Distribution Agreement

Essays on housing and macroeconomics

By

Tianhao Zhao
Doctor of Philosophy

Economics

Vivian Yue, Ph.D.
Advisor

Kaiji Chen, Ph.D.
Committee Member

Elena Pesavento, Ph.D.
Committee Member

Tao Zha, Ph.D.
Committee Member

Accepted:

Kimberly Jacob Arriola, Ph.D.
Vice Provost for Graduate Affairs & Dean, James T. Laney School of Graduate Studies

Date

Essays on housing and macroeconomics

By

Tianhao Zhao
M.A., Emory University, GA, 2022
B.A., Beihang University, China, 2018

Advisor: Vivian Yue, Ph.D.

An abstract of
A dissertation submitted to the Faculty of the
James T. Laney School of Graduate Studies of Emory University
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy
in Economics
2025

Abstract

Essays on housing and macroeconomics
By Tianhao Zhao

The U.S. economy after the Great Recession shows a set of persistent and puzzling macroeconomic patterns, including a sharp and prolonged decline in aggregate consumption, a slow recovery in house prices and employment, and substantial regional variation in these economic outcomes. These facts have raised ongoing academic and policy debates about the underlying mechanisms driving the depth and persistence of the economic downturn, as well as the slow and uneven path to recovery. Existing studies highlight the role of financial frictions such as collateral constraints, and nominal rigidities, particularly downward nominal wage rigidity (DNWR). However, these two classes of frictions have often been studied in isolation. This dissertation proposes and systematically investigates a unified theory of friction interaction, which emphasizes how the simultaneous presence of financial and nominal frictions can jointly amplify the effects of aggregate shocks and delay recovery dynamics in a nonlinear and persistent manner.

The first chapter provides empirical support for this theory by estimating the heterogeneous effects of net worth shocks across the U.S. counties using a newly constructed dataset, *CountyPlus*, covering the period from 2003 to 2019. The analysis focuses on two key frictions: collateral constraints and DNWR. It uncovers substantial spatial heterogeneity in the response of consumption, unemployment, and housing outcomes to net worth shocks. Using a semi-varying coefficient model, this chapter identifies significant amplification and interaction effects in the transmission of shocks: counties facing both tighter collateral constraints and more binding downward wage rigidity experience deeper and more persistent downturns. These effects are understated by standard models. The empirical findings are rationalized by a tractable two-agent general equilibrium model, which demonstrates how the interaction of these frictions can endogenously produce the observed heterogeneity and persistence, especially through their impact on household deleveraging, consumption cuts, and labor market slack.

Building on this evidence, the second chapter develops a full-scale quantitative heterogeneous agent model to formally study the amplification effects of friction interaction. The model features households with liquid savings, illiquid housing wealth, fixed mortgage debt payment, and portfolio adjustment decisions, combined with collateral and DNWR constraints. The equilibrium is solved globally, and the model is calibrated to match key macroeconomic outcomes before the Great Recession. When subject to an adverse aggregate shock, the model reproduces the joint dynamics of the U.S. economy after the Great Recession, including the deep drop and slow recovery in consumption, employment, house prices etc. Comparative statistics demonstrate that the interaction of collateral constraint and DNWR generates the strongest and most persistent responses across all major macro variables. Removing either friction significantly weakens the amplification channel. A welfare analysis further quantifies

the economic cost of each friction and their interaction, offering insights into targeted stabilization policies.

Together, these chapters provide a coherent and empirically grounded explanation for the uneven and prolonged recovery from the Great Recession. By highlighting the amplification effects arising from the interaction of financial and nominal frictions, this dissertation contributes to a deeper understanding of the transmission of aggregate shocks in frictional economies and informs the design of more effective macroeconomic stabilization policies.

Essays on housing and macroeconomics

By

Tianhao Zhao
M.A., Emory University, GA, 2022
B.A., Beihang University, China, 2018

Advisor: Vivian Yue, Ph.D.

A dissertation submitted to the Faculty of the
James T. Laney School of Graduate Studies of Emory University
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy
in Economics
2025

Acknowledgments

I would like to express my deepest gratitude to Professor Vivian Yue, my advisor, for her exceptional mentorship, unwavering support, and constant encouragement throughout my PhD journey. Her intellectual guidance, patience, and generosity have been instrumental in shaping not only this dissertation but also my academic career. I feel incredibly fortunate to have had her as my advisor.

I am also sincerely thankful to my committee members: Professor Tao Zha, Professor Kaiji Chen, and Professor Elena Pesavento. Their deep insights, critical feedback, and continued support have pushed me to improve my work at every stage. Their advice has profoundly influenced both the structure and substance of this dissertation. I am truly grateful for the time and thought they have invested in my training.

I thank the Department of Economics at Emory University for providing a supportive and stimulating academic environment over the past six years. I would like to extend special thanks to the three Directors of Graduate Studies I have worked with: Professor Ian McCarthy, Professor David Jacho-Chávez, and Professor Sara Markowitz, for their thoughtful guidance and administrative support. I also want to thank Professor Esfandiar Maasoumi, whose suggestions played an important role in shaping the first chapter of this dissertation. I am also deeply grateful to our graduate program coordinator, Renee Sevy-Hasterok, for her consistent and patient assistance throughout my studies.

To my peers and friends, I thank Cheng Ding for his collaboration and companionship, and my roommate Cheng Liu for the many thoughtful conversations and everyday support that helped me stay grounded. Most of all, I thank my family, my girlfriend Yichen Xu, and my friends for their endless love, patience, and belief in me. Their presence has been my greatest source of strength.

To all of you, I offer my heartfelt thanks.

Contents

1 Frictions, Net Worth Shocks, and Heterogeneous Impacts 1

1.1 Introduction 1

1.2 A simple two-agent model 5

1.3 Data: CountyPlus 11

1.4 Estimating the heterogeneous effects 20

1.4.1 Linear LP with independent heterogeneous effects 20

1.4.2 A semi-varying coefficient model 22

1.5 Conclusion 34

2 Downward Nominal Wage Rigidity and Collateral Constraints: A Theory for Post-Great Recession U.S. Economy 36

2.1 Introduction 36

2.2 Model 42

2.3 Computation algorithm 47

2.4 Calibration 51

2.5 Quantitative results 53

2.6 Conclusion 59

Appendix A Chapter 1 61

A.1 Data documentation 62

A.1.1 Measurement error of consumption 62

A.1.2 Supplementary figures 63

A.2 Theory model 64

A.2.1 Model system 64

A.2.2 Proof: Proposition 1.2.1 66

A.2.3 Proof: Proposition 1.2.2 67

A.2.4 A friction-less economy 74

A.2.5 Deterministic steady state 74

A.2.6 Baseline scenario for illustration 75

A.3 Robustness check: Degree selection 77

A.4 Robustness check: Profile likelihood ratio test 79

A.5 Robustness check: Sensitivity analysis 82

A.6 Robustness check: spatial spillover effects 89

A.7 Robustness check: Panel unit root tests 93

Appendix B Chapter 2 **94**

B.1 Numerical algorithm: household problem 94

Bibliography **98**

List of Figures

1.1 Baseline shock effects 9
1.2 Net worth and net worth shocks 15
1.3 Comparing with Flow of Funds 16
1.4 Collateral constraint intensity over time 17
1.5 FWCP over time 19
1.6 Geographical variation of the frictions 19
1.7 Linear LP with linear independent heterogeneous effects 23
1.8 Results of the varying coefficient model 26
1.9 Comparing the ATE of linear LP and varying coefficient LP 27
1.10 MPC estimates 30
1.11 Impulse response by friction constraint 31
1.12 IRF by income group 32

2.1 Deterministic steady state 52
2.2 Impulse response of the one-shot-deviation shock 54
2.3 Comparative statistics 57

A.1 Friction distribution by income decile 64
A.2 Non-linear heterogeneous effects 76
A.3 Order = 3 77
A.4 Order = 4 78

A.5 Local linear regression 81
A.6 Sensitivity contour plots: point estimate of ATE 85
A.7 Sensitivity contour plots: t-value of ATE 86
A.8 Sensitivity contour plots: point estimate of ATE 87
A.9 Sensitivity contour plots: t-value of ATE 88
A.10 Average Spillover Effects of net worth shocks 90
A.11 Net worth shock effects: inverse distance weighting 91
A.12 Net worth shock effects: 1st neighbor adjacency weighting 92

List of Tables

1.1 Descriptive statistics for regressions 12

1.2 F-test of non-linear heterogeneous effects in the linear LP model . . . 34

2.1 Baseline parameters 53

2.3 Welfare analysis 59

A.1 Baseline parameters 76

A.2 Profile Likelihood Ratio test results 80

A.3 Sensitivity measures 84

A.4 Panel unit root tests 93

List of Algorithms

1 Solving the deterministic steady state 49

2 Solving the approximated equilibrium globally 50

Chapter 1

Frictions, Net Worth Shocks, and Heterogeneous Impacts

1.1 Introduction

The wealth effect is a key mechanism through which economic shocks propagate through the economy, influencing household consumption, unemployment, and other economic outcomes. In the analysis of 2008 Great Recession, Bernanke (2018) emphasized how disruptions in credit supply, including those stemming from declines in housing wealth, significantly impacted economic activity. This perspective continues to shape policy discussions today, with Federal Reserve officials consistently highlighting the importance of housing market dynamics and their potential impacts on the broader economy (Powell, 2022, 2023).

The recovery from the Great Recession revealed significant geographical disparities. Yagan (2019) documents that workers in harder-hit areas faced persistent employment losses even six years after the shock, and Beraja et al. (2019) shows how regional differences in wage flexibility led to varying unemployment responses. These regional patterns extend beyond crisis periods. Guren et al. (2021) reveals substantial geographic differences in housing wealth effects across cities over a 40-year period. These findings suggest that local economic conditions play a crucial role in determining both the initial impact and the persistence of aggregate shocks.

This paper aims to uncover the mechanisms that drive disparate economic outcomes in response to similar aggregate shocks by examining the heterogeneous effects of net worth shocks across U.S. counties. We focus specifically on local collateral constraints, downward nominal wage rigidity (DNWR), and their interactions. A growing body of research indicates that the presence of financial and nominal frictions can amplify the effects of net worth shocks and impede the recovery process. Financial frictions, including collateral constraints and borrowing limits, can constrain households' ability to smooth consumption and limit firms' investment capacity, thereby leading to more severe and persistent downturns (Mian and Sufi, 2014). Similarly, nominal frictions, such as downward nominal wage rigidity, may hinder wage adjustments during recessions, resulting in elevated unemployment and slower recovery (Schmitt-Grohé and Uribe, 2016). While these frictions have been examined in isolation, there is a lack of empirical evidence on their interaction and collective influence on shock propagation and recovery dynamics.

This gap in our understanding has significant policy implications, as the effectiveness of macroeconomic interventions may vary substantially across regions with different financial and labor market characteristics. While the aggregate effects of wealth shocks are well-documented, their manifestation across diverse local economies, each with its own financial and labor market characteristics, remains less understood. Regions may exhibit markedly different dynamics of consumption, unemployment, and housing prices in response to the same aggregate shock, necessitating tailored policy responses. For instance, areas with tighter collateral constraints might require targeted credit easing measures, while regions with higher DNWR might benefit more from active labor market policies.

We fill this gap by presenting new empirical evidence of the persistent and heterogeneous effects of net worth shocks throughout the United States. We have developed a novel county-level dataset, named *CountyPlus*, covering 3,058 US counties spanning the years 2003 to 2019. This granular data allows us to study the impact of shocks at a local scale and examine how the effects vary depending on the severity of collateral constraints and DNWR within each county. We posit that these interactions may lead to non-linear effects, where the impact of shocks varies disproportionately with the intensity of local frictions. To capture the nonlinear and heterogeneous effects of net worth shocks, we employ a semi-varying coefficient model. This approach allows us to uncover significant non-linearities in the impact of net worth shocks, which standard linear models, such as linear local projections (Jordà, 2005), fail to capture. The linear local projections that assume linear heterogeneous effects across counties (Cloyne et al., 2023), thereby underestimate the potential heterogeneity in the effects of net worth shocks arising from the interaction of financial and nominal frictions.

Our empirical analysis reveals several key findings. Firstly, we observe significant heterogeneity in the impact of net worth shocks across counties, with the magnitude of effects varying based on the degree of local financial and nominal frictions. Counties with tighter collateral constraints and greater prevalence of DNWR tend to endure more enduring and pronounced downturns subsequent to adverse net worth shocks. Furthermore, employing a semi-varying coefficient model, we unveil noteworthy non-linearities in these heterogeneous effects, suggesting that the impact of net worth shocks can be disproportionately amplified when both collateral constraints and DNWR are binding.

To rationalize these empirical findings, we develop a tractable two-agent general equilibrium model that integrates collateral constraints and DNWR. In the model, a negative net worth shock induces households to increase precautionary savings and deleverage in response to tightened collateral constraints. This adjustment process is prolonged by the illiquidity of housing wealth, leading to a persistent decline in consumption. In the meanwhile, the drop in aggregate demand caused by deleveraging results in higher unemployment due to DNWR. The model provides a conceptual framework for understanding the slow recovery observed after the Great Recession and the role of financial and nominal frictions in propagating the effects of shocks.

Our paper contributes to several strands of the literature. We add to the growing body of work concerning the impact of balance sheet shocks on economic outcomes, extending the influential studies conducted by Mian et al. (2013) and Mian and Sufi (2014). Recent research in this domain has explored the role of housing wealth during the Great Recession (Kaplan et al., 2020a; Guren et al., 2021; Kaplan et al., 2020b). We complement these studies by providing new evidence on the heterogeneous and persistent effects of net worth shocks across US counties while also emphasizing the role of local financial and nominal frictions in shaping these outcomes. Furthermore, our findings contribute to the literature on the importance of financial and nominal frictions in amplifying economic shocks. In situations where borrowing constraints exist, households with high debt positions exhibit heightened sensitivity to income fluctuations (Baker, 2018), while this effect demonstrates differentiated impacts on consumption during boom and bust years (Guerrieri and Iacoviello, 2017a). On the firm side, financial frictions in the form of borrowing and liquidity constraints also contribute to reduced employment growth (Siemer, 2019). Several recent theoretical studies have explored how collateral constraints lead to a slow deleveraging process following a crisis (Guerrieri et al., 2020c; Berger et al., 2018a) in partial equilibrium models. Concurrently, recent empirical research, such as Jones et al. (2022), has emphasized the role of credit shocks in decelerating recovery due to the gradual household deleveraging process. In the meanwhile, theoretical work has explored the consequences of DNWR for the labor market and the macroeconomy (Schmitt-Grohé and Uribe, 2016, 2017). Empirical works have examined the relationship between

DNWR and unemployment in the US, demonstrating the significant negative impact on the labor market (Fallick et al., 2016; Daly and Hobijn, 2014; Elsby et al., 2016). Additionally, research has shown that DNWR can affect other macroeconomic variables and policy multipliers (Shen and Yang, 2018). We provide empirical evidence on the interaction between these two types of frictions, emphasizing their joint role in generating non-linear and persistent responses to shocks.

The remainder of the paper is structured as follows. In Section 1.2, we present a straightforward theoretical framework that accounts for the slow recovery following the Great Recession, employing collateral constraint and DNWR. Section 1.3 outlines the construction of the CountyPlus dataset, while Section 1.4 provides the estimated heterogeneous impact of net worth shocks under financial and nominal frictions.

1.2 A simple two-agent model

This section proposes an analytical two-agent general equilibrium model, following the approach outlined by Bocola and Lorenzoni (2020a), to illustrate our mechanism. The model is in discrete time with an infinite horizon starting from $t = 0$. Uncertainty is encapsulated by a Markov chain s_t, with transition probabilities denoted by $\pi(s_{t+1}|s_t)$. Let $s^t := (s_t, s_{t-1}, \dots)$ denote the history. The economy is populated by two types of homogeneous agents, entrepreneurs (borrowers) and households (lenders). Households supply one unit of labor inelastically and receive wage income. They enjoy non-durable goods consumption and housing services, denoted as c^l and h^l respectively. Furthermore, households can allocate their resources into two types of assets: liquid savings a_t and liquid housing wealth h^l. Equation (1.1) outlines the household's lifetime problem, where $q(s^{t+1}|s^t)$ represents the price of liquid bonds, $p(s^t)$ denotes house prices, $w(s^t)$ signifies wage rates, and $l(s^t)$ indicates labor supply.

$$V^l(s^t) = \max_{c^l(s^t),\{a(s^{t+1})\},h^l(s^t)} c^l(s^t) + \gamma \log h^l(s^t) + \beta \mathbb{E} V^l(s^{t+1}) \tag{1.1}$$

$$c^l + \sum_{s^{t+1}} q(s^{t+1}|s^t)a(s^{t+1}) + p(s^t)[h^l(s^t) - h^l(s^{t-1})] = w(s^t)l(s^t) + a(s^t)$$

The entrepreneurs function as producers of consumption goods, utilizing their housing wealth and hiring labor from households. They employ technology, as represented in Equation (1.2), to produce consumption goods and pay households with wages. The market for consumption goods is perfectly competitive, , with producers earning zero profits. However, the labor market encounters friction due to downward nominal wage rigidity (DNWR). DNWR is modeled, as shown in Equation (1.3), by imposing a lower bound δ on the growth rate of nominal wages, following the formulation in Schmitt-Grohé and Uribe (2016). The slackness condition outlined in Equation (1.4) signifies the presence of involuntary unemployment when the DNWR constraint becomes binding.

$$Y(s^t) = Al^{\alpha}(s^t)[u(s^t)h(s^t)]^{1-\alpha} \tag{1.2}$$

$$w(s^t) \geq \delta w(s^{t-1}) \tag{1.3}$$

$$(1 - l(s^t))(w(s^t) - \delta w(s^{t-1})) = 0 \tag{1.4}$$

Entrepreneurs have the option to obtain a loan $b(s^{t+1})$ to finance their house purchase. However, they encounter friction in the financial market, leading to state-contingent collateral constraints governing the extent of debt they can secure. Specifically, the amount of raised debt cannot exceed θ fraction of the purchased housing value[1].

[1]WLOG, an alternative model equivalently assumes experts face a "lottery" for borrowing adjustments, winning with probability $\lambda(s^{t+1})$. The empirical measure of collateral constraints in later sections aligns conceptually with this λ.

Equation (1.5) gives the formulation of the entrepreneur's lifetime problem, wherein $u(s^t)$ denotes the housing productivity shock[2], the sole source of uncertainty in this economic framework, and $n(s^t)$ represents net worth. Further details regarding the model and proofs are presented in Appendix A.2.

$$V^b(s^t) = \max_{c(s^t),\{b(s^{t+1})\},h(s^t),l(s^t)} \log c(s^t) + \beta \mathbb{E} V^b(s^{t+1}) \tag{1.5}$$

$$c(s^t) + p(s^t)h(s^t) = n(s^t) + \sum_{s^{t+1}} q(s^{t+1}|s^t)b(s^{t+1})$$

$$n(s^t) := p(s^t)h(s^{t-1}) + y(s^t) - b(s^t)$$

$$y(s^t) := Y(s^t) - w(s^t)l(s^t)$$

$$b(s^{t+1}) \leq \theta p(s^{t+1})h(s^t), \forall s^{t+1}$$

We adopt a general equilibrium in our analysis. This framework enables us to explore the interaction between nominal and financial frictions. In equilibrium, the liquid bond market must clear, ensuring that assets equal liabilities, expressed as $a(s^t) = b(s^t)$, given bond prices $\{q(s^{t+1}|s^t)\}$. In the housing market, with a constant supply denoted as H, equilibrium is achieved when the sum of housing and liquid housing wealth equals the total supply, represented by $h(s^t) + h^l(s^t) = H$, at the equilibrium price level $p(s^t)$. The labor market, however, does not necessarily clear due to DNWR. Nonetheless, when it does, labor supply equals 1, denoted as $l(s^t) = 1$, and entrepreneurs remunerate households at competitive wage rates. We examine a specific case of one-shot deviation in our economy, where all $u_t \equiv 1$ except for u_1. The housing productivity shock u_1 is drawn from a distribution over the support $(0, \bar{u}]$. Consequently, the realized economy follows a deterministic trajectory from $t = 2$. In a frictionless version of this model, the shock u_1 would exert no influence

[2]This shock can be translated into a shock to household wealth, as its impact on household net worth can be analytically determined within this model.

on consumption and unemployment, and the economy would remain on an unbinding equilibrium trajectory.

Proposition 1.2.1 (Persistent effect). *There exist a unique continuation equilibrium that depends on the states $(u_1, h_0, b_1(u_1))$. In the continuation equilibrium, the collateral constraint is binding for a finite number of periods J, with $J = 0$ iff $n_1(u_1) \geq \bar{n}_1 := \bar{p}\bar{h}(1-\beta\theta)/\beta$, where $\bar{p}$ and $\bar{h}$ are jointly determined by*

$$(1-\beta)\bar{p} = \frac{\gamma}{H - \bar{h}} \tag{1.6}$$

$$\bar{p}(1-\beta) = \beta(1-\alpha)\bar{h}^{-\alpha} \tag{1.7}$$

Proposition 1.2.2 (Heterogeneous effect). *There exist levels of the entrepreneur's financial friction parameter θ and the DNWR parameter δ, such that if*

$$\frac{w_0}{n_0} \geq \frac{\alpha}{\delta(1-\alpha)}\left[1 + (1-\theta)(1-\alpha)(\frac{p_0 h_0^l}{\gamma} - 1)\right] \tag{1.8}$$

then, in equilibrium, the date 1 collateral constraint and DNWR both bind when u_1 is in a non-empty interval $[\hat{u}_0, u_0^*]$ where

$$u_0^* = \left(\frac{\delta w_0}{\alpha A}\right)^{\frac{1}{1-\alpha}} h_0^{-1} \tag{1.9}$$

$$\hat{u}_0 = \frac{n_0}{A(1-\alpha)h_0}\left(\frac{\delta w_0}{\alpha A}\right)^{\frac{\alpha}{1-\alpha}}\left[1 + (1-\theta)(1-\alpha)\left(\frac{p_0 h_0^l}{\gamma} - 1\right)\right] \tag{1.10}$$

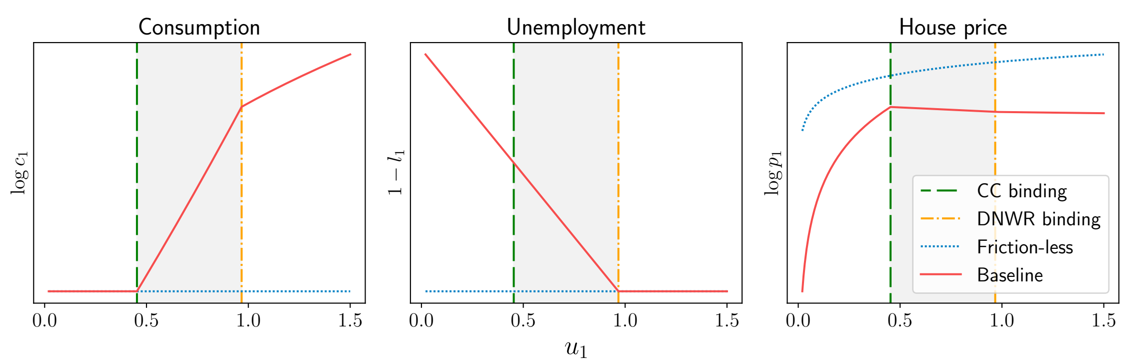
, and the u_1 shock effects:

$$\frac{\partial c_1}{\partial u_1} > 0,\ \frac{\partial l_1}{\partial u_1} > 0,\ \frac{\partial p_1}{\partial u_1} > 0 \text{ or } < 0 \text{ or } = 0 \text{ depends} \tag{1.11}$$

which are also non-linear functions of θ and δ.

Proposition 1.2.1 provides the formal statement regarding the potential persistent effect of the one-deviation shock u_1[3]. In a frictionless economy, state-contingent claims enable borrowers to fully insure their consumption against any magnitude of the one-deviation shock u_1, which is a standard conclusion in complete market models. However, the presence of collateral constraints impedes borrowers from immediately adjusting to maintain the ideal net worth level. Consequently, when a significant net worth loss occurs due to the shock u_1, borrowers are unable to borrow sufficiently to promptly revert to an unbinding equilibrium trajectory but must instead wait for J periods. As a result, they endure an extended period of consumption cut. This slow recovery process is influenced by both economic frictions. Notably, the threshold of net worth required for instant adjustment decreases with the collateral constraint parameter θ, implying that even minor negative shocks can induce persistent effects in regions characterized by stringent collateral constraints. Additionally, the magnitude of net worth loss is depending on the DNWR parameter δ, indirectly contributing to the persistence of effects.

Figure 1.1: Baseline shock effects



The financial and nominal frictions not only divert the economy from an unbinding equilibrium path over time, but also generate heterogeneous effects regarding the size of the u_1 shock. Consider the scenario at $t = 1$. Let $f_1 := f_1(u_1)$ represent a period-1 economic outcome (e.g., consumption, unemployment, and house price). Quantita-

[3]See Appendix A.2.2 for the proposition proof.

tively, $f_1'(u_1)$ indicates the impact of the u_1 shock. If $f_1'(u_1)$ exhibits dependence on θ or δ, then we denote the presence of heterogeneous effect(s) of the financial or nominal frictions. Moreover, we refer to such heterogeneous effects as *non-linear* if $f_1'(u_1)$ varies for equal-size deviations in θ or δ from the baseline level. Proposition 1.2.2 addresses these non-linear heterogeneous effects. Under specific initial conditions, the effects of the u_1 shock on period-1 consumption, unemployment, and house price are highly non-linear functions of θ and δ. Depending on varying initial conditions and whether the collateral constraint and DNWR constraint bind, the effects of the u_1 shock at the same u_1 level differ. Figure 1.1 offers an illustrative depiction of the u_1 shock effects, contrasting the baseline economy satisfying Equation (1.8) with a friction-less version of the model introduced in Appendix A.2.4. To further demonstrate the heterogeneity and non-linearity of these effects concerning the frictions, consider the point $u_1 = \hat{u}_0$ as an example. At this shock level $\hat{u}_0$, the collateral constraint exactly binds while $c_1'(u_1)$ equals zero under the baseline θ level. Such a shock effect on consumption would be positive for a lower θ, indicating a tighter collateral constraint. However, it would remain zero for larger θ. This discrepancy in the u_1 shock effect across different levels of the financial friction underscores its heterogeneous nature, with this heterogeneity being non-linear relative to the baseline level of θ. Furthermore, the threshold level $\hat{u}_0$ is also depending on the nominal friction parameter δ, indicating complex interaction effects between the financial friction and nominal friction. A more direct illustration of such non-linear heterogeneous effects is depicted in Figure A.2, which showcases the results of simple sensitivity analysis concerning the θ and δ parameters. Expanding on this, the interval of amplification effects of the u_1 shock in Figure 1.1 may vanish if the initial condition of Equation (1.8) is not met. In such cases, the functional shapes of the u_1 shock effects would further alter. In Section A.2.3 of Appendix A.2, we engage in further theoretical discussions concerning the global properties of our model.

So far, we have presented a basic two-agent model to explore the dynamics of financial and nominal frictions and their interactive impact on the economy, echoing the large heterogeneous economic fluctuations and sluggish recovery observed during and post the Great Recession. Certainly, our model is oversimplified, lacking several quantitatively significant features such as incomplete markets, realistic balance sheet structures, long-term mortgages, and illiquid housing wealth. The frictions are reduced to two parameters for analytical tractability, whereas recent research has highlighted a more intricate relationship between unemployment and financial frictions (Siemer, 2019). Nonetheless, our simplified model unveils complex non-linear heterogeneous effects of shocks to net worth. In the following sections, we quantitatively estimate these effects using a novel dataset, employing an empirical strategy adapted to accommodate such non-linear heterogeneity.

1.3 Data: CountyPlus

We construct a county-level panel dataset (CountyPlus) spanning the period from 2003 to 2019, covering about 3,058 counties in the United States[4]. This dataset comprises a broad collection of variables relating to household balance sheets, local economies, and demographics. Its high replicability stems from the fact that all data sources are publicly available and accessible free of charge. In this section, we outline the definition of core variables and detail the methodology employed for dataset construction[5]. Table 1.1 presents the descriptive statistics of the principal variables in the sub-sample used in the analysis of this paper.

We adhere to the methodology outlined in Mian et al. (2013) for constructing household asset and liability variables. The primary variable under consideration in our study is household net worth. We define household net worth for county i in year

[4]Github repository: github.com/Clpr/CountyPlus

[5]See Appendix A.1 for a comprehensive list of available variables.

Table 1.1: Descriptive statistics for regressions

Variable	**N**	**25% qt**	**Median**	**Mean**	**Std. Dev**	**75% qt**
Logarithm real net worth per capita, log million $	25974	-9.18	-8.59	-8.69	1.00	-8.11
Logarithm real consumption per capita, log $	27640	4.36	4.73	4.71	0.58	5.08
Unemployment rate, pct	27751	4.10	5.50	6.10	2.68	7.60
Logarithm real house price index	27751	-0.30	-0.02	0.05	0.49	0.32
Fraction of Wage Cuts Prevented (FWCP), pct	25009	-31.46	-11.03	-26.86	57.65	-1.29
Share of mortgage loan denials due to lack of collateral, pct	27744	10.17	13.64	14.87	6.72	18.56
Logarithm housing wealth share to net worth	25974	0.54	0.78	0.88	2.19	1.17
Share of tradable sector employment, pct	27751	0.00	1.03	4.48	8.00	5.80
Share of construction sector employment, pct	27751	8.25	11.68	13.45	9.95	16.47
Logarithm housing units per capita	27751	-0.88	-0.79	-0.78	0.38	-0.67

Notes: 1. The year spans from 2013 to 2019. 2. Because of the availability of consumption data, only 1700 counties are reported in this table which is consistent with the baseline regressions. 3. Real variables are deflated by CPI All Urban Consumers (1982-1984=100)

t as $NW_{it} = S_{it} + B_{it} + H_{it} - D_{it}$, where S_{it} represents equity asset holdings, B_{it} denotes fixed-income holdings, H_{it} signifies housing asset holdings, and D_{it} indicates the debt position. Our estimation of equity and fixed-income holdings works under the assumption that households hold the market portfolio. This assumption implies that the income share from these asset classes aligns with the proportion of asset holdings. To determine the income derived from equity and fixed-income holdings, we utilize data on ordinary dividends and taxable interest sourced from the Survey of Income (SOI)[6] published by the Internal Revenue Service (IRS). This approach enables us to allocate national holdings for households and non-government organizations across individual counties. The aggregate data used for this purpose is obtained from the Federal Reserve's Flow of Funds, which is regularly updated on a quarterly basis[7]. For each year, we derive the average holding value. Specifically, equity holdings consist of directly held and indirectly held equity, while fixed-income holdings encompass both directly held and indirectly held debt securities[8]. Equation (1.12) and (1.13) outline the formulas employed to estimate $S_{i,t}$ and $B_{i,t}$.

[6] IRS: Survey of Income - County Data
[7] Fed Flow of Funds: Balance Sheet of Households and Nonprofit Organizations, 1952 - 2022
[8] In this way, the holding of mutual funds is split into the two asset categories.

$$S_{i,t} = \frac{\text{County dividend income}_{i,t}}{\sum_j \text{County dividend income}_{j,t}} \times \text{National total equity of household}_t \quad (1.12)$$

$$B_{i,t} = \frac{\text{County interest income}_{i,t}}{\sum_j \text{County interest income}_{j,t}} \times \text{National total debt security of household}_t \quad (1.13)$$

To derive the aggregate household debt position for county i in year t, we multiply the debt-to-income ratio with the adjusted gross income (AGI) sourced from the SOI. The county-level debt-to-income ratio is accessible through the Enhanced Financial Accounts of the Fed's Flow of Funds[9]. The formula to calculate debt positions is outlined in Equation (1.14).

$$D_{i,t} = \text{Household debt-to-income ratio}_{i,t} \times \text{AGI}_{i,t} \quad (1.14)$$

Estimating household housing wealth holdings involves employing more intricate techniques. In Mian et al. (2013), the assumption was made that changes in housing wealth value were solely attributed to fluctuations in housing prices, utilizing the 2006 housing wealth exposure and the Core Logic house price index. However, due to improved data availability today, we are now able to relax this assumption. Equation (1.15) defines the formula for estimating household housing wealth for each year. Specifically, the exposure is estimated by considering the total number of housing units estimated by the Census Bureau[10] and a parameter representing the average number of housing units per house. As illustrated in Figure 1.3, this parameter is set to 1.8 to align with the trajectory of household balance sheet structure. County-level housing prices are projected using the House Price Index (HPI)[11] published by the Federal Housing Finance Agency (FHFA), alongside the cross-sectional distribution

[9] Fed Flow of Funds: Enhanced Financial Accounts, Household Debt by State, County, and MSA
[10] Census Bureau: Population and Housing Unit Estimates Datasets
[11] Federal Housing Finance Agency: House Price Index Datasets

of median housing values in 2019 estimated from Ameican Community Survey (ACS) data.

$$H_{i,t} = \frac{\text{Total housing units}_{i,t}}{\text{Average housing units per house}} \times \text{Median house value}_{i,2019} \times \frac{\text{HPI}_{i,t}}{\text{HPI}_{i,2019}} \tag{1.15}$$

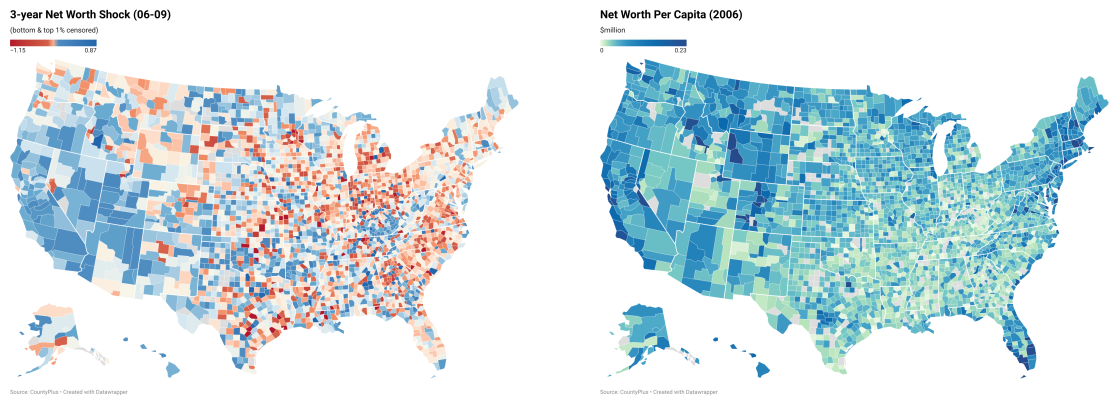
The net worth shock is identified as a Bartik-style shock[12] in Equation (1.16) which alleviates the endogeneity concern. Here, $s^j_{i,t-1}$ represents the one-year lagged exposure of asset type j, while $g^j_{t-1,t}$ means the one-year growth rate of the asset price. To illustrate, the net worth shock from 2008 to 2009 is formulated by interacting the 2008 asset exposures with the 2008-2009 growth rate of aggregate price indices. Such a constructed shock is considered exogenous to the county's outcome variables and other predetermined factors. Specifically, we utilize the NASDAQ composite index as the price index for equity holdings, the ICE BoA Corporate Bond Index for fixed income holdings, and the logarithmic growth of state leave-one-out average House Price Index (HPI) for housing asset holdings. Figure 1.2 displays the large geographical variation of both net worth and net worth shocks across the U.S.

$$x_{i,t} := \sum_{j \in \{S,B,H\}} s^j_{i,t-1} g^j_{t-1,t} \tag{1.16}$$

To estimate county consumption, we adopt a methodology akin to Zhou and Carroll (2012) and Case et al. (2011), leveraging sales tax data. Sales tax in the United States varies across states, principally applying to tangible consumption and some services for final consumers. Equation (1.17) outlines the formula for estimating county consumption within state s. This method distributes state aggregate consumption, obtained from personal consumption expenditure (PCE) data by the Bureau of Eco-

[12] Appendix A.7 performs panel unit root tests for the identified shock.

Figure 1.2: Net worth and net worth shocks



(a) Net worth shock during the Great Recession

(b) Net worth before the Great Recession

nomic Analysis (BEA) and population estimates by the Census Bureau, to counties within the state. The distribution share is determined by the taxable sales in each county[13]. We collect tax data from local departments of revenue in 27 states, encompassing 1700 counties[14].

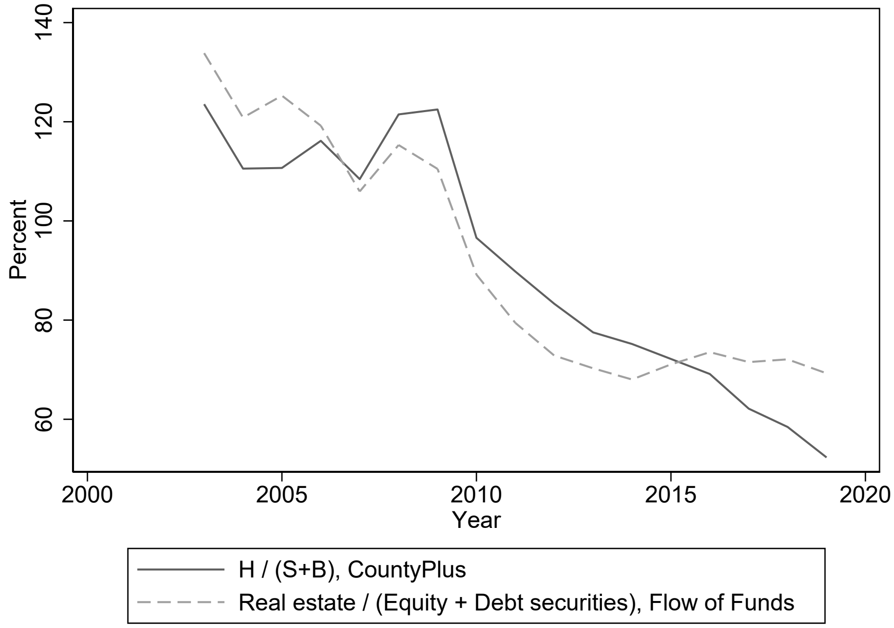
$$C_{i,t} = \text{PCE}_{s,t} \times \text{Population}_{s,t} \times \frac{\text{Taxable sales}_{i,t}}{\sum_{j \in s} \text{Taxable sales}_{j,t}} \tag{1.17}$$

To measure the magnitude of financial friction, particularly in the form of collateral constraints, we construct an indicator termed as the proportion of mortgage loan denials attributable to insufficient collateral in relation to total denial cases ($DENI$) in each county. The data for this indicator is sourced from the public version of the Home Mortgage Disclosure Act (HMDA) transaction data. Figure 1.4 illustrates the overall pattern of financial friction intensity over time. Notably, the share of collateral constraints experienced a substantial surge during the Great Recession, followed by a

[13]Some states directly report gross/taxable sales (consumption), while some states report tax revenue only. For those states only reporting tax revenue, we adjust the distribution with tax rates to correct the measurement error. In our regression analysis, state fixed effects are also added to further control the potential measurement errors. See Appendix A.1.1 for details.

[14]See Appendix A.1 for a list of available states and discussions about important institutional details.

Figure 1.3: Comparing with Flow of Funds

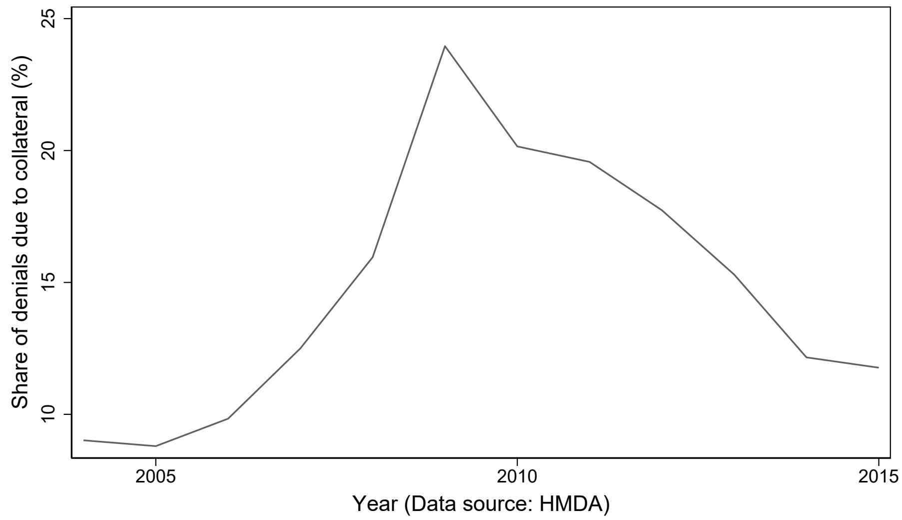


protracted period to revert to pre-crisis levels. Meanwhile, Figure 1.6a reveals there is also a large geographical variation of the financial friction across the county, which allows us to empirical examine the heterogeneity of shock propagation.

Local labor markets are investigated through wage and employment data by industry, classified into tradable, non-tradable, and construction sectors at the 4-digit NAICS level, utilizing the County Business Pattern (CBP) dataset and following the methodology of Mian and Sufi (2014). Hourly wages are estimated using an equation where total salaries and wages are sourced from the Statistics of Income (SOI), and state-level weekly working hours are obtained from the Bureau of Labor Statistics' Current Employment Statistics (CES)[15]. Furthermore, the Herfindahl-Hirschman Index (HHI) is computed for each industry annually, defined as the sum of the squared employment share percentages across counties.

[15]Bureau of Labor Statistics: Current Employment Statistics

Figure 1.4: Collateral constraint intensity over time



$$W_{i,t} = \frac{\text{Total salaries and wages}_{i,t}}{\text{Weekly working hours}_{s,t} \times 52} \tag{1.18}$$

To measure DNWR, we adopt the approach of Holden and Wulfsberg (2009). We assess DNWR in counties by contrasting the actual nominal wage growth distribution with a rigidity-free notional distribution, using data from county-industry pairs by assuming average employees. The nominal wage growth for industry j in county i during year t is represented by $\Delta w_{j,i,t}$, while $G_{i,t}$ means the assumed notional distribution of nominal wage growth for county i in year t. We parameterize $G_{i,t}$ using a location parameter and a dispersion parameter, $(\mu_{i,t}, \sigma_{i,t})$, with $\mu_{i,t}$ indicating the median nominal wage growth for the county, and $\sigma_{i,t}$ representing the span between the 35th and 75th quantiles, $q_{i,t}^{35}$ and $q_{i,t}^{75}$, respectively. This model allows variations in $G_{i,t}$ across different counties and years. To construct $G_{i,t} = G(\mu_{i,t}, \sigma_{i,t})$, we first estimate an empirical normalized distribution $G(0, 1)$, then re-scale it to $G(\mu_{i,t}, \sigma_{i,t})$, assuming uniformity in the shape of notional distributions across counties. Specifically, we take a sub-sample from all counties with upper 25% wage growth in a given year to estimate a normalized empirical distribution $G(0,1)$. These upper-quarter observations are considered unbinding to downward nominal wage rigidity such that Equation (1.19) can be seen as realizations from $G(0,1)$.

$$x_{j,m,t} = \frac{\Delta w_{j,m,t} - \mu_{i,t}}{q^{75}_{m,t} - q^{35}_{m,t}}, m = 1, \ldots, N_t^{\text{top25\%}}, x_{j,m,t} \sim G(0,1) \tag{1.19}$$

With the empirical distribution $G(0,1)$ estimated from $\{x_{j,m,t}\}$, we construct a rigidity-free notional distribution $G_{i,t}$ for all counties using Equation (1.20), where $\mu_{i,t}$ represents the median nominal wage growth in county i, and X is a random variable that follows $G(0,1)$. The realizations of the random variable $Z_{i,t}$ then define an empirical notional distribution $G_{i,t}$ for county i in year t.

$$Z_{i,t} = X(q^{75}_{i,t} - q^{35}_{i,t}) + \mu_{i,t}, X \sim G(0,1) \tag{1.20}$$

In Holden and Wulfsberg (2009), the fraction of wage cuts prevented (FWCP) is used in Equation (1.21) as a measure of the extent of DNWR, where π_t represents the inflation rate, $N_{i,t}$ denotes the number of observations in county i, $\tilde{p}_{i,t}$ is the notional incidence rate of a nominal wage cut as defined in Equation (1.22), and $p_{i,t}$ means the actual incidence rate of nominal wage cuts in $G_{i,t}$ as outlined in Equation (1.23). Figure 1.5 illustrates the proportion of counties with a positive FWCP each year. A higher proportion reflects a more pronounced DNWR. It is evident that DNWR was more restrictive during the Great Recession and then decreased to pre-crisis levels. In the meantime, Figure 1.6b displays the large degree of geographical variation of the DNWR.

Figure 1.5: FWCP over time

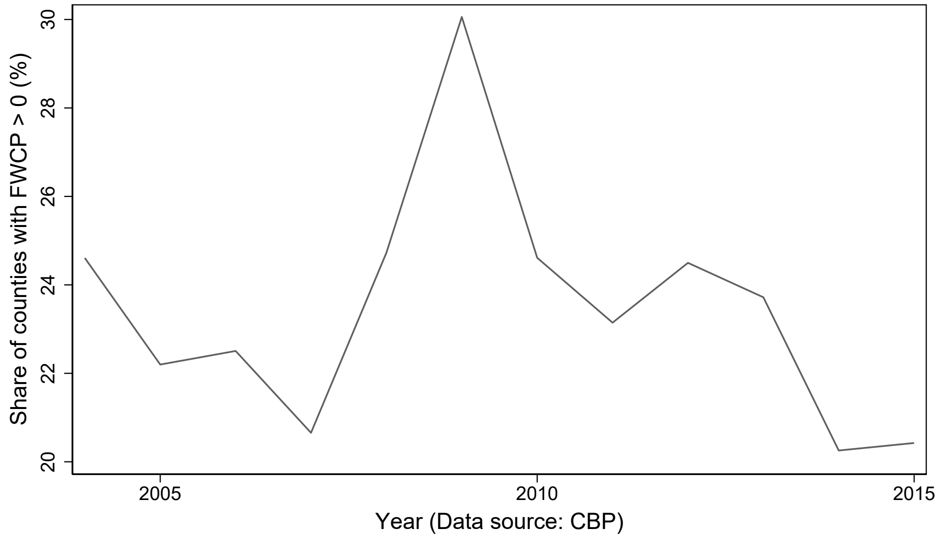
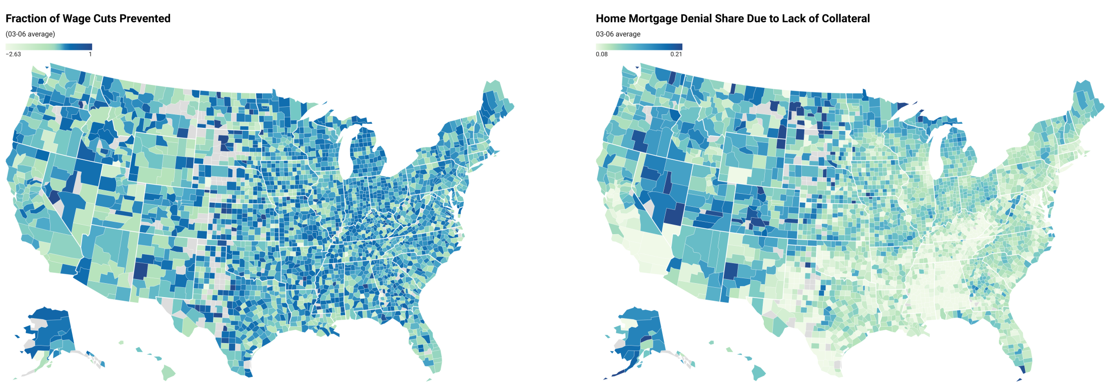


Figure 1.6: Geographical variation of the frictions



(a) FWCP (b) DENI

$$\mathrm{FWCP}_{i,t} = 1 - p_{i,t}/\tilde{p}_{i,t} \tag{1.21}$$

$$\tilde{p}_{i,t} := \frac{\#\{Z_{i,t} < 0\}}{N_t^{\text{top } 25\%}} \tag{1.22}$$

$$p_{i,t} := \frac{\#\{\Delta w_{j,i,t} < 0\}}{N_{i,t}} \tag{1.23}$$

1.4 Estimating the heterogeneous effects

This section outlines the empirical framework for analyzing the persistent and non-linear heterogeneity in the effects of net worth shocks. Initially, we discuss the estimation results from linear local projections that allow for linear heterogeneous effects (Cloyne et al., 2023). These results reveal the presence of heterogeneous effects due to financial (collateral constraint) and nominal (DNWR) frictions. Then, we introduce our baseline model: a semi-varying coefficient model (Fan and Huang, 2005) which is a non-linear extension of Jorda's local projections. This model helps the estimation of complex, heterogeneous shock effects across counties, influenced concurrently by the two economic frictions. It accounts for potential non-linear heterogeneous relationships in the shock's effects and the interaction between economic frictions. Furthermore, we conduct F-tests to assess the semi-varying coefficient model against the linear LP specification.

1.4.1 Linear LP with independent heterogeneous effects

Jorda's local projection method is a widely used approach in empirical macroeconomics to estimate the potential persistent effects of identified shocks. The LP method is favored for its robustness of the average treatment effects estimation against potential misspecifications. A recent study by Cloyne et al. (2023) has expanded this framework to include linear heterogeneous effects. In the context of unanticipated continuous shocks, this parametric model facilitates analysis by extending the average marginal effects to encompass heterogeneous effects. These effects are assumed to be linear and independent of each other. The causality argument is further reinforced by the Kitagawa-Oaxaca-Blinder (KOB) decomposition. In our theoretical model, as discussed in Section 1.2, we investigate not only the linear heterogeneous effects of DNWR and collateral constraints but also identify potential non-linearity and complex interaction between these two economic frictions. To preliminarily assess the presence of non-linearity and friction interaction, we estimate Equation (1.24) following Cloyne et al. (2023).

$$y_{i,t+h} = \alpha_h + x_{i,t}\beta_h + x_{i,t}\Delta\mathbf{Z}'_{i,t}\begin{bmatrix}\gamma_h^{fwcp} \\ \gamma_h^{deni}\end{bmatrix} + \Delta\mathbf{Z}'_{i,t}\boldsymbol{\delta}_h \tag{1.24}$$

$$+ g(N_{i,t-1}) + \mathbf{W}_{i,t}\lambda_h + \iota_{i\in s} + \nu_t + \varepsilon_{i,t+h}, h = 0, \ldots, H \tag{1.25}$$

$$\Delta\mathbf{Z}_{i,t} := (\Delta z_{i,t}^{fwcp}, \Delta z_{i,t}^{deni})' = \begin{pmatrix}\text{fwcp}_{i,t} - \overline{\text{fwcp}}_i & \text{deni}_{i,t} - \overline{\text{deni}}_i\end{pmatrix}' \tag{1.26}$$

In the model equation, β_h represents the average treatment effect (ATE) of the net worth shock $x_{i,t}$, and $\Delta\mathbf{Z}_{i,t}$ represents the deviation of *FWCP* and *DENI* from their mean levels. The coefficients γ_h^{fwcp} and γ_h^{deni} reflect the heterogeneous effects, while $\boldsymbol{\delta}_h$ accounts for the composition effects of heterogeneity deviations. The function $g(N_{i,t-1})$ controls predetermined economic conditions using logarithm lagged household net worth, and the vector $\mathbf{W}_{i,t}$ includes the remaining control variables. Fixed effects for state s and year t are denoted by $\iota_{i\in s}$ and ν_t respectively. Following Mian et al. (2013); Mian and Sufi (2014), the shock to household net worth is identified as a Bartik-style net worth shock, as delineated in Equation (1.16), ensuring that $x_{i,t}$ is exogenous to county i by using lagged exposure and aggregate price growth [16]. When the causality identification assumptions outlined in Cloyne et al. (2023) hold true, analyzing the coefficients γ_h^{fwcp} and γ_h^{deni} allows for the exploration of potential heterogeneous effects arising from the nominal and financial frictions.

With exogenous and pre-determined covariates, the OLS estimator is consistent. Figure 1.7 illustrates the yearly projection horizon against the model coefficients, including the impulse response function (IRF) of the shock. The IRF estimates

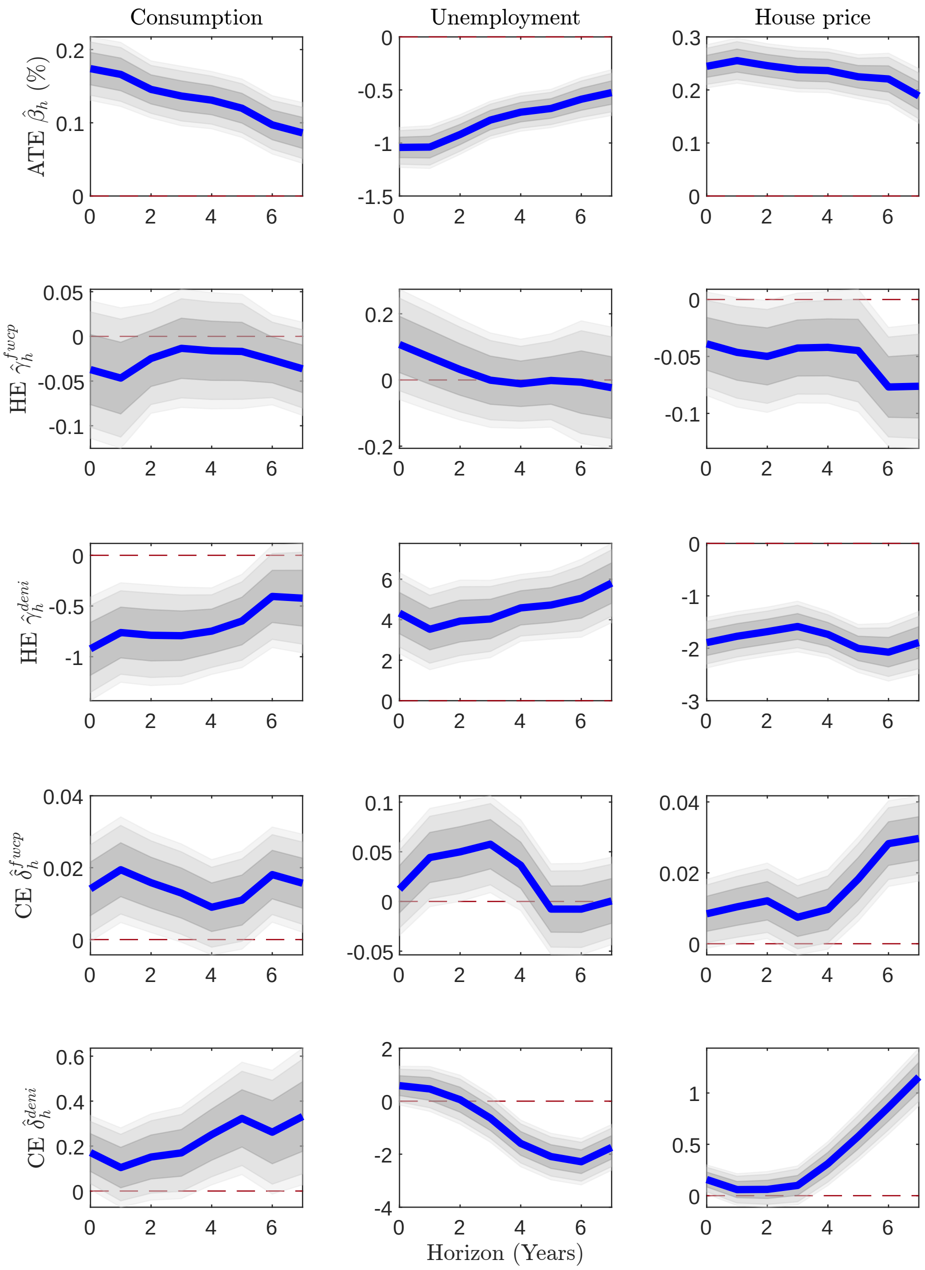
[16] Appendix A.7 performs panel unit root tests which shows that there is no common trend between outcomes variables and the shock.

reveal that net worth shocks have statistically significant and persistent impacts on key economic indicators: logarithm real consumption per capita, unemployment rate, and logarithm real house price index, as shown in the first row of the figure matrix. This finding supports the intuition that positive shocks increase consumption and house prices while decreasing unemployment, and the opposite for negative shocks, consistent with findings from theoretical studies (Berger et al., 2018a; Guerrieri et al., 2020c). The last two rows of Figure 1.7 present the estimated coefficients of the composition effects of $FWCP$ and $DENI$. They clearly indicate that deviations of these two types of frictions from their average levels can directly affect outcomes. A deeper analysis into the second and third rows reveals the heterogeneous effects of DNWR and collateral constraints on these economic outcomes. Regions with more severe financial frictions experience less growth in consumption and house prices following a positive net worth shock compared to regions with fewer frictions. This detail is in line with theoretical expectations and emphasizes the varying impact of financial frictions on local economies. Moreover, the inference on $\hat{\gamma}_h^{fwcp}$ suggests significant heterogeneous effects on house prices, with counties facing a higher degree of DNWR showing a smaller house price response to net worth shocks. However, the results of linear LP largely underestimate the heterogeneous effect of DNWR. The next section, which is our baseline model, reveals the necessity of modeling the non-linear heterogeneous effects.

1.4.2 A semi-varying coefficient model

The parametric KBO extension terms in Equation (1.24) offer an intuitive inference of the heterogeneous effects of financial and nominal frictions. However, the model's assumption of linear and independent heterogeneous effect terms precludes the possibility of heterogeneous effects arising from interactions between frictions, as well as potential non-linear heterogeneous effects. Our theoretical model in Section 1.2

Figure 1.7: Linear LP with linear independent heterogeneous effects



Notes: 1. The control variable $\mathbf{W}_{i,t}$ includes total housing units, share of housing wealth in household net worth, share of tradable sector employment in total employment, and share of construction sector employment in total employment. 2. The non-parametric control $g(N_{i,t-1})$ is approximated by a cubic polynomial. 3. The regression is weighted by total population of each county, and the standard errors are clustered at state level. 4. The sample size used in each horizon's estimation varies from 12208 to 22784 where about 1700 counties with constructed consumption data are included, and the time spans from 2004 to 2019. 5. State and year fixed effects are controlled. 6. The scales of $x_{i,t}$, $\Delta z_{i,t}^{fwcp}$ and $\Delta z_{i,t}^{deni}$ are in digits. 7. "HE" means heterogeneous effect, "IE" means the heterogeneous effect of friction interaction. 8. The blue solid line is the value of coefficient point estimates, the darkest gray shadow band is the one-sigma confidence interval, the middle gray shadow band is the 90% confidence interval, and the light gray shadow band is the 95% confidence interval.

indicates interactions between the two types of frictions, as well as a certain degree of non-linearity in these heterogeneous effects. Consequently, it is beneficial to investigate the interactions of frictions by further relaxing the assumptions of usual local projections. The semi-varying coefficient model discussed by Fan and Huang (2005), Zhang et al. (2002) and Li et al. (2002), provides a robust and flexible method to accommodate potential higher-order and non-linear effects of shocks. In this section, we estimate non-linear local projections using a non-parametric coefficient $\beta_h(\Delta\mathbf{Z}_{i,t})$:

$$y_{i,t+h} = \alpha_h + x_{i,t} \cdot \beta_h(\Delta\mathbf{Z}_{i,t}) + \Delta\mathbf{Z}'_{i,t}\boldsymbol{\delta}_h + g(N_{i,t-1}) + \mathbf{W}_{i,t}\lambda_h + \iota_{i\in s} + \nu_t + \varepsilon_{i,t+h} \tag{1.27}$$

$$\tag{1.28}$$

where $\beta_h(\Delta\mathbf{Z}_{i,t})$ is the coefficient of shock effect which depends on county-specific heterogeneities $\Delta\mathbf{Z}_{i,t}$; $\boldsymbol{\delta}_h$ is the composition effects of friction levels; $g(N_{i,t-1})$ is a functional control of pre-determined economic conditions; $\mathbf{W}_{i,t}$ is a vector of the left control variables; $\iota_{i\in s}$ is the state fixed effect; and ν_t is the year fixed effect. This model is a natural extension of the linear local projections with heterogeneous effects (Gourieroux and Lee, 2023; Cloyne et al., 2023). However, the causal interpretation of $\beta_h(\mathbf{Z}_{i,t})$ requires some key assumptions given our semi-parametric context. By assuming the true coefficient $\beta_h(\mathbf{Z}_{i,t})$ is right continuous (Potter, 2000), we define the following marginal response function for the infinitesimal shock:

$$\mathrm{MR}_h(\Delta\mathbf{Z}_{i,t}) = \lim_{\delta\to 0^+} \frac{1}{\delta}\mathbb{E}_x\Big\{y_{i,t+h}(x_{i,t}+\delta, \Delta\mathbf{Z}_{i,t}|N_{i,t-1}, \mathbf{W}_{i,t}) \tag{1.29}$$

$$-y_{i,t+h}(x_{i,t}, \Delta\mathbf{Z}_{i,t}|N_{i,t-1}, \mathbf{W}_{i,t})\Big\} \tag{1.30}$$

which is a function of the friction heterogeneities $\Delta\mathbf{Z}_{i,t}$. With the non-anticipated and randomly assigned shock assumptions, the marginal response function then has causality interpretations (Rambachan and Shephard, 2019a; Gonçalves et al., 2024; Bojinov and Shephard, 2019; Rambachan and Shephard, 2019b). In this case, the coefficient $\beta_h(\Delta\mathbf{Z}_{i,t})$ is the conditional treatment effect of the shock $x_{i,t}$.

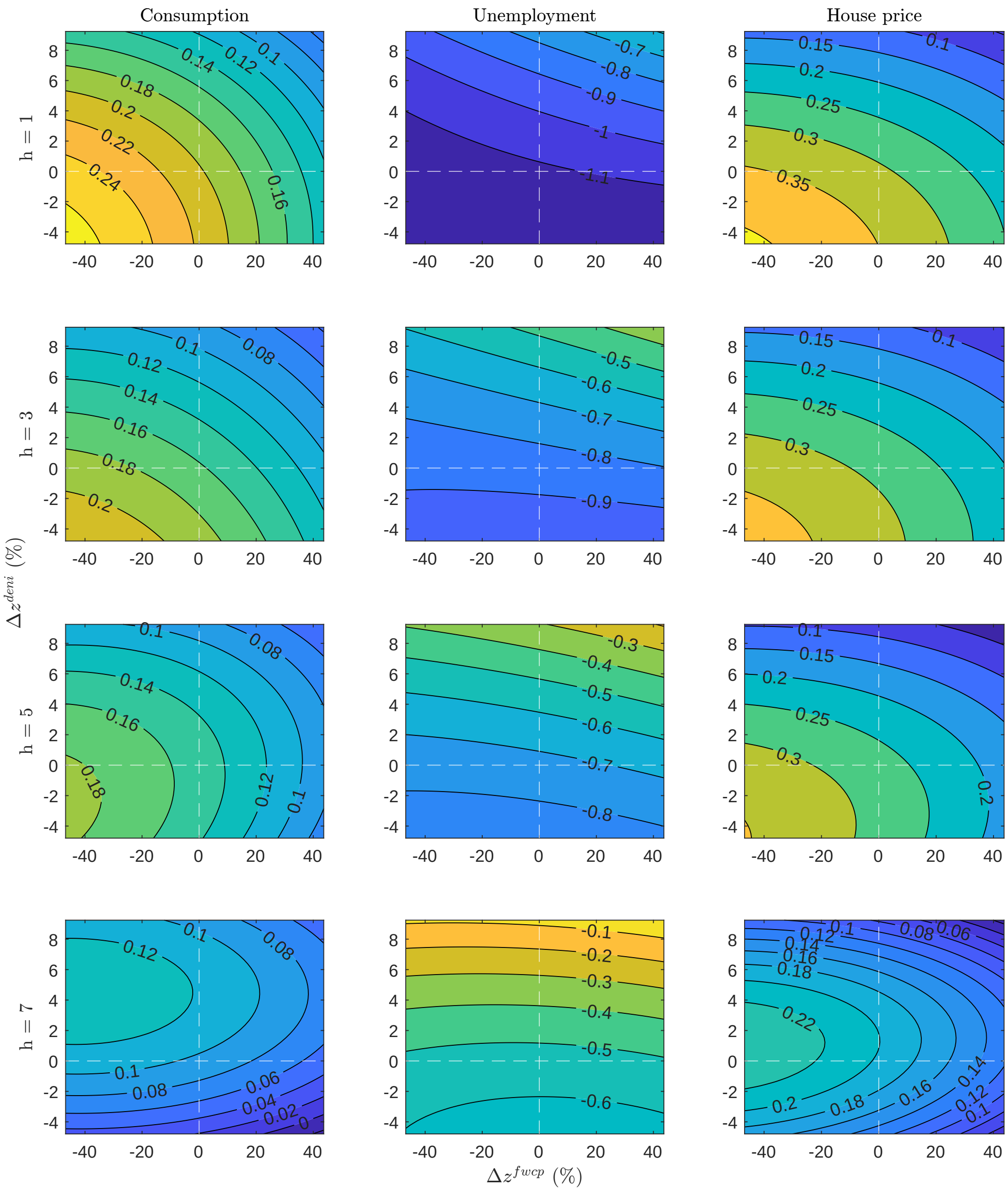
In our baseline results, we use a sieve estimator by approximating the effects β_h with the following quadratic polynomial globally, which expands the coefficient functional at $\mathbf{Z}_{i,t} = \bar{\mathbf{Z}}_i$:

$$\beta_h(\Delta\mathbf{Z}_{i,t}) \approx b_h^0 + b_h^1 \Delta z_{i,t}^{fwcp} + b_h^2 \Delta z_{i,t}^{deni} + b_h^3 \Delta z_{i,t}^{fwcp} \Delta z_{i,t}^{deni} \tag{1.31}$$

$$+ b_h^4 (\Delta z_{i,t}^{fwcp})^2 + b_h^5 (\Delta z_{i,t}^{deni})^2 \tag{1.32}$$

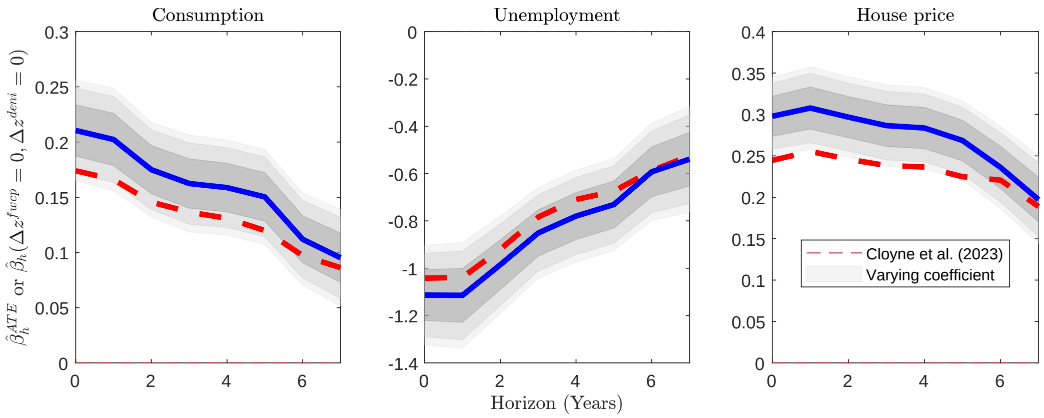
Subsequently, we employ an OLS estimator for Equation (1.27). According to Chen (2007), this estimator is consistent. In Appendix A.3, we conduct robustness checks on the sensitivity of our polynomial choices, revealing that higher approximation orders do not introduce new patterns but may heighten the risk of overfitting. In Appendix A.4, the results of a local kernel-polynomial estimator (Zhang et al., 2002) are also provided as robustness check. The results show the local patterns are also consistent with our global approximation. Meanwhile, in Appendix A.5, we perform the sensitivity analysis on our $\beta_h(\Delta\mathbf{Z}_{i,t} = 0)$ estimates against any potential leftover confounding effects, e.g. policy interventions, that were not captured by the fixed effects. The results suggest that our baseline estimates are robust against the omitted variable bias. Furthermore, Appendix A.6 performs the robustness check on potential spatial spillover effects of the net worth shocks. The estimates of average spillover effects reveals statistically significant spillover effects of the shocks on unemployment whereas it does not largely change the baseline estimates of $\beta_h(\Delta\mathbf{Z})$. Figure 1.9

Figure 1.8: Results of the varying coefficient model



Notes: 1. The control variable $\mathbf{W}_{i,t}$ includes total housing units, share of housing wealth in household net worth, share of tradable sector employment in total employment, and share of construction sector employment in total employment. 2. The function $g(N_{i,t-1})$ is approximated by a cubic polynomial. 2. The regression is weighted by total population of each county, and the standard errors are clustered at state level. 3. The sample size used in each horizon's estimation varies from 12208 to 22784 where about 1700 counties with constructed consumption data are included, and the time spans from 2004 to 2019. 4. State and year fixed effects are controlled. 5. The scales of *FWCP* and *DENI* are in digits while they are visualized as percentage points in the figure.

Figure 1.9: Comparing the ATE of linear LP and varying coefficient LP



Notes: 1. The blue solid line is the point estimate of $\hat{\beta}_h(\Delta z^{fwcp} = 0, \Delta z^{deni} = 0)$ in the varying coefficient model. The gray shaded areas are the corresponding confidence interval in which t-testings are performed for points $\hat{\beta}_h(\Delta z^{fwcp} = 0, \Delta z^{deni} = 0)$. 2. The red dashed line is the point estimate of $\hat{\beta}_h$ in the linear LP model, as a contrast to our varying coefficient model estimates. 3. The two models share the same control variable set, standard error cluster and weights.

compares the β_h estimate from the linear LP model with the $\beta_h(\Delta \mathbf{Z}_{i,t} = 0)$ point estimates from the varying coefficient model. This comparison indicates that for a "representative" county, the average treatment effect estimates from both models are in alignment.

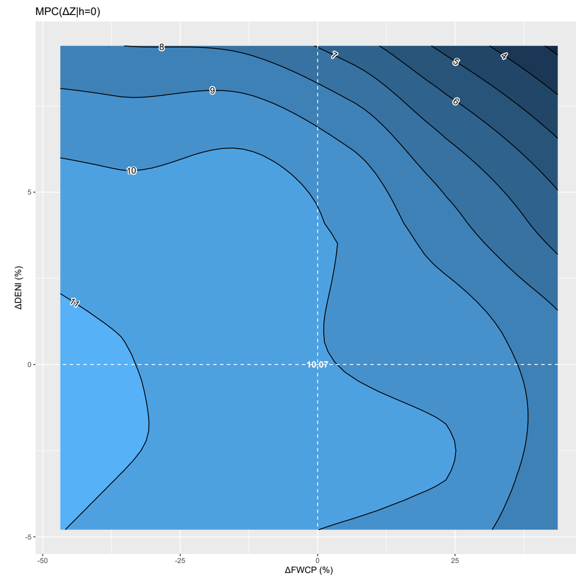
The heterogeneous effects can then be qualitatively analyzed by examining the estimated function $\beta_h(\Delta \mathbf{Z}_{i,t})$ across the space of $(\Delta z_{i,t}^{fwcp}, \Delta z_{i,t}^{deni})$. Figure 1.8 presents the estimated $\beta_h(\Delta \mathbf{Z}_{i,t})$ as a contour diagram, which allows for visually checking the heterogeneous effects. The individual subplots show the estimated $\beta_h(\Delta \mathbf{Z}_{i,t})$ against $(\Delta z_{i,t}^{fwcp}, \Delta z_{i,t}^{deni})$. This figure visually characterizes the heterogeneous effects across nominal and financial friction heterogeneities. The intersection of the white dashed lines indicates the ATE point estimate in Figure 1.9. In the matrix of figures, each row corresponds to the estimated $\beta_h(\Delta \mathbf{Z}_{i,t})$ at different projection horizons h, and each column gives the estimates for each outcome variable. In the absence of heterogeneous effects, one would expect these contour diagrams to appear as flat surfaces across all values of $(\Delta z_{i,t}^{fwcp}, \Delta z_{i,t}^{deni})$. However, the diagrams for all outcome variables and across different horizons are not flat, indicating the presence of heterogeneous effects. Specifically, for consumption as the outcome variable, the estimates suggest that counties with higher financial or nominal frictions experience a reduced impact from positive net worth shocks. This implies that economic frictions generally mitigate the effects of net worth shocks on consumption, as supported by (Christiano et al., 2015). A similar pattern is observed with house prices, where economic frictions diminish the impact of net worth shocks. Regarding unemployment, the estimates reveal heterogeneous effects of financial friction. This discrepancy highlights the significant impact of financial friction on labor market outcomes (Branch et al., 2016), a conclusion that is consistent with the linear LP model's estimates. There, linear heterogeneous effects across the dimensions of financial and nominal friction heterogeneity would suggest that equal deviations of $\Delta z_{i,t}^{fwcp}$ or $\Delta z_{i,t}^{deni}$ from zero would result in similar changes in effect level or magnitude. However, the observed variation in effect level changes across these subplots reveals a non-linearity in the heterogeneous effects, further emphasizing the complexity of the economic dynamics under examination. Upon reconfirming the presence of heterogeneous effects, we turn our attention to the interaction between financial and nominal frictions. In the absence of such interaction, contour diagrams would display parallel straight lines. However, the curvature observed in these diagrams' level lines indicates a clear interaction between the heterogeneous effects of financial and nominal frictions. In the end of this section, we confirm the above observations with statistical inference.

Beyond qualitative observations, baseline estimates yield significant quantitative results. The direct interpretation of the IRF $\beta_h(\Delta\mathbf{Z})$ is elasticity: the relative percentage change of the outcome variables regarding one percent relative change in net worth shocks. The marginal propensity to consume (MPC) out of wealth is another crucial indicator for various policies. With our baseline specification, one can estimate MPC out of the wealth by simply multiplying the estimated coefficient $\beta_0(\Delta\mathbf{Z})$ with the ratio between the conditional average consumption and household net worth[17]. The average MPC is 10.1 cent per dollar which is larger but still aligns with the estimate of around 7 percent per dollar out of housing wealth using the US data (Mian et al., 2013; Aladangady, 2017; Angrisani et al., 2019). Figure 1.10 illustrates the significant variation in the MPC across different levels of local frictions. As either Δz^{fwcp} or Δz^{deni} increases, the MPC decreases, reflecting the intuitive idea that households consume a smaller fraction of each additional dollar of wealth when there is larger unemployment risk or precautionary saving motive. MPC can reach as high as 11 cents per dollar in counties with minimal frictions, yet drop to 3 in counties with the most frictions. This broad range of MPC estimates offers a plausible rationale for the disparate MPC estimates derived from a variety of structural models, which incorporate varying degrees of frictions based on their assumptions and demographic structures. Additionally, the MPC profile across $\Delta\mathbf{Z}$ serves as a valuable instrument for model calibration.

The counterfactual IRFs for four scenarios, derived from our tractable model, are presented in Figure 1.11, allowing for an intuitive interpretation of the estimation results. We define the scenarios based on the quantiles of $\Delta\mathbf{Z}$, corresponding to: neither friction constraint binding, either the collateral constraint or DNWR binding, and both constraints binding. The results indicate that net worth shocks have the most significant impact on counties with minimal economic frictions, aligning with the theoretical predictions of the model. Specifically, consumption, unemployment, and house prices each exhibit varying sensitivities to net worth shocks. For consumption, a 1% relative increase in net worth (e.g., from 5% to 5.05%) can lead to a 0.25% rise in consumption, reflecting high sensitivity in this area. Unemployment, however, is less responsive; reducing the unemployment rate by approximately one percentage point would require a doubling of household net worth. This lower sensitivity is likely

[17] The identification comes from differentiating both sides of the baseline specification with respect to household net worth at the moment t of the shock.

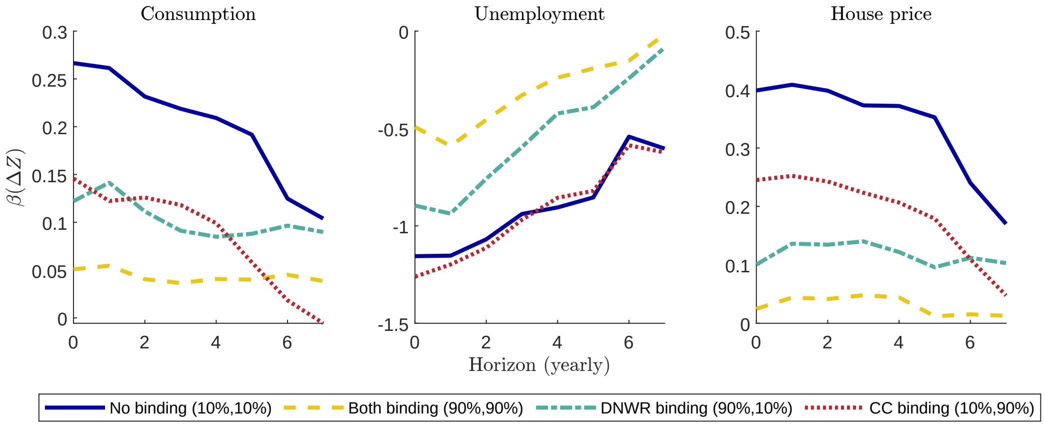
Figure 1.10: MPC estimates



Notes: The long-term conditional expectations of consumption and net worth are estimated using locally estimated scatterplot smoothing in which the bandwidth is selected using bias-corrected AIC criterion.

due to unemployment being influenced by broader, more aggregate factors than the other variables. House prices show even greater sensitivity than consumption, with a 1% relative change in net worth shock resulting in a 0.4% fluctuation in house prices. In contrast, when either friction constraint is binding, the effects of a positive shock are generally reduced. Notably, when only the collateral constraint is binding, the reduction in unemployment is minimal, which aligns with the theoretical relationship between DNWR and unemployment, emphasizing DNWR's essential role in these dynamics. Meanwhile, in the case where both constraints are binding, the effects of net worth shocks are substantially suppressed. The magnitude of these effects is reduced by a factor of 2 to 5 compared to the scenario with the least binding frictions. These findings provide additional support for the theoretical results. These results from the counterfactual IRF carry significant policy implications. When the government seeks to stimulate the economy through policies that increase household wealth—such as universal basic income or stimulus checks—the presence of economic frictions can sub-

Figure 1.11: Impulse response by friction constraint

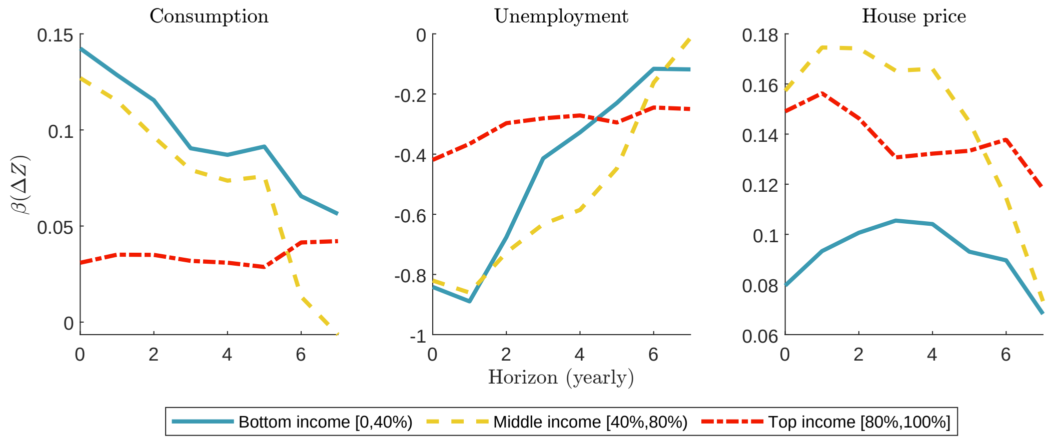


Notes: 1. Cases are defined by the quantiles of $\Delta\mathbf{Z}_{i,t}$. For example, a $(90\%, 10\%)$ represents the point estimate for counties locating at the right tail of the FWCP marginal distribution and the left tail of the DENI marginal distribution. Visually, it is the "bottom-right corner" in a panel of Figure 1.8.

stantially dampen the effectiveness of such interventions. Policymakers must account for these frictions when evaluating the potential outcomes of wealth-based stimulus policies to ensure accurate assessments of their impact. Moreover, to maximize the effectiveness of these stimulus policies, it is crucial for policymakers to implement complementary measures aimed at reducing local economic frictions, such as tightening labor markets or increasing credit supply. By addressing these frictions, the overall economic response to wealth-based stimulus policies can be enhanced, leading to more robust outcomes.

The heterogeneous effects of net worth shocks across regions, as demonstrated by our analysis of financial and nominal frictions, highlight the importance of local economic conditions in shock transmission. A key dimension of local economic variation is income disparity, which is substantial across U.S. counties. Figure A.1 illustrates that counties with different household income levels experience varying degrees of economic friction. The impact of net worth shocks should vary among counties based on their income levels. To empirically test this, we categorize all counties into three income groups: the bottom 40%, the middle 40%, and the top 20%. We then apply

Figure 1.12: IRF by income group



Notes: 1. Each line is an IRF at $\Delta\mathbf{Z} = (0,0)$ within the corresponding income group. 2. The cut line of income groups are determined using the 2006 cross-sectional distribution of county real median household income.

our baseline model, which properly handles the non-linear heterogeneous effects of the frictions, to these sub-samples. Figure 1.12 illustrates the IRF assessed at the average counties (i.e., $\Delta\mathbf{Z} = (0,0)$) for each income group. Regarding consumption, it is observed that lower and middle income groups are more vulnerable to net worth shocks and endure a longer recovery period post-shock. Conversely, the top income group exhibits minimal response less thant 0.05% to such changes in their wealth. This could be attributed to the marginal propensity to consume decreasing as income rises. In terms of unemployment, the bottom and middle income groups display comparable sensitivity to net worth shocks, whereas the top income group shows little reaction. This disparity may stem from occupational differences among the income brackets: higher income counties likely have a greater proportion of firm owners, entrepreneurs, and other occupations with increased job security (Clark and Postel-Vinay, 2009). With respect to house prices, the middle and top income groups react more significantly to net worth shocks, while the bottom income group shows less sensitivity of value 0.08% which is half of the other two groups. This variation could be due to the diversity in household balance sheet exposures across these counties:

those in the lower income bracket may possess a smaller share of secondary or investment housing wealth. Coupled with their generally lower income levels, households in these counties tend to engage less frequently in buying or selling properties.

The heterogeneous responses across income groups, along with the varying prevalence of economic frictions, hold significant implications for both the transmission of macroeconomic shocks and the design of stabilization policies. These findings suggest that broad, country-wide interventions may have uneven effects across the income distribution, potentially exacerbating existing inequalities. For instance, policies such as lowering interest rates for large national banks like Chase or Bank of America may not be as effective as implementing differentiated policies for local financial institutions, which could have a more direct impact on lower-income households. To optimize the effectiveness of stimulus measures, policymakers should account for the differing sensitivities of outcome variables across regions. For example, lowering interest rates on credit card or consumption loans could be more beneficial in lower-income regions, while reducing mortgage rates might yield better outcomes in higher-income areas. These tailored approaches could enhance the overall impact of economic stimulus. Future research could delve deeper into understanding the mechanisms behind these differential responses and their implications for the design of optimal policies.

To statistically test for potential interaction and non-linear heterogeneous effects, one must conduct inference on the semi-varying coefficient model. Typically, this involves point testings such as profile likelihood ratio tests (Fan and Huang, 2005; Härdle et al., 1998) and bootstrapping-based tests (Cai et al., 2000; Wang and Xue, 2011; Belloni et al., 2019). Moreover, the body of literature on the uniform inferences on functional coefficients is rapidly expanding (Hu, 2024). In this paper, we can conveniently conduct an F-test on the semi-varying coefficient model utilizing the fact that we approximate $\beta_h(\Delta \mathbf{Z}_{i,t})$ with a polynomial globally. Specifically, the null hypothesis is that the polynomial approximation coefficients for the non-linear terms in Equation (1.31) are jointly zero, indicating no interaction nor non-linear heterogeneous effects. It is important to note that such F-test does not necessitate the form of non-linearity and interaction to be identical to the quadratic polynomials in Equation (1.31). Table 1.2 presents the F-statistics and corresponding p-values. The null hypotheses are rejected in most cases. In Appendix A.4, we also perform the profile likelihood ratio test (Fan and Huang, 2005) as robustness check. The results also align with the F tests.

Table 1.2: F-test of non-linear heterogeneous effects in the linear LP model

$h =$	0	1	2	3	4	5	6	7
Consumption	9.680	9.709	9.392	8.086	8.226	11.013	8.152	5.830
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)
Unemployment	5.919	3.874	2.551	2.963	3.453	3.292	2.532	1.627
	(0.001)	(0.009)	(0.054)	(0.031)	(0.016)	(0.020)	(0.056)	(0.181)
House price	24.967	23.961	22.215	21.083	22.661	19.744	14.116	11.973
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Notes: 1. *H0*: $0 = b_h^3 = b_h^4 = b_h^5$. 2. The numbers with stars are F-statistic; the numbers between parentheses are the p-values.

1.5 Conclusion

By constructing a comprehensive new county-level dataset, we empirically estimated the heterogeneous effects of net worth shocks in the US. The findings suggest that counties with tighter financial friction (collateral constraints) and more severe nominal friction would experience a more severe recession and prolonged economic recovery following negative net worth shocks. Our estimation results also reveal non-linear heterogeneous effects of the shock, which underscores the complex interactions between financial constraints and wage rigidity. Moreover, we proposed a two-agent general equilibrium model that provides a simple theoretical framework to understand how the financial friction and the nominal friction interactively shape the heterogeneity among the net worth shock effects. This work contributes to the existing literature by offering new empirical evidence of how financial and nominal frictions may amplify the effects of economic shocks. It highlights the necessity for policy interventions addressing structural issues related to financial access and labor market rigidities.

Chapter 2

Downward Nominal Wage Rigidity and Collateral Constraints: A Theory for Post-Great Recession U.S. Economy

2.1 Introduction

The U.S. economy suffered a severe slump during the Great Recession, followed by a prolonged housing deleveraging process and recovery (Powell, 2023). A noticeable feature of this recovery is the slow rebound of key macroeconomic indicators (Guerrieri et al., 2020a). For instance, aggregate consumption growth remained subdued for nearly a decade, only gradually returning to its pre-crisis trend until 2015. The unemployment rate also sharply increased at the beginning of the period then gradually fell down. Similar slow recoveries were observed in housing prices and a wide range of asset and labor markets (Powell, 2023; Favilukis et al., 2017). These patterns have raised important questions about the fundamental forces shaping the post-crisis dynamics: what factors contributed to such a protracted recovery? What mechanisms can account for the observed joint sluggish adjustments across sectors? These questions remain central in both academic and policy discussions. As evidence of the continuing relevance of this topic, Federal Reserve officials have highlighted the need to understand the structural causes of the post-crisis stagnation (Bernanke, 2018), emphasizing that "the crisis left lasting scars on demand and productive capacity." In response, a growing body of research has sought to explain the post-crisis trajectory by emphasizing the role of economic frictions, such as financial constraints, nominal rigidities, and asset illiquidity that may have amplified the initial shock and slowed the return to equilibrium.

Financial frictions that restrict the household credit availability has been shown to be important in explaining the households response during and after the Great Recession (Christiano et al., 2015). These frictions, particularly collateral constraints, play a critical role when interacting with the illiquidity of housing wealth. Housing wealth accounts for approximately 60% of total assets for U.S. families, making it a central component of the household balance sheet. In an incomplete market economy, precautionary motives generate strong demand for savings (Berger and Vavra, 2015; Berger et al., 2018b). However, the illiquidity of housing wealth makes it difficult for households to adjust their portfolios quickly in response to shocks (Garriga and Hedlund, 2020). When asset liquidation is infeasible or costly, borrowing becomes an important channel for consumption smoothing. During the crisis, as house prices declined substantially, many households attempted to borrow more to offset income or asset losses. At the same time, a major financial friction emerged. Large-scale borrowing typically requires collateral, and the value of collateral directly determines the borrowing limit. For most households, the collateral consists of their homes. As house prices dropped significantly during the Great Recession, the borrowing capacity of many households fell sharply, preventing them from obtaining the amount of funding they needed (Guerrieri and Iacoviello, 2017b). Consequently, they had to cut consumption, save more, and gradually deleverage by selling housing assets to rebuild the necessary buffer for portfolio adjustment. When the house price remains low for an extended period, this deleveraging and consumption adjustment process becomes long and drawn-out (Guerrieri et al., 2020a). As house prices recover over time, more households accumulate sufficient savings and complete their portfolio adjustment. This leads to a gradual rebound in aggregate consumption and a slow recovery of debt after the Great Recession.

Nominal frictions have also been widely recognized as important contributors to the dynamics of key macroeconomic variables following the Great Recession (Kaplan et al., 2018; Schmitt-Grohé and Uribe, 2016). The large body of New Keynesian literature has developed a coherent framework to study these frictions, most notably through the Phillips curve, which jointly analyzes inflation and unemployment. Among the various types of nominal rigidities, downward nominal wage rigidity (DNWR) offers a particularly compelling mechanism for understanding the asymmetric behavior of unemployment relative to wage adjustments during downturns (Schmitt-Grohé and Uribe, 2016). Recent theoretical and empirical work has extended this insight by deriving non-linear Phillips curves under DNWR, which offer a clear explanation for the observed phenomenon of missing inflation during the long recovery (Schmitt-Grohé and Uribe, 2022; Aktug, 2025). In the context of the post-crisis period, DNWR helps explain why unemployment remained elevated despite limited declines in nominal wages. Specifically, as aggregate demand contracted due to widespread household consumption cuts, firms faced declining revenues and needed to reduce production. In a frictionless labor market, this would translate into falling wages to restore equilibrium. However, when wages are downwardly rigid, employers cannot lower wages in response to weaker demand. Instead, they reduce labor input by cutting jobs or freezing hiring, resulting in involuntary unemployment despite an ample labor supply. This mechanism illustrates how DNWR contributes to persistent slack in the labor market and a slower recovery in aggregate output. Therefore, DNWR provides a powerful and intuitive explanation for several puzzling features of the post-recession macroeconomic environment and complements the role of financial frictions in shaping the broader adjustment path.

In this paper, I study the interaction between financial frictions and nominal frictions, and argue that their joint presence provides a unified explanation for the observed dynamics of consumption, unemployment, and house prices during and after the Great Recession. In particular, I emphasize the persistent nature of the recovery and show that the interaction of these two frictions plays a central role in shaping the adjustment process. When both friction constraints are simultaneously binding, I find that they generate a significant amplification effect, which allows the model to qualitatively and quantitatively match the U.S. data. Specifically, an adverse aggregate shock, such as a decline in land supply or total factor productivity, depresses house prices and tightens collateral constraints. As households approach their borrowing limits, especially those with high leverage, the precautionary saving motive induces consumption cuts and deleveraging. In the aggregate, this widespread reduction in consumption leads to a fall in the aggregate demand. With downward wage rigidity, producers respond by reducing employment, which results in involuntary unemployment. At the household level, increased unemployment risk raises idiosyncratic income uncertainty, further strengthening precautionary saving behavior. This leads to additional consumption cuts, more house selling, and continued downward pressure on house prices. The illiquidity of housing wealth prolongs this feedback loop, making the recovery even more slow and persistent. By combining these two frictions within a coherent framework, my analysis captures the co-movement and persistence of key macroeconomic indicators and offers a elegant account of the U.S. economy's adjustment in the period after the Great Recession.

The first contribution of this paper is to provide a simple but unified explanation for the joint trends of key macroeconomic indicators in the post-crisis period by slightly extending the workhorse heterogeneous agent framework. I build a baseline model in the spirit of the workhorse model in this literature, featuring a continuum of heterogeneous households. Each household holds a realistic balance sheet composed of liquid savings, illiquid housing wealth with convex adjustment costs, and mortgage debt that requires fixed periodic payments. In addition to standard savings decisions, households choose how much housing wealth to allocate to a mutual fund that transforms housing wealth into productive capital. This capital is then used by firms to produce consumption goods. On the production side, firms hire capital and labor by taking prices as given. Importantly, the labor market features DNWR, which prevents the wage rate from falling below a fraction of the previous period's level. To analyze the effect of aggregate shocks, I solve the model globally and simulate the transition paths of the economy following an one-shot-deviation adverse productivity shock[1]. The baseline model replicates several central features of the U.S. macroeconomic dynamics following the Great Recession with notable accuracy. It captures the sharp initial contraction in aggregate consumption, the spike in unemployment, and the steep decline in house prices, all of which align closely with the magnitude and timing observed in the data. The model also reproduces key dynamics of output and household leverage, including the delayed peak in leverage and its gradual unwinding, consistent with empirical patterns. Importantly, the model explains a large fraction of the observed movements in house prices, highlighting its strength in capturing asset price sensitivity to household balance sheets. Notably, the model achieves this without relying on detailed institutional assumptions or additional sector-specific mechanisms, demonstrating the explanatory power of a parsimonious framework that

[1] When household idiosyncratic income uncertainty depends on aggregate dynamics, the transition paths obtained from the first and second order perturbations tend to underestimate the strength of the precautionary saving motive, as argued by Reiter (2023) and Auclert et al. (2021).

integrates key financial and nominal frictions. While some differences in persistence remain, due in part to real-world policy responses and structural frictions not modeled, the baseline framework offers a quantitatively compelling account of the crisis dynamics and provides a strong foundation for further policy analysis.

In addition to the analysis of the baseline model, I examine a set of model variants that selectively remove one or both of the key frictions. These counterfactual experiments allow us to isolate the individual and joint contributions of the collateral constraint and downward nominal wage rigidity. The comparative statics analysis demonstrates that the baseline model, which includes both collateral constraints and DNWR, most effectively replicates the magnitude and persistence of observed macroeconomic responses to large shocks. Across consumption, unemployment, and house prices, the baseline scenario consistently shows the largest responses, highlighting the importance of friction interactions in amplifying economic fluctuations. The collateral constraint alone accounts for a substantial share of the declines in consumption and house prices, while DNWR is essential for generating unemployment dynamics and further amplifies the effects when combined with financial frictions. These findings underscore the model's strength in capturing key empirical patterns and support the central role of economic frictions—especially their interaction—in explaining the severity of macroeconomic downturns. Beyond the dynamic responses, I also conduct a welfare analysis across the different model specifications. This analysis compares social welfare under alternative scenarios and highlights the costs associated with each friction. The findings suggest that reducing the severity of collateral constraint and DNWR can lead to substantial improvements in household welfare in the meaning of consumption equivalence variation. These results offer important policy implications for the design of effective stimulus and stabilization policies aimed at addressing local economic distress in the period after large aggregate shocks.

The second contribution of this paper is the development of a new and efficient algorithm for computing the global solution of high-dimensional heterogeneous agent models with occasionally binding constraints and implicit asset prices. The algorithm is designed to enforce these constraints while accurately capturing the economy's equilibrium responses. The primary challenge lies in the high dimensionality of the problem, which requires advanced numerical techniques to ensure both computational feasibility and accuracy. Standard approaches often struggle with issues such as divergence in value function iteration due to non-convexities and the lack of shape preservation in high-dimensional approximations (Bohn, 2018). To overcome these challenges, I adopt a sequential programming algorithm to solve the deterministic steady state by progressively narrowing the search space. In addition, I introduce a bounded rationality assumption on house price expectations to enhance the efficiency and numerical stability of updating the house price profile. This approach enables efficient computation of the equilibrium while preserving essential nonlinearities and binding constraints, and it broadens the applications of finite moment methods in models with rich heterogeneity and frictional features.

In Section 2, I present the baseline model and define the equilibrium. Section 3 and 4 outline the numerical algorithm and the calibration strategy. In Section 4, I analyze the dynamic response of the baseline economy by computing the transition paths following an one-shot-deviation from the deterministic steady state. I then conduct comparative statics by selectively removing each friction and evaluate the resulting welfare implications.

2.2 Model

I consider an economy with heterogeneous agents, illiquid housing wealth, fixed debt payment, and a collection of frictions. The economy is in discrete time and starts from $t = 0$. There lives a continuum of ex ante homogeneous households whose distribution measure is $\Phi_t(x)$ of total mass 1.

Firm. There is a representative competitive firm that is owned by households through a mutual fund. It produces consumption goods using capital K_t and using labor hired from the households. The production technology is Cobb-Douglas in Equation (2.10).

$$Y_t = e^{Z_t} K_t^{\alpha} [N_t \cdot (1 - \bar{\xi})]^{1-\alpha} \tag{2.1}$$

where Z_t is an aggregate uncertainty of total productivity that follows an exogenous AR(1) process. The unemployment in this model is distinguished to be two types: in addition to $N_t \in [0, 1]$, $\bar{\xi} \in (0, 1)$ is the natural rate of unemployment due to some frictions in the labor market (Schmitt-Grohé and Uribe, 2022). In each period, the firm chooses (K_t, N_t) by taking the wage rate w_t as given. After paying the wage to the households, the left profit $\Pi_t := Y_t - w_t N_t (1 - \bar{\xi})$ is paid to the mutual fund which has the technology of converting housing wealth to capital and supplies the capital to the firm.

DNWR. In the economy, the labor market is not necessary to be cleared. The wage rate w_t is downward rigid which prevents it falls below a δ fraction of the last period's wage rate w_{t-1}

$$w_t \geq \delta w_{t-1} \tag{2.2}$$

$$(w_t - \delta w_{t-1})(N_t - 1) = 0 \tag{2.3}$$

When the downward nominal wage rigidity constraint of Equation (2.2) is binding, the optimality condition of the firm gives a labor demand $N_t < 1$ which generates involuntary unemployment. The slackness condition in Equation (2.3) characterizes the relationship between the wage rate and the involuntary unemployment.

Mutual fund. The households own the firm by holding the shares of a revenue-maximization competitive mutual fund. The mutual fund collects housing wealth from the households and use technology $\kappa(h)$ to convert them into productive capital.

$$\kappa(h) := Z_h h^{\tau}, \tau \in (0, 1] \tag{2.4}$$

When receiving the profit Π_t from the firm, the mutual fund pays returns R_t to the households per unit of housing wealth.

Households. Households are differentiable and the law of large numbers holds. Each individual household has a vector of individual states

$$x_t := (a_{t-1}, b_{t-1}, h_{t-1}, s_{t-1}, \xi_t) \tag{2.5}$$

where a_{t-1} denotes liquid asset holdings at the beginning of period t; b_{t-1} is the outstanding mortgage debt balance; h_{t-1} is the value of housing wealth; s_{t-1} represents the share of h_{t-1} allocated to a mutual fund, determined in the previous period; and ξ_t captures idiosyncratic frictional unemployment, which follows an exogenous AR(1) process over a finite support $[\bar{\xi} - \Delta\xi, \bar{\xi} + \Delta\xi]$ (Schmitt-Grohé and Uribe, 2022). In period t, each household inelastically supplies one unit of labor to the market and receives a wage from the firm. Households have access to three types of assets: liquid savings $a_t \geq 0$, which are lent to a neutral external borrower at a risk-free interest rate r; perpetual mortgage debt $b_t \geq 0$, borrowed from a neutral external lender at a mortgage rate $r_b \geq r$; and illiquid housing wealth h_t, valued at the market house price p_t, which is endogenously determined in equilibrium. In addition, households can allocate a share $1 - s_t \in [0, 1]$ of their housing wealth h_t to a mutual fund that yields a return R_{t+1}. The remaining share s_t is retained for residential use and provides instantaneous utility jointly with consumption. In each period, the household chooses consumption $c_t \geq 0$, new mortgage debt holdings $b_t \geq 0$, the housing wealth growth rate $h_t \geq -1$, and the share $s_t \in [0, 1]$ of housing wealth allocated to the mutual fund for the following period. In making decisions, households face several frictions. First, housing wealth is illiquid: any adjustment to the housing stock incurs a quadratic adjustment cost, denoted by $\varphi_h(\cdot)$. Second, the financial market is imperfect, and households are subject to a collateral constraint that limits borrowing to a fraction θ of the housing wealth allocated for residential use. In addition, households must pay a convex maintenance cost $\varphi_s(\cdot)$ for consuming residential housing services valued at $s_{t-1}h_{t-1}$. [2] The Bellman equation of the individual household is then as follows:

$$v(x_t; X_t) = \max_{\mathcal{C}_t} u(c_t, s_{t-1}h_{t-1}) + \beta \mathbb{E}_t\{v(x_{t+1}; X_{t+1})|x_t, X_{t+1}\} \tag{2.6}$$

s.t.

$$x_t := (a_{t-1}, b_{t-1}, h_{t-1}, s_{t-1}, \xi_t)$$

$$X_t := (\Phi_{t-1}(x), w_{t-1}, Z_t)$$

$$\mathcal{C}_t := (c_t, b_t, \mu_t^h, s_t)$$

$$a_t = \mathcal{Y}_t(x_t, X_t) - \mathcal{E}_t(x_t, X_t, \mathcal{C}_t)$$

$$h_t = (1 + \mu_t^h)h_{t-1}$$

$$b_t \leq \theta p(X_t) \cdot (s_t h_t)$$

$$c_t \geq 0, b_t \geq 0, \mu_t^h \geq -1, s_t \in [0, 1]$$

$$\xi_{t+1} \sim \text{AR}(1)[\bar{\xi} - \Delta\xi, \bar{\xi} + \Delta\xi]$$

$$\mathcal{Y}_t(x_t, X_t) := (1 + r)a_{t-1} - r_b b_{t-1} + R(X_t)(1 - s_{t-1})h_{t-1} + w(X_t)(1 - \xi_t)N(X_t)$$

$$\mathcal{E}_t(x_t, X_t, \mathcal{C}_t) := c_t + p(X_t)h_{t-1}[\mu_t^h + \varphi_h(\mu_t^h)] + b_{t-1} - b_t + \varphi_s(s_{t-1})$$

$$\varphi_h(\mu_t^h) := \frac{\psi_h}{2}(\mu_t^h)^2$$

$$\varphi_s(s_{t-1}) := \frac{\psi_s}{2}s_{t-1}^2 p_t h_{t-1}$$

where β is the utility discounting factor; θ is the loan-to-value ratio; ψ_h and ψ_s are

[2] For computational simplicity, I impose an additional solvency constraint $a_t \geq r_b b_t$ to rule out default, which is not explicitly modeled in this framework.

the adjustment cost coefficients; $\mathcal{C}_t$ is the vector of control variables; $\mathcal{Y}_t$ is the flow income function; $\mathcal{E}_t$ is the flow expenditure function; and X_t is the vector of aggregate states. The equilibrium profiles $Q(X) := (p(X), w(X), R(X), N(X))$ are functions of the aggregate variables X_t.

Aggregation. Aggregating the individual households over the distribution $\Phi_t(x)$ leads to

$$\begin{aligned}
\mathcal{A}(X_t) &:= \int a_t(x; X_t)\mathrm{d}\Phi_{t-1}(x) \\
\mathcal{B}(X_t) &:= \int b_t(x; X_t)\mathrm{d}\Phi_{t-1}(x) \\
\mathcal{H}(X_t) &:= \int h_t(x; X_t)\mathrm{d}\Phi_{t-1}(x) \\
\mathcal{S}(X_t) &:= \frac{1}{\mathcal{H}(X_t)} \cdot \int s_t(x; X_t)h_t(x; X_t)\mathrm{d}\Phi_{t-1}(x) \\
\bar{\xi} &\equiv \int \xi_t(x; X_t)\mathrm{d}\Phi_{t-1}(x)
\end{aligned} \tag{2.7}$$

while the distribution $\Phi_t(x)$ evolves according to Kolmogorov forward equation.

Equilibrium. There is a constant supply $\bar{H} \in \mathbb{R}$ of housing wealth in each period. The house price p_t is a function of the aggregate states X_t which solves the following market clearing condition.

$$\bar{H} = \mathcal{H}(X_t), t = 0, 1, \ldots \tag{2.8}$$

A recursive equilibrium is defined as a collection of functions: the household value function $v(x, X)$, the policy functions $\mathcal{C}(x, X)$, the distribution $\Phi(x)$, and the equilibrium price profiles $Q(X)$. In equilibrium, the household's value and policy functions solve the dynamic optimization problem given the equilibrium prices. The distribution $\Phi(x)$ aggregates to satisfy all market clearing conditions except for the labor market. In addition, the DNWR constraint and the associated slackness condition are satisfied.

2.3 Computation algorithm

I solve the approximate equilibrium of the model by modifying the algorithm proposed in Krusell and Smith (1998). This section begins by briefly discussing the key numerical challenges, followed by an explanation of the sequential programming algorithm used to compute the deterministic steady state. It then describes the global solution method based on approximating the joint dynamics of (p_t, Z_t). Appendix B.1 explains how I solved the high-dimensional individual household's problem in detail where I combine a set of advanced numerical techniques.

Solving the equilibrium of the model is computationally intensive, particularly for the baseline specification. To address the challenges described below, I employ a set of advanced numerical techniques to compute an approximate equilibrium.

1. *High dimensionality*: The model includes five individual state variables and at least five aggregate state variables, even under a first-order finite-moment approximation. As a result, solving the household's problem involves a 10-dimensional dynamic programming problem, which is impractical using standard numerical algorithms.

2. *Multiple control variables*: Each period involves solving a portfolio allocation problem with four control variables. The presence of multiple controls increases the likelihood of numerical instability during iteration, particularly when the gradient of the value function is small in magnitude.

3. *Intra-temporal house price determination*: Unlike wages w_t and returns R_t, the house price p_t lacks a closed-form solution and must be determined numerically by solving the equation $\bar{H} = \mathcal{H}(X_t)$ for each X_t. This creates an "outer" or "nested" fixed-point problem, which is common in the literature of two-agent models (Bocola and Lorenzoni, 2020b). This issue still exists even in solving the deterministic steady state.

4. *Occasionally binding constraints*: The model features two occasionally binding constraints: collateral and DNWR. These inequality constraints introduce kinks in the policy functions, rendering standard perturbation methods ineffective. Given the model's high dimensionality, widely used approaches such as the endogenous grid method are not applicable.

The deterministic steady state serves as the reference point for computing impulse responses and defining the computational state space. A central challenge in solving for the steady state, including the stationary distribution of households, lies in determining the equilibrium house price $\bar{p}$. This difficulty stems from the nature of the housing market clearing condition, where housing demand depends implicitly and nonlinearly on the price profile $p(X)$. Specifically, housing demand aggregates over individual households, whose decisions are functions of $p(X)$ itself. Since evaluating this demand requires solving the household problem under a given $p(X)$, which is an infinite-dimensional object, computing its functional derivative is practically infeasible. In the steady state, the functional $p(X)$ simplifies to a scalar $\bar{p}$, but it still must be jointly determined with other equilibrium objects. In the baseline model, this involves simultaneously guessing $(\bar{S}, \bar{p})$ and updating them. While $\bar{S}$ can be directly updated through aggregation, no explicit equation guides the update of $\bar{p}$, which remains embedded in the housing demand. Therefore, a numerical search over a bounded domain is required to solve for $\bar{p}$. To reduce the computational burden associated with repeatedly solving the household problem, I adopt a sequential programming approach detailed in Algorithm 1. This procedure exploits the structure of the $p(X)$ profile and strategically samples the state space to limit the number of required household problem evaluations[3].

In addition to solving the aggregate law of motion $X'(X)$, computing the global

[3]For simplicity and to ensure uniqueness, I approximate the $(\bar{p}, \bar{S})$ profiles using linear functions. A quadratic approximation would correspond to the traditional sequential quadratic programming (SQP) framework in optimization theory.

Algorithm 1: Solving the deterministic steady state

Set up a bounded space $[p_{\min}, p_{\max}] \times [S_{\min}, S_{\max}]$ for $(\bar{p}, \bar{S})$ and sample total N points uniformly ;

while $(\bar{p}, \bar{S})$ *not converge* **do**

- For every sample point $(\bar{p}_j, \bar{S}_j)$ where $j = 1, \dots, N$, solve the individual household problem taking them as given ;
- Aggregate individual households to compute the distance $\Delta S_j := |\int s'(x|\bar{p}_j, \bar{S}_j)dx - \bar{S}_j|$ for each j ;
- Aggregate the individual households to compute the residual of the house market clearing condition $\Delta H_j := |\int h'(x|\bar{p}_j, \bar{S}_j)dx - \bar{H}|$ for each j ;
- Fit the linear system $\begin{bmatrix} \Delta S_j \\ \Delta H_j \end{bmatrix} = M_1 \cdot \begin{bmatrix} S_j \\ p_j \end{bmatrix} + M_2$;
- Solve the root $(\bar{p}^*, \bar{S}^*)$ of $M_1 \cdot \begin{bmatrix} S_j \\ p_j \end{bmatrix} + M_2 = 0$;
- Resolve the stationary model using $(\bar{p}^*, \bar{S}^*)$ and check the corresponding $(\Delta S, \Delta H)$. If the error is small enough, then quit the algorithm. Otherwise, set up a new bounded space for $(\bar{p}, \bar{S})$ by centering at $(\bar{p}^*, \bar{S}^*)$ and using smaller radius ;
- Repeat the above procedures until the bounded space converges;

end

solution of the model requires numerically solving for the house price profile $p(X)$, which mathematically corresponds to searching for an optimal functional. This task is challenging for two main reasons. First, the infinite-dimensional nature of $p(X)$ necessitates updating the function at every grid point of the aggregate state space. A naive update strategy, which feeds the entire approximated profile into the individual household problems, often leads to non-convergence in the dynamic programming step and introduces numerical instability in the fixed-point iteration. Second, when using the standard Krusell–Smith simulation-based procedure to update forecasting rules, the realized simulation paths may not sufficiently explore the full state space, leading to poorly identified or unstable approximations. To address these issues, I impose an additional bounded rationality assumption on top of the standard finite-moment method. Specifically, households do not observe the full $p(X)$ profile but instead treat p_t as an exogenous stochastic process. They form expectations based on a joint

Markov process for (p_t, Z_t), which summarizes aggregate uncertainty[4]. Algorithm 2 outlines the global solution procedure under this assumption. In particular, the algorithm begins with an initial guess of the $p(X)$ profile, simulates the aggregate law of motion, estimates a bi-variate AR(1) process for (p_t, Z_t), and then discretizes the process using the method of Tauchen (1986).

Algorithm 2: Solving the approximated equilibrium globally

Solve the deterministic steady state using Algorithm 1, then set up the computational domain centered at the steady state ;
Discretize the distribution $\Phi(x, X)$ with its first order moment and denote the new aggregate states as X ;
Guess a collection of aggregate profile approximations for $(X'(X), p(X))$, while the profiles for $(w(X), R(X))$ are analytical ;
Denote the approximation coefficient vector in iteration j with θ_j ;
while *θ_j does not converge* **do**
    Taking $(X'_j(X), p_j(X))$ as given, simulate the aggregate states for a long period to get $\{p_t, Z_t\}$ sample paths ;
    Estimate an AR(1) process of (p_t, Z_t) with covariance, then discretize the process as a bi-variate finite state Markov chain that has total M states ;
    Solve the individual household problem $v(x, X, p, Z)$ with bounded rationality[5] ;
    Uniformly sample N individual households from the state space of x to adequately cover the state space of X ;
    for *(p^i_j, Z^i_j) in the state space of the Markov chain (p, Z)* **do**
        Apply the individual policy functions $\mathcal{C}(x, X, p^i_j, Z^i_j)$ to all individual households ;
        Aggregate the total N individual households to obtain the aggregate housing wealth demand and the other aggregate variables ;
        Compute the residual of the house market clearing condition ΔH^i_j ;
    end
    For every possible discrete Z state, choose $p^*_j := \arg\min H^i_j$ conditional on Z as the updated value of the $p(X)$ profiles. Only the valid observations are kept ;
    Update the approximated $(X'(X), p(X))$ profiles using the filtered observations. Check θ_{j+1} until convergence ;
end

[4] The covariance between p_t and Z_t should be included to correctly form the expectation.

2.4 Calibration

To simulate the economy, I calibrate the model at the deterministic steady state using the parameters listed in Table 2.1. The stationary distribution is shown in Figure 2.1. One model period corresponds to one quarter for a representative U.S. county. The instantaneous utility of households is specified by Equation (2.9), following Guerrieri et al. (2020b). Consistent with standard practice, the coefficient of relative risk aversion is set to $\gamma = 2$, while the consumption preference for housing, $\eta = 0.82$, is calibrated based on Guerrieri et al. (2020c).

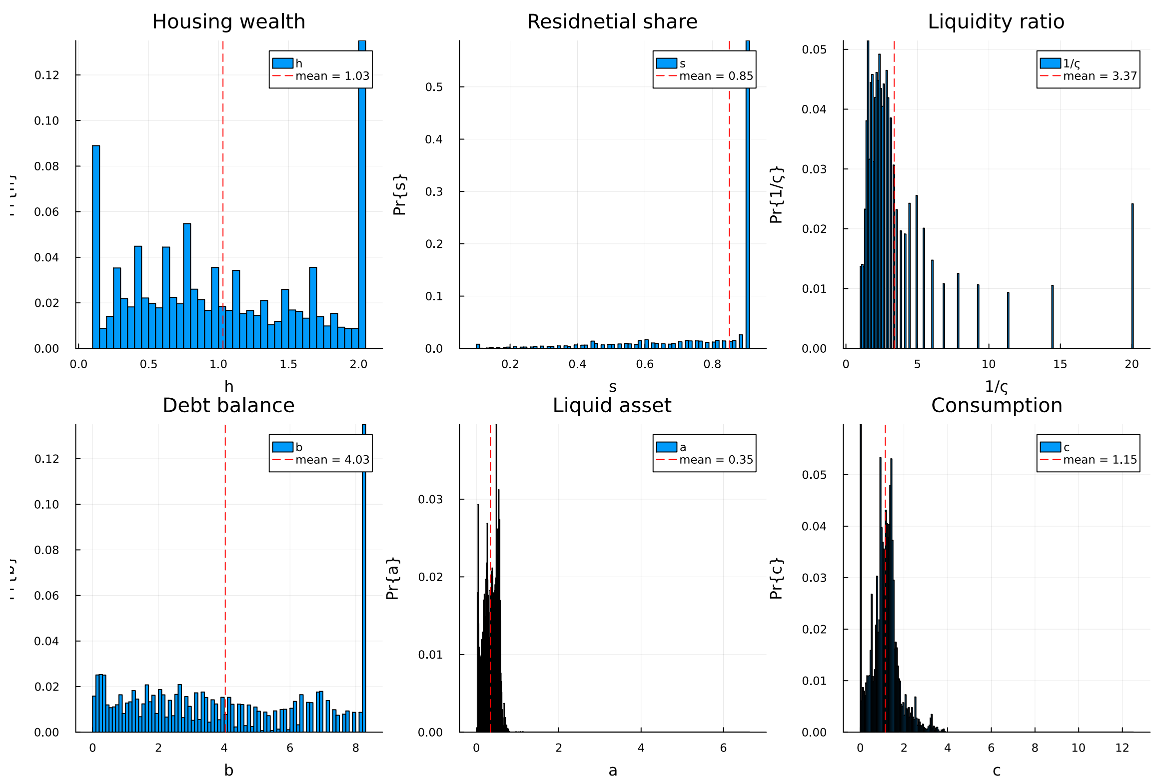
$$u(c_t, s_{t-1}, h_{t-1}) := \frac{1}{1-\gamma}[c_t^\eta (s_{t-1}h_{t-1})^{1-\eta}]^{1-\gamma} \tag{2.9}$$

The production function follows a Cobb–Douglas specification as shown in Equation (2.10), with the capital share set to the conventional value of $\alpha = 0.3$. The average frictional unemployment rate is set to $\bar{\xi} = 0.04$, consistent with U.S. data (Schmitt-Grohé and Uribe, 2022). The downward nominal wage rigidity parameter δ is calibrated to 0.99, following the literature (Schmitt-Grohé and Uribe, 2016).

$$Y_t = e^{Z_t} K_{t-1}^{\alpha} [N_t(1-\bar{\xi})]^{1-\alpha} \tag{2.10}$$

The bounded rationality algorithm requires approximating the productivity process Z_t, idiosyncratic unemployment ξ_t, and the house price p_t using finite-state Markov chains. In the baseline model, the AR(1) process for Z_t is discretized into a two-state Markov chain following the method of Tauchen (1986). Similarly, the idiosyncratic unemployment process is discretized into two states, 0 and 0.08, centered around the average $\bar{\xi} = 0.04$, with a stationary distribution assigning equal probability to each state. To reduce approximation error in the housing market clearing

Figure 2.1: Deterministic steady state



Notes: The liquidity ratio is defined as the ratio between the liquid saving a and the debt balance b.

condition, the house price process p_t is discretized using five states, which provide sufficient resolution to capture the relevant variation in the state space.

Table 2.1: Baseline parameters

Parameter	Value	Definition
α	0.3	Capital income share
$\bar{\xi}$	0.04	Average frictional unemployment
$\bar{Z}, N_Z, N_Z^\sigma$	0,2,1	Productivity
$\bar{H}$	1	House supply
δ	0.99	DNWR
Z_h	1	House-to-capital technology
τ	1	
r	0.04	Risk-free rate on the liquid saving
r_b	0.04	Mortgage rate
ψ_h	0.04	Housing wealth adjustment cost
ψ_s	1	Housing maintenance cost
β	0.96	Utility discounting
θ	0.8	Loan-to-value ratio
γ	2	CRRA coefficient
η	0.82	Consumption preference
$N_\xi, \Pr\{\xi' = 0\|0\}, \Pr\{\xi' = 1\|1\}$	2, 0.9, 0.9	Unemployment process
ξ	$\in \{0, 0.08\}$	
N_p, N_p^σ	5,2	House price process

Notes: The notations N_Z^σ and N_p^σ represent how many standard deviation to apply in the Tauchen (1986) algorithm.

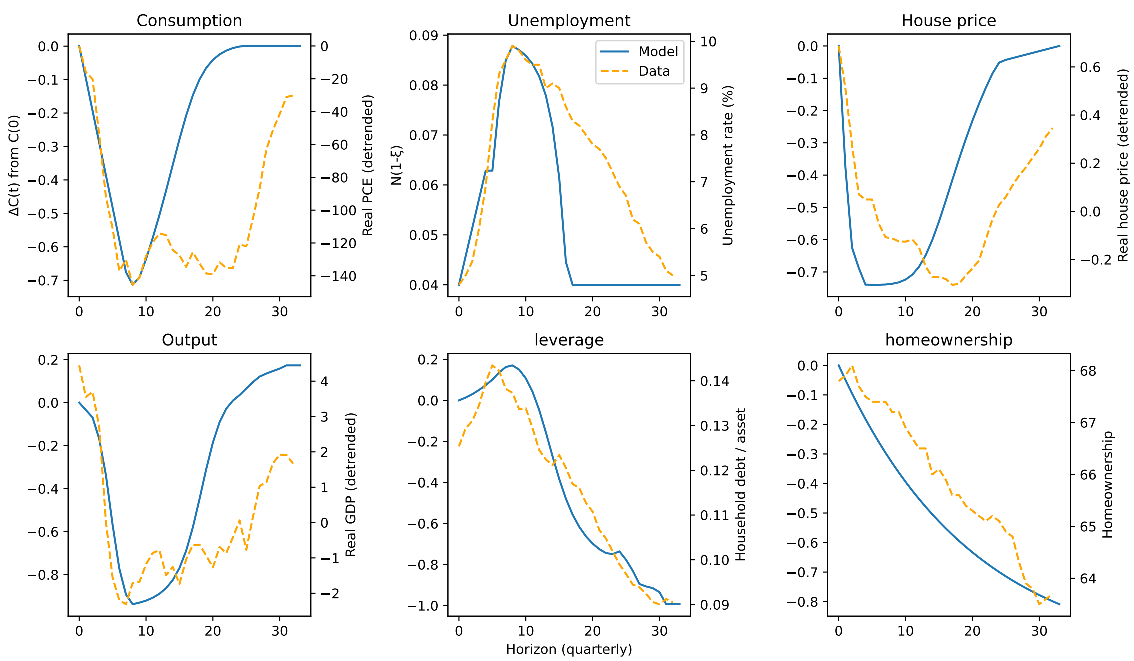
2.5 Quantitative results

Figure 2.2 presents the nonlinear impulse responses of selected aggregate variables to a one-time, 20% deviation shock, starting from the deterministic steady state. Each panel compares the model's response to corresponding U.S. data from 2007Q1 to 2015Q4, displayed on the secondary vertical axis.[6] The model predictions and the data show similar pattern and persistence, especially in the early periods after the shock. Panel 1 shows the percentage deviation of aggregate consumption C_t from the

[6]Certain empirical series are detrended to remove long-term growth components, as the model does not incorporate trend growth.

steady state, benchmarked against real personal consumption expenditures.[7] Panel 2 plots the unemployment rate alongside actual U.S. unemployment data. Panel 3 reports the percentage change in the model-implied house price p_t compared to observed house prices. Panel 4 illustrates the percentage change in output Y_t, aligned with U.S. real GDP over the same period. Panel 5 plots the change in household leverage, measured as $B_t/(p_t\bar{H})$, and compares it with the debt-to-asset ratio of households and nonprofit organizations from the Federal Reserve's Flow of Funds. Finally, Panel 6 presents the change in the model-generated homeownership rate S_t, alongside the empirical homeownership rate reported by the U.S. Census Bureau.

Figure 2.2: Impulse response of the one-shot-deviation shock



Facing the large one-shot-deviation shock, aggregate consumption drops by approximately 70% over the first 9 quarters, as shown by the blue solid line. This sharp decline is substantially larger than the drop observed in the actual data (orange dashed line), which reflects a more moderate contraction in real personal consumption expenditure. The exaggerated response in the model is partly due to the

[7] A value of 0.01 on the vertical axis represents a 1% deviation.

absence of long-term growth, which causes the simulated consumption path to deviate more significantly from trend-relative measures. After reaching its trough, the model predicts a gradual recovery, with consumption returning to the steady state around quarter 25. In contrast, the data exhibit much more persistent weakness, with consumption remaining below pre-shock levels throughout the sample period. This difference in persistence may stem from the model's use of a one-time shock, whereas in reality the economy experienced more sustained or recurring shocks. Additionally, real-world dynamics such as institutional frictions may further delay recovery in actual consumption behavior.

The model predicts that the unemployment rate rises sharply from 4% to just under 9% within 8 quarters, closely matching the peak level and timing observed in the actual data, as shown in the figure. However, while the initial rise is similar, the model-generated unemployment rate quickly reverts to its steady-state level after about 16 quarters. In contrast, the observed unemployment rate remains elevated for a considerably longer period, declining only gradually over the entire sample horizon. This difference in persistence likely reflects the greater persistence of shocks in reality, as the model assumes a one-time shock with no further propagation. Moreover, additional labor market frictions, such as slow job matching, skill mismatch, or discouraged workers exiting the labor force, could have contributed to the slower decline in actual unemployment (Branch et al., 2016). Notably, the stagnation in labor force growth during the post-crisis period may have further impeded recovery in the data, a feature not captured in the current model framework.

Following the one-shot-deviation shock, the model predicts that house prices fall sharply by more than 70% within the first 6 quarters, as shown by the blue solid line. In contrast, the actual housing price decline, illustrated by the orange dashed line, is both smaller in magnitude and more gradual, with the trough occurring several quarters later. Even though the model-generated response is more immediate and volatile than the empirical counterpart, the model-generated response essentially explain a large fraction of the pattern in the data. This discrepancy may reflect real-world policy interventions that buffered the housing market during the crisis period, such as government-supported mortgage programs and housing market stabilization efforts, which delayed or softened price adjustments. Additionally, housing is a financial asset subject to regulatory oversight, and post-crisis financial market reforms, along with ongoing frictions in credit access and refinancing, likely contributed to the prolonged and smoother adjustment path observed in the data. These features are abstracted from in the model, which helps explain the faster and deeper response of house prices in the simulated economy.

The model's output response closely mirrors the data in the immediate aftermath of the shock, with both series showing a sharp decline followed by a trough around quarter 10. However, the recovery in the data is noticeably slower, a pattern consistent with the persistence observed in consumption dynamics. This difference may again be attributed to more persistent real-world shocks and additional frictions not captured in the model. For household leverage, the model replicates the observed dynamics well: leverage rises for several quarters post-shock, peaking around quarter 8, and then gradually declines over time. The homeownership rate also exhibits a qualitatively similar downward trend in both the model and the data. However, the model predicts a substantially larger decline. This overreaction may stem from the modeling choice: homeownership is proxied by the share s_t of housing wealth allocated to residential use. In the model, households endogenously choose between residential and investment use of housing. In reality, according to ACS data, most U.S. households own only one home, and secondary homeownership is relatively rare[8].

To evaluate the individual contributions of the mechanism proposed in the paper, as well as the roles of the collateral constraint and DNWR, I conduct comparative

[8] `https://eyeonhousing.org/2024/09/the-nations-stock-of-second-homes-2/`

statics by selectively removing each friction and both simultaneously. This yields four scenarios: the baseline economy, an economy with only the collateral constraint (CC only), an economy with only DNWR (DNWR only), and a frictionless economy. For comparability, all four economies begin from the same initial state and are subjected to an identical one-shot deviation shock to Z_t. The resulting impulse responses of the three main outcome variables are presented in Figure 2.3.

Figure 2.3: Comparative statistics

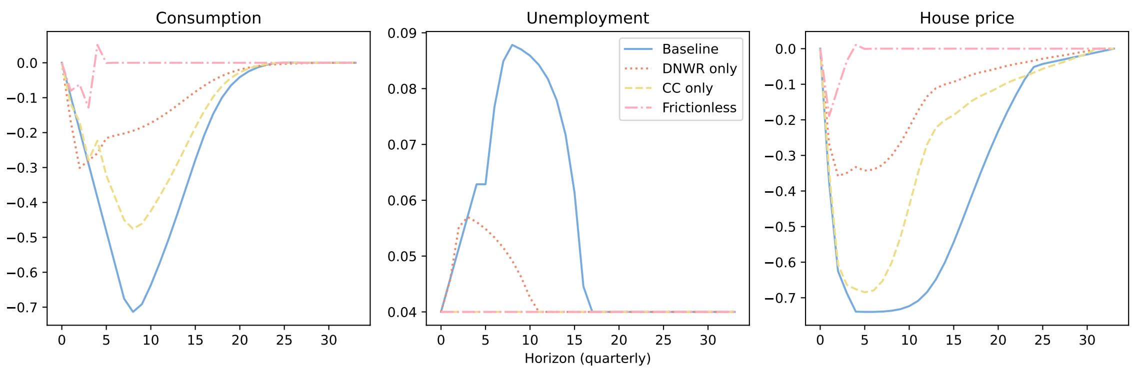


Figure 2.3 presents the comparative impulse responses of aggregate consumption under the four model specifications. Among them, the baseline economy exhibits the largest drop in consumption, highlighting the compounded effect of both frictions. In contrast, the frictionless economy shows no persistent deviation in consumption following the shock, indicating sufficient consumption smoothing and perfect insurance in the absence of frictions. The economy with only the collateral constraint active (CC only) shows the second-largest decline in consumption, around 45%, underscoring the critical role of financial market frictions in amplifying the effects of shocks. The economy with only DNWR active exhibits a more moderate decline of about 30%, suggesting a smaller standalone contribution of nominal wage rigidities. These comparative results reinforce the discussion in Chapter 1 regarding the importance of nominal wage rigidity in shaping consumption dynamics, particularly through its interaction with collateral constraints.

The figure displays the comparative impulse responses of the unemployment rate across the four economies. The baseline economy exhibits the largest response to the shock, with unemployment rising by approximately 5 percentage points before gradually returning to steady state. In contrast, both the frictionless economy and the economy with only the collateral constraint show no increase in unemployment, reflecting the absence of wage rigidity, which eliminates involuntary unemployment in these settings. The DNWR-only economy generates a moderate increase in unemployment of about 1.5 percentage points, indicating that wage rigidity alone accounts for a limited portion of the baseline response. These results not only reinforce the justification in Chapter 1 regarding the importance of nominal wage rigidity, but also highlight the amplification effect of the collateral constraint. Specifically, the interaction between DNWR and financial frictions amplifies both the magnitude and persistence of unemployment dynamics in the baseline economy, relative to the case with wage rigidity alone.

The figure shows the comparative impulse responses of house prices across the four economies. The baseline economy exhibits the largest decline in house prices, with a peak drop of approximately 70%, highlighting the combined impact of financial and nominal rigidities. The frictionless economy displays only a temporary and modest drop of about 20%, suggesting limited amplification in the absence of frictions. The collateral constraint–only economy experiences a sharp decline of around 65%, underscoring the sensitivity of asset prices to household balance sheet conditions and the role of financial market frictions. The DNWR-only economy shows a more moderate decline of about 30%, indicating that nominal wage rigidity alone contributes less significantly to house price dynamics. These results are consistent with the empirical evidence discussed in Chapter 1, which documents large house price responses to aggregate shocks. The findings here further support the presence of amplification effects arising from the interaction between financial and nominal frictions.

Finally, I summarize the welfare implications under the four model specifications. Table 2.3 reports two welfare measures: ex-post discounted utility and consumption equivalent variation (CEV). The ex-post discounted utility is computed by plugging the realized paths of c_t, s_t, and h_t into the utility function and discounting back to the beginning of the simulation. The CEV is defined as the percentage change in consumption that would equate social welfare between a given economy and the frictionless benchmark, holding (s_t, h_t) fixed. In the baseline economy, the ex-post discounted utility reaches the lowest level at -23.227, corresponding to a CEV of 516.0%, indicating substantial welfare losses due to the combined frictions. In the economy with only the collateral constraint (CC only), the ex-post utility improves to -11.942, and the associated CEV is reduced to 173.7%, suggesting that financial frictions alone account for a large share of the welfare loss. The DNWR-only economy yields an ex-post utility of -6.745 and a CEV of 36.4%, reflecting a more limited welfare impact from nominal wage rigidity in isolation. As expected, the frictionless economy produces the highest welfare level with an ex-post utility of -5.230 and serves as the reference point with a CEV of 0.0%. These results confirm that both types of frictions contribute to welfare deterioration, with financial frictions playing the dominant role.

Table 2.3: Welfare analysis

	Baseline	CC only	DNWR only	Frictionless
Ex-post discounted utility	-23.227	-11.942	-6.745	-5.230
CEV (%)	516.0	173.7	36.4	0.0

2.6 Conclusion

In this chapter, I construct a quantitative heterogeneous agent model and solve for the recursive equilibrium globally to examine the role of two key frictions, collateral constraints and DNWR, in shaping the macroeconomic dynamics of the U.S. economy following the Great Recession. The model successfully replicates several empirical patterns observed in the data, including the depth and persistence of declines in consumption, house prices, and unemployment. Through a series of comparative statics exercises, I evaluate the relative contributions of each friction individually and in combination, and assess the associated welfare implications under each scenario.

The findings provide a coherent and quantitatively grounded explanation for the macroeconomic dynamics observed in the period after the Great Recession. In particular, the results highlight the critical role of the interaction between collateral constraints and DNWR in amplifying and propagating the effects of large shocks. This interaction generates substantial and persistent deviations from steady state, consistent with empirical evidence. The analysis underscores the importance of incorporating economic frictions in macroeconomic models, both to improve empirical fit and to guide policy design. These insights have meaningful implications for the formulation of stabilization policies aimed at mitigating the impact of adverse shocks in frictional economies.

Appendix A

Chapter 1

A.1 Data documentation

This appendix provides detailed documentation for the construction and contents of a new US county-level panel dataset spanning from 2003 to 2019. The dataset includes various economic and demographic variables that can be used to analyze a range of topics related to local economies, household behavior, and housing markets. This documentation provides a detailed account of data sources, construction, and limitations.

For further details, please check our data repository *CountyPlus* on GitHub.

A.1.1 Measurement error of consumption

In some states, only sales tax revenue data are available while the taxable sales (consumption) are not. Directly estimating the in-state distribution of consumption using tax revenue leads to measurement error. In year t, households living in county $i = 1, \ldots, I$ may consume J types of goods. Denote the true value of consumption by $C_{j,i,t}$ for goods j. The true consumption distribution $\tilde{S}_{i,t}$ of county i is defined as:

$$\tilde{S}_{i,t} := \frac{C_{i,t}}{\sum_{m=1}^{I} C_{m,t}} = \frac{\sum_{j=1}^{J} C_{j,i,t}}{\sum_{m=1}^{I} \sum_{j=1}^{J} C_{j,m,t}} \tag{A.1}$$

If we estimate the consumption distribution using tax revenue, then the observed consumption distribution $S_{i,t}$ is:

$$S_{i,t} := \frac{T_{i,t}}{\sum_{m=1}^{I} T_{m,t}} = \frac{\sum_{j=1}^{J} C_{j,i,t}\tau_{j,t}}{\sum_{m=1}^{I} \sum_{j=1}^{J} C_{j,m,t}\tau_{j,t}} \tag{A.2}$$

where $T_{i,t}$ is the sales tax revenue of county i, $\tau_{j,t}$ is the tax rate applied to goods type j. The measurement error is then defined as:

$$e_{i,t} := S_{i,t} - \tilde{S}_{i,t} \tag{A.3}$$

One can see that the measurement error $e_{i,t}$ is zero if the same tax rate applies to tall types of goods. Such a measurement error is raised by the differential tax rate among goods. To further check it, we define the following county average tax rate and state average tax rate

$$\bar{\tau}_{i,t} = \frac{\sum_{j=1}^{J} C_{j,i,t}\tau_{j,t}}{\sum_{j=1}^{J} C_{j,i,t}} \tag{A.4}$$

$$\bar{\tau}_{t} = \frac{\sum_{m=1}^{I}\sum_{j=1}^{J} C_{j,m,t}\tau_{j,t}}{\sum_{m=1}^{I}\sum_{j=1}^{J} C_{j,m,t}} \tag{A.5}$$

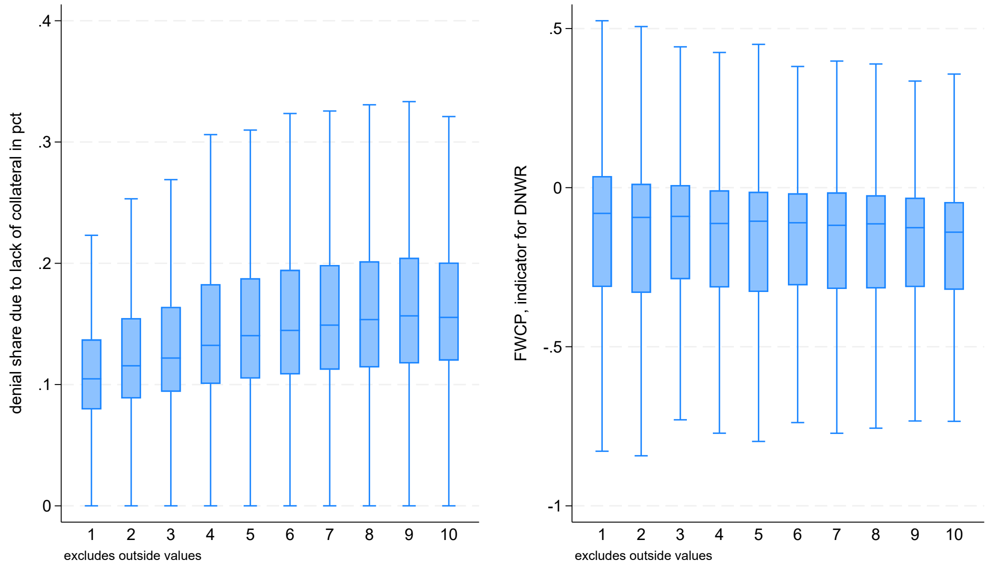
which are average tax rates weighted by consumption. Thus, the observed distribution is proportional to the true distribution

$$S_{i,t} = \frac{\bar{\tau}_{i,t}\sum_{j=1}^{J} C_{j,i,t}}{\bar{\tau}_{t}\sum_{m=1}^{I}\sum_{j=1}^{J} C_{j,m,t}} = \frac{\bar{\tau}_{i,t}}{\bar{\tau}_{t}}\tilde{S}_{i,t} \tag{A.6}$$

The measurement error $e_{i,t} = \left(\frac{\bar{\tau}_{i,t}}{\bar{\tau}_{t}} - 1\right)\tilde{S}_{i,t}$ is a non-classical measurement error. Therefore, for those states reporting tax revenue only, we correct such measurement error due to tax rate by multiplying $\bar{\tau}_{t}/\bar{\tau}_{i,t}$ on the observed consumption distribution.

A.1.2 Supplementary figures

Figure A.1: Friction distribution by income decile



Notes: 1. The x-axis is the decile groups of county's real median household income. For example, 1 represents the counties with real median household income locating in the $[0\%, 10\%)$ quantile range. 2. Outliers are dropped in the visualization but not in computing the statistics. 3. The sample is the same as the baseline estimation.

A.2 Theory model

A.2.1 Model system

Solving the lifetime problems of the households and entrepreneurs, the system of equilibrium conditions is listed from Equation (A.7) to Equation (A.25). Additionally, the optimal consumption policy $c(s^t)$, which is easy to derive, is a $1-\beta$ fraction of the net worth $n(s^t)$.

$$\forall s^t, \tag{A.7}$$

$$a(s^t) = b(s^t) \tag{A.8}$$

$$h^l(s^t) + h(s^t) = H \tag{A.9}$$

$$c^l(s^t) + c(s^t) = Y_t \tag{A.10}$$

$$Y_t = Al_t^{\alpha}(u_t h_{t-1})^{1-\alpha} \tag{A.11}$$

$$w_t = \alpha A l_t^{\alpha-1}(u_t h_{t-1})^{1-\alpha} \tag{A.12}$$

$$w_t \geq \delta w_{t-1} \tag{A.13}$$

$$(w_t - \delta w_{t-1})(1 - l_t) = 0 \tag{A.14}$$

$$n(s^t) := p(s^t)h(s^{t-1}) + y(s^t) - b(s^t) \tag{A.15}$$

$$c(s^t) + p(s^t)h(s^t) = n(s^t) + \sum_{s^{t+1}} q(s^{t+1}|s^t)b(s^{t+1}) \tag{A.16}$$

$$c^l(s^t) + \sum_{s^{t+1}} q(s^{t+1}|s^t)a(s^{t+1}) + p(s^t)[h^l(s^t) - h^l(s^{t-1})] = w(s^t)l(s^t) + a(s^t) \tag{A.17}$$

$$l(s^t) = \begin{cases} 1, else \\ (\frac{\alpha A}{\delta w_{t-1}})^{\frac{1}{1-\alpha}} h_{t-1}u_t, u_t < (\frac{\delta w_{t-1}}{\alpha A})^{\frac{1}{1-\alpha}} h_{t-1}^{-1}, \end{cases} \tag{A.18}$$

$$\text{and } \forall s^{t+1}, \tag{A.19}$$

$$\beta\pi(s^{t+1}|s^t) = q(s^{t+1}|s^t) \tag{A.20}$$

$$\frac{\gamma}{h^l(s^t)} + \beta\pi(s^{t+1}|s^t)p(s^{t+1}) = p(s^t) \tag{A.21}$$

$$\frac{q(s^{t+1}|s^t)}{c(s^t)} = \frac{\beta\pi(s^{t+1}|s^t)}{c(s^{t+1})} + \beta\pi(s^{t+1}|s^t)\mu(s^{t+1}) \tag{A.22}$$

$$\frac{p(s^t)}{c(s^t)} = \frac{\beta\pi(s^{t+1}|s^t)}{c(s^{t+1})}[p(s^{t+1}) + (1-\alpha)^2 A l(s^{t+1})^{\alpha} u(s^{t+1})^{1-\alpha} h(s^t)^{-\alpha}]$$

$$+ \theta\beta\pi(s^{t+1}|s^t)\mu(s^{t+1})p(s^{t+1}) \tag{A.23}$$

$$b(s^{t+1}) \leq \theta p(s^{t+1})h(s^t) \tag{A.24}$$

$$y(s^t) = (1-\alpha)A l(s^t)^{\alpha}(u(s^t)\bar{h}(s^{t-1}))^{1-\alpha} \tag{A.25}$$

Combining Equation (A.21), Equation (A.23), and Lemma A.2.2, there is risk sharing condition $\forall s^{t+1}$:

$$\frac{1}{c(s^t)} \frac{1}{1 - \frac{\gamma}{p(s^t)h^l(s^t)}} = \frac{1}{c(s^{t+1})}\{1 + (1-\alpha)^2 A[l(s^{t+1})]^{\alpha}[u(s^{t+1})]^{1-\alpha}$$

$$[h(s^t)]^{-\alpha}[p(s^{t+1})]^{-1}\} + \theta\mu(s^{t+1}) \tag{A.26}$$

A.2.2 Proof: Proposition 1.2.1

Proposition (Persistent effect)**.** *There exist a unique continuation equilibrium that depends on the states* $(u_1, h_0, b_1(u_1))$*. In the continuation equilibrium, the collateral constraint is binding for a finite number of periods* J*, with* $J = 0$ *iff* $n_1(u_1) \geq \bar{n}_1 := \bar{p}\bar{h}(1-\beta\theta)/\beta$*, where* $\bar{p}$ *and* $\bar{h}$ *are jointly determined by*

$$(1-\beta)\bar{p} = \frac{\gamma}{H-\bar{h}} \tag{A.27}$$

$$(1-\beta)\bar{p} = \beta(1-\alpha)\bar{h}^{-\alpha} \tag{A.28}$$

Proof. First, we define a continuation equilibrium since date 2, then we show the threshold for the collateral constraint to bind. We assume that u_t is drawn from a degenerated distribution after date 1 i.e. there is no uncertainty. The economy follows a deterministic path which can be characterized as follows:

$$h = \frac{\beta n}{p - \theta\beta p'} \tag{A.29}$$

$$n' = (1-\theta)p'h + (1-\alpha)h^{1-\alpha} \tag{A.30}$$

$$p = \frac{\gamma}{H-h} + \beta p' \tag{A.31}$$

$$\frac{p}{n} = \frac{\beta}{n'}[p' + A(1-\alpha)^2 h^{-\alpha}] + \theta\beta p'(\frac{1}{n} - \frac{1}{n'}) \tag{A.32}$$

Knowing net worth n at the beginning of the current period, then the whole path is determined then. We recursively return to date 1 and look at the definition of net worth n_1:

$$n_1(u_1) = p_1(u_1) \cdot h_0 + (1-\alpha)Ah_0^{1-\alpha}(l_1(u_1))^{\alpha}u_1^{1-\alpha} - b_1(u_1) \tag{A.33}$$

When knowing states u_1, $b_1(u_1)$ and h_0, the deterministic path is determined then. If collateral constraint is unbinding, then net worth must be fully insured and keep constant across time. The Equation (A.29) suggests that if net worth n_1 is above such a fully insured level $\bar{n}_1 := \bar{p}\bar{h}(1-\beta\theta)/\beta$, then borrower saved enough to instantly adjust net worth to the fully insured level. Otherwise, the collaterl constraint will bind at least one another period. □

A.2.3 Proof: Proposition 1.2.2

Lemma A.2.1 (Entrepreneur's consumption policy)**.** $\forall s^t, c(s^t) = (1-\beta)n(s^t)$.

Lemma A.2.2 (Household's housing wealth lower bound)**.** *For all* s^t, $p(s^t)h^l(s^t) > \gamma$

Proof. Rearranging Equation (A.21), then for all s^t, $\beta\pi(s^{t+1}|s^t) = \frac{p(s^t)-\gamma/h^l(s^t)}{p(s^{t+1})} > 0$ must hold on the support of s^{t+1}. Knowing $p(s^{t+1}) > 0$, then $p(s^t)h^l(s^t) > \gamma$ □

Lemma A.2.3 (Non-negative net worth with collateral constraint)**.** *Iff* $0 \leq \theta < \frac{1}{1-\frac{\gamma}{p(s^t)h^l(s^t)}}$ *and* $n(s^t) > 0$*, then* $n(s^{t+1}) > 0$*.*

Proof. Plugging entrepreneur's consumption policy and Equation (A.22) into the risk sharing condition of Equation (A.26), one can find that $n(s^{t+1}) > 0$ *iff* $\theta(1 - \frac{\gamma}{p(s^t)h^l(s^t)}) < 1$. By Lemma A.2.2, one can get the range of θ. □

Lemma A.2.4 (Involuntary unemployment)**.** *There exists a threshold* $u^*(s^t)$ *of* $u(s^{t+1})$ *such that if* $u(s^{t+1}) < u^*(s^t)$*, then there is involuntary unemployment in equilibrium. The equilibrium employment is:*

$$l(s^{t+1}) = \begin{cases} 1, u(s^{t+1}) \geq u^*(s^t) \\ \left(\frac{\alpha A}{\delta w(s^t)}\right)^{\frac{1}{1-\alpha}} h(s^t)u(s^{t+1}), \textit{else} \end{cases} \tag{A.34}$$

$$u^*(s^t) := \left(\frac{\delta w(s^t)}{\alpha A}\right)^{\frac{1}{1-\alpha}} [h(s^t)]^{-1} \tag{A.35}$$

Proof. When the DNWR constraint is binding, one can solve the equilibrium employment by solving the profit maximization problem of entrepreneurs. When DNWR constraint is exactly binding, $l(s^{t+1}) = 1$ such that a threshold $u^*(s^t)$ can be solved. □

Lemma A.2.5 (Condition of binding collateral constraint)**.** *With θ satisfying the condition in Lemma A.2.3 and $\alpha > 0$, for all s^t, there exists a unique $\hat{u}(s^{t+1})$ defined in Equation (A.43), such that, if $u(s^{t+1}) \geq \hat{u}(s^{t+1})$, then $b(s^{t+1}) = \theta p(s^{t+1})h(s^t)$ in equilibrium.*

Proof. Let's consider an arbitrary state $u(s^{t+1})$ where the $t+1$ collateral constraint is not binding. By entrepreneur's Euler equation of Equation (A.22), consumption is fully insured iff the collateral constraint is not binding such that $n(s^{t+1}) = n(s^t)$. Applying Lemma A.2.1, the risk sharing condition of Equation (A.26) leads to

$$\begin{aligned} p(s^{t+1}) = & \left[\frac{p(s^t)h^l(s^t)}{\gamma} - 1\right](1-\alpha)^2 A[l(s^{t+1})]^{\alpha}[u(s^{t+1})]^{1-\alpha}[h(s^t)]^{-\alpha} \\ =&(1-\alpha)^2 \frac{1}{h(s^t)}\left[\frac{p(s^t)h^l(s^t)}{\gamma} - 1\right] Y(s^{t+1}) \end{aligned} \tag{A.36}$$

Plugging it into the net worth definition of Equation (A.15), there is

$$b(s^{t+1}) = (1-\alpha)Y(s^{t+1})\left[\left(\frac{p(s^t)h^l(s^t)}{\gamma} - 1\right)(1-\alpha) + 1\right] - n(s^t) \tag{A.37}$$

$$\frac{\partial b(s^{t+1})}{\partial u(s^{t+1})} = (1-\alpha)\left[\left(\frac{p(s^t)h^l(s^t)}{\gamma} - 1\right)(1-\alpha) + 1\right]\frac{\partial Y(s^{t+1})}{\partial u(s^{t+1})} \tag{A.38}$$

Equation (A.36) leads to the expression of the borrowing limit:

$$\theta p(s^{t+1})h(s^t) = \theta(1-\alpha)^2\left(\frac{p(s^t)h^l(s^t)}{\gamma} - 1\right)Y(s^{t+1}) \tag{A.39}$$

$$\frac{\partial[\theta p(s^{t+1})h(s^t)]}{\partial u(s^{t+1})} = \theta(1-\alpha)^2\left(\frac{p(s^t)h^l(s^t)}{\gamma} - 1\right)\frac{\partial Y(s^{t+1})}{\partial u(s^{t+1})} \tag{A.40}$$

Knowing that $\frac{\partial Y(s^{t+1})}{\partial u(s^{t+1})} > 0$, and $p(s^t)h^l(s^t) > \gamma$ (Lemma A.2.2), the relative speed of debt growing and borrowing limit growing by $u(s^{t+1})$ is:

$$\frac{\partial b(s^{t+1})/\partial u(s^{t+1})}{\partial[\theta p(s^{t+1})h(s^t)]/\partial u(s^{t+1})} = \frac{1}{\theta}\frac{1 + (1-\alpha)\left(\frac{p(s^t)h^l(s^t)}{\gamma} - 1\right)}{(1-\alpha)\left(\frac{p(s^t)h^l(s^t)}{\gamma} - 1\right)} \tag{A.41}$$

To assure a non-negative net worth $n(s^{t+1}) > 0$, θ must satisfy the condition in Lemma A.2.3. Combining with Lemma A.2.2 and parameter restriction $\alpha > 0$, the RHS of the relative speed must satisfy:

$$\begin{aligned}\text{RHS} >& \frac{1+(1-\alpha)\left(\frac{p(s^t)h^l(s^t)}{\gamma}-1\right)}{(1-\alpha)\left(\frac{p(s^t)h^l(s^t)}{\gamma}-1\right)}\left(1-\frac{\gamma}{p(s^t)h^l(s^t)}\right)\\ =& 1+\left(\frac{1}{1-\alpha}-1\right)\frac{\gamma}{p(s^t)h^l(s^t)} > 1\end{aligned} \tag{A.42}$$

The above inequality tells that debt always changes faster than the borrowing limit by $u(s^{t+1})$. Thus, there must be a unique threshold $\hat{u}$ such that for all $u(s^{t+1}) \geq \hat{u}$, the collateral constraint binds. To obtain the value of $\hat{u}$, let's consider the point $u(s^{t+1}) = \hat{u}$ where the collateral constraint exactly binds. Equation (A.36) still holds and Equation (A.37) leads to the following equation if DNWR is unbinding:

$$n(s^t) = A(1-\alpha)[h(s^t)]^{1-\alpha}\left[1+(1-\theta)(1-\alpha)\left(\frac{p(s^t)h^l(s^t)}{\gamma}-1\right)\right]\cdot[\hat{u}(s^{t+1})]^{1-\alpha} \tag{A.43}$$

Specially, if DNWR is binding at $\hat{u}(s^{t+1})$:

$$n(s^t) = A(1-\alpha)h(s^t)\left[1+(1-\theta)(1-\alpha)\left(\frac{p(s^t)h^l(s^t)}{\gamma}-1\right)\right]\cdot\left(\frac{\alpha A}{\delta w(s^t)}\right)^{\frac{\alpha}{1-\alpha}}\hat{u}(s^{t+1}) \tag{A.44}$$

□

Lemma A.2.6 (Amplification effect on $n_1(u_1)$ in Case 4 and its iff conditions)**.** *If initial condition of Equation (A.50) holds, then* $[\hat{u}_1, u_0^*]$ *is not empty. For all* u_1 *in this interval, there is* $\varepsilon_1^p < 1$ *and* $n_1'(u_1) > 0$.

Proof. Assume DNWR and collateral constraints are both binding and $[\hat{u}_1, u_0^*] \neq \emptyset$. By Lemma A.2.4, there is $\varepsilon_1^l = 1$. Plugging ε_1^l and l_1 level into the equilibrium shock effect of Equation (A.57), the effect degenerates to:

$$n_1'(u_1) = M_0 p_1^{-1}(1-\varepsilon_1^p) \tag{A.45}$$

$$M_0 := n_0(1-\alpha)^2 A \left(\frac{\alpha A}{\delta w_0}\right)^{\frac{\alpha}{1-\alpha}} \left[\frac{1}{1-\frac{\gamma}{p_0 h_0^l}} - \theta\right]^{-1} > 0 \tag{A.46}$$

Thus, $n_1'(u_1) > 0$ (amplification effect) iff $\varepsilon_1^p < 1$. To show this, taking derivatives about u_1 on both sides of Equation (A.60) and plugging ε and l_1 in:

$$M_0 p_1^{-1}(1-\varepsilon_1^p) = (1-\theta)\frac{p_1 h_0}{u_1}\varepsilon_1^p + (1-\alpha)A\left(\frac{\alpha A}{\delta w_0}\right)^{\frac{\alpha}{1-\alpha}} h_0 \tag{A.47}$$

For convenience, let

$$X_0 := \left[\frac{1}{1-\frac{\gamma}{p_0 h_0^l}} - \theta\right]^{-1} > 0$$

then we solve ε_1^p as:

$$\begin{aligned}\varepsilon_1^p =& \frac{M_0}{M_0 + \frac{(1-\theta)h_0}{u_1}p_1^2}\left[1 - \frac{p_1 h_0}{X_0 n_0}\frac{1}{1-\alpha}\right] \\ =& \underbrace{\frac{M_0}{M_0 + \frac{(1-\theta)h_0}{u_1}p_1^2}}_{<1} \underbrace{\left[1-\alpha-\frac{p_1 h_0}{X_0 n_0}\right]\frac{1}{1-\alpha}}_{<1}\end{aligned} \tag{A.48}$$

It is obvious that $\varepsilon_1^p < 1$. Thus, $n_1'(u_1) > 0$ for all $u_1 \in [\hat{u}_1, u_0^*]$.

To ensure $[\hat{u}_1, u_0^*]$ is not empty, an iff condition is $\hat{u}_1 \leq u_0^*$. Recalling Lemma A.2.5 and Lemma A.2.4 and plugging in l_1 and b_1, there is

$$\begin{aligned}&\frac{n_0}{A(1-\alpha)h_0}\left(\frac{\delta w_0}{\alpha A}\right)^{\frac{\alpha}{1-\alpha}} \left[1 + (1-\theta)(1-\alpha)\left(\frac{p_0 h_0^l}{\gamma} - 1\right)\right]^{-1} \\ \leq& \left(\frac{\delta w_0}{\alpha A}\right)^{\frac{1}{1-\alpha}} h_0^{-1}\end{aligned} \tag{A.49}$$

which implies

$$\frac{w_0}{n_0} \geq \frac{\alpha}{\delta(1-\alpha)}\left[1+(1-\theta)(1-\alpha)(\frac{p_0 h_0^l}{\gamma}-1)\right]^{-1} \tag{A.50}$$

□

Proposition (Non-linear heterogeneous effect)**.** *There exist levels of the entrepreneur's financial friction parameter θ and the DNWR parameter δ, such that if*

$$\frac{w_0}{n_0} \geq \frac{\alpha}{\delta(1-\alpha)}\left[1+(1-\theta)(1-\alpha)(\frac{p_0 h_0^l}{\gamma}-1)\right]^{-1} \tag{A.51}$$

then, in equilibrium, the date 1 *collateral constraint and DNWR both bind when u_1 is in a non-empty interval $[\hat{u}_0, u_0^*]$ where*

$$u_0^* = \left(\frac{\delta w_0}{\alpha A}\right)^{\frac{1}{1-\alpha}} h_0^{-1} \tag{A.52}$$

$$\hat{u}_0 = \frac{n_0}{A(1-\alpha)h_0}\left(\frac{\delta w_0}{\alpha A}\right)^{\frac{\alpha}{1-\alpha}}\left[1+(1-\theta)(1-\alpha)\left(\frac{p_0 h_0^l}{\gamma}-1\right)\right] \tag{A.53}$$

, and the u_1 shock effects:

$$\frac{\partial c_1}{\partial u_1} > 0 \qquad \frac{\partial l_1}{\partial u_1} > 0 \qquad \frac{\partial p_1}{\partial u_1} > 0 \textit{ or } < 0 \textit{ or } = 0 \textit{ depends} \tag{A.54}$$

which are also non-linear functions of θ and δ.

Proof. We do the proof in two steps. Firstly, we derive the shock effect $n_1'(u_1)$ in equilibrium. Then, we show the conditions for different cases in which $n_1(u_1)$ (non-linear heterogeneous effect).

Let's start from the risk sharing condition at date 1 of Equation (A.26):

$$\frac{1}{c_0}\frac{1}{1-\frac{\gamma}{p_0 h_0^l}} = \frac{1}{c_1}\left[1+(1-\alpha)^2 A l_1^{\alpha} u_1^{1-\alpha} h_0^{-\alpha} p_1^{-1}\right] + \theta\mu_1 \tag{A.55}$$

Rearranging the equation and plugging in consumption policy, and Equation (A.22), we obtain the equilibrium n_1 regardless if DNWR and collateral constraints are binding:

$$n_1 = n_0 \left[\frac{1}{1 - \frac{\gamma}{p_0 h_0^l}} - \theta \right]^{-1} \left[(1-\theta) + (1-\alpha)^2 A l_1^\alpha u_1^{1-\alpha} h_0^{-\alpha} p_1^{-1} \right] \tag{A.56}$$

Taking derivatives with respect to u_1 except two special points[1], there is equilibrium shock effect:

$$n_1'(u_1) = n_0 \left[\frac{1}{1 - \frac{\gamma}{p_0 h_0^l}} - \theta \right]^{-1} (1-\alpha)^2 A h_0^{-\alpha} \cdot (l_1^\alpha u_1^{-\alpha} p_t^{-1})(\alpha \varepsilon_1^l - \alpha + 1 - \varepsilon_1^p) \tag{A.57}$$

$$\varepsilon_1^l := \frac{u_1}{l_1} l_1'(u_1) \tag{A.58}$$

$$\varepsilon_1^p := \frac{u_1}{p_1} p_1'(u_1) \tag{A.59}$$

where ε_1^l is the shock elasticity of employment, and ε_1^p is the shock elasticity of house price. Lemma A.2.4 tells the level of $l_1(u_1)$, and suggests that $\varepsilon_1^l = 0$ if the DNWR is not binding while $\varepsilon_1^l = 1$ if the DNWR is binding. To obtain the value of p_1 and ε_1^p in equilibrium, we equate the equilibrium net worth of Equation (A.56) and the net worth definition of Equation (A.15).

$$\begin{aligned} & n_0 \left[\frac{1}{1 - \frac{\gamma}{p_0 h_0^l}} - \theta \right]^{-1} \left[(1-\theta) + (1-\alpha)^2 A l_1^\alpha u_1^{1-\alpha} h_0^{-\alpha} p_1^{-1} \right] \\ & = p_1 h_0 + (1-\alpha) A h_0^{1-\alpha} l_1^\alpha u_1^{1-\alpha} - b_1 \end{aligned} \tag{A.60}$$

Specially, as long as the collateral constraint is binding, there is a quadratic equation[2] that determines equilibrium house price p_1 the house price h_1 can be solved from the

[1]Specifically, the point where DNWR exactly binds (Lemma A.2.4) and the point where b_1 exactly binds. All other differentials in this paper involving l_1 and p_1 follows the same exclusion rule as well.

[2]One can show that the larger root of the quadratic equation is the only feasible solution.

following quadratic equation:

$$h_0 p_1^2 + \left[\frac{1-\alpha}{1-\theta}Y_1 - X_0 n_0\right] p_1 - \frac{X_0 n_0}{1-\theta}(1-\alpha)^2 \frac{1}{h_0} Y_1 = 0 \qquad \text{(A.61)}$$

where X_0 is defined in Lemma A.2.6.

To obtain the value of ε_1^p, we take derivatives about u_1 on both sides of Equation (A.60), then re-arrange terms to get the equation.

With formulas of the equilibrium shock effect and its components, now let's discuss the non-linear heterogeneity of the effect and derive conditions of each case. Let's use $n_1(u_1)$, or equivalently $c_1(u_1)$, as an example. In general, if $n_1'(u_1)$ varies by θ and/or δ, then there is heterogeneous effects. If such variation is different by the level of θ and/or δ, then there is non-linear heterogeneous effects. Lemma A.2.4 and Lemma A.2.5 suggests the following four possible cases of DNWR and collateral constraints:

- **Case 1**: DNWR not binding ($u_1 \geq u_0^*$), collateral constraint not binding ($u_1 \leq \hat{u}_1$)
- **Case 2**: DNWR binding ($u_1 < u_0^*$), collateral constraint not binding ($u_1 \leq \hat{u}_1$)
- **Case 3**: DNWR not binding ($u_1 \geq u_0^*$), collateral constraint binding ($u_1 > \hat{u}_1$)
- **Case 4**: DNWR binding ($u_1 < u_0^*$), collateral constraint binding ($u_1 > \hat{u}_1$)

In Case 1, the shock effect on consumption is equivalent to the friction-less economy in Section A.2.4 such that $n_1'(u_1) \equiv 0$ over the entire non-empty (u_1, θ, δ) subspace that satisfies Case 1. In Case 2, $n_1'(u_1)$ go the same as Case 1 while the shock effect on employment has non-linear heterogeneous effect over different level of δ (Lemma A.2.4). In Case 3, $n_1'(u_1) > 0$ iff $\varepsilon_1^p < 1-\alpha$ in which ε_1^p is a highly non-linear function of (θ, δ) simultaneously. One can show that $\frac{\partial n_1'(u_1)}{\partial \theta}$ and $\frac{\partial n_1'(u_1)}{\partial \delta}$ are not constants such that there is non-linear heterogeneous effects. The situation of $\varepsilon_1^p > 1-\alpha$

follows the same idea. In Case 4, $n_1'(u_1) > 0$ is always true. Lemma A.2.6 gives the proof and the condition for the interval $[\hat{u}_0, u_0^*]$ being non-empty.

□

A.2.4 A friction-less economy

In order to illustrate the non-linear heterogeneous effect of the frictions, we compute comparative statistics of the frictional economy in Section A.2.1 and a friction-less economy. In such a friction-less economy, there is no DNWR and collateral constraint such that the agents have perfect consumption insurance. In this case, allocations keep constant over time and there is no involuntary unemployment. The house price is determined by the risk sharing condition across households and entrepreneurs via combining their Euler equations about housing wealth:

$$\forall s^{t+1}, p(s^{t+1}) = \left[\frac{p(s^t)(H - h(s^t))}{\gamma} - 1\right](1-\alpha)^2 \frac{1}{h(s^t)} Y(s^{t+1}) \tag{A.62}$$

And, the optimal borrowing $b(s^{t+1})$ can be solved from the net worth definition by having $n(s^{t+1}) = n(s^t)$:

$$\forall s^{t+1}, b(s^{t+1}) = (1-\alpha)\left[(1-\alpha)\left(\frac{p(s^t)(H - h(s^t))}{\gamma} - 1\right) + 1\right] Y(s^{t+1}) - n(s^t) \tag{A.63}$$

A.2.5 Deterministic steady state

To set up the baseline scenario of our comparative statistics for illustration purpose, we assume the economy starts from a steady state defined in a deterministic version model. In such a deterministic version model, the distribution of u_t degenerates such that $\pi(s^{t+1}|s^t) \equiv 1$. Let's assume the mean of u_t is 1 for simplicity. In steady state, neither DNWR and collateral constraints binds, and the housing productivity shock

$u \equiv 1$. Eliminating the time notation, the system equations in steady state are listed as Equation (A.64) to Equation (A.66)[3].

$$\bar{b} < \theta \bar{p}\bar{h}, \bar{l} = 1, \bar{u} = 1 \tag{A.64}$$

$$(1-\beta)\bar{p} = \frac{\gamma}{H - \bar{h}} \tag{A.65}$$

$$(1-\beta)\bar{p} = \beta A(1-\alpha)^2 \bar{h}^{-\alpha} \tag{A.66}$$

Lemma A.2.7 (A continuum of deterministic steady states)**.** *Equation (A.64) to Equation (A.66) can uniquely determines a continuum of deterministic steady states indexed by steady state LTV ratio* $\bar{\nu} := \frac{\bar{b}}{\bar{p}\bar{h}} < \theta$. *In these steady states, either collateral constraint and DNWR is not binding.*

A.2.6 Baseline scenario for illustration

We consider a scenario that the economy starts from an unbinding deterministic steady state while the collateral constraint is almost binding. It depends on initial steady state LTV ratio $\nu_0 = \bar{\nu} < \theta$. Table A.1 lists all parameters values.

[3]The deterministic version model is different from a friction-less (stochastic) model in Section A.2.4, which leads to different house pricing. Specifically, in the deterministic version model, the $\beta\pi(s^{t+1}|s^t)$

Figure A.2: Non-linear heterogeneous effects

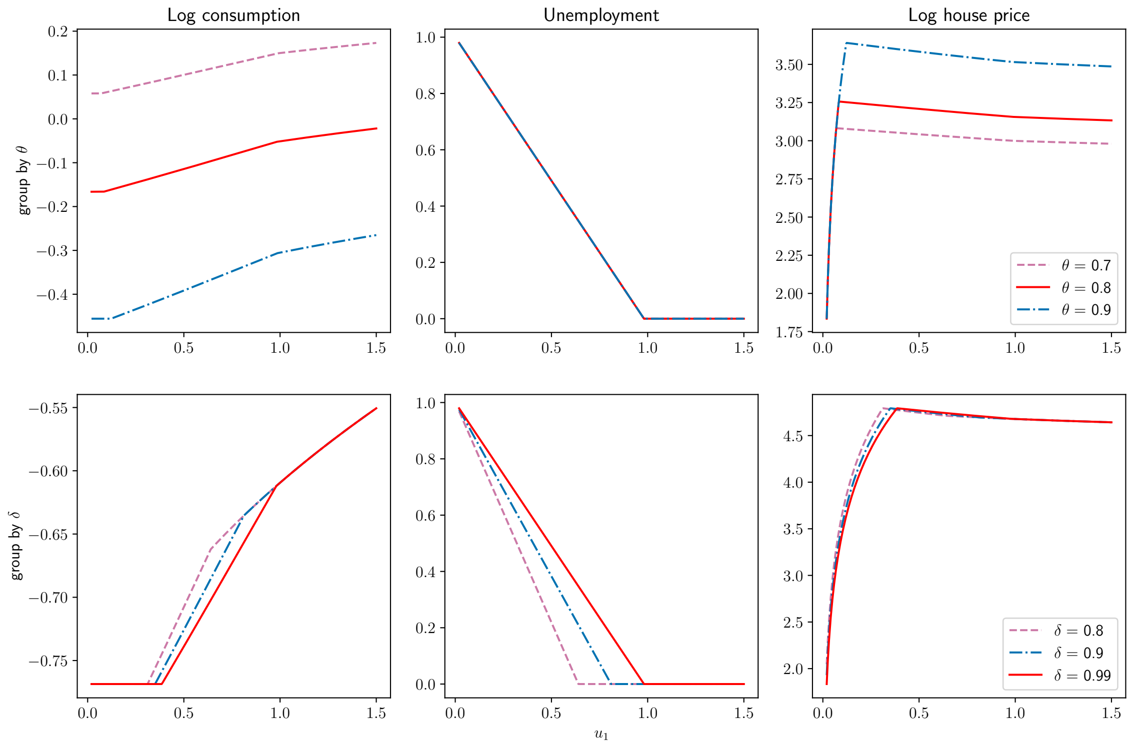


Table A.1: Baseline parameters

Parameter	Definition	Value
β	Utility discounting factor	0.9
α	Labor income share	0.7
δ	Parameter of DNWR	0.99
θ	Collateral constraint as LTV ratio	0.8
A	Technology level	1
$\bar{\nu}$	Steady state LTV ratio	0.79
γ	Housing preference	0.8
H	House supply	30

A.3 Robustness check: Degree selection

The baseline results use a second-order polynomial for the sieve estimator. To determine if there are potential higher-order effects that the approximation should capture, we do the estimation using third and fourth-order polynomials as robustness check. Figures A.3 and A.4 present the β estimates, which do not reveal new patterns beyond those captured by the baseline estimates.

Figure A.3: Order = 3

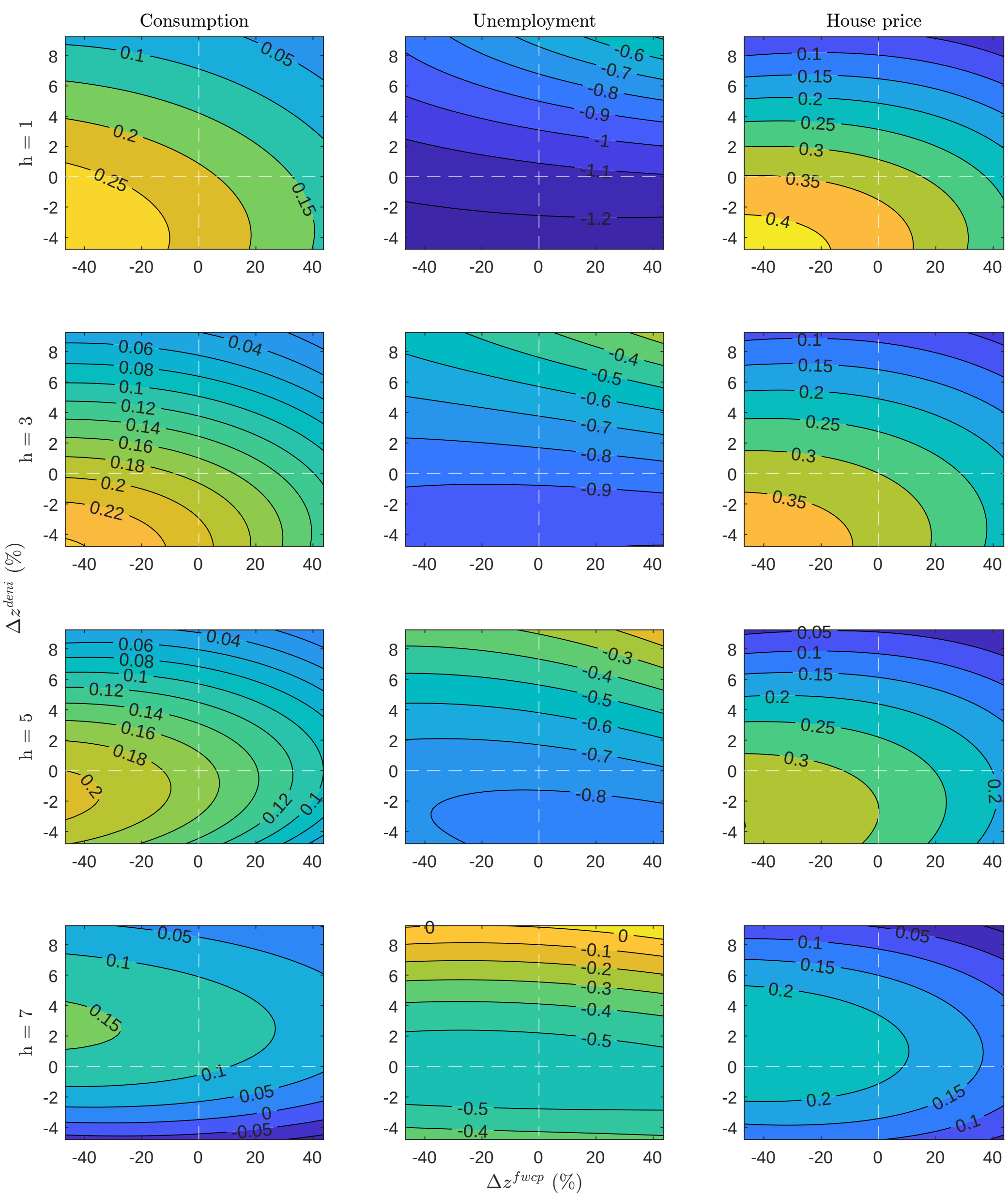
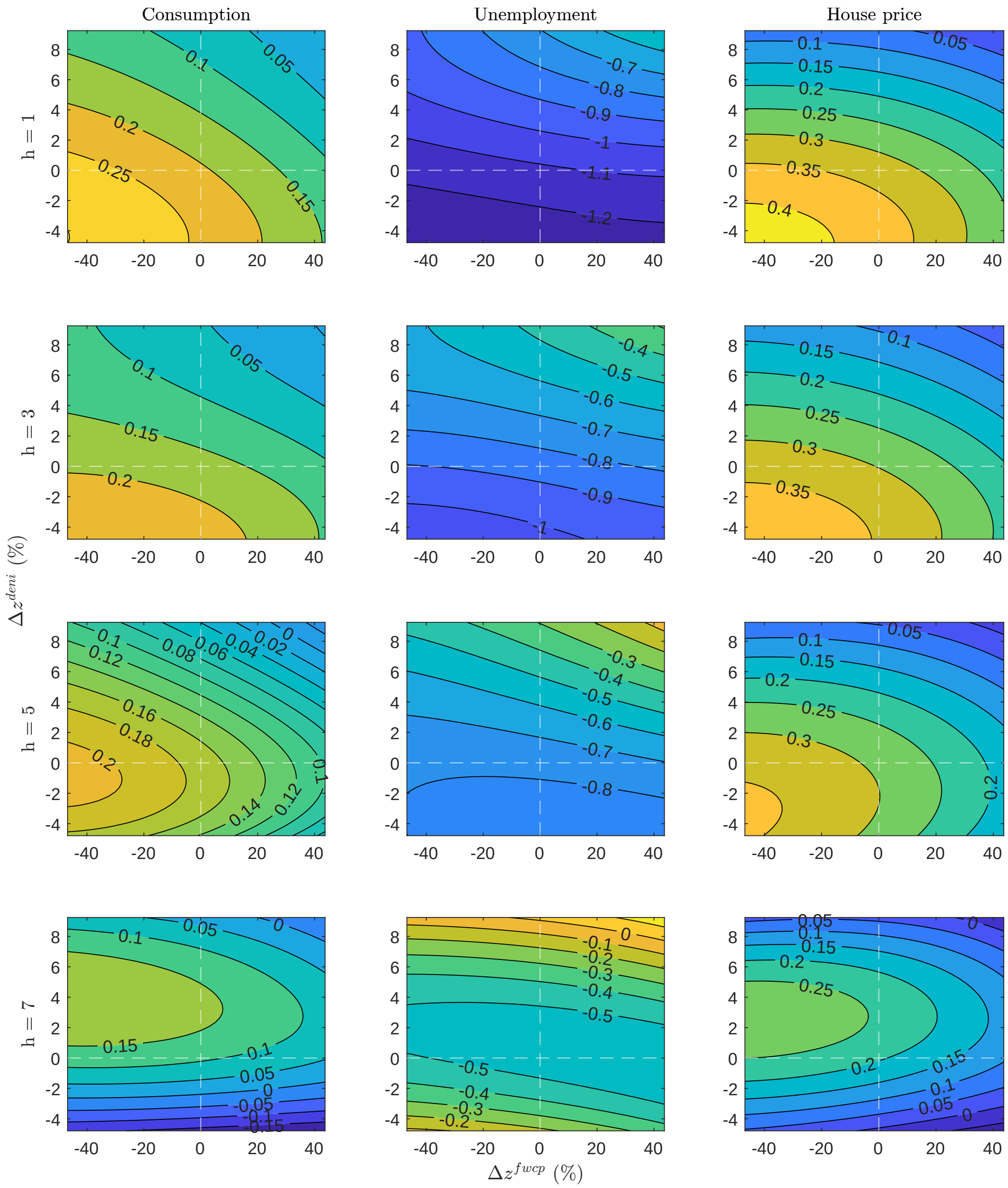


Figure A.4: Order = 4



A.4 Robustness check: Profile likelihood ratio test

The sieve estimator in Equation (1.31) provides a flexible characterization of heterogeneity, allowing a direct F test within our semi-varying coefficient model. For robustness, we use a local kernel polynomial estimator as described in Zhang et al. (2002) and conduct the profile likelihood ratio test as proposed by Fan and Huang (2005) and Fan et al. (2001) to further verify our hypothesis testing outcomes.

In this exercise, as an alternative to the global polynomial approximation, we apply Gaussian kernel smoothing to $\beta(\Delta z^{fwcp}, \Delta z^{deni})$. The estimation follows a two-step procedure. Initially, a varying coefficient model, as described by Hastie and Tibshirani (1993), is estimated using local linear regressions. The bandwidth for this initial stage is set at half of the plug-in bandwidth estimator suggested by Yang and Tschernig (1999). Subsequently, all coefficient profiles, with the exception of the net worth shock, are averaged across the quantile knots within the space of $(\Delta z^{fwcp}, \Delta z^{deni})$, which act as the point estimates for the non-varying coefficient terms. In the second stage, the "residuals", i.e. the outcome after deducting the effects of these non-varying coefficient terms, are regressed on the net worth shock in another varying coefficient model. This second stage estimation also employs the kernel polynomial estimator, with the bandwidth being the plug-in bandwidth.

Figure A.5 presents the estimates of $\beta(\Delta z^{fwcp}, \Delta z^{deni})$. These results are consistent with our baseline findings from the sieve estimator and also uncover more detailed local patterns. Upon reviewing these estimates, we perform the profile likelihood ratio test and report the results in Table A.2. The null hypothesis H_0 is that if the overall treatment effect β is dependent on Δz^{fwcp} and Δz^{deni} and the Equation (1.27) is correctly specified, then it equals to the estimates from the linear LP model as shown in Equation (1.24). Our findings reject this hypothesis across all outcomes and time horizons, indicating that the true effect is different from the linear local

Table A.2: Profile Likelihood Ratio test results

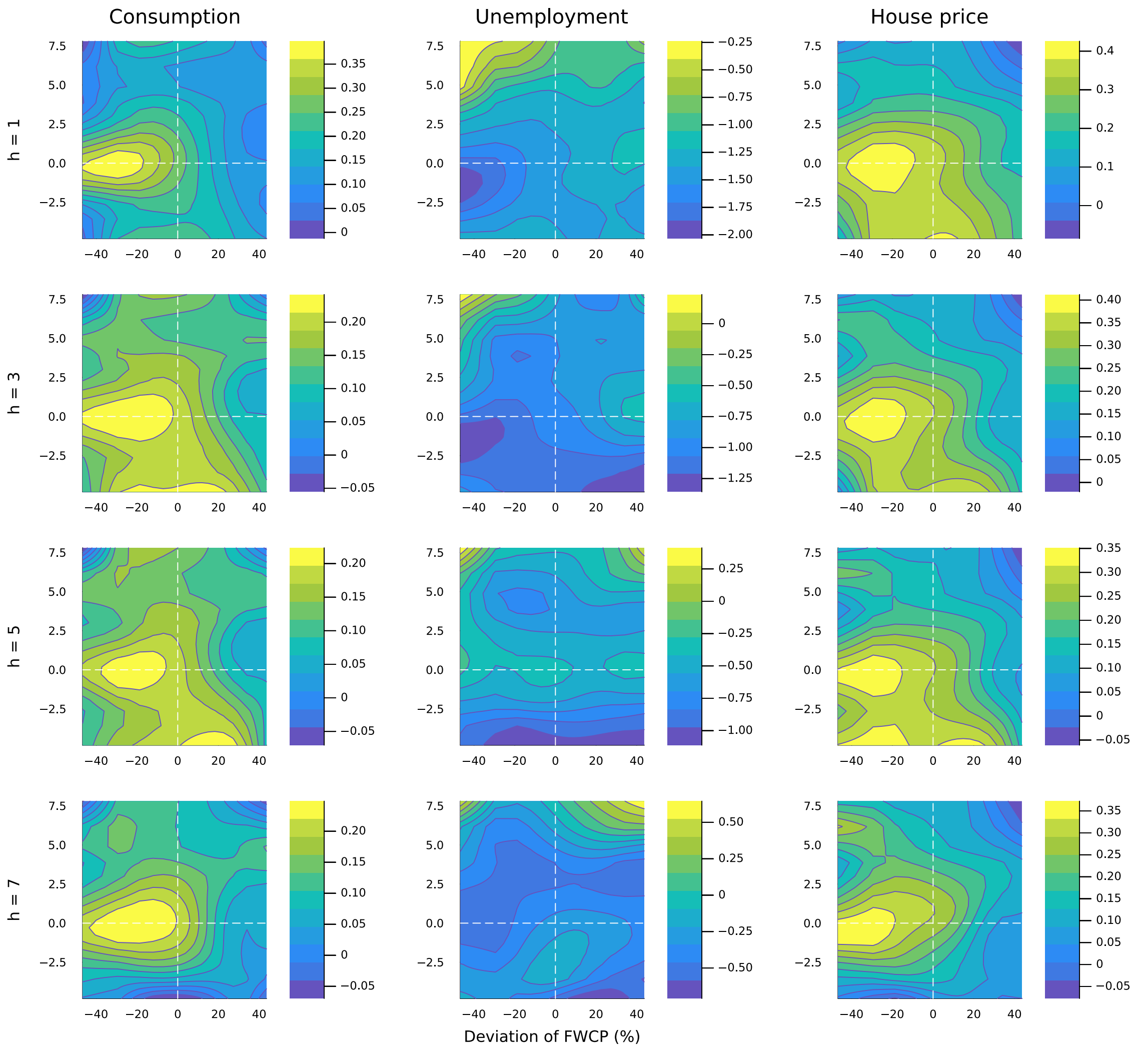
Horizon	Consumption	Unemployment	House price
0	3178.98***	328.46***	1596.15***
	(0.1503)	(0.1503)	(0.1503)
1	2880.26***	355.91***	1166.61***
	(0.1504)	(0.1504)	(0.1504)
2	3132.23***	1301.31***	1230.62***
	(0.1504)	(0.1504)	(0.1504)
3	2840.75***	1684.84***	1127.61***
	(0.1504)	(0.1504)	(0.1504)
4	2378.04***	1605.61***	589.91***
	(0.1504)	(0.1504)	(0.1504)
5	2051.58***	1829.66***	770.64***
	(0.1504)	(0.1504)	(0.1504)
6	1719.26***	1837.51***	841.8***
	(0.1503)	(0.1503)	(0.1503)
7	1616.08***	1799.62***	935.84***
	(0.1502)	(0.1502)	(0.1502)

Notes: 1. *H0*: $\beta(\Delta z^{fwcp}, \Delta z^{deni}) = \hat{\beta} + \hat{\gamma}^{fwcp}\Delta z^{fwcp} + \hat{\gamma}^{deni}\Delta z^{deni}$, where $(\hat{\beta}, \hat{\gamma}^{fwcp}, \hat{\gamma}^{deni})$ are the linear LP estimates from Equation (1.24). 2. The number with stars are the generalized likelihood ratio statistic T_0, the number in parenthesis is δ_n the degree of freedom of the asymptotic $\chi^2_{\delta_n}$ distribution, the other asymptotic parameter $r_K \approx 0.51579$ for our Gaussian kernel. 3. ***: $r_K T_0 > T_{99}$, **: $r_K T_0 > T_{95}$, *: $r_K T_0 > T_{90}$, where T_{90}, T_{95} and T_{99} are the critical values of significance level $\alpha = 10\%, 5\%, 1\%$ respectively.

projection estimates[4].

[4]However, caution is important when interpreting such a "point inference" at a specific point in the function space of $\beta(\cdot)$. Rejecting the null hypothesis does not necessarily disprove the linearity of $\beta(\cdot)$ in Δz^{fwcp} and/or Δz^{deni}. It merely suggests that the linear LP estimates do not align with the data patterns as closely as the kernel polynomial method does. For additional testing of the linearity of heterogeneous effects, one might consult the emerging literature on uniform inference for functional coefficients (Hu, 2024) in which a uniform inference tests if an estimated $\beta(\cdot)$ belongs to a family of functions (such as linear functions).

Figure A.5: Local linear regression



Notes: 1. The horizontal axis of each panel denotes Δz^{fwcp} in percentage points, the vertical axis denotes Δz^{deni} in percentage points. 2. The cross of the white dashed lines denote the point $\hat{\beta}(0,0)$. 3. The contour plot is created by a piece-wise linear interpolation of $\hat{\beta}$ over the along-each-dimension $10\% \sim 90\%$ quantile ranges of Δz^{fwcp} and Δz^{fwcp}. 4. The quantile reference grid knots are chosen by every 5% quantile such that there are $17 \times 17 = 289$ knots. 5. The distance for the kernel function is Euclidean distance where both reference variables are normalized to $[0, 1]$ in order to eliminate the impact of scales. 6. The bandwidth selection is done by the plug-in estimator in Yang and Tschernig (1999).

A.5 Robustness check: Sensitivity analysis

Our baseline results covered the important time period of the Great Recession where the US government largely intervened the economy[5]. Such large scale of policy intervention may potentially weaken our estimates by introducing the omitted variable issue due to unobserved policy-related confounder(s) that were not captured by the year and state fixed effects. Meanwhile, these confounders may affect the baseline results in non-linear and interacting fashions. To examine the sensitivity of our ATE estimates against any potential leftover confounding effects, we perform sensitivity analysis using the framework introduced by Cinelli and Hazlett (2020). This framework uses the partial R^2 of the treatment (net worth shock) $R^2_{Y\sim D|\mathbf{X}}$ in the baseline result, and an observed covariate (the 2008 year fixed effect) as the benchmark for potential confounders, to test the robustness of our estimates against potential omitted variable issue, and how plausible that any proposed confounder(s) Z might be. Specifically, the sensitivity analysis shows how the baseline estimates could be biased given the unobserved confounder(s) proposed relationship with the treatment (parameterized by $R^2_{D\sim Z|\mathbf{X}}$) and the outcome (parameterized by $R^2_{Y\sim Z|D,\mathbf{X}}$) respectively[6].

In Table A.3, we report the main sensitivity measures: the *robustness value* (RV) for the point estimate of ATE, the *robustness value* ($RV_{\alpha=0.05}$) for its t-value at significance level 0.05, and partial R square $R^2_{Y\sim D|\mathbf{X}}$ of the treatment. The robustness value RV measures how sufficiently strong the association of unobserved confounder(s) with the treatment and outcome, which is denoted by the two scale-free

[5]For example, these policy interventions included the Emergency Economic Stabilization Act of 2008, which established the Troubled Asset Relief Program (TARP) to purchase toxic assets and inject capital into banks. Additionally, the Federal Reserve cut interest rates to near zero and implemented quantitative easing to increase liquidity. The American Recovery and Reinvestment Act of 2009 injected $787 billion into the economy through tax cuts, unemployment benefits, and public works projects. These interventions aimed to restore confidence, stimulate economic activity, and mitigate the impacts of the recession.

[6]Intuitively, the partial R-square $R^2_{D\sim Z|\mathbf{X}}$ summarizes the size of potential confounding effects, and $R^2_{Y\sim Z|D,\mathbf{X}}$ summarizes the improvement by adding such potential confounder(s) to the baseline model.

parameters $R^2_{D\sim Z|\mathbf{X}}$ and $R^2_{Y\sim Z|D,\mathbf{X}}$, must be to reduce our baseline point estimate of ATE to 0. Any stronger relationship implies sign flip of the estimates. Another robustness value $RV_{\alpha=0.05}$ measures how strong that association must be to not reject the null hypothesis of the t test of ATE estimates. Meanwhile, the partial R square of the treatment serves as an extreme scenario that how much residual variance of the outcome must the confounder(s) to explain in order to eliminate our baseline ATE estimates. Additionally, we report the bounds of the strength of the confounder(s) by using the 2008 year fixed effect as benchmark[7]. If a confounder is as "strong" as the benchmark covariate, then whether it would overturn the baseline estimates. Then for any proposed confounder, one could answer the question that how strong the proposed confounder is compared to the benchmark and whether it is possible. In our sensitivity analysis, both RV and $RV_{\alpha=0.05}$ are high which imply that our ATE estimates are invulnerable against omitted variable issue in most horizon's projections. For example, the RV for 1 horizon of consumption as outcome is 65.5%. It implies that a potential confounder must have at least $> 65.5\%$ partial R square simultaneously with the net worth shock and consumption to flip our point estimate's sign. The $RV_{\alpha=0.05}$ is 56.2%, which implies that a confounder must have a high association to fail the t-testing. In the most extreme scenario, the potential confounders must be able to explain 55.4% residual variance of the outcome to overturn our estimates. Given that we have controlled both time and state fixed effects, it is less likely to have such strong confounding effects that eliminate our baseline results. However, one may observe that our estimates is less invulnerable by projection horizon increasing. It suggests that if there are strong policy related confounders not captured by our fixed effects and other control variables, then they are possibly affecting the ATE estimates at longer horizons rather than closer horizons. We also give two visualizations Figure

[7]The choice of benchmark covariate requires domain knowledge about potential confounder(s) and how they might be related to an observed covariate in the baseline model. In our context, the 2008 year fixed effect is more likely to be conceptually associated with unobserved policy interventions than the other covariates.

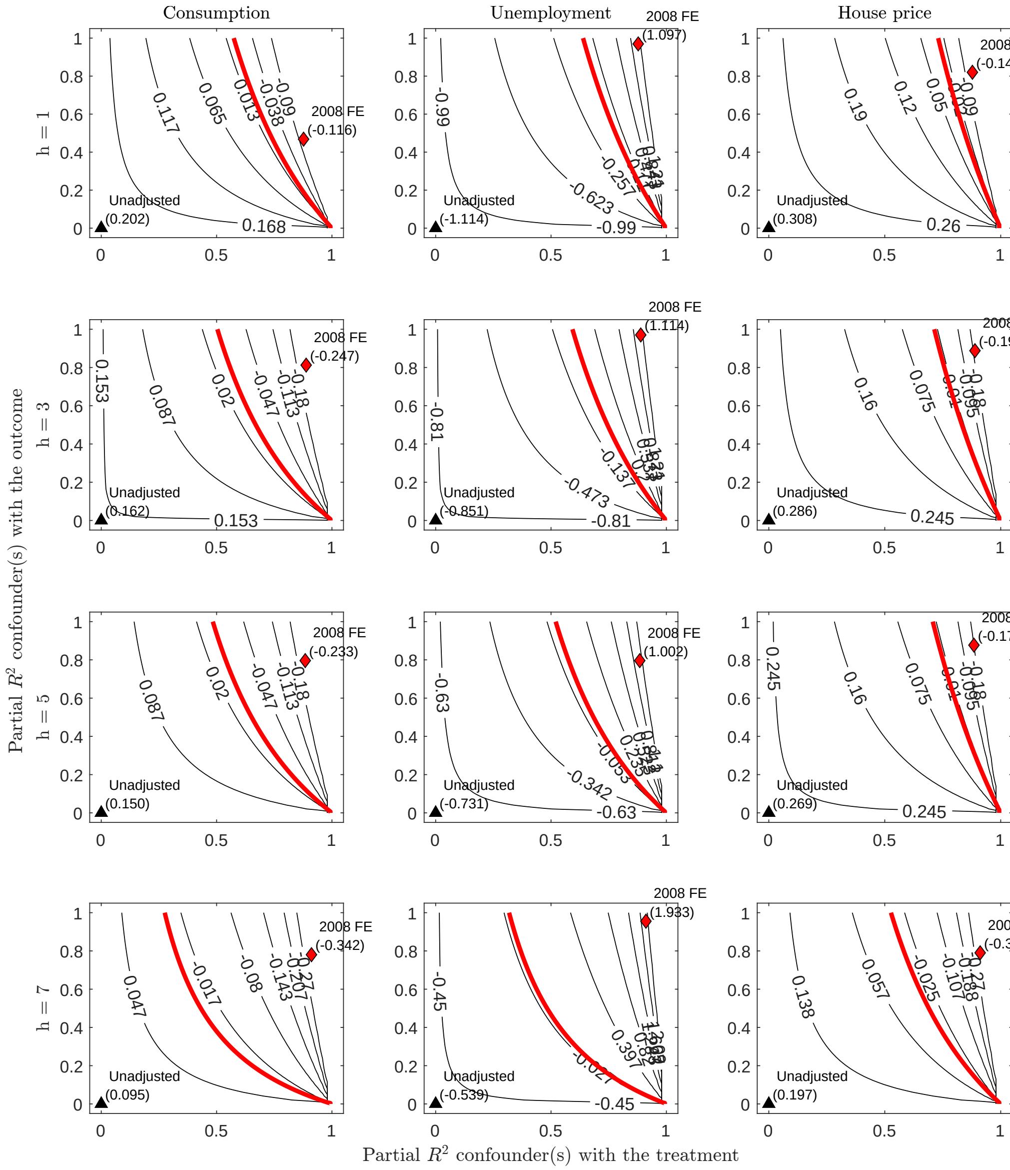
A.6 and Figure A.7 to comprehensively illustrate the whole profile of our sensitivity analysis results[8].

Table A.3: Sensitivity measures

	Outcome: Log(real consumption per capita)							Bound (Z as strong as 2008 year FE)	
Horizon	Estimate	SE	t-value	$R^2_{Y\sim D\|\mathbf{X}}$ (%)	RV (%)	$RV_{\alpha=0.05}$ (%)	df	$R^2_{Y\sim Z\|D,\mathbf{X}}$ (%)	$R^2_{D\sim Z\|\mathbf{X}}$ (%)
0	0.210	0.023	9.07	59.9	68.6	60.4	55	1.2	87.5
1	0.202	0.024	8.53	57.4	66.9	58.0	54	46.8	87.7
2	0.175	0.022	8.01	54.8	65.0	55.5	53	79.3	87.9
3	0.162	0.022	7.26	50.3	62.0	51.3	52	81.2	88.8
4	0.159	0.022	7.22	50.5	62.2	51.4	51	80.7	88.0
5	0.150	0.022	6.84	48.4	60.7	49.2	50	79.6	88.4
6	0.112	0.021	5.25	36.0	52.0	37.2	49	78.5	89.7
7	0.095	0.022	4.27	27.5	45.5	28.2	48	78.0	91.1
	Outcome: Unemployment rate in percentage points								
0	-1.114	0.107	-10.38	66.2	72.9	66.1	55	56.7	87.5
1	-1.114	0.114	-9.78	63.9	71.3	63.9	54	97.0	87.8
2	-0.985	0.105	-9.40	62.5	70.3	62.5	53	97.4	87.9
3	-0.851	0.098	-8.70	59.3	68.1	59.5	52	97.0	88.9
4	-0.780	0.099	-7.86	54.8	65.1	55.3	51	93.6	88.1
5	-0.731	0.099	-7.36	52.0	63.2	52.6	50	79.6	88.5
6	-0.592	0.105	-5.64	39.4	54.4	40.5	49	95.2	89.8
7	-0.539	0.114	-4.73	31.8	48.9	32.8	48	95.5	91.1
	Outcome: Log(real house price index)								
0	0.298	0.024	12.33	73.4	78.0	72.9	55	76.0	87.5
1	0.308	0.025	12.10	73.0	77.7	72.4	54	82.0	87.8
2	0.297	0.025	11.95	72.9	77.6	72.3	53	86.4	87.9
3	0.286	0.025	11.38	71.4	76.5	70.7	52	88.8	88.9
4	0.284	0.025	11.38	71.7	76.8	71.0	51	88.8	88.1
5	0.269	0.024	10.99	70.7	76.0	70.0	50	87.7	88.5
6	0.236	0.025	9.27	63.7	71.2	63.3	49	85.2	89.8
7	0.197	0.027	7.31	52.7	63.6	53.0	48	79.0	91.1

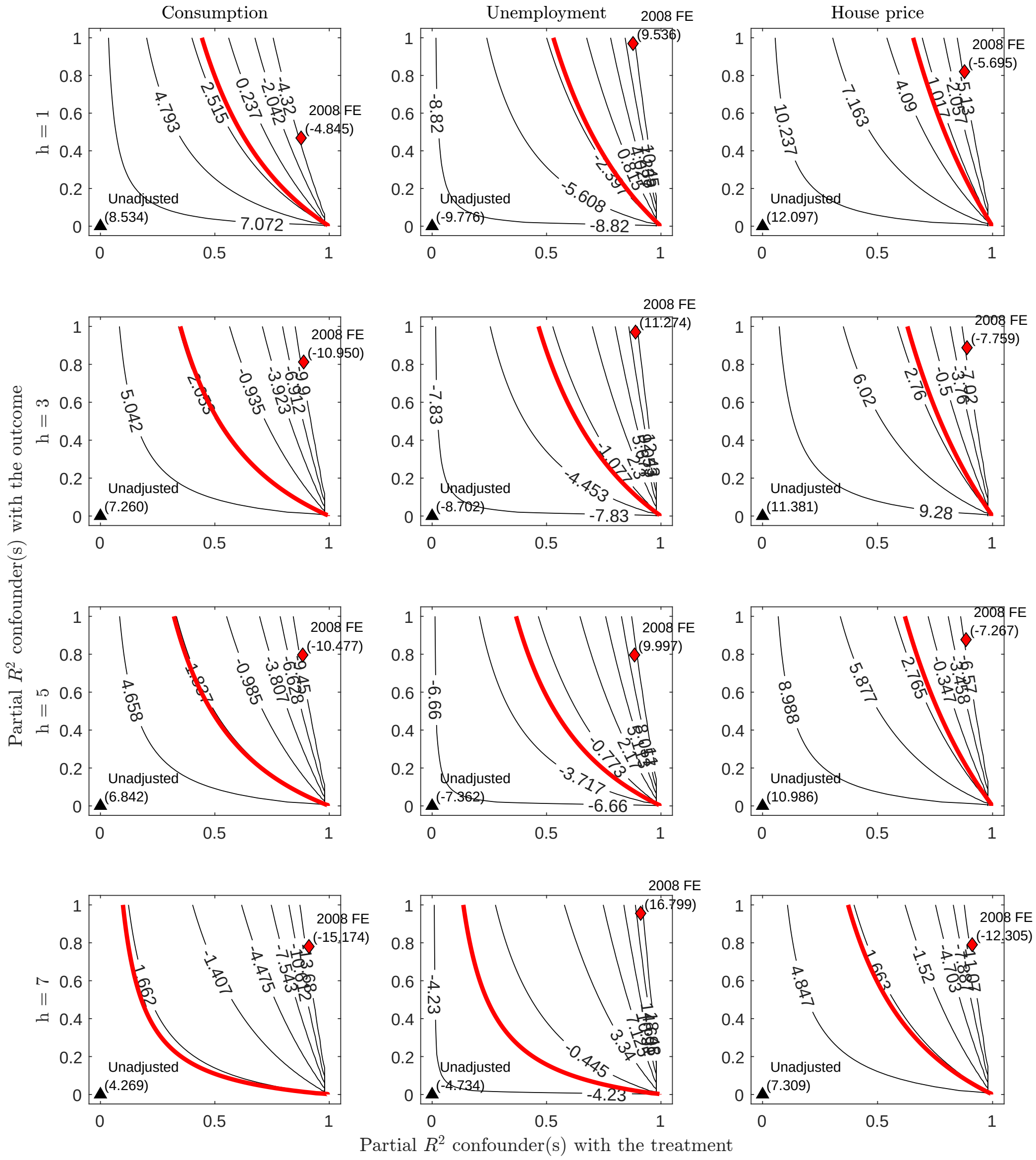
[8] As a contrast, we also provide sensitivity analysis to the baseline Cloyne et al. (2023) model in Figure A.8 and Figure A.9. The comparison suggests that the linear LP model is more vulnerable to potential confounding effects at long horizon of the projection.

Figure A.6: Sensitivity contour plots: point estimate of ATE



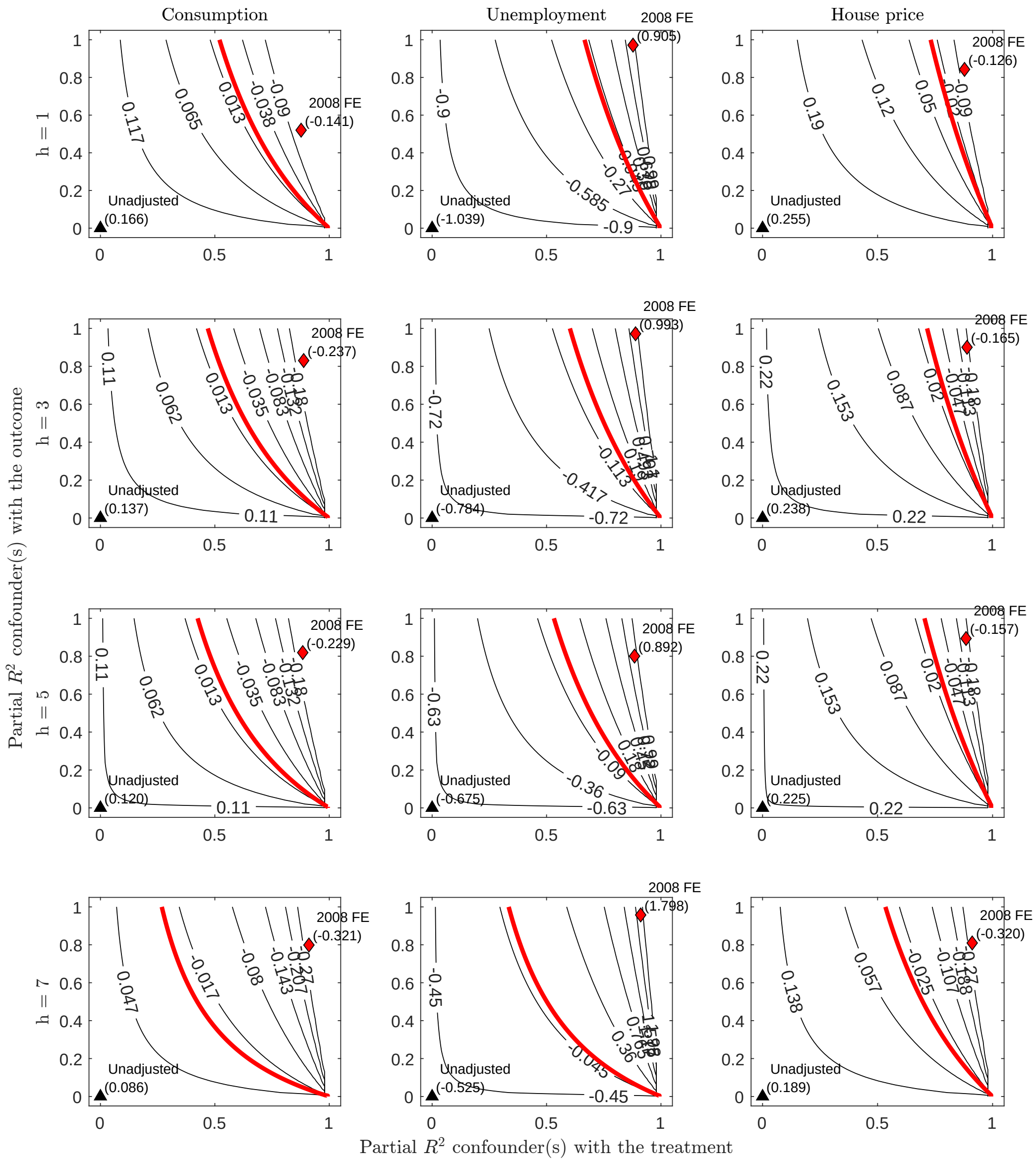
Notes: 1. The horizontal axis denotes $R^2_{D\sim Z|\mathbf{X}}$, the vertical axis denotes $R^2_{Y\sim Z|D,\mathbf{X}}$. 2. The dark triangle mark at the origin denotes the baseline ATE point estimate in which the value is labelled in the parenthesis. 3. The red diamond mark denotes the point estimate if there is confounder(s) as strong as the benchmark covariate (2008 year fixed effect); The new point estimate is labelled in the parenthesis. 4. The red thick curve marks the zero line. Point estimates beyond this line means sign flips.

Figure A.7: Sensitivity contour plots: t-value of ATE



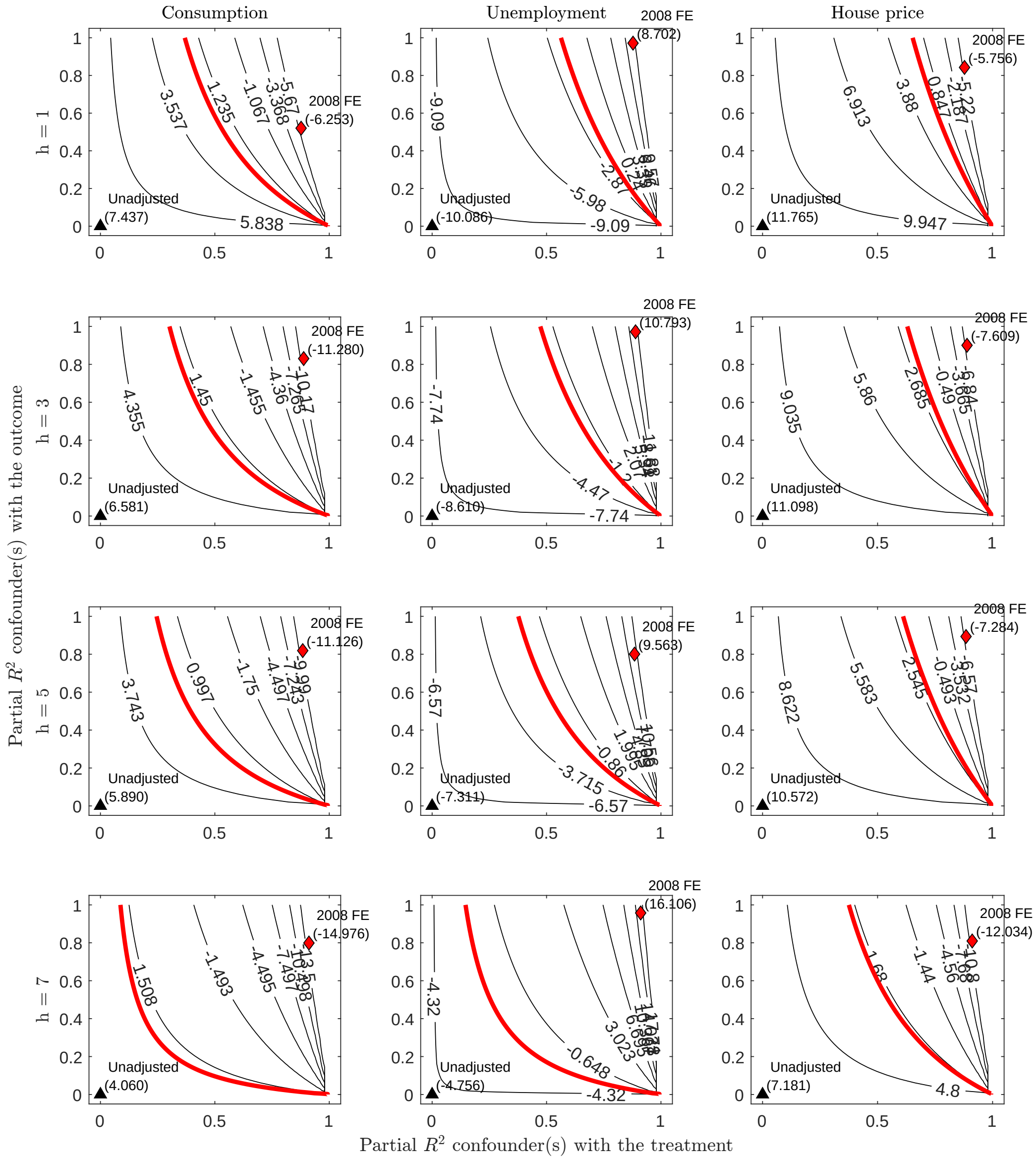
Notes: 1. The horizontal axis denotes $R^2_{D\sim Z|\mathbf{X}}$, the vertical axis denotes $R^2_{Y\sim Z|D,\mathbf{X}}$. 2. The dark triangle mark at the origin denotes the baseline ATE t-value in which the value is labelled in the parenthesis. 3. The red diamond mark denotes the t-value if there is confounder(s) as strong as the benchmark covariate (2008 year fixed effect); The new value is labelled in the parenthesis. 4. The red thick curve marks the 1.96 critical line. Point estimates beyond this line means not rejecting the H_0.

Figure A.8: Sensitivity contour plots: point estimate of ATE



Notes: 1. The horizontal axis denotes $R^2_{D\sim Z|\mathbf{X}}$, the vertical axis denotes $R^2_{Y\sim Z|D,\mathbf{X}}$. 2. The dark triangle mark at the origin denotes the baseline ATE point estimate in which the value is labelled in the parenthesis. 3. The red diamond mark denotes the point estimate if there is confounder(s) as strong as the benchmark covariate (2008 year fixed effect); The new point estimate is labelled in the parenthesis. 4. The red thick curve marks the zero line. Point estimates beyond this line means sign flips.

Figure A.9: Sensitivity contour plots: t-value of ATE

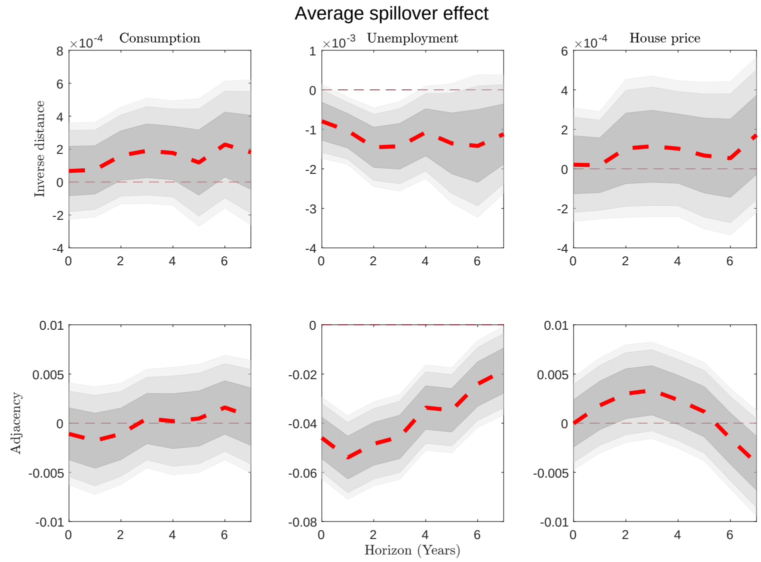


Notes: 1. The horizontal axis denotes $R^2_{D\sim Z|\mathbf{X}}$, the vertical axis denotes $R^2_{Y\sim Z|D,\mathbf{X}}$. 2. The dark triangle mark at the origin denotes the baseline ATE t-value in which the value is labelled in the parenthesis. 3. The red diamond mark denotes the t-value if there is confounder(s) as strong as the benchmark covariate (2008 year fixed effect); The new value is labelled in the parenthesis. 4. The red thick curve marks the 1.96 critical line. Point estimates beyond this line means not rejecting the H_0.

A.6 Robustness check: spatial spillover effects

It is well understood that local economies are closely interconnected. Beyond the geographic variation of economic variables used in our baseline analysis, the uneven distribution of these variables indicates the potential for significant spillover effects of net worth shocks. Figure 1.2b illustrates various wealth clusters across the U.S (e.g. West Coast, New England, and Florida). Concurrently, Figure 1.2a shows clearer spatial clustering of net worth shocks, with the Great Lakes Region and the South being most adversely affected during the Great Recession. To examine how robust of our baseline results against spatial spillover effects of net worth shocks, we re-estimate the baseline model of Equation (1.27) by adding a spatial Durbin term $\eta_h \cdot \mathbf{W}\mathbf{X}_t$ while not assuming spatial dependency of the outcome variables nor the error. In such a term, $\mathbf{W}$ is the zero-diagonal spatial weight matrix, $\mathbf{X}_t$ is the stacked net worth shocks in year t, and η_h is the coefficient of spillover/indirect effects. In such a simplified Spatial Durbin Model, the average spillover effects (ASE) of net worth shocks defined by LeSage and Pace (2009) can be fully represented by the coefficient η_h, while the inference of ASE is simply implemented by the regular t-test.

Figure A.10: Average Spillover Effects of net worth shocks



Notes: 1. The value of Average Spillover Effect is represented by η_h. The data sample is the same as the baseline regressions. 2. The first row of the figure matrix represents the ASE estimates using inverse distance weight matrix, in which the distance between two counties are computed using Haversine formula. 3. The second row of the figure matrix represents the ASE estimates using the 1st-neighbor adjacency matrix. The adjacency data is published by Census Bureau. 4. There are three confidence interval bands. By the color being lighter: 1-σ, 90% and 95%. The SE estimator is the same as the baseline regressions.

In this exercise, we explore two types of spatial weight matrices: inverse distance and 1st-neighbor adjacency, with each assuming different rates of shock decay as distance increases. Figure A.10 presents the ASE estimates for net worth shocks across projection horizons. Statistically significant spillover effects on local unemployment are observed, whereas local consumption and housing prices show no significant effects. The negative η_h value associated with unemployment indicates that net worth shocks in other counties can impact the local labor market, likely due to interconnected local labor markets on a broader scale, such as metropolitan areas and states. Figures A.11 show the $\beta_h(\Delta\mathbf{Z})$ estimates under both spatial weightings. While the magnitude of shock effects remains consistent across most of the $\Delta\mathbf{Z}$ space, accounting for spatial spillover effects yields a more negative β_h for unemployment in regions where collateral constraints and DNWR are likely to be binding simultaneously. This indicates that the impact of net worth shocks in counties with high frictions is mitigated by their distribution across neighboring counties. It also implies a greater non-linearity in the heterogeneous effects of net worth shocks, further verifying our baseline results.

Figure A.11: Net worth shock effects: inverse distance weighting

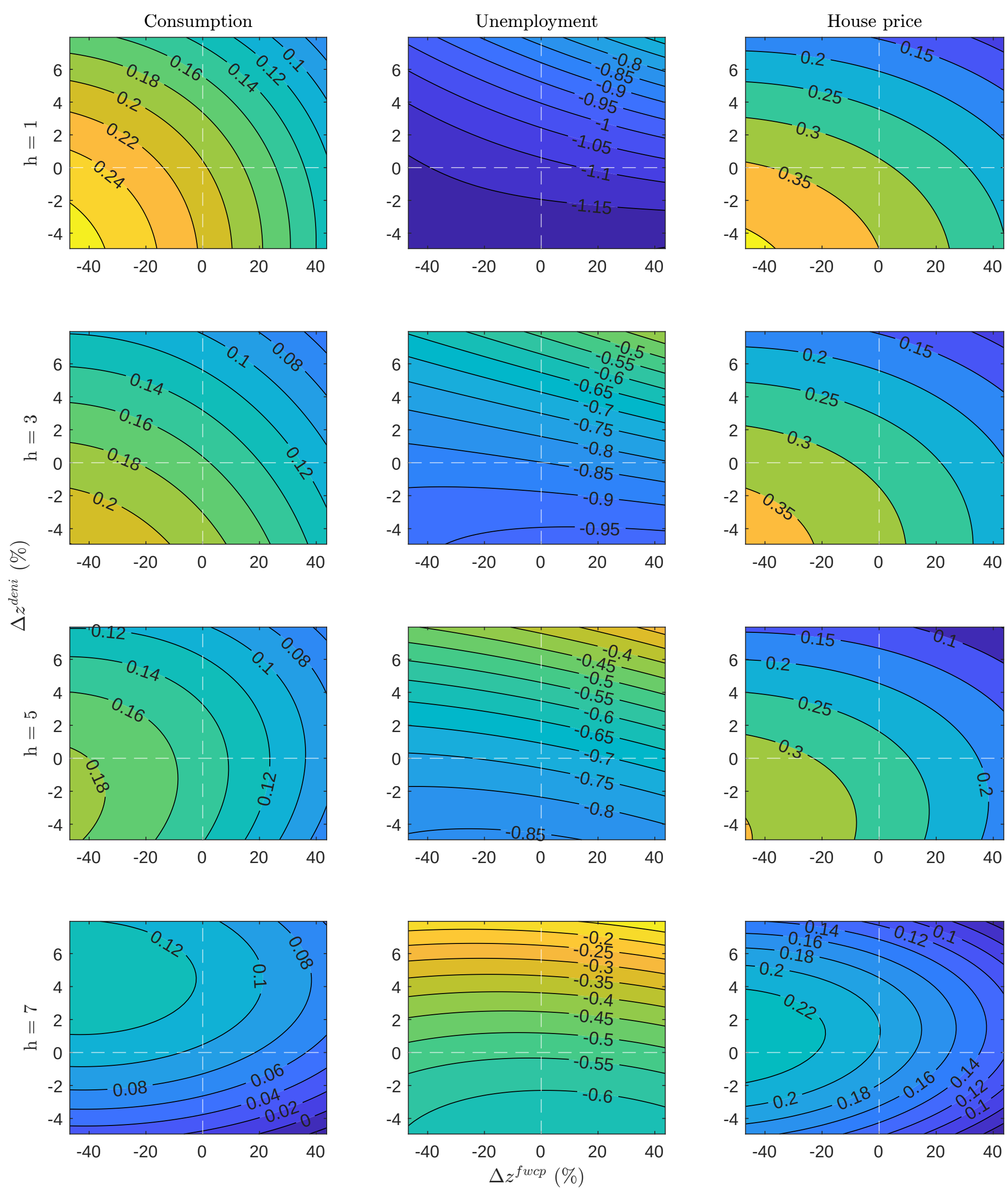
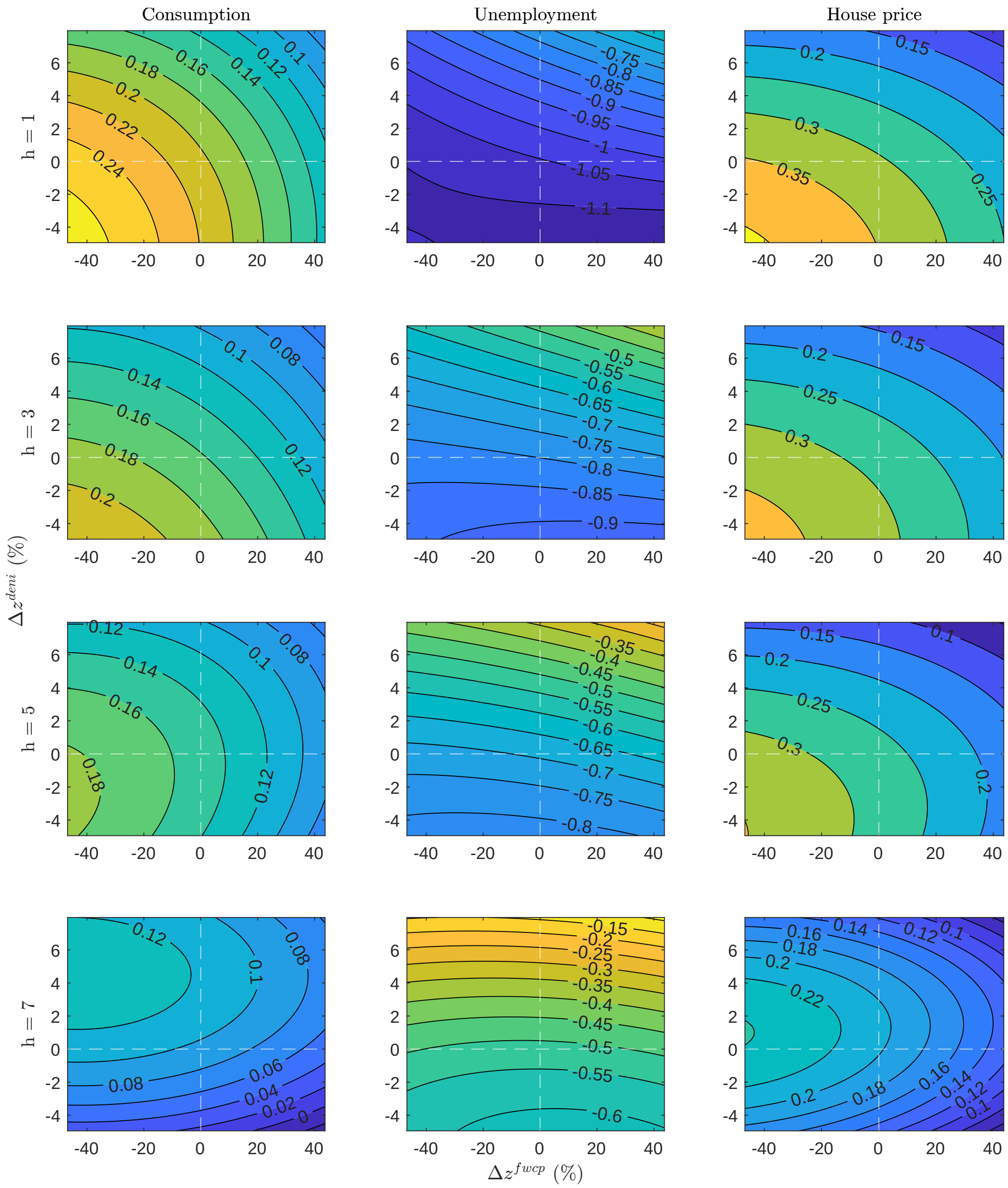


Figure A.12: Net worth shock effects: 1st neighbor adjacency weighting



A.7 Robustness check: Panel unit root tests

Unit roots can result in dubious or spurious estimates, which are particularly problematic when analyzing the longitudinal behavior of data. In our baseline model, the net worth shock is identified at the first order (growth of net worth); however, it is still crucial to check for the presence of unit roots for the shocks. Even if time fixed effects are controlled, the existence of unit roots may indicate a potential misidentification of the shock or necessitate the use of error correction models (ECM). Considering the unbalanced wide panel nature and large heterogeneity among counties, we use the Im-Pesaran-Shin (IPS) test (Im et al., 2003) and the ADF-based Fisher-type test (Choi, 2001) to check the panel unit roots of the shock $x_{i,t}$. Table A.4 reports the testing results in which the null hypothesis are rejected by both tests. Thus, our baseline model is not suffering severe issue of spurious estimates.

Table A.4: Panel unit root tests

Test		Statistic	p-value
IPS (Im et al., 2003)	$\bar{t}$	-41.0955	< 0.01
Fisher-type (Choi, 2001)	Inverse χ^2	8597.715	< 0.01
	Inverse normal	-41.7423	< 0.01
	Inverse logit t	-48.5836	< 0.01
	Modified inv. χ^2	63.0316	< 0.01

Notes: 1. *H0*: All panels contain unit roots. 2. The maximum lag is 2 by convention. 3. Demeaning is performed due to spatial dependence and the Great Recession period. 4. No trend term added.

Appendix B

Chapter 2

B.1 Numerical algorithm: household problem

Solving the individual household problems constitutes the main performance bottleneck and is the most time-consuming part of the algorithm. Several challenges contribute to this. The primary challenge is the high dimensionality of the state space. Even in computing the deterministic steady state, the problem involves five individual state variables, making traditional methods based on uniform dense grids computationally infeasible. To address this, I adopt the sparse grid technique developed for economic applications by Griebel (1998), Brumm and Scheidegger (2017), and Brumm et al. (2021). Specifically, I implement an isotropic regular sparse grid and develop the computation pipeline as a Julia package, `AdaptiveSG.jl`, which is publicly available on our GitHub repository.

The second challenge is the irregular shape of the admissible state space. Due to the collateral constraint and fixed mortgage debt payments, the admissible domain of $x = (a, b, h, s, \xi)$ is non-rectangular and evolves endogenously across iterations. In certain regions of the feasible space, no admissible control variables exist. While economists have developed techniques such as the endogenous grid method

(Mendoza and Villalvazo, 2020), these approaches are typically applicable only in low-dimensional settings. To address this issue in the deterministic steady state, I transform and normalize the individual state variables into an alternative hyper-rectangle:

$$\tilde{x}_t := (h_{t-1}, s_{t-1}, \ell_{t-1}, \varsigma_{t-1}, \xi_t) \tag{B.1}$$

where $\ell_{t-1} := \frac{b_{t-1}}{\theta p_t s_{t-1} h_{t-1}} \in [0,1]$ is the leverage ratio and $\varsigma_{t-1} := \frac{r_b b_{t-1}}{a_t} \in (0,1]$ is the liquidity ratio. These two transformed variables automatically satisfy the collateral and solvency constraints under standard box constraints. I then re-write the household's Bellman equation and solve the problem within this rectangularized state space. When solving the global solution of the model, the above transformation is no longer applicable because the house price p_t becomes time-varying. To address this, I use a high-accuracy sparse grid and dynamically filter the admissible grid points when updating the aggregate profiles $(X'(X), p(X))$ using least squares fitting and parametric approximations.

The third challenge is numerical oscillation during value function iteration, caused by the non-smoothness and non-monotonicity of high-dimensional value function approximations. In high-dimensional settings with sparse grids, grid-based methods such as multi-linear sparse grid interpolation (Schaab and Zhang, 2022) often suffer from the vanishing phenomenon[1] in the "vacuum" regions where grid nodes are sparse. On the other hand, global approximation methods such as radial basis function interpolation, Gaussian process regression, and Smolyak polynomials (Judd et al., 2014; Brumm and Scheidegger, 2017) avoid vanishing but suffer from the Runge phenomenon, which introduces strong oscillations, breaks monotonicity, and generates many local extrema and saddle points. These approximation issues result in non-

[1]The vanishing phenomenon refers to situations where the predicted function value is close to zero because the point being evaluated is far from supporting grid nodes, causing most basis functions to be zero.

smooth and non-monotonic value functions, making it difficult to accurately solve the household's maximization problem. The associated numerical errors accumulate across iterations, leading to inconsistent solutions, oscillations, or even divergence. To address this, I adopt generalized additive model (GAM) approximations, which are parametric and pseudo-linear. The proposed method is called vector-valued least square generalized additive model (VLS-GAM) For example, Equation (B.2)

$$v(x) \approx m_0 + \sum_{i=1}^{N} m_i \cdot g_i(x) \tag{B.2}$$

$$g_1(x) = h; g_2(x) = s; g_3(x) = \ell; g_4(x) = \varsigma; g_5(x) = \xi$$

$$g_6(x) = h^{-1}; g_7(x) = s^{-1}; \dots$$

$$g_{11}(x) = h \cdot s; g_{12}(x) = h \cdot \ell; \dots$$

gives a GAM approximation of $v(x)$ consists of an intercept term, linear terms that capture monotonicity, inverse power-law terms that reflect concavity and risk aversion[2], and interaction terms that account for first-order correlations between state variables. The model is estimated using a least squares method over sparse grid node points. When fitting policy functions with this method, I impose an additional constraint to bound the predicted values within the feasible state space, introducing nonlinearity into the model. To ensure monotonicity of the value function approximation, I select monotonic additive functionals and perform model selection by comparing the pseudo R^2 values of the converged value function approximations. This VLS-GAM procedure yields a smooth and monotonic approximation that closely fits the true value function. Meanwhile, because the policy function approximations do not require the shape preserving property, I use dimension-specific Chebyshev polynomials and linear interaction terms to accurately capture the local shapes of the policy func-

[2]This specification is motivated by classical results on the exponential utility family, which includes the CRRA utility used in this model.

tions. The VLS-GAM method is highly efficient. Even without supplying analytical Jacobians or Hessians, it achieves a high degree of accuracy—requiring fewer than 1,000 grid node points while attaining an R^2 above 0.97.

Bibliography

Aktug, E. (2025). Downward nominal wage rigidity and the optimal inflation target. *Macroeconomic Dynamics*, 29:e108.

Aladangady, A. (2017). Housing wealth and consumption: Evidence from geographically-linked microdata. *American Economic Review*, 107(11):3415–46.

Angrisani, M., Hurd, M., and Rohwedder, S. (2019). The effect of housing wealth losses on spending in the great recession. *Economic inquiry*, 57(2):972–996.

Auclert, A., Bardóczy, B., Rognlie, M., and Straub, L. (2021). Using the sequence-space Jacobian to solve and estimate heterogeneous-agent models. *Econometrica*, 89(5):2375–2408. Publisher: Wiley Online Library.

Baker, S. R. (2018). Debt and the response to household income shocks: Validation and application of linked financial account data. *Journal of Political Economy*, 126(4):1504–1557.

Belloni, A., Chernozhukov, V., Chetverikov, D., and Fernández-Val, I. (2019). Conditional quantile processes based on series or many regressors. *Journal of Econometrics*, 213(1):4–29.

Beraja, M., Hurst, E., and Ospina, J. (2019). The aggregate implications of regional business cycles. *Econometrica*, 87(6):1789–1833.

Berger, D., Guerrieri, V., Lorenzoni, G., and Vavra, J. (2018a). House prices and consumer spending. *The Review of Economic Studies*, 85(3):1502–1542.

Berger, D., Guerrieri, V., Lorenzoni, G., and Vavra, J. (2018b). House prices and consumer spending. *The Review of Economic Studies*, 85(3):1502–1542. Publisher: Oxford University Press.

Berger, D. and Vavra, J. (2015). Consumption dynamics during recessions. *Econometrica*, 83(1):101–154. Publisher: Wiley Online Library.

Bernanke, B. S. (2018). The real effects of disrupted credit: Evidence from the global financial crisis. *Brookings Papers on Economic Activity*, 2018(2):251–342.

Bocola, L. and Lorenzoni, G. (2020a). Risk sharing externalities. Technical report, National Bureau of Economic Research.

Bocola, L. and Lorenzoni, G. (2020b). Risk sharing externalities. Technical report, National Bureau of Economic Research.

Bohn, B. (2018). *On the convergence rate of sparse grid least squares regression*. Springer.

Bojinov, I. and Shephard, N. (2019). Time series experiments and causal estimands: exact randomization tests and trading. *Journal of the American Statistical Association*.

Branch, W. A., Petrosky-Nadeau, N., and Rocheteau, G. (2016). Financial frictions, the housing market, and unemployment. *Journal of Economic Theory*, 164:101–135.

Brumm, J., Krause, C., Schaab, A., and Scheidegger, S. (2021). Sparse grids for dynamic economic models. *Available at SSRN 3979412*.

Brumm, J. and Scheidegger, S. (2017). Using Adaptive Sparse Grids to Solve High-Dimensional Dynamic Models. *Econometrica*, 85(5):1575–1612.

Cai, Z., Fan, J., and Li, R. (2000). Efficient estimation and inferences for varying-coefficient models. *Journal of the American Statistical Association*, 95(451):888–902.

Case, K. E., Quigley, J. M., and Shiller, R. J. (2011). Wealth effects revisited 1978-2009. Technical report, National Bureau of Economic Research.

Chen, X. (2007). Large sample sieve estimation of semi-nonparametric models. *Handbook of econometrics*, 6:5549–5632.

Choi, I. (2001). Unit root tests for panel data. *Journal of international money and Finance*, 20(2):249–272.

Christiano, L. J., Eichenbaum, M. S., and Trabandt, M. (2015). Understanding the great recession. *American Economic Journal: Macroeconomics*, 7(1):110–167.

Cinelli, C. and Hazlett, C. (2020). Making sense of sensitivity: Extending omitted variable bias. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, 82(1):39–67.

Clark, A. and Postel-Vinay, F. (2009). Job security and job protection. *Oxford Economic Papers*, 61(2):207–239.

Cloyne, J., Jordà, Ò., and Taylor, A. M. (2023). State-dependent local projections: Understanding impulse response heterogeneity. Technical report, National Bureau of Economic Research.

Daly, M. C. and Hobijn, B. (2014). Downward nominal wage rigidities bend the phillips curve. *Journal of Money, Credit and Banking*, 46(S2):51–93.

Elsby, M. W., Shin, D., and Solon, G. (2016). Wage adjustment in the great recession and other downturns: Evidence from the united states and great britain. *Journal of Labor Economics*, 34(S1):S249–S291.

Fallick, B., Lettau, M., and Wascher, W. (2016). Downward nominal wage rigidity in the united states during and after the great recession. *SSRN*.

Fan, J. and Huang, T. (2005). Profile likelihood inferences on semiparametric varying-coefficient partially linear models. *Bernoulli*, 11(6):1031–1057.

Fan, J., Zhang, C., and Zhang, J. (2001). Generalized likelihood ratio statistics and wilks phenomenon. *The Annals of statistics*, 29(1):153–193.

Favilukis, J., Ludvigson, S. C., and Van Nieuwerburgh, S. (2017). The macroeconomic effects of housing wealth, housing finance, and limited risk sharing in general equilibrium. *Journal of Political Economy*, 125(1):140–223. Publisher: University of Chicago Press Chicago, IL.

Garriga, C. and Hedlund, A. (2020). Mortgage debt, consumption, and illiquid housing markets in the great recession. *American Economic Review*, 110(6):1603–1634.

Gonçalves, S., Herrera, A. M., Kilian, L., and Pesavento, E. (2024). State-dependent local projections. *Journal of Econometrics*, page 105702.

Gourieroux, C. and Lee, Q. (2023). Nonlinear impulse response functions and local projections. *arXiv preprint arXiv:2305.18145*.

Griebel, M. (1998). Adaptive sparse grid multilevel methods for elliptic PDEs based on finite differences. *Computing*, 61(2):151–179.

Guerrieri, L. and Iacoviello, M. (2017a). Collateral constraints and macroeconomic asymmetries. *Journal of Monetary Economics*, 90:28–49.

Guerrieri, L. and Iacoviello, M. (2017b). Collateral constraints and macroeconomic asymmetries. *Journal of Monetary Economics*, 90:28–49. Publisher: Elsevier.

Guerrieri, V., Lorenzoni, G., and Prato, M. (2020a). Schumpeter lecture 2019: Slow household deleveraging. *Journal of the European Economic Association*, 18(6):2755–2775. Publisher: Oxford University Press.

Guerrieri, V., Lorenzoni, G., and Prato, M. (2020b). Schumpeter lecture 2019: Slow household deleveraging. *Journal of the European Economic Association*, 18(6):2755–2775.

Guerrieri, V., Lorenzoni, G., and Prato, M. (2020c). Slow household deleveraging. *Journal of the European Economic Association*, 18(6):2755–2775.

Guren, A. M., McKay, A., Nakamura, E., and Steinsson, J. (2021). Housing wealth effects: The long view. *The Review of Economic Studies*, 88(2):669–707.

Härdle, W., Mammen, E., and Müller, M. (1998). Testing parametric versus semiparametric modeling in generalized linear models. *Journal of the American Statistical Association*, 93(444):1461–1474.

Hastie, T. and Tibshirani, R. (1993). Varying-coefficient models. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, 55(4):757–779.

Holden, S. and Wulfsberg, F. (2009). How strong is the macroeconomic case for downward real wage rigidity? *Journal of monetary Economics*, 56(4):605–615.

Hu, J. (2024). *Uniform inference on functional coefficients quantile regression model with endogeneity and its application*. Dissertation, Beihang University, Beijing, China.

Im, K. S., Pesaran, M. H., and Shin, Y. (2003). Testing for unit roots in heterogeneous panels. *Journal of econometrics*, 115(1):53–74.

Jones, C., Midrigan, V., and Philippon, T. (2022). Household leverage and the recession. *Econometrica*, 90(5):2471–2505.

Jordà, Ò. (2005). Estimation and inference of impulse responses by local projections. *American economic review*, 95(1):161–182.

Judd, K. L., Maliar, L., Maliar, S., and Valero, R. (2014). Smolyak method for solving dynamic economic models: Lagrange interpolation, anisotropic grid and adaptive domain. *Journal of Economic Dynamics and Control*, 44:92–123.

Kaplan, G., Mitman, K., and Violante, G. L. (2020a). The housing boom and bust: Model meets evidence. *Journal of Political Economy*, 128(9):3285–3345.

Kaplan, G., Mitman, K., and Violante, G. L. (2020b). Non-durable consumption and housing net worth in the great recession: Evidence from easily accessible data. *Journal of Public Economics*, 189:104176.

Kaplan, G., Moll, B., and Violante, G. L. (2018). Monetary Policy According to HANK. *American Economic Review*, 108(3):697–743.

Krusell, P. and Smith, Jr., A. (1998). Income and Wealth Heterogeneity in the Macroeconomy. *Journal of Political Economy*, 106(5):867–896.

LeSage, J. and Pace, R. K. (2009). *Introduction to spatial econometrics*. Chapman and Hall/CRC.

Li, Q., Huang, C. J., Li, D., and Fu, T.-T. (2002). Semiparametric smooth coefficient models. *Journal of Business & Economic Statistics*, 20(3):412–422.

Mendoza, E. G. and Villalvazo, S. (2020). FiPIt: A simple, fast global method for solving models with two endogenous states & occasionally binding constraints. *Review of Economic Dynamics*, 37:81–102. Publisher: Elsevier.

Mian, A., Rao, K., and Sufi, A. (2013). Household balance sheets, consumption, and the economic slump. *The Quarterly Journal of Economics*, 128(4):1687–1726.

Mian, A. and Sufi, A. (2014). What explains the 2007–2009 drop in employment? *Econometrica*, 82(6):2197–2223.

Potter, S. M. (2000). Nonlinear impulse response functions. *Journal of Economic Dynamics and Control*, 24(10):1425–1446.

Powell, J. H. (2022). Inflation and the labor market. Federal Reserve Board. Speech at the Hutchins Center on Fiscal and Monetary Policy, Brookings Institution, Washington, D.C.

Powell, J. H. (2023). Semiannual monetary policy report to the congress. Federal Reserve Board. Testimony before the Committee on Banking, Housing, and Urban Affairs, U.S. Senate, Washington, D.C. Identical remarks submitted to the Committee on Financial Services, U.S. House of Representatives, on March 8, 2023.

Rambachan, A. and Shephard, N. (2019a). Econometric analysis of potential outcomes time series: instruments, shocks, linearity and the causal response function. *arXiv preprint arXiv:1903.01637*.

Rambachan, A. and Shephard, N. (2019b). A nonparametric dynamic causal model for macroeconometrics. *available at SSRN 3345325*.

Reiter, M. (2023). State reduction and second-order perturbations of heterogeneous agent models. Technical report, IHS Working Paper.

Schaab, A. and Zhang, A. (2022). Dynamic Programming in Continuous Time with Adaptive Sparse Grids.

Schmitt-Grohé, S. and Uribe, M. (2016). Downward nominal wage rigidity, currency pegs, and involuntary unemployment. *Journal of Political Economy*, 124(5):1466–1514.

Schmitt-Grohé, S. and Uribe, M. (2017). Liquidity traps and jobless recoveries. *American Economic Journal: Macroeconomics*, 9(1):165–204.

Schmitt-Grohé, S. and Uribe, M. (2022). Heterogeneous Downward Nominal Wage Rigidity: Foundations of a Nonlinear Phillips Curve. Technical report, National Bureau of Economic Research.

Shen, W. and Yang, S.-C. S. (2018). Downward nominal wage rigidity and state-dependent government spending multipliers. *Journal of Monetary Economics*, 98:11–26.

Siemer, M. (2019). Employment effects of financial constraints during the great recession. *Review of Economics and Statistics*, 101(1):16–29.

Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters*, 20(2):177–181.

Wang, Q. and Xue, L. (2011). Statistical inference in partially-varying-coefficient single-index model. *Journal of Multivariate Analysis*, 102(1):1–19.

Yagan, D. (2019). Employment hysteresis from the great recession. *Journal of Political Economy*, 127(5):2505–2558.

Yang, L. and Tschernig, R. (1999). Multivariate bandwidth selection for local linear regression. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, 61(4):793–815.

Zhang, W., Lee, S.-Y., and Song, X. (2002). Local polynomial fitting in semivarying coefficient model. *Journal of Multivariate Analysis*, 82(1):166–188.

Zhou, X. and Carroll, C. D. (2012). Dynamics of wealth and consumption: new and improved measures for us states. *The BE Journal of Macroeconomics*, 12(2).