The Associations between Ordinality and Mathematical Development

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Abstract

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In the modern world, mathematical competence is key to the competiveness of countries as well as individuals. Unfortunately, the mathematical abilities of American students lag significantly behind those of other industrialized nations (OECD, 2016). Thus, this dissertation was motivated by the broad goals of informing theoretical debates about the cognitive foundations of mathematics but also the practical importance of improving education in mathematics. Recent research suggests links between mathematical development and foundational abilities such as reasoning about ordinality. However, questions concerning the mechanisms underlying these links remain largely unresolved.

To answer these questions, this dissertation tested how a specific component of ordinality, namely rank, was related to early mathematical competence. In Paper 1, I present a study that tested the developmental relation between rank and exact number representations in 3- and 4-year-olds. Results showed that children who were better at tracking the rank of an item within a sequence also acquired more number words. Moreover, children who could reliably name the next number word in the count list also had a better grasp of numerical equality and a greater repertoire of number words. These findings suggest the ability to extract rank information from the count list is critical for the acquisition of number words and exact number representations. In Paper 2, I present a study that tested the developmental relation between rank-based operations and symbolic arithmetic. Results showed that children who were more proficient in making inferences based on inter-item distance between letters were better at solving symbolic arithmetic problems. Moreover, children who understood how rank should be updated after item insertion or removal also showed better arithmetic performance. The findings of Paper 2 suggest rank-based operations are recruited for the computation of addition and subtraction. Together, these two studies help to pinpoint the relations between specific ordinal abilities and early mathematical competence, which is critical for understanding the nature of the associations between ordinality and the development of mathematical competence.

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Introduction

Being able to both understand numbers and perform computations is of tremendous importance in modern society. To function normally, humans must process numerical values on a regular basis, including when we engage in monetary transactions, a variety of measurement activities, and when we take standardized math tests. Moreover, unique components of technological human culture such as science, engineering, and architecture all require precise numerical computations. The general public is concerned that US students' mathematical abilities are lagging behind students from other industrialized countries (OECD, 2016). To improve math education, we will need to better understand what underlies mathematical development.

Recent studies have shown that mathematical development may be related to the ability to memorize order information or judge the order of a sequence. For example, these studies found that deficits in memorizing order was related to developmental dyscalculia, a disorder that affects the ability to acquire knowledge about numbers and arithmetic skills (Attout & Majerus, 2015; Butterworth, 2010; Butterworth, Varm, & Laurillard, 2011; De Visscher, Szmalec, Van Der Linden, & Noël, 2015; M Piazza et al., 2010). These studies have also found that on order judgment tasks, individuals with dyscalculia show differential brain activation and behavioral patterns in comparison to people without dyscalculia (Kaufmann, Vogel, Starke, Kremser, & Schocke, 2009; Rubinsten & Sury, 2011). In the healthy population, there is evidence that children's ability to process order information is related to their arithmetic performance (Attout, Noël, & Majerus, 2014; Lyons & Ansari, 2015; Lyons, Price, Vaessen, Blomert, & Ansari, 2014). Moreover, performance in order judgment tasks is also a strong predictor of adults' symbolic arithmetic performance (Lyons & Beilock, 2011). In addition, some earlier works following Piaget's paradigms had also demonstrated that children who showed better ordinal abilities also showed better arithmetic understanding (e.g., Lemoyne & Favreau, 1981). Although these studies are clearly suggestive of an association between ordinal abilities and arithmetic competence, there are many unresolved issues. For example, is ordinality associated with arithmetic only, or does it influence mathematical abilities more broadly? What do children know about ordinality? How does the understanding of ordinal concepts contribute to the learning of mathematics? In an effort to answer these questions, this dissertation was designed to investigate the development of ordinality and its relation with other symbolic number abilities, as described in more detail below.

Different Aspects of Ordinality

Ordinality is not a monolithic concept (Lyons, Vogel, & Ansari, 2016). In the current developmental literature, terms such as "ordinal relation" or "ordinal concepts" may refer to different aspects of ordinality (see Table 1 for examples and descriptions of these concepts). Researchers in the field often conflate these different aspects because of the confusing terminology. As discussed in the latter sections of the dissertation, some problems in the existing literature are caused by overlooking the differences between these related, but different, components of ordinality. Thus, I will first explain the different aspects of ordinality by suggesting a classification framework. The suggested framework is modelled after Uttal et al. (2013) who proposed a categorization system for spatial abilities that is based on two orthogonal dimensions. Here, I propose a similar system that differentiates ordinality based on two independent dimensions. Crossing these two dimensions generates four categories of ordinal concepts (see Table 2).

The first dimension is concerned with whether an ordinal concept is contingent on the magnitude. When contingent on magnitude, ordinality is used for comparing the magnitude between items (i.e., less/more relation) or describing how items are arranged based on their relative magnitude (i.e., ascending/descending order). When not contingent on magnitude, ordinality is used to pinpoint the position of items in a sequence (i.e., rank) or to describe the arrangement pattern of items in a sequence (i.e., forward or backward order). Crucially, this distinction is grounded in psychological reality. Empirical research demonstrates different patterns of neural activation for ordinal concepts that are based on magnitude versus those that are not (Cheng, Tang, Walsh, Butterworth, & Cappelletti, 2013; Turconi et al., 2004).

Concept	Description	Examples of Citations
Less/more relation	For comparing the relative magnitude of items	Brannon and Van de Walle (2001); vanMarle (2013)
Ascending/ descending order	For describing whether all items of a sequence consistently increase or decrease across the sequence	Lyons and Beilock (2011); Macchi Cassia, Picozzi, Girelli, and de Hevia (2012)
Rank	Position of an item in a sequence	Colomé and Noël (2012); Lewkowicz (2013)
Before/after relation	Relation that is based on the position of items in a sequence	Gevers, Reynvoet, and Fias (2003); Turconi, Jemel, Rossion, and Seron (2004)
Forward/backward order	For describing whether the overall arrangement of the items follows a particular pattern	Berteletti, Lucangeli, and Zorzi (2012); Previtali, de Hevia, and Girelli (2010)

Table 1. Different ordinal concepts mentioned in the developmental literature.

Another dimension pertains to the level of description, namely whether it can be considered global or local. Some ordinal concepts focus on the global level description that applies to the sequence as a whole. Concepts such as ascending/descending or forward/backward order belong to this category. To decide whether a numerical sequence is in ascending order or not, one needs to consider all the items in the sequence. Moreover, the description of “ascending order” applies to the sequence as a whole, not to any specific item. By contrast, some ordinal concepts focus on the local level of description. Concepts such as rank and less/more relations belong to this category. For example, the rank of the letter *A* is 1 because it is the first letter in the alphabet. The rank 1 applies to the letter *A* specifically rather than the sequence as a whole.

	Magnitude	Non-magnitude
Local	Less/More	Before/After (i.e., rank)
Global	Ascending/Descending Order	Forward/backward Order

Table 2. A classification framework of ordinal concepts.

In this dissertation, I will focus on the local component of ordinality that is not based on magnitude. The concept is specifically referred to as rank. Rank is concerned with the position of items within a sequence (refered to as "ordinal position" in Nieder, 2005; Wiese, 2003). Given that magnitude constitutes an important aspect of number, it may seem odd to expect a non-magnitude aspect of ordinality to be associated with mathematical development. However, a reason to expect such an association is that, in the natural number system, the cardinal values of numbers are systematically mapped to the ranks of numbers in the number word sequence (Sarnecka & Carey, 2008). Cardinal value is the quantity that is represented by a specific number, which pertains to the question of “how many?” (Nieder, 2005; Wiese, 2003). Cardinal value is also a kind of numerical magnitude. The cardinality of a number and its rank in the number word sequence are mapped in the following way:

1) The smaller/larger relations between numerical values correspond to the before/after relations among number words (Carey, 2004).

2) Adjacent numbers always have a difference of one, so that moving one forward (i.e., moving from 2 to 3) or backward (i.e., 3 to 2) in the number list corresponds to addition or subtraction, respectively, of one (Carey, 2004).

3) The cardinal value of a number word is determined by its ordinal position in the sequence of counting words (Sarnecka & Carey, 2008).

This systematic mapping guarantees that when the counting procedure is executed correctly (i.e., following the principles outlined by Gallistel and Gelman [1978]), the last number word uttered by the speaker is the cardinal value of the set. This makes the counting word sequence a potential tool to exact enumeration. In addition, the systematic mapping makes the number word sequence useful for tracking change in numerical values when performing the computations of addition and subtraction (Frank, Fedorenko, Lai, Saxe, & Gibson, 2012).

Indeed, the importance of connecting the ordinal position of number words with their numerical values has been recognized by researchers as early as Piaget (1952; 1951/1975). In one experiment, Piaget and colleagues studied whether children could make use of ordinal information to infer cardinal value. They presented children with a series of 10 cards (referred to as cards A through K by Piaget). The cards were designed such that card A represented one unit of area. Card B was twice the area of card A (i.e., B

= 2A); card C was three times the area of card A (i.e., C = 3A), and so on (i.e., D = 4A; E = 5A; … K = 10A). In other words, the size of the card was associated with its ordinal position in the sequence (N^{th} card had the area of N units). This was analogous to the number word sequence, as the cardinal value of a number word is determined by its ordinal position. Piaget believed that if children understood the relation between ordinal position and cardinal value, then they should be able to infer the area of the cards by using their ordinal positions. In this study, the participants that was used by Piaget as examples for showing this mature understanding were all 6-year-yolds. However, more recent studies typically found children acquire the cardinality principle around 4 years of age (Le Corre & Carey, 2007; Sarnecka & Carey, 2008; Wynn, 1990, 1992). Nevertheless, it should be noted that Piaget's goal was not to pinpoint the age at which children attained this understanding, so he did not provide the age range of participants who had achieved this mature understanding about ordinal position and cardinal value. Moreover, Piaget's task was not a typical counting task. The system of cards used in Piaget's experiment was novel to children. In contrast, children have much more experience counting and reciting the sequence of number words. To summarize, even though Piaget's paradigm may have underestimated children's numerical understanding, his work illustrated the importance of integrating cardinal and ordinal concepts about numbers.

In the next section, I will describe the state of the current literature on this issue of the role of ordinality in mathematical development, then I will explain how my work specifically is in a position to address important unresolved issues.

The role of ordinality in mathematical development: state of the current literature

Even though the discussion on how ordinality is related to mathematical development can be traced back to Piaget (1952; 1951/1975), this issue has not been the focus of research until recent years. To the best of my knowledge, researchers have yet to formulate any official theory that explains the mechanism that underlies the association between ordinality and mathematical development. Nevertheless, there are two lines of research that attempt to reveal the mechanism underlying the connection between ordinality and mathematics. One line of research focuses on working memory. The other focuses on symbol-symbol associations. Both lines of research provide important insights for understanding the role of ordinality in mathematical development. In this section, I will provide a brief description for each line of research.

Role of working memory. This line of research aims to explain how a specific type of working memory (WM), namely order-WM, is related to mathematics. Order-WM refers to the WM for storing items' positions in a list. By contrast, item-WM refers to the WM for storing the identity of items in the memorized list (Attout, Van der Kaa, George, & Majerus, 2012; Majerus, Poncelet, Van der Linden, & Weekes, 2008; see also Burgess & Hitch, 1999; Henson, Hartley, Burgess, Hitch, & Flude, 2003; Nairne & Kelley, 2004). For example, to memorize the letters "A, E, D" in the presented order, order-WM allows for keeping track of the mapping between letters and positions, whereas item-WM keeps track of the letters that are part of the list (i.e., A, D, and E). In the context of mathematical development, the two types of WM show different associations with arithmetic abilities. In a longitudinal study that followed a group of students from kindergarten to Grade 3, Attout et al. (2014) found that kindergarteners' order-WM capacity, but not item-WM capacity, predicted future math achievement in grade 1 and grade 2. This association was replicated in a sample that contained both dyscalculic and typically-developing students (8- to 12-years-old; Attout & Majerus, 2015). Moreover, the researchers found that dyscalculic students had deficits in order-WM, as their performance memorizing serial order was significantly worse than that of typically-developing children. This deficit in order-WM was also found in dyscalculic adults (De Visscher et al., 2015).

Why is order-WM related to arithmetic performance? There are at least two possibilities. First, to perform arithmetic computation, one needs to keep track of the sequential steps in the computational process, such that order-WM supports this process (Attout et al., 2014). Second, arithmetic facts may be stored in sequential format (De Visscher et al., 2015). For example, one may remember the multiplication table by a sequence that modifies one operant at a time (e.g., $2 \times 2 = 4$; $2 \times 3 = 6$, etc.). Thus, deficits in order-WM may hinder the ability to retain and retrieve arithmetic facts.

This line of research provides concrete evidence that order-WM is related to symbolic number abilities. The findings reveal how a domain general process, namely WM, is related to the processing of numerical information. The focus on WM also means that the mechanism mainly explains how information is stored or retrieved. While this approach is valuable for understanding the cognitive processes that underlie arithmetic, such an approach is less useful in understanding the conceptual changes that underlie mathematical development. For example, what does it take to understand numbers? What do children know about ordinality? These important questions are not currently addressed in the extant literature that it is set up to. Thus, to fully understand the developmental relation between ordinality and mathematics, it is necessary to study the development of these concepts themselves.

Symbol-symbol associations. This line of research focuses on the potential contribution of ordinality to the symbolic number system. Research in this area often emphasizes the difference between ordinality and an intuitive system for processing numerical magnitude information, namely the Approximate Number System (ANS). The ANS is functional from birth (de Hevia, Izard, Coubart, Spelke, & Streri, 2014; Xu, Spelke, & Goddard, 2005), but its representations for numerical magnitude are noisy and imprecise (Feigenson, Dehaene, & Spelke, 2004; Halberda, Ly, Wilmer, Naiman, & Germine, 2012). Researchers in this area of research believe that the ANS is crucial for supporting the learning of symbolic numbers. However, with more experience in symbolic number, it has been suggested that the meaning of number gradually shifts from numerical magnitude to associations between numerical symbols, which critically rests on ordinality (Lyons & Beilock, 2011; Reynvoet & Sasanguie, 2016). Lyons, Ansari, and Beilock (2012) illustrated this idea by asking readers to consider numbers that are extremely large, such as 1,000,000. They argued that it is difficult to get an intuitive sense of the magnitude for this extremely large number. However, it is possible to understand this number by drawing on its ordinal relation with other numbers. For example, we can understand the number in the sense that 1,000,000 is larger than 999,999, but smaller than 1,000,001. Lyons et al. thus concluded that it is possible to understand a number without fully grasping its numerical magnitude. They use the following evidence to support this claim.

Order judgment predicts children's and adults' performance in symbolic arithmetic tests. In a large scale cross-sectional study, Lyons et al. (2014) showed that the association between order judgment ability and symbolic arithmetic increased across development. Even though order judgment was a poor predictor in grades 1 and 2, it became a significant predictor beginning in grade 3 and eventually becoming the strongest predictor in grade 6. In adults, researchers have also found significant associations between an order judgment ability and symbolic arithmetic performance (Goffin & Ansari, 2016; Lyons & Beilock, 2011). Together, these findings support the notion that ordinality is related to symbolic arithmetic.

Order judgment fully mediates the relation between the ANS and symbolic arithmetic. Not only has ordinality been shown to relate to symbolic arithmetic, but there is also evidence that it mediates the relation between other known links to symbolic arithmetic. In an adult study, Lyons and Beilock (2011) showed that the ability to make ascending order judgments fully mediated the relation between ANS and symbolic arithmetic. This finding can be interpreted as the ANS having no direct involvement in the psychological process of symbolic arithmetic. Instead, the ANS contributes to the development of ordinal ability, which in turn contributes to the development of symbolic arithmetic.

Critique. This above-mentioned research provides evidence for a role of ordinal ability in mathematics development. However, the suggestion that the ANS, a system that supports magnitude meaning, would gradually be overshadowed by ordinality is debatable. First, as described in the classification scheme above, some ordinal concepts such as less/more are contingent on magnitude. Thus, these magnitude-concepts may also be supported by the ANS, making it difficult to interpret how the ANS could be completely replaced by ordinality. Second, there are good reasons to believe that magnitude and ordinality both constitute important aspects of symbolic number ability. Studies have shown that the cardinal value of symbolic numbers are mapped onto the ANS in both adults (Izard & Dehaene, 2008) and children (Le Corre & Carey, 2007; Lipton & Spelke, 2005). Moreover, neuroimaging studies have showed that processing of symbolic number implicate the intraparietal sulcus, an area that is typically associated with the processing of magnitude information (Eger et al., 2009; Notebaert, Nelis, & Reynvoet, 2010; Piazza, Pinel, Le Bihan, & Dehaene, 2007; for meta-analysis, see Arsalidou & Taylor, 2011; Dehaene, Piazza, Pinel, & Cohen, 2003). Thus, it is reasonable to expect that numerical magnitude, and the ANS that supports it, should be involved in the processing of symbolic number.

If both magnitude and symbolic relations are important, then how to explain the finding that ordinal ability completely mediated the effect of ANS on symbolic arithmetic (Lyons & Beilock, 2011)? In the study that demonstrated this mediation effect, an ascending order judgment task was used to assess participants' ordinal ability. However, to decide whether a sequence is in ascending order, one must consider both the magnitude and arrangement of numbers in the sequence. Thus, magnitude processes may support both the order judgment task and the task for assessing ANS acuity, making it difficult to determine whether ANS is directly involved in the psychological process of symbolic arithmetic based on the results of mediation analyses. In summary, there are good reasons to believe that magnitude and ordinality both constitute important aspects of symbolic number ability. The approach of establishing the importance of ordinality by denying the influence of ANS is debatable.

Another issue in this line of research concerns the vagueness of the claim about symbol-symbol relations. Indeed, ordinality can be used to describe the relations among numerical symbols, but how does the understanding of such relations contribute to other mathematical abilities? This question is particularly pertinent to the association between ascending order and arithmetic. Moreover, even though we can understand that 999,999 < 1,000,000 < 1,000,001, how does this understanding relate to other mathematical abilities? The exact contributions of these ordinal relations have yet to be well articulated.

These difficulties are partly due to the fact that this line of research only utilizes the order judgment task as the paradigm for measuring ordinal ability. As described above, ascending/descending order only constitutes one of the many aspects of ordinality. Given the way that cardinal value and rank are mapped within the natural numbers, a narrow focus on ascending/ descending order judgment may not be conducive to revealing the mechanisms that underlie the connection between ordinality and mathematics.

Unresolved issues. The two lines of research discussed above provide important groundwork for understanding the role of ordinality in mathematical development. However, there are still a number of important issues that need to be addressed.

Development of ordinality. The first issue is concerned with the development of ordinality. Both lines of research take it for granted that children understand ordinality. The development of ordinality itself is largely ignored. Even though infant studies have shown that preverbal infants have some rudimentary ordinal abilities (Brannon, 2002; Brannon & Van de Walle, 2001; Lewkowicz, 2013; Lewkowicz & Berent, 2009; Macchi Cassia et al., 2012; Suanda, Tompson, & Brannon, 2008), there is very limited information on children's explicit knowledge about ordinality in the current literature. One exception is the work by Colomé and Noël (2012), which demonstrated that children master cardinal counting (i.e., counting for set size) before they master ordinal counting (i.e., counting for the rank of a specific item). This study illustrates that it should not be taken for granted that children understand all concepts of ordinality. Thus, to fully understand the developmental relation between ordinality and mathematics, it is important to understand the developmental trajectory of ordinal understanding itself.

Relation to symbolic numbers. The second issue is concerned with the specific contributions of ordinality to the understanding of symbolic numbers. Although the line of research on symbol-symbol relations has been somewhat promising on this issue, the precise contribution of ordinality to symbolic number understanding is largely unclear. Moreover, previous research has mainly focused on the association between symbolic arithmetic and the ability to judge or memorize order, leaving unexplored other potential associations between ordinality and symbolic numbers. As discussed above, there is a systematic mapping between numerical values and the rank of number words in the natural number system. Thus, exploring the role of rank in mathematical development may provide new insights to the question of how ordinality contributes to symbol-symbol relations among symbolic numbers.

Researchers who have focused on the development of symbol-symbol relations also suggested that, with development, the meaning of numbers shifts from numerical magnitude to a relation among numerical symbols. Thus, the importance of the ANS would gradually be overshadowed by ordinality, which characterizes the relations among the numerical symbols. However, some ordinal concepts, such as less/more and ascending/descending are actually based on relative magnitude, making the distinction between ordinal and magnitude processing difficult to interpret. Thus, the developmental relations between magnitude processes, ordinal processes, and mathematics is an important theoretical question that needs to be addressed.

Overview of the dissertation

In this dissertation, I provide a systematic account for understanding the developmental relations between ordinality and mathematics. Above, I have provided a conceptual framework for understanding different aspects of ordinality and have summarized the key issues that will need to be addressed. In the two papers that constitute the subsequent sections of this dissertation, I will explain how one aspect of ordinality, namely rank, may contribute to two important mathematical achievements, namely the acquisition of exact number representations and symbolic arithmetic, in young children. To support these claims, I studied children's explicit knowledge of rank and the relations to their mathematical abilities.

Paper 1 addresses how rank may contribute to the acquisition of exact number representations. Before children understand the meaning of number words, numerical values that are larger than four can only be represented as approximate numerical magnitude with poor precision (Feigenson et al., 2004). I propose that rank contributes to children's acquisition of number words, which enables children to represent any numerical values with precision. Paper 1 of the dissertation put this proposal to the test. To anticipate the results, the findings suggest that 3- and 4-year-olds' ability to represent rank information in a sequence, as well as their knowledge about rank-based relation among number words, are both related to children's ability to represent exact numbers.

Paper 2 addresses the issue of how rank may contribute to the development of symbolic arithmetic, specifically addition and subtraction. I propose that rank-based operations may be recruited in the computational procedures of addition and subtraction because such procedures typically involve movement along the numerical sequence. Paper 2 of the dissertation tested whether this proposal was supported by empirical data. To anticipate the results, the findings suggested that certain types of rank-based operations are related to 5- and 6-year-olds' performance in symbolic addition and subtraction tests. Moreover, a mediation analysis showed that the influence of ANS is not mediated by rank-based abilities.

In the conclusion portion of this dissertation, I discuss the implications of the findings in Papers 1 and 2, as well as future directions for research.

The first number is one: The associations between the ordinal concept of rank and cardinality

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Abstract

Despite accumulating evidence for an association between ordinal concepts and numerical abilities, the mechanism that underlies this connection remains largely unclear. The current study examined the potential contributions of ordinality to the acquisition of children's exact number representations. Exact number representations entail the ability to establish numerical equity between any numbers and an understanding of the cardinality principle (CP) that enables exact numeration of any number. The current study tested 3- to 4-year-olds to determine whether the ability to represent absolute rank (i.e., the ordinal position of an item in a sequence, e.g., first, second), as well as the understanding of rank-based relations among numbers (e.g., "two" comes after "one") contribute to the acquisition of numerical equity and CP. Results suggested that children who were better at tracking absolute rank showed greater understanding of CP. Moreover, children who showed better understanding of rank-based relations among numbers also showed better understanding of CP and numerical equality. These findings suggest that the ability to extract rank information from the count list is an important step in the acquisition of exact number representations. I argue that the vital concepts for exact number representations are built on the rank information about the number words. But such rank information is not available to children when they first learn to recite the count list. Thus, an extra step of extracting the embedded rank information from the list is critical for rendering the rank information useable for building exact number concepts.

There has long been interest in the psychological foundations of mathematical development. Understanding the basis of early mathematical thinking has important implications for psychology as well as education. Previous studies have focused on the Approximate Number System (ANS) as a foundational ability. The ANS is an intuitive, nonverbal sense of numerical magnitude, and accumulating evidence suggests a link between the precision of the ANS and the development of formal mathematics (e.g., Halberda, Mazzocco, & Feigenson, 2008; Lourenco, Bonny, Fernandez, & Rao, 2012; M Piazza, Pica, Izard, Spelke, & Dehaene, 2013). Less attention has been paid to another potentially foundational ability, namely the representation of ordinality (Rubinsten & Sury, 2011). Ordinality concerns the position of items within a sequence (Nieder, 2005; Wiese, 2003) and, importantly, it applies to both numerical and non-numerical sequences. Within the natural number system, the properties of magnitude and ordinal position are deeply intertwined, yet behavioral and neural data suggest that they dissociate in the mind and brain (Cheng, Tang, Walsh, Butterworth, & Cappelletti, 2013; Franklin & Jonides, 2008; Lyons & Beilock, 2013; Turconi, Campbell, & Seron, 2006). That these properties are dissociable is critical because it allows for testing their independent contributions to mathematical development. In the current research, I focus specifically on the potential contribution of ordinality to the acquisition of exact number representations.

Ordinality as position

Ordinality is not a single, monolithic construct but, rather, it incorporates multiple, inter-related concepts. The current study focuses on the positional aspect of ordinality, which can be defined specifically as the rank of items in a sequence (e.g., the 5^{th} letter in the alphabet is *E*; Nieder, 2005; Wiese, 2003). Rank can be further defined according to the reference point used for defining position. In the current study, I focused on the development of two types of rank information, namely absolute rank and rank-based relations, described further below.

Absolute rank. Absolute rank refers to the absolute position occupied by each item within a sequence and with reference to the beginning of the sequence (e.g., first, second, etc.). One way to represent this type of information is to associate a nonverbal placeholder with an object or event (cf. Lewkowicz & Berent, 2009). Information such as "A is the first letter" can be represented by associating the letter "A" with the nonverbal placeholder that stands for "first". Without language, though, the number of distinct ordinal positions that might be indexed are necessarily limited by one's attentional resources (Nieder, 2009).

Rank-based relations. Rank-based relations refers to the before/after relations among items in a sequence. In contrast to absolute rank, which is defined with reference to the beginning of the sequence (e.g., B is the 2^{nd} letter because it is the 2^{nd} item from the beginning of the list), rank-based relations can be defined with reference to any item in a sequence (e.g., B comes before C but after A—both C and A are reference points to B).

Rank information in the number word sequence. In the context of learning number words, the ability to represent absolute rank pertains to the understanding that *one* is the first word in the counting list, *two* is the second word, and so on. The ability to represent rank-based relations pertains to the understanding that *two* comes after *one,* that *three* comes after *two*, and so on. However, children who can recite numbers do not necessary understand the rank information that is embedded in the routine. This is analogous to the case that children do not always understand the cardinal meaning of the number words they can recite (Wynn, 1990, 1992). To adults, number words are discrete entities that can be easily discriminated from one another. Nevertheless, Fuson (1988) found that some children may go through an "unbreakable chain" stage, in which they treat the number word sequence as a single undifferentiated structure, rather than a list made up of fully separated words. Fuson suggested that children need to go through a protracted process, which can span from 4 to 8 years of age, to gain access to the rank-based relations among the number words. With development, children eventually come to treat each number word as a discrete entity that takes up a fixed and unique position in the number word sequence (see also C. Xu & LeFevre, 2016).

How might ordinality contribute to representations of exact number?

Before humans acquire and understand symbolic numbers, representations of number are processed by two nonverbal systems. One of these systems is known as the object file system and it is characterized by its precision in tracking up to four objects (Feigenson & Carey, 2003; Trick & Pylyshyn, 1994; Uller, Carey, Huntley-Fenner, & Klatt, 1999). The other system is known as the Approximate Number System (ANS, Dehaene, 2011; Feigenson, Dehaene, & Spelke, 2004; M. Piazza, 2010; F. Xu & Spelke, 2000), which does not have an upper limit for numerical value, but its precision decreases as the numerical value increases. The ANS represents numerical quantity as continuous magnitude. The system is constrained by Weber's law such that the discriminability of two numbers is determined by their ratio, not the absolute distance. Since there are no discrete boundaries between adjacent values (e.g., 100 and 101 are highly overlapping), it can be impossible to differentiate between two numbers that are close in value.

Moreover, when representing large numbers of the same values (e.g., 100 and 100), the noise inherent in the ANS makes it impossible to ascertain whether the two values are identical or just highly similar (e.g., 100 ≈ 101). Thus the ANS is ill-equipped for representing exact numbers.

In contrast to the two nonverbal number systems, number words are better suited to representing a full range of exact numbers. In contrast to the continuous nature of the ANS representations, number words are discrete entities. There is no overlap between the word "nine" and the word "ten." In contrast to the crude representations of the ANS that fail to represent small changes (versus no change) in numerical value, number words can represent all changes in numerical values accurately. This is because each number word refers to a unique and exact numerical value in the natural number system (Condry & Spelke, 2008; Lipton & Spelke, 2006). A different number word should be used to describe a set even if its numerical value is changed (e.g., by addition or subtraction) by the minimal possible amount of one (Izard, Pica, Spelke, & Dehaene, 2008; Izard, Streri, & Spelke, 2014; Sarnecka & Wright, 2013), whereas the same number word still applies to a post-transformed set if such a transformation does not involve changes in numerical value, such as stretching out a line of objects. And when a sufficient set of number words is acquired, even numbers that are beyond the set size limitation of object file system can be represented with precision. Thus, comparing to the two intuitive, nonverbal number systems, number words provide a more advanced representational system that enable more complex numerical reasoning.

Researchers suggest that the acquisition of number words is critical for developing exact number representations (Carey, 2009; Condry & Spelke, 2008;

Sarnecka, 2015; but see Butterworth et al., 2008; Leslie, Gelman, & Gallistel, 2008). In the process of learning number words, children gradually acquire two vital concepts that are required for fully-fledged exact number representations, namely numerical equality (refered as "exact equality" in Izard et al., 2008; Izard et al., 2014) and the cardinality principle (CP; Gallistel & Gelman, 1978; Sarnecka, 2015; Wynn, 1990, 1992). In the next section, I explain how rank information may contribute to the development of exact number by facilitating the understanding of number words in children.

Numerical Equality. By definition, numerical equality refers to a dichotomous state of equivalence, namely equal or not equal. Among the two nonverbal numerical systems, the object file system supports numerical equality because it represents numerical values as discrete entities. By contrast, the ANS does not support numerical equality because it represents numerical values with continuous representations that are imprecise. Given that the object file system has a set size limit of four, what causes children to believe numerical equality applies to all numerical values? Research has shown that a mature understanding of numerical equality depends on the acquisition of number words (Condry & Spelke, 2008; Lipton & Spelke, 2006). I propose that acquiring number words, depends on an understanding of rank information.

Understanding absolute rank in the context of number words may contribute to the acquisition of numerical equality. As discussed above, when children first learn how to recite number words, they treat this sequence as an undifferentiated structure (Fusion, 1988), not a list of discrete entities that occupy unique positions in the sequence. Thus, developing the ability to extract absolute rank from a sequence may critically aid in differentiating the individual number words, which, in turn, may facilitate the understanding of numerical equality.

The understanding of rank-based relations (i.e., before/after) among numbers may also contribute to the acquisition of numerical equality. Although the ANS fails to represent the qualitative difference between equality (e.g., 100 = 100) and similarity (e.g., 100 ≈ 101), adults can easily tell that the value represented by the words "one hundred" does not equal that of "one hundred and one", because they can draw on the knowledge that "one hundred" comes before "one hundred and one" in the number word sequence. The rank-based relation among number words helps to highlight the clear boundaries between adjacent numerical values. It also provides an alternative system for representing the less/more relation, which can be buried by the imprecision of the ANS.

Cardinality Principle (CP). Another vital concept for exact number representations is the cardinality principle (CP), which states that the last number referenced in counting represents the total number of items in a set (Gallistel & Gelman, 1978; Wynn, 1990, 1992). This principle is important because it makes counting a functional tool for exact enumeration. Children who master this principle can associate an exact numerical value to the appropriate number word without the set-size constraint associated with the object file system. Sarnecka and Carey (2008) suggested that the acquisition of CP is related to the implicit understanding that the quantity represented by a number word corresponds to its absolute rank in the sequence (i.e., the first word in the number word list represents the quantity of one, the second word represents the quantity of two, and so on). However, this understanding is not possible if children represent the count sequence as an undifferentiated structure. As discussed above, information about absolute rank in the count sequence may not be readily available to children when they first learn the number words. Thus, the ability to represent the absolute rank of number words within the count sequence may be crucial in establishing the correspondence between quantity and the rank of the numbers.

Another reason to believe that the acquisition of CP is related to rank information comes from research showing a developmental association between CP and the successor function. The successor function indicates that the number immediately following *N* is *N* + 1 (Gallistel & Gelman, 1978). Sarnecka and Carey (2008) showed that only children who were near acquisition of CP or had already acquired CP showed robust understanding that larger numbers come after smaller numbers in the count sequence. Moreover, it has been shown that only CP-knowers understand that if exactly one object is added to a set, then the new set size should be the next word in the number word sequence, as opposed to any other number word. These findings suggest that knowing CP implies an implicit understanding of the successor function. As described above, the conceptualization of successor function is rooted in rank-based relations. More specifically, it is built on the ability to understand the "after" relation among number words. Thus, the understanding of rank-based relations among the number word may contribute to the acquisition of the successor function, which, in turn, may support learning of CP.

Current Study

The current study investigated whether children's ordinal abilities were related to their acquisition of exact numbers, as characterized by the concepts of equality and the cardinality principle (CP). To this end, I focused on the two ordinal abilities of absolute rank and rank-based relations, and I examined the links between individual differences associated with these abilities and children's emerging understanding of equality and cardinality.

I designed two separate tasks to capture inter-individual variability in 3-to 5-year-olds' understanding of absolute rank and rank-based relations. The Ordinal Position task required that children identify a target character on the basis of its absolute rank—that is, its ordinal position in a sequence of other characters. Three different characters (all rabbits) showed up sequentially in a fixed order. Children were tasked with selecting the target character when it appeared. The only way in which to identify the target character was on the basis of its absolute rank. The Rank-based Relations task required that children name the number that came after a specific number (e.g., "3, 4, what's next?"). This task allowed for testing whether children have explicit access to the rank-based relations among number words.

There were also two separate tasks to assess children's understanding of exact numbers—one that captured their understanding of equality and the other of CP. The first task was the Transform-sets task, adapted from Sarnecka and Gelman (2004), in which children judged equality. In this task, children witnessed a transformation that either altered the number of items in a box or left number unaltered. Then, without counting or seeing the objects in the box, children were asked whether the same number word still applied to the unseen objects in the box after the transformation. The second task was a variant of the Give-a-Number (GAN) task, adapted from Patro and Haman (2012), in which I assessed the level of CP understanding in children. In this task, children are asked to produce a set of objects that match the number requested by an experimenter. I also included a set of other tasks that assessed children's abilities to recite number words, ANS acuity, visual-spatial working memory (WM) span, and expressive vocabulary. If children's ordinal abilities are related to their exact number representations, then better performance in Ordinal Position and Rank-based Relations tasks should be accompanied by better performance in the Transform-sets and GAN tasks. Moreover, and crucially, these correlations should reflect specificity between the tested abilities such that the effects should hold when accounting for performance on other numerical measures (e.g., number word recitation, ANS acuity) and general cognitive functions such as WM and linguistic competence.

Method

Participants

Sixty-two preschoolers (33 male; age range: 3 years 3 months – 4 years 8 months, M_{age} = 3 years 11 months) participated in this study. Four additional children (range: 3 years 6 months – 4 years 0 month) were tested but excluded from statistical analyses for failing to follow instructions. All children received a small gift for participating. Informed consent was obtained on behalf of each child by a parent or legal guardian. Experimental procedures were approved by the local ethics committee.

Tasks and Procedure

Participants were tested individually in a university laboratory. All ten tasks were administered in a single experimental session (see Table 1), lasting approximately 90 minutes. Task order was randomized across participants with the constraint that the tasks assessing the abilities of interest (i.e., ordinal and exact number concepts) were administrated before the control tasks (i.e., general intelligence and ANS acuity).

Task Name	Construct Assessed
Ordinal Position task	Ordinality: *Absolute rank*
Rank-based Relations task	Ordinality: *Rank-based relations*
Transform-sets task	Exact Number: Numerical Equality
Give-A-Number task	Exact Number: Cardinality principle
Recitation task	Recitation of number words
Picture Recognition task	Nonverbal working memory span
ANS task	Acuity of nonverbal number representations
DVAP	Expressive vocabulary

Table 1. List of tasks administered to children in the current study

While children participated in the study their parents filled out the Developmental Vocabulary Assessment for Parents (DVAP, Libertus et al., 2013), a parental measure of expressive vocabulary. Parents were instructed to select the words that they had heard their children utter. The score of DVAP is the total number of selected words.

Ordinal Position Task. This computerized task was designed to tap children's sensitivity to absolute rank. Specifically, this task assessed whether children could recognize the rank of an item in a temporal sequence. On each trial, children were told that three different cartoon rabbits would emerge from behind a rock in a fixed order (i.e., "It's always the brown bunny, then the pink bunny, then the gray bunny"), and that their task was to "catch" the target rabbit when it appeared by tapping the touchscreen. The target rabbit was indicated to the child at the beginning of each trial with a "Wanted!" poster (see Figure 1). In addition, the background color of the computer program changed to that of the target rabbit to serve as a reminder throughout the trial. Children were given

6 training trials in which the rabbits always appeared in the following order: brown, then pink, then gray. On these trials, the color of the rabbits was always visible. Twelve test trials followed. These trials were identical to training except that the cartoon rabbits were now all the same color. Children were told that “the rabbits covered themselves with a rainbow blanket.” This manipulation ensured that a successful response would be based on the rank of the rabbit in the temporal sequence (i.e., first, second, or third). After each rabbit emerged from behind the rock, children were asked “Is that the right bunny?”. If the children thought it was the target, then they tapped the rabbit. Each trial was followed by corrective feedback (see Figure 1). Accuracy was measured by the total number of correct answers.

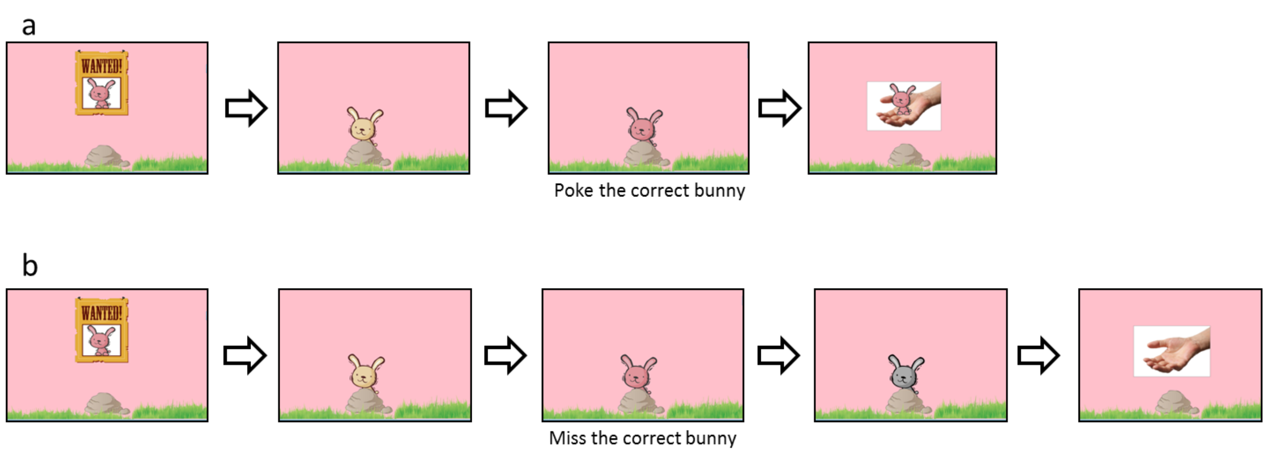

Figure 1. Screen shots of a practice trial. The upper part of the figure is an example of a correct trial. In this example trial, the target is the pink rabbit, which is shown as part of the “Wanted” poster at the beginning of the trial. Then, the rabbits rose up from behind the rock one after another. In this example, the target bunny was tapped when it appeared, therefore a feedback screen for a correct answer is shown subsequently. The lower part of the figure is an example of an incorrect trial, in which the participant failed to poke the target bunny. The feedback for incorrect answer is shown.

Rank-based Relations task. This task was designed to evaluate children’s understanding of rank-based relations between number words. It was inspired by a similar task by Fuson, Richards, and Briars (1982), which assessed preschoolers' ability to name numbers that came either before or after a given number word.

In the practice trials, participants saw that a puppet experienced difficulty when reciting numbers (i.e., "1, 2, umm… What's next?") or letters (i.e., "A, B, umm… What's next?"). The experimenter invited the participant to help the puppet by telling it what the next number or letter should be. Corrective feedback was given on practice trials. The test trials (15 trials) were the same as the practice trials, except that the questions were exclusively about numbers and no corrective feedback was provided. Two types of cues (one-word/ two-word cue) were used in the prompt. One-word cues contained only one number word (e.g., "two … what's next?"; 8 trials), and two-word cues contained two number words (e.g., "two, three… what's next?"; 7 trials). The number cues ranged from 1 to 9, and the correct answers ranged from 2 to 10. The order of questions was randomized across participants. Accuracy was measured by the total number of correct answers.

Give-a-Number task. This task assessed children's understanding of the cardinality principle (CP; Wynn, 1990, 1992). Following Patro and Haman (2012), children were prompted to place a designated number of coins into a plastic box (i.e., "Put *N* coins into the box."). When participants completed the request, the experimenter asked for confirmation ("Is that *N*?"). If the child replied no, then the experimenter made the request again (i.e., "Please put *N* coins into the box."). The first trial always began with a request for one coin. The subsequent values of *N* (ranging from 1 to 9) were contingent on the child's performance. Following a correct trial, the value of *N* increased by two. Following an incorrect trial, the value of *N* decreased by one. The maximum number of trials for each *N* was three. The task ended when participant met one of the following conditions: 1) succeeded in giving the highest number (i.e., 9) twice; 2) failed the lowest number (i.e., 1) twice; 3) succeeded in giving *N* twice and failed *N*+1 twice. Following previous research (e.g., Le Corre, Van de Walle, Brannon, & Carey, 2006; Sarnecka & Carey, 2008; Wynn, 1990, 1992), children were classified into six groups based on the highest number they answered correctly. A child whose highest number was greater than five was classified as a CP knower. A child whose highest number ranged from one to four was classified as a one-knower, two-knower, three-knower, or four-knower, respectively. Children who failed to give the lowest number were classified as a pre-numeral-knower.

Transform-Sets task. This task was used to evaluate children's understanding of numerical equality and it was adapted from a study by Sarnecka and Gelman (2004). In this task, children were asked to decide whether the same number word still applied to the set of unseen objects after witnessing a transformation that either altered (set-size changed condition) or did not alter (set-size unchanged condition) the quantity of objects in a covered box. In the set-size changed condition (4 trials), an object was either added to or removed from the box, changing the number of objects in the box by one. In the set-size unchanged condition (4 trials), the box was either shaken or rotated; there was no change to the number of objects in the box in this condition. To ensure that children possessed the required information for inference, the experimenter always confirmed that children knew the number. The number was repeated if children forgot the correct quantity.

The task began with four practice trials and was followed by eight test trials (random order). In the practice trials, only one object was put in the box and participants were asked to report the name of the object in the box after a transformation (either rotating the box, shaking the box, or swapping out the content of the box, e.g., replacing the bird in the box with a frog). In the test trials, the experimenter put either 2, 3, 5, or 6 objects in a box, then the experimenter performed the condition-relevant transformation. The post-transformation set size ranged from two to six, and participants were provided with two choices to choose from (e.g., "Is that 3 or 4?"). The range of the choices was two to six. Performance was measured by the total number of correct answers.

Recitation task. This task was adapted from Sarnecka and Carey (2008) for assessing children's ability to recite number words from one to ten. The experimenter asked participants to recite the count list by saying "Let's count. Can you count to ten?" If the child did not respond, then the experimenter suggested that they first count together and then by him- or herself[1]. The highest number recited in correct order was used as the performance measure of this task.

Picture Recognition Task. Participants' nonverbal working memory (WM) span was assessed by the Picture Recognition subtest in the Woodcock-Johnson III Test of Cognitive Abilities (Woodcock, Mather, et al., 2001). On each trial, children were tasked with memorizing a list of target pictures presented for five seconds (list length: 1 to 4). Then, children were asked to identify the targets from an array that contained both target and non-target pictures. Performance was measured using standard scores for this task.

ANS task. This was a computerized task designed to assess the precision of one's ANS and it was adapted from similar studies in the literature (e.g., Bonny & Lourenco, 2013; Libertus et al., 2011; Piazza et al., 2010). Each trial was initiated by the experimenter who pressed a virtual red star presented centrally onscreen. Two arrays of heterogeneous rectangles were shown horizontally onscreen in front of two characters (i.e., Big Bird and Grover). After a delay of two seconds, the arrays were replaced by the full body pictures of Big Bird and Grover, respectively. The experimenter then instructed the child to touch the character that had more "boxes."

There were two practice trials and 16 test trials. Both types of trials followed the same procedure except that there was no corrective feedback during the test trials. The positions of correct answers were counterbalanced across trials. The number of rectangles in each array varied from 4 to 21. Four ratio bins were used (2:1, 3:2, 4:3 and 7:6) with four trials in each bin. Following the protocol used in the study of Libertus et al. (2011), continuous properties of the arrays were varied in one of two ways. On half the trials, the cumulative area for the two arrays were equated within trial; in this case, average element size was inversely related to number. On the other half of the trials, the average element size was equated within trial; in this case, cumulative area was positively related to number. Performance of this task was measured by the total number of correct trials.

Results

Internal consistencies for Ordinal Position, Rank-based Relations, Transform-sets, ANS, Picture Recognition and DVAP[2] were assessed using split-half correlations (Spearman-Brown formula). Analyses revealed acceptable reliabilities for all tasks, with split-half correlations ranging from .56 to .98 (see Table 2).

Task Name	Split-half correlations
Ordinal Position task	.57
Rank-based Relations task	.91
Transform-sets task	.56
ANS task	.60
Picture Recognition task	.98
DVAP	.98

Table 2. Split-half correlations (Spearman-brown formula) for all measures used in the current study.

Preliminary analyses: Performance by Task

Ordinal Position task. An analysis of accuracy for 3- and 4-year-olds revealed that 4-year-olds (M = 71.83%, SD = 26.58%) performed significantly better than 3-year-olds (M = 50.00%, SD = 23.93%), $t(60) = 3.40$, $p = .001$, $d = 0.87$. Despite the age difference, both groups performed significantly above the chance level of 33.33% on this task[3]: 3-year-olds, $t(32) = 4.00$, $p < .001$, $d = 0.70$; 4-year-olds: $t(28) = 7.80$, $p < .001$, $d = 1.45$. Thus, by 3 years of age, children order items in a temporal sequence by their absolute ranks with improvement in this ability between 3 and 4 years of age.

Rank-based Relations task. Accuracy[4] on this task was analyzed using a mixed design ANOVA, with cue type (1- or 2-word cue) as the within-subjects factor and age (3- or 4-years-old) as the between-subjects factor. The main effect of age was significant, $F(1, 59) = 5.06$, $p = .028$, $\eta_p^2 = .08$, as 4-year-olds (M = 77.01%, SD = 24.85%) performed significantly better than 3-year-olds (M = 59.38%, SD = 34.64%). The main effect of cue type was also significant, $F(1, 59) = 11.85$, $p = .001$, $\eta_p^2 = .17$, as accuracy was significantly better on the 2-word cue condition (M = 72.83%, SD = 33.27%) than the 1-word cue condition (M = 63.32%, SD = 33.06%), consistent with Fuson et al. (1982). The interaction between cue type and age was not significant, $F(1, 59) = 0.28$, $p > .6$, suggesting that the effect of cue type was comparable for both age groups. In summary, children's understanding of rank-based relations improves across 3 and 4 years of age, and both age groups showed better performance when prompted with a longer cue.

Give-a-number (GAN) Task. There were 39 children (62.9%, $\text{range}_{\text{age}}$= 3 years 3 months to 4 years 8 months, M_{age} = 4 years 0 month) who were classified as CP-knowers (see Table 3 for the number and age distribution of knower levels) on the GAN task. The age distribution was comparable to those reported in previous studies (e.g., Patro & Haman, 2012; Sarnecka & Carey, 2008).

Knower Level	*N*	Mean Age	Range
1-knower	4	3; 9	3; 8 to 3; 10
2-knower	13	3; 9	3; 6 to 4; 4
3-knower	5	3; 8	3; 6 to 4; 0
4-knower	1	3; 9	-
CP-knower	39	4; 0	3; 3 to 4; 8

Table 3. Age distribution of knower level on the GAN task. Children's ages are reported in the format of "*year*; *months*". The number before the semicolon(;) denotes the year, the number after the semicolon denotes the months

Transform-Sets Task. Four-year-olds' (M = 86.21%, SD = 15.07%) performance on this task was marginally better than that of 3-year-olds (M = 78.41%, SD = 17.48%), $t(60) = 1.87$, $p = .067$, $d = 0.48$. When compared to the chance level of 50%, both 3-year-olds, $t(32) = 9.34$, $p < .001$, $d = 1.63$, and 4-year-olds, $t(28) = 12.94$, $p < .001$, $d = 2.4$, performed significantly above chance. Thus, by age 3 years, children understand how number words represent numerical equality with improvement in this ability between 3 and 4 years of age.

Recitation Task. No significant age difference was observed in the ability to recite the numbers 1 to 10 (3-year-olds: $M = 9.52$, $SD = 1.48$; 4-year-olds: $M = 9.59$, $SD = 1.21$), $t(60) = 0.21$, $p = .838$, which was likely the result of a ceiling effect. More specifically, the majority of children in this study (54 participants, 87.1%, $\text{range}_{\text{age}} =$ 3 years 6 months to 4 years 8 months, $M_{\text{age}} =$ 3 years 11 months) recited numbers from 1 to 10 correctly (see Table 4 for information about age and performance of all children).

Highest recited number	*N*	Mean Age	Range
10	54	3; 11	3; 6 to 4; 8
8/9	3	3; 9	3; 3 to 4; 0
5/6	4	3; 11	3; 6 to 4; 4
4	1	3; 10	-

Table 4. Age distribution based on recitation performance on the Recitation task. Children's ages are reported in the format of "*year*; *months*". The number before the semicolon(;) denotes the year, the number after the semicolon denotes the months

ANS Task. One typical finding in the ANS task is the ratio effect (e.g., Bonny & Lourenco, 2013; Libertus, Feigenson, & Halberda, 2011), which is that better performance is associated with larger ratios. As expected, a significant ratio effect, $F(3, 180) = 7.79$, $p < .001$, $\eta_p^2 = .12$ (linear trend: $F[1, 60] = 24.27$, $p < .001$, $\eta_p^2 = .29$) was found using a mixed design ANOVA on accuracy, with ratio (2:1, 3:2, 4:3 and 7:6) as the within-subjects factor and age (3 or 4 years old) as the between-subjects factor. The effect of age, and the interaction between age and ratio were not significant (*p*s > .07). When compared to the chance level of 50%, children's performance with the two easier ratios was significantly above chance (2:1 ratio: $M = 69.35\%$, $SD = 27.69\%$, $t[61] = 5.50$, $p < .001$, $d = 0.70$; 3:2 ratio: $M = 64.11\%$, $SD = 26.64\%$, $t[61] = 4.17$, $p < .001$, $d = 0.53$), whereas their performance with the two more difficult ratios did not differ significantly from chance (4:3 ratio: M = 52.42%, SD = 25.49%, t[61] = .74, p > .4; 7:6 ratio: M = 54.03%, SD = 26.08%, t[61] = .1.22, p > .2). In summary, our results confirmed involvement of the ANS on this task, as revealed by children's general success and modulation by ratio.

Other control tasks. Children's scores indicate that their performance was consistent with the typical level for both the Picture Recognition task and the DVAP. Children's mean standardized score on the Picture Recognition task[5] was 109.75 (SD = 21.83), and children's mean expressive vocabulary on the DVAP[6] was 98.87 (SD = 26.92).

Relations between Ordinality and Exact Number Representations

	2	3	4	5	6	7	8	9
1. Age (month)	.499**	.225	.268*	.442**	.061	.062	.343**	.171
2. Ordinal Position	—	.298*	.420**	.616**	.323*	.309*	.180	.442**
3. Rank-based Relations		—	.492**	.564**	.505**	.448**	.079	.161
4. Transform-set			—	.599**	.473**	.337**	.234	.223
5. Give-A-Number				—	.401**	.347**	.175	.295*
6. Recitation					—	.335*	.032	.293*
7. Pic Recognition						—	.279*	.237
8. ANS							—	.170
9. DVAP								—

Table 5. Zero order Spearman correlations between all experimental tasks in this study (N = 59).
*p < .05, **p < .01

I next examined the links between children's ordinal abilities and their understanding of exact numbers. Preliminary examination of the data suggested non-linear relations among the variables of interest. I thus used Spearman correlations in the subsequent analyses as it can detect monotonic relations without the requirement of linearity (Siegel, 1988). I also used partial correlation analyses to ensure the specificity of the links between children's ordinal abilities and their understanding of exact numbers (i.e., numerical equality and CP).

Ordinal abilities and Numerical Equality. I first focused on the concept of numerical equality, which was measured by the Transform-set task. If ordinal abilities support the acquisition of numerical equality, then performance on the Transform-sets task should be positively correlated with that of the Ordinal Position task (which assessed absolute rank) and Rank-based Relations task (which accessed rank-based relations). Analyses revealed a significant zero-order correlation between Transform-sets and Ordinal Position tasks, $r_s(57) = .42$, $p < .001$, as well as between Transform-sets and Rank-based Relations tasks, $r_s(57) = .49$, $p < .001$. To test whether these links were driven by other numerical abilities, I partialled out performance on the Recitation task, ANS acuity, and children's chronological age. The partial correlation between the Transform-sets and Rank-based Relations tasks remained statistically significant, $r_s(54) = .30$, $p = .027$ (the partial correlation between Transform-sets and Ordinal Position tasks was no longer statistically significant, $r_s[54] = .22$, $p = .100$). To test whether the correlation between the Transform-sets and Rank-based Relations tasks was due to general cognitive abilities, I also partialled out the effect of nonverbal working memory span (Picture Recognition task), size of expressive vocabulary (DVAP), and chronological age. Again, the correlation remained significant, $r_s(54) = 0.37$, $p = .005$. These findings suggest that children who have a better understanding of the rank-based relations among number words are also better with the concept of equality and this is not likely explained by individual differences in general cognitive abilities. Moreover, these findings also suggest that the effect of rank-based relations among number words cannot be reduced to children's the ability to recite the number words.

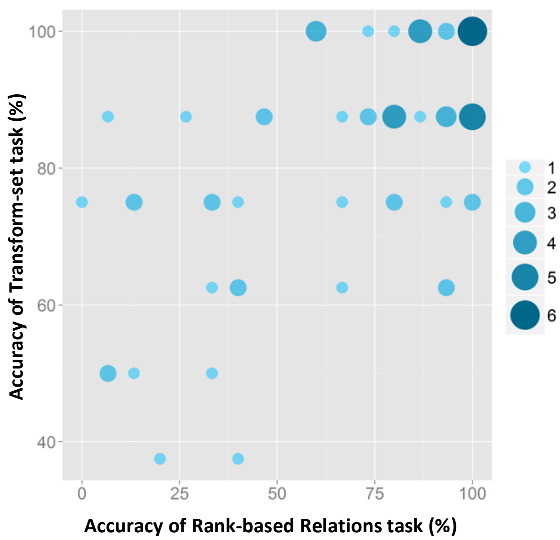

Figure 2. Children's accuracy on the Transform-sets task as a function of their accuracy on the Rank-based Relations tasks. Size and color of each point represent the frequency of participants.

Ordinal abilities and Cardinality Principle. The understanding of cardinality principle (CP) was measured by the GAN task. If ordinal abilities support the acquisition of CP, then participants who are better in the two ordinal tasks should also be better in the GAN task. Consistent with this suggestion, correlational analyses revealed significant zero-order correlations between GAN and Ordinal Position tasks, $r_s(57) = .62$, $p < .001$, and between GAN and Rank-based Relations tasks, $r_s(57) = .56$, $p < .001$. To test whether these relations were driven by other number-related abilities, I partialled out the performance in Recitation task, ANS acuity, and chronological age. The results remained statistically significant for the partial correlation between GAN and Ordinal Position tasks, $r_s(54) = .43$, $p = .001$, as well as the partial correlation between GAN and Rank-based Relations tasks, $r_s(54) = .41$, $p = .002$, suggesting that the relation between the understanding of cardinality was not driven by other numerical abilities. I also tested whether these associations could be explained by general cognitive abilities by partialling out nonverbal WM span (Picture Recognition task), size of expressive vocabulary (DVAP), and chronological age. Children's performance on the GAN task remained significantly correlated with their performance on the Ordinal Position task, $r_s(54) = .41$, $p = .002$, and with their performance on the Rank-based Relations task, $r_s(54) = .45$, $p = .001$, confirming a relation between CP and ordinal abilities that cannot be accounted for by general cognitive functioning.

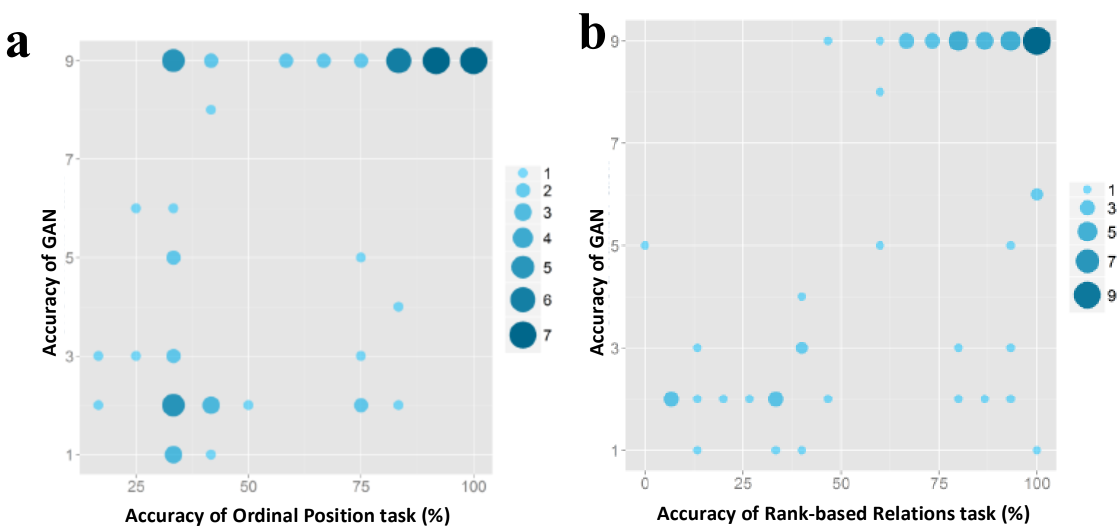

Figure 3. Children's scores on the GAN task as a function of their accuracy on the (a) Ordinal Position and (b) Rank-based Relations tasks. Size and color of each point represent the frequency of participants.

Given that both ordinal tasks were significantly correlated with the GAN task, one may ask whether these correlations were driven by shared ordinal processes. Indeed, the zero-order correlation between the two ordinal tasks was statistically significant, $r_s(57) = .30$, $p = .022$. Thus, to answer this question, I tested whether the two ordinal tasks explained unique variance in the GAN task by partialling out the effect of the other ordinal task and chronological age. If there are some common abilities that drive the correlations between GAN and both ordinal tasks, then these partial correlations should fail to reach significance when the other ordinal task was partialled out. Nevertheless, all correlations with GAN task remained statistically significant: Ordinal Position task: $r_s(55) = .47$, $p < .001$; Rank-based Relations task: $r_s(55) = .50$, $p < .001$. Taken together, these findings suggest that the ability to represent absolute rank and rank-based relations may each contribute uniquely to the individual differences in children's understanding of CP.

Discussion

Previous studies have suggested the possibility that ordinality may aid in the acquisition of exact numbers (e.g., Attout, Noël, & Majerus, 2014; Lyons & Beilock, 2011; Spaepen, Coppola, Spelke, Carey, & Goldin-Meadow, 2011). The current study pinpoints the specific ordinal abilities that are associated with this important mathematical achievement. Specifically, I found that children who were better at recognizing the absolute rank in a temporal sequence also showed more advanced understanding of the cardinality principle (CP), even when accounting for children's ability to recite the number word sequence, as well as their ANS acuity and general cognitive abilities. Moreover, I found that children who had a better understanding of rank-based relations among number words also showed more advanced exact number representations. In particular, 3- and 4-year-olds who were better at naming the next number in the count list showed better understanding of both numerical equality and CP. Taken together, these findings support the hypothesized claims about the role of ordinal abilities in the acquisition of exact number representations.

The findings also suggest the extraction of rank information from the count list is a critical step in the acquisition of number words. When children first learn how to recite number words, the rank information associated with these words was embedded in the recitation routine, which is not immediately available to direct access. In the current study, most of the participants were perfect in reciting numbers 1 to 10, but their performance in naming the next number in the count list was far from perfect, suggesting that their ability to recite the numbers is not equivalent to having access to rank information. The need to extract rank information from the count list may be one of the reasons why there is a delay for young children to understand the meaning of number words after they first learn how to recite the numbers. The delay may be caused by the need to develop new abilities for extracting ordinal information embedded in the word list, which enables young children to access the pertinent ordinal information for developing numerical concepts such as CP. Moreover, the need to extract rank information from the count list also explains a limitation in children's number knowledge before they acquire CP. Condry and Spelke (2008) investigated whether sub-set knowers understood that larger numbers come after smaller numbers in the count list. These subset-knowers could recite numbers correctly up to ten, but they had not acquire CP and did not know the meaning of the number words "five" and above. The task required children to identify the larger number word in a pair. When presented with numbers that they knew (e.g., three and one), subset-knowers performed consistently above chance. When presented with number words that were beyond their knower level (e.g., five and ten), these subset-knowers performed at chance, suggesting they had not mapped the rank-based relations (i.e., before/after) among number words onto the magnitude-based relation (i.e., less/ more) among numerical values. The finding of Condry and Spelke is consistent with the findings from the current study. The subset-knowers in the current study also had less knowledge about the rank-based relations among the number words. This association may explain why Condry and Spelke found subset-knowers in general experienced more difficulty in mapping the rank-based relations onto the magnitude-based relations.

In the current study, the finding of the association between absolute rank and CP is particularly intriguing. The task that assessed absolute rank (i.e., Ordinal Position task) did not involve any number words, yet the ability to track the rank of rabbit was related to the understanding about counting (CP). It is unlikely that this association was due to the use of counting in the tracking of rabbits. At this age, children have yet to acquire ordinal number words (Miller, Major, Shu, & Zhang, 2000). Moreover, they had difficulty in using counting to find the rank of an item within a sequence (Colomé & Noël, 2012). It is possible that children may process the rabbit sequence as a word sequence ("brown bunny, pink bunny and grey bunny") or picture sequence (i.e., picture of brown, pink and grey bunnies). Either way, this finding suggests that the process that supports absolute rank of number words is not specific to the count list--such a process also supports a similar function in other types of sequences.

The association between absolute rank and CP observed in the current study also converges with a suggestion made by Sarnecka and Carey (2008). They argued that the acquisition of CP requires an implicit understanding that "a numeral's cardinal meaning is determined by its ordinal position in the list" (p. 665, Sarnecka & Carey, 2008). However, in their study, Sarnecka and Carey tested this claim by examining CP- and non-

CP-knowers' knowledge of rank-based relations, not the absolute rank of number words (referred to as ordinal position in the quote above). In the current study, the understanding of absolute rank and rank-based relations was not related, prompting the question of whether it is appropriate to use knowledge about rank-based relations to support their claim. Though the current study confirms the claim by the significant correlation between the understanding of absolute rank and CP, this did not change the fact that Sarnecka and Carey's interpretation of their own results was inaccurate. Researchers should be mindful about the difference between absolute rank and rank-based relation in future studies.

The findings from the current study also provide support for Spaepen et al.'s (2011) suggestion that the ordinal structure of the number word list may be critical for exact number representations. In their study, Spaepen et al. examined hearing impaired adults who used homesigns (i.e., nonconventional gestures) for communication. Since they did not know any spoken language or conventional sign language, they had no access to the conventional number system and had to use their own homesigns for labelling and communicating numerical values. The homesigns scaled with the presented set size when homesigners provided estimation for the number of items in a set. Nevertheless, these homesigners failed to associate consistent homesigns to the same set size if the number was outside the subitizing range (i.e., 1 to 4), suggesting that labels per se were not sufficient for exact number representations. These homesigners also failed to perform exact addition or subtraction on large numbers. Crucially, Spaepen et al. (2011) noted that homesigns lack the ordinal structure that is typically found in the conventional number system, which is the stable order of number words that embed the principle of the successor function. Our data are consistent with this claim. I showed that children's ability to represent exact numbers is related to their abilities to extract and use rank information, suggesting the ordinal structure of number words is indeed important for the acquisition of exact numbers. Our results suggest that even when children are exposed to a conventional number system, if they lack the ability to extract the rank information that is embedded in the sequence, then they benefit less from the ordinal structure of the number word sequence, and they were less capable of representing exact numbers. In the case of homesign, there is no ordinal structure for homesigners to draw on. Without this ordinal structure, homesigners could not perform exact enumeration, nor could they carry out computation by moving up or down the number word list. Therefore, they failed to develop fully-fledged exact number representations.

Footnote

[1] In the cases in which the experimenter recited with the children, the extra step of reciting together helped children to overcome their shyness in speaking and was effective in eliciting responses. Since the sequence to be recited is long (1 to 10), it is highly unlikely that children could memorize such a long sequence without drawing on prior knowledge.

[2] Split-half correlations could not be computed on GAN and Recitation tasks.

[3] Chance was set at 33.33% because there were three characters to choose from (1/3 = .3333). It can be argued that the chance level should be 25% because there were actually four possible choices (i.e., picking 1st, 2nd, 3rd or picking no bunny). I set the chance level at 33.33% instead of 25% for two reasons: 1) This was more consistent with the task instructions, which implied three choices; 2) This criterion was more stringent.

[4] One participant (3 years 6 months) did not complete the task so her data were excluded from the analyses of this task and subsequent correlational analyses.

[5] One participant (4 years 1 months) refused to attempt this task so she did not contribute data to the analyses of this task or to subsequent correlational analyses.

[6] Two participants' (3 year 11 months and 3 years 6 months) guardians did not fill out the DVAP questionnaires, so these participants were not included in the analyses of DVAP and subsequent correlation analyses.

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The relation between ordinal concepts and children's arithmetic competence

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Abstract

Accumulating evidence suggests a relation between ordinal ability and competence with symbolic arithmetic. Yet the nature of this association remains unresolved. The current study addressed this issue in two directions. First, I studied children's ability to manipulate rank information. I assessed 5- to 6-year-olds' knowledge about three different types of rank-based operations. This provided important background information for understanding the role of rank knowledge in arithmetic. Second, I tested the proposal that the computation procedure of arithmetic is rooted in the manipulation of rank information—because addition or subtraction can be solved by reciting numbers forward or backward in a specific number of steps. Importantly, this way of solving arithmetic problem can be conceptualize as manipulation of rank information about number words in the count list. To test the proposal, the current study tested the associations between different rank-based operations and symbolic arithmetic in 5- to 6-year-olds. I found that children who were better at making inferences based on inter-item distance between letters were better at solving arithmetic problems. The study also found that children who showed better understanding of how insertion or removal affects the rank of items in a sequence also showed higher arithmetic competence. Together, these findings suggest rank-based operations are recruited in the computation of arithmetic. By pinpointing the specific aspects of ordinality that are related to arithmetic, the current study provided important groundwork for understanding the nature of the association between ordinal ability and arithmetic competence .

A recent large scale study found that the mathematical abilities of American students lag behind those of other industrialized countries (OECD, 2016). Thus, understanding the psychological processes that underlie mathematical thinking will be crucial in helping to improve the quality of mathematics education. A new and promising direction pertains to the role of ordinal abilities in arithmetic. Studies have found that deficits in ordinal abilities may be related to developmental dyscalculia (Attout & Majerus, 2015; Kaufmann, Vogel, Starke, Kremser, & Schocke, 2009; Rubinsten & Sury, 2011). Moreover, in a typically-developing population, performance on order judgment tasks has proven predictive of arithmetic performance in both adults (Lyons & Beilock, 2011) and children (Lyons & Ansari, 2015; Lyons, Price, Vaessen, Blomert, & Ansari, 2014). Nevertheless, it remains largely unclear why such associations exist. In the current study, I investigated the contributions of ordinality to the development of arithmetic understanding among children beginning to learn formal arithmetic.

Connections between Ordinality and Arithmetic

The procedure for arithmetic computations such as addition and subtraction can be conceived as movement along the sequence of number words (i.e., counting up or down the sequence). Consider the so-called *min* procedure, an algorithm that is commonly used by children (Groen & Parkman, 1972; Siegler, 1987; Siegler & Jenkins, 1989). This algorithm handles addition by counting from the larger addend in the number of steps specified by the smaller addend. For example, to solve "3 + 2", one counts from three and then moves forward by two steps (i.e., "three", then "four and five"). An analogous procedure can be used for subtraction such that the starting point would always be the minuend, and the directional movement would be backward instead of forward.

Conceptually, the operations of moving along a sequence can be viewed as ordinal operations because these operations can be performed on any ordered sequence, including those without numerical magnitude such as the letters of the alphabet or months in a year. For instance, to find the month that is two ahead from March, one could recite the month list from March and move two steps forward, producing the answer May. Even though there are parallels between arithmetic computations and ordinal operations, addition and subtraction are typically viewed as operations on numerical magnitude. Addition of natural numbers results in an increase in numerical magnitude, whereas subtraction results in a decrease in magnitude. Yet it is an open question whether ordinal processes are recruited when computing addition and subtraction problems.

The distinction between magnitude and ordinality in mathematical processing is an important and complex issue. One source of the complexity is the diversity of meaning associated with the concept of ordinality (or “ordinal relations”). The ordinal terminology can be used for describing relative numerical magnitude, such as the less/more relation (e.g., 3 is more than 2; e.g., Brannon & Van de Walle, 2001; vanMarle, 2013). However, the same ordinal terminology could also be used to describe the position of items within a sequence. One such example is the before/after relation (e.g., 3 comes after 2; e.g., Gevers, Reynvoet, & Fias, 2003; Turconi, Jemel, Rossion, & Seron, 2004), though unlike the less/more relation, before/after is not specific to magnitude and applies more generally to all kinds of sequences. In the context of natural numbers, the less/more and before/after relations are largely synonymous. Less/more and before/after perfectly mapped to each other such that smaller numbers always come before larger numbers in the natural number list. Nevertheless, research has shown that these analogue relations are at least partially dissociated from each other neurally and behaviorally (Cheng, Tang, Walsh, Butterworth, & Cappelletti, 2013; Turconi et al., 2004). Thus, it is important to test whether magnitude-based and non-magnitude-based ordinal concepts make different contributions to mathematical development.

This conceptual distinction brings a new perspective to the discussion on the origins of formal mathematical ability. There is an ongoing debate about whether, and how, the Approximate Number System (ANS), an intuitive representational system for numerical magnitude, contributes to the development of mathematical ability such as the understanding of symbolic arithmetic (for meta-analyses, see Chen & Li, 2014; Fazio, Bailey, Thompson, & Siegler, 2014; Schneider et al., 2016). One account suggests a direct association between ANS acuity and math ability, such that the ANS should be directly involved in mathematic operations on symbolic numbers. This account is supported by findings that show symbolic numbers are mapped onto ANS representations so that these symbols can acquire their meaning in numerical value (Halberda, Mazzocco, & Feigenson, 2008; Lourenco, Ayzenberg, & Lyu, 2016; Piazza, Pinel, Le Bihan, & Dehaene, 2007). On another account, the ANS only helps to jump start the learning of formal mathematics. Once this learning begins, the magnitude component (supported by the ANS) gradually loses its importance. The meaning of number shifts to the relation between symbols in the formal number system (Lyons, Ansari, & Beilock, 2012; Reynvoet & Sasanguie, 2016). On this account, ordinality plays an intermediary role in the causal relation between the ANS and symbolic number competence because ordinal concepts are critical for organizing the abstract relations between symbols in the number system. The mediating role of ordinality is supported by research showing that ordinal ability completely mediates the relation between ANS acuity and performance in symbolic arithmetic among adults (Lyons & Beilock, 2011). This finding is consistent with the view that the ANS does not directly support symbolic arithmetic (Reynvoet & Sasanguie, 2016). Since the finding on the mediating role of ordinality is important for understanding the relation between the ANS and math competence in adults, I wanted to test whether ordinality also mediates the relation between ANS and symbolic arithmetic in school-aged children. Moreover, I also wanted to address the distinction between magnitude and ordinality in Lyons and Beilock's mediation study. In their study, ordinal ability was assessed by an order judgment task, in which participants judged whether the presented number triads were in ascending order (e.g., 3, 1, 7). Such a judgment requires one to consider both numerical magnitude and the arrangement of items, raising the question of whether magnitude processing in ascending order judgments is responsible for the mediation (see also Franklin & Jonides, 2009; Kaufmann et al., 2009).

In the current paper, I specifically focus on the non-magnitude component of ordinality and its role in children's developing arithmetic competence. In particular, in this work, I attempted to isolate non-magnitude ordinality by examining the understanding of rank, which concerns the position occupied by an item in a sequence (Nieder, 2005; Wiese, 2003). Following the approach in the studies described above, I examined whether the understanding of rank was predictive of children's performance on addition and subtraction problems. I chose to focus on rank in this work because of its relevance to the computational procedure in addition and subtraction described above. Reciting numbers forward or backward is essentially moving from one position to another position in the number-word list. I refer to manipulations of positional information as rank-based operations. These manipulations can take different forms and below I describe the rank-based operations examined in the current study:

1) Rank Deduction: This type of operation involves movement from one position to another position within an ordered sequence. To correctly execute such operations, it is necessary to keep track of the number of steps involved in the movement, the movement's direction, and the updated position after each step. As described above, solving addition and subtraction problems by reciting the number list from one of the addends involves the same operation.
2) Sequence Reversal: This type of operation involves a systematic transformation of rank to create a reversal of the sequence (e.g., the first item in a list becomes the last item when the sequence is reversed). If the original order of the sequence is "A, B, C", then the reversed sequence is "C, B, A". Subtraction can be solved by reciting the number sequence backwards, analogous to the operation of sequence reversal.
3) Sequence Modification: This type of operation involves the updating of rank information to accommodate specifically the insertion or removal of an item(s) in a sequence. By inserting an item into a sequence, the rank of all items following the inserted item is moved backward by one step. Item ranks before the inserted items are unaffected. Analogous effects occur when an item is removed from the sequence. Importantly, sequence modification is not a procedural analogue of addition and subtraction. Specifically, numerical magnitude is not affected by the location of insertion/removal. Insertion of an item always results in an increase in

numerical magnitude, whereas removal of an item always results in a decrease in magnitude.

Current study

Despite much interest in the cognitive foundations of formal mathematics, we know very little about the individual differences that contribute to mathematical development. Here I focus on children's ordinal abilities and their potential role in early arithmetic competence. The existing developmental literature has very little information about children's understanding of rank (except Colomé & Noël, 2012; Miller, Major, Shu, & Zhang, 2000). It is unclear whether children understand the above-mentioned rank-based operations when they learn arithmetic, making it difficult to determine the developmental relation between the two. The current study aimed to examine children's understanding of rank-based operations and to directly test whether these abilities are related to arithmetic performance. To this end, the work I conducted in this study begins with an investigation of the development of specific rank-based abilities, namely rank deduction, sequence reversal, and sequence modification. I then turned to whether rank-based operations might mediate the relation between the ANS and arithmetic competence. If these operations indeed mediate the relation between the ANS and arithmetic, then one can rule out the role of common magnitude processing because these rank-based operations do not implicate numerical magnitude. Such results would support the idea that ordinality mediates the causal pathway by its role in organizing relations between numerical symbols.

To achieve these goals, the current study includes assessments of arithmetic ability and the understanding of rank-based operations at 5 and 6 years of age. In addition, I also included an ANS task so that I could directly test the possible role of rank-based operations in the causal relation between ANS acuity and arithmetic competence. Moreover, I also measured a collection of general cognitive abilities to control for effects not specific to rank-based operations.

Method

Participants

Seventy-seven preschoolers (38 male; age range: 5 years 0 months – 6 years 11 months, M_{age} = 5 years 11 months) participated in this study. One additional participant (6 years 2 months) was tested but excluded from statistical analyses for failing to follow instructions. All children received a small gift for participating in the study. Informed consent was obtained on behalf of each child by a parent or legal guardian. Experimental procedures were approved by the local ethics committee.

Tasks and Procedure

Participants were tested individually in a university laboratory. All ten tasks were administered in a single experimental session (see Table 1), lasting approximately 90 minutes. Task order was randomized across participants with the constraint that the tasks assessing the abilities of interest (i.e., rank-based operations and symbolic arithmetic) were administrated before the control tasks (i.e., general intelligence and ANS acuity).

Task Name	Construct	Specific Ability
Rank Deduction	Rank-based operations	Movement along a sequence, e.g., reciting the letters forward/ backward in specific number of steps
Sequence Reversal	Rank-based operations	Systematic conversion of rank for reversing the order of a sequence
Sequence Modification	Rank-based operations	Updating rank information after an item is added or removed from a sequence
Arithmetic	Symbolic arithmetic	Arithmetic operations of addition and subtraction
ANS	Nonverbal numerical magnitude	Acuity of the Approximate Number System (ANS)
Inhibition	General intelligence	Inhibition ability
Picture Recognition	General intelligence	Nonverbal working memory span
Memory for Words	General intelligence	Verbal working memory span
Picture Vocabulary	General intelligence	Vocabulary
Decision Speed	General intelligence	Processing speed

Table 1. List of constructs and the corresponding tasks in the study.

Rank Deduction Task. This tasked was designed to test children's ability to transform rank information, specifically in relation to an exact number of steps (e.g., start at the 2^{nd} position of the sequence, then move two steps forward to reach the 4^{th} position). It is particularly important to use non-numerical stimuli in this task because this type of rank-based operation is analogous to the computation of addition and subtraction. Using numerical stimuli would essentially turn the task into an arithmetic task. Thus, letter stimuli were used in the current task. Children were required to perform such operations on a subset of the alphabet sequence (i.e., start at *B*, then move two steps forward to reach *D*). Only children who successfully recognized printed letters and could successfully recite the alphabet were included in this task. All children were screened for these abilities by requiring that they identify the letters A to K when printed on individual cards (order randomized) and that they recite these letters in correct order. Four children (1 male; range: 5 years 1 month — 6 years 2 months) failed to pass the screening and thus did not contribute data related to this task in subsequent analyses.

This task began with two practice trials. In the first practice trial, the experimenter placed the letter cards A to K on a table and explained that the letters were arranged in “the way we say our letters” (i.e., alphabetical order). Then, the experimenter covered all the cards except one, which acted as the anchor. Children were then asked to name one of the covered cards, which the experimenter pointed to. To answer correctly, children had to use not only the letter on the anchor card but also the inter-item distance between the anchor and covered target cards. Corrective feedback was provided by uncovering the letters on the covered cards. The same procedure was used in the second practice trial and the six test trials, except that these trials were computerized. The eight test trials were generated by completely crossing four levels of inter-item distance (i.e., 1 to 4, e.g., the inter-item distance of A and B is 1) and two levels of operation direction (i.e., forward vs. backward direction). Operation direction was specific to the relative position between the anchor and the target card. For example, if the anchor card was E, and the target card came after the anchor card, then the answer could be determined by reciting the letter sequence forward from the letter E. However, if the target card came before the anchor card, then one could recite the letters backwards to find the answer. Performance was measured as children’s accuracy across the test trials.

Sequence Reversal Task. This task was designed to test children’s understanding of reversal in the context of an ordered sequence. Specifically, children had to perform a systematic conversion of rank to reverse the order of a sequence of items. On each trial, children were presented with two identical rows of cards (see Figure 1). Paper cards were used in the first practice trial to familiarize children with the task procedure. Subsequent trials were computerized. The content of the top row of cards remained visible throughout the trial to serve as a memory aid for the original order. The bottom row was used to demonstrate the reversal operation and to elicit a response. After explaining to children that the top and bottom rows were identical, the experimenter reversed the order of the bottom row and covered all but the rightmost (last) card of the row. The rightmost card was left uncovered to emphasize the reversal. Children were asked to name the content on the covered cards one by one.

There were two test trials followed by six test trials. Test and practice trials were identical, except that in the test trials, the cards were covered before the reversal action. On the test trials, the first half of trials involved sets of three cards and the last half involved sets of five cards. Trials included letter and picture sequences. I had also included numbers to examine whether the type of rank-based operations differed across different types of sequences. Despite the difference in sequence type (i.e., letters, shapes, numbers), all trials required that children respond on the basis item position after a sequence was reversed. The six test trials were generated by completely crossing three levels of sequence length (i.e., 3 cards vs 5 cards) and three levels of sequence type (i.e., letters, shapes, numbers). Performance was measured as children’s accuracy across the test trials.

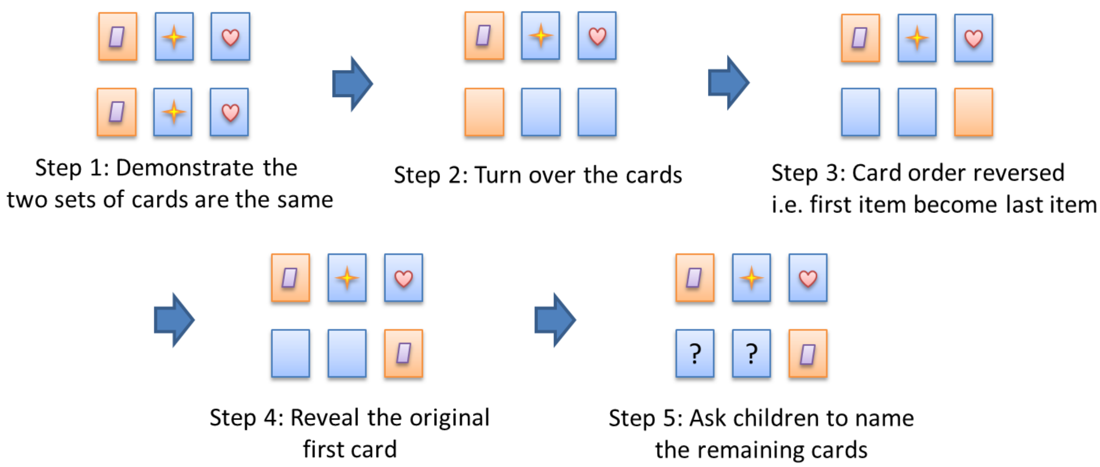

Figure 1. Procedure of the Rank Deduction task

Sequence Modification Task. This task was designed to test whether children understand how to update rank information to accommodate the insertion or removal of an item(s) in a sequence. Specifically, this computerized task required an understanding that inserting an item into a sequence changes the rank of all items following it, but not the items preceding it. On each test trial, children were shown a line of nine animals waiting in front of a cashier (see Figure 2). The experimenter then indicated to children the rank of the target animal by counting from the first animal (next to the cashier) to the target animal. Following this demonstration, two occluders appeared and covered all animals before and after the target animal, leaving only the target animal visible. A memory check of the target animal's rank was immediately performed by requiring children to report its rank. If children failed the memory check, then the experimenter removed the occluders and performed the counting demonstration again. This step was repeated until participants correctly reported the rank of the target animal. Children were then shown an animation, in which an animal either joined or left the line before or after the target animal, so that the rank of the target animal was either altered or remained the same. Children were then asked to report the rank of the target animal. Since all animals that came before or after the target were covered by occluders, children could not use counting to determine the answer. Instead, they had to infer the answer based on the change shown in the animation.

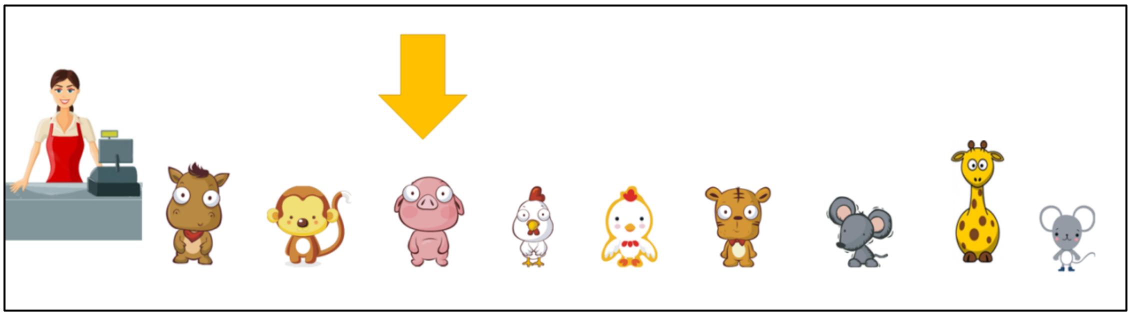

Figure 2. Screenshot of the beginning of a trial from the Sequence Modification task. In this task, children were shown a line of nine animals waiting in front of a cashier, and the experimenter indicated to children the rank of the target animal (yellow arrow included). Then, the animals before and after the target were covered by an occluder and an animal was either added or removed from behind the occluder (either before or after the target animal). Children were asked to report the rank of the target after witnessing this event.

The task began with two practice trials and was followed by eight test trials. Practice and test trials were identical except that there was corrective feedback during practice but none during test. Test trials were generated by fully crossing the type of operation (insertion or removal of animal) and location (before or after the target animal). The rank of the target animals (start of the trial) ranged from three to seven. Performance was measured as children's accuracy across the test trials.

Arithmetic Task. This task assessed participants' symbolic arithmetic ability. The task began with two practices questions, in which the experimenter worked with the children to solve the problems using real objects. For example, to illustrate the practice problem of 2 + 1, the experimenter showed children that there were two coins in a box, then the box was covered and one more coin was added to the box; children were then asked to indicate the number of coins in the box. After the practice questions, children completed 18 test trials presented in paper-and-pencil format, all as Arabic numerals. All test trials were arithmetic problems that involved single-digit operands (half addition and half subtraction). No feedback was given on the test trials. Performance was measured as children's accuracy across the test trials.

ANS task. This task assessed children's ANS acuity, which is the precision of their nonsymbolic representations of numerical magnitude. This computerized task was adapted from similar studies in the literature (e.g., Bonny & Lourenco, 2013; Libertus et al., 2011; Piazza et al., 2010). On each trial, two arrays of rectangles were shown side-by-side on the screen in front of two characters (i.e., Big Bird and Grover). Both arrays were made up of rectangles of different sizes. After 2 s, the arrays were replaced by full body pictures of Big Bird and Grover. Children were asked to indicate which array was larger in numerosity (i.e., "Who had more boxes?) by touching the corresponding characters onscreen (side of correct answer counterbalanced across trials). Each trial was initiated by the experimenter who touched a virtual red star on the computer touchscreen.

Five different ratios were used in the 20 test trials (2:1, 4:3, 6:5, 11.5:10 and 11:10), with the stimulus number ranging from 10 to 39. Easier ratios (2:1 and 4:3) and smaller numbers (2 to 4) were used in the two practice trials to help children understand the task. Corrective feedback was provided during the practice trials, but not during the test trials. The non-numerical properties of these arrays were controlled using the procedure of Libertus et al. (2011). In particular, on half the trials, cumulative area was matched across the two arrays and, on the other half, average element size was matched across the two arrays. Performance was measured as children's accuracy across the test trials.

Decision Speed Task. Individual differences in processing speed were assessed with the Decision Speed Task of the Woodcock-Johnson III Tests of Cognitive Abilities (Woodcock, Mather, McGrew, & Schrank, 2001). Children were given a worksheet that contained 40 test items. Each item consisted of seven different pictures, two of which belonged to the same category (e.g., sun and moon). Children were instructed to circle the pictures that belonged to the same category. Children completed as many trials as possible in three minutes. Performance was measured using standard scores for this task (scaled based on $M = 100$ and $SD = 15$).

Inhibition Task. As a measure of inhibitory control I used the Inhibition subtest from the Developmental Neuropsychological Assessment-2nd Edition (Korkman, Kirk, & Kemp, 2007). On this task, children labeled shapes (circle vs. square) or the direction of arrows (up vs. down) as quickly as possible such that responses were either congruent (e.g., say "circle" if the stimulus was a circle; Naming condition) or incongruent (e.g., say "circle" if the stimulus was a square; Inhibition condition) with the presented stimulus. Performance for this task was measured by a contrast score, which is a scale score that is based on both Naming and Inhibition scores.

Picture Recognition Task. Participants' nonverbal working memory (WM) span was assessed by the Picture Recognition subtest in the Woodcock-Johnson III Test of Cognitive Abilities (Woodcock, Mather, et al., 2001). On each trial, children were tasked with memorizing a list of target pictures presented for five seconds (list length: 1 to 4).

Then, children were asked to identify the targets from an array that contained both target and non-target pictures. Performance was measured using standard scores for this task (scaled based on *M* = 100 and *SD* = 15).

Picture Vocabulary Task. Children's vocabulary was assessed by the Picture Vocabulary subtest of the Woodcock-Johnson III Test of Achievement (Woodcock, McGrew, & Mather, 2001). Children were required to name pictures of objects in a test booklet with multiple pictures on each page. On each trial, the experimenter pointed to a picture on the page and children were required to provide the name of the referenced object. This task used an adaptive procedure, such that trials were presented in order of increasing difficulty, and termination of the task depended on the participant's performance. Performance was measured using standard scores (scaled based on *M* = 100 and *SD* = 15).

Memory for Words Task. Verbal WM capacity was assessed by the Memory for Words subtest of the Woodcock-Johnson III Tests of Cognitive Abilities (Woodcock, Mather, et al., 2001). On each trial, children were presented with an audio recording of a word list that they were asked to remember. They then were asked to repeat the words in the order in which they were presented. The length of word lists varied from 1 to 7 words (3 lists each per length of word). This task used an adaptive procedure, such that trials were presented in increasing difficulty and termination depended on the participant's performance. Performance was measured using standard scores (scaled based on *M* = 100 and *SD* = 15).

Results

Internal consistencies of experimental tasks were assessed using split-half correlations (Spearman-Brown formula). Analyses revealed acceptable reliability for all tasks, with split-half correlations ranging from .55 to 87 (see Table 2).

In this section, I first analyzed children's performance on each of the tasks administered to them. These tasks included all measures designed to assess rank-based operations, arithmetic, ANS acuity and general cognitive abilities. Second, I analyzed the relation between children's performance on each of the ordinal tasks and their ability to solve arithmetic problems. Third, I specifically tested the potential mediating role of rank-based operations in the relation between ANS and symbolic arithmetic.

Task Name	Split-half correlations
Rank Deduction	.55
Sequence Modification	.80
Sequence Reversal	.57
Arithmetic	.87
ANS	.57
Inhibition	.82
Picture Recognition	.72
Memory for Words	.78
Decision Speed	.87

Table 2. Split-half correlations (Spearman-brown formula) of tasks in the current study. For standardized tasks, the correlations are reported values in the manuals.

Performance on Individual Measures

Rank Deduction Task. Accuracy ($M_{overall}$ = 65.41%, *SD* = 19.15%) on this task was analyzed using a mixed factor analysis of variance (ANOVA), with direction (forward or backward) and inter-item distance (1 to 4) as the within-subjects factors and age (5- or 6-year- olds) as the between-subjects factor. There was a significant main effect of age, $F(1, 71) = 6.77$, $p = .011$, $\eta_p^2 = .09$, such that 6-year-olds (*M* = 70.95%, *SD*

= 15.61%) performed better than 5-year-olds (*M* = 59.72%, *SD* = 20.94%). There were also significant main effects of direction, $F(1, 71) = 84.65$, $p < .001$, $\eta_p^2 = .54$, and inter-item distance, $F(2.61, 185.6) = 47.06$ (Greenhouse-Geisser corrected), $p < .001$, $\eta_p^2 = .40$, as well as a significant interaction between these two factors, $F(2.61, 185.16) = 13.25$ (Greenhouse-Geisser corrected), $p < .001$, $\eta_p^2 = .157$. Pairwise comparisons revealed that performance on the forward trials was better than on the backward trials at all distances (*p*s < .001, Bonferroni adjusted $\alpha = .05/4 = .0125$), except inter-item distance one (p = .714, $M_{\text{Forward}} = 93.1\%$, $SD_{\text{Forward}} = 2.99\%$; $M_{\text{Backrward}} = 94.5\%$, $SD_{\text{Backward}} = 2.68\%$). No other interactions reached statistical significance (*p*s > .05). Taken together, these results suggest that children of this age are highly accurate in deducing rank with letters that are adjacent each other. Moreover, children experience greater difficulty in moving backward in a sequence, which is consistent with Fuson's (1988) findings that reciting numbers in a backwards direction is more difficult than reciting numbers in a forward direction.

Previous studies have shown reverse distance effects in ordinal judgment tasks; that is, better performance for sequences with smaller inter-item distance as compared to those with larger inter-item distance (Lyons & Ansari, 2015; Lyons & Beilock, 2013; Turconi, Campbell, & Seron, 2006). To investigate whether the current task showed similar effects, I analyzed children's accuracy on forward and backward trials using separate mixed-design ANOVAs, with inter-item distance (1 to 4) as the within-subjects factor and age (5 or 6-year-olds) as the between-subject factor. The effect of inter-item distance was significant in both analyses: forward trials, $F(2.49, 176.76) = 13.36$ (Greenhouse-Geisser corrected), $p < .001$, $\eta_p^2 = .16$; backward trials, $F(2.52, 178.72) = 44.03$ (Greenhouse-Geisser corrected), $p < .001$, $\eta_p^2 = .38$. The effect of age was also significant in both analyses: forward trials, $F(1, 71) = 4.47$, $p = .038$, $\eta_p^2 = .06$; backward trials, $F(1, 71) = 3.99$, $p = .050$, $\eta_p^2 = .053$. The interaction of the two factors was not significant in either analysis (*p*s > .3). Trend analyses revealed significant linear trends in both analyses: forward trials, $F(1, 71) = 29.51$, $p < .001$, $\eta_p^2 = .29$; backward trials, $F(1, 71) = 192.24$, $p < .001$, $\eta_p^2 = .73$ (see Figure 3). However, a quadratic trend was also significant in backward, $F(1, 71) = 11.18$, $p = .001$, $\eta_p^2 = .14$, but not forward ($p > .07$), trials. These findings suggest that although reverse distance effects exist in both directions of operations, the pattern of performance differed based on the operation direction. For the forward direction, performance decreased gradually at the same rate across inter-item distance from one to four. For the backward direction, there was a sharp drop in performance from inter-item distance one and two, which was followed by a gradual decrease across inter-time distance two to four.

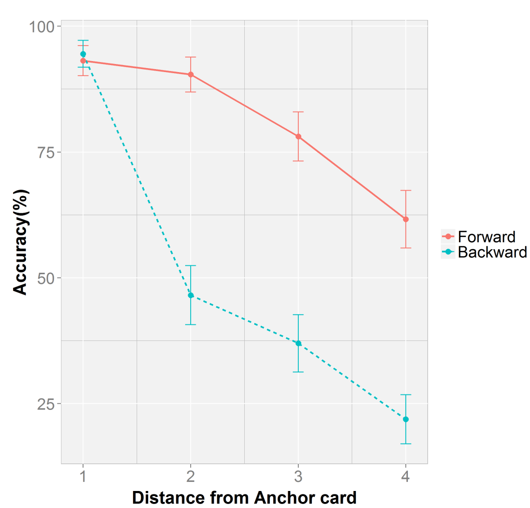

Figure 3. Accuracy of Rank Deduction task in forward and backward trials as a function of distance from anchor card. The downward lines were consistent with a reverse distance effect, as the accuracy decreased as the inter-time distance increased. Error bar represents standard error of the mean.

In subsequent analyses, I examined children's response patterns in an effort to shed light on the processes underlying rank deduction. In particular, I analyzed how well children's answers corresponded to the actual operations required by the task. To this end, I compared the relation between two variables. I refer to one variable as the *target vector* – the relative distance and direction between target and anchor letters. For example, if the letter on the anchor card was *C* (rank = 3) and the letter on the target card was *A* (rank = 1), then the value of the target vector for this trial was -2 (i.e., 1 - 3 = -2). A negative value indicates that the target letter came before the anchor letter, whereas a positive value indicates that the target letter came after the anchor letter. I refer to the other variable in this analysis as the *response vector* – the relative distance and direction between response and anchor letters. Following from the previous example, if the child responded with *E* (rank = 5) instead of A (rank = 1), then the value of the response vector was 2 (i.e., 5 - 3 = 2). A positive value indicates that the response letter came after the anchor letter, whereas a negative value indicates that the response letter came before the anchor letter. In the case of perfect performance, target and response vector variables are perfectly correlated. By contrast, in the case of random responding, the correlation would be close to zero.

Separate analyses for the 5- and 6-year-olds revealed that both age groups showed significant correlations between target and response vectors on the forward trials (5-year-olds: $M_r = .90$, $t[35] = 24.51$, $p < .001$; 6-year-olds: $M_r = .95$, $t[36] = 45.34$, $p < .001$). This result suggests that children in both age groups understood the rule for rank deduction in the forward direction. Errors tended to be in the correct direction and were not far off from the correct answers. Analyses of the backward trials revealed a similar pattern. In particular, separate analyses for the 5- and 6-year-olds, again, revealed that both age groups showed correlations on the backward trials that were significantly different from zero (5-year-olds: $M_r = .33$, $t[35] = 3.29$, $p = .002$; 6-year-olds: $M_r = .58$, $t[36] = 6.40$, $p < .001$), suggesting that even though 5- and 6-year-olds' understanding of rank deduction in the backward direction was far from perfect, they were not performing randomly. In summary, children have some rudimentary understanding of forward and backward operations such that their responses scale in both direction and magnitude with the correct answers. Nevertheless, these correlations were lower for backwarck than forward trials (5-year-olds: $t[35] = 5.41$, $p <.001$; 6-year-olds: $t[36] = 4.56$, $p < .001$), which was consistent with the accuracy analyses above.

Sequence Reversal Task. Accuracy ($M_{\text{Overall}} = 67.75\%$, $SD_{\text{Overall}} = 27.33\%$) on this task was analyzed using a mixed factor ANOVA, with sequence type (letter, shape, or number sequences) and sequence length (three or five) as the within-subjects factors and age (5- or 6- year-olds) as the between-subjects factor. There was a significant main effect of sequence length, $F(1, 75) = 23.03$, $p < .001$, $\eta_p^2 = .24$, such that children's performance for shorter sequences ($M = 77.16\%$, $SD = 28.94\%$) was better than for longer sequences ($M = 57.92\%$, $SD = 36.36\%$). There were also significant main effects of sequence type, $F(2, 150) = 3.67$, $p = .028$, $\eta_p^2 = .05$, and age, $F(1, 75) = 6.36$, $p = .014$, $\eta_p^2 = .08$, as well as a significant interaction between these two factors, $F(2, 150) = 3.09$, $p = .049$, $\eta_p^2 = .04$. To further investigate this interaction, I analyzed the performance of 5- and 6-year-olds using separate within-subjects ANOVAs, with sequence type (letter, shape or number sequences) and sequence length (three or five) as factors. The effect of sequence length was significant in both age groups: 5-year-olds, $F(1, 38) = 17.83$, $p <$

.001, η_p^2 = .32; 6-year-olds, *F*(1, 37) = 6.29, *p* = .017, η_p^2 = .15. However, the effect of sequence type was only significant in 5-year-olds, *F*(2, 76) = 5.85, *p* = .004, η_p^2 = .13 (6-year-olds: *F*[2, 74] = 0.53, *p* = .592). The effect of sequence type among 5-year-olds was further investigated using pairwise comparisons. Performance for the number sequence (*M* = 69.23%, *SD* = 4.70%) was significantly better than for the letter sequence (*M* = 52.24%, *SD* = 5.47%; *p* = .001, Bonferroni adjusted *α* = .05/4 = .0125) and marginally better than for the shape sequence (*M* = 59.29%, *SD* = 5.52%, *p* = .035). No other comparison reached significance (*ps* >.2). These results suggest that sequence reversal may be first acquired for numerical sequences, which is later generalized to other types of sequences.

Sequence Modification Task. Accuracy on this task ($M_{overall}$ = 57.78%, *SD* = 22.12%) was analyzed using a mixed-factor ANOVA, with operation (insertion or removal) and location (before or after the target) as the within-subjects factors, and age (5- or 6-year-olds) as the between-subjects factor. There was a significant main effect of location, *F*(1, 75) = 5.61, *p* = .020, η_p^2 = .07, such that children performed worse when the operations occurred after (*M* = 48.38%, *SD* = 44.11%) than before (*M* = 67.21%, *SD* = 40.00%) the target. There was also a significant main effect of age, *F*(1, 75) = 7.42, *p* = .008, η_p^2 = .09, such that 6-year-olds (*M* = 64.47%, *SD* = 22.80%) performed better than 5-year-olds (*M* = 51.28%, *SD* = 19.62%). No other main effect or interactions reached statistical significance (*p*s > .06).

Following the analyses performed above on the Rank Deduction task, I examined how children's answers corresponded to the actual operations required by the task using regression analysis. For each participant, I regressed their responses on two predictors, one was the original rank of the target animal and the other was the actual operation[2]. The value of the actual operation ranged from -1 to +1, with -1 indicating the rank of the target animal moving closer to the beginning of the sequence by one step, and +1 indicating the target's rank moving away from the beginning of the sequence by one step. If children provided perfect responses on all trials, then the coefficient of the actual operation should be significantly greater than zero. However, if children provided random responses, then the actual operation would not be a significant predictor of children's responses. Given that there was a significant effect of location (i.e., operation occurred before vs. after the target) in the ANOVA above, separate analyses should be conducted on trials in which changes occurred before versus after the target. However, when changes occurred after the target, the actual operation was always is zero, making it impossible to calculation the effect of this predictor. I thus only performed the regression analysis on trials in which changes occurred before the target.

Actual operation was a significant predictor of children's responses for both age groups (5-year-olds: $M = 0.54$, $SD = 0.53$, $t(38) = 6.47$, $p < .001$; 6-year-olds: $M = 0.82$, $SD = 0.36$, $t[37] = 13.85$, $p < .001$). In addition, the coefficient of the actual operation for 6-year-olds was significantly higher than for 5-year-olds, $t(67.48) = 2.61$ (degrees of freedom corrected for unequal variance), $p = .01$, which is consistent with the age difference found in the ANOVA conducted above. In summary, although 5- and 6-year-olds had difficulty estimating rank when changes occurred before the target, they did not respond randomly. Their responses scaled in both direction and magnitude with the correct answers.

Arithmetic Task. Accuracy on this task (M_{Overall} = 69.70%, SD_{Overall} = 29.21%) was analyzed using a mixed-factor ANOVA, with question type (addition or subtraction) as the within-subjects factor and age (5- or 6-year-olds) as the between-subjects factor. The only significant effect was the main effect of age, $F(1,75) = 21.71$, $p < .001$, $\eta_p^2 = .23$, such that 6-year-olds (M = 83.63%, SD = 21.34%) performed better than 5-year-olds (M = 56.13%, SD = 29.67%), as would be expected given the known developmental improvement in arithmetic competence over this age range. I found no difference with respect to question type, $F(1,75) = 0.40$, $p > .5$. In other words, children were equally good at addition and subtraction problems, perhaps because the harder addition problems involved double digit answers, whereas subtraction always resulted in single digit answers, which may have made the two types of problems more comparable in difficulty.

ANS Task. Accuracy on this task (M_{Overall} = 75.79%, SD_{Overall} = 11.62%) was analyzed using a mixed-factor ANOVA, with ratio (2:1, 4:3, 6:5, 11.5:10, and 11:10) as the within-subjects factor and age (5- or 6-year-olds) as the between-subjects factor. As expected, I found a significant ratio effect, $F(3.287, 246.49) = 39.91$ (Greenhouse-Geisser corrected), $p < .001$, $\eta_p^2 = .35$ (linear trend analysis: $F(1, 75) = 162.34$, $p < .001$, $\eta_p^2 = .68$), consistent with much extant findings showing that magnitude precision follows Weber’s law (Dehaene, 2003; Halberda & Feigenson, 2008; Lourenco, Bonny, Fernandez, & Rao, 2012). The effect of age and the interaction between age and ratio was not significant (ps > .1). Performance was significantly above the chance level of 50% at all ratios (ps < .001) except for the most difficult one (11:10, ps > .07). This pattern of performance was found for both age groups.

Other Control tasks. The standardized scores of the remaining control tasks (Decision Speed, Inhibition, Picture Recognition, Picture Vocabulary and Memory for Words) are presented in Table 4.

Tasks	5-year-olds (*SD*)	6-year-olds (*SD*)
Decision Speed	106.67 (14.71)	107.55 (14.07)
Inhibition	10.69 (2.68)	10.16 (2.69)
Picture Recognition	113.23 (10.35)	111.55 (11.68)
Picture Vocabulary	106.15 (11.14)	108.79 (10.54)
Memory for Word	110.69 (14.94)	111.71 (16.80)

Table 4. Age-related performance on control tasks.

Relations between arithmetic and ordinal abilities

	2	3	4	5	6	7	8	9	10	11
1. Age (month)	.265*	.276*	.331*	.624*	.097	-.058	-.005	-.008	.066	-.049
2. Rank Deduction	—	.108	.162	.455*	.261*	.137	.111	.197	.100	.243*
3. Seq Reversal		—	.301*	.311*	.074	.349*	-.040	.050	.207	.199
4. Seq Mod			—	.517*	.264*	.100	.161	.116	.351*	.206
5. Arithmetic task				—	.391*	.141	-.044	.250*	.346*	.192
6. ANS					—	.212	.260*	.233	.211	.251*
7. Inhibition						—	.019	.157	.134	.219
8. Pic Recognition							—	.146	.147	.082
9. Word Memory								—	.260*	.084
10. Pic Vocab									—	.234
11. Decision Speed										—

Table 5. Zero order correlations among experimental tasks and age ($N = 69$).
*$p < .05$

To answer the question of whether the ability to understand rank-based operations is related to arithmetic performance, I used Pearson correlational analyses. Participants who failed to complete all tasks were excluded from these analyses[1]. To ensure these correlations were not unduly driven by outliers, cases in which any score deviated from the task mean by 3 SDs were also excluded from the analyses (final $N = 69$). Performance on all tasks, except the Arithmetic, Sequence Reversal, Picture Recognition, and ANS tasks, had acceptable skewness or kurtosis (i.e., did not deviate from the skewness and kurtosis of normal distribution by 2 SEs, Tabachnick & Fidell, 1996). Tasks in which performance did not yield acceptable skewness or kurtosis were transformed using a Rank-based inverse Normal (RIN) transformation (Bishara & Hittner, 2012); skewness and kurtosis were within an acceptable range post-transformation for the transformed tasks.

All three tasks that assessed rank-based abilities (Sequence Modification, Rank Deduction, and Sequence Reversal) were significantly correlated with the Arithmetic task (see Table 5). When partialling out age (in months) and general cognitive abilities (i.e., Inhibition, Picture Recognition, Memory for Words, Decision Speed, and Picture Vocabulary), arithmetic performance remained significantly correlated with performance on Sequence Modification, $r(61) = .33$, $p = .008$, and Rank Deduction, $r(61) = .33$, $p = .009$, suggesting the relations were not driven by general cognitive abilities. However, the correlation between arithmetic and the Sequence Reversal task was no longer significant, $r(61) = .03$, $p = .801$.

I also tested whether ANS acuity could account for the relation between the ordinal tasks and arithmetic performance by partialling out the effect of ANS acuity and age. The partial correlations of arithmetic with Sequence Modification, $r(65) = .36$, $p = .003$, and Rank Deduction, $r(65) = .32$, $p = .009$, remained statistically significant (Sequence Reversal, $r(65) = .18$, $p = .143$). In summary, these results suggest that the understanding of how rank is affected by the location of insertion or removal (Sequence Modification task), as well as the ability to move forward and backward along an ordinal sequence (Rank Deduction task) are related to solving arithmetic problems, and these associations cannot be explained by age, general cognitive abilities, or ANS acuity.

Sequence Modification and Rank Deduction tasks were designed to assess specific types of rank-based abilities. Thus, one might ask whether the two tasks were actually measuring the same cognitive process, accounting for correlations with arithmetic. Though this claim is not consistent with the non-significant zero-order correlation between Sequence Modification and Rank Deduction, $r(69) = .16$, $p = .184$, direct evidence is needed to ensure that each is uniquely related to arithmetic competence. Partialling out the effect of Rank Deduction and age did not change the significance of the correlation between children's performance on the arithmetic task and Sequence Modification, $r(65) = .42$, $p < .001$. Similarly, partialling out the effect of Sequence Modification and age did not change the correlation between children's performance on the arithmetic task and Rank Deduction, $r(65) = .39$, $p = .001$. These results suggest that the two types of rank-based abilities are uniquely related to children's understanding of symbolic arithmetic.

I also examined whether the three types of rank-based abilities were related to each other. Of the zero-order correlations among the three rank-based operation tasks, none except the correlation between Sequence Reversal and Sequence Modification was significant, $r(67) = .30$, $p = .012$. However, if general cognitive abilities (i.e., inhibition, decision speed, vocabulary and working memory) and age are taken into account, no correlation remained statistically significant, *r*s < .15, *p*s > .2, confirming the uniqueness of the rank-based abilities assessed in the current study.

The relation between ANS acuity, ordinal abilities and arithmetic ability

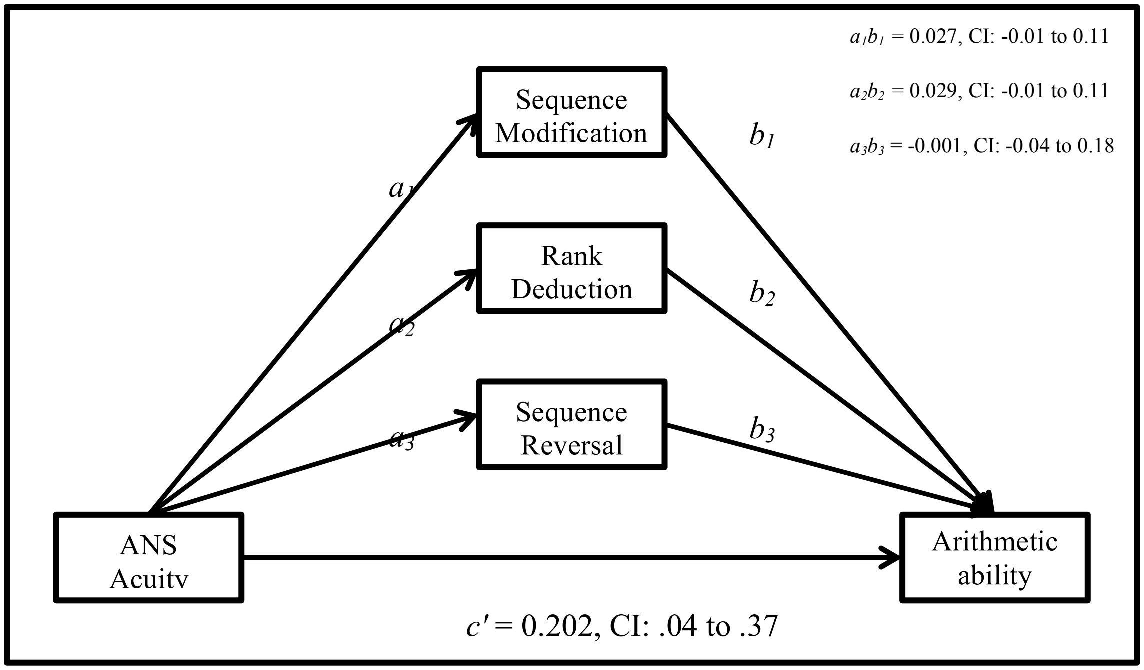

Figure 4. Conceptual representation of the mediation model tested. Control variables were included in the model but are not depicted (see main text). The analysis found a significant direct effect of ANS acuity on arithmetic ability but there was no evidence of indirect effects, suggesting no mediation by any of the tested variables.

Consistent with extant findings (Attout, Noël, & Majerus, 2014; Lyons & Beilock, 2011; Rubinsten & Sury, 2011), I found that ANS acuity was significantly correlated with arithmetic performance, $r(67) = .39$, $p < .001$ (see Table 5). However, Lyons and Beilock (2011) showed that the association between ANS acuity and arithmetic performance was mediated by performance on an order judgment task. However, as I discussed in the Introduction, magnitude is inherent to such a judgment, making it unclear whether ordinality per se or magnitude processing mediated the relation between ANS acuity and arithmetic. I tested whether rank-based abilities (i.e., Sequence Modification, Rank Deduction and Sequence Reversal) mediated the influence of ANS acuity on arithmetic abilities (see Figure 4 for the conceptual model). Although the zero-order correlation between Sequence Reversal and ANS tasks was not significant, $r(67) = .07$, $p = .57$ (see Table 5), I included Sequence Reversal as a mediator because it is theoretically possible for this ability to have a significant indirect effect despite the non-significant zero-order correlation (Hayes, 2013). Control variables including age (in months) and general cognitive measures (i.e., Inhibition, Memory of Word, Picture Recognition, Decision Speed and Picture Vocabulary tasks) were included in the model. The analysis found no evidence of significant indirect effects[2], since zero was included in the 95% confidence interval of all three indirect effects: Sequence Modification (a_1b_1 = 0.027, 95% CI: -0.01 to 0.11), Rank Deduction (a_2b_2 = 0.029, 95% CI: <-0.01 to 0.11) and Sequence Reversal (a_3b_3 = -0.001, 95% CI: -0.04 to 0.02). In contrast, the direct effect of ANS acuity on arithmetic ability was significant, $c' = 0.20$, $p = .017$, 95% CI: 0.04 to 0.37. Thus, this analysis does not support the notion that rank-based abilities mediated the relation between ANS acuity and arithmetic performance. Together with the finding in the previous section, showing a relation between rank deduction and sequence modification, these results suggest that ANS acuity and an understanding of rank-based operations each exert a unique influence on symbolic arithmetic.

Discussion

The current study examined the role of ordinal abilities in early arithmetic competence. I focused on one aspect of ordinality, namely rank, in an effort to isolate the non-magnitude component of ordinality, and to test whether symbolic arithmetic draws on non-magnitude ordinal processes. I found that 5- and 6-year-olds' performance in two tasks designed to assess rank-based operations, namely rank deduction (moving forward/

backward along a sequence) and sequence modification (insertion/ removal of item in a sequence), was related to competence with symbolic arithmetic, even when controlling for general cognitive abilities or ANS acuity. I also tested whether ANS exerted its influence on symbolic arithmetic via rank-based operations and only found a direct effect of the ANS in children aged 5 and 6 years. Our results suggest that ANS and rank-based operations may each have unique contributions to early symbolic arithmetic competence. The implications of these findings are discussed below.

The development of rank-based abilities

The current study examined 5- to 6-year-olds' abilities to perform three types of rank-based operations, namely rank deduction, sequence reversal, and sequence modification. Novel tasks were designed to test each of these abilities. I tested whether children's understanding of forward and backward movements along a sequence (letters of the alphabet) in our Rank Deduction task. The findings suggested 5- and 6-year-olds do have some, albeit not perfect, understanding of these operations. They were better at determining the letter that comes after a given letter as compared to finding a letter that comes before a given letter. Analyses revealed that 5- to 6-year-olds were highly accurate in deducing rank that involved adjacent letters, but their performance decreased as the number of steps involved increased, suggesting a reverse distance effect. This is consistent with findings on a numerical order judgment, in which performance is typically better for sequences with smaller inter-item distance as compared to those with larger inter-item distance (Lyons & Ansari, 2015; Lyons & Beilock, 2013; Turconi et al., 2006). Lyons and Beilock (2013) suggested the count list may be an important factor driving the reverse distance effect. However, no number words were used in the Rank

Deduction task. Thus, I suggest that this effect is driven by ordered sequence more generally rather than the number word list specifically.

Children's ability to systematically engage in reversals of rank information was examined in the Sequence Reversal task. Analyses revealed that the type of sequence (i.e., number, letter or shape sequence) affected 5-year-olds', but not 6-year-olds', performance, which suggests increasing generalization with development. In addition, 5-year-olds performed better with number than letter and shape sequences, suggesting children may first acquire sequence reversal ability in number sequence, and then generalize this ability to other types of sequence. This developmental pattern mirrors the findings of the linearization of the mental number line (MNL). When young children are asked to map numbers with locations on a physical line, they have a tendency to overestimate the distance between small numbers and to underestimate the distance between large numbers, which suggests compressive representations (Siegler & Booth, 2004; Siegler & Opfer, 2003). Such representations become increasingly linear across development. Researchers found that this shift from compressive to linear representations also occurs in letter sequences (Berteletti et al., 2012; Hurst et al., 2014). Moreover, the linearization of the MNL occurred before the linearization of letter sequences, suggesting generalization from numerical sequences. Together, the developmental pattern of sequence reversal and MNL linearization suggests that children's experience with numerical sequences may aid in their understanding of ordinal properties within other, non-numerical sequences.

Sequence modification concerns the rule that the action of insertion or removal only affects the rank of items that comes after the changed item(s). Children's knowledge of this rule was assessed by the Sequence Modification task. Both 5- and 6-year-olds showed some, albeit not perfect, understanding of how rank should be updated when changes occurred before the target item because their responses were not random but systematically corresponded to the correct operation. Nevertheless, both age groups performed significantly worse when the change occurred after, as compared to before, the item of interest. The result suggests children first acquire the rule for updating ranks that are supposed to change; the knowledge about when the rank of item should stay unchanged emerges later in development.

Relation between rank-based relations and symbolic arithmetic

In the current study, I found that children's arithmetic performance was related to their abilities to deduce rank and modify sequences, specifically, these operations involve moving along a sequence and updating the rank of an item after insertion or removal of items, respectively. Moreover, these relations cannot be explained by general cognitive abilities or ANS acuity, suggesting that rank deduction and sequence modification each have unique relations with the development of symbolic arithmetic, though I acknowledge that the casual role in these relations remain to be established.

Rank deduction and the computational procedures of addition and subtraction both involve movements along a sequence. In the current study, letters were used in the rank deduction task to isolate rank-based operations, which may have been intermixed with magnitude processes in addition and subtraction. The significant correlation between Rank Deduction and Arithmetic tasks provides some support for the claim that rank-based processes are recruited when solving symbolic addition and subtraction problems.

In contrast to rank deduction, sequence modification is not a procedural analogue of addition and subtraction. A key difference between the two is that the location of insertion or removal determines whether the rank an item is affected by the change. Location, however, is irrelevant in the case of numerical magnitude. Insertion always results in an increase in numerical magnitude while removal always results in a decrease in numerical magnitude. Despite this difference, sequence modification was significantly correlated with symbolic arithmetic. This relation is consistent with the development of counting. Colomé and Noël (2012) found that children's knowledge about cardinal counting (i.e., counting for set size) was correlated with their knowledge about ordinal counting (i.e., counting for the rank of a specific item). Together with the findings of the existing studies, the current study suggests that the connection between rank-based abilities and symbolic arithmetic goes beyond procedural similarity. The ability to understand rank may be intrinsically important for the symbolic number system (Cheung & Lourenco, in prep). This is because magnitude and rank information of numbers are intertwined in the natural number system. For example, the rank and value (magnitude) of a number are systematically mapped to one another such that the first number represents the value one, the second number represents the value of two, and so on (Sarnecka & Carey, 2008). Also, the successor function guarantees that moving forward one step in the number words sequence is equivalent to adding one in terms of numerical value (Gallistel & Gelman, 1978; Sarnecka & Carey, 2008). Moreover, the rank-based relation of before/after is mapped onto the less/more relation of numerical magnitude, such that smaller numbers always comes before larger numbers in the numerical sequence (Carey, 2009). Thus, even though rank-based processes could be used independently for processing non-numerical sequences (e.g., Rank Deduction task in the current study), they may also constitute an indispensable part of mathematical thinking.

The current study shows that rank-based operations are related to symbolic arithmetic. However, since magnitude and rank information of numbers are intertwined in the natural number system, it is possible that the developmental relation between the two is reciprocal in nature. This argument actually does not go against my hypothesis. What I have argued is that rank-based processes are recruited in symbolic arithmetic, such that both magnitude and rank-based processes constitute the ability to perform addition and subtraction. One of the major goals of the current study is to single out the rank-based processes that may have been conflated with magnitude process in the past, so that I can achieve a more accurate picture of the mechanisms that underlie symbolic number abilities. If rank-based operations constituted the cognitive mechanism for arithmetic computation, then deficits in rank-based operations would lead to deficits in arithmetic. Indeed, existing research has already shown that dyscalculic adults and children have deficits in ordinal ability (Attout & Majerus, 2015; Kaufmann, Vogel, Starke, Kremser, & Schocke, 2009; Rubinsten & Sury, 2011). Nevertheless, future research is needed to specify the details of the causal mechanism underlying ordinal and mathematical development.

The role of ordinality in the causal pathway between ANS and symbolic arithmetic

The current study also examined whether the three rank-based abilities mediated the relation between ANS and arithmetic. I found ANS had a significant direct effect on symbolic arithmetic, but no indirect effect via the rank-based abilities. This finding is different from that of Lyons and Beilock (2011), in which they found that ordinality completely mediated the influence of ANS on symbolic arithmetic. What might account for the discrepancy between the two studies? One possibility concerns with difference in participants' age. Lyons and Beilock tested adult participants while I tested 5- and 6-year-olds. However, this is not likely the only reason for the difference because Lyons et al. (2014) examined children and found that ordinality was not a significant predictor for arithmetic performance in grades 1 and 2. Another possibility is that the purported ordinal tasks across the two studies actually measured different ordinal abilities. As discussed in the Introduction, Lyons and Beilock (2011) measured ordinal ability using an order judgment task, which required that participants consider both the arrangement (order) and magnitude of the numbers. In contrast, the current study focused on rank-based operations, which are not contingent on the numerical magnitude of the stimuli. This difference may be responsible for the absence of a mediation effect in the current study. These findings illustrate the importance of the conceptual distinction between magnitude (e.g., ascending order, less/more) and non-magnitude- (e.g., forward order, before/ after) based ordinal concepts. Although the distinction between these two types of ordinal concepts is often overlooked, both behavioral and neural evidence suggests that they are dissociable (Cheng et al., 2013; Turconi et al., 2004).

In summary, the current study contributes to the developmental literature by identifying different types of rank-based abilities, by proposing how specific rank-based abilities may be related to symbolic arithmetic in children, and by providing empirical evidence to support the specific associations between rank-based abilities and understanding of early symbolic arithmetic.

Footnotes

[1]Four participants were not tested on the Rank Deduction task because they failed the screening. One participant refused to finish the Decision Speed task.

[2]I used the bootstrapping procedure implemented by Hayes (2013) in SPSS to test the hypothesis. A confidence interval was calculated for each indirect effect using bootstrapping, a significant indirect effect is indicated by a confidence interval that does not include zero.

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Conclusions

The major goal of this dissertation was to investigate the developmental relation between ordinality and mathematics. The two papers in this dissertation add to the existing literature by addressing unresolved questions about why ordinality is related to mathematical competence. In the current dissertation, I tackled this question by addressing two unresolved issues. The first issue concerns with the development of ordinality. This issue is largely ignored by the studies that investigate the associations between ordinality and mathematics. This seriously limits our ability to understand the nature of the associations between the two kinds of abilities. To address this issue, I designed novel tasks to assess 3- to 6-year-olds' knowledge of rank. The second issue concerns how ordinality contributes to mathematical development by characterizing the relations among numerical symbols. Ordinality can be used to describe the relations among numerical symbols, but it remains largely unclear how these relations contribute to other mathematical abilities. To address this issue, I outlined two ways in which ordinality might be related to mathematical development. The first proposal pertains to the role of rank in the acquisition of exact number representations. The second proposal concerns with the specific associations between rank-based operations and children's understanding of symbolic arithmetic. In the following sections, I will discuss how the findings of this dissertation help address these unresolved issues.

Issue 1: Development of ordinality

Knowledge of rank information (e.g., January is the first month of a year) is critical for resolving everyday problems. For example, when waiting in line, it is important to know that someone cutting the line before you will increase your wait time, but someone joining the line after you will not affect you at all. Knowledge of rank is also pertinent to mathematical thinking. As discussed elsewhere in this dissertation, rank knowledge is important for the acquisition of exact number representations, as well as symbolic arithmetic. Nevertheless, the literature is limited in its understanding of the origins and development of rank knowledge. The nuances and complexity of this concept is often overlooked by researchers in the field. The current study highlighted three important aspects of rank—namely, absolute rank (e.g., first, second), rank-based relations (i.e., before/after) and rank-based operations (e.g., rank deduction, sequence reversal and sequence modification)—and provide empirical data on young children's knowledge of these aspects.

In Paper 1, I examined preschoolers' understanding of absolute rank. The findings suggested that 3-year-olds can identify the absolute rank of items in a temporal sequence and this ability improves between 3 and 4 years of age. It should be noted that most children of this age have yet to acquire ordinal number words (e.g., first, second, Miller, Major, Shu, & Zhang, 2000), and they have difficulty using counting to identify the verbal label for rank (Colomé & Noël, 2012). Thus, it is unlikely that they used number words to keep track of the absolute rank in the task. Previous studies suggest a rudimentary ability for representing absolute rank information in infants (e.g., Lewkowicz, 2013; Lewkowicz & Berent, 2009). The current study provided evidence that explicit understanding of absolute rank has emerged by 3 years of age.

Paper 1 also examined preschoolers' knowledge of rank-based relations. Specifically, I studied children's ability to name the next number when prompted with number cues. I found age-related improvement among 3- and 4-year-olds. Moreover, both age groups performed better when they were prompted with longer number cues (e.g., “2, 3, what’s next?” elicit better results than “3, what’s next?”), which is consistent with earlier findings by Fuson and colleagues (Fuson, 1988; Fuson, Richards, & Briars, 1982), suggesting that children may still rely on the recitation routine to make sense of the “after” relation. To illustrate how a routine may assist understanding, consider this everyday example. There is a password that I use to log onto my computer. Sometimes I forget the exact combination of numbers and letters in the password, but I can type it out when I place my hand on a keyboard. Thus, I can use my own typing action to figure out the actual password. This example illustrates that information can be embedded in a routine such that it is not available to conscious awareness (c.f. Karmiloff-Smith, 1992). In this situation, reenacting the routine may help to make such information explicit. Children may go through a similar process when they encounter the task of naming the next number. The rank-based relations among the numbers are embedded in the recitation routine, which is not immediately available to children. They may need to reenact part of the recitation routine to discover the answer, just like I need to use my typing action to figure out the actual password. Fuson (1988) suggested that a longer cue may be helpful to children because it serves as a “running start” of the recitation routine, this is helpful to children because they have yet acquired the flexibility to recite the numbers from any point of the sequence. In the current study, both age groups of children benefitted from a longer cue when prompted to name the next number, suggesting continued reliance on the recitation routine to process the rank-based information among numbers throughout 3 and 4 years of age.

In Paper 2, I examined 5- and 6-year-olds' understanding of three types of rank-based operations—namely, rank deduction, sequence modification and sequence reversal—by testing their performance on tasks designed to tap each of these operations. Rank deduction refers to the ability to move forward or backward along a sequence in a specific number of steps. For example, to find the weekday two days from Tuesday, I can start reciting the weekdays from Tuesday and move forward in two steps. Sequence modification is concerned with how the rank of items should be updated after other items are inserted or removed form a sequence. The impact of the change is contingent on location. Insertion or removal does not affect the rank of the items that come before the changed location, but rather, affect all the items that follow the change. Paper 2 found that even though 5- and 6-year-olds were far from perfect in the operations of rank deduction and sequence modification, their responses were systematic in that they corresponded to the direction of the correct answers. Paper 2 also studied children's ability to perform sequence reversal, which is the systematic conversion of rank to reverse the order of the sequence. Whereas 6-year-olds performed comparably regardless of sequence type, 5-year-olds showed better performance for number then letter and shape sequences, suggesting that children may acquire the ability of sequence reversal from numerical sequence, then generalizing to other sequences.

Papers 1 and 2 also provided important information about how these different aspects of rank are related to one another. Interestingly, even though these aspects are conceptually related, I found no behavioral evidence of associations across the respective tasks. Among 3- and 4-year-olds, the ability to identify absolute rank in a sequence (Ordinal Position task) was not related to the understanding of rank-based related among number words (Number Words Relations task). Among 5- and 6-year-olds, none of the rank-based operations were related to one another, suggesting different psychological processes may be recruited for different types of rank-based operations.

In summary, the current dissertation adds to the literature by highlighting the importance of rank in cognitive development, pointing out the different aspects involved in this concept, and providing empirical data vital to understanding how this general, though multifaceted, phenomenon develops in children aged 3 to 6 years. The study on rank-based operations is particularly novel because it is not typical to treat rank as an object to be operated on. The current dissertation examined three different types of rank-based operations and described the rules for these operations. This new way of conceptualizing rank opens doors for understanding more complex aspects of ordinality.

Issue 2: The specific contributions of ordinality to the understanding of symbolic numbers

Paper 1 focused on the potential contributions of rank knowledge to the acquisition of exact number representations. Before acquiring number words, children can only represent exact numbers for values under four using the object file system (Feigenson & Carey, 2003; Trick & Pylyshyn, 1994; Uller, Carey, Huntley-Fenner, & Klatt, 1999). The preverbal numerical system for larger numbers, namely the ANS, is inherently noisy and thus unable to represent numbers as discrete and unique values (Dehaene, 2011; Feigenson et al., 2004; Piazza, 2010; Xu & Spelke, 2000). To acquire exact number representations, children must learn two vital concepts: one is numerical equality (Izard, Pica, Spelke, & Dehaene, 2008; Izard, Streri, & Spelke, 2014) and the other is cardinality principle (CP; Gallistel & Gelman, 1978; Sarnecka, 2015; Wynn, 1990, 1992). Previous research showed that the acquisition of number words is critical for the development of these two concepts (Izard et al., 2008; Sarnecka, 2015). In Paper 1, I hypothesized that two aspects of ordinality, namely absolute rank and rank-based relations, support the acquisition of number words and, thus, representations of exact numbers, with findings from this study supporting this hypothesis. Absolute rank was shown to be related to the understanding of CP. In addition, rank-based relations among numbers were shown to be related to both the understanding of numerical equality and CP. Together, these findings confirmed Piaget's (1952; 1951/1975) insight that the integration of cardinal and ordinal properties of numbers is an important step in the development of numerical concept. More importantly, my dissertation goes beyond Piaget's work in the following way. Piaget did not address how the concept of exact, discrete numerical values emerges from the mapping between cardinal and ordinal properties of numbers. In his work, it is unclear why this kind of mapping would result in the mature understanding that each number word represents a unique and discrete numerical value. In the years after Piaget's work, research has demonstrated the importance of number word acquisition in exact number representations (Carey, 2009; Condry & Spelke, 2008; Sarnecka, 2015), as well as children's limitation in extracting rank information about the number words (Fuson, 1988). The current dissertation demonstrates specifically how the ordinal information about number words may contribute to the acquisition of vital exact number concepts.

Paper 2 focused on the associations between rank-based operations and symbolic arithmetic. The computational procedure associates with both addition and subtraction can be viewed as a rank-based operation because it involves reciting numbers forward or backward in a specific number of steps (Attout et al., 2014). Yet, whether ordinal processes are recruited in the process of computation is an open question. Paper 2 demonstrated the association between symbolic arithmetic and two types of rank-based operations, namely rank deduction and sequence modification, supporting the idea that arithmetic problem-solving also recruits rank-based operations.

In a previous study, Lyons and Beilock (2011) found that order judgment ability mediated the influence of ANS on symbolic arithmetic. This finding can be used as evidence to support the proposal that the meaning of number would gradually shift from magnitude (supported by ANS) to symbol-symbol relations (based on ordinality). One concern with this interpretation is that magnitude processes may be recruited in the order judgment task and the ANS task. If this is the case, then it is not appropriate to conclude magnitude processes have no direct effect on arithmetic. Paper 2 tested whether rank-based operations mediated the influence of the ANS on symbolic arithmetic. The rationale is that since rank-based operations are not contingent on magnitude, if the mediation were significant, then the finding would provide support to the proposal of a developmental shift in numerical meaning. Nevertheless, none of the rank-based operations was a significant mediator, so the results do not provide such support. But this nonsignificant result cannot be taken as counter evidence to the proposal either, because I only included one aspect of ordinality, namely rank-based operations, in the mediation analysis. As discussed in the Introduction, rank is just one of the many aspects of ordinality, thus my mediation analysis did not rule out the possibility that other non-magnitude ordinal concepts might mediate the effect between ANS and arithmetic. Thus, the question of whether symbol-symbol relations completely mediate the effect of ANS on arithmetic should be further investigated in future research.

Limitations and future research directions

Both Paper 1 and 2 used an individual differences approach to establish associations between specific types of rank knowledge with specific types of mathematical competence. This method is suitable for the exploratory nature of the current studies. However, to further understand the causal relation between rank and mathematics, experimental designs such as those that involve a training component would be useful. For example, in future training study, children should be randomly assigned to a rank training group or non-rank training group. If the group that received rank training showed greater improvement in mathematics, then the finding would provide strong support to the contribution of rank knowledge in mathematical development.

The current dissertation focused on how ordinality was related to mathematical development. Nevertheless, math is not the only academic discipline that draws on ordinality. Another important STEM subject, namely computer programming, is also built on ordinal concepts. A large part of programming involves arranging actions in a correct order so that a program can perform its function. Data structure is another important aspect of programming that draws heavily on ordinal understanding. Data structure concerns with how information should be organized so that it can be accessed efficiently and takes up least amount of memory. What is referred to as "array" in programming is a way of organizing data as an ordered list. To understand how an array works, one needs to understand ordinal concepts such as absolute rank. Programing skills have become increasingly important for the development of science and technology. Some countries, such as Finland, have already integrated computer programming into elementary school curricula (Deruy, 2017). Thus, studying how ordinality contributes to the learning of computer programming is one promising direction for future research.

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