typically accomplished by finding the difference in information criterion values (ΔIC) between a model of interest and the model with the lowest IC value (which is the "best" model). In general, a ΔIC value less than 10 indicates that the model of interest is within the margin of error of the "best" model (Burnham & Anderson, 2002; Navakatikyan, 2007) and should not be considered a worse fitting model.

Klapes et al. (2018) used the data from Experiments 3A and 3B of Critchfield et al.'s (2003) study to test the quantitative superiority of the generalized subtractive model (Equation

3b) over all of its predecessors. Using McArdle et al.'s (2007) model comparison technique, they found that the generalized subtractive model (Equation 3b) had the lowest AICc and BIC values of the models from Figure 1 when fitted to the data. Additionally, they found the ΔIC values for most of the models compared to Equation 3b were much higher than 10. However, the ΔIC value for the comparison of the GML (Equation 1b) to Equation 3b was only 7.5, meaning that the GML could not be ruled out as the best model of the set. In summary, Klapes et al. showed that Equation 3b did not convincingly outperform the GML in describing concurrent choice behavior in the presence of superimposed concurrent punishment schedules. However, as mentioned on p. 5, the GML is a theoretically insufficient explanation of behavior under punishing contingencies. Thus, new matching-law-based punishment models are required.

## Developing Contemporary Matching-Law-Based Punishment Models

### 2.1. Model Constraints

Klapes et al. (2018) asserted that (at least) seven theoretical constraints should be applied when developing new matching-law-based punishment models. 1) New matching-law-based punishment models should incorporate both reinforcers (stimuli that result in an increase in behavior) and punishers (stimuli that result in a decrease in behavior). 2) When there is no punishment delivered, these models should reduce to the GML. 3) Models should be able to handle data with different units. 4) Models should be able to tolerate acquired punishment rates that exceed acquired reinforcement rates and 5) account for qualitatively different reinforcers and punishers. 6) It should not matter which alternative is in the numerator. 7) Models should have significant quantitative superiority over predecessor models (assessed by the information-theoretic technique developed by McArdle et al., 2007). Klapes et al. noted that no current matching-law-based model (i.e., those found in Figure 1) complies with all the above criteria. The new set of matching-law-based punishment models built in this dissertation will be developed within the confines of these constraints.

## 2.2. Building the Candidate Models

### *An Intuitive Approach to Developing Contemporary Additive and Subtractive Models*

As seen in Figure 1, the lineage of previous matching-law-based models has a conspicuous omission and obvious extension: Critchfield et al.'s (2003) generalized additive and subtractive models (Equations 2b and 3b) with the addition of the "rate-of-exchange" parameter. Just as Critchfield et al. developed Equations 2b and 3b, these models can be formed by simply adding the rate-of-exchange parameter to the expressions housed within the logarithm:

$$\log\left(\frac{B_1}{B_2}\right) = a\log\left(\frac{R_1 + c\,P_2}{R_2 + c\,P_1}\right) + \log(b) \qquad (2b')$$

and

$$\log\left(\frac{B_1}{B_2}\right) = a\log\left(\frac{R_1 - c\,P_1}{R_2 - c\,P_2}\right) + \log(b). \qquad (3b')$$

These two models will be referred to as the generalized rate-of-exchange additive and subtractive models, respectively.

Both Equations 2b′ and 3b′ satisfy all of Klapes et al.'s (2018) theoretical criteria. However, these models exhibit certain peculiarities, specifically related to the sensitivity parameter ($a$). As noted by Klapes et al., these versions of the additive and subtractive models entail the same sensitivity parameter for both the acquired reinforcement and punishment rate.

As noted on p. 4, the sensitivity parameter attempts to account for the subject's sensitivity to the acquired reinforcement rate ratio and would thus appear theoretically problematic if it also applied to the acquired punishment rate. Nevertheless, these models deserve to be tested as potential heirs to the matching-law-based punishment model throne.

***Theoretically-Sound Additive and Subtractive Models***

As alluded to above, the simple addition of the "rate-of-exchange" parameter to the generalized additive (Equation 2b) and subtractive (Equation 3b) in creating Equations 2b′ and 3b′ represents a theoretical lackadaisicalness. The main intent of Klapes et al.'s (2018) reanalysis was to show that this approach (i.e., a reliance on the readily-available, rather than the carefully-constructed) is deleterious to successful model development. Thus, another set of additive and subtractive punishment models should be built using the sturdy theoretical foundation of the matching law.

These new models utilize the concatenated generalized matching law (cGML; Davison & McCarthy, 1988), which incorporates other reinforcer attributes multiplicatively (Killeen, 1972; Rachlin, 1971) into the GML:

$$\frac{B_1}{B_2} = \left(\frac{R_1}{R_2}\right)^{a_R} \left(\frac{M_1}{M_2}\right)^{a_M} \left(\frac{I_1}{I_2}\right)^{a_I} \left(\frac{X_1}{X_2}\right)^{a_X},$$

where $M_1$ and $M_2$ are the scheduled reinforcer magnitude (i.e., the quantity, or quality, of reinforcers per reinforcer delivery) on the first and second alternatives, $I_1$ and $I_2$ are the immediacy of reinforcer delivery (i.e., the inverse of the delay from when the reinforcer is signaled to be delivered and when the subject actually acquires the reinforcer) on the first alternative and second alternatives, $X_1$ and $X_2$ are estimates of all other reinforcer attributes affecting behavior that are not rate, amount, and immediacy on the first and second alternative, and $a_R$, $a_M$, $a_I$, and $a_X$ are sensitivity estimates with respect to each reinforcer attribute. The GML is, in a sense, a special case of the cGML; when the scheduled reinforcement rate is the only experimentally manipulated variable, all other variables can be subsumed into $X_1$ and $X_2$ because this reinforcer attribute is inherently non-operationally defined. The bias parameter ($b$) can then be substituted for this all-inclusive $\left(\frac{X_1}{X_2}\right)^{a_X}$ term, resulting in the familiar GML:

$$\frac{B_1}{B_2} = b\left(\frac{R_1}{R_2}\right)^{a_R}. \tag{1b}$$

The most common version of the cGML is a bivariate rate-magnitude version (e.g., Cording et al., 2011; Davison & McCarthy, 1988; Landon et al., 2008; McDowell et al., 2012; Schneider, 1973):

$$\frac{B_1}{B_2} = b\left(\frac{R_1}{R_2}\right)^{a_R}\left(\frac{M_1}{M_2}\right)^{a_M}. \tag{1c}$$

From this model, we can now build a more theoretically-sound additive and subtractive models. First, we can distribute the expressions across the numerators and denominator to create a joint rate/magnitude expression for each alternative:

$$\frac{B_1}{B_2} = b\left(\frac{{R_1}^{a_R}{M_1}^{a_M}}{{R_2}^{a_R}{M_2}^{a_M}}\right).$$

Next, we add the joint rate/magnitude punishment expression for the additive and subtractive theories as

$$\frac{B_1}{B_2} = b\left(\frac{{R_1}^{a_R}{M_{R_1}}^{a_{M_R}} + {P_2}^{a_P}{M_{P_2}}^{a_{M_P}}}{{R_2}^{a_R}{M_{R_2}}^{a_{M_R}} + {P_1}^{a_P}{M_{P_1}}^{a_{M_P}}}\right)$$

and

$$\frac{B_1}{B_2} = b\left(\frac{{R_1}^{a_R}{M_{R_1}}^{a_{M_R}} - {P_1}^{a_P}{M_{P_1}}^{a_{M_P}}}{{R_2}^{a_R}{M_{R_2}}^{a_{M_R}} - {P_2}^{a_P}{M_{P_2}}^{a_{M_P}}}\right),$$

respectively, where $M_{R_1}$ and $M_{R_2}$ represent the reinforcer magnitudes for the first and second alternative, $a_{M_R}$ is the sensitivity parameter estimate for reinforcer magnitude, $a_P$ is the sensitivity parameter estimate for punishment rate, $M_{P_1}$ and $M_{P_2}$ are the punisher magnitudes for the first and second alternative, $a_{M_P}$ is the sensitivity parameter estimate for punisher magnitude.

Typically, the same stimulus is used as a reinforcer and punisher across the two alternatives (e.g., the reinforcer is 3 s of grain presentation for both the first and second alternatives, while the punisher is a 4-mA shock for both the first and second alternatives). Thus, if we are to assume this experimental scenario, we can remove the numerical subscripts from the $M_R$ and $M_P$ expressions. We can then multiply both the numerator and denominator by $[1/(M_R)^{aM_R}]$ (i.e., the inverse of the reinforcer magnitude expression) to attain

$$\frac{B_1}{B_2} = b\left[\frac{{R_1}^{a_R} + {P_2}^{a_P}\left(\frac{{M_P}^{a_{M_P}}}{{M_R}^{a_{M_R}}}\right)}{{R_2}^{a_R} + {P_1}^{a_P}\left(\frac{{M_P}^{a_{M_P}}}{{M_R}^{a_{M_R}}}\right)}\right]$$

and

$$\frac{B_1}{B_2} = b\left[\frac{{R_1}^{a_R} - {P_1}^{a_P}\left(\frac{{M_P}^{a_{M_P}}}{{M_R}^{a_{M_R}}}\right)}{{R_2}^{a_R} - {P_2}^{a_P}\left(\frac{{M_P}^{a_{M_P}}}{{M_R}^{a_{M_R}}}\right)}\right].$$

As noted on p. 10, the rate-of-exchange parameter attempts to reconcile the subjective difference between reinforcer and punisher magnitudes. Thus, this parameter, in theory, can be represented mathematically by the quotient of the punishment and reinforcement magnitudes. That is,

$$c = \left(\frac{{M_P}^{a_{M_P}}}{{M_R}^{a_{M_R}}}\right).$$

Wonderfully, our derived theoretical statement of $c$ appears in the equations! Substitution of this expression results in:

$$\frac{B_1}{B_2} = b\left(\frac{R_1{}^{a_R} + cP_2{}^{a_P}}{R_2{}^{a_R} + cP_1{}^{a_P}}\right) \tag{2c'}$$

and

$$\frac{B_1}{B_2} = b\left(\frac{R_1{}^{a_R} - cP_1{}^{a_P}}{R_2{}^{a_R} - cP_2{}^{a_P}}\right), \tag{3c'}$$

which are the final forms for the theoretically-sound additive and subtractive models.

***Another cGML-Based Punishment Model: Neither Additive nor Subtractive***

Critchfield et al. (2003) suggested that, for some of their participants, the acquired punishment rate may have been more influential on response allocation than the acquired reinforcement rate. This hypothesis led Critchfield et al. to speculate about a GML-like model that utilized the acquired punishers as the sole predictor of response allocation:

$$\log\left(\frac{B_1}{B_2}\right) = a_P \log\left(\frac{P_2}{P_1}\right) + \log(b)\ .$$

Rather, it may be possible to incorporate both the acquired reinforcement rate and the acquired punishment rate into a new version of the cGML. That is, instead of a bivariate reinforcement rate-magnitude cGML, there could be a bivariate reinforcement rate-punishment rate cGML:

$$\log\left(\frac{B_1}{B_2}\right) = a_R \log\left(\frac{R_1}{R_2}\right) + a_P \log\left(\frac{P_2}{P_1}\right) + \log(b)\ .$$

This cGML-based punishment model can be strengthened by incorporating two additional elements. First, the current model predicts that increasing the overall punishment rate, while keeping the proportion of scheduled punishment across the two alternatives constant, would have no effect on the response ratio. This scenario is both theoretically problematic and does not follow the empirical data (Experiment 2 of Farley, 1980). Including the ratio of total acquired punishment to total acquired reinforcement ($P_T/R_T$) as a coefficient to the punishment rate sensitivity parameter could alleviate this issue:

$$\log\left(\frac{B_1}{B_2}\right) = a_R \log\left(\frac{R_1}{R_2}\right) + \left[a_P\left(\frac{P_T}{R_T}\right)\right]\log\left(\frac{P_2}{P_1}\right) + \log(b)\ . \qquad (4c)$$

Second, McDowell and Klapes (2019) hypothesized that the reinforcement context (i.e., whether the alternative upon which the punishment schedule was superimposed is the richer or leaner of the two alternatives) applies a strong influence on punishment's efficacy. This aspect can be incorporated into Equation 4c by using the acquired reinforcement rate ratio as a coefficient of the acquired punishment ratio:

$$\log\left(\frac{B_1}{B_2}\right) = a_R \log\left(\frac{R_1}{R_2}\right) + \left[a_P\left(\frac{P_T}{R_T}\right)\right]\log\left[\left(\frac{R_1}{R_2}\right)\left(\frac{P_2}{P_1}\right)\right] + \log(b)\ . \qquad (4c')$$

Notably, the inclusion of the reinforcement context in the punishment expression of Equation 4c′ allows for the model to make the same qualitative predictions as the subtractive theory (i.e., increase in sensitivity parameter estimates in constant and equal punishment conditions and no change in sensitivity parameter estimates in proportional conditions) that were supported by de Villiers' (1980), Farley's (1980), and Critchfield et al.'s (2003) studies; the punishment rate can now directly affect the reinforcement rate sensitivity parameter ($a_R$) in Equation 4c′. In the "constant and equal" punishment condition (i.e., the scheduled $P_1 = P_2$; e.g., Experiment 1 of Farley, 1980), the equation reduces to

$$\log\left(\frac{B_1}{B_2}\right) = a_R \log\left(\frac{R_1}{R_2}\right) + \left[a_P\left(\frac{P_T}{R_T}\right)\right]\log\left[\left(\frac{R_1}{R_2}\right)(1)\right] + \log(b),$$

which, when distributing the exponent across common log expressions, results in

$$\log\left(\frac{B_1}{B_2}\right) = \left[a_R + a_P\left(\frac{P_T}{R_T}\right)\right]\log\left(\frac{R_1}{R_2}\right) + \log(b)\ .$$

In the "proportional" condition (i.e., $R_1/R_2 = P_1/P_2$; e.g., Experiment 3B of Critchfield et al., 2003), the equation reduces to

$$\log\left(\frac{B_1}{B_2}\right) = a_R\log\left(\frac{R_1}{R_2}\right) + \left[a_P\left(\frac{P_T}{R_T}\right)\right]\log(1) + \log(b)\ ,$$

which, with log(1) = 0, results in the GML:

$$\log\left(\frac{B_1}{B_2}\right) = a_R\log\left(\frac{R_1}{R_2}\right) + \log(b)\ .$$

**2.3. A Theoretical Comparison of the New Models**

The cGML-based punishment model (Equation 4c′) has two distinct advantages over the additive and subtractive models (viz., Equations 2b′, 3b′, 2c′, and 3c′). First, there are no theoretical constraints on the parameters (e.g., the "rate-of-exchange" parameter in the other models). This freedom alleviates many of the annoyances involved with fitting the other models to data (e.g., making sure that one calculates the minimum *R/P* ratio so that there are not negative values in the subtractive models; making sure that *c* is always a positive value for both additive and subtractive models). Second, Equation 4c′ is linear in its parameters. That is, no parameter is estimated simultaneously with another parameter. Equations 2b′, 3b′, 2c′, and 3c′ are not linear in their parameters, as *c* is always estimated as a function of either *a* (Equations 2b′ and 3b′) or $a_P$ (Equations 2c′ and 3c′).

The cGML-based punishment model (Equation 4c′) does have a flaw that is not observed in the other models. The use of the ($P_T/R_T$) ratio as a coefficient for $a_P$ theoretically alleviates situations where there is no punishment at all (i.e., punishment expression completely drops out and the GML is left; requirement 2 on p. 20). In practice, however, the model will break down whenever the acquired punishment rate on either or both alternatives is 0. To uphold the theoretical basis of the model, Equation 4c′ can still be fitted to data in which there is no punishment (i.e., a baseline condition) by simply dropping the punishment expression and only fitting the GML to the data. Unfortunately, it is impossible to fit Equation 4c′ to data obtained from concurrent reinforcement schedules upon which punishment is superimposed on only one of the alternatives. Notably, this same flaw is prominent in the GML with respect to reinforcement schedules (i.e., if there is no acquired reinforcement on one of the alternatives, the GML will be unable to fit the data).

## Experiment 1: Fitting New Models to Available Data

As seen above, the new matching-law-based models built in this dissertation inherently entail more complexity. That is, to be able to account for the data significantly better than the GML, the new models are likely to need more free parameters. More complex models (i.e., models with more free parameters) are likely to be severely discounted by the information criteria when fitted to datasets of the relatively small size seen in the literature. This discounting is represented by the right-most expressions in the calculation of AICc and BIC (for reference, see p. 19). To test this hypothesis, the first experiment of this dissertation entailed fitting the newly-built additive, subtractive, and cGML-based punishment models (viz., Equations 2b′, 2c′, 3b′, 3c′, and 4c′) to previously collected continuous choice data with superimposed concurrent punishment schedules and comparing their descriptive accuracy to the GML.

### 3.1. Methods

The same data inclusion criteria used by Klapes et al. (2018) were used for this reanalysis. Specifically, each included dataset was required to have at least five unique conc VI VI or conc RI RI schedule of reinforcement, and at least one of these schedules needed to have a superimposed conc VI VI or conc RI RI punishment schedules. In order to fit the subtractive models that did not include a "rate-of-exchange" parameter (Equations 3a and 3b), Klapes et al. required every schedule in their reanalysis to have more acquired reinforcers than acquired punishers. As such, Experiment 3 of Critchfield et al.'s (2003) study was the sole data series that conformed to these requirements. Because the new models are required to account for scenarios in which there are more punishers than reinforcers acquired during the schedule presentation (see p. 20; Requirement 4), two additional data series that were excluded from Klapes et al.'s reanalysis can now be used.

Datasets from 37 subjects were used for this reanalysis. Data came from Farley's (1980) Experiment 1 (three pigeon subjects; described on pp. 13-14), a study by Reed and Yoshino (2005; 21 rat subjects),[7] and Critchfield et al.'s (2003) Experiment 3 (13 human participants; described on pp. 16-18). Each dataset consisted of response, acquired reinforcement, and acquired punishment rate data for between seven and 12 concurrent schedules. All models developed in the theoretical component of the dissertation and the GML (which serves as the

[7] Reed and Yoshino (2005) stratified four rats to six groups: two levels of reinforcement rate density (lean: VI 450-s VI 150-s, VI 350-s VI 150-s, VI 150-s VI 350-s, and VI 150-s VI 450-s; rich: VI 90-s VI 30-s, VI 60-s VI 30-s, VI 30-s VI 60-s, and VI 30-s VI 90-s) across three different food deprivation levels (80% of their free-feeding weight; 90% of their free-feeding weight; 100% of their free-feeding weight). The rats were exposed to one baseline condition with reinforcement (delivery of one 45-g food pellet) only (i.e., no punishment) and one "constant and equal" condition involving a superimposed VI 60-s VI 60-s punishment (a 1-s long contingent tone presentation) on all four conc reinforcement schedules. Three of the rats' data were not available (see McDowell & Klapes, 2019, for details). Thus, 21 rats were used in this analysis.

predecessor model to all new models) were fitted to each subject's dataset via nonlinear regression. This process was performed in Microsoft Excel using iterative parameter estimation via the Generalized Reduced Gradient (GRG) Nonlinear Solving Method (Lasdon et al., 1978) in the Solver Add-In. AICc and BIC values were calculated across the multiple datasets using McArdle et al.'s (2007) method. The "best" model was determined as the model with the lowest AICc and BIC values in the set, and the other models were compared to it by ΔIC values. If no model comparison produced a ΔIC value under 10, then the best model is convincingly better than all other models that are being tested.

### 3.2. Results

Table A1 of Appendix A shows the number of data points (i.e., concurrent schedules) to which each model was fitted, and the parameter estimates, AIC values, and BIC values for each model's fit to each subject's data. Table 1 presents a summary of each model's descriptive accuracy across the entirety of the data series. In the second column, the median PVAF and 95% confidence interval (CI) around the median (Campbell & Gardner, 1988) are presented. There was no significant difference in the amount of variance explained across the different models. The AICc, ΔAICc, BIC, ΔBIC values for each model are presented in third through sixth columns of Table 1, respectively. The GML (Equation 1b) was the "best" model of the set by both AICc and BIC. Additionally, the GML significantly outperformed all other models when accounting for complexity; the next closest model, the cGML-based punishment model (Equation 4c′), had ΔIC values that were much greater than 10.

### 3.3. Discussion

All fitted models in Experiment 1 accounted for a large amount of variance in the data (all model's median PVAF > 0.90). This result is somewhat surprising, particularly as it relates to the GML. The GML does not incorporate punishment rates into its prediction of response allocation; theoretically, it should not account for a large amount of the variance in a dataset including punishers. Although very similar and not significantly different, the punishment models did explain slightly more of the variance in the data than the GML. Nevertheless, the GML dominated the new matching-law-based punishment models when accounting for complexity. Thus, as hypothesized, the small number of data points used in each fit (i.e., all datasets contained less than 13 data points each) may have unfairly discounted the new matching-law-based models when comparing via information criteria.

The results of Experiment 1 suggest that a new data series with many more data points per dataset must be collected before any of the new matching-law-based models of punishment can be fairly judged against their less-complex predecessor (i.e., the GML). This analysis can be performed on a data series generated by the Evolutionary Theory of Behavior Dynamics (ETBD; McDowell, 2004). The ETBD is a computational theory that can generate simulated operant behavior datasets of any desired size. If the ETBD is a correct account of the behavior of live organisms, then the model that provides the best account of continuous choice data under both reinforcing and punishing contingencies that is generated by the theory will also provide the best account of data from live organisms.

## Experiment 2: An ETBD Prediction of Model Superiority

The ETBD utilizes selectionist principles (selection, reproduction, and mutation) and a genetic algorithm to generate simulated operant behavior by artificial organisms (AOs). The ETBD operates as a complexity theory (McDowell & Popa, 2009), whereby the interaction of low-level rules results in emergent phenomena. Emergent phenomena cannot be predicted solely based on the theory's rules, or on mathematical manipulations of those rules, but rather must be observed by analyzing the resulting data once the computation has been completed. These phenomena can then be compared to data collected from living organisms. The ETBD has accounted for and predicted a plethora of results, producing patterns of behavior that are virtually indistinguishable from analogous live-organism data (McDowell, 2013b, 2019). A selection of these results includes continuous choice behavior on conc RI RI schedules with varying reinforcement rates (McDowell et al., 2008), conc RI RI schedules with covarying reinforcement rates and magnitudes (McDowell et al., 2012), conc RI RI schedules with rapidly changing reinforcement rates within the same session (Kulubekova & McDowell, 2013), and conc random-ratio random-ratio (RR RR) schedules with equal and unequal ratios in the components (McDowell & Klapes, 2018).

The algorithmic implementation of the ETBD begins with a population of potential behaviors. This population is usually made up of 100 behaviors. Each potential behavior has two representations: a genotype and a phenotype. The genotype is a string of 10 binary digits (i.e., 0s and 1s). When taken together, these digits, or bits, can be converted to an integer value. This value is the phenotype of the behavior. For example, a string of bits might be 0000010000. Converting this string from the binary to an integer value gives the number 16. Thus, this behavior's genotype is 0000010000 and its phenotype is 16.

Ranges of integer values represent a class of behaviors that may result in the same operant behavior. For example, a behavior with the phenotype of 16 could be thought of as a rat using its right paw to press a lever, while a behavior with the phenotype of 17 could be thought of as a rat using its left paw to press that same lever. Both phenotypes would constitute a press of the lever but are slightly different in detail. Two ranges of integers can be used to represent a two-alternative system. In most studies of continuous choice using the ETBD, these two ranges, or target classes, extend from 471 through 511 and 512 through 552.

### *Low-Level Rules of the ETBD*

Populations of potential behaviors are acted upon by the ETBD's rules of selection, reproduction, and mutation. These rules create a new population of behaviors at each time tick, or generation. The rules are implemented many times, with most studies churning through 20,500 generations per reinforcement schedule. Generations act as a time-keeping device in the ETBD. To implement an interval schedule, reinforcers can be set up after a certain number of generations have elapsed. For example, a RI 10 schedule of reinforcement would, on average, set up a reinforcer delivery approximately 10 generations after the last reinforcer was delivered.

**Selection.** The ETBD algorithm begins with the emission of a randomly chosen behavior from the starting population. If this behavior was emitted from one of the target classes, and it produced a benefit (i.e., resulted in an acquired reinforcer), then selection occurs. In this case, behaviors with phenotypes that are closer to that of the emitted behavior will be more likely to contribute to the next population of potential behaviors (i.e., these behaviors' traits have a greater fitness for survival to the next generation). After many generations, these contributors will become increasingly represented in the population; in other words, clumping of behaviors in and around the target classes will occur. If the emitted behavior does not produce a benefit, then the behaviors that contribute to the next population are chosen at random.

**Reproduction.** Regardless of how the contributors were obtained from the population, the next generation of behaviors is generated by the reproduction of the contributors. Like sexual reproduction in natural evolution, two contributing behaviors act as parent behaviors and give portions of their genotypes to a child behavior in the new population. A process called bitwise recombination is typically used. Each bit in the first contributing behavior's string has a 50% chance of being incorporated into the new behavior. If it is not incorporated, then the new behavior's bit at that point in the string will be that of the second contributor. This recombination results in the new behavior being similar to each of the contributors, but unlikely to be identical to either (like children of parents).

**Mutation.** After reproduction occurs, mutation is applied to the new population of behaviors. This occurs by randomly choosing a percentage of behaviors (usually 10%) and then randomly choosing one of the 10 bits in their genotypes to flip from 0 to 1 or 1 to 0. Depending on the bit being flipped, this could have small or large effects on the behavior's phenotype. For example, it has already been discussed that a genotype of 0000010000 would result in a phenotype of 16. If the first bit of this genotype is mutated, then the result will be a genotype of 1000010000 and a phenotype of 528. However, if the last bit is mutated instead, then the result will be a genotype of 0000010001 and a phenotype of 17. After mutation has occurred, a behavior is emitted from the new population and the cycle is repeated for as many generations as the researcher specifies for that reinforcement schedule.

### *Punishment in the ETBD*

McDowell and Klapes (2019) proposed a mechanism by which punishment schedules can be implemented within the ETBD. This mechanism is called forced mutation; when a punisher is delivered, all behaviors in the target class are probabilistically subjected to an additional round of mutation. That is, every behavior currently in the punished target class has a chance of having one of its bits changed from 0 to 1 or 1 to 0. This probability is determined by the reinforcement density context. The richer the reinforcement context on the punished alternative, the less likely a behavior will be forced to mutate; the leaner the reinforcement context, the more likely a behavior will be forced to mutate. It is important to note that the forced mutation implementation of punishment in the ETBD does not alter any of the theory's existing mechanisms of action. Rather, it is an addition to the rules of selection, reproduction, and (most importantly) mutation.

To test whether the forced mutation implementation of punishment works within the ETBD, McDowell and Klapes (2019) performed three studies that replicated live organism findings of continuous choice behavior in the face of superimposed punishment schedules. First, they replicated Critchfield et al.'s (2003) Experiment 1 to show that the method effectively suppresses behavior. They exposed 30 AOs to a conc RI 10 RI 10 schedule of reinforcement as a baseline. They then arranged three punishment conditions in which the AOs were exposed to the same schedule of reinforcement, but with superimposed RI punishment schedules of various RI values on only one alternative. In all three punishment conditions, there was a decrease in behavior on the punished alternative and an increase in behavior on the unpunished alternative compared to the baseline condition.[8] Thus, Critchfield et al.'s Experiment 1 was replicated, lending support for a forced mutation event to be considered an effective punisher.

In Experiment 2, McDowell and Klapes (2019) replicated the "constant and equal" punishment experiment to see if the AOs' preference for the richer alternative increased under punishment (i.e., shifted toward overmatching). This finding has been observed in experiments with pigeons (Experiment 1 of Farley, 1980), rats (Reed & Yoshino, 2005), and human participants (Experiments 2A and 3A of Critchfield et al., 2003).[9] In all conditions, they used a set of reinforcement schedules with RI values that are typically used in continuous choice studies with the ETBD (e.g., McDowell et al., 2008). McDowell and Klapes exposed 30 AOs to six sets of these reinforcement schedules. The first set served as a baseline condition, with no superimposed punishment schedules. The five remaining sets served as punishment conditions, during which concurrent punishment schedules with equal RI values on the two alternatives were superimposed and those values were increased across conditions (i.e., the first condition had superimposed conc RI 10 RI 10 punishment schedules, while the second condition had superimposed conc RI 20 RI 20 punishment schedules, and so on). The GML was fitted to the

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[8] In one of the punishment conditions (RI 20), there was a decrease in the punished alternative's response rate and increase in the unpunished alternative's response rate. However, there was overlap in the 95% CI for the means of these two values (i.e., a non-significant difference).

[9] McDowell and Klapes (2019) noticed that most analyses on the shift towards overmatching in the presence of superimposed "constant and equal" punishment schedules were performed via visually inspecting the plots (e.g., Critchfield et al., 2003; de Villiers, 1980; Farley, 1980). For a more objective assessment of the extent to which participants shifted towards overmatching, they compared within-subject differences between the *a* parameters (which account for the amount of under- or overmatching exhibited in the data) of GML fits to the data from baseline and punished conditions. The search for all "constant and equal" studies in which both the baseline and punished conditions included at least four distinct conc VI VI schedules resulted in 54 datasets from 27 subjects/participants. Twenty-one of the 27 subjects/participants' *a* parameter estimates increased from the punished to the unpunished conditions.

data in each of these conditions and the *a* parameters of those fits were compared across conditions. In the punishment conditions, the *a* parameter estimate from the GML fitted to the data exceeded that of the *a* parameter estimated from the GML fit to the baseline condition. This result indicated that AOs' preference towards the richer alternative did indeed increase (i.e., they shifted toward overmatching) in the presence of constant and equal punishment.

In Experiment 3, McDowell and Klapes (2019) replicated the "proportional" punishment experiment to see if the AOs' preference remained invariant under these conditions, which is the finding that is observed in experiments with human participants (Experiments 2B and 3B of Critchfield et al., 2003). They again used the standard 11 concurrent schedule sequence seen in most ETBD continuous choice studies. They used the baseline condition from Experiment 2 as the baseline condition for Experiment 3. They then exposed the 30 AOs to five sets of these reinforcement schedules, where concurrent punishment schedules with RI values proportional to the RI values of the reinforcement schedule on that alternative were superimposed. The punishment values were increased across conditions (i.e., the first condition had superimposed punishment schedules with RI values that were twice the RI values of the reinforcement schedule on that alternative, while the second condition had superimposed punishment schedules with RI values that were equal to the RI values of the reinforcement schedule on that alternative, and so on). The GML was fitted to the data from each of these conditions and the *a* parameters of those fits were compared across conditions. In all five punishment conditions, the *a* parameter estimated from the GML fitted to the data did not differ from the *a* parameter estimated from the GML fitted to the no-punishment baseline condition. This result indicated that the AOs' response allocation generally did not change in the presence of proportional punishment.

The results of the McDowell and Klapes' (2019) experiments indicate that the forced mutation implementation of punishment allows for the ETBD to effectively simulate continuous choice under superimposed punishing contingencies. The size of the datasets presented in McDowell and Klapes' Experiment 2 (five "constant and equal" conditions) and Experiment 3 (five "proportional conditions") would allow for the fair comparison of new matching-law-based punishment models to the GML (i.e., 11 conditions, including the baseline condition, resulting in 121 data points per fit). Thus, as performed in Experiment 1, the five new matching-law-based models developed above (viz., Equations 2b′, 2c′, 3b′, 3c′, and 4c′) were fitted to this dataset and compared against the GML. This experiment would be a bold test of the ETBD, as the prediction generated by this experiment would be made without any currently available live organism data to support it (cf., McDowell & Calvin, 2015).

**4.1. Methods**

Thirty AOs animated by the ETBD served as the participants for this experiment. They were exposed to the 11 conditions described in Experiments 2 and 3 of McDowell and Klapes' (2019) study (summarized in Table 2). All conditions involved the standard 11 schedules of conc RI RI schedules for ETBD studies of continuous choice (e.g., McDowell et al., 2008). The first condition served as a baseline; there were no superimposed punishment schedules during this condition. Following the baseline condition were the five "constant and equal" conditions. These conditions superimposed RI punishment schedules with equal RI values for both alternatives across all 11 concurrent. The five "proportional" conditions were presented last. This condition again used the concurrent reinforcement schedules seen in the baseline condition, but superimposed RI punishment schedules that are equal to the RI values of the reinforcement schedules upon which they were superimposed multiplied by some factor.

As performed in Experiment 1, all models developed in this dissertation and the GML (which served as the predecessor model to all new models) were fitted to each of the 30 AOs' 121 data points in Microsoft Excel using the GRG Nonlinear Solving Method in the Solver Add-In. AICc and BIC values were calculated across the multiple datasets using McArdle et al.'s (2007) method. The "best" model was determined as the model with the lowest AICc and BIC values in the set, and the other models were compared to it by ΔIC values. If no model comparison produced a ΔIC value under 10, then the best model is convincingly better than all other models that are being tested; in this case, the best model of this set would be the one predicted by the ETBD to be the superior matching-law-based model of punishment.

### 4.2. Results

Summaries of each model's descriptive accuracy across the entire dataset are presented in Table 3. As with Table 1, the second column displays the median PVAF and 95% CI around the median. There was no statistically significant difference in the amount of variance explained across the different models. The AICc, ΔAICc, BIC, and ΔBIC values for each model are presented in third through sixth columns of Table 3, respectively. As in Experiment 1, the GML (Equation 1b) was the "best" model of the set by both AICc and BIC. Additionally, the GML significantly outperformed all other models when accounting for complexity. This outperformance was more drastic in terms of BIC than AICc; the next closest models by AICc, the generalized "rate-of-exchange" subtractive model (Equation 3b′) and the cGML-based punishment model (Equation 4c′), had ΔAICc values that were only slightly larger than 10, while these same comparisons via ΔBIC resulted in values much greater than 10.

### 4.3. Discussion

All models fitted to data collected during Experiment 2 accounted for almost all the variance in the data (median PVAF for all models = 0.98). As with Experiment 1, the GML (Equation 1b) did surprisingly well in describing the data, achieving the lowest IC values in the set for both AICc and BIC. The ΔIC values for this experiment, although slightly different in scale, converge on the same conclusion; AICc says that the GML is convincingly better than the other models, while BIC says that the GML decimated the other models. This result represents a prediction generated by the ETBD about live organism continuous choice performance under punishing contingencies: the GML will be the "best" model in the set (i.e., has the lowest IC value for both AICc and BIC) and convincingly outperform all other models (i.e., ΔIC will be greater than 10). A replication with live organisms will confirm or refute this prediction.

## Experiment 3: A Test of the ETBD Prediction with Human Participants

The Procedure for Rapidly Establishing Steady-State Behavior (PRESS-B; Klapes et al., 2020) is a discriminated rapid-acquisition procedure for human operant performance. It capitalizes on organisms' propensity to rapidly acquire a response allocation pattern that adheres to the matching law under certain reinforcing contingencies (Baum & Davison, 2000). During PRESS-B, the participant is presented with a computerized operant response panel (Figure 2; video). The participant interacts with the environment via a standard computer keyboard. When participants press the space bar, the response button is simultaneously pressed on the screen. Pressing the response button allows them to acquire available (i.e., baited) stimuli from the system. When participants press the control key, the change-over button[10] is simultaneously pressed on the screen. The lights at the top of the screen act as ordinally-related, schedule-correlated discriminative stimuli (i.e., when a specific reinforcement schedule is underway, a specific light in the row is illuminated), while the response button's color (blue or yellow) acts as an indication of the active component of the concurrent schedule (first alternative and second alternative, respectively). When reinforcers are delivered, the green light to the left of the response button briefly illuminates and a "ding" sound is played through the computer's speakers. When punishers are delivered, the red light to the right of the response button briefly illuminates and a "womp" sound is played through the computer's speakers.

Klapes et al. (2020) tested the quality of human continuous choice data generated from PRESS-B. They recruited 27 undergraduate students via an online recruitment system for students to receive class credit in their introductory psychology course. The participants were exposed to nine conc RI RI reinforcement schedules. The accrual of points served as the reinforcer in the study; when a reinforcer was delivered the counter would increment by a point. Additionally, participants were presented with an arcade-style leaderboard at the start of the experiment. If they acquired enough points during the session, their score would be placed on the board along with a three-character expression of their choosing. Each concurrent schedule was

[10] PRESS-B implements a special type of concurrent schedule of reinforcement called a switching-key (Findley, 1958) concurrent schedule. In this type of schedule, there is a single operandum that detects behavior (i.e., the response button). There is another button that switches that operandum from being active on one alternative to the other. These schedules do not significantly differ in their ability to estimate behavior allocation across the two alternatives from the standard two-key concurrent schedule (Catania, 1963). A switching-key concurrent schedule allows for easier quantification of the amount of time allocated to the two alternatives, which has also been used as the dependent variable in Equations 1a and 1b (Baum & Rachlin, 1969).

presented for 200 s and had a 2-s changeover delay[11] (COD; both experimental aspects also used by Popa, 2013). The RI values for the reinforcement schedules ranged from 1 to 3 s (a similar, yet slightly wider, range as was used by Popa, 2013) for all 27 participants.

Overall, PRESS-B generated participant behavior that was well described by the GML (median PVAF of 0.94), despite the brevity of the sessions. The *a* parameter estimates (median *a* of 0.67) were similar to those found in most other human concurrent choice experiments (~0.7; Kollins et al., 1997). The participants showed no significant bias, evidenced by the overall median of the log(*b*) parameter estimates being approximately 0. These results constituted a vast improvement over previous human rapid-acquisition continuous choice tasks (e.g., Bull et al., 2015; Lie et al., 2009; Popa, 2013), suggesting that PRESS-B is a viable procedure for the quick collection of high-quality human continuous choice data.

### *The Issue of Exclusive Preference in PRESS-B*

Klapes et al.'s (2020) study with PRESS-B did present one major limitation of the procedure; occasionally, no responses were emitted on one of the alternatives during a concurrent schedule presentation (i.e., participants exhibited "exclusive preference" for one alternative). Of the 243 concurrent schedules presented to the participants, 27 (11%) fell into this category. Sixteen of the participants (59%) had at least one schedule of this nature. Klapes et al.

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[11] CODs are implemented because when independent concurrent schedules are being employed, it is likely that the interval on the non-active component will elapse while the participant is operating on the active component. A reinforcer will then be cued for delivery on the first emitted behavior after switching, reinforcing the behavior of switching between alternatives. When a COD is in effect, the participant will not receive the scheduled reinforcer on that alternative after switching from the other alternative, but rather needs to emit a behavior on that alternative after the specified COD length has elapsed to acquire the cued reinforcer. This disrupts the pairing of switching and acquired reinforcement, allowing for the participant to allocate their behavior to the alternatives' differential reinforcing contingencies instead. CODs are commonplace in experiments that arrange concurrent schedules of reinforcement.

noted that these schedules were not able to be integrated into the GML fits to the participants' data; the ratio nature of the GML results in an undefined prediction when a zero value is in the denominator, and the logarithmic nature of the GML results in an undefined prediction when a zero value is in the numerator. Schedules during which participants exhibited exclusive preference are problematic because it is unclear how these schedules would affect the GML fit if they were somehow able to be incorporated into the dataset.

Klapes et al. (2020) suggested that experimental manipulations could potentially remedy the issue of exclusive preference. Possible manipulations include shortening the COD, implementing short black-out periods (i.e., timeouts from the procedure) between schedule presentations, and widening the range of reinforcement ratios across the two alternatives. Preliminary work on these modifications has been very promising. During the Spring 2018 semester, 120 undergraduate students were exposed to nine conc RI RI reinforcement schedules using PRESS-B and all other experimental variables used in Klapes et al.'s study. The participants were split into four groups of 30, with each group assigned a different COD condition: 0-s, 0.5-s, 1.0-s, and 2.0-s (the value used by Klapes et al.) COD. The percent of participants with no instances of exclusive preference during the study was 66%, 80%, 70%, and 47% for each COD condition, respectively (summarized in Figure 3). During the Fall 2018 and Spring 2019 semesters, 60 undergraduate students were exposed to nine conc RI RI reinforcement schedules that ranged from 1 to 7 s using PRESS-B, a 0.5-s COD (based on the results of the experiment above), and all other experimental variables used in Klapes et al.'s study. The participants were split into two groups of 30: one with two 90-s breaks and one with nine 5-s breaks after every schedule presentation. The percentages of participants with no instances of exclusive preference during this study were 93% and 90%, respectively (as seen in

Figure 3). Thus, decreasing the COD, extending the reinforcement range, and implementing short blackout periods between schedule presentations significantly decreased the number of schedules produced by PRESS-B during which participants exhibited exclusive preference.

### *Punishment in PRESS-B*

Punishment schedules have also been used in PRESS-B. Klapes (2016) wished to replicate and extend a finding that matching-based hedonic scaling (Miller, 1976) could be used to estimate a participant's level of loss aversion (Rasmussen & Newland, 2008). Like Critchfield et al. (2003), Klapes needed to show that the aversive stimulation he was using was an effective punisher. He recruited 21 undergraduate students using the same recruitment system used by Popa (2013) and Klapes et al. (2020). The participants were exposed to three sets of three concurrent schedules of reinforcement (same reinforcement system of point accrual and leaderboards as was used by Klapes et al.) using PRESS-B. Each concurrent schedule was presented for 200 s and had a 2-s COD. For each set of schedules, the same conc RI RI reinforcement schedules with equivalent RI values across the two alternatives were used (conc RI 1.0-s RI 1.0-s, conc RI 2.0-s RI 2.0-s, and conc RI 3.0-s RI 3.0-s for each set, respectively).

The first schedule of each set served as a baseline condition (i.e., no superimposed punishment). For the second schedule in each set, a RI schedule of punishment with a value of either 2 (50%; P50) or 1.25 (80%; P80) times greater than the RI value of the reinforcement schedules was randomly assigned and superimposed on one of the alternatives. The other punishment schedule was then presented on the same alternative for the third schedule in the set. In all sets, there was a decrease in responses on the punished alternative and an increase in responses on the unpunished alternative compared to the baseline condition. This pattern was observed in both the P50 and P80 conditions. Thus, these results indicate that point-loss is an effective punisher in PRESS-B.

The results of Klapes' (2016) experiment indicate that a replication of Experiment 2 with human participants can be successfully completed using PRESS-B. The major advantage of this procedure over other human operant procedures (e.g., Bradshaw et al., 1976; Madden & Perone, 1999) is the ability for a researcher to acquire high-quality continuous choice data (i.e., data to which the GML conforms excellently) in a very short period. The procedure's brevity significantly reduces participant burden, allowing them to complete many more experimental schedules without experiencing fatigue or boredom. The modifications to the procedure explained on pp. 43-44 appear to be alleviating issues associated with exclusive preference, which would significantly increase the number of usable data points in the dataset. Thus, an appropriately designed study using PRESS-B could feasibly replicate Experiment 2 in search for the superior matching-law-based punishment model.

**5.1. Methods**

Thirty undergraduate students served as the participants for Experiment 3. They were recruited using the same online system as was used by Popa (2013), Klapes (2016), and Klapes et al. (2020) during the Spring 2020 semester. Each participant came to the laboratory for two sessions. During each study visit, they performed two experimental conditions during which they were exposed to concurrent schedules of reinforcement (one point being added to the reinforcement counter) using PRESS-B (i.e., four conditions total; Table 4). Each concurrent schedule was presented in random order (without replacement) for 200 s and operated under a 0.5-s COD. Blackout periods were implemented by forcing the participants to take a 5-s break after each schedule presentation. Punishment (one point being deducted from the reinforcement counter) schedules, of varying RI values, were superimposed on the reinforcement schedules for the three punishment conditions.

During the first visit, the participants were always presented with the baseline condition first; no superimposed punishment schedules were used during this condition. The RI values for the reinforcement schedules were RI 7.3-s RI 1.2-s, RI 5.0-s RI 1.3-s, RI 3.5-s RI 1.4-s, RI 2.6-s RI 1.6-s, RI 2-s RI 2-s, RI 1.6-s RI 2.6-s, RI 1.4-s RI 3.5-s, RI 1.3-s RI 5.0-s, and RI 1.2-s RI 7.3-s. These values are approximately within the 1 to 7-s range for the RI reinforcement values, which elicited far fewer unanalyzable schedules (compared to the 1 to 3-s range) in the unpublished experiments described on pp. 43-44. However, this schedule set also holds the overall scheduled reinforcement rate constant across all reinforcement schedules, an experimental aspect that was overlooked in previous experiments using PRESS-B.

Following the baseline condition, the three punishment conditions (viz., “constant and equal,” “proportional,” or “changing punishment”) were presented in random order (without replacement) to the participants. One punishment condition occurred after the baseline condition on the first visit and the other two occurred during the second visit. During the constant and equal condition, RI punishment schedules of RI 8-s RI 8-s (i.e., four times the RI value of the median RI value of the reinforcement schedules) were superimposed across all nine concurrent reinforcement schedules. During the proportional condition, the participants were exposed to punishment schedules that had an RI value twice the RI values of the reinforcement schedules upon which they are superimposed (e.g., RI 2.3-s RI 14.6-s for the RI 1.2-s RI 7.3-s reinforcement schedule). During the changing punishment condition, punishment schedules were the same superimposed RI punishment schedules as used in the proportional condition, but the reinforcement schedule was held constant (RI 2-s RI 2-s) across all nine concurrent reinforcement schedules.

Each participant produced a dataset with 36 data points across the four conditions of the experiment. This data series is a vast improvement in terms of the number of data points per participant compared to previous studies. As in Experiments 1 and 2, the models developed in this dissertation (viz., Equations 2b′, 2c′, 3b′, 3c′, and 4c′) and the GML were fitted to each of the 30 participants' data points in Microsoft Excel using the GRG Nonlinear Solving Method within the Solver Add-In. AICc and BIC values were calculated across the multiple datasets using McArdle et al.'s (2007) method. The "best" model was determined as the model with the lowest IC value in the set. All other models were compared to the best model.

**5.2. Results**

Table A2 of Appendix A shows the number of data points (i.e, concurrent schedules) to which each model was fitted, and the parameter estimates, AIC values, and BIC values for each model's fit to each participant's data. Summaries of each model's descriptive accuracy across the entirety of the data series are presented in Table 5. As with Tables 1 and 3, the second column displays the median PVAF and 95% CI around the median. There was no statistically significant difference in the amount of variance explained across the different models. The AICc, ΔAICc, BIC, and ΔBIC values for each model are presented in third through sixth columns, respectively. By AICc, the cGML-based punishment model (Equation 4c′) was the "best" model of the set; by BIC, the GML (Equation 1b) was the "best" model of the set. In terms of AICc, Equation 4c′ significantly outperformed the GML and all other punishment models accounting for complexity. In terms of BIC, the GML significantly outperformed all but Equation 4c′ when accounting for complexity; the ΔBIC value for the comparison between the GML and Equation 4c′ was less than 10.

**5.3. Discussion**

Of the 1080 concurrent schedules presented, the participants exhibited exclusive preference in 91 (8.4%) of them. This rate is slightly lower than that seen in Klapes et al.'s (2020) original PRESS-B study (11%) but much higher than those from the unpublished studies designed to limit exclusive preference (described on pp. 43-44; 1.3% for Fall 2018/Spring 2019 study). For a more direct comparison, 3.7% of the baseline schedules showed exclusive preference. The punishment conditions contributed to the higher exclusive preference rate; the "constant and equal" condition elicited more schedules during which participants exhibited exclusive preference (11.5%) than the "proportional" (8.1%) and "changing punishment" (8.5%) conditions. Regardless, all participants had at least 22 schedules to which the models could be fitted, representing a vast improvement over previously collected live organism continuous choice datasets incorporating punishment.

Unlike the two previous experiments, AICc and BIC gave two different results for the "best" model in this experiment. When AICc and BIC agree, then the conclusion is very straightforward: the "best" model is consistently supported. However, when the information criteria disagree as to which model in the set is superior, the researcher must analyze the aims and experimental design of their study to arrive at the appropriate conclusion. Brewer et al. (2016), in a simulation study, concluded that AICc is more accurate than BIC in picking the "best" model within a set when there are strong effects, highly correlated covariates, and small numbers of data points per dataset. This experiment would appear to fit these criteria; all models presented in this dissertation explained a large amount of the variance in the data, the acquired punishment rates are inherently strongly correlated with the acquired reinforcement rates (i.e., both stimuli are contingent upon the same response), and (when compared to between-subject designs) the number of data points per fit was relatively small. As such, AICc could be considered as the preferable information criterion with which to draw a conclusion.

By AICc, three of the punishment models — the subtractive models, Equations 3b′ and 3c′, and the cGML-based punishment model, Equation 4c′ — convincingly outperformed the GML (ΔAICc values were 89.35, 96.87, and 132.96, respectively). Thus, the qualitative prediction made by the ETBD that the GML would consistently be the best model was not replicated in this experiment. More importantly, Equation 4c′ also convincingly outperformed all other punishment models in this experiment by AICc. Thus, Equation 4c′ can be considered the contemporary quantitative model of punishment.

## General Discussion

When only interpreting AICc values, the three experiments in this dissertation come to different conclusions. The data from Experiment 1 strongly supported the GML over all candidate punishment models. As mentioned on p. 31, the results of Experiment 1 are likely due to the information criteria unfairly discounting the more complex models when fitting to such small datasets. The data from Experiment 2, which was designed to eliminate this unfairness by fitting each model to more data points, also supported the GML over all candidate models (although not quite as strongly as in Experiment 1). The data from Experiment 3, however, supported the cGML-based punishment model (Equation 4c′) over the GML and other punishment models. Both Experiments 2 and 3 had many more datapoints per fit, which presumably alleviated the heavy discounting due to complexity. Why, then, wasn't the ETBD prediction confirmed with human participants using PRESS-B?

## 6.1. Failure to Replicate the ETBD Qualitative Prediction with PRESS-B

There are many possible explanations for the failed replication. However, comparisons of Tables 2 and 4 show that Experiment 3 was not a direct replication of Experiment 2. Thus, it is likely that methodological differences between the two experiments are the reason for the discrepancy in the results.

### *The "Changing Punishment" Condition*

The main methodological difference between the two experiments was the inclusion of the "changing punishment" condition in Experiment 3. This condition being included in the experiment should have eliminated the GML from contention; when the reinforcement rate is constant and equal across both alternatives, then the GML would predict response allocation to be constant and equal across both alternatives, too. With differential punishment rates on the two alternatives, it would seem very unlikely that a participant would allocate the same number of responses to two alternatives that produced equal reinforcement but different amounts of punishment. This condition's data being included in the fit should have deflated the GML's overall descriptive accuracy, giving an advantage to the punishment models (i.e., the ones that incorporate punishment rates into the response allocation prediction).

The data tell a different story. Table 6 shows the median parameter estimates and PVAF for GML fits to the data from each condition in Experiments 2 and 3. The GML fit the data produced from Experiment 3's "changing punishment" condition excellently (last row of Table 6). On face value, this result is perplexing; this experimental design should have resulted in terrible fits of the GML. However, a clarifying factor for this result is that the GML is fitted to the “acquired” reinforcement rate, not the “scheduled” reinforcement rate. As such, when responses were driven away from the more punished alternative, the acquired reinforcement rate also decreased. Thus, even though the scheduled rates of reinforcement were equivalent across the two alternatives, the acquired reinforcement rates were not. The changing punishment condition may have, in fact, given a slight advantage to the GML; most participants’ response ratios closely followed their acquired reinforcement ratios in this condition. Consequently, the changing punishment condition is unlikely to be the cause of the difference in results generated from Experiments 2 and 3.

### ***Relatively Weak Punishment in Experiment 2***

The GML fit the data from Experiment 2 exceptionally well in all conditions (median PVAF for all conditions = 0.99; Table 6). Additionally, one can see that two of the “constant and equal” conditions (“Constant and Equal 1” and “Constant and Equal 2”) did not show the expected shift toward overmatching (i.e., an increase in the sensitivity parameter estimate compared to the baseline condition); that is, the GML *a* parameter estimates during these conditions were not significantly different than the *a* parameter estimate during the baseline condition (compare median and 95% CI of *a* in columns 1, 2, and 3 of Table 6), while the *a* parameter estimates during the other three constant and equal conditions were significantly greater than the baseline *a* parameter estimate. These two empirical results suggest that the punishing contingencies may have been relatively weak during Experiment 2.

This hypothesis was followed up with an additional empirical investigation. Figure 4 compares two “single-alternative” conditions (e.g., Experiment 1 of Critchfield et al., 2003) when using the reinforcement schedules similar to those used in Experiments 2 and 3. For Figure 4*A*, a RI 70 RI 70 reinforcement schedule (i.e., the reinforcement schedule that is the median of the set used during Experiment 2) was implemented in an ETBD study as the baseline upon which the P50 (RI 140) and P80 (RI 87.5) punishment schedules are superimposed. Figure 4*B* represents a replotting of data from Klapes' (2016) single-alternative study, focusing on the RI 2-s RI 2-s reinforcement schedule condition (i.e., the reinforcement schedule that is the median of the set used during Experiment 3). Neither the P50 punishment schedule nor the P80 punishment schedule showed a significant suppressive effect when superimposed on the conc RI 70 RI 70 reinforcement schedule in the ETBD study. In contrast, the suppression effects of the RI 4-s (P50) and RI 2.5-s (P80) punishment schedules on the conc RI 2-s RI 2-s reinforcement were quite extreme. Thus, the GML was likely unfairly advantaged in Experiment 2 due to relatively weak punishment used in the experiment.

## 6.2. Future Directions

### *Continued Developments for Subtractive Theory-Based Punishment Models*

As evidenced by de Villiers (1980), Farley (1980), Critchfield et al. (2003), Reed and Yoshino (2005), McDowell and Klapes (2019), and the experiments in this dissertation, the subtractive theory's qualitative predictions (i.e., that in "constant and equal" conditions there will be a shift towards overmatching, while in "proportional" conditions there will be no change in the reinforcement rate sensitivity) are widely supported over the additive theory. Additionally, some version of the subtractive theory would bear a striking resemblance to Tversky and Kahneman's (1992) cumulative prospect theory (CPT) model, which is often used to estimate discrete-trial behavior under reinforcing and punishing contingencies (e.g., in mixed-gambles tasks; see pp. 10-11 for discussion). This model is typically presented as a two-part (one part for gains and one part for losses) power function,

$$v(x) = \begin{cases} x^{\alpha} & if\ \mathrm{x} \geq 0 \\ -\lambda\,(-x)^{\beta} & if\ x < 0 \end{cases},$$

where *v* is the value of the choice, *x* is the objective outcome of the choice, α and β are freely estimated exponents for gains and losses, respectively, and λ is a coefficient that accounts for loss aversion. Thus, with more consideration, it is possible that some other model based on the subtractive theory could eventually supplant the cGML-based punishment model (Equation 4c′) as the superior quantitative punishment model in the future.

### *An Alternative to the GML: The Contingency-Discriminability Model (CDM)*

Although the GML is generally touted as the superior mathematical statement of continuous choice, there are alternative models. One of these models is the contingency-discriminability model (CDM; Davison & Jenkins, 1985):

$$\frac{B_1}{B_2} = b\left(\frac{d_r R_1 + R_2}{d_r R_2 + R_1}\right),$$

where $d_r$ represents the amount of discriminability between the two alternatives. This parameter can take on values ranging from 1 to ∞; when $d_r$ is 1, there is no discriminability of the two alternatives (i.e., the subject cannot tell the two alternatives apart) and when $d_r$ is ∞, there is complete discriminability of the two alternatives. Through algebraic manipulation, Davison and Jones (1995) present a more intuitive version of the CDM:

$$\frac{B_1}{B_2} = b\left(\frac{R_1 - pR_1 + pR_2}{R_2 - pR_2 + pR_2}\right),$$

where *p* is the inverse of $d_r$; values of *p* range from 0 (complete discriminability) to 0.5 (no discriminability). The CDM has many advantages over the GML (M. Davison, personal communication, May 23rd, 2020); namely, it can be easily fitted to single-alternative data (i.e., when either $R_1$ or $R_2 = 0$, the model can still be fitted to data; see pp. 28 and 43 for reference), it predicts non-linearity in scenarios with highly discrepant reinforcement rates on the two alternatives (e.g., $R_1 >>> R_2$; Davison & Jones, 1995, 1998), and has parameters with meaning (i.e., $d_r$ and $p$ are rooted in a theoretical rationale, while the GML's $a$ has no theoretical basis).

In either form above, CDM is unable to predict the relatively uncommon, but occasionally observed, experimental scenario of overmatching. To account for this shortcoming, Davison and McCarthy (1994) proposed a version of the CDM as

$$\frac{B_1}{B_2} = b\left(\frac{d_r R_1 + R_2 \ - w}{d_r R_2 + R_1 \ - w}\right),$$

where *w* is the magnitude of punishment associated with response cost (e.g., effort or energy expenditure for emitting the operant behavior that constitutes the response). One can see the similarities of this model (in principle and by design) to de Villiers' (1977) subtractive model (Equation 3a); as noted in many instances above, the subtraction applied to both alternatives (e.g., a "constant and equal" punishment condition) will result in a shift towards overmatching. Thus, a CDM-based subtractive punishment model (i.e., a model that incorporates both acquired reinforcement and punishment rates) could be attainable. Based on the placement of *w* in the above model, that model may look like:

$$\frac{B_1}{B_2} = b\left[\frac{d_r R_1 + R_2 \ - c(d_p P_1 - \ P_2)}{d_r R_2 + R_1 \ - c(d_p P_2 \ - P_1)}\right],$$

or, in the preferred (Davison & Jones, 1995) form,

$$\frac{B_1}{B_2} = \ c\left[\frac{R_1 - p_R R_1 + p_R R_2 - c(P_1 - p_P P_1 + p_P P_2)}{R_2 - p_R R_2 + p_R R_1 - c(P_1 - p_P P_1 + p_P P_2)}\right],$$

where $d_p$ and $p_p$ are the discriminability and inverse discriminability, respectively, for punishment.

The support for the cGML-based punishment model (Equation 4c′) in this dissertation suggests that a “concatenated” CDM (cCDM) could be used to develop a CDM-based punishment model. M. Davison (personal communication, May 23rd, 2020) has proposed two versions of this prospective cCDM: one with the same $d_r$ for reinforcement rate and magnitude,

$$\frac{B_1}{B_2} = b\left(\frac{d_r R_1 M_1 + R_2 M_2}{d_r R_2 M_2 + R_1 M_1}\right),$$

and one model with separate discriminability parameters for rate ($d_r$) and magnitude ($d_{r_m}$),

$$\frac{B_1}{B_2} = b\left(\frac{d_r R_1 + R_2}{d_r R_2 + R_1}\right)\left(\frac{d_{r_m} M_1 + M_2}{d_{r_m} M_2 + M_1}\right),$$

Although neither has been empirically studied or supported, both models represent promise for future research involving the CDM and the development of CDM-based punishment models.

## 6.3. Conclusion

Based on a reanalysis of applicable previously collected live organism data (Experiment 1), none of the novel matching-law-based punishment models developed in this dissertation were supported over the GML. This result was likely due to the small number of data points used to fit each model. Studies using a well-regarded computational theory of operant behavior (ETBD; Experiment 2) and a recently developed rapid-acquisition procedure for human continuous choice (PRESS-B; Experiment 3) initially resulted in two divergent conclusions: the ETBD predicted that the GML was the superior model, while data generated by PRESS-B showed that a cGML-based punishment model (Equation 4c′) was better than the GML and the other punishment models. Upon closer inspection, Experiment 2 was found to have relatively weak punishing contingencies, which was the likely source of the discrepant conclusions. Thus, Equation 4c′ is the presumptive contemporary quantitative model of punishment.

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**Table 1.**

*Experiment 1 Results*

| ***Model*** | ***PVAF*** | ***AICc*** | ***ΔAICc*** | ***BIC*** | ***ΔBIC*** |
|---|---|---|---|---|---|
| *Equation 1b* | 0.92 [0.89, 0.94] | **-1191.05** | - | **-993.88** | - |
| *Equation 2b′* | 0.93 [0.90, 0.95] | -1069.29 | 121.76 | -853.66 | 140.22 |
| *Equation 3b′* | 0.93 [0.91, 0.95] | -1039.61 | 151.44 | -823.98 | 169.89 |
| *Equation 2c′* | 0.93 [0.90, 0.95] | -757.91 | 433.13 | -647.55 | 346.33 |
| *Equation 3c′* | 0.92 [0.91, 0.95] | -787.97 | 403.07 | -677.61 | 316.27 |
| *Equation 4c′* | **0.93** [0.91, 0.95] | -1077.31 | 113.74 | 861.68 | 132.30 |

*Note.* PVAF = proportion variance accounted for (median and 95% confidence interval around median); bolded values indicate “best” model

**Table 2.**

*Experiment 2 Study Design* (McDowell & Klapes, 2019)

| **Condition** | **RI values for punishment schedules** |
|---|---|
| *Baseline* | – |
| *Constant and Equal 1* | RI 120 RI 120 for all schedules |
| *Constant and Equal 2* | RI 60 RI 60 for all schedules |
| *Constant and Equal 3* | RI 40 RI 40 for all schedules |
| *Constant and Equal 4* | RI 20 RI 20 for all schedules |
| *Constant and Equal 5* | RI 10 RI 10 for all schedules |
| *Proportional 1* | 2x reinforcement RI values |
| *Proportional 2* | 1x reinforcement RI values |
| *Proportional 3* | 0.5x reinforcement RI values |
| *Proportional 4* | 0.25x reinforcement RI values |
| *Proportional 5* | 0.125x reinforcement RI values |

**Table 3.**

*Experiment 2 Results*

| ***Model*** | ***PVAF*** | ***AICc*** | ***ΔAICc*** | ***BICc*** | ***ΔBIC*** |
|---|---|---|---|---|---|
| *Equation 1b* | 0.98 [0.98, 0.99] | **-20455.87** | - | **-20087.10** | - |
| *Equation 2b′* | 0.98 [0.98, 0.99] | -20393.44 | 62.43 | -19841.87 | 245.23 |
| *Equation 3b′* | 0.98 [0.98, 0.99] | -20445.67 | 10.20 | -19894.10 | 193.00 |
| *Equation 2c′* | 0.98 [0.98, 0.99] | -20329.70 | 126.22 | -19596.36 | 490.74 |
| *Equation 3c′* | **0.98** [0.98, 0.99] | -20424.45 | 31.42 | -19691.16 | 395.94 |
| *Equation 4c′* | 0.98 [0.98, 0.99] | -20443.29 | 12.58 | -19891.72 | 195.38 |

*Note.* PVAF = proportion variance accounted for (median and 95% confidence interval around median); bolded values indicate “best” model

**Table 4.**

*Experiment 3 Study Design*

| **Condition** | **RI values for punishment** |
|---|---|
| *Baseline* | – |
| *Constant and Equal* | RI 8-s RI 8-s |
| *Proportional* | 2x reinforcement RI values |
| *Changing Punishment** | Same as *Proportional* |

*Note.* *unlike the other conditions, scheduled reinforcement was kept constant and equal (RI 2-s RI 2-s) for all schedules

**Table 5.**

*Experiment 3 Results*

| ***Model*** | ***PVAF*** | ***AICc*** | ***ΔAICc*** | ***BIC*** | ***ΔBIC*** |
|---|---|---|---|---|---|
| *Equation 1b* | 0.85 [0.78, 0.89] | -2995.84 | 132.96 | **-2714.28** | - |
| *Equation 2b′* | 0.85 [0.78, 0.89] | -2940.56 | 188.24 | -2525.23 | 189.05 |
| *Equation 3b′* | 0.86 [0.79, 0.89] | -3085.19 | 43.61 | -2669.87 | 44.42 |
| *Equation 2c′* | 0.85 [0.78, 0.89] | -2872.76 | 256.04 | -2329.03 | 385.25 |
| *Equation 3c′* | **0.88** [0.80, 0.90] | -3092.71 | 36.09 | -2548.98 | 165.30 |
| *Equation 4c′* | 0.87 [0.80, 0.90] | **-3128.80** | - | -2713.98 | 0.81 |

*Note.* PVAF = proportion variance accounted for (median and 95% confidence interval around median); bolded values indicate "best" model

**Table 6.**

*GML Fits by Condition*

| Condition | *a* | log(*b*) | PVAF |
|---|---|---|---|
| ***Experiment 2*** | | | |
| *Baseline* | 0.85 [0.84, 0.87] | -0.01 [-0.02, 0.01] | 0.99 [0.99, 0.99] |
| *Constant and Equal 1* | 0.85 [0.83, 0.87] | 0.00 [0.00, 0.01] | 0.99 [0.99, 0.99] |
| *Constant and Equal 2* | 0.86 [0.85, 0.87] | 0.00 [-0.01, 0.01] | 0.99 [0.99, 0.99] |
| *Constant and Equal 3* | 0.90 [0.88, 0.91] | 0.01 [0.00, 0.01] | 0.99 [0.99, 0.99] |
| *Constant and Equal 4* | 0.92 [0.91, 0.94] | 0.00 [-0.01, 0.00] | 0.99 [0.99, 0.99] |
| *Constant and Equal 5* | 0.95 [0.93, 0.95] | 0.00 [0.00, 0.01] | 0.99 [0.99, 0.99] |
| *Proportional 1* | 0.84 [0.83, 0.87] | 0.00 [0.00, 0.01] | 0.99 [0.98, 0.99] |
| *Proportional 2* | 0.85 [0.82, 0.86] | 0.00 [-0.01, 0.01] | 0.99 [0.98, 0.99] |
| *Proportional 3* | 0.85 [0.83, 0.87] | 0.00 [-0.01, 0.01] | 0.99 [0.98, 0.99] |
| *Proportional 4* | 0.87 [0.85, 0.89] | 0.00 [-0.01, 0.01] | 0.99 [0.99, 0.99] |
| *Proportional 5* | 0.87 [0.86, 0.88] | 0.00 [-0.01, 0.01] | 0.99 [0.98, 0.99] |
| ***Experiment 3*** | | | |
| *Baseline* | 0.50 [0.46, 0.61] | -0.02 [-0.04, 0.01] | 0.92 [0.81, 0.94] |
| *Constant and Equal* | 0.67 [0.58, 0.72] | 0.01 [-0.03, 0.03] | 0.93 [0.87, 0.95] |
| *Proportional* | 0.45 [0.29, 0.65] | -0.01 [-0.04, 0.01] | 0.68 [0.53, 0.79] |
| *Changing Punishment* | 0.95 [0.89, 0.99] | -0.01 [-0.03, 0.02] | 0.97 [0.95, 0.97] |

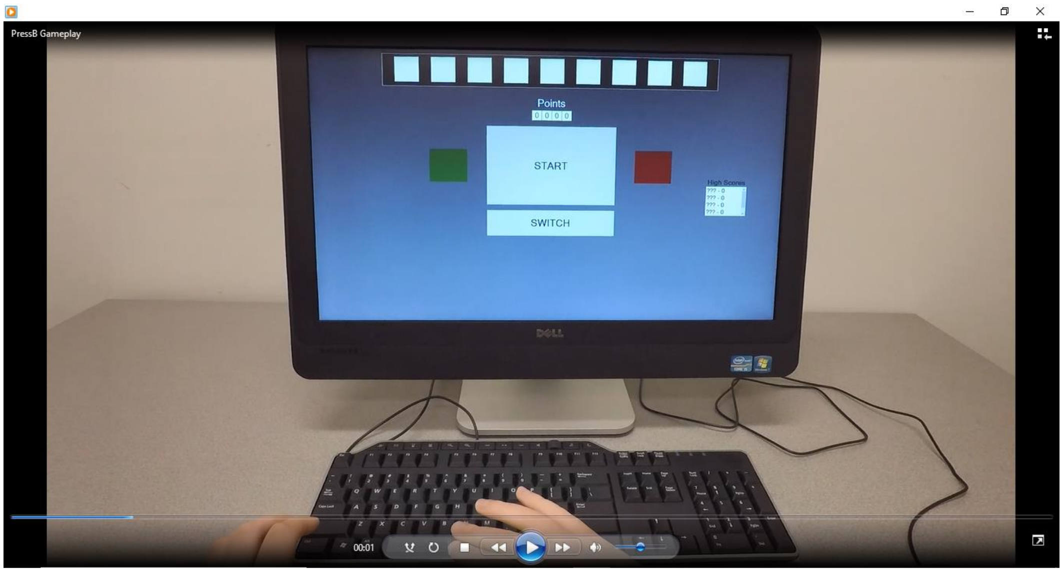
**Figure 1.**

*Previously Published Matching-Law-Based Punishment Models*

| Original Models | | Generalized Models | |
|---|---|---|---|
| No-punishment Models | | | |
| (1a) | $\log\left(\frac{B_1}{B_2}\right) = \log\left(\frac{R_1}{R_2}\right)$ | (1b) | $\log\left(\frac{B_1}{B_2}\right) = a\log\left(\frac{R_1}{R_2}\right) + \log(b)$ |
| Additive Models | | | |
| (2a) | $\log\left(\frac{B_1}{B_2}\right) = \log\left(\frac{R_1 + P_2}{R_2 + P_1}\right)$ | (2b) | $\log\left(\frac{B_1}{B_2}\right) = a\log\left(\frac{R_1 + P_2}{R_2 + P_1}\right) + \log(b)$ |
| (2a′) | $\log\left(\frac{B_1}{B_2}\right) = \log\left(\frac{R_1 + c\,P_2}{R_2 + c\,P_1}\right)$ | | |
| Subtractive Models | | | |
| (3a) | $\log\left(\frac{B_1}{B_2}\right) = \log\left(\frac{R_1 - P_1}{R_2 - P_2}\right)$ | (3b) | $\log\left(\frac{B_1}{B_2}\right) = a\log\left(\frac{R_1 - P_1}{R_2 - P_2}\right) + \log(b)$ |
| (3a′) | $\log\left(\frac{B_1}{B_2}\right) = \log\left(\frac{R_1 - c\,P_1}{R_2 - c\,P_2}\right)$ | | |

**Figure 2.**

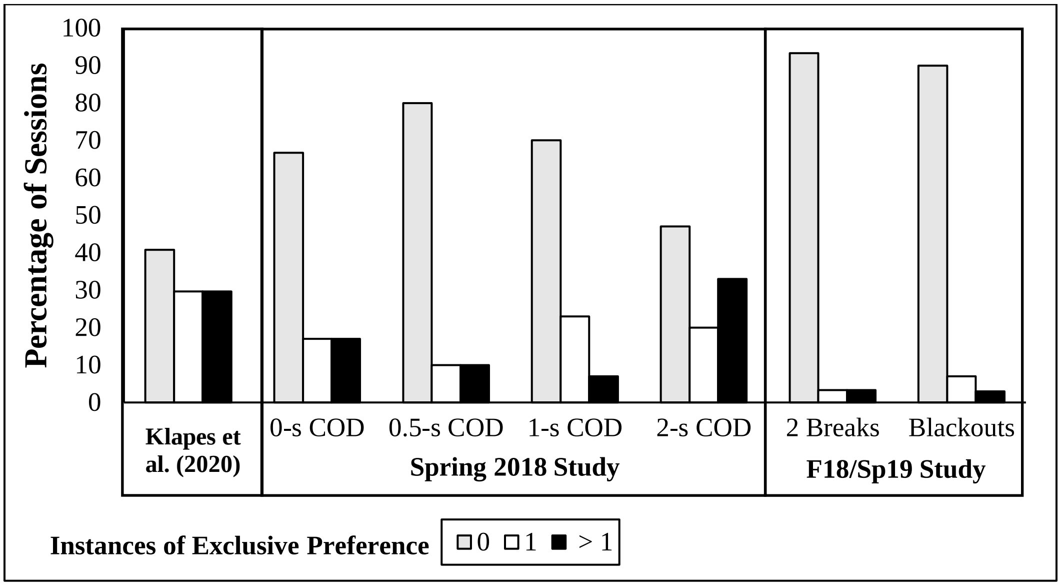
*Video Gameplay of PRESS-B*




*Note.* Screenshot of video; please play via supplemental material file link

**Figure 3.**

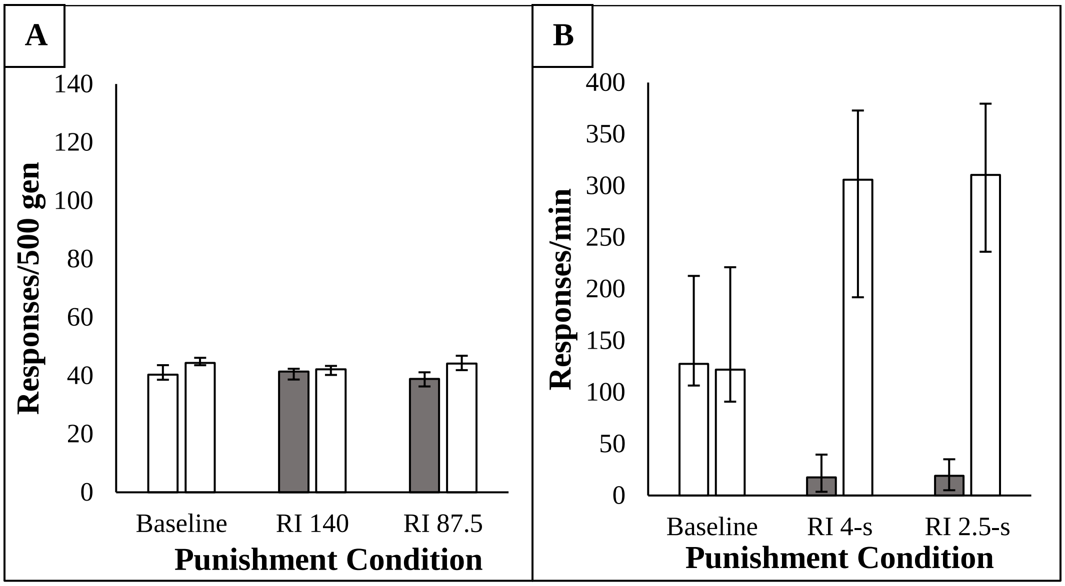
*Percentage of Sessions with Exclusive Preference*




*Note.* F18/Sp19 = Fall 2018/Spring 2019

**Figure 4.**

*Response Rates across Punishment Conditions: ETBD* (A) *vs. PRESS-B* (B)



## Appendix A

**Table A1.**

*Experiment 1: Number of schedules (n), parameter estimates, and fit quality metrics (AICc and BIC) for each model when fitted to each subject's data*

| **Study** *Subject* | | *n* | $a_R$ | log(*b*) | *c* | $a_P$ | *AICc* | *BIC* |
|---|---|---|---|---|---|---|---|---|
| **Farley (1980)** | | | | | | | | |
| *Bird 1* | | 10 | | | | | | |
| | Equation 1b | | 1.16 | 0.05 | - | - | -34.13 | -35.24 |
| | Equation 2b′ | | 1.16 | 0.05 | 0.00 | - | -29.85 | -32.94 |
| | Equation 3b′ | | 1.09 | 0.05 | 0.02 | - | -30.85 | -33.95 |
| | Equation 2c′ | | 1.16 | 0.05 | 0.00 | 1.00 | -23.85 | -30.64 |
| | Equation 3c′ | | 1.02 | 0.03 | 0.00* | 6.21 | -31.09 | -37.88 |
| | Equation 4c′ | | 0.99 | 0.04 | - | 0.09 | -34.75 | -37.84 |
| *Bird 2* | | 10 | | | | | | |
| | Equation 1b | | 1.16 | 0.07 | - | - | -34.18 | -35.28 |
| | Equation 2b′ | | 1.16 | 0.07 | 0.00 | - | -29.89 | -32.98 |
| | Equation 3b′ | | 1.16 | 0.07 | 0.00* | - | -29.90 | -32.99 |
| | Equation 2c′ | | 1.16 | 0.07 | 0.00 | 1.00 | -23.89 | -30.68 |
| | Equation 3c′ | | 1.16 | 0.07 | 0.00* | 1.38 | -23.78 | -30.57 |
| | Equation 4c′ | | 1.09 | 0.07 | - | 0.04 | -30.49 | -33.58 |
| *Bird 3* | | 10 | | | | | | |
| | Equation 1b | | 0.91 | -0.06 | - | - | -46.05 | -47.16 |
| | Equation 2b′ | | 0.91 | -0.06 | 0.00 | - | -41.77 | -44.86 |
| | Equation 3b′ | | 0.84 | -0.06 | 0.02 | - | -43.39 | -46.49 |
| | Equation 2c′ | | 0.91 | -0.06 | 0.00 | 1.00 | -35.77 | -42.56 |
| | Equation 3c′ | | 0.91 | -0.06 | 0.00 | 1.00 | -35.67 | -42.46 |
| | Equation 4c′ | | 0.84 | -0.06 | - | 0.03 | -43.51 | -46.61 |
| **Critchfield et al. (2003)** | | | | | | | | |
| *Subject 209* | | 12 | | | | | | |
| | Equation 1b | | 0.32 | 0.01 | - | - | -23.00 | -23.36 |
| | Equation 2b′ | | 0.62 | 0.01 | 4.33 | - | -46.77 | -48.31 |
| | Equation 3b′ | | 0.32 | 0.01 | 0.00 | - | -19.33 | -20.87 |
| | Equation 2c′ | | 0.62 | 0.00 | 0.01 | 1.80 | -42.38 | -46.15 |
| | Equation 3c′ | | 0.58 | -0.02 | 0.00* | 2.34 | -35.72 | -39.49 |
| | Equation 4c′ | | 0.43 | 0.00 | - | -3.81 | -24.00 | -25.54 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *Subject 210* | 7 | | | | | | |
| Equation 1b | | 0.83 | -0.19 | - | - | -28.18 | -31.29 |
| Equation 2b′ | | 0.87 | -0.23 | 0.01 | - | -22.78 | -30.95 |
| Equation 3b′ | | 0.83 | -0.19 | 0.00 | - | -21.18 | -29.34 |
| Equation 2c′ | | 0.87 | -0.23 | 0.19 | 0.34 | -8.97 | -29.19 |
| Equation 3c′ | | 0.83 | -0.19 | 0.00 | 1.00 | -7.18 | -27.40 |
| Equation 4c′ | | 0.84 | -0.20 | - | -0.35 | -21.34 | -29.50 |
| *Subject 243* | 10 | | | | | | |
| Equation 1b | | 0.23 | 0.02 | - | - | -21.92 | -23.03 |
| Equation 2b′ | | 0.46 | 0.03 | 6.57 | - | -39.84 | -42.94 |
| Equation 3b′ | | 0.23 | 0.02 | 0.00 | - | -17.63 | -20.72 |
| Equation 2c′ | | 0.46 | 0.03 | 0.83 | 0.70 | -34.09 | -40.88 |
| Equation 3c′ | | 0.42 | -0.01 | 0.19 | 0.79 | -30.02 | -36.81 |
| Equation 4c′ | | 0.29 | 0.07 | - | -2.19 | -18.99 | -22.08 |
| *Subject 252* | 12 | | | | | | |
| Equation 1b | | 0.26 | -0.03 | - | - | -47.47 | -47.84 |
| Equation 2b′ | | 0.35 | -0.03 | 0.43 | - | -55.60 | -57.14 |
| Equation 3b′ | | 0.26 | -0.03 | 0.00 | - | -43.81 | -45.35 |
| Equation 2c′ | | 0.35 | -0.03 | 0.43 | 0.35 | -48.67 | -52.44 |
| Equation 3c′ | | 0.33 | -0.03 | 0.01 | 1.07 | -43.98 | -47.75 |
| Equation 4c′ | | 0.33 | -0.01 | - | -1.66 | -58.09 | -59.63 |
| *Subject 253* | 10 | | | | | | |
| Equation 1b | | 1.09 | -0.23 | - | - | -26.85 | -27.96 |
| Equation 2b′ | | 1.09 | -0.23 | 0.00 | - | -22.57 | -25.66 |
| Equation 3b′ | | 1.09 | -0.23 | 0.00 | - | -22.57 | -25.66 |
| Equation 2c′ | | 1.09 | -0.23 | 0.00 | 1.00 | -16.57 | -23.36 |
| Equation 3c′ | | 1.04 | -0.23 | 1.21 | 0.46 | -17.25 | -24.04 |
| Equation 4c′ | | 1.10 | -0.22 | - | -1.12 | -22.81 | -25.90 |
| *Subject 254* | 12 | | | | | | |
| Equation 1b | | 0.62 | 0.02 | - | - | -37.75 | -38.11 |
| Equation 2b′ | | 0.63 | 0.02 | 0.02 | - | -34.22 | -35.76 |
| Equation 3b′ | | 0.62 | 0.02 | 0.00 | - | -34.08 | -35.63 |
| Equation 2c′ | | 0.62 | 0.02 | 0.02 | 0.50 | -29.38 | -33.16 |
| Equation 3c′ | | 0.62 | 0.02 | 0.00 | 1.00 | -29.37 | -33.14 |
| Equation 4c′ | | 0.64 | 0.03 | - | -0.81 | -34.69 | -36.23 |
| *Subject 265* | 7 | | | | | | |
| Equation 1b | | 0.68 | 0.18 | - | - | -16.09 | -19.20 |
| Equation 2b′ | | 0.68 | 0.18 | 0.00 | - | -9.09 | -17.26 |
| Equation 3b′ | | 0.46 | 0.34 | 1.49 | - | -15.28 | -23.44 |
| Equation 2c′ | | 0.68 | 0.18 | 0.00 | 1.00 | 4.91 | -15.31 |
| Equation 3c′ | | 0.66 | 0.26 | 1.16 | 0.69 | -1.49 | -21.70 |
| Equation 4c′ | | 0.46 | 0.36 | - | 11.6 | -13.84 | -22.00 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *Subject 267* | 7 | | | | | | |
| Equation 1b | | 1.01 | -0.07 | - | - | -7.18 | -10.29 |
| Equation 2b′ | | 1.23 | -0.09 | 0.14 | - | -4.95 | -13.12 |
| Equation 3b′ | | 1.04 | -0.09 | 0.75 | - | -0.36 | -8.52 |
| Equation 2c′ | | 1.24 | -0.09 | 0.13 | 1.19 | 9.58 | -10.64 |
| Equation 3c′ | | 1.27 | -0.13 | 0.07 | 1.93 | 5.20 | -15.02 |
| Equation 4c′ | | 1.08 | -0.12 | - | 2.33 | -0.73 | -8.89 |
| *Subject 268* | 8 | | | | | | |
| Equation 1b | | 0.86 | 0.06 | - | - | -11.66 | -13.90 |
| Equation 2b′ | | 0.89 | 0.05 | 0.02 | - | -6.30 | -12.06 |
| Equation 3b′ | | 0.86 | 0.06 | 0.00 | - | -6.06 | -11.82 |
| Equation 2c′ | | 0.90 | 0.05 | 0.00* | 3.34 | 2.87 | -10.14 |
| Equation 3c′ | | 0.90 | 0.04 | 0.00* | 4.30 | 2.93 | -10.09 |
| Equation 4c′ | | 0.88 | 0.04 | - | -0.81 | -6.22 | -11.98 |
| *Subject 512* | 10 | | | | | | |
| Equation 1b | | 0.78 | -0.02 | - | - | -24.17 | -25.28 |
| Equation 2b′ | | 0.78 | -0.02 | 0.00 | - | -19.88 | -22.97 |
| Equation 3b′ | | 0.59 | 0.03 | 1.14 | - | -23.74 | -26.83 |
| Equation 2c′ | | 0.78 | -0.02 | 0.00 | 1.00 | -13.88 | -20.67 |
| Equation 3c′ | | 0.61 | 0.04 | 0.69 | 0.74 | -17.54 | -24.33 |
| Equation 4c′ | | 0.64 | 0.03 | - | 2.78 | -22.94 | -26.03 |
| *Subject 513* | 8 | | | | | | |
| Equation 1b | | 0.68 | -0.07 | - | - | -11.03 | -13.27 |
| Equation 2b′ | | 0.68 | -0.07 | 0.00 | - | -5.43 | -11.19 |
| Equation 3b′ | | 0.54 | -0.07 | 0.75 | - | -6.69 | -12.45 |
| Equation 2c′ | | 0.68 | -0.07 | 0.00 | 1.00 | 3.90 | -9.11 |
| Equation 3c′ | | 0.49 | 0.04 | 0.00* | 2.96 | -5.37 | -18.38 |
| Equation 4c′ | | 0.41 | -0.06 | - | 2.47 | -8.81 | -14.57 |
| *Subject 514* | 10 | | | | | | |
| Equation 1b | | 0.69 | 0.08 | - | - | -35.90 | -37.01 |
| Equation 2b′ | | 0.69 | 0.08 | 0.00 | - | -31.61 | -34.71 |
| Equation 3b′ | | 0.55 | 0.11 | 0.70 | - | -42.76 | -45.85 |
| Equation 2c′ | | 0.69 | 0.08 | 0.00 | 1.00 | -25.61 | -32.40 |
| Equation 3c′ | | 0.51 | 0.10 | 0.68 | 0.48 | -42.80 | -49.59 |
| Equation 4c′ | | 0.52 | 0.08 | - | 1.56 | -49.00 | -52.09 |
| *Subject 515* | 10 | | | | | | |
| Equation 1b | | 0.48 | 0.00 | - | - | -30.60 | -31.71 |
| Equation 2b′ | | 0.48 | 0.00 | 0.00 | - | -26.32 | -29.41 |
| Equation 3b′ | | 0.46 | 0.01 | 0.47 | - | -26.54 | -29.64 |
| Equation 2c′ | | 0.48 | 0.00 | 0.00 | 1.00 | -20.32 | -27.10 |
| Equation 3c′ | | 0.48 | 0.00 | 0.05 | 0.62 | -20.38 | -27.17 |
| Equation 4c′ | | 0.47 | 0.00 | - | 0.30 | -26.40 | -29.49 |

**Reed & Yoshino (2005)**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *Rat 1 – Rich, 80% FFW* | 8 | | | | | | |
| Equation 1b | | 0.44 | -0.01 | - | - | -52.20 | -54.44 |
| Equation 2b′ | | 0.44 | -0.01 | 0.00 | - | -46.60 | -52.36 |
| Equation 3b′ | | 0.41 | -0.01 | 0.20 | - | -49.08 | -54.84 |
| Equation 2c′ | | 0.44 | -0.01 | 0.00 | 1.00 | -37.26 | -50.28 |
| Equation 3c′ | | 0.42 | -0.01 | 0.30 | 2.18 | -44.68 | -57.70 |
| Equation 4c′ | | 0.41 | -0.01 | - | 0.10 | -48.82 | -54.58 |
| *Rat 3 – Rich, 80% FFW* | 8 | | | | | | |
| Equation 1b | | 0.68 | -0.07 | - | - | -30.72 | -32.97 |
| Equation 2b′ | | 0.75 | -0.06 | 0.31 | - | -26.13 | -31.89 |
| Equation 3b′ | | 0.68 | -0.07 | 0.00 | - | -25.12 | -30.89 |
| Equation 2c′ | | 0.75 | -0.06 | 0.25 | 2.81 | -16.79 | -29.80 |
| Equation 3c′ | | 0.68 | -0.07 | 0.00 | 1.00 | -15.79 | -28.81 |
| Equation 4c′ | | 0.75 | -0.07 | - | -0.33 | -26.25 | -32.01 |
| *Rat 4 – Rich, 80% FFW* | 8 | | | | | | |
| Equation 1b | | 0.86 | 0.16 | - | - | -28.33 | -30.57 |
| Equation 2b′ | | 1.00 | 0.17 | 0.55 | - | -26.92 | -32.68 |
| Equation 3b′ | | 0.86 | 0.16 | 0.00 | - | -22.73 | -28.49 |
| Equation 2c′ | | 1.02 | 0.17 | 1.16 | 16.7 | -27.61 | -40.63 |
| Equation 3c′ | | 0.86 | 0.16 | 0.00 | 1.00 | -13.39 | -26.41 |
| Equation 4c′ | | 0.94 | 0.16 | - | -0.32 | -23.74 | -29.50 |
| *Rat 1 – Rich, 90% FFW* | 8 | | | | | | |
| Equation 1b | | 0.56 | 0.19 | - | - | -33.09 | -35.33 |
| Equation 2b′ | | 0.56 | 0.19 | 0.00 | - | -27.49 | -33.25 |
| Equation 3b′ | | 0.45 | 0.19 | 0.46 | - | -30.66 | -36.42 |
| Equation 2c′ | | 0.56 | 0.19 | 0.00 | 1.00 | -18.15 | -31.17 |
| Equation 3c′ | | 0.42 | 0.19 | 0.46 | 0.35 | -21.99 | -35.01 |
| Equation 4c′ | | 0.43 | 0.19 | - | 0.46 | -31.40 | -37.16 |
| *Rat 2 – Rich, 90% FFW* | 8 | | | | | | |
| Equation 1b | | 0.64 | 0.06 | - | - | -32.09 | -34.33 |
| Equation 2b′ | | 0.64 | 0.06 | 0.00 | - | -26.49 | -32.25 |
| Equation 3b′ | | 0.59 | 0.06 | 0.21 | - | -26.97 | -32.73 |
| Equation 2c′ | | 0.64 | 0.06 | 0.00 | 1.00 | -17.15 | -30.17 |
| Equation 3c′ | | 0.64 | 0.06 | 0.00 | 1.00 | -17.15 | -30.17 |
| Equation 4c′ | | 0.57 | 0.06 | - | 0.24 | -27.22 | -32.98 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *Rat 3 – Rich, 90% FFW* | 8 | | | | | | |
| Equation 1b | | 0.46 | -0.29 | - | - | -37.59 | -39.83 |
| Equation 2b′ | | 0.46 | -0.29 | 0.00 | - | -31.99 | -37.75 |
| Equation 3b′ | | 0.41 | -0.29 | 0.30 | - | -33.54 | -39.30 |
| Equation 2c′ | | 0.46 | -0.29 | 0.00 | 1.00 | -22.66 | -35.67 |
| Equation 3c′ | | 0.40 | -0.29 | 0.30 | 0.41 | -24.54 | -37.56 |
| Equation 4c′ | | 0.40 | -0.29 | - | 0.22 | -33.79 | -39.55 |
| *Rat 4 – Rich, 90% FFW* | 8 | | | | | | |
| Equation 1b | | 0.63 | 0.06 | - | - | -27.81 | -30.05 |
| Equation 2b′ | | 0.63 | 0.06 | 0.00 | - | -22.21 | -27.98 |
| Equation 3b′ | | 0.56 | 0.06 | 0.29 | - | -22.78 | -28.54 |
| Equation 2c′ | | 0.63 | 0.06 | 0.00 | 1.00 | -12.88 | -25.90 |
| Equation 3c′ | | 0.63 | 0.06 | 0.00 | 1.00 | -12.88 | -25.90 |
| Equation 4c′ | | 0.54 | 0.06 | - | 0.33 | -22.99 | -28.76 |
| *Rat 1 – Rich, FFW* | 8 | | | | | | |
| Equation 1b | | 0.64 | 0.10 | - | - | -38.28 | -40.52 |
| Equation 2b′ | | 0.64 | 0.10 | 0.00 | - | -32.68 | -38.45 |
| Equation 3b′ | | 0.63 | 0.10 | 0.06 | - | -32.75 | -38.51 |
| Equation 2c′ | | 0.64 | 0.10 | 0.00 | 1.00 | -23.35 | -36.37 |
| Equation 3c′ | | 0.64 | 0.10 | 0.00 | 1.00 | -23.35 | -36.37 |
| Equation 4c′ | | 0.62 | 0.10 | - | 0.06 | -32.77 | -38.53 |
| *Rat 2 – Rich, FFW* | 8 | | | | | | |
| Equation 1b | | 0.87 | 0.10 | - | - | -25.51 | -27.75 |
| Equation 2b′ | | 0.88 | 0.10 | 0.03 | - | -19.92 | -25.68 |
| Equation 3b′ | | 0.87 | 0.10 | 0.00 | - | -19.91 | -25.67 |
| Equation 2c′ | | 0.87 | 0.10 | 0.00 | 1.00 | -10.57 | -23.59 |
| Equation 3c′ | | 0.87 | 0.10 | 0.00 | 1.00 | -10.57 | -23.59 |
| Equation 4c′ | | 0.93 | 0.09 | - | -0.24 | -20.27 | -26.03 |
| *Rat 3 – Rich, FFW* | 8 | | | | | | |
| Equation 1b | | 0.43 | -0.03 | - | - | -50.30 | -52.54 |
| Equation 2b′ | | 0.43 | -0.03 | 0.00 | - | -44.70 | -50.46 |
| Equation 3b′ | | 0.42 | -0.03 | 0.08 | - | -44.92 | -50.68 |
| Equation 2c′ | | 0.43 | -0.03 | 0.00 | 1.00 | -35.37 | -48.38 |
| Equation 3c′ | | 0.43 | -0.03 | 0.00 | 1.00 | -35.37 | -48.38 |
| Equation 4c′ | | 0.42 | -0.03 | - | 0.04 | -44.88 | -50.64 |
| *Rat 4 – Rich, FFW* | 8 | | | | | | |
| Equation 1b | | 0.81 | 0.18 | - | - | -32.50 | -34.74 |
| Equation 2b′ | | 0.81 | 0.18 | 0.00 | - | -26.90 | -32.66 |
| Equation 3b′ | | 0.81 | 0.18 | 0.00 | - | -26.90 | -32.66 |
| Equation 2c′ | | 0.81 | 0.18 | 0.00 | 1.00 | -17.57 | -30.58 |
| Equation 3c′ | | 0.81 | 0.18 | 0.00 | 1.00 | -17.57 | -30.58 |
| Equation 4c′ | | 0.82 | 0.18 | - | -0.03 | -26.91 | -32.67 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *Rat 2 – Lean, 80% FFW* | 8 | | | | | | |
| Equation 1b | | 0.37 | 0.15 | - | - | -38.70 | -40.94 |
| Equation 2b′ | | 0.37 | 0.15 | 0.00 | - | -33.10 | -38.86 |
| Equation 3b′ | | 0.37 | 0.15 | 0.00 | - | -33.10 | -38.86 |
| Equation 2c′ | | 0.37 | 0.15 | 0.00 | 1.00 | -23.76 | -36.78 |
| Equation 3c′ | | 0.37 | 0.15 | 0.00 | 1.00 | -23.76 | -36.78 |
| Equation 4c′ | | 0.37 | 0.15 | - | 0.00 | -33.10 | -38.86 |
| *Rat 3 – Lean, 80% FFW* | 8 | | | | | | |
| Equation 1b | | 0.39 | -0.05 | - | - | -42.27 | -44.51 |
| Equation 2b′ | | 0.39 | -0.05 | 0.00 | - | -36.67 | -42.43 |
| Equation 3b′ | | 0.36 | -0.06 | 0.04 | - | -37.08 | -42.84 |
| Equation 2c′ | | 0.39 | -0.05 | 0.00 | 1.00 | -27.33 | -40.35 |
| Equation 3c′ | | 0.39 | -0.05 | 0.00 | 1.00 | -27.33 | -40.35 |
| Equation 4c′ | | 0.36 | -0.05 | - | 0.02 | -36.93 | -42.69 |
| *Rat 4 – Lean, 80% FFW* | 8 | | | | | | |
| Equation 1b | | 0.47 | 0.25 | - | - | -30.38 | -32.62 |
| Equation 2b′ | | 0.47 | 0.25 | 0.00 | - | -24.78 | -30.55 |
| Equation 3b′ | | 0.47 | 0.25 | 0.00 | - | -24.78 | -30.55 |
| Equation 2c′ | | 0.47 | 0.25 | 0.00 | 1.00 | -15.45 | -28.47 |
| Equation 3c′ | | 0.47 | 0.25 | 0.00 | 1.00 | -15.45 | -28.47 |
| Equation 4c′ | | 0.47 | 0.25 | - | 0.00 | -24.78 | -30.55 |
| *Rat 1 – Lean, 90% FFW* | 8 | | | | | | |
| Equation 1b | | 0.75 | 0.16 | - | - | -43.87 | -46.11 |
| Equation 2b′ | | 0.77 | 0.16 | 0.01 | - | -38.38 | -44.14 |
| Equation 3b′ | | 0.75 | 0.16 | 0.00 | - | -38.27 | -44.04 |
| Equation 2c′ | | 0.75 | 0.16 | 0.00 | 1.00 | -28.94 | -41.96 |
| Equation 3c′ | | 0.75 | 0.16 | 0.00 | 1.00 | -28.94 | -41.96 |
| Equation 4c′ | | 0.76 | 0.16 | - | -0.01 | -38.33 | -44.10 |
| *Rat 2 – Lean, 90% FFW* | 8 | | | | | | |
| Equation 1b | | 0.72 | 0.26 | - | - | -39.84 | -42.08 |
| Equation 2b′ | | 0.72 | 0.26 | 0.00 | - | -34.24 | -40.00 |
| Equation 3b′ | | 0.72 | 0.26 | 0.00 | - | -34.24 | -40.00 |
| Equation 2c′ | | 0.72 | 0.26 | 0.00 | 1.00 | -24.90 | -37.92 |
| Equation 3c′ | | 0.72 | 0.26 | 0.00 | 1.00 | -24.90 | -37.92 |
| Equation 4c′ | | 0.72 | 0.26 | - | 0.00 | -34.24 | -40.00 |
| *Rat 3 – Lean, 90% FFW* | 8 | | | | | | |
| Equation 1b | | 0.78 | -0.16 | - | - | -35.57 | -37.81 |
| Equation 2b′ | | 0.79 | -0.16 | 0.01 | - | -29.99 | -35.76 |
| Equation 3b′ | | 0.78 | -0.16 | 0.00 | - | -29.97 | -35.73 |
| Equation 2c′ | | 0.79 | -0.16 | 0.02 | 11.5 | -20.75 | -33.76 |
| Equation 3c′ | | 0.78 | -0.16 | 0.00 | 1.00 | -20.64 | -33.66 |
| Equation 4c′ | | 0.79 | -0.16 | - | -0.01 | -30.00 | -35.76 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *Rat 4 – Lean, 90% FFW* | 8 | | | | | | |
| Equation 1b | | 0.69 | 0.38 | - | - | -29.29 | -31.53 |
| Equation 2b′ | | 0.69 | 0.38 | 0.00 | - | -23.69 | -29.45 |
| Equation 3b′ | | 0.57 | 0.40 | 0.09 | - | -25.39 | -31.15 |
| Equation 2c′ | | 0.69 | 0.38 | 0.00 | 1.00 | -14.35 | -27.37 |
| Equation 3c′ | | 0.59 | 0.39 | 0.15 | 0.67 | -15.88 | -28.90 |
| Equation 4c′ | | 0.59 | 0.39 | - | 0.08 | -25.04 | -30.80 |
| *Rat 2 – Lean, FFW* | 8 | | | | | | |
| Equation 1b | | 0.46 | 0.11 | - | - | -35.48 | -37.72 |
| Equation 2b′ | | 0.46 | 0.11 | 0.00 | - | -29.88 | -35.64 |
| Equation 3b′ | | 0.38 | 0.12 | 0.12 | - | -35.76 | -41.52 |
| Equation 2c′ | | 0.46 | 0.11 | 0.00 | 1.00 | -20.54 | -33.56 |
| Equation 3c′ | | 0.46 | 0.11 | 0.00 | 1.00 | -20.54 | -33.56 |
| Equation 4c′ | | 0.36 | 0.12 | - | 0.11 | -36.60 | -42.36 |
| *Rat 3 – Lean, FFW* | 8 | | | | | | |
| Equation 1b | | 0.51 | -0.11 | - | - | -39.65 | -41.89 |
| Equation 2b′ | | 0.51 | -0.11 | 0.00 | - | -34.05 | -39.81 |
| Equation 3b′ | | 0.47 | -0.11 | 0.08 | - | -35.90 | -41.66 |
| Equation 2c′ | | 0.51 | -0.11 | 0.00 | 1.00 | -24.72 | -37.73 |
| Equation 3c′ | | 0.51 | -0.11 | 0.00 | 1.00 | -24.72 | -37.73 |
| Equation 4c′ | | 0.48 | -0.11 | - | 0.05 | -35.84 | -41.60 |
| *Rat 4 – Lean, FFW* | 8 | | | | | | |
| Equation 1b | | 0.59 | 0.06 | - | - | -31.92 | -34.16 |
| Equation 2b′ | | 0.59 | 0.06 | 0.00 | - | -26.32 | -32.08 |
| Equation 3b′ | | 0.59 | 0.06 | 0.00 | - | -26.32 | -32.08 |
| Equation 2c′ | | 0.59 | 0.06 | 0.00 | 1.00 | -16.98 | -30.00 |
| Equation 3c′ | | 0.59 | 0.06 | 0.00 | 1.00 | -16.98 | -30.00 |
| Equation 4c′ | | 0.63 | 0.05 | - | -0.05 | -26.81 | -32.57 |

*Note.* * denotes values that are rounded to 0.00, but not equal to 0. FFW = free feeding weight.

**Table A2.**

*Experiment 3: Number of schedules (n), parameter estimates, and fit quality metrics (AICc and BIC) for each model when fitted to each subject's data*

| *Participant* | | ***n*** | $a_R$ | **log(*b*)** | ***c*** | $a_P$ | ***AICc*** | ***BIC*** |
|---|---|---|---|---|---|---|---|---|
| *476* | | 34 | | | | | | |
| | Equation 1b | | 0.63 | -0.01 | - | - | -98.67 | -96.00 |
| | Equation 2b′ | | 0.71 | 0.00 | 0.11 | - | -96.32 | -92.54 |
| | Equation 3b′ | | 0.58 | -0.01 | 0.20 | - | -93.68 | -88.95 |
| | Equation 2c′ | | 0.63 | -0.01 | 0.00 | 0.12 | -94.88 | -90.16 |
| | Equation 3c′ | | 0.61 | -0.01 | 0.26 | 0.77 | -93.68 | -88.95 |
| | Equation 4c′ | | 0.67 | -0.01 | - | 0.48 | -98.13 | -94.35 |
| *477* | | 36 | | | | | | |
| | Equation 1b | | 0.63 | -0.01 | - | - | -130.20 | -127.40 |
| | Equation 2b′ | | 0.71 | 0.00 | 0.11 | - | -129.52 | -125.52 |
| | Equation 3b′ | | 0.58 | -0.01 | 0.37 | - | -133.08 | -129.08 |
| | Equation 2c′ | | 0.63 | -0.01 | 0.00 | 0.12 | -125.28 | -120.23 |
| | Equation 3c′ | | 0.61 | -0.01 | 0.26 | 0.77 | -130.95 | -125.91 |
| | Equation 4c′ | | 0.60 | -0.01 | - | 0.41 | -132.13 | -128.13 |
| *478* | | 30 | | | | | | |
| | Equation 1b | | 0.49 | 0.05 | - | - | -78.55 | -76.19 |
| | Equation 2b′ | | 0.53 | 0.05 | 0.10 | - | -73.55 | -69.54 |
| | Equation 3b′ | | 0.46 | 0.05 | 0.53 | - | -76.54 | -73.26 |
| | Equation 2c′ | | 0.53 | 0.04 | 0.03 | 0.92 | -76.61 | -73.33 |
| | Equation 3c′ | | 0.49 | 0.05 | 0.20 | 0.59 | -73.67 | -69.67 |
| | Equation 4c′ | | 0.49 | 0.05 | - | 0.05 | -76.08 | -72.80 |
| *480* | | 35 | | | | | | |
| | Equation 1b | | 0.62 | 0.00 | - | - | -102.57 | -99.83 |
| | Equation 2b′ | | 0.65 | 0.00 | 0.02 | - | -97.81 | -92.92 |
| | Equation 3b′ | | 0.62 | 0.00 | 0.06 | - | -100.30 | -96.41 |
| | Equation 2c′ | | 0.65 | 0.00 | 0.02 | 0.86 | -100.20 | -96.31 |
| | Equation 3c′ | | 0.63 | 0.00 | 0.01 | 1.66 | -97.74 | -92.85 |
| | Equation 4c′ | | 0.61 | 0.00 | - | 0.14 | -100.34 | -96.45 |
| *481* | | 36 | | | | | | |
| | Equation 1b | | 0.25 | 0.00 | - | - | -175.62 | -172.81 |
| | Equation 2b′ | | 0.34 | 0.00 | 0.73 | - | -182.44 | -178.44 |
| | Equation 3b′ | | 0.20 | 0.00 | 0.72 | - | -191.21 | -187.21 |
| | Equation 2c′ | | 0.32 | 0.00 | 0.01 | 1.54 | -186.89 | -181.84 |
| | Equation 3c′ | | 0.22 | 0.00 | 0.31 | 0.37 | -194.30 | -189.26 |
| | Equation 4c′ | | 0.19 | 0.00 | - | 0.37 | -194.33 | -190.33 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| *482* | | 30 | | | | | | |
| | Equation 1b | | 0.71 | 0.03 | - | - | -78.57 | -76.22 |
| | Equation 2b′ | | 0.71 | 0.03 | 0.00 | - | -76.10 | -72.82 |
| | Equation 3b′ | | 0.57 | 0.05 | 0.49 | - | -85.75 | -82.47 |
| | Equation 2c′ | | 0.71 | 0.03 | 0.00 | 1.00 | -73.42 | -69.41 |
| | Equation 3c′ | | 0.56 | 0.05 | 0.53 | 0.29 | -85.90 | -81.90 |
| | Equation 4c′ | | 0.64 | 0.02 | - | 0.81 | -81.18 | -77.90 |
| *483* | | 22 | | | | | | |
| | Equation 1b | | 0.63 | -0.01 | - | - | -38.02 | -36.47 |
| | Equation 2b′ | | 0.71 | 0.00 | 0.11 | - | -30.48 | -28.54 |
| | Equation 3b′ | | 0.58 | -0.01 | 0.37 | - | -37.06 | -35.12 |
| | Equation 2c′ | | 0.63 | -0.01 | 0.00 | 1.00 | -32.30 | -30.29 |
| | Equation 3c′ | | 0.61 | -0.01 | 0.26 | 0.77 | -32.16 | -30.15 |
| | Equation 4c′ | | 0.72 | -0.07 | - | 0.40 | -39.70 | -37.76 |
| *485* | | 35 | | | | | | |
| | Equation 1b | | 0.64 | -0.02 | - | - | -78.73 | -76.00 |
| | Equation 2b′ | | 0.64 | -0.02 | 0.00 | - | -76.34 | -72.44 |
| | Equation 3b′ | | 0.57 | -0.02 | 0.40 | - | -77.74 | -73.85 |
| | Equation 2c′ | | 0.64 | -0.02 | 0.00 | 1.00 | -73.78 | -68.89 |
| | Equation 3c′ | | 0.73 | 0.00 | 0.01 | 2.89 | -78.76 | -73.87 |
| | Equation 4c′ | | 0.60 | -0.02 | - | 0.27 | -77.04 | -73.15 |
| *486* | | 34 | | | | | | |
| | Equation 1b | | 0.62 | 0.01 | - | - | -89.08 | -86.41 |
| | Equation 2b′ | | 0.67 | 0.01 | 0.05 | - | -87.43 | -83.65 |
| | Equation 3b′ | | 0.54 | 0.00 | 0.45 | - | -95.64 | -91.86 |
| | Equation 2c′ | | 0.62 | 0.01 | 0.00 | 1.00 | -84.08 | -79.36 |
| | Equation 3c′ | | 0.54 | 0.01 | 0.52 | 0.67 | -103.89 | -99.17 |
| | Equation 4c′ | | 0.52 | 0.01 | - | 1.20 | -99.85 | -96.07 |
| *487* | | 35 | | | | | | |
| | Equation 1b | | 0.70 | -0.02 | - | - | -121.08 | -118.35 |
| | Equation 2b′ | | 0.70 | -0.02 | 0.00 | - | -118.68 | -114.79 |
| | Equation 3b′ | | 0.68 | -0.02 | 0.13 | - | -118.95 | -115.06 |
| | Equation 2c′ | | 0.70 | -0.02 | 0.00 | 1.00 | -116.12 | -111.23 |
| | Equation 3c′ | | 0.67 | -0.02 | 0.14 | 0.38 | -116.49 | -111.60 |
| | Equation 4c′ | | 0.67 | -0.03 | - | 0.39 | -120.38 | -116.49 |
| *488* | | 36 | | | | | | |
| | Equation 1b | | 0.61 | 0.00 | - | - | -118.73 | -115.92 |
| | Equation 2b′ | | 0.78 | 0.00 | 0.43 | - | -123.82 | -119.82 |
| | Equation 3b′ | | 0.47 | -0.01 | 0.66 | - | -136.65 | -132.65 |
| | Equation 2c′ | | 0.61 | 0.00 | 0.00 | 1.00 | -113.80 | -108.76 |
| | Equation 3c′ | | 0.54 | -0.01 | 0.25 | 0.95 | -151.64 | -146.60 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Equation 4c′ | | 0.41 | -0.01 | - | 1.19 | -139.66 | -135.66 |
| *489* | | 33 | | | | | | |
| | Equation 1b | | 0.63 | 0.00 | - | - | -88.04 | -85.45 |
| | Equation 2b′ | | 0.66 | -0.01 | 0.03 | - | -85.92 | -82.26 |
| | Equation 3b′ | | 0.58 | 0.01 | 0.38 | - | -88.39 | -84.72 |
| | Equation 2c′ | | 0.68 | 0.00 | 0.00* | 4.56 | -91.43 | -86.87 |
| | Equation 3c′ | | 0.63 | 0.01 | 0.32 | 0.89 | -87.77 | -83.22 |
| | Equation 4c′ | | 0.60 | 0.00 | - | 0.49 | -86.94 | -83.28 |
| *490* | | 36 | | | | | | |
| | Equation 1b | | 0.63 | 0.00 | - | - | -100.88 | -98.08 |
| | Equation 2b′ | | 0.63 | 0.00 | 0.00 | - | -98.50 | -94.50 |
| | Equation 3b′ | | 0.53 | -0.01 | 0.52 | - | -105.44 | -101.44 |
| | Equation 2c′ | | 0.63 | 0.00 | 0.00 | 1.00 | -95.96 | -90.91 |
| | Equation 3c′ | | 0.28 | 0.00 | 0.90 | 0.19 | -107.00 | -101.96 |
| | Equation 4c′ | | 0.47 | -0.01 | - | 0.98 | -106.91 | -102.91 |
| *491* | | 27 | | | | | | |
| | Equation 1b | | 0.80 | -0.04 | - | - | -80.88 | -78.79 |
| | Equation 2b′ | | 0.80 | -0.04 | 0.00 | - | -78.34 | -75.50 |
| | Equation 3b′ | | 0.72 | -0.03 | 0.70 | - | -89.08 | -86.24 |
| | Equation 2c′ | | 0.80 | -0.04 | 0.00 | 1.00 | -75.57 | -72.20 |
| | Equation 3c′ | | 0.53 | -0.03 | 0.96 | 0.18 | -90.94 | -87.58 |
| | Equation 4c′ | | 0.74 | -0.04 | - | 1.33 | -88.40 | -85.56 |
| *492* | | 34 | | | | | | |
| | Equation 1b | | 0.75 | -0.02 | - | - | -108.95 | -106.29 |
| | Equation 2b′ | | 0.75 | -0.02 | 0.00 | - | -106.54 | -102.76 |
| | Equation 3b′ | | 0.71 | -0.03 | 0.29 | - | -110.59 | -106.81 |
| | Equation 2c′ | | 0.75 | -0.02 | 0.00 | 1.00 | -103.96 | -99.24 |
| | Equation 3c′ | | 0.72 | -0.03 | 0.24 | 0.97 | -110.23 | -105.50 |
| | Equation 4c′ | | 0.69 | -0.03 | - | 0.78 | -116.83 | -113.05 |
| *493* | | 36 | | | | | | |
| | Equation 1b | | 0.29 | 0.01 | - | - | -109.82 | -107.01 |
| | Equation 2b′ | | 0.29 | 0.01 | 0.01 | - | -107.43 | -103.43 |
| | Equation 3b′ | | 0.29 | 0.01 | 0.00 | - | -107.43 | -103.43 |
| | Equation 2c′ | | 0.29 | 0.01 | 0.00 | 1.00 | -104.89 | -99.85 |
| | Equation 3c′ | | 0.29 | 0.01 | 0.00 | 1.00 | -104.89 | -99.85 |
| | Equation 4c′ | | 0.31 | 0.01 | - | -0.35 | -109.22 | -105.22 |
| *494* | | 35 | | | | | | |
| | Equation 1b | | 0.73 | 0.02 | - | - | -112.97 | -110.24 |
| | Equation 2b′ | | 0.73 | 0.02 | 0.00 | - | -110.60 | -106.71 |
| | Equation 3b′ | | 0.66 | 0.01 | 0.35 | - | -115.06 | -111.17 |
| | Equation 2c′ | | 0.73 | 0.02 | 0.00 | 1.00 | -108.01 | -103.13 |
| | Equation 3c′ | | 0.63 | 0.01 | 0.43 | 0.69 | -115.01 | -110.13 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Equation 4c′ | | 0.63 | 0.02 | - | 0.92 | -117.71 | -113.82 |
| *495* | | 36 | | | | | | |
| | Equation 1b | | 0.45 | 0.00 | - | - | -143.37 | -140.57 |
| | Equation 2b′ | | 0.45 | 0.00 | 0.00 | - | -140.99 | -136.98 |
| | Equation 3b′ | | 0.43 | 0.00 | 0.30 | - | -142.11 | -138.11 |
| | Equation 2c′ | | 0.48 | 0.00 | 0.08 | 0.42 | -138.73 | -133.69 |
| | Equation 3c′ | | 0.46 | 0.00 | 0.06 | 0.88 | -139.84 | -134.80 |
| | Equation 4c′ | | 0.43 | 0.00 | - | 0.15 | -142.09 | -138.09 |
| *496* | | 26 | | | | | | |
| | Equation 1b | | 0.75 | -0.10 | - | - | -64.31 | -62.32 |
| | Equation 2b′ | | 0.75 | -0.10 | 0.00 | - | -61.75 | -59.06 |
| | Equation 3b′ | | 0.75 | -0.10 | 0.00 | - | -61.75 | -59.06 |
| | Equation 2c′ | | 0.75 | -0.10 | 0.00 | 1.00 | -58.93 | -55.80 |
| | Equation 3c′ | | 0.75 | -0.10 | 0.00 | 1.00 | -58.93 | -55.80 |
| | Equation 4c′ | | 0.79 | -0.07 | - | -0.94 | -67.03 | -64.35 |
| *497* | | 35 | | | | | | |
| | Equation 1b | | 0.55 | -0.03 | - | - | -113.66 | -110.92 |
| | Equation 2b′ | | 0.57 | -0.03 | 0.03 | - | -111.34 | -107.45 |
| | Equation 3b′ | | 0.48 | -0.02 | 0.38 | - | -120.20 | -116.31 |
| | Equation 2c′ | | 0.55 | -0.03 | 0.00 | 1.00 | -108.70 | -103.81 |
| | Equation 3c′ | | 0.34 | -0.02 | 0.69 | 0.30 | -125.38 | -120.49 |
| | Equation 4c′ | | 0.41 | -0.02 | - | 0.99 | -126.36 | -122.47 |
| *498* | | 36 | | | | | | |
| | Equation 1b | | 0.63 | -0.01 | - | - | -84.20 | -81.39 |
| | Equation 2b′ | | 0.63 | -0.01 | 0.00 | - | -81.81 | -77.81 |
| | Equation 3b′ | | 0.63 | -0.01 | 0.00 | - | -81.81 | -77.81 |
| | Equation 2c′ | | 0.63 | -0.01 | 0.00 | 1.00 | -79.27 | -74.23 |
| | Equation 3c′ | | 0.63 | -0.01 | 0.00 | 1.00 | -79.27 | -74.23 |
| | Equation 4c′ | | 0.67 | 0.00 | - | -0.99 | -89.22 | -85.22 |
| *499* | | 36 | | | | | | |
| | Equation 1b | | 0.97 | -0.01 | - | - | -109.71 | -106.91 |
| | Equation 2b′ | | 0.97 | -0.01 | 0.00 | - | -107.32 | -103.32 |
| | Equation 3b′ | | 0.76 | -0.01 | 0.55 | - | -139.81 | -135.81 |
| | Equation 2c′ | | 0.97 | -0.01 | 0.00 | 1.00 | -104.78 | -99.74 |
| | Equation 3c′ | | 0.66 | -0.01 | 0.66 | 0.61 | -145.32 | -140.28 |
| | Equation 4c′ | | 0.83 | 0.00 | - | 1.22 | -122.25 | -118.25 |
| *500* | | 26 | | | | | | |
| | Equation 1b | | 0.75 | -0.02 | - | - | -81.15 | -79.16 |
| | Equation 2b′ | | 0.75 | -0.02 | 0.00 | - | -78.59 | -75.90 |
| | Equation 3b′ | | 0.69 | -0.03 | 0.22 | - | -80.35 | -77.66 |
| | Equation 2c′ | | 0.75 | -0.02 | 0.00 | 1.00 | -75.77 | -72.64 |
| | Equation 3c′ | | 0.66 | -0.03 | 0.29 | 0.49 | -78.27 | -75.15 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Equation 4c′ | | 0.75 | -0.02 | - | 0.02 | -78.59 | -75.91 |
| *501* | | 35 | | | | | | |
| | Equation 1b | | 0.73 | -0.07 | - | - | -85.42 | -82.69 |
| | Equation 2b′ | | 0.73 | -0.07 | 0.00 | - | -83.03 | -79.13 |
| | Equation 3b′ | | 0.71 | -0.07 | 0.07 | - | -83.23 | -79.34 |
| | Equation 2c′ | | 0.73 | -0.07 | 0.00 | 1.00 | -80.47 | -75.58 |
| | Equation 3c′ | | 0.75 | -0.08 | 0.00* | 3.45 | -83.57 | -78.68 |
| | Equation 4c′ | | 0.69 | -0.07 | - | 0.32 | -83.90 | -80.01 |
| *502* | | 31 | | | | | | |
| | Equation 1b | | 0.75 | 0.02 | - | - | -88.83 | -86.39 |
| | Equation 2b′ | | 0.75 | 0.02 | 0.00 | - | -86.37 | -82.95 |
| | Equation 3b′ | | 0.68 | 0.00 | 0.27 | - | -93.61 | -90.19 |
| | Equation 2c′ | | 0.75 | 0.02 | 0.00 | 1.00 | -83.72 | -79.52 |
| | Equation 3c′ | | 0.72 | 0.01 | 0.29 | 1.09 | -102.87 | -98.67 |
| | Equation 4c′ | | 0.60 | 0.01 | - | 1.72 | -103.37 | -99.95 |
| *503* | | 34 | | | | | | |
| | Equation 1b | | 0.80 | 0.01 | - | - | -92.14 | -89.48 |
| | Equation 2b′ | | 0.80 | 0.01 | 0.00 | - | -89.73 | -85.95 |
| | Equation 3b′ | | 0.76 | 0.01 | 0.22 | - | -91.54 | -87.76 |
| | Equation 2c′ | | 0.80 | 0.01 | 0.00 | 1.00 | -87.15 | -82.42 |
| | Equation 3c′ | | 0.74 | 0.01 | 0.28 | 0.78 | -89.74 | -85.01 |
| | Equation 4c′ | | 0.67 | 0.03 | - | 1.37 | -99.26 | -95.48 |
| *504* | | 32 | | | | | | |
| | Equation 1b | | 0.66 | 0.00 | - | - | -96.30 | -93.79 |
| | Equation 2b′ | | 0.70 | 0.00 | 0.01 | - | -94.65 | -91.11 |
| | Equation 3b′ | | 0.64 | -0.01 | 0.13 | - | -94.36 | -90.82 |
| | Equation 2c′ | | 0.66 | 0.00 | 0.00 | 1.00 | -91.24 | -86.86 |
| | Equation 3c′ | | 0.62 | -0.01 | 0.23 | 0.59 | -92.46 | -88.08 |
| | Equation 4c′ | | 0.63 | -0.01 | - | 0.32 | -95.66 | -92.12 |
| *505* | | 34 | | | | | | |
| | Equation 1b | | 0.82 | 0.00 | - | - | -118.60 | -115.93 |
| | Equation 2b′ | | 0.84 | 0.00 | 0.01 | - | -116.43 | -112.65 |
| | Equation 3b′ | | 0.75 | 0.00 | 0.41 | - | -123.86 | -120.08 |
| | Equation 2c′ | | 0.82 | 0.00 | 0.00 | 1.00 | -113.61 | -108.88 |
| | Equation 3c′ | | 0.77 | -0.01 | 0.34 | 0.90 | -124.34 | -119.62 |
| | Equation 4c′ | | 0.74 | 0.00 | - | 1.00 | -131.30 | -127.52 |
| *506* | | 36 | | | | | | |
| | Equation 1b | | 0.42 | -0.04 | - | - | -133.29 | -130.49 |
| | Equation 2b′ | | 0.47 | -0.05 | 0.17 | - | -132.32 | -128.32 |
| | Equation 3b′ | | 0.34 | -0.03 | 0.39 | - | -136.49 | -132.49 |
| | Equation 2c′ | | 0.47 | -0.04 | 0.05 | 0.88 | -130.40 | -125.36 |
| | Equation 3c′ | | 0.42 | -0.04 | 0.02 | 1.05 | -129.13 | -124.09 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Equation 4c′ | | 0.42 | -0.04 | - | 0.01 | -130.91 | -126.91 |
| *507* | | 28 | | | | | | |
| | Equation 1b | | 0.42 | -0.01 | - | - | -73.47 | -71.29 |
| | Equation 2b′ | | 0.42 | -0.01 | 0.00 | - | -70.95 | -67.95 |
| | Equation 3b′ | | 0.42 | -0.01 | 0.00 | - | -70.95 | -67.95 |
| | Equation 2c′ | | 0.42 | -0.01 | 0.00 | 1.00 | -68.21 | -64.62 |
| | Equation 3c′ | | 0.42 | -0.01 | 0.00 | 1.00 | -68.21 | -64.62 |
| | Equation 4c′ | | 0.49 | 0.00 | - | -0.51 | -74.02 | -71.02 |

*Note*. * denotes values that are rounded to 0.00, but not equal to 0.