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How does uncertainty over a critical asset with the potential to change future balance of power affect conflict and communication patterns in multilateral bargaining?

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Abstract

How does uncertainty over a critical asset with the potential to change future balance of power affect conflict and communication patterns in multilateral bargaining?

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Artificial intelligence (AI) has emerged as a transformative technology with significant implications for both economic development and military power. As a result, control over its hardware, particularly the semiconductor and chip industry in Taiwan, has become a central geopolitical concern within the China-Taiwan-U.S. triangular relationship. However, the future of AI technology remains uncertain, which may stem from either imperfect information regarding its prospects or incomplete information that Taiwan possesses regarding its potential. Furthermore, if AI is deemed critical in altering the future balance of power between the United States and China, the U.S. is likely to intervene preemptively when China employs force against Taiwan. Drawing upon Fearon's (1995) rationalist framework for interstate conflict—specifically the commitment problem and private information—this study employs formal modeling to analyze how uncertainty surrounding a vital asset with the potential to shift the future balance of power can reshape crisis bargaining among three major powers. In scenarios where information concerning the asset's prospects is imperfect, a third party may be encouraged to disclose their type truthfully in private to its ally, even when incentivized to freeride on an ally's wartime efforts. Additionally, the asset's prospects positively correlate with the likelihood of multilateral war. However, when the asset is too imperative in altering the future balance of power, the rising power is likely to consistently avoid actions that risk war. When such information is exclusive to the target state, it will refrain from sending cooperative signals to the third party and will reject any low offers, aiming to entangle the third party in a multilateral war. In this scenario, the likelihood of war increases and may proliferate when prior beliefs indicate that the asset is substantial.

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1 Introduction

AI is considered one the emerging technologies with the greatest potential to reshape the existing geopolitical balance. In the field of International Relations, scholars argue that "artificial intelligence will have a large and potentially deterministic influence on global politics and the balance of power" (Horowitz, 2018, 38). Horowitz classifies AI as an enabling technology, since "it can be party of many specific technologies, analogous to the internal combustion engine as well as electricity" (Horowitz, 2018, p. 41). This "dual-use" of AI in both the economic and military realm is exceptional. As Gill puts it, "there is no other sector, with the exception perhaps of the cyber domain, where the commercial stakes are so high and so intertwined with the security stakes" (Gill, 2019, 171). Given that AI is not as yet fully dominated by any country or company, the country that ends up possessing the most advanced AI technology may not triumph only in economic development, but even in military supremacy. However, contrary to popular belief, the success of AI has not been primarily driven by its software (learning algorithms) but rather by the computing power on which these algorithms are trained. Since AI hardware requires highly specialized GPU (graphical processing units), the regions which can produce microprocessors may become a potential source of conflict, as occupying them will generate rents that are essential for AI development.

It turns out that an area which is already a source of major geopolitical tension in the escalating competition between the U.S. and China, Taiwan, is also the dominant specialist in the manufacture of AI-specialized semiconductors and chips. The Taiwanese private company, TSMC, accounts for "around 55 % of the global market for contract chip fabrication," including "semiconductors used in F-35 fighters and a wide range of "military-grade" devices used by US Department of Defense" (Liber, 2023, 44). China on the other hand "does not yet have the capabilities of manufacturing the world's most advanced semiconductors to enhance its AI technologies" as "microchips are currently the product of US giant NVIDIA, and are manufactured by TSMC" (Bega, 2023, 89). It could, therefore, be very beneficial for China to seize AI-enabling manufacturing capacity or technology by occupying Taiwan.

In the broader realm of Political Science, Fearon has proposed rational explanations for war (Fearon, 1995), with the two most imperative being the commitment problem and the problem of private information. The commitment problem suggests that whenever there is a drastic power shift, the rising state cannot credibly convince other parties to resist taking advantage of their improved capacity in the future against the declining power. This incentivizes the latter to initiate preventive war. The problem of private information refers to the fact that a state may misrepresent or miscalculate the other state's willingness or capability to fight.

It is possible for both problems of commitment and of private information to be present in the scenario of multilateral relations where major states are involved and one state possesses a crucial asset. For the state with the crucial asset, they either have private information or are perhaps uncertain about how such an asset will change the balance of power between major powers. This may lead to miscalculations when weighing the benefits of a war with another major power. Regarding the commitment problem, in a similar scenario, the third state will fear that if their opponent successfully captures or partially controls the state with the important asset, the opponent will have sufficient capacity to challenge the third party. This may lead to the proliferation of war due to the perceived benefit of the third state interceding preventively to prevent changes in the future balance of power with respect to the other major power.

It is therefore worthwhile studying how the possession of a crucial asset by a state affects conflict patterns as well as communication patterns when both a commitment problem and a private information problem are present. In this research, I will first present a three-player crisis bargaining model with incomplete information to demonstrate the possible conflict and communication patterns under such conditions. In addition to the "ongoing" case study of China's potential invasion of Taiwan, I also utilize two additional case studies, 1954-55 Taiwan Strait Crisis and Gulf War, to examine the external validity of this model.

In the baseline model I demonstrate that, when there is incomplete information over the willingness to fight for the third party and imperfect information on the future return of the asset, the third party will not truthfully reveal their cost of intervention publicly. They will seek to manipulate the outcome through public diplomacy, and there is a fully informative equilibrium when the signal is private to their ally. However, in a fully informative equilibrium where players lack information about the asset's potential value, multilateral war is more likely when the expected future returns are high. I also demonstrate that when the prospect of the asset is extremely high, the rising power would rather pacify other parties than risk war due to the exceptionally high opportunity cost of multilateral war. Additionally, I show that a third-party state will not choose to freeride on their ally's war effort and will provide cooperative private information when its policy preferences align more closely with its opponent's. This is because of the increased risk of freeriding on an ally's war effort, since the stakes are higher where players in our model are concerned not only with the current policy outcome, but also the future balance of power. In the extension model, I illustrate that there will be no cooperative equilibrium when the player possessing the asset has private information regarding the asset's prospects. Such a player will always reject the offer when the rising power proposes a low offer, signaling that the asset is worthwhile to defend even when the prospect of the asset is actually low. This strategy entangles the third party in a multilateral war to achieve an optimal outcome by increasing the probability of victory. Since the defending state has the incentive to withhold information and misrepresent it, the third party will not rely on the signal, but will use their prior beliefs to decide whether to intervene or remain neutral. Furthermore, under such circumstances, multilateral war is likely.

2 Past Models on Commitment Problem, Private Information, and Crisis Bargaining

There is an abundant literature on both the commitment problem and private information as causes of international conflict that inform this endeavor. Powell describes the commitment problem as occurring when, "...in the anarchy of international politics, states may be unable to commit themselves to following through on an agreement and may also have incentives to renege on it" (Powell, 2006, p. 170). Fearon (Fearon, 1999, 401-408) further specifies three ways in which commitment problems lead to wars: (1) there is a concern in the declining state about the future increase in power of the rising state, leading it to attack preventatively; (2) there is a general offensive advantage in staging preemptive war; and (3) there are issues at stake that will affect future bargaining power.

Levy expands the concept of preventive war beyond dyadic relationships to multilateral conflicts, arguing that the target of a threat posed by a rising power can also be a state's allies, and that the source of a rising threat can be a coalition of states (Levy, 2008, 7-8). Regarding the specific multilateral case of a third-party in the shadow of a conflict with a commitment problem, Powell finds that the third-party's decision to enter the war with the rising state is weighed up by its assessment of the future threat posed by the rising state and the potential return to scale in aggregation of military capabilities (Powell, 1999, 195). Regarding private information, Powell argues that in situations of asymmetric information, the declining state is always uncertain about how much concession will be needed to appease the rising power to avert war (Powell, 1999, 117). Reed, meanwhile, shows that the positive probability of conflict increases under information asymmetry, while the probability of conflict must be zero in an ultimatum bargaining model where war is costly and information is complete. Furthermore, he demonstrates that the closer two states are toward power parity, the higher will be the private information problem, leading to increasing probability of conflict (Reed, 2003).

However, Leventoğlu and Tarar find that private information alone does not incite war in a bargaining model, so long as there is a greater array of choice available to players (Leventoğlu and Tarar, 2008). Powell argues that war also occurs "when a state becomes convinced it is facing an adversary it would rather fight than accommodate," in contrast to Reed's assertion that a state will never choose to fight when there is complete information (Powell, 2006, 194).

This research contributes to the existing International Relations literature that focuses on formal theory about the commitment problem, private information, and crisis bargaining. Fearon (Fearon, 1996) provides a baseline formal model for commitment problems where two players are bargaining over some asset that can change the balance of power, and both sides are aware of the future prospect of controlling the asset. Krainin and Schub (Krainin and Schub, 2021) designs a formal model with a commitment problem, with the intention to address how prospective changes in balance of power may affect alliance arrangement. Smith (Smith, 2021) introduces a formal model for demonstrating when the communication facilitates military cooperation, including in the model the scenario where the private information about a third party's cost for joining the war is only known to that party, but not to the target or proposer. Alexandre and Nuno (Alexandre and Nuno, 2014) incorporate both the commitment problem and private information in a model where they explore how the imperfect information about military investment affects an opponent's decision of whether to launch a preventive war. Wolford et.al (Wolford et.al, 2011) uses a formal model to explore how the presence of both the commitment problem and private information affect the conflict outcomes in a two-player game where the proposer is uncertain of the target's battle cost, and the target will have an exogenous shift in power that will change its probability in winning battles in each round of a repeated game. Wolford's model (Wolford, 2020) includes both a commitment problem and private information, studying the possibility of soft balancing a state that would potentially become or not become a threat to a third party in a future where the proposer has a hidden level of aggressiveness, namely, the extent to which they can pose a threat to the third player in the future. Another formal model from Wolford (Wolford, 2014) aims to explore the conflict pattern in a scenario where the proposer faces a tradeoff between signaling military capability to its opponent while maintaining support from allies.

Given the focus on private information and commitment problems in the existing literature, and the inconsistencies in different models, this research aims to provide a model that captures both variables as causes of conflict in a multilateral setting. Specifically, despite the many formal modelling studies that exist, the existing literature has not yet considered a scenario like the one leveraged in this paper: a multilateral conflict setting where the sender of the signal is a target state and where the players (including or excluding the target state) are uncertain about the prospect of the asset they are bargaining over. Hence, I contribute to the formal theory and IR literatures by exploring the possible conflict patterns of a multilateral bargaining scenario with both a commitment problem and private information, combined with uncertainty about an asset that will change both short-term utility and the long-term balance of power of the major states as well as the target state.

3 Case Study: The Prospect of AI technology and the Taiwan dilemma

As mentioned, our model has been inspired by an "ongoing" case study: the multilateral dynamics around China, the U.S., and Taiwan – particularly given Taiwan's unique possession of AI manufacturing assets.

The Taiwan Strait conflict has become increasingly high-stakes because Taiwan is home to TSMC, an essential actor in the global chip manufacturing supply chain. Taiwan has hence become an area of increasing strategic importance in the competition between the U.S. and China. To understand the prospect of AI technology as we model this dilemma, it is important to set out some background about TSMC and the geopolitics of the chip industry.

TSMC has pursued a business strategy that makes it an indispensable destination for offshoring semiconductor manufacturing, given its comparative advantage in fabrication. As Miller details, it would be costly for any chip-designing company to build and operate its own fabs. Instead, a more efficient and profitable approach is for these companies to allocate resources to chip design while offshoring manufacturing to companies with existing fabs (Miller, 2022, 210). With the explosive growth of fabless firms starting in the 1980s, TSMC's business became an essential actor to one of the world's most promising industries. TSMC's visionary leader, Morris Chang, saw the fabless revolution as an opportunity to enhance competitiveness by forming a "Grand Alliance" and becoming a coordinator to set global industry standards. Other companies in the chip industry had little choice but to comply with TSMC's standards, as none possessed the capability to manufacture independently. Consequently, compatibility with TSMC's manufacturing processes became crucial for chip design. Additionally, equipment and material suppliers in the supply chain also rely on TSMC, as it is their largest customer (Miller, 2022, 219-220). Currently TSMC in Taiwan accounts for a substantial share of global chip production: According to Miller, "Taiwan produces 11 percent of the world's memory chips. More importantly, it fabricates 37 percent of the world's logic chips" (Miller, 2022, 339-340). Many companies in the industry rely on and align with TSMC, while those that do not struggle to compete. As a result, gaining access to or even exerting control over TSMC would be highly advantageous, particularly for countries lagging in the industry.

The semiconductor and chip industry is not only essential for economic growth but also for military power. Specifically, dependence on foreign semiconductor suppliers could potentially limit China's military and economic capabilities as a rising power. A pertinent example is Russia's situation in the Ukraine War. Economically, this dependence "has given the United States and its allies a powerful point of leverage," causing "Russia's manufacturing sector to experience wrenching disruptions, with a substantial portion of Russian auto production knocked offline" after the U.S. imposed strict restrictions on chip sales to Russia (Miller, 2022, 342-343). Militarily, Russia's weak semiconductor industry contributed to its failure to achieve its initial military goals. Even before the war, "high-priority defense projects in Russia struggled to acquire the chips they needed" (Miller, 2022, 343). After the U.S. imposed an embargo following the war in Ukraine, "Russia faced shortages of guided cruise missiles within several weeks of attacking Ukraine," whereas Ukraine "received huge stockpiles of guided munitions from the West, such as Javelin anti-tank missiles that rely on over 200 semiconductors each" (Miller, 2022, 343). Unsurprisingly, since taking power in 2012, Chinese President Xi Jinping has consistently emphasized the chip industry as a "core industry." In a 2016 conference on "cybersecurity and informatization," Xi urged China to prioritize breakthroughs in core technology, particularly semiconductors (Miller, 2022, 247). He further stated, "We must promote strong alliances and attack strategic passes in a coordinated manner. We must concentrate the most powerful forces to act together, compose shock brigades, and special forces to storm the passes" (Miller, 2022, 247-248). This military rhetoric highlights Xi's determination to strengthen China's chip industry and reflects "the precariousness of China's technological position" as "the chip industry was changing in ways that weren't favorable to China" (Miller, 2022, 248).

To mitigate the risks of dependency, the Chinese government has implemented various policies to support its domestic chip industry, including "vast government subsidies, state-backed theft of trade secrets, and leveraging access to the world's second-largest consumer market to compel foreign firms to comply" (Miller, 2022, 252). This approach has seen partial success, particularly with Huawei, which, "by the end of the 2010s, was designing some of the world's most complex chips for smartphones and had become TSMC's second-largest customer" (Miller, 2022, 275). Huawei also made significant advancements in 5G technology, positioning itself as a challenger to the American monopoly on chip design (Miller, 2022, 279).

However, while China has improved its design capabilities, it still lags technologically in other essential stages of chip manufacturing, leaving numerous chokepoints under U.S. and Taiwanese control. According to Miller, producing the most advanced chips requires access to software, fabs, and EUV lithography machines, which are all controlled by the U.S. and its allies (Miller, 2022, 315). Without access to these elements, China's chip industry cannot produce cutting-edge chips, even with the designs. A vivid example was the U.S. sanctions on Huawei in 2020, for the stated purpose of national security. The U.S. Commerce Department not only halted sales of U.S.-produced goods to Huawei but also "restricted any goods made with U.S. technology from being sold to Huawei" (Miller, 2022, 316). This policy severely impacted Huawei, as "TSMC couldn't fabricate advanced chips for Huawei without using U.S. manufacturing equipment," and even China's leading foundry, SMIC, "relies extensively on U.S. tools" (Miller, 2022, 316-317). As a result, Huawei had to divest parts of its smartphone and server businesses due to its inability to access the necessary chips (Miller, 2022, 317). This sanction highlighted the vulnerability of the Chinese chip industry and underscored the lack of large-scale, advanced semiconductor manufacturing infrastructure in China. Thus, China faces a choice: invest more heavily in domestic manufacturing or attempt to exert control over TSMC by force.

Achieving complete self-sufficiency in domestic chip production would be extremely costly and unlikely to yield cutting-edge technology comparable to TSMC's. As Miller notes, "the global chip industry spends over $100 billion annually on capital expenditures. China would need to replicate this spending, along with building the expertise and facilities it currently lacks." This makes total self-reliance implausible. Instead, China could reduce reliance on the U.S. in specific areas and increase its innovation and overall activity in the chip industry, gradually freeing itself from key chokepoints (Miller, 2022, 322-323). One approach could involve producing non-cutting-edge chips, giving China more leverage in demanding technology transfer and reducing the cost of U.S. export restrictions, while expanding its pool of skilled workers (Miller, 2022, 324-325). However, the effectivess of this strategy is likely to be limited, as it does little to enhance China's ability to manufacture the advanced hardware crucial for AI technology, leaving the U.S. with a much stronger position in this area.

With Taiwan's advanced chip manufacturing infrastructure located just across the strait—a region China claims as its own—it is conceivable, even likely, that China could attempt to pressure Taiwan into ceding some control of TSMC. If China succeeds in gaining some level of control over TSMC, it would effectively cripple the U.S. chip industry, as the U.S. relies heavily on TSMC for the production of advanced chips. "It would take years to replicate Taiwan's chip-making capacity in other countries" (Miller, 2022, 339). Furthermore, if the U.S. continues to rely on Taiwan, China "could gain influence or control over the only fabs with the technological capability and production capacity to produce the chips the U.S. depends on" (Miller, 2022, 339). In the event of a conflict, the destruction of some fabs would be devastating to the global chip industry and carry immense costs for both sides.

Given the U.S. policy of strategic ambiguity toward Taiwan, how would the U.S. respond if China sought control over TSMC? Both the U.S. and China face a commitment problem, as neither can credibly assure the other that they won't weaponize AI technology in the future. Therefore, each side is preoccupied with the potential impact of AI on the balance of power. If the U.S. adopts an appeasement strategy, it could avoid the immediate cost of war but risk a shift in power due to future AI advancements. Conversely, military intervention would address long-term concerns but incur substantial costs. Each side's cost-benefit analysis is based on uncertain AI technology outcomes, making their decisions reliant on their assessments about the future. What strategies would be rational under this uncertainty? Moreover, as Taiwan possesses valuable information about the potential of AI technology and its impact on chip production, the nature of communication during this crisis bargaining is crucial. Would Taiwan truthfully disclose the potential of AI technology to the U.S., or would it has the incentive to misrepresent this information to entangle U.S. into a multilateral war?

This research aims to examine the patterns of communication and conflict in such three-player crisis bargaining scenarios (China as player 1, Taiwan as player 2, and the U.S. as player 3), with a focus on the commitment problem and the problem of private information over a critical asset that could alter the future balance of power.

4 Baseline Model

The formal model we use will be a three-player dynamic game with incomplete information, following Smith (Smith, 2021). Consider a world with two states, player 1 and player 2, bargaining over an international issue in the shadow of the intervention from a third state, player 3. Player 3 is friendly to player 2 and is the potential opponent of player 1. Player 1 and player 2 are bargaining over some international policy denoted as x in policy space $X \in [0,1]$. The ideal policy for each player is denoted as $\hat{x_i}$, is $\hat{x_1} = 0$, $\hat{x_2} = 1$,

$\hat{x_3} \in [0,1]$ respectively. The utility for each player with respect to x immediately after the crisis bargaining outcome will be negative quadratic $u_i = -(\hat{x}_i - x)^2$.

However, unlike Smith's model (Smith, 2021), each player in this model will be concerned not only with their utility from the policy in the short-run, but also the influence of that policy on the balance of power in the long-run. The future balance of power is determined by the future prospect of such international policy x, and the amount each player compromises, which is $-(\hat{x}_i - x)^2$ for player 1 and 2. The prospect of the international policy is a type chosen by Nature from the type space $T_\theta = \{\underline{\theta}, \overline{\theta}\}$, and every player has the common prior belief $P[\theta = \underline{\theta}] = q$. $\overline{\theta}$ that such international policy is lucrative and can significantly improve future capabilities. For player 1, their long-run utility is $u_1 = -(\hat{x_1} - x)^2(q\underline{\theta} + (1-q)\overline{\theta})$, and for player 2, their long-run utility is $u_2 = -(\hat{x_2} - x)^2(q\underline{\theta} + (1-q)\overline{\theta})$. Player 3, who is the third party, also has concerns about the effect of player 1 becoming more capable in the future, which would make player 3 worse off in the long run. Hence, player 3's long-run utility is misaligned with player 1's, which is $u_3 = +(\hat{x_1} - x)^2(q\underline{\theta} + (1-q)\overline{\theta})$.

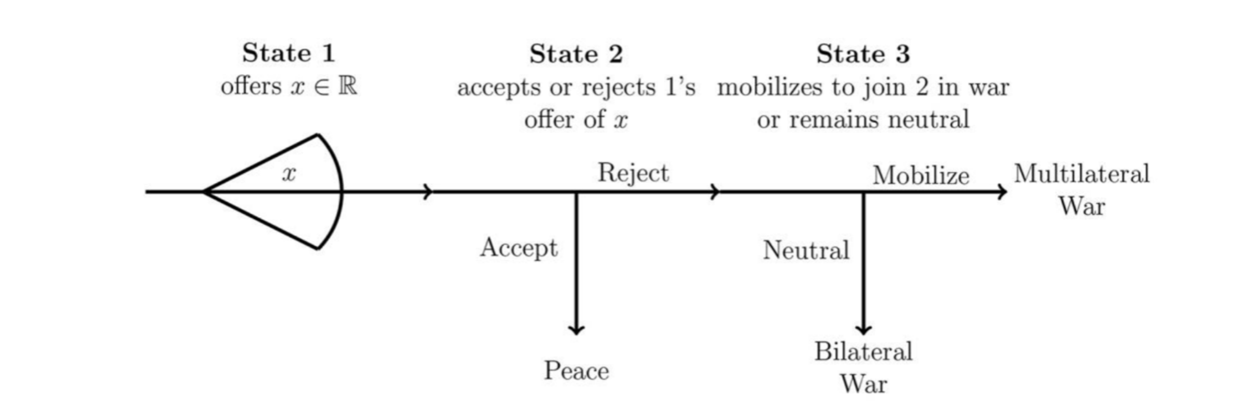

Figure 1: Timeline of the model after messages are sent (Smith, 2021, 1372)

At stage 1, Nature will choose both the type of the prospect of the policy from the type space $T_\theta = \{\underline{\theta}, \overline{\theta}\}$ as well as the type of the cost of war for player 3 from the type space $T_c = \{\underline{c_3}, \overline{c_3}\}$ where $0 < \underline{c_3} < \overline{c_3}$. In the baseline model, none of the players observe the future prospect of the policy, and only player 3 observes their cost of war while others hold the prior belief $P[c_3 = \underline{c_3}] = \phi$.

At stage 2, player 3 can choose to communicate with other players by sending a public cheap talk message or a private cheap talk message $m_3 \in T_c$. If a public cheap talk message is sent, then both player 1 and 2 are able to update their belief. If a private cheap talk message is sent, only player 2 – who is friendly to player 3 – is able to update their belief, while player 1 still possesses their prior belief about the cost of war for player 3. Note that similar to Crawford and Sobel (1982), such cheap talk message does not affect the payoff of each player directly.

At stage 3, player 3 proposes compromise policy $x \in [0,1]$. Next, at stage 4, player 2 responds by choosing to reject or to accept the compromise policy. If player 2 chooses acquiescence, the crisis bargaining will end in peace, where each player will receive the payoff with respect to the compromise strategy x player 1 chose. But if, at stage 4, player 2 chooses to reject the compromise policy, then the interaction will move to stage 5, at which point player 3 will subsequently choose to mobilize militarily or to remain neutral. If player 3 remains neutral, then there will be a bilateral war between only player 1 and player 2 where player 2 has the probability p to win the war. If player 3 chooses to mobilize, then there will be a multilateral war where a cost of war will be imposed on player 3 $\underline{c_3}$ or $\overline{c_3}$, and player 2 will have the probability $p' > p$ to win the war. In either case, the cost of war $c_1 > 0$ or $c_2 > 0$ will be imposed on player 1 and player 2 respectively. The winner of the war will implement their ideal policy $\hat{x_i}$. Furthermore, if there is a bilateral war, the future prospect of the policy will be discounted by r, and if there is a multilateral war, the future prospect of the policy will be discounted by $0 < r' < r$. The parameter r can be thought of as a portion of the intended asset that will be destroyed in the war or the fact that players will become less patient in the future (will elaborate in Key Features section), with the future return on the balance of power for possessing the asset reduced. Table 1 presents the payoff for each player under each scenario.

	Multilateral War
Player 1	$-p' - c_1 - r'p'(q\underline{\theta} + (1-q)\overline{\theta})$
Player 2	$-(1-p') - c_2 - r'(1-p')(q\underline{\theta} + (1-q)\overline{\theta})$
Player 3	$-p'(\hat{x_3} - \hat{x_2})^2 - (1-p')(\hat{x_3} - \hat{x_1})^2 - c_3 + r'p'(q\underline{\theta} + (1-q)\overline{\theta})$

	Bilateral War
Player 1	$-p - c_1 - rp(q\underline{\theta} + (1-q)\overline{\theta})$
Player 2	$-(1-p) - c_2 - r(1-p)(q\underline{\theta} + (1-q)\overline{\theta})$
Player 3	$-p(\hat{x_3} - \hat{x_2})^2 - (1-p)(\hat{x_3} - \hat{x_1})^2 + rp(q\underline{\theta} + (1-q)\overline{\theta})$

	Peace
Player 1	$-(\hat{x_1} - x)^2 - (\hat{x_1} - x)^2(q\underline{\theta} + (1-q)\overline{\theta})$
Player 2	$-(\hat{x_2} - x)^2 - (\hat{x_2} - x)^2(q\underline{\theta} + (1-q)\overline{\theta})$
Player 3	$-(\hat{x_3} - x)^2 + (\hat{x_1} - x)^2(q\underline{\theta} + (1-q)\overline{\theta})$

Table 1: Expected utility for each player by cases in baseline model

5 Extension Model

In the extension, we aim to test what will be the communication and conflict pattern when player 2 has private information on the capacity of the asset to change the future balance of power. This is in contrast to the baseline model where it was assumed that none of the players knows the type of future prospect of the policy.

In the extension model, only player 2 is capable of observing the type of θ. Consequently, at stage 2, player 2's strategy of deciding whether to accept or reject the offer proposed by player 1 has the potential to signal player 3, as it discloses information regarding the type of the asset. Specifically, player 2's decision to reject informs player 3 that player 1's offer does not surpass the expected value from engaging in conflict, implying the asset is worthwhile to defend; thus, player 3 can infer the type of the asset. Therefore, different from the baseline, the extension model will not contain the stage where player 2 sends a cheap talk message at the beginning of the game.

Furthermore, for simplicity, the willingness of player 3 to participate in multilateral war c_3 is common knowledge to all players in the extension, and there is only one type of such cost. This is different from the baseline model, where player 3 possesses private information about the cost of war, and there are two types of such cost. However, other elements, including the overall timeline of the model, the strategy space for each player at every stage of the model, and each player's payoff function, will be similar compared with the baseline. Table 2 summarizes the payoff for each player under peace, bilateral war, and multilateral war in extension.

	Multilateral War
Player 1	$-p' - c_1 - r'p'\theta$
Player 2	$-(1-p') - c_2 - r'(1-p')\theta$
Player 3	$-p'(\hat{x_3} - \hat{x_2})^2 - (1-p')(\hat{x_3} - \hat{x_1})^2 - c_3 + r'p'\theta$

	Bilateral War
Player 1	$-p - c_1 - rp\theta$
Player 2	$-(1-p) - c_2 - r(1-p)\theta$
Player 3	$-p(\hat{x_3} - \hat{x_2})^2 - (1-p)(\hat{x_3} - \hat{x_1})^2 + rp\theta$

	Peace
Player 1	$-(\hat{x_1}-x)^2-(\hat{x_1}-x)^2\theta$
Player 2	$-(\hat{x_2}-x)^2-(\hat{x_2}-x)^2\theta$
Player 3	$-(\hat{x_3}-x)^2+(\hat{x_1}-x)^2\theta$

Table 2: Expected utility for each player by cases in extension model

6 Key Features

In the basic setup of the model, the commitment problem arises because none of the three players can credibly commit to not exploiting the future advantage of possessing the critical asset. As a result, each player is concerned about the future payoff based on the current policy outcome. As stated above, the basic assumption is that players 1 and 2 will be worse off as there is a higher disparity between their optimal policy and the actual policy outcome. Player 3, on the other hand, will benefit in the future if their adversary, player 1, experiences a high disparity. However, the extent to which each player benefits or suffers depends on two additional factors: the expected value of the asset's future potential and the discount factor.

The calculation of the expected value of the prospect of the asset is straightforward where it is based only on private information. In the baseline model, none of the players has information about the asset's future potential, so they estimate the expected value based on their prior belief, q. In the extension model, however, players 3 can update their beliefs depending on player 2's communication strategy, leading to different optimal strategies compared to the baseline model. To refer to our "live" case study, if Taiwan were to produce information available to the U.S. concerning the significant impacts that the semiconductor industry will bring to AI technology in the future by costly signaling that the asset is worthwhile to defend, the U.S. will likely be more determined to defend Taiwan; it benefits if China has less access to such technology. Conversely, if Taiwan costly signals that the asset is with low prospect by accepting the offer, the game will simply end with peace being achieved. The model formally analyzes the patterns of communication and conflict under such a scenario.

Regarding the discount factors, their values vary based on the outcome of the crisis bargaining. If the game ends peacefully, the future value is not discounted. However, if it concludes in a multilateral or bilateral war, the future value is discounted, with a greater discount applied in the case of multilateral war. This setup is based on two fundamental principles. The first concerns the capital destruction that occurs during war. Multilateral war is particularly destructive, leading to unintentional or intentional damage (e.g., through scorched earth tactics) to essential physical capital. Additionally, war can lead to an outflow of human capital, as specialists and skilled workers crucial to producing or improving the critical asset may be displaced. Note that this is different from the cost of war, because the discount factor is imposed on all players uniformly given the eroded prospect of the critical asset. Thus, if an interstate war occurs, the asset's future quantity or quality will be lower than in a peaceful scenario. In general, multilateral war is also more destructive than bilateral war.

Second, the discount factor is also a reflection of players' patience toward potential future gains. The benefits from possessing the critical asset are not immediate; for instance, developing advanced AI technology or extracting natural resources takes time. In peacetime, countries are more patient because the perceived threat is less urgent, leading them to value the future more. In contrast, during wartime, countries prioritize immediate gains over the long-term benefits of possessing a critical asset, as they cannot wait for these benefits to materialize. Intuitively, the threat of being involved in a multilateral war is more immediate than in a bilateral war.

In the model presented above, we set the value of $\hat{x_3}$ between 0 and 1, without equating it to the ideal policy value of player 2, to maintain generality. It is plausible, for example, that the optimal U.S. position on controlling Taiwan's semiconductor industry does not involve Taiwan entirely withholding technology or manufacturing capability from China, because (1) American industries remain economically dependent on China and Chinese-designed chips; and (2) such an action would provoke the Chinese government, exacerbating the U.S.-China relationship in an undesirable way. However, since players 2 and 3 are allied states, the ideal policy positions of players 2 and 3 may be similar. Therefore, the scenario where $\hat{x_2} = \hat{x_3}$ is also tested in this research.

7 Analysis of Baseline

To evaluate how the prospect of the critical asset influences communication and conflict patterns in multilateral crisis bargaining, I will first analyze why player 3's communication strategy remains decisive in shaping both the conflict dynamics and the model's outcomes. Next, I will outline the fully informative equilibrium of the model, detailing the conditions upon which such equilibrium depends. Finally, I will compare my results to Smith's model (Smith, 2021) to explore how incorporating players' calculations regarding the future prospects of an asset alters the equilibrium outcomes.

In the analysis, we adopt a methodology consistent with Smith's framework (Smith, 2021, 1372), focusing on the fully informative perfect Bayesian equilibrium (PBE). This model operates within a cheap talk framework with incomplete information on player 3's cost of intervening, equivalent to their willingness to proliferate the conflict into a multilateral war, and where players update their beliefs based on the signals they receive. Also, different from Smith's model, there is imperfect information concerning the critical asset possessed by player 2. Our primary interest lies in identifying the sequentially rational and incentive-compatible PBE. A fully informative PBE is defined as one in which player 3 employs a separating strategy, sending cheap talk messages that faithfully correspond to their type with probability 1 ($m_3(\underline{c}) = \underline{c}$, $m_3(\overline{c}) = \overline{c}$, where $P[m_3 = \underline{c} \mid \underline{c}] = 1$, $P[m_3 = \overline{c} \mid \overline{c}] = 1$).

In order to identify the communication and conflict patterns that arise when private information and the commitment problem are present, it is also necessary to determine the conditions under which fully cooperative behavior between potential military allies can exist. Therefore, we aim to find an equilibrium in which players send honest cheap talk signals. Additionally, we focus on equilibria where signaling is influential, meaning that the receiver's behavior differs when a signal is sent compared to when it is not.

Following Smith's approach (Smith, 2021, 1372), we assume that the prior belief about player 3's probability of being a low-cost type is sufficiently low, $\phi < \phi^*$, which implies that there is a significant probability that player 3's cost of war is high (willingness of intervene is low). In this case, player 1 is willing to risk war if no signal is sent. If this assumption is not satisfied, the signal would not be influential, as player 2 would always accept a high offer from player 1, knowing that player 3 has little incentive to intervene in a conflict between player 1 and player 2.

7.1 Powerfulness of Player 3's Signal

In the baseline model, both the information shared by player 3 and their chosen communication response play a critical role in shaping both the conflict pattern and the overall outcome of the game, consistent with Smith's findings (Smith, 2021). This influence arises because the information conveyed by player 3 signals their willingness to intervene. Consequently, the signal can affect player 1's proposal if the message is public, as well as player 2's decision to accept or reject either if the message is public or private.

Intuitively, if player 3 publicly presents themselves as a low-cost type—indicating that they would intervene should player 2 reject the low proposal—player 1 is likely to propose a more generous deal, while player 2 will raise their threshold for accepting the offer. Conversely, if player 3 publicly signals themselves as a high-cost type, player 1 would propose a deal that is more favorable to themselves, and player 2 would lower their acceptance threshold. However, if player 3 chooses to privately communicate with their ally, player 2, the signal will only influence player 2's acceptance threshold, while player 1 continues to base their strategy on prior beliefs. Thus, we first present the condition under which player 3 would choose to mobilize.

Lemma 1. *Player 3 will choose to mobilize when*

$$c_3 \leq (p' - p)[(\hat{x_3} - \hat{x_1})^2 - (\hat{x_3} - \hat{x_2})^2] + (r'p' - rp)A \equiv c^*,$$

or otherwise, player 3 will remain neutral.

Lemma 1 establishes the threshold c, which determines whether player 3 will mobilize or remain neutral. The decision depends on player 3's cost of engaging in a multilateral war; they will mobilize if this cost is sufficiently low. A larger c indicates a broader range of c_3 values that meet the mobilization condition, meaning player 3 is more likely to intervene. Additionally, the threshold c^* depends on three key factors: $(p' - p)$, $\hat{x_3}$, and $(r'p' - rp)A$, reflecting the third party's cost-benefit calculation regarding intervention.

First, $(p' - p)$ represents the effectiveness of player 3's intervention in altering the outcome of the war. If player 3's military capabilities significantly increase player 2's probability of victory ($(p' - p)$ increases), player 3 becomes more likely to intervene (c^* increases). This shows that the third party's decision to intervene is not solely based on whether they can win the multilateral war but also on their ability to influence the conflict's outcome. If the impact of a third party's intervention is marginal, it will likely freeride on their ally's efforts, as the conflict's outcome would remain largely unchanged while the third party avoids the costs of military participation.

This dynamic highlights a common challenge in military alliances: weaker states often have incentives to remain neutral and freeride on the efforts of more militarily capable allies. Such behavior reflects a broader pattern in international relations, where the disparity in military strength among allies influences their willingness to commit to joint actions.

Regarding the effect of $\hat{x_3}$ on c, if player 3's ideal policy is more closely aligned with player 2's ideal policy, player 3 will be more inclined to intervene ($\hat{x_3}$ increases, c increases). This relationship is intuitive, as greater policy alignment incentivizes allies to cooperate. Finally, $(r'p' - rp)A$ represents the additional expected value of the critical asset that player 3 can extract in the future by intervening, which includes the probability of winning the war. If the future value of the critical asset gained through intervention

$(r'p' - rp)A$ increases, player 3 becomes more likely to intervene (c^* increases).

The logic underlying this relationship is similar to that of player 3's capacity to alter the war outcome. If player 3's involvement does not significantly enhance the likelihood of securing the critical asset (e.g., multilateral war results in substantial destruction of the asset compared to bilateral war, or player 3's involvement cannot sufficiently alter battlefield dynamics), then player 3 will tend to freeride on their ally's efforts to achieve a better outcome for the future balance of power. Thus, player 3 is more motivated to intervene directly when they believe their participation will meaningfully affect the conflict's outcome.

Comparing this to Smith's threshold $c^* = (p' - p)[(\hat{x_3} - \hat{x_1})^2 - (\hat{x_3} - \hat{x_2})^2]$, we observe that when players consider the prospect of the critical asset, player 3 generally has a wider range of c_3 values that satisfy the mobilization condition. This result is unsurprising because, under the new assumptions, player 3 is concerned not only with how closely the current policy aligns with their ideal policy but also with the future balance of power between player 1 and player 3. The worse off player 1 becomes, the better off player 3 will be, except in a case where multilateral war is exceptionally destructive thus causing players become significantly less patient after the war, which reduces r' to a point where $(r'p' - rp) < 0$. Furthermore, as A (the expected value of the critical asset) increases, player 3's willingness to intervene grows, holding other parameters constant.

This analysis suggests that unless the collateral damage of multilateral war is extraordinarily higher than that of bilateral war, the third party will generally be more inclined to intervene after observing their ally engage in conflict with a potential opponent, especially when considering the high prospect of the critical asset. In the Taiwan case, the model predicts that if the U.S. perceives AI technology as highly likely to be strategically vital in the future, it would be much more likely to intervene militarily after observing a failure in crisis bargaining between China and Taiwan.

We have demonstrated that player 3's strategy at the end of the game tree determines the type of conflict the three countries will enter, significantly affecting the ultimate payoffs for all players. Player 3 employs a threshold strategy based on their cost of intervention when selecting its response. Consequently, if player 3 can credibly signal this cost at the beginning of the game, other players will adjust their strategies accordingly. This underscores the powerful role of cheap talk messaging. Lemma 2 further validates this speculation regarding the influence of cheap talk in shaping player 1's offer.

Lemma 2. *Under complete information, player 1 would offer:*

$$\begin{cases} \overline{x} = \hat{x_2} - \sqrt{\frac{(1-p')(\hat{x_2}-\hat{x_1})^2(1+r'A)+c_2}{1+A}} & \text{if } c_3 \leq c^* \\ \underline{x} = \hat{x_2} - \sqrt{\frac{(1-p)(\hat{x_2}-\hat{x_1})^2(1+rA)+c_2}{1+A}} & \text{if } c_3 > c^* \end{cases}$$

Lemma 2 illustrates the offers player 1 would propose under complete information over player 3's type. When player 3's cost of war is low and they are more inclined to intervene, backward induction suggests that player 1, knowing that player 3 is likely to join the war, would anticipate player 2 having a wider rejection interval for the proposal x. Consequently, player 1 would propose a higher offer $\overline{x}$ to induce player 2 to accept and thus avoid multilateral war, while still maximizing their own payoff in peace.

A similar rationale applies when player 3's cost of war is high and they are more likely to acquiesce. In this case, player 1 would propose a lower offer $\underline{x}$, reflecting the reduced likelihood of player 3's intervention and player 2's narrower rejection interval.

The complete information game suggests that in the context of an incomplete information model, if player 3 sends an informative message to other players, these players will use the message to infer player 3's true type. Using this information, they will adjust their strategies accordingly, applying the same logic to revise their offers or responses to proposals. This demonstrates the power of cheap talk messages in changing the strategy and the outcome of crisis bargaining.

7.2 Equilibrium Circumstances

Given that player 3's message significantly influences the behavior of players 1 and 2, it is reasonable to assume that player 3 may not present information truthfully, as doing so allows them to manipulate the behavior of other players to achieve an optimal outcome for themselves. In this section, we analyze the conditions under which a fully informative equilibrium, where player 3 sends truthful signals, may arise. Note that player 3 has two communication strategies: to reveal their type publicly to both their opponent and allies (players 1 and 2); or to communicate privately with only their ally (player 2). We examine the fully informative PBE for each case, starting with public communication.

Proposition 1. *When player 3 sends a cheap talk message publicly to both player 1 and player 2, there will be no fully informative equilibrium. This result holds regardless of whether players have assessed the future prospect of the asset.*

The outcome of Proposition 1 aligns with Smith's findings, which similarly conclude that player 3 will never send truthful signals when communication is public. The reasoning, consistent with Smith's explanation (Smith, 2021, 1375), is that public communication gives player 3 the opportunity to manipulate both players' behavior. As a result, player 3 is incentivized to misrepresent their type to achieve an outcome closer to their ideal policy and cost of war as dictated by nature. Thus, a separating strategy—where player 3 truthfully signals their type publicly—is not incentive-compatible in this case.

Moreover, when player 3's ideal policy aligns closely with that of player 2, a high-cost type player 3 will always misrepresent themselves as a low-cost type, which emphasizes their determination to intervene militarily. This signaling leads player 2 to reject any low offer from player 1, as player 2 perceives that player 3 is committed to intervening. Consequently, player 1 is forced to propose a deal more favorable to player 2 to avoid war. Player 3 benefits from this outcome by securing a policy closer to their ideal while avoiding war. Similarly, when player 3's ideal policy is more moderate relative to player 2, a low-cost type player 3 will misrepresent themselves as a high-cost type, thus downplaying their willingness to intervene. This causes player 2 to show less resistance to player 1's proposals, as they perceive that player 3 will not intervene. In turn, player 1 can propose a deal more favorable to themselves without risking conflict. For player 3, this outcome aligns with their ideal policy and avoids the costs of war.

Because the third party consistently has incentives to misrepresent their type in public communications, such signaling lacks credibility for players 1 and 2. This is especially true when the allies' ideal policies are aligned, as the potential opponent (player 1) has strong incentives to believe that player 3 is bluffing. This pattern is even more pronounced in scenarios where the future prospect of the critical asset is included in players' calculations. In this context, player 3's overall payoff depends not only on the current policy outcome but also on how much worse off player 1 becomes. Consequently, player 3 has stronger incentives to bluff, exaggerating their willingness to intervene to induce player 1 to propose a deal further from their own ideal policy. Recognizing player 3's incentives, player 1 may discount the credibility of public diplomacy and instead rely on prior beliefs when proposing a deal, even at the risk of war.

A historical example provided by Smith (Smith, 2021, 1374) illustrates this dynamic. In 1939, Adolf Hitler interpreted Britain's public statements of support for Poland as a bluff intended to deter German aggression. Consequently, these signals did not alter Hitler's plans to invade Poland, demonstrating the limited credibility of public signals in such scenarios.

Given the low credibility of public signals, particularly when players account for the future value of the critical asset, we next analyze whether a fully informative PBE exists when player 3 communicates privately with their ally.

Proposition 2. *When player 3 sends the cheap talk message privately to player 2 only, there will be a fully informative equilibrium iff*

$$\hat{x_3} \in \left[\frac{p'(1-r'A) - \underline{x}^2(1-A) + \underline{c}}{2(p'-\underline{x})}, \frac{p(1-rA) - \underline{x}^2(1-A)}{2(p-\underline{x})} \right],$$

and in such equilibrium, there will be a multilateral war when:

$$c_3 = \underline{c},\ x = \underline{x}, \text{ and } \phi \leq \frac{(1+A)[(\hat{x_1}-\underline{x})^2 - (\hat{x_1}-\overline{x})^2]}{p'+c_1+r'p'A-(\hat{x_1}-\underline{x})^2(1+A)}$$

The result from Proposition 2 aligns with Smith's findings (Smith, 2021, 1376). It demonstrates that when player 3's ideal policy falls within the specified range, a fully informative PBE can arise where the private message sent by player 3 is credible. However, in this equilibrium, player 1 remains uninformed and relies solely on prior beliefs about player 3. As a result, player 1 may occasionally risk war by proposing an offer more favorable to themselves, particularly when the probability of player 3 being a low-cost type is low (indicating player 3 is less likely to intervene). This finding reinforces the idea that private information serves as a rational explanation for war.

While credible private signaling between allies enhances cooperative action among military allies, thereby optimizing outcomes given player 1's strategy, it does not eliminate the risk of war if prior beliefs are inaccurate. Specifically, if player 1 believes there is a low probability that player 3 is a low-cost type but player 3's true type is indeed low-cost, war may still occur due to the misalignment between player 1's prior belief and the reality of player 3's cost type.

Combining the results of Propositions 1 and 2 further illustrates the inherent challenges of multilateral crisis bargaining with signaling. If player 3 publicly signals their type, war can theoretically be avoided, provided the signal is credible and other players update their beliefs and adjust their strategies accordingly, thereby resolving the private information problem. However, because player 3 consistently has incentives to deviate and bluff—particularly when the future value of the critical asset is factored in, as the opponent player 1 is unlikely to trust public signals. In practice, player 1 would conceivably ignore the diplomacy and instead rely on prior beliefs, which could lead to war if those beliefs are inaccurate in predicting player 3's type.

Conversely, if the signal is private, the private information problem persists, as player 1 retains an incentive to gamble by proposing a riskier offer when they perceive that player 3 is unlikely to intervene. Thus, in either scenario, war cannot be efficiently avoided through diplomatic signaling alone.

7.3 Comparative Statistics

Having derived the equilibrium when players account for the prospect of the critical asset, which aligns with Smith's findings (Smith, 2021), we now address two key questions. Does this consideration make war more likely? And how does it influence each player's optimal strategy? In this section, we present a comparative analysis to illustrate the effects of the critical asset's prospect.

Proposition 3. *When players account for the prospect of the critical asset, compared to Smith's work (Smith, 2021), the offering of* $\underline{x}$ *and* $\overline{x}$ *will be higher. Additionally, there is a positive correlation between the expected value of the asset's prospect and the values of* $\underline{x}$ *and* $\overline{x}$*.*

The result of Proposition 3 is straightforward and builds upon Lemma 1. When players consider the future returns of the critical asset, player 3 is more inclined to intervene, provided that the multilateral war is not excessively destructive, as shown in Lemma 1. Consequently, through backward induction, player 2 becomes more likely to reject low offers from player 1, knowing that player 3 would be predisposed to intervene. This improves player 2's bargaining power, as the probability of victory in a multilateral war is higher than in a bilateral war. To pacify player 2 and induce them to accept the offer, player 1 is compelled to offer more.

Following the same logic, as the asset's prospect increases, player 2 is even more willing to risk war. In turn, this forces player 1 to make increasingly generous offers. Thus, when players account for the critical asset's prospect and its implications for the future balance of power, player 1 ultimately ends up with a less favorable outcome. This dynamic arises because the players in the model face not only the private information problem but also a commitment problem. None of the players can credibly assure the others that they will refrain from leveraging the critical asset to develop stronger military or economic capabilities that could threaten their opponents.

As a result, as a third party, player 3's decision to intervene can be interpreted as a form of ”preventive war,” aiming to stop a rising power from gaining control of the critical asset. Recognizing the commitment problem and player 3's increasing resolve to act preventively, player 2's bargaining power increases, which enables them to demand more from their potential opponent, player 1, at the bargaining table.

Proposition 4. *When players are concerned about the prospect of the critical asset (cf. Smith, 2021):*

1. *When the communication is private, there exists a fully informative equilibrium where the lower bound for $\hat{x}_3$ will be higher for certain compared to the equilibrium where players are not concerned about the prospect of the asset when A and p is high, and $r'p'$ is low.*

2. *The upper bound for $\hat{x}_3$ will be higher for certain scenarios compared to the equilibrium in which players are not concerned about the prospect of the asset when A and p is high, and rp is low.*

Proposition 5. *When players are concerned about the prospect of the critical asset (cf. Smith 2021):*

1. *In a fully informative equilibrium where communication is private, player 3's ideal policy range for sending the truthful signal can be higher or lower.*

2. *The correlation between the expected value of the prospect A and player 3's ideal policy range for sending the truthful signal is ambiguous.*

Propositions 4 and 5 provide a comparative analysis of the equilibrium circumstances in which player 3 will send a private cheap talk signal to player 2. Proposition 4 examines the upper and lower bounds of the interval for $\hat{x}_3$ to determine whether including the prospect of the critical asset excludes certain third parties with different alignment interests from sending cooperative signals. For example, a higher upper bound would indicate that third parties whose ideal policies align more closely with their ally's are more likely to reveal their type truthfully. Proposition 5 examines the range of the interval compared to Smith's results and analyzes the correlation between the prospect of the asset and the range. Proposition 5 therefore aims to explore whether the prospect of the critical asset alters player 3's overall incentive to send cooperative signals. A larger range would indicate that player 3 is more motivated to truthfully reveal their type to their ally.

The findings of Proposition 5 reveal some ambiguity, as there is no definitive increasing or decreasing relationship between the interval and the expected value of the asset's prospect, A. Similarly, there is no conclusive evidence that considering the future prospect of the asset incentivizes player 3 to use cooperative signals. However, under certain heterogeneous parameter conditions analyzed in Proposition 4, we observe scenarios where the bounds of the interval for $\hat{x}_3$ are higher compared to Smith's work.

The first statement of Proposition 4 demonstrates that when the asset's prospect (A) and the probability of winning a bilateral war (p) are high, while the expected future change in the balance of power in favor of third party in a multilateral war is low ($r'p'$), third parties with less aligned interests are more likely to falsely signal their type to their ally. Under such circumstances, low-cost type players are incentivized to feign being high-cost types, as low $r'p'$ makes mobilization less appealing, even for low-cost types. This finding is consistent with Lemma 1, where the consideration of a high future balance of power (A) further motivates such behavior, as player 3 would prefer to secure some critical assets today rather than risk having fewer assets in the future by engaging in a multilateral war.

The second statement of Proposition 4 highlights that when player 3's ideal policy aligns more closely with player 2's, they are more likely to send cooperative signals when A and p are high, and rp is low. In such cases, if player 3's cost of war is low, they will faithfully reveal this information to their ally privately, as they can bear the cost of war if player 1 chooses to risk it, while achieving the highest payoff if player 1 chooses peace. Even when player 3's cost of war is high, they will still reveal their type truthfully, opting not to freeride on their ally's war effort. This is because, under low rp, ending the game in a bilateral war and freeriding on player 2's efforts is not so advantageous to player 3, especially when the stakes in the future balance of power (high A) are significant.

A similar logic extends to Proposition 6, which states the following:

Proposition 6. *When player 2 and 3's ideal policies match, $\hat{x}_2 = \hat{x}_3$, there will exist a fully informative equilibrium through private communication.*

Proposition 6 provides evidence that differs from Smith's results (Smith, 2021, pp. 1373–1374). It demonstrates that a fully informative equilibrium can exist when the allies' ideal policies are aligned. This divergence arises because, in my model, the risk of freeriding is higher. In the scenario where player 3 misrepresents themselves as a low-cost type and player 1, based on this misrepresentation, proposes a low offer that risks war, player 3 stands to lose significantly more if player 2 is defeated in a bilateral war. This is because player 2's and player 3's future interests are aligned; both seek to limit the growing power of player 1.

Therefore, when the third party's policy interests align with those of their ally, and they prioritize the future balance of power, the communication dynamics change. In this scenario, there exists an equilibrium where the third party faithfully reveals their type privately to their ally. This cooperative signaling helps avoid the higher risks associated with freeriding on their ally's efforts.

Proposition 7. *When players are concerned about the prospect of the critical asset, in the fully informative equilibrium where player 3 communicate privately (cf. Smith, 2021):*

1. *There is still the probability of war where player 1 will have a wider interval of ϕ to risk multilateral war unless the value of $r'p'$ is excessively high.*

2. *The range of ϕ is positively correlated with the expected value of the future prospect of the asset when the value of $r'p'$ is not excessively high and the starting value of A is low. Otherwise, the relationship is ambiguous.*

Proposition 7 highlights the conflict patterns specific to our model. In contrast to Smith's result (Smith, 2021), the probability of war increases when players account for the prospect of the critical asset. In this model, the future returns associated with the critical asset provide additional utility to player 1 when player 3 is a high-cost type, as this implies player 3 will not intervene. Consequently, player 1 is more willing to gamble on player 3's type, choosing a lower offer and thereby making multilateral war more likely in such scenarios.

Moreover, when the expected value of the critical asset's prospect increases, the likelihood of war also increases. However, this positive correlation only holds when the initial expected value of the asset's prospect is low, reflecting the diminishing marginal returns of the prospect of the asset on the probability of war. When the asset's prospect is already excessively high, player 1 finds it more beneficial to secure part of the asset through a deal, as the risk of losing everything in a war becomes too great. In such cases, the opportunity cost of gambling on war is high. Conversely, when the prospect is initially low, a marginal increase in the asset's value incentivizes player 1 to risk war, as the opportunity cost of losing remains much lower.

Furthermore, for this positive correlation to hold, the value of $r'p'$ cannot be excessively high. When $r'p'$ is very large, player 1 is less inclined to risk war because the likelihood of losing a multilateral war (due to high p') is substantial. Additionally, if player 1 loses the war, the change in the balance of power will be significant, as a high r' ensures that the critical asset's value is not sufficiently diminished by the conflict. Thus, the model demonstrates that even in a fully informative equilibrium, war remains possible. Moreover, the consideration of the asset's future prospect increases the likelihood of multilateral war by incentivizing the rising power to gamble for an optimal outcome.

7.4 Baseline Model Conclusion

Overall, these results reinforce the external validity of Fearon's rational explanation of war. In a crisis bargaining model with three players that incorporates both private information and a commitment problem, private information can render multilateral war more likely. Furthermore, the model's finding of diminishing marginal returns of the critical asset on the probability of war contributes to the broader literature on conflict patterns in multilateral war settings. The results indicate that assets with moderate effects on the long-term balance of power are more likely to incite multilateral war. Paradoxically, an excessively indispensable critical asset does not necessarily sustain this positive correlation, as the rising power is more inclined to cut a deal and secure part of the asset to avoid the high opportunity cost of losing a multilateral war.

To conclude, in the baseline model, we have shown that when players account for the prospect of the critical asset, and when signals remain public, no fully informative equilibria exist. In this case, as a third party player 3 always has an incentive to misrepresent their willingness to fight in order to manipulate the other players and achieve the highest utility outcome for them. However, fully informative equilibria do exist when signals remain private. In such scenarios, war is more likely compared to situations where players do not consider the future balance of power. Additionally, there is a positive correlation between the expected value of the critical asset's prospect and the probability of multilateral war, unless the risks associated with war are too high for player 1, the rising power. Finally, when the ideal policy points of players 2 and 3 are aligned, specific parameter values can ensure that player 3 sends cooperative signals privately.

8 Analysis of Extension

To examine how player 2's private information regarding the critical asset influences communication and conflict patterns in multilateral crisis bargaining, I will first determine whether player 2's signaling decisively impacts the strategic decisions of other players and the conflict outcome, paralleling the baseline analysis. Subsequently, I will explain why cooperative signaling does not occur in this extension and describe the non-cooperative PBE of this model. Finally, I will compare the extension's outcomes with the baseline results to understand how changes in information endowment allow player 2 to strategically shift the bargaining outcome in their favor.

Recall that in the extension, the incomplete information is over the prospect of the asset θ where only player 2, the owner of the asset, possesses such information where other players are relying on the prior belief or the signal from player 2. Different from the baseline model, for simplicity, there is no longer uncertainty over the cost of war for player 3, where such information is commonly known by all players. The sequence of play of the extension as well as the payoff function is similar to the baseline, except that player 2 can only use the costly signal of reject or accept the offer to inform player 3 regarding the type of the asset where there is no cheap talk message given by player 2.

In contrast to the baseline, the extension does not involve sending cheap talk messages; thus, there is no fully informative PBE where players send cheap talk messages accurately reflecting their true type. Instead, this extension focuses on PBEs in which player 2 behaves cooperatively. Within this model, a cooperative signal is defined as the strategy that player 2 would adopt under complete information equilibrium conditions. Conversely, any strategy that diverges from the complete information equilibrium strategy is classified as non-cooperative, since such deviations allow player 2 to manipulate player 3 and freeride on player 3's war efforts. Under complete information, in this PBE, when player 1 makes a low offer $\underline{x}$, player 2 employs a separating strategy that accepts if the asset type is low and rejects if the asset type is high ($P[s_2 = A|\theta = \underline{\theta}] = 1, P[s_2 = A|\theta = \underline{\theta}] = 1$). When player 1 makes a high offer $\bar{x}$, player 2 adopts a pooling strategy, always accepting regardless of asset type ($P[s_2 = A|\theta = \underline{\theta}] = 1, P[s_2 = A|\theta = \underline{\theta}] = 1$).

Similar to the baseline scenario, it is assumed that the prior belief about the asset type being low is sufficiently low ($q < q$) when $r'p' < \bar{x}^2$, and sufficiently high ($q > q$) when $r'p' > \bar{x}^2$ (such condition is derived in the proof of Proposition 9). Given this assumption, player 1 is willing to risk war in the absence of a signal. If this assumption does not hold, signaling becomes ineffective, as player 2 would consistently accept a high offer from player 1, knowing that player 3 has little incentive to intervene in a conflict between players 1 and 2.

8.1 Powerfulness of Player 2's Signal

In the extension, the expected prospect of the asset significantly influences other players' strategic decisions, thereby altering conflict dynamics. This impact arises because when anticipated future changes in the balance of power favor the third party more in a multilateral war than in a bilateral war ($r'p' > rp$), player 3 mobilizes only if the asset is perceived to have substantial potential. Consequently, player 1 is compelled to extend a more favorable offer to player 2 to ensure their acceptance, and vice versa. However, unlike in the baseline scenario, the exact influence of the costly signal on player 3's decision to intervene remains ambiguous based solely on the complete information results. Therefore, further analysis is necessary to evaluate the effectiveness of player 2's signaling under conditions of incomplete information.

Lemma 3. *Under complete information, if $r'p' > rp$, Player 3 will choose to mobilize when*

$$\theta \geq \frac{(p'-p)[(\hat{x_3}-\hat{x_2})^2-(\hat{x_3}-\hat{x_1})^2]}{(r'p'-rp)} \equiv \theta^*$$

Otherwise, player 3 remains neutral.

Lemma 3 demonstrates that under complete information, the prospect of the asset can effectively influence player 3's intervention decision, where player 3 relies on threshold θ^* to decide between intervention and neutrality. A higher threshold implies player 3 is more likely to remain neutral.

Analyzing this inequality reveals that as $\hat{x_3}$ increases, the threshold decreases, making player 3 more inclined to mobilize. Furthermore, if $\hat{x_3}$ surpasses 0.5, indicating player 3's foreign policy preference aligns closely with player 2, player 3 will mobilize regardless of other parameter values. This behavior occurs because mobilization supports player 3's short-term objective when aligned with player 2's goals, as a multilateral war victory helps achieve player 2's preferred foreign policy. Conversely, when $\hat{x_3}$ is below 0.5, indicating player 3's foreign policy preference aligns closer to a potential adversary, an increase in p' or r' enhances player 3's willingness to mobilize. This finding aligns with results from the baseline scenario, highlighting the free-rider problem in alliance theory, where states benefit from their allies' efforts without equally contributing.

Having established player 3's optimal response at the game's final stage, Lemma 4, derived via backward induction, determines the offer player 1 should present initially, demonstrating how the asset's prospect affects player 1's initial proposal.

Lemma 4. *Under complete information, when $r'p' > rp$, player 1 would offer*

$$\begin{cases} \overline{x} = \hat{x_2} - \sqrt{\frac{(1-p')(\hat{x_2}-\hat{x_1})^2(1+r'\overline{\theta})+c_2}{1+\overline{\theta}}} & \text{if } \theta = \overline{\theta} \geq \theta^* \\ \underline{x} = \hat{x_2} - \sqrt{\frac{(1-p)(\hat{x_2}-\hat{x_1})^2(1+r\underline{\theta})+c_2}{1+\underline{\theta}}} & \text{if } \theta = \underline{\theta} < \theta^* \end{cases}$$

Lemma 4 addresses crisis bargaining dynamics in complete information equilibrium. When the asset's prospect is high, player 3 is more inclined to mobilize in the final stage. Consequently, player 2 is motivated to reject the offer, anticipating a higher probability of victory in a multilateral war. Therefore, player 1 must offer a higher amount $\overline{x}$ to secure player 2's acceptance at the second stage, maximizing player 1's payoff by avoiding costly multilateral conflict.

Similarly, when the asset's prospect is low and player 3 prefers neutrality, player 1 proposes a lower offer $\underline{x}$, aligning with player 2's narrower rejection range due to player 3's anticipated non-intervention.

We assume expected future shifts in the balance of power favor the third party more in multilateral war ($r'p' > rp$), since deriving a generalized optimal strategy for player 1 becomes impractical under complete information when $r'p' < rp$. In such scenarios, player 1's optimal offers vary case by case, as short-term advantages for players 2 and 3 might conflict with their long-term interests. Specifically, multilateral conflict engagement may be less beneficial in the long term compared to bilateral conflict but advantageous in the short term. Thus, for simplicity and broader applicability, this extension's results assume $r'p' > rp$.

However, the complete information model alone does not clearly indicate whether player 2's costly signal (accepting or rejecting the offer) strongly influences player 3's decision. Nonetheless, one might intuitively infer that player 2 rejecting a low offer signals the asset's significant future value outweighing war costs, marking a vital asset worth defending through conflict. Yet, player 2 could still adopt a babbling strategy under private information conditions, potentially misleading player 3 into conflict regardless of the asset's true value. This scenario may be particularly relevant when expected future shifts in the balance of power favor the third party more strongly in multilateral conflict than bilateral conflict ($r'p' > rp$). The subsequent analysis will clarify equilibrium outcomes under incomplete information.

8.2 Equilibrium Circumstance

Following the results from the complete information version of the extension, we postulate that, although the potential of the asset is indeed significant in altering the strategic dynamics, the costly signal from player 2 may convey no information to their ally, player 3, in the incomplete information version of the model. It is important to note that player 1 within the model cannot utilize the signal to update their beliefs and subsequently adjust their strategy, as the signal is communicated in the second stage of the game while player 1 acts in the first stage. Consequently, player 1 relies on their prior belief regarding the type of the asset's potential when deciding whether to extend a high offer $\overline{x}$, or a low offer $\underline{x}$, which is similar to the baseline model where player 3 sends a private signal. As a result, we can partition the game into two subgames: player 1 offers high offer or a low offer in the first stage. This segmentation facilitates a detailed examination of the strategic interactions between player 2 and 3, thereby analyzing the effectiveness of the signal in influencing the outcome of the conflict.

Proposition 8. *Player 2 will cooperate with player 3 when sending the costly signal in second stage only when player 1 offers $\overline{x}$. If player 1 offers $\underline{x}$, player 2 will not cooperate but use a pooling strategy $s_2 = R, \forall\theta$ when the prior belief of the likelihood of the type of asset is a low type is low ($q < q^* = \frac{(\hat{x_3}-\hat{x_2})^2(p'-p)+(\hat{x_3}-\hat{x_1})^2(p-p')+\overline{\theta}(rp-r'p')}{(r'p'-rp)(\underline{\theta}-\overline{\theta})}$) to entangle player 3 into multilateral war and free ride on player 3's war effort.*

The findings of Proposition 8 confirm that player 2's costly signaling does not reveal any information to player 3 in either subgame, as player 2 consistently possesses the incentive to adopt a pooling strategy. In this context, player 3's belief will not be effectively updated, and they will continue to rely on their prior belief regarding the type of the asset's prospects to determine whether to mobilize or remain neutral. Furthermore, no cooperative Perfect Bayesian Equilibrium (PBE) exists in this extension since the best response for player 2 when the offer is low is to always reject to drag player 3 into a multilateral war. However, note that such strategy is a best response only if the prior belief is low enough ($q < q^*$) where player 3 is willing to risk multilateral war.

Firstly, when player 1 makes a high offer $\bar{x}$, player 2 will accept this offer under all circumstances, thereby concluding the game amicably. This behavior aligns with the strategy employed in the complete information version of the game. The rationale behind this lies in the fact that an offer of $\bar{x}$ can satisfy even the most demanding player 2, whose rejection range is extensive due to the significant incentive for player 3 to intervene, along with the considerable potential of the asset. Consequently, irrespective of whether the information is complete or incomplete, such an offer has the capacity to placate any player 2, leading to behavior that mirrors that outlined in the complete information model. This indicates that player 2's signal is cooperative, as they lack the incentive to alter player 3's behavior by deviating from the strategy adopted in the complete information game.

However, when player 1 proposes a low offer $\underline{x}$, it is incentive-incompatible for player 2 to act cooperatively by accepting when the asset's prospect is low and rejecting when high. Instead, player 2 prefers a pooling strategy that obscures information from player 3 if player 3 is likely to intervene based on prior beliefs. This preference emerges because player 3's prior belief of a high asset potential ($q < q^*$) motivates their mobilization even without updated beliefs. Hence, player 2's optimal response is to reject all offers regardless of the asset's true type, inevitably involving player 3 in conflict. Assuming $r'p' > rp$, player 2 benefits more from multilateral conflict by freeriding on player 3's efforts. Thus, deviating from cooperative signaling becomes advantageous for player 2 when player 1 is prepared to risk war, and the asset initially appears valuable. This dynamic underscores international relations theory suggesting private information fosters conflict, as player 2 can exploit informational asymmetry to manipulate player 3's involvement, exacerbating war.

Having identified the best response of player 2 in each subgame, we are able to derive the PBE of the extension model as follows:

Proposition 9. *Assume the likelihood of the asset is likely to be a high type $q < q^*$. When the cost of war for player 1 is high and the prior belief suggests the asset is likely to be a high value type, there is a PBE that player 1 will offer $\bar{x}$, player 2 plays a pooling strategy of accept regardless of the type of the asset, resulting peace. When the cost of war for player 1 is low and the prior belief suggests the asset is likely to be a high value type, player 1 will offer $\underline{x}$, player 2 plays a pooling strategy of reject regardless of the type of the asset, and player 3 mobilize resulting a multilateral war. Furthermore, in such PBE, when $r'p' < \bar{x}^2$, player 1 will risk war when $q < \frac{-(p'-\bar{x}^2)-\bar{\theta}(r'p'-\underline{x}^2)-c_1}{(\underline{\theta}-\bar{\theta})(r'p'-\bar{x}^2)}$. When $r'p' > \bar{x}^2$, player 1 will risk war when $q > \frac{-(p'-\bar{x}^2)-\bar{\theta}(r'p'-\underline{x}^2)-c_1}{(\underline{\theta}-\bar{\theta})(r'p'-\bar{x}^2)}$.*

Proposition 9 details the equilibrium condition of the extension model. In such PBE, there is a positive probability of multilateral war. Similar to the baseline, such war will take place when player 1 offers a low offer $\underline{x}$ at the first stage to risk war, and player 1 uses the prior belief q to decide which offer to propose.

Initially, player 1's decision to propose a value of $\underline{x}$ and risk war depends on their prior belief regarding the potential of the asset in question. When the anticipated alteration in the future balance of power notably favors their adversary (player 3) during a multilateral war is high ($r'p' > \bar{x}^2$), player 1 is likely to exhibit greater restraint in pursuing warfare and may opt to extend a more generous offer to appease player 2, particularly if the asset demonstrates a high probability of being lucrative in the future (indicated by a low q). On the other hand, when multilateral war is not that favorable for player 3 (low $r'p'$), player 1 may find themselves motivated to assume the risk of war and present a lower offer to compel player 2 to reject it, thus provoking conflict if the asset is deemed likely to yield significant future profits (characterized by a high q). Such a result is similar with the pattern delineated in Proposition 7 of the baseline model. The expected value of the critical asset's prospect is positively correlated with the probability that player 1 is inclined to gamble on the true nature of the asset, provided that losing the war does not result in a considerable shift in power favoring their opponent (as represented by a low $r'p'$). However, should $r'p'$ reach excessively high levels, player 1 would prefer to pursue peace and secure a portion of the asset, given the substantial repercussions associated with a potential defeat in war.

Nonetheless, in contrast to the baseline, the decision to engage in warfare is not associated with the "true type" but is solely linked to the prior belief concerning the type. This phenomenon occurs because player 2 tends to find that withholding information and employing a pooling strategy is more advantageous when conveying the costly signal. In this context, the true type of the asset does not preclude player 2 from invariably rejecting the proposal. Thus, the true type of the asset merely provides a definitive range for player 2 to determine which proposals from Player 1 to decline, while still leaving players 1 and 3 infer the true type of the asset based on the prior belief. Furthermore, the decision to initiate warfare is correlated with the offer made by player 1, and the potential proliferation of warfare is influenced by the intervention or neutral decision rendered by player 3. Consequently, when the prior belief meets the warfare condition outlined in Proposition 9, a state of war will ensue irrespective of the true type of the asset.

This suggests that the communication from player 2 regarding the significance of the asset in the future balance of power, particularly the costly signal that indicates a willingness to resort to military action to protect such an asset, is limited in its ability to influence the conflict's outcome. This is primarily because player 3 knows that for any true value of the asset, should player 1 extend a low offer, player 2 would be inclined to risk engaging in warfare and would misrepresent the asset as being of high value in order to entangle player 3 in a multilateral conflict, given the conditions of $r'p' > rp$ and $q < q^*$. An implication is that the third party does not need to concern itself with how its ally responds to the opponent but should instead utilize the prior knowledge regarding the significance of certain allies' assets in altering the future balance of power. This is because allies generally possess the incentive to decline offers and involve the third party in conflict if their opponents do not present sufficiently high proposals.

Thus, such strategic interaction indicates that the private information held by player 2 regarding the true value of the asset's prospects incites conflict. In the complete information scenario depicted in Lemmas 3 and 4, despite the existence of a commitment problem, in which players are concerned that their opponents may exploit the asset to alter the future balance of power but have no means to credibly commit not to exploit, no conflict occurs. The Nash Equilibrium of this game indicates that it will conclude in the second stage, wherein player 2 invariably accepts the proposed agreement. This suggests that the commitment issue can be managed to some degree through transparency regarding how current hostile or non-hostile outcomes might influence future power dynamics. Nevertheless, it is player 2's private information concerning this uncertainty that incentivizes the rejection of the offer and the subsequent escalation into multilateral conflict.

8.3 Comparative Statistics

As we developed the extension model, it became evident that the conflict dynamics and signaling patterns differ significantly from the baseline. In this section, we seek to address a pivotal question: why does player 3 in the baseline offer a cooperative signal, while player 2 in the extension does not signal cooperatively? I argue that the difference primarily stems from the variation in the sender's goal and the incentive for deviating.

To begin with, it is essential to recognize that the overall payoff for all participants in both models is influenced by two components: the short-term policy outcome, which is represented by the disparity between the ideal foreign policy position and the policy that is ultimately implemented following the conclusion of crisis bargaining, and the long-term evolution of the balance of power, which is related to both the present policy outcome and the prospect of the asset. For player 3 in baseline, their optimal payoff is flexible since their short-term goal can differ from the long-term goal. In the short run, player 3 needs to adopt a policy that is close to their ideal policy, $\hat{x_3}$, which ranges between 0 and 1. However, to achieve an optimal long-term payoff, player 3 must secure as many critical assets as possible (close to 1). Consequently, the disparity between the long-run and short-run goals leads player 3 to exhibit different signaling preferences based on varying ideal policy points in baseline. If player 3's ideal policy point is close to that of player 1 (indicating a high disparity between short-run and long-run goals), they will always prefer to signal reluctance to intervene (pooling on a high-cost type) to discourage player 2 from entering a war, thus balancing their short-run and long-run goals. Conversely, if player 3's ideal policy point is close to player 2 (showing alignment between short-run and long-run goals), they will signal their willingness to intervene (pooling on a low-cost type) to encourage player 2 to engage in war, allowing player 3's short-run and long-run goals to be better fulfilled. Therefore, for some moderate player 3s, as depicted in the $\hat{x_3}$ truth-telling range in Proposition 2, they will adopt an informative separating strategy, having found that neither persistently signaling willingness nor unwillingness to intervene dominates the other strategies.

But for player 2 in extension, their short-term and long-term interests are uniform, as they always desire the policy to be set at 1. Consequently, when player 1 proposes a low offer, rejecting it and signaling that the asset is worth defending (θ is high type) in order to entangle player 3 into a multilateral war is always profitable for player 2 compared with accepting the low offer. This is because they can benefit from player 3's military efforts while increasing their chances of winning the war, which can establish a foreign policy of $x = 1$, optimal for both player 2's short-term and long-term interests. Therefore, player 2 is unlikely to prefer cooperative behavior, instead opting to entangle player 3 in warfare, independent of the true type of the asset. Thus, one explanation for the disparity in signaling behavior across the two models is that the difference in flexibility of optimal payoffs for the signal sender leads to non-flexible senders having a dominant strategy when signaling while flexible senders do not have a dominant strategy (at least for the sender within some ideal policy range). In future research, it is necessary to test what the communication strategy will be when there is a variation between player 2's short-term and long-term goals.

Secondly, another rationale that elucidates the disparity in communication patterns may be approached from a different perspective: why do players opt to diverge from sending cooperative signals? From the analysis of the baseline model, we found that, consistent with Smith's findings (Smith, 2021, 1376), when player 3's genuine cost of engagement in warfare is high and player 3's ideal policy point closely aligns with that of player 2, player 3 will deviate from sending faithful signal, misrepresenting their type as low to persuade player 2 to reject the low offer. Following player 2's rejection of the low offer, player 3 will refrain from intervening, thereby allowing player 2 to contend independently in a bilateral war in order to freeride on player 2's military efforts without incurring the costs associated with warfare. Consequently, player 3's motivation to diverge and misrepresent their type predominantly stems from the fact that this strategy enables player 3 to secure equivalent benefits if player 2 wins the bilateral war compared with the circumstance that player 3 intervenes and wins the multilateral war, while avoiding the costs of war. It is noteworthy to mention that player 3's deviation is not because it could increase the likelihood of winning the war, since winning a multilateral war is more likely than winning a bilateral war ($p' > p$), implying player 3's deviation is actually gambling on player 2's probability to win the bilateral war.

However, for player 2 in the second model, their incentive to deviate is solely based on the fact that the expected payoff from multilateral war is higher than from bilateral war (See the proof of Proposition 8. When $\theta = \underline{\theta}$, the expected utility of sending a cooperative signal and accepting the low offer is congruent to the expected utility of rejecting the offer and entering bilateral war, which is lower than the expected utility of deviating and entering multilateral war). Regarding the long-term balance of power, given the presumption $r'p' > rp$, multilateral war is more desirable since the expected value of the prospect of the asset after multilateral war is higher. Even regarding the short-term goal, multilateral war is preferable since the probability of winning the multilateral war is higher than the probability of winning the bilateral war ($p' > p$). Therefore, it can be concluded that player 2's deviation serves to directly increase the overall payoff. However, for player 3 in the baseline, deviation from cooperation does not ensure an increase in the overall payoff for player 3 since, although freeriding helps circumvent the cost of war, it also increases the risk of losing the war and receiving nothing, which is undesirable for the long-term balance of power for player 3. As a result, player 3 in the baseline model will be more restrained from deviation, as it is not always beneficial to do so, while player 2 in the extension is incentivized to deviate from cooperation, since it would surely increase the overall utility by entangling player 3 into multilateral war.

8.4 Extension Model Conclusion

The extension model is designed to analyze the communication and conflict patterns that arise when player 2 possesses private information regarding the extent to which asset ownership may alter the future balance of power. The results indicate that there exists no cooperative PBE where player 2 consistently has the incentive to signal that the asset is worth defending by rejecting offers and engaging in warfare; this is especially pertinent when the prior belief suggests that the asset is likely to be crucial in the long term and when player 1 risks war by proposing a low initial offer.

This model reveals a divergence in communication patterns when compared to the baseline, attributable to the heightened incentive for the sender (player 2) to deviate, coupled with the inflexibility of the sender's ideal policy position. This pattern suggests that the possession of private information can lead to the proliferation of warfare. Within an alliance, states may utilize the possession of private information to misrepresent the true value of the asset bargaining in an effort to entangle allies into conflict, thereby increasing the probability of victory, even when they know the asset is not lucrative.

Furthermore, similar to the baseline, the results from the extension also predict that private information would incentivize player 1 to risk engaging in war. This observation aligns with Fearon's rational explanation of war, wherein the possession of private information concerning a critical asset increases the likelihood of conflict, particularly when contrasted with the complete information version of the same model. In contrast to the baseline, it is noteworthy that, in equilibrium, the probability of war is not correlated with the actual type of the asset; rather, it is influenced by the commonly known prior belief regarding the asset. This dynamic arises because the optimal signaling strategy for player 2 is to adopt a pooling strategy that reject any low offer irrespective of the asset's true type. Consequently, when player 2 possesses private information about the critical asset, the signal from player 2 becomes trivial for their ally in determining whether to intervene; thus, they must rely on their own judgment regarding the benefits of intervention. The findings indicate that as the asset appears increasingly likely to prompt intervention, player 3 becomes incentivized to intervene, player 2 is motivated to withhold information and risk the potential for war. Additionally, player 1 demonstrates a greater willingness to engage in war under circumstances where the anticipated alteration of the balance of power in favor of the opponent is perceived to be low (denoted as $r'p'$ being low). Consequently, a positive correlation exists between the likelihood of the asset being of a high type and the probability of war, under specified parameter values.

9 Implication

This research has presented the conflict and communication patterns in two different models: imperfect information and private information regarding the asset's prospects. In this section, we will illustrate how the results of this research predict the case of Taiwanese possession of AI technology to demonstrate the external validity of this research.

Firstly, when the U.S., as a third party, possesses private information regarding its willingness to intervene, it will faithfully disclose such information to Taiwan, where cooperation occurs due to the potential of the critical asset to alter the future balance of power. The baseline model, specifically Proposition 5 and 6, suggests that the presence of the asset generally increases the likelihood of a third party with a policy point similar to that of its ally to signal truthfully, as the incentive to freeride on the ally's war efforts is reduced. In the context of the U.S.-Taiwan relationship, although the U.S. employs strategic ambiguity regarding Taiwan, indicating that its ideal policy point does not align perfectly with Taiwan's, the American ideal policy point remains closer to Taiwan's than to China's. Consequently, with the integration of AI technology in the analysis, the U.S. would be hesitant to feign its willingness to intervene as high when its actual willingness is low, in order to avoid entangling Taiwan in a bilateral conflict and freeriding on Taiwan's war outcomes. This is because the risk of freeriding increases when accounting for the influence of AI as a long-term factor on the balance of power. Should the U.S. choose to freeride, it might avoid the costs of war; however, the likelihood of Taiwan winning a bilateral war is significantly lower than that of a U.S.-Taiwan coalition succeeding in a multilateral conflict. Therefore, the U.S. would genuinely communicate its willingness to intervene privately to Taiwan, while we would observe that when the U.S. signals a high willingness to intervene, if China subsequently incites a conflict, the U.S. would intervene, resulting in a multilateral war.

Such a pattern was observed between the United States and Saudi Arabia prior to the outbreak of the Gulf War. In this context, the critical asset was the oil fields owned by Saudi Arabia. The ascending power, identified as Iraq (player 1), posed a threat to Saudi Arabia (player 2) as well as to the United States (player 3), which was a third party (UVA, 2011, 22; UVA, 2000, 134; Jhaveri, 2004, 3-5). Following Iraq's successful control over Kuwait, officials within the U.S. government expressed concerns that Iraq might invade or subsequently establish dominance over Saudi Arabia. Evidence suggesting such an operation was already apparent, as satellites at that time indicated that two Iraqi divisions were positioned near the Saudi border (*Frontline* 1996, at 25:43-25:48). Moreover, the CIA reported that Saudi officials were contemplating the possibility of purchasing Saddam off where Saddam's capture of Saudi Arabia would enable him to control approximately 40 percent of the world's oil supply (*Frontline* 1996, at 25:48-26:07). During a briefing at Camp David regarding the defense strategy for Saudi Arabia, General Norman Schwarzkopf, the U.S. Commander, emphasized the urgency of deploying troops to Saudi Arabia, stating that "it was necessary to guarantee a defense of Saudi Arabia and the rest of the Gulf oil fields" (*Frontline* 1996, at 23:08-23:42). Also, Secretary of Defense Richard Cheney described that Iraq's control of the oil fields poses a strategic threat to the U.S. because it can generate enormous wealth and be used for harmful purposes, ultimately allowing it to dominate the region (UVA, 2000, 134). Consequently, on August 6, 1990, Richard Cheney, alongside a high-level delegation, was dispatched to Saudi Arabia to persuade its leadership to permit the deployment of U.S. troops on Saudi soil. Remarkably, the leadership of Saudi Arabia consented to this proposal during the meeting (*Frontline* 1996, at 26:07-27:28). In this context, the American proposal to deploy troops can be interpreted as a signal of the U.S. willingness to intervene in the event of a conflict, indicating that the nation is prepared to intervene due to the significance of the region's oil fields to U.S. interests and the long-term balance of power in the area. Notably, this signal was indeed informative, as during the Gulf War, the United States and Saudi Arabia cooperated in their military efforts against Iraq, thereby further corroborating the theoretical implications derived from the baseline model that the third party would be more willing to faithfully reveal their incentive to intervene when bargaining over the asset that would change the future balance of power.

Nevertheless, while AI – the critical asset capable of altering the future balance of power – may foster cooperation between the United States and Taiwan, it concurrently heightens the likelihood of a multilateral conflict. This is due to the increasing readiness of China, in this context, to engage in warfare in order to secure additional assets that would enhance its long-term position within the balance of power. A similar pattern is observable in the Gulf War, where one of the primary rationales for Saddam Hussein's decision to invade and subsequently refuse to withdraw from Kuwait was the economic downturn facing Iraq (Meierding, 2020, 144-145). Consequently, it was advantageous for Iraq to establish control over Kuwait, as doing so would provide greater leverage in influencing global oil prices while extracting revenue from Kuwait's oil fields. Therefore, the long-term significance of this asset incentivized Saddam to maintain control over Kuwait, equivalent to engaging in the risks of warfare and proposing minimal offers in our model, which ultimately resulted in the occurrence of a multilateral war.

Nonetheless, as articulated in Proposition 7, when the anticipated value of the asset is significantly elevated, the rising power is likely to refrain from engaging in war due to the associated high risks. This power has the incentive to secure at least a portion of the asset to mitigate potential risk in the future balance of power in the event of a military defeat. In the context of the China-US-Taiwan relationship, this implies that if the AI technology is deemed exceedingly critical in the foreseeable future—similar to the influence of nuclear weapons that could drastically alter the balance of power—China is unlikely to instigate conflict initially, given that the stakes are excessively high. They would prefer to acquire some share of the asset rather than none, thereby fostering an optimistic outlook for the future of this multilateral relationship.

However, the extension model suggests that the states possessing private information regarding the potential value of the asset—in this context, Taiwan—will consistently reject any low offers from the rising power where this rejection signals that the asset is deemed worthy of defense. Furthermore, it indicates that a third party may become entangled in a multilateral conflict should it perceive the asset's potential value as significant. Consequently, given that Taiwan will invariably withhold vital information and reject offers, it implies that the United States will determine whether to intervene not based on Taiwan's signals concerning its willingness to defend the industry, but rather on its own assessments of the future significance of AI technology. Therefore, any signals sent privately to the United States would be futile, as the United States recognizes that unless China sufficiently pacifies Taiwan, Taiwan will continuously reject any offer and ultimately enter into multilateral warfare, thus heightening the risk of a proliferation of war. Such a pattern is observed in the former 1954-55 Taiwan Strait Crisis.

Beginning in September 1954, the People's Republic of China (PRC) and the Chinese Nationalist government in Taiwan became embroiled in a conflict concerning the offshore islands controlled by the Nationalists. In December of 1954, the United States entered into a formal mutual defense treaty with the Nationalist government; however, this treaty does not extend to the disputed offshore islands (Beckley, 2015, 27-28). In this scenario, the rising power PRC (player 1) constitutes a threat to the Nationalist government (player 2), while the United States (player 3), as a third party, deliberates whether to intervene or maintain neutrality. The critical asset in this case is the strategic importance of the off-shore island as well as the troops on these island where Beckley notes that ”the Eisenhower administration believed that these troops were vital to Taiwan's security, even if the islands they were based on were not, and therefore attempted to compel the Nationalists to 'redeploy and consolidate' these forces on Taiwan" (Beckley, 2015, 28). To ensure that the United States would provide a robust security guarantee, the Nationalist government consistently rejected American pledges, even declining an American offer to blockade the Chinese coast opposite Taiwan in exchange (Beckley, 2015, 28). The costly signaling in this circumstance manifests through Taiwan's insistence on deploying troops to the offshore islands, thereby enhancing these islands' strategic importance to incentivize U.S. military intervention. This behavior aligns with the findings from the extension, which assert that a state possessing private information regarding the asset's prospects is motivated to signal its worthiness for defense to compel intervention from a third party for optimal outcomes. Ultimately, however, war was averted for two primary reasons. Firstly, the PRC refrained from further escalation and proposed peace talks, partially out of concern for potential American retaliation and intervention (Beckley, 2015, 28-29). This pattern conforms to Proposition 9, wherein a heightened military capacity of the third party (high $r'p'$) can deter the ascendant power from pursuing war. Secondly, U.S. officials recognized the significant international audience cost associated with intervention, as European allies expressed horror at the prospect of the U.S. risking conflict with China over minor islands, and British leaders even threatened to withdraw support for UN neutralization efforts in the Taiwan Strait (Beckley, 2015, 29). Therefore, it is evident that the United States relied not only on the signals from Taiwan but also on a cost-benefit analysis informed by pre-existing beliefs regarding the prospects of this critical asset which leaders ultimately concluded that the prospect is not that imperative where the cost is tremendous, consistent with Proposition 8's findings—that the third party does not solely depend on signals but also on prior beliefs when deciding whether to intervene, given Player 2's strong incentives to emphasize the asset's importance. Thus, paralleling Beckley's findings (Beckley, 2015, 47), this research also suggests that the risks associated with entanglement are somewhat manageable, as the decision to intervene is ultimately influenced not by allies' signals but by leaders' perceptions of their nation's core interests.

Nonetheless, we recognize that the existing tensions surrounding Taiwan are considerably more intricate than the dynamics presented in the research, as the situation extends beyond the semiconductor industry and its potential advancements in artificial intelligence technology. For instance, the Chinese government may regard Taiwan as indivisible since they have strategically deploy propaganda of indivisibility among the Chinese population, which their hands are tied by such a maximalist demand on the bargaining table (Braniff, 2018, 105-106). In scenarios where stakeholders confront an indivisibility dilemma, we would hypothesize that the positive correlation between the prospect of the asset and the likelihood of war would be more salient compared with issue divisible models in this research, since player 1 (China) possesses a heightened incentive to assert complete control over the asset. Meanwhile, player 2 (Taiwan) is more likely to reject the offer while player 3 (U.S.) will be increasingly motivated to intervene. This phenomenon arises from the fact that acceptance of the proposal would yield zero long-term benefits for players 2 and 3 while risking the war and gambling on the likelihood of winning can provide them with some probability of securing the entirety of the asset. Subsequent research could explore this hypothesis by integrating the issue of indivisibility into this model.

10 Conclusion

This research investigates how uncertainty regarding a critical asset, which could alter the future balance of power, affects conflict and communication patterns in multilateral crisis bargaining, extending the framework proposed by Smith (Smith, 2021). We identify two types of uncertainty concerning critical assets: imperfect information, where no player knows the asset's true nature, and incomplete information, where the asset holder has private knowledge about its type. Our findings indicate that under imperfect information conditions, there is a positive correlation between the likelihood of multilateral war and the asset's expected value, as rising powers become increasingly incentivized to risk conflict to maximize potential gains. However, when the asset's expected value becomes exceedingly high, rising powers consistently avoid risking war due to prohibitively high conflict costs, preferring instead to secure a guaranteed share of the asset. Regarding communication patterns, consistent with Smith's findings (Smith, 2021), an informative Perfect Bayesian Equilibrium (PBE) emerges only in scenarios involving private communication, as public messaging incentivizes the third party to deviate and manipulate other players. Nevertheless, in contrast to Smith's conclusions, our analysis reveals that when future shifts in the balance of power are taken into consideration, the third party is motivated to truthfully disclose their type even if incentivized to freeride on an ally's wartime efforts, as the long-term risks associated with freeriding become more pronounced under these conditions.

However, when the information regarding the prospect of the asset is private to the player who owns the asset, they have no incentive to send a costly cooperative signal to the third party. Instead, they will always withhold the information and reject any low offer from the rising power if the prior belief indicates that the asset is likely to be substantial in the future. This strategy aims to entangle the third party into a multilateral war to maximize their payoff. Given such an incentive, war is more likely to occur and proliferate when the prior belief suggests that the asset seems substantial. We argue that the difference in the communication pattern in this model compared to the baseline stems from the increased incentive for the sender (player 2) to deviate, along with the inflexibility of the sender's ideal policy position.

This research contributes to the extensive international relations literature on military alliances and the causes of war. Our results align with Fearon's explanation of conflict causation, wherein private information and commitment issues provoke and proliferate the war. Furthermore, our findings suggest that within an alliance, states may exploit private information to misrepresent the true value of assets being negotiated, aiming to draw allies into conflict, thereby increasing the probability of victory.

However, this research does have limitations. First, we assume that the short-term and long-term goals of the state possessing the asset are uniform, which potentially creates a strong incentive for this player to entangle the third party into the conflict. Future research could challenge this assumption and examine whether this pattern persists.

Additionally, this research only investigates the scenario in which the player possessing private information about the type of asset can solely signal privately to the third party. However, it would be intriguing to explore the conflict and communication dynamics when this player publicly informs both the rising power and the third party, given that this player faces a dilemma where signaling a high type may encourage the third party to intervene, while simultaneously incentivizing the rising power to risk war in the first place. It would be worthwhile to examine how the player possessing private information leverages and manipulates other players to achieve an optimal payoff.

Moreover, this research is predicated on the presumption that the critical asset has the potential to alter the universal balance of power. However, we acknowledge that certain assets, such as fortresses, may solely enhance one's defensive capabilities without augmenting offensive capabilities. Therefore, the commitment problem may be undermined in such instances, as possessing additional critical assets at the current stage would not be perceived as more threatening. Future research could thus relax this assumption and explore the communication and conflict patterns in which the critical asset increases the likelihood of success in defensive engagements while leaving offensive capabilities intact.

Furthermore, it is plausible that in scenarios where an asset possesses significant potential, the adversaries, perceiving a diminished likelihood of success in the conflict, may intentionally compromise a portion of the asset during the warfare as a preemptive measure to avert a detrimental alteration in the future balance of power. Consequently, it is reasonable to infer that such conduct will motivate other stakeholders to pursue a peaceful resolution in the bargaining stage, even when their likelihood of winning the war is substantially elevated. Therefore, subsequent research may incorporate an additional strategic option whereby players are enabled to employ the scorched earth tactic, aimed at undermining a portion of the critical asset in instances where the initial bargaining efforts fail.

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13 Appendix

13.1 Proofs for the Baseline

In the baseline model, recall that the utility for each player is the following:

	Multilateral War
Player 1	$-p' - c_1 - r'p'(q\underline{\theta} + (1-q)\overline{\theta})$
Player 2	$-(1-p') - c_2 - r'(1-p')(q\underline{\theta} + (1-q)\overline{\theta})$
Player 3	$-p'(\hat{x}_3 - \hat{x}_2)^2 - (1-p')(\hat{x}_3 - \hat{x}_1)^2 - c_3 + r'p'(q\underline{\theta} + (1-q)\overline{\theta})$

	Bilateral War
Player 1	$-p-c_1-rp(q\underline{\theta}+(1-q)\overline{\theta})$
Player 2	$-(1-p)-c_2-r(1-p)(q\underline{\theta}+(1-q)\overline{\theta})$
Player 3	$-p(\hat{x_3}-\hat{x_2})^2-(1-p)(\hat{x_3}-\hat{x_1})^2+rp(q\underline{\theta}+(1-q)\overline{\theta})$

	Peace
Player 1	$-(\hat{x_1}-x)^2-(\hat{x_1}-x)^2(q\underline{\theta}+(1-q)\overline{\theta})$
Player 2	$-(\hat{x_2}-x)^2-(\hat{x_2}-x)^2(q\underline{\theta}+(1-q)\overline{\theta})$
Player 3	$-(\hat{x_3}-x)^2+(\hat{x_1}-x)^2(q\underline{\theta}+(1-q)\overline{\theta})$

To simplify the notation in the calculation, we write $(q\underline{\theta}+(1-q)\overline{\theta})=A$

Lemma 1. Player 3 will choose to mobilize when

$$c_3 \leq (p'-p)[(\hat{x_3}-\hat{x_1})^2-(\hat{x_3}-\hat{x_2})^2]+(r'p'-rp)A \equiv c^*$$

or otherwise, player 3 will remain neutral.

Proof:

Player 3's utility when choosing to mobilize is greater than remaining neutral when

$$EU_3(Multilateral) \geq EU_3(Bilateral)$$

$$-p(\hat{x_3}-\hat{x_2})^2-(1-p)(\hat{x_3}-\hat{x_1})^2+r'p'A \geq p(\hat{x_3}-\hat{x_2})^2-(1-p)(\hat{x_3}-\hat{x_1})^2+rpA$$

Simplifying the inequality, we get

$$c_3 \leq (p'-p)[(\hat{x_3}-\hat{x_1})^2-(\hat{x_3}-\hat{x_2})^2]+(r'p'-rp)A$$

QED

Lemma 2. Under complete information, player 1 would offer

$$\begin{cases} \overline{x}=\hat{x_2}-\sqrt{\frac{(1-p')(\hat{x_2}-\hat{x_1})^2(1+r'A)+c_2}{1+A}} & \text{if } c_3 \leq c^* \\ \underline{x}=\hat{x_2}-\sqrt{\frac{(1-p)(\hat{x_2}-\hat{x_1})^2(1+rA)+c_2}{1+A}} & \text{if } c_3 > c^* \end{cases}$$

Proof:

From Lemma 1, we assessed the threshold for player 3 choosing to mobilize. When the cost of war for player 3 is sufficiently low, then player 2 would reject crisis bargaining and player 3 would mobilize, with the game resulting in a multilateral war. In such case, player 2 would receive

$$EU_2(Multilateral) = -(1-p') - c_2 - r'(1-p')A$$

Similarly, when the cost of war for player 3 is high, then player 3 would remain neutral, such that if player 2 rejects crisis bargaining then the game will result in a bilateral war. In such a case, player 2 would receive

$$EU_2(Bilateral) = \underbrace{-(1-p)}_{<-(1-p')} - c_2 \underbrace{-r(1-p)A}_{<r'(1-p')A}$$

Hence, the expected utility for player 2 in multilateral war is higher than in bilateral war.

$$EU_2(Multilateral) > EU_2(Bilateral)$$

To maximize player 1's own payoff, they will offer some x to make player 2 indifferent, in which assumption, player 2 will choose to accept. Therefore, in order to lead player 2 to accept, player 1 needs to offer some $\overline{x}$ when c_3 is low and offer some $\underline{x}$ when c_3 is high where $\overline{x} > \underline{x}$

When $c_3 > (p'-p)[(\hat{x_3}-\hat{x_1})^2 - (\hat{x_3}-\hat{x_2})^2] + (r'p'-rp)A$, player 1 would offer $\underline{x}$ that satisfies

$$EU_2(Accept) = EU_2(Reject)$$
$$-(\hat{x_2}-\underline{x})^2 - (\hat{x_2}-\underline{x})^2 A = -(1-p) - c_2 - r(1-p)A$$
$$\underline{x} = \hat{x_2} - \sqrt{\frac{(1-p)(\hat{x_2}-\hat{x_1})^2(1+rA)+c_2}{1+A}}$$

Similarly, we get

$$\overline{x} = \hat{x_2} - \sqrt{\frac{(1-p')(\hat{x_2}-\hat{x_1})^2(1+r'A)+c_2}{1+A}} \text{ when } c_3 \leq (p'-p)[(\hat{x_3}-\hat{x_1})^2 - (\hat{x_3}-\hat{x_2})^2] + (r'p'-rp)A$$

QED

Proposition 1. When player 3 sends a cheap talk message publicly to both player 1 and player 2, there will be no fully informative equilibrium. This result holds regardless of whether players have assessed the future prospect of the asset.

Proof:

When the message is fully informative, the cheap talk strategy of player 3 is the following separating strategy

$$m_3(\underline{c}) = \underline{c},\, m_3(\overline{c}) = \overline{c}, \text{ where}$$
$$P[m_3 = \underline{c} \mid \underline{c}] = 1,\, P[m_3 = \overline{c} \mid \overline{c}] = 1$$

Since the communication is public, both player 1 and 2 update their beliefs as a result of the cheap talk message. The posterior beliefs are the following

$$\mu^*(m_3 = \underline{c}) = P[\underline{c} \mid m_3 = \underline{c}] = \frac{\phi \cdot 1}{\phi \cdot 1 + (1-\phi)0} = 1$$
$$\mu^*(m_3 = \overline{c}) = P[\underline{c} \mid m_3 = \overline{c}] = \frac{\phi \cdot 0}{\phi \cdot 0 + (1-\phi)1} = 0$$

As player 1 and 2 updated the beliefs based on the signal, the game will become a complete information game where player 1 will propose $\underline{x}$ when $m_3 = \overline{c}$ or $\overline{x}$ when $m_3 = \underline{c}$ respectively, and player 2 will choose to accept. Hence the equilibrium is sequentially rational. Then, we need to check the incentive compatibility for player 3 in each scenario with different types of cost.

When $c_3 = \overline{c}$,

if player 3 signals $m_3 = \overline{c}$, player 1 will offer $\underline{x}$

$$EU_3(m = \overline{c}) = -(\hat{x_3} - \underline{x})^2 + (\hat{x_1} - \underline{x})^2 A$$

if player 3 signals $m_3 = \underline{c}$, player 1 will offer $\overline{x}$

$$EU_3(m = \underline{c}) = -(\hat{x_3} - \overline{x})^2 + (\hat{x_1} - \overline{x})^2 A$$

In order to satisfy incentive compatibility,

$$EU_3(m = \overline{c}) \geq EU_3(m = \underline{c})$$
$$-(\hat{x_3} - \underline{x})^2 + (\hat{x_1} - \underline{x})^2 A \geq -(\hat{x_3} - \overline{x})^2 + (\hat{x_1} - \overline{x})^2 A$$

When $c_3 = \underline{c}$,

if player 3 signals $m_3 = \underline{c}$, player 1 will offer $\overline{x}$

$$EU_3(m = \underline{c}) = -(\hat{x_3} - \overline{x})^2 + (\hat{x_1} - \overline{x})^2 A$$

if player 3 signals $m_3 = \overline{c}$, player 1 will offer $\underline{x}$

$$EU_3(m = \overline{c}) = -(\hat{x_3} - \underline{x})^2 + (\hat{x_1} - \underline{x})^2 A$$

In order to satisfy incentive compatibility,

$$EU_3(m = \underline{c}) \geq EU_3(m = \overline{c})$$
$$-(\hat{x_3} - \overline{x})^2 + (\hat{x_1} - \overline{x})^2 A \geq -(\hat{x_3} - \underline{x})^2 + (\hat{x_1} - \underline{x})^2 A$$

Hence, we observe that in order to satisfy the incentive compatibility, $\hat{x_3}$ needs to satisfy

$$-(\hat{x_3} - \overline{x})^2 + (\hat{x_1} - \overline{x})^2 A = -(\hat{x_3} - \underline{x})^2 + (\hat{x_1} - \underline{x})^2 A$$

However, we know that $\underline{x} < \overline{x}$, so other than in a knife-edge case of $\hat{x_3}$, $LHS \neq RHS$. Therefore, there will be no separating PBE where player 3 communicates faithfully. This is consistent with Smith's result (Smith, 2021, 1375).

QED

Proposition 2. When player 3 sends the cheap talk message privately to player 2 only, there will be a fully informative equilibrium *iff*

$$\hat{x_3} \in \left[\frac{p'(1 - r'A) - \underline{x}^2(1 - A) + \underline{c}}{2(p' - \underline{x})}, \frac{p(1 - rA) - \underline{x}^2(1 - A)}{2(p - \underline{x})} \right],$$

and in such equilibrium, there will be a multilateral war when

$$c_3 = \underline{c},\ x = \underline{x},\ \text{and}\ \phi \leq \frac{(1+A)[(\hat{x_1} - \underline{x})^2 - (\hat{x_1} - \overline{x})^2]}{p' + c_1 + r'p'A - (\hat{x_1} - \underline{x})^2(1+A)}$$

Proof:

Similar to Proposition 1, the cheap talk strategy of player 3 when the message is fully informative is the following separating strategy

$$m_3(\underline{c}) = \underline{c},\ m_3(\overline{c}) = \overline{c},\ \text{where}$$
$$P[m_3 = \underline{c} \mid \underline{c}] = 1,\ P[m_3 = \overline{c} \mid \overline{c}] = 1$$

However, note that in this circumstance only player 2 is able to update their belief based on the cheap talk information, while player 1 still maintains their prior belief $p[c_3 = \underline{c}] = \phi$ since the communication is private. Therefore, the posterior belief of player 2 is the same as in Proposition 1.

$$\mu_2^*(m_3 = \underline{c}) = P[\underline{c} \mid m_3 = \underline{c}] = \frac{\phi \cdot 1}{\phi \cdot 1 + (1 - \phi)0} = 1$$
$$\mu_2^*(m_3 = \overline{c}) = P[\underline{c} \mid m_3 = \overline{c}] = \frac{\phi \cdot 0}{\phi \cdot 0 + (1 - \phi)1} = 0$$

To validate such a strategy imposed by player 3, first we will examine the sequential rational strategy for the other two players. We aim to deduce the value that player 1 will propose and player 2's response by cases. There are three cases in total we examine: $x < \underline{x}$, $x \in [\underline{x}, \bar{x})$, $x \geq \bar{x}$.

First, consider the case where $x < \underline{x}$. In such case, player 2 will always reject the offer regardless of the cheap talk message received. Hence the utility player 1 will receive by proposing $x < \underline{x}$ is $EU_1(Bilateral)$ with probability $1-\phi$ or $EU_1(Multilateral)$ with probability ϕ. Hence, the utility of proposing $x < \underline{x}$ is

$$EU_1(x < \underline{x}) = \phi EU_1(Multilateral) + (1-\phi)EU_1(Bilateral)$$

On the other hand, if player 1 offers $x = \underline{x}$, then player 2 will reject with probability ϕ and accept with probability $1-\phi$. The utility of proposing $\underline{x}$ is

$$EU_1(x = \underline{x}) = \phi EU_1(Multilateral) + (1-\phi)EU_1(Peace)$$

$$EU_1(Peace) = -(\hat{x_3} - \underline{x})^2 + (\hat{x_1} - \underline{x})^2 A$$

$$= -(1+A) + 2\sqrt{(1+rA)(1-p)+c_2} \cdot \sqrt{1+A} - (1-p)(1+rA) - c_2$$

Hence, to show $EU_1(x < \underline{x}) < EU_1(x = \underline{x})$, is equivalent to showing $EU_1(Bilateral) < EU_1(Peace)$. Simplifying, we aim to show that

$$-(1+A) + 2\sqrt{(1+rA)(1-p)+c_2} \cdot \sqrt{1+A} - (1-p)(1+rA) - c_2 + c_1 + p + rpA > 0$$

Suppose $\alpha = \sqrt{(1+rA)(1-p)+c_2}$, $\beta = \sqrt{1+A}$,

$$-(\alpha - \beta)^2 = -(1+A) + 2\alpha\beta - (1-p)(1+rA) - c_2$$

Hence,

$$LHS = -\underbrace{(\alpha-\beta)^2}_{\geq 0} + \underbrace{c_1 + p + rpA}_{>0}$$

The inequalities that $LHS > 0$ would hold true as long as the squared difference $-(\alpha-\beta)^2$ cannot outweigh the constant on the right. As the value of r and p are relatively confined, the squared term cannot be excessively large except for some extreme cases about c_2 and c_3, because for $(1+rA)(1-p)$ in $\alpha = \sqrt{(1+rA)(1-p)+c_2}$, its value will be lower than $1+A$ in $\beta = \sqrt{1+A}$ based on $0 < p < 1, 0 < r < 1$. This implies that if the squared term is without the parameter c_2 (in another word, $c_2 = 0$), then the lowest value for $-(\alpha-\beta)^2$ is $-1-A$ where the inequality will be satisfied if $c_1 > (1-p) + A(1-rp)$. With the parameter c_2 included, $-(\alpha-\beta)^2$ will be higher than $-1-A$ where the acceptance range for c_1 will increase, leaving more possible c_2 value. Unless c_2 is excessively large where $\sqrt{(1+rA)(1-p)+c_2} > 2\sqrt{1+A}$, $c_2 > \frac{4(1+A)}{(1+rA)(1-p)}$ where such an extreme condition implies that player 2's cost of war would be roughly 4 times the future prospect of the technology plus another 1, the expected utility from peace is higher than the expected utility from bilateral war for player 1, implying that for player 1 $x = \underline{x}$ strictly dominates $x < \underline{x}$.

Second, consider the case where player 1 offers $x \in [\underline{x}, \overline{x})$. In such range, player 2 will reject with probability ϕ, and accept with probability $1-\phi$. The expected utility can be written as

$$EU_1(x \in [\underline{x}, \overline{x})) = \phi EU_1(Multilateral) + (1-\phi)EU_1(Peace)$$

However, the $EU_1(Peace)$ would be maximized if player 1 chooses $x = \underline{x}$ in this interval while the utility from $EU_1(Multilateral)$ remains the same. Therefore, for player 1 $x = \underline{x}$ strictly dominates $x \in (\underline{x}, \overline{x})$.

Finally, consider the case where player 1 offers $x \geq \overline{x}$, player 2 will accept with probability 1 in this circumstance, where the expected utility for player 1 is

$$EU_1(x \geq \overline{x}) = EU_1(Peace)$$

But for player 1, their utility from offering $x = \overline{x}$ is strictly larger than offering $x \geq \overline{x}$. Hence $x = \overline{x}$ strictly dominates $x \geq \overline{x}$. Hence we derive that player 1 would offer two sequential rational strategies in the equilibrium: $\underline{x}, \overline{x}$.

However, player 1 does not have a strict preference over these two strategies since although the expected utility from maintaining peace while offering $\underline{x}$ is larger than $\overline{x}$, player 1 risks war when offering $\underline{x}$ where their expected utility will be lower than offering $\overline{x}$ with probability ϕ. Player 1 will choose $\underline{x}$ and risk war when

$$EU_1(\underline{x}) \geq EU_1(\overline{x})$$
$$-(\hat{x_1}-\underline{x})^2(1+A)(1-\phi) + \phi(-p'-c_1-r'p'A) \geq -(\hat{x_1}-\overline{x})^2(1+A)$$
$$\phi \leq \frac{(1+A)[(\hat{x_1}-\overline{x})^2-(\hat{x_1}-\underline{x})^2]}{p'+c_1+r'p'A-(\hat{x_1}-\underline{x})^2(1+A)}$$

Hence, when $\phi \leq \frac{(1+A)[(\hat{x_1}-\overline{x})^2-(\hat{x_1}-\underline{x})^2]}{p'+c_1+r'p'A-(\hat{x_1}-\underline{x})^2(1+A)}$, then player 1 will choose to propose $\underline{x}$. Meanwhile, if the type of player 3 is $\underline{c}$ and player 3 chooses to privately communicate such a private message truthfully to player 2, player 2 will reject player 1's offer and a multilateral war will take place. Otherwise, when

$\phi > \frac{(1+A)[(\hat{x_1}-\underline{x})^2-(\hat{x_1}-\overline{x})^2]}{p'+c_1+r'p'A-(\hat{x_1}-\underline{x})^2(1+A)}$, player 1 will propose $\overline{x}$ which player 2 will certainly accept, and the three states will remain at peace.

Since we have derived the sequential rational strategies for each player, now we need to check the incentive compatibility for player 3.

When $c_3 = \underline{c}$,

If player 3 notifies their ally player 2 truthfully through a cheap talk message $m = \underline{c}$, then player 2 after updating their belief will reject any offer lower than $\overline{x}$. If player 2 rejects, player 3 will choose to mobilize, resulting in multilateral war. If player 3 chooses to deviate from the separating strategy and send the cheap talk message $m = \overline{c}$, then player 2 will reject any offer lower than $\underline{x}$. If player 2 rejects, player 3 will choose to mobilize, resulting in multilateral war. Note that to be consistent with Smith's work (Smith, 2021), we are only focusing on $\phi \leq \phi^*$ where player 1 will always choose $\underline{x}$. Player 3 will choose to stick to the separating strategy when

$$EU_3(m = \underline{c}) \geq EU_3(m = \overline{c})$$
$$-p'(\hat{x_3} - \hat{x_2})^2 - (1-p')(\hat{x_3} - \hat{x_1})^2 - \underline{c} + r'p'A \geq -(\hat{x_3} - \underline{x})^2 + (\hat{x_1} - \underline{x})^2 A$$
$$\hat{x_3} \geq \frac{p'(1-r'A)-\underline{x}^2(1-A)+\underline{c}}{2(p'-\underline{x})}$$

When $c_3 = \overline{c}$,

Following a similar logic to the above, if player 3 faithfully reveals its type through $m = \overline{x}$, then player 2 will reject any offer lower than $\underline{x}$. If player 2 rejects, player 3 will choose to remain neutral, resulting in bilateral war. If player 3 chooses to deviate from the separating strategy and sends the cheap talk message $m = \overline{c}$, then player 2 will reject any offer lower than $\overline{x}$. If player 2 rejects, player 3 will choose to remain neutral, resulting in bilateral war. Player 3 will choose to stick to the separating strategy when

$$EU_3(m = \overline{c}) \geq EU_3(m = \underline{c})$$
$$-(\hat{x_3} - \underline{x})^2 + (\hat{x_1} - \underline{x})^2 A \geq -p(\hat{x_3} - \hat{x_2})^2 - (1-p)(\hat{x_3} - \hat{x_1})^2 + rpA$$
$$\hat{x_3} \leq \frac{p(1-rA)-\underline{x}^2(1-A)}{2(p-\underline{x})}$$

Combining the two inequalities, we have proved when player 3 sends the cheap talk message privately to player 2 only, there will be a fully informative equilibrium *iff*

$$\hat{x_3} \in \left[\frac{p'(1-r'A)-\underline{x}^2(1-A)+\underline{c}}{2(p'-\underline{x})}, \frac{p(1-rA)-\underline{x}^2(1-A)}{2(p-\underline{x})}\right]$$

QED

Proposition 3. When players account for the prospect of the critical asset, compared to Smith's work (Smith, 2021), the offering of $\underline{x}$ and $\overline{x}$ will be higher. Additionally, there is a positive correlation between the expected value of the asset's prospect and the values of $\underline{x}$ and $\overline{x}$.

Proof:

When players are not concerned about the prospect of the critical asset, from Smith's work,

$$\underline{x} = \hat{x_2} - \sqrt{(1-p)(\hat{x_2}-\hat{x_1})^2 + c_2}$$
$$\overline{x} = \hat{x_2} - \sqrt{(1-p')(\hat{x_2}-\hat{x_1})^2 + c_2}$$

For $\underline{x}$, comparing with

$$\underline{x} = \hat{x_2} - \sqrt{\frac{(1-p)(\hat{x_2}-\hat{x_1})^2(1+rA)+c_2}{1+A}},$$

For the fraction, the $1+A$ in the denominator helps to decrease the value of the fraction, while the $(1+rA)$ part in the numerator helps to increase the value of the fraction. However, as $(1+A) \geq (1+rA)$, the denominator will increase at a faster rate compared to the numerator. Therefore, the $\sqrt{\frac{(1-p)(\hat{x_2}-\hat{x_1})^2(1+rA)+c_2}{1+A}} < \sqrt{(1-p)(\hat{x_2}-\hat{x_1})^2 + c_2}$. Hence, the value of $\underline{x}$ will be higher when players are concerned about the prospect of the critical asset. Similarly, the value of $\overline{x}$ will be higher when players are concerned about the prospect of the critical asset.

Furthermore, taking the derivative of $\underline{x}$, we get

$$\frac{\partial \underline{x}}{\partial A} = \frac{(1-p)(1-r)+c_2}{2(1+A)^{\frac{3}{2}}\sqrt{(1-p)(1+rA)+c_2}}$$

Given the assumption of the parameters, $\frac{\partial \underline{x}}{\partial A}$ is positive, entailing a positive correlation between the expected value of the prospect of the asset and the value of $\underline{x}$. Similarly, such a relationship is also true for $\overline{x}$.

QED

Proposition 4. When players are concerned about the prospect of the critical asset, comparing with Smith's work (Smith, 2021),

1. When the communication is private, there exists a fully informative equilibrium where the lower bound for $\hat{x_3}$ will be higher for certain compared with the equilibrium where players are not concerned about the prospect of the asset when A and p is high, and $r'p'$ is low.

2. The upper bound for $\hat{x_3}$ will be higher for certain compared with the equilibrium in which players are not concerned about the prospect of the asset when A and p is high, and rp is low.

Proof: For the first statement, according to Smith's result (Smith, 2021, 1376), when players are not concerned about the future return from the asset, there exists a fully informative equilibrium when

$$\hat{x_3} \in [\frac{p'-\underline{x}^2+\underline{c}}{2(p'-\underline{x})}, \frac{p-\underline{x}^2}{2(p-\underline{x})}]$$

From proposition 2, when players care about the future return of the asset, and player 3 privately conveys the information, the fully informative equilibrium exists when

$$\hat{x_3} \in \left[\frac{\overbrace{p'(1-r'A)}^{\downarrow} - \overbrace{\underline{x}^2}^{\uparrow} \overbrace{(1-A)}^{\downarrow} + \underline{c}}{\underbrace{2(p'-\underline{x})}_{\downarrow}}, \frac{p(1-rA)-\underline{x}^2(1-A)}{2(p-\underline{x})} \right]$$

First, we will focus on the lower bound $\frac{p'(1-r'A)-\underline{x}^2(1-A)+\underline{c}}{2(p'-\underline{x})}$. For the denominator of the fraction, the latter is lower than the prior as $\underline{x}$ is higher when players are concerned about the prospect. For the numerator of the fraction, suppose $\underline{x_L} = \hat{x_2} - \sqrt{(1-p)(\hat{x_2}-\hat{x_1})^2+c_2}$ is the value in the former interval and $\underline{x_H} = \hat{x_2} - \sqrt{\frac{(1-p)(\hat{x_2}-\hat{x_1})^2(1+rA)+c_2}{1+A}}$ is the value in the latter interval where $\underline{x_H} > \underline{x_L}$. It is higher than the former one if

$$p'(1-r'A) - \underline{x_H}^2(1-A) + \underline{c} > p' - \underline{x_L}^2 + \underline{c}$$
$$-r'p'A - \underline{x_H}^2 + \underline{x_H}^2 A > -\underline{x_L}^2$$

Insert the value for $\underline{x_L}$ and $\underline{x_H}$. To simplify, suppose

$$\sqrt{\frac{(1-p)(\hat{x_2}-\hat{x_1})^2(1+rA)+c_2}{1+A}} = S_H,\ \sqrt{(1-p)(\hat{x_2}-\hat{x_1})^2+c_2} = S_L$$

$$\text{where } \underline{x_L} = 1-S_L,\ \underline{x_H} = 1-S_H,$$

$$S_L > S_H$$

Then, we get

$$\underbrace{A(1-r'p')}_{>0} + (1-A)\underbrace{(2S_H - S_H^2)}_{>0}\underbrace{-(2S_L - S_L^2)}_{<0} > 0$$

Note when p or A increases, then the value for S_H decreases, and when p increases, then the value for S_L decreases. From the inequality, we observe that the sign for the second term is ambiguous depending on the sign of $(1-A)$. Hence, we need to examine the cases of $0 < A < 1$ and $A > 1$ separately.

When $0 < A < 1$, the second term is positive. To satisfy the inequality, the third term needs to be low, and the first and second term needs to be high. Considering the property that $(2S_H - S_H^2) < (2S_L - S_L^2)$. The second term cannot sufficiently triumph the third term even when $A = 0$. Hence the value of A needs to be high and the value of $r'p'$ needs to be low to maximize the first term and the value of p needs to be high to minimize the third term.

When $A > 1$, the second term is negative. To satisfy the inequality, the second and the third term needs to be low, and the first needs to be high. Hence, the value A needs to be high for certain to maximize the first term and reduce the second and the third term. Also, the value of $r'p'$ needs to be low to maximize the first term and the value of p needs to be high to minimize the second and the third term.

Hence, combining the result from the two conditions, the lower bound for $\hat{x_3}$ will be higher for certain compared with the equilibrium in which players are not concerned about the prospect of the asset when A and p is high, and $r'p'$ is low.

Similarly, for the second statement regarding the upper bound $\frac{p(1-rA)-\underline{x}^2(1-A)}{2(p-\underline{x})}$, the denominator is lower, and in order to let numerator become higher, it needs to satisfy

$$\underbrace{A(1-rp)}_{>0} + (1-A)\underbrace{(2S_H - S_H^2)}_{>0}\underbrace{-(2S_L - S_L^2)}_{<0} > 0$$

Similarly, when $0 < A < 1$, the second term is positive. Following the same logic above, the value of A needs to be high and the value of rp needs to be low to maximize the first term and the value of p needs to be high to minimize the third term.

When $A > 1$, the second term is negative. Similarly, the value A needs to be high for certain to maximize the first term and reduce the second and the third term. Also, the value of rp needs to be low to maximize the first term and the value of p needs to be high to minimize the second and the third term.

Hence, combining the result from two conditions, the upper bound for $\hat{x_3}$ will be higher for certain compared with the equilibrium in which players are not concerned about the prospect of the asset when A and p is high, and rp is low.

QED

Proposition 5. When players are concerned about the prospect of the critical asset (cf. Smith 2021):

1. In a fully informative equilibrium where communication is private, player 3's ideal policy range for sending the truthful signal can be higher or lower.

2. The correlation between the expected value of the prospect A and player 3's ideal policy range for sending the truthful signal is ambiguous.

Proof:

If the range is higher, then we need to prove

$$\frac{p(1-rA)-\underline{x_H}^2(1-A)}{2(p-\underline{x_H})} - \frac{p'(1-r'A)-\underline{x_H}^2(1-A)+\underline{c}}{2(p'-\underline{x_H})} > \frac{p-\underline{x_L}^2}{2(p-\underline{x_L})} - \frac{p'-\underline{x_L}^2+\underline{c}}{2(p'-\underline{x_L})}$$

However, given that the lower bound and higher bound are all higher than in Smith's work (as shown in proposition 4), we cannot derive such a result. From the proof of the third statement below, we also show that the effect of changing A has ambiguous effect on the value in LHS. Hence, in such an equilibrium, player 3's satisfying range for sending the truthful signal can be higher or lower.

For the third statement, first define $F(A)$:

$$F(A) = \frac{p(1-rA)-\underline{x_H}^2(1-A)}{2(p-\underline{x_H})} - \frac{p'(1-r'A)-\underline{x_H}^2(1-A)+\underline{c}}{2(p'-\underline{x_H})}$$

Then, the derivative of the function is

$$F'(A) = \frac{\left(-pr-2\underline{x_H}\,\underline{x_H}'(1-A)+\underline{x_H}^2\right)\cdot 2(p-\underline{x_H})+2\underline{x_H}'\left(p(1-rA)-\underline{x_H}^2(1-A)\right)}{4(p-\underline{x_H})^2}$$
$$-\frac{\left(-p'r'-2\underline{x_H}\,\underline{x_H}'(1-A)+\underline{x_H}^2\right)\cdot 2(p'-\underline{x_H})+2\underline{x_H}'\left(p'(1-r'A)-\underline{x_H}^2(1-A)+\underline{c}\right)}{4(p'-\underline{x_H})^2}$$

where

$$\underline{x_H} = 1 - \sqrt{\frac{(1-p)(1+rA)+c_2}{1+A}}$$

and

$$\underline{x_H}' = \frac{(1-p)(1-r)+c_2}{2(1+A)^{3/2}\sqrt{(1-p)(1+rA)+c_2}}$$

From the derivative of the function, we can observe that the value could be positive or negative. Regarding the denominator, as it is equivalent to the squared distance between p and A or p' and A there can be three possible cases: $0 < p < p' < \underline{x_H}$, $0 < \underline{x_H} < p < p'$, and $0 < p < \underline{x_H} < p'$.

When $0 < p < p' < \underline{x_H}$, the denominator of the first term is higher than the second term. Therefore when the numerator of the first term is lower than the second term, the derivative is negative for certain. Considering the term $\left(-p'r' - 2\underline{x_H}\,\underline{x_H}'(1-A) + \underline{x_H}^2\right) \cdot 2(p' - \underline{x_H})$. As $0 < p < p' < \underline{x_H}$, the term $2(p - \underline{x_H})$ and $2(p' - \underline{x_H})$ is negative. As other elements except the $-r'p'$ and $-rp$ remain the same, $r'p' > rp$ in order to make the second term larger. Next, considering the term $2\underline{x_H}'\left(p'(1-r'A) - \underline{x_H}^2(1-A) + \underline{c}\right)$. Other elements are the same except $\underline{c}$ and $p'(1-r'A)$. In order to achieve a higher second term, $\underline{c}$ needs to be high. Under such a condition, there is a negative correlation between A and the range of the interval of $\hat{x}_3$ where player 3 will signal truthfully.

When $0 < \underline{x_H} < p < p'$, the denominator of the first term is lower than the second term. Therefore when the numerator of the first term is higher than the second term, the derivative is negative for certain. Considering the term $\left(-p'r' - 2\underline{x_H}\,\underline{x_H}'(1-A) + \underline{x_H}^2\right) \cdot 2(p' - \underline{x_H})$, as $0 < \underline{x_H} < p < p'$, the term $2(p - \underline{x_H})$ and $2(p' - \underline{x_H})$ is positive. As other elements except the $-r'p'$ and $-rp$ remain the same, $r'p' > rp$ in order to make the second term lower. Next, considering the term $2\underline{x_H}'\left(p'(1-r'A) - \underline{x_H}^2(1-A) + \underline{c}\right)$. Other elements are the same except $\underline{c}$ and $p'(1-r'A)$. In order to achieve a lower second term, $\underline{c}$ needs to be low. Under such a condition, there is a positive correlation between A and the range of the interval of $\hat{x}_3$ in which player 3 will signal truthfully.

When $0 < p < \underline{x_H} < p'$, the relationship between two terms are more ambiguous. Regarding the denominator, which term is higher or lower depends on comparing the intimacy between p' and $\underline{x_H}$ with between p and $\underline{x_H}$. Moreover, regarding the numerator, for the term $\left(-p'r' - 2\underline{x_H}\,\underline{x_H}'(1-A) + \underline{x_H}^2\right) \cdot 2(p' - \underline{x_H})$, the term $2(p - \underline{x_H})$ is positive and $2(p' - \underline{x_H})$ is negative. As a result, it renders the direct comparison ambiguous such that the difference between the two terms can be positive or negative.

To further elucidate the outcomes presented in Propositions 4 and 5, Figures 2, 3, and 4 visualize the relationship between the independent variable, the Expected Prospect of the Asset (A), and the dependent variables, which include the Higher Bound, Lower Bound, and Range of player 3's fully informative signaling equilibrium under certain heterogeneous parameter assumptions. The results of the following figures are based on the assumption of $p' = 0.41, p = 0.4, r' = 0.2, r = 0.3, c_1 = 0.1, c_2 = 0.1$ to satisfy all circumstances of the heterogeneous parameters in Propositions 4 and 5 as closely as possible.

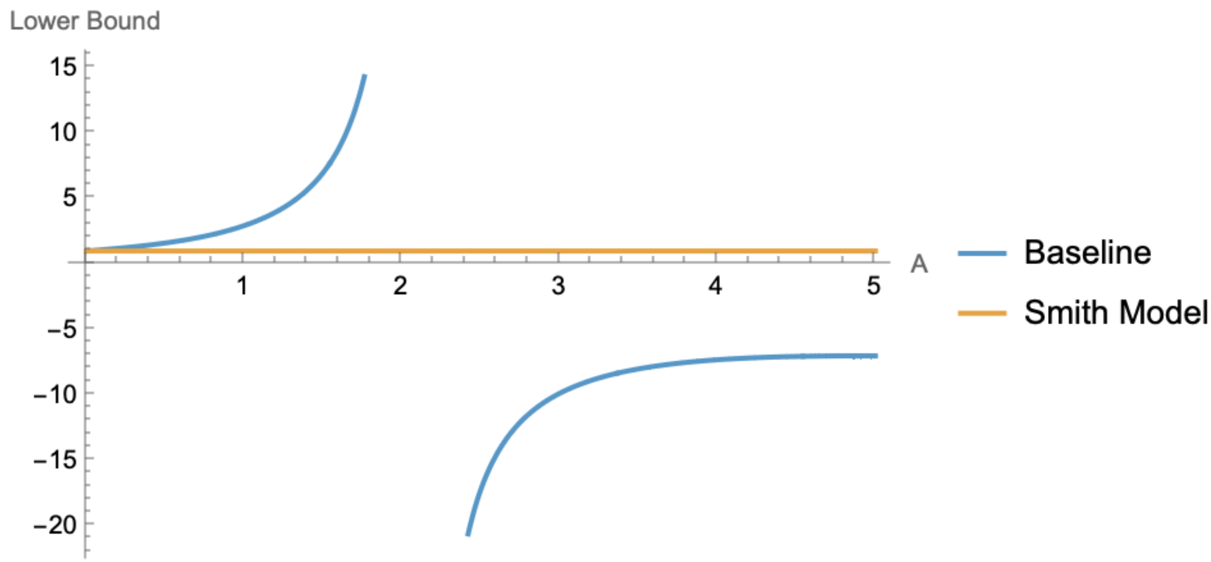

Figure 2: Relationship between the lower bound of the interval and the prospect of the asset

Consistent with Proposition 4, Figures 2 and 3 demonstrate a positive correlation between the prospect of the asset and both the lower and upper bounds of the truth-telling interval, provided that the initial value of the asset prospect is low. However, the figures indicate that when the starting value of the prospect of the asset is extremely high (around twice the current value of the asset), such positive correlation does not hold, as both the upper and lower bounds of the interval remain negative. This shows that when player 3's interests are more aligned with their ally player 2's interests, a high-cost type will always misrepresent as a low-cost type to freeride on player 2's war effort. Moreover, when player 3's interests are more aligned with player 1's, player 3 will always faithfully reveal their type, whereas a low-cost type pretends to be a high-cost type to prevent war will not take place.

I argue that this pattern occurs because when the anticipated prospect of the asset is extremely high,

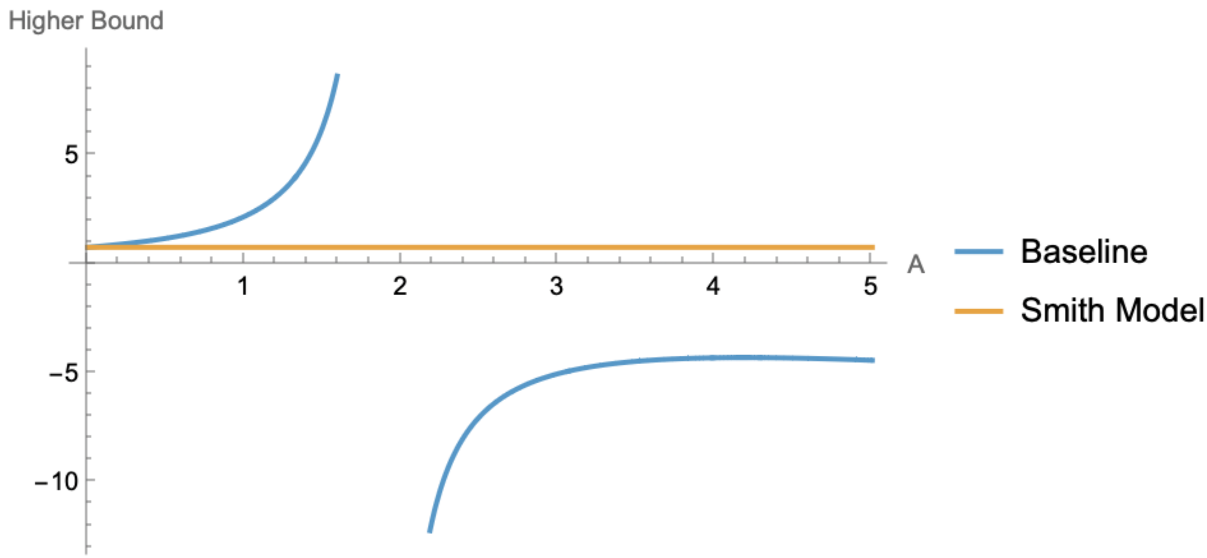

Figure 3: Relationship between the higher bound of the interval and the prospect of the asset

player 3 will never identify themselves as a high-cost type in order to avoid peace with high opportunity costs. For player 3 with aligned interests with player 2, as the analysis of Lemma 1 indicates, when the expected prospect of the asset is extremely high, the threshold for player 3's mobilization or neutral decision will also be extremely high. Consequently, this model exhibits a selection effect whereby if the prospect of the asset is extremely high and nature endows player 3 as a high type, that player 3 will face an overwhelmingly high cost of war, making them less motivated to intervene. However, an extremely high prospect of the asset also incentivizes player 3 to manipulate player 2 into rejecting the offer, as the expected value from bilateral war correspondingly increases. Thus, player 3 has a strong incentive to misrepresent as a low-cost type in order to freeride on player 2's war effort.

For players whose interests align more with player 1, when the prospect of the asset is overwhelmingly high, player 3 will be more concerned with long-term interests compared to short-term interests. Note that player 3 with aligned interest with player 1 originally would prefer to deviate and signal a high-cost type since they wish to fulfill their short-term goals and accept an offer that benefits player 1 while pacifying player 2. However, in this scenario, such incentives do not exist; thus, player 3 will always signal cooperatively when the prospect of the asset is extremely high.

QED

Proposition 6. When player 2 and 3's ideal policies match, $\hat{x_2} = \hat{x_3}$, there will exist a fully informative equilibrium through private communication.

Proof:

When the cheap talk message is private to player 2, the sequential rational strategy is shown in proposition 2. Now we need to check the incentive compatibility for player 3:

When $c_3 = \overline{c}$,

Following the same logic as in proposition 2, player 3 will choose to stick to the separating strategy when

$$EU_3(m=\overline{c}) \geq EU_3(m=\underline{c})$$
$$-(\hat{x_3}-\underline{x})^2+(\hat{x_1}-\underline{x})^2A \geq -p(\hat{x_3}-\hat{x_2})^2-(1-p)(\hat{x_3}-\hat{x_1})^2+rpA$$
$$1 \leq \frac{p(1-rA)-\underline{x}^2(1-A)}{2(p-\underline{x})}$$

When $c_3 = \underline{c}$,

Following the same logic as in proposition 2, player 3 will choose to stick to the separating strategy when

$$EU_3(m=\underline{c}) \geq EU_3(m=\overline{c})$$
$$-p'(\hat{x_3}-\hat{x_2})^2-(1-p')(\hat{x_3}-\hat{x_1})^2-\underline{c}+r'p'A \geq -(\hat{x_3}-\underline{x})^2+(\hat{x_1}-\underline{x})^2A$$
$$1 \geq \frac{p'(1-r'A)-\underline{x}^2(1-A)+\underline{c}}{2(p'-\underline{x})}$$

However, different from Smith's result, there is no such contradiction where $c_2 \leq 0$ where in this case, the inequality will be satisfied when

$$c_2 \geq \frac{(1+A)(A+\sqrt{A^2-(A-1)[A(1-rp)+(1-p)]}^2}{((A-1)^2} - (1-p)(1+rA)$$

Hence, there is some A, p, p', r, r', $\underline{c}$, c_2 such that

$$1 \in \left[\frac{p'(1-r'A)-\underline{x}^2(1-A)+\underline{c}}{2(p'-\underline{x})}, \frac{p(1-rA)-\underline{x}^2(1-A)}{2(p-\underline{x})}\right],$$

This result is consistent with propositions 4 and 5 where we substantiated that the upper bound and lower bound in our model can be higher in some specific cases compared to Smith's model. In return, for some specific cases it is conceivable that the interval will include 1.

Combining the result in figure 2 and 3, figure 4 demonstrates the correlation between the range and the expected prospect of the asset.

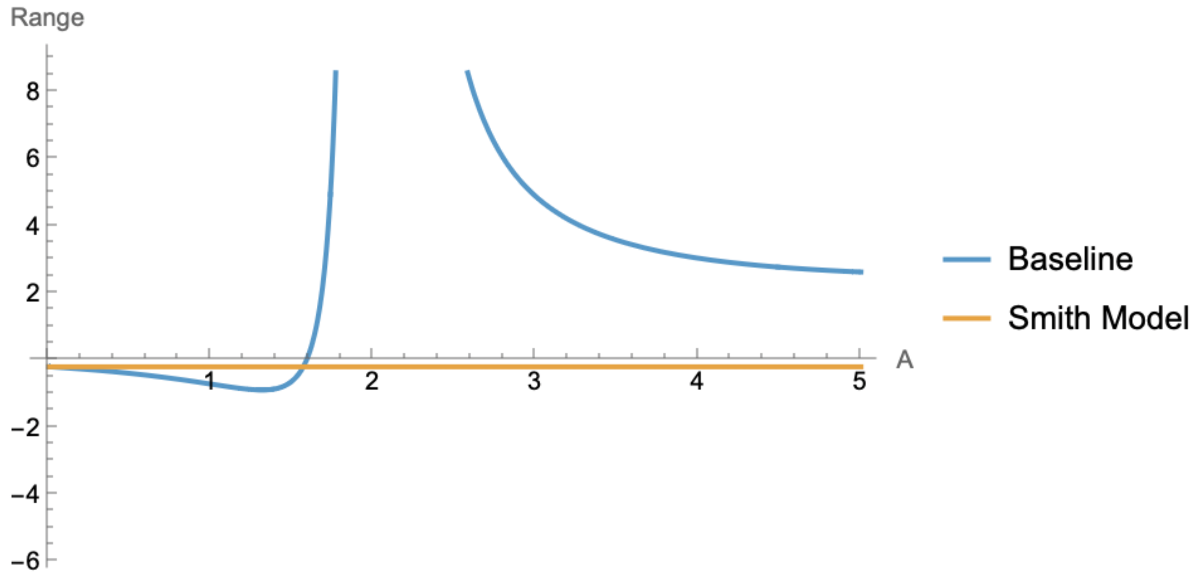

Figure 4: Relationship between the range of the interval and the prospect of the asset

Figure 4, consistent with Proposition 5, illustrates that there is an ambiguous relationship between the asset's prospect and the incentive for player 3 to send the cooperative signal. The curve on the right can be disregarded because when the asset's prospect is extremely high, both the upper and lower bounds of the interval are negative, indicating that player 3 will never signal cooperatively. However, the pattern does provide evidence supporting Proposition 6, showing that for some values of each exogenous parameter, there exists an interval that includes the point $\hat{x_3} = 1$, as the range of the curve on the left can exceed 1.

QED

Proposition 7. When players are concerned about the prospect of the critical asset, in the fully informative equilibrium where player 3 communicate privately (cf. Smith, 2021):

1. There is still the probability of war where player 1 will have a wider interval of ϕ to risk multilateral war unless the value of $r'p'$ is excessively high.

2. The range of ϕ is positively correlated with the expected value of the future prospect of the asset when the value of $r'p'$ is not excessively high and the starting value of A is low. Otherwise, the relationship is ambiguous.

Proof: For the first statement, from Smith's work, player 1 will choose to risk war and offer $\underline{x}$ when

$$\phi \leq \frac{(\hat{x_1}-\overline{x})^2-(\hat{x_1}-\underline{x})^2}{p'+c_1-(\hat{x_1}-\underline{x})^2}$$

However, when players are concerned about the future return of controlling the asset, we have

$$\phi \leq \frac{\overbrace{(1+A)}^{\uparrow}[(\hat{x_1}-\overline{x})^2-(\hat{x_1}-\underline{x})^2]}{p'+c_1+\underbrace{r'p'A}_{\uparrow}-\underbrace{(\hat{x_1}-\underline{x})^2(1+A)}_{\uparrow}}$$

Compared with Smith's result, the numerator of the function is larger. However, for the denominator of the function, it is lower unless $r'p' > \underline{x}^2$. Therefore, player 1 will have a wider interval of ϕ to choose $\underline{x}$ and risk war unless the value for r' and p' is excessively large.

For the second statement, take the derivative of ϕ over A, get

$$\frac{\partial \phi}{\partial A} = \frac{\overbrace{\left(\overline{x}^2-\underline{x}^2+(1+A)\left[2\overline{x}\frac{d\overline{x}}{dA}-2\underline{x}\frac{d\underline{x}}{dA}\right]\right)}^{>0}\left(p'+c_1+r'p'A-\underline{x}^2(1+A)\right)}{\underbrace{\left(p'+c_1+r'p'A-\underline{x}^2(1+A)\right)^2}_{>0}}$$

$$-\frac{\overbrace{(1+A)(\overline{x}^2-\underline{x}^2)}^{>0}\left(r'p'-\underline{x}^2-2\underline{x}(1+A)\frac{d\underline{x}}{dA}\right)}{\underbrace{\left(p'+c_1+r'p'A-\underline{x}^2(1+A)\right)^2}_{>0}}$$

In order to achieve a positive $\frac{\partial \phi}{\partial A}$ for certain,

$$\begin{cases} p'+c_1+r'p'A-\underline{x}^2(1+A) > 0 \\ r'p'-\underline{x}^2-2\underline{x}(1+A)\dfrac{d\underline{x}}{dA} < 0 \end{cases}$$

For the second inequality, it will be always be true if $\underline{x}^2 > r'p'$. Interestingly, note that this property is also used in proving the first statement where we show that if $\underline{x}^2 > r'p'$, there will be a wider interval of ϕ for player 1 to risk war.

Simplifying the first inequality, we get

$$p' + c_1 + A\underbrace{(r'p' - \underline{x}^2)}_{<0} - \underline{x}^2 > 0$$

Using the result from the second inequality, the term $A(r'p' - \underline{x}^2)$ is negative. Therefore, in order to satisfy the inequality, the term $A(r'p' - \underline{x}^2)$ and $\underline{x}$ needs to be minimized. From proposition 3, we proved the positive correlation between $\underline{x}$ and A, so a low A will result in a low $\underline{x}$ which in return minimizes both $A(r'p' - \underline{x}^2)$ and $\underline{x}$.

Hence, combining the result from the analysis of both inequalities, we conclude that there will be a positive correlation between ϕ and A when $\underline{x}^2 > r'p'$ ($r'p'$ is not excessively high) and A is low. Otherwise the relationship is ambiguous such that it could be a positive or negative correlation.

QED

13.2 Proofs for the Extension

In the extension model, recall that the utility for each player is the following:

	Multilateral War
Player 1	$-p' - c_1 - r'p'\theta$
Player 2	$-(1-p') - c_2 - r'(1-p')\theta$
Player 3	$-p'(\hat{x}_3 - \hat{x}_2)^2 - (1-p')(\hat{x}_3 - \hat{x}_1)^2 - c_3 + r'p'\theta$

	Bilateral War
Player 1	$-p - c_1 - rp\theta$
Player 2	$-(1-p) - c_2 - r(1-p)\theta$
Player 3	$-p(\hat{x}_3 - \hat{x}_2)^2 - (1-p)(\hat{x}_3 - \hat{x}_1)^2 + rp\theta$

	Peace
Player 1	$-(\hat{x}_1 - x)^2 - (\hat{x}_1 - x)^2\theta$
Player 2	$-(\hat{x}_2 - x)^2 - (\hat{x}_2 - x)^2\theta$
Player 3	$-(\hat{x}_3 - x)^2 + (\hat{x}_1 - x)^2\theta$

Lemma 3. Under complete information, if $r'p' > rp$, Player 3 will choose to mobilize when

$$\theta \geq \frac{(p'-p)[(\hat{x}_3-\hat{x}_2)^2 - (\hat{x}_3-\hat{x}_1)^2]}{(r'p'-rp)} \equiv \theta^*$$

or otherwise, player 3 will remain neutral.

Proof:

Player 3's utility when choosing to mobilize is greater than remaining neutral when

$$EU_3(Multilateral) \geq EU_3(Bilateral)$$
$$-p(\hat{x_3}-\hat{x_2})^2-(1-p)(\hat{x_3}-\hat{x_1})^2+r'p'\theta \geq p(\hat{x_3}-\hat{x_2})^2-(1-p)(\hat{x_3}-\hat{x_1})^2+rp\theta$$

Simplifying the inequality, we get

$$\theta \geq \frac{(p'-p)[(\hat{x_3}-\hat{x_2})^2-(\hat{x_3}-\hat{x_1})^2]}{(r'p'-rp)} \equiv \theta^*$$

QED

Lemma 4. Under complete information, when $r'p' > rp$, player 1 would offer

$$\begin{cases} \overline{x} = \hat{x_2} - \sqrt{\frac{(1-p')(\hat{x_2}-\hat{x_1})^2(1+r'\overline{\theta})+c_2}{1+\overline{\theta}}} & \text{if } \theta = \overline{\theta} \geq \theta^* \\ \underline{x} = \hat{x_2} - \sqrt{\frac{(1-p)(\hat{x_2}-\hat{x_1})^2(1+r\underline{\theta})+c_2}{1+\underline{\theta}}} & \text{if } \theta = \underline{\theta} < \theta^* \end{cases}$$

Proof:

From Lemma 1, we assessed the threshold for player 3 choosing to mobilize. When the prospect of the asset is sufficiently high, then player 2 would reject crisis bargaining and player 3 would mobilize, with the game resulting in a multilateral war. In such case, player 2 would receive

$$EU_2(Multilateral) = -(1-p') - c_2 - r'(1-p')\overline{\theta}$$

Similarly, when the prospect of the asset is low, then player 3 would remain neutral, such that if player 2 rejects crisis bargaining then the game will result in a bilateral war. In such a case, player 2 would receive

$$EU_2(Bilateral) = -(1-p) - c_2 - r(1-p)\underline{\theta}$$

To maximize player 1's own payoff, they will offer some x to make player 2 indifferent, in which assumption, player 2 will choose to accept. Therefore, in order to lead player 2 to accept, player 1 needs to offer some $\overline{x}$ when θ is high and offer some $\underline{x}$ when θ is low where $\overline{x} > \underline{x}$

When θ is high where $\theta \geq \frac{(p'-p)[(\hat{x_3}-\hat{x_2})^2-(\hat{x_3}-\hat{x_1})^2]}{(r'p'-rp)}$, player 1 would offer $\overline{x}$ that satisfies

$$EU_2(Accept) = EU_2(Reject, Multilateral)$$
$$-(\hat{x_2}-\overline{x})^2-(\hat{x_2}-\overline{x})^2\overline{\theta} = -(1-p') - c_2 - r'(1-p')\overline{\theta}$$
$$\overline{x} = \hat{x_2} - \sqrt{\frac{(1-p')(\hat{x_2}-\hat{x_1})^2(1+r'\overline{\theta})+c_2}{1+\overline{\theta}}}$$

Similarly, when θ is low where $\theta < \frac{(p'-p)[(\hat{x_3}-\hat{x_2})^2-(\hat{x_3}-\hat{x_1})^2]}{(r'p'-rp)}$, we get

$$EU_2(Accept) = EU_2(Reject, Bilateral)$$
$$-(\hat{x_2}-\underline{x})^2 - (\hat{x_2}-\underline{x})^2\underline{\theta} = -(1-p) - c_2 - r(1-p)\underline{\theta}$$
$$\underline{x} = \hat{x_2} - \sqrt{\frac{(1-p)(\hat{x_2}-\hat{x_1})^2(1+r\underline{\theta})+c_2}{1+\underline{\theta}}}$$

Then, we need to further prove $\bar{x} > \underline{x}$ to ensure player 1 has the incentive to always offer a higher deal when the prospect of the asset is high for all circumstances. Otherwise, player 1's best response will be ambiguous which the sequential rational strategy of player 1 after receiving the signal from player 2 cannot be uniformly derived and case dependent on the specific value of each parameters.

Note that if the fraction part of the $\underline{x}$ is greater than the fraction part of the $\bar{x}$, then the $\bar{x} > \underline{x}$ will be satisfied.

First, consider the denominator of each offer. Since $\overline{\theta} > \underline{\theta}$, the denominator of $\underline{x}$ is lower than $\bar{x}$.

Secondly, for the nominator of each offer, since $p' > p$, then $(1-p) > (1-p')$. Therefore, the ordering of the nominator is reliant on the value of $\frac{1+r\underline{\theta}}{1+\underline{\theta}}$ and $\frac{1+r'\overline{\theta}}{1+\overline{\theta}}$. If $\frac{1+r\underline{\theta}}{1+\underline{\theta}} > \frac{1+r'\overline{\theta}}{1+\overline{\theta}}$, then we can conclude $\bar{x} > \underline{x}$.

Suppose $F(m,n) = \frac{1+nm}{1+m}$, $m > 0$ and $0 < n < 1$

$$\frac{\partial F}{\partial n} = \frac{m}{1+m} > 0$$
$$\frac{\partial F}{\partial m} = \frac{n-1}{(1+m)^2} < 0$$

Hence, it implies that the function is increasing with n and decreasing with m. Therefore, the following properties can be derived:

$$F(r', \underline{\theta}) < F(r, \underline{\theta}),$$
$$F(r', \overline{\theta}) < F(r', \underline{\theta})$$

Reorder the inequalities, we have

$$F(r', \overline{\theta}) < F(r, \underline{\theta})$$
$$\text{Equivalent to } \frac{1+r\underline{\theta}}{1+\underline{\theta}} > \frac{1+r'\overline{\theta}}{1+\overline{\theta}}$$

Therefore, we proved $\bar{x} > \underline{x}$ when $r'p' > rp$

Furthermore, note that when $r'p' < rp$,

Player 3 will mobilize when

$$\theta \leq \frac{(p'-p)[(\hat{x_3}-\hat{x_2})^2-(\hat{x_3}-\hat{x_1})^2]}{(r'p'-rp)} \equiv \theta^*$$

This implies that when the asset is extremely valuable in the future $\theta = \overline{\theta}$, player 3 will freeride on player 2's war effort since engaging in multilateral war is not as beneficial. When the asset is not valuable where $\theta = \underline{\theta}$, it is surprising that Player 3 would mobilize as they do not care about the prospect of the asset and seek to satisfy their short-term foreign policy goal. Furthermore, when the ideal policy point of player 3 $\hat{x_3} < 0.5$, player 3 will never mobilize since even a high type of prospect cannot out-balance the foreign policy need in the short term.

Moreover, we derive the offer proposed from player 1 under such condition

$$\overline{x} = \hat{x_2} - \sqrt{\frac{(1-p')(\hat{x_2}-\hat{x_1})^2(1+r'\underline{\theta})+c_2}{1+\underline{\theta}}}$$
$$\underline{x} = \hat{x_2} - \sqrt{\frac{(1-p)(\hat{x_2}-\hat{x_1})^2(1+r\overline{\theta})+c_2}{1+\overline{\theta}}}$$

However, we cannot derive which offer is strictly larger than the other because for $\overline{x}$, the denominator of the fraction part is low but the nominator is also low comparing with $\underline{x}$, showing that player 1's best response is ambiguous and is different if the value of the parameter is different. As the best response of player 1 when $r'p' < rp$ cannot be generalized under such condition, we assume $r'p' > rp$ throughout the examination of the extension model.

QED

Proposition 8. Player 2 will cooperate with player 3 when sending the costly signal in second stage only when player 1 offers $\overline{x}$. If player 1 offers $\underline{x}$, player 2 will not cooperate but use a pooling strategy $s_2 = R, \forall\theta$ when the prior belief of the likelihood of the type of asset is a low type is low ($q < q^* = \frac{(\hat{x_3}-\hat{x_2})^2(p'-p)+(\hat{x_3}-\hat{x_1})^2(p-p')+\overline{\theta}(rp-r'p')}{(r'p'-rp)(\underline{\theta}-\overline{\theta})}$) to entangle player 3 into multilateral war and free ride on player 3's war effort.

Proof:

To derive the strategy sustained for player 2 in the second stage given the player 1's strategy in the first stage, we divide the game into two subgames where in the first case player 1 offers $\overline{x}$ and in the second case player 1 offers $\underline{x}$ in the first stage. We will test whether such offering sustains with player 2's strategy in stage 2 later.

From Lemma 3, the sequential rational strategy for player 3 is: When player 2 rejects, if the updated belief is that the type of the asset is $\overline{\theta}$, player 3 will mobilize and enter a multilateral war. If the updated belief is that the type of the asset is $\underline{\theta}$, player 3 will stay neutral and there will be a bilateral war between player 1 and 2.

Also derived from Lemma 4, when there is complete information over the type of the asset, the best response of player 2 is:

If player 1 offers $\underline{x}$,

$$s_2 = \begin{cases} A & \text{if } \theta = \underline{\theta} \\ R & \text{if } \theta = \overline{\theta} \end{cases}$$

If player 1 offers $\bar{x}$

$$s_2 = A, \forall \theta$$

Then, we will first examine whether these two strategies sustain in the incomplete information model.

Subgame 1: Player 1 offers $\bar{x}$ in the first stage

If player 2 chooses the strategy $s_2 = A, \forall \theta$, then

$$P[s_2 = A | \theta = \underline{\theta}] = 1$$
$$P[s_2 = R | \theta = \underline{\theta}] = 0$$

The updated belief for player 3 is:

$$\mu^*(A) = p[\theta = \underline{\theta} | s_2 = A] = \tfrac{q \cdot 1}{q \cdot 1 + (1-q) \cdot 1} = q$$
$$\mu^*(R) = p[\theta = \underline{\theta} | s_2 = R] = \tfrac{q \cdot 0}{q \cdot 0 + (1-q) \cdot 0} = \alpha \in [0, 1]$$

Then, derive the sequentially rational strategy for player 3 after player 1 rejects given the updated belief. Note there is no need to derive the sequentially rational strategy for player 3 after player 1 accepts given the updated belief since the game ends.

$$EU_3(s_3 = Mobilize | s_2 = R)$$
$$= \alpha[-p'(\hat{x_3} - \hat{x_2})^2 - (1-p')(\hat{x_3} - \hat{x_1})^2 - c_3 + r'p'\underline{\theta}] + (1-\alpha)[-p'(\hat{x_3} - \hat{x_2})^2 - (1-p')(\hat{x_3} - \hat{x_1})^2 - c_3 + r'p'\overline{\theta}]$$
$$EU_3(s_3 = Neutral | s_2 = R)$$
$$= \alpha[-p(\hat{x_3} - \hat{x_2})^2 - (1-p)(\hat{x_3} - \hat{x_1})^2 - c_3 + rp\underline{\theta}] + (1-\alpha)[-p(\hat{x_3} - \hat{x_2})^2 - (1-p)(\hat{x_3} - \hat{x_1})^2 - c_3 + rp\overline{\theta}]$$

Simplfy, player 3 will choose to mobilize when

$$\alpha < \frac{(\hat{x_3}-\hat{x_2})^2(p'-p)+(\hat{x_3}-\hat{x_2})^2(p-p')+\overline{\theta}(rp-r'p')}{(r'p'-rp)(\underline{\theta}-\overline{\theta})} \equiv \alpha^*$$

Note that the fraction is positive (both nominator and denominator are negative) due to the assumption $r'p' > rp$.

Next, we substantiate player 2's strategy is incentive compatible.

When $\theta = \underline{\theta}$,

$$EU_2(R,\mu^*) = \begin{cases} -(1-p') - c_2 - r'(1-p')\underline{\theta} & \text{if } \alpha < \alpha^* \\ -(1-p) - c_2 - r(1-p)\underline{\theta} & \text{if } \alpha > \alpha^* \end{cases}$$

$$EU_2(A,\mu^*) = -(\hat{x_2}-\overline{x})^2 - (\hat{x_2}-\overline{x})^2\underline{\theta}$$

Due to the properties of the parameters, we have

$$EU_2(A,\mu^*) = -(\hat{x_2}-\overline{x})^2 - (\hat{x_2}-\overline{x})^2\underline{\theta} > -(\hat{x_2}-\underline{x})^2 - (\hat{x_2}-\underline{x})^2\underline{\theta}$$

Moreover, from the proof of Lemma 4, we know that

$$-(\hat{x_2}-\underline{x})^2 - (\hat{x_2}-\underline{x})^2\underline{\theta} = -(1-p) - c_2 - r(1-p)\underline{\theta} = EU_2(R,\mu^*,\alpha > \alpha^*)$$

Therefore, $EU_2(A,\mu^*) > EU_2(R,\mu^*,\alpha > \alpha^*)$, implying that the strategy is incentive compatible when $\alpha > \alpha^*$ for the condition $\theta = \underline{\theta}$

When $\theta = \overline{\theta}$,

$$EU_2(R,\mu^*) = \begin{cases} -(1-p') - c_2 - r'(1-p')\overline{\theta} & \text{if } \alpha < \alpha^* \\ -(1-p) - c_2 - r(1-p)\overline{\theta} & \text{if } \alpha > \alpha^* \end{cases}$$

$$EU_2(A,\mu^*) = -(\hat{x_2}-\overline{x})^2 - (\hat{x_2}-\overline{x})^2\overline{\theta}$$

From the proof of Lemma 4, we know that

$$EU_2(A,\mu^*) = -(\hat{x_2}-\overline{x})^2 - (\hat{x_2}-\overline{x})^2\overline{\theta} = -(1-p') - c_2 - r'(1-p')\overline{\theta} = EU_2(Reject,\mu^*,\alpha < \alpha^*)$$

Due to the properties of each parameters, we have

$$EU_2(Reject,\mu^*,\alpha < \alpha^*) = -(1-p') - c_2 - r'(1-p')\overline{\theta} > -(1-p) - c_2 - r(1-p)\overline{\theta} = EU_2(R,\mu^*,\alpha > \alpha^*)$$

Therefore, $EU_2(A,\mu^*) > EU_2(R,\mu^*)$, implying that the strategy is always incentive compatible when $\theta = \overline{\theta}$

To conclude, in the subgame that player 1 offers $\bar{x}$, the pooling strategy of $s_2 = A, \forall\theta$ sustains when $\alpha < \alpha^*$.

Subgame 2: Player 1 offers $\underline{x}$ in the first stage

If player 2 chooses the strategy

$$s_2 = \begin{cases} A & \text{if } \theta = \underline{\theta} \\ R & \text{if } \theta = \overline{\theta} \end{cases}$$

Hence,

$$P[s_2 = A|\theta = \underline{\theta}] = 1, P[s_2 = A|\theta = \underline{\theta}] = 1$$

The update belief of player 3 is

$$\mu^*(s_2 = A) = P[\theta = \underline{\theta}|s_2 = A] = \frac{p\cdot 1}{p\cdot 1+(1-p)\cdot 0} = 1$$
$$\mu^*(s_2 = R) = P[\theta = \underline{\theta}|s_2 = R] = \frac{p\cdot 0}{p\cdot 0+(1-p)\cdot 1} = 0$$

According to Lemma 3, the sequential rational strategy for player 3 is to mobilize when player 2 rejects.

Then, we need to check the incentive compatibility of such strategy:

When $\theta = \underline{\theta}$,

$$EU_2(A,\mu^*) = -(\hat{x_2} - \underline{x})^2 - (\hat{x_2} - \underline{x})^2\underline{\theta}$$
$$EU_2(R,\mu^*) = -(1-p') - c_2 - r'(1-p')\underline{\theta}$$

From the proof of Lemma 4, we know that

$$EU_2(A,\mu^*) = -(\hat{x_2} - \underline{x})^2 - (\hat{x_2} - \underline{x})^2\underline{\theta} = -(1-p) - c_2 - r(1-p)\underline{\theta}$$

However, from the properties of parameters,

$$EU_2(A,\mu^*) = -(1-p) - c_2 - r(1-p)\underline{\theta} < -(1-p') - c_2 - r'(1-p')\underline{\theta} = EU_2(R,\mu^*)$$

Therefore, such strategy does not satisfy the incentive compatibility. As a result, this cooperative separating strategy does not sustain in the equilibrium. This implies that when player 1 offers a low proposal, player 2 has the incentive to drag player 3 into war to free ride on their war effort, even when the offer from player 1 is enough to pacify player 2. Hence, next we will test whether the pooling strategy of always choosing reject when player 1 offers a low proposal regardless of the future value of asset.

If player 2 chooses the pooling strategy $s_2 = R, \forall\theta$, which $P[s_2 = A|\theta = \underline{\theta}] = 0$, $P[s_2 = R|\theta = \underline{\theta}] = 1$

Hence, the updated belief for player 3 is that

$$\mu^*(A) = P[\theta = \underline{\theta}|s_2 = A] = \frac{q\cdot 0}{q\cdot 0+(1-q)\cdot 0} = \beta \in [0,1]$$
$$\mu^*(R) = P[\theta = \underline{\theta}|s_2 = R] = \frac{q\cdot 1}{q\cdot 1+(1-q)\cdot 1} = q$$

Then, derive the sequentially rational strategy for player 3 after player 1 rejects given the updated belief.

$$EU_3(s_3 = Mobilize|s_2 = R)$$
$$= q[-p'(\hat{x_3}-\hat{x_2})^2-(1-p')(\hat{x_3}-\hat{x_1})^2-c_3+r'p'\underline{\theta}]+(1-q)[-p'(\hat{x_3}-\hat{x_2})^2-(1-p')(\hat{x_3}-\hat{x_1})^2-c_3+r'p'\overline{\theta}]$$
$$EU_3(s_3 = Neutral|s_2 = R)$$
$$= q[-p(\hat{x_3}-\hat{x_2})^2-(1-p)(\hat{x_3}-\hat{x_1})^2-c_3+rp\underline{\theta}]+(1-q)[-p(\hat{x_3}-\hat{x_2})^2-(1-p)(\hat{x_3}-\hat{x_1})^2-c_3+rp\overline{\theta}]$$

Simplfy, player 3 will choose to mobilize when

$$q < \frac{(\hat{x_3}-\hat{x_2})^2(p'-p)+(\hat{x_3}-\hat{x_1})^2(p-p')+\overline{\theta}(rp-r'p')}{(r'p'-rp)(\underline{\theta}-\overline{\theta})} \equiv q^*$$

Note that the fraction is positive (both nominator and denominator are negative) due to the assumption $r'p' > rp$.

Next, we substantiate player 2's strategy is incentive compatible.

When $\theta = \underline{\theta}$,

$$EU_2(R,\mu^*) = \begin{cases} -(1-p')-c_2-r'(1-p')\underline{\theta} & \text{if } q < q^* \\ -(1-p)-c_2-r(1-p)\underline{\theta} & \text{if } q > q^* \end{cases}$$
$$EU_2(A,\mu^*) = -(\hat{x_2}-\underline{x})^2-(\hat{x_2}-\underline{x})^2\underline{\theta}$$

Due to the properties of the parameters, we have

$$EU_2(R,\mu^*,q < q^*) > EU_2(R,\mu^*,q > q^*)$$

Moreover, from the proof of Lemma 4, we know that

$$EU_2(A,\mu^*) = -(\hat{x_2}-\underline{x})^2 - (\hat{x_2}-\underline{x})^2\underline{\theta} = -(1-p) - c_2 - r(1-p)\underline{\theta} = EU_2(R,\mu^*,q>q^*)$$

Therefore, $EU_2(R,\mu^*,q<q^*) > EU_2(A,\mu^*) = EU_2(R,\mu^*,q>q^*)$, implying that the strategy is incentive compatible when $q<q^*$ for the condition $\theta=\underline{\theta}$

When $\theta=\overline{\theta}$,

$$EU_2(R,\mu^*) = \begin{cases} -(1-p') - c_2 - r'(1-p')\overline{\theta} & \text{if } q<q^* \\ -(1-p) - c_2 - r(1-p)\overline{\theta} & \text{if } q>q^* \end{cases}$$

$$EU_2(A,\mu^*) = -(\hat{x_2}-\underline{x})^2 - (\hat{x_2}-\underline{x})^2\overline{\theta}$$

Due to the properties of each parameters, we have

$$EU_2(A,\mu^*) = EU_2(A,\mu^*) = -(\hat{x_2}-\underline{x})^2 - (\hat{x_2}-\underline{x})^2\overline{\theta} < -(\hat{x_2}-\overline{x})^2 - (\hat{x_2}-\overline{x})^2\overline{\theta}$$

From the proof of Lemma 4, we know that

$$-(\hat{x_2}-\overline{x})^2 - (\hat{x_2}-\overline{x})^2\overline{\theta} = -(1-p') - c_2 - r'(1-p')\overline{\theta} = EU_2(Reject,\mu^*,q<q^*) > EU_2(Reject,\mu^*,q>q^*)$$

Therefore, $EU_2(R,\mu^*,q<q^*) > EU_2(A,\mu^*)$, implying that the strategy is incentive compatible if $q<q^*$ when $\theta=\overline{\theta}$

To conclude, in the subgame that player 1 offers $\overline{x}$, the pooling strategy of $s_2=R,\forall\theta$ sustains when $q<q^*$.

Finally, we need to show that given player 2's sustained strategy and player 3's sequential rational strategy, player 1 will still offer $\overline{x}$, $\underline{x}$ in each case. The logic is similar with the proof proposition 2.

Consider an offer $x>\overline{x}$, player 2 will always accept the offer. However, $EU_1(A,x>\overline{x}) < EU_1(A,x=\overline{x})$. Hence the offer $x>\overline{x}$ is strictly dominated.

Consider an offer $x\in[\underline{x},\overline{x})$, the offer will be always rejected when player 2 plays sustained strategy. Hence the offer $x\in(\underline{x},\overline{x})$ is weakly dominated by $\underline{x}$. Even for the strategy not sustained, where $x\in[\underline{x},\overline{x})$, $x\in(\underline{x},\overline{x})$ is strictly dominated by $\underline{x}$ since

$$EU_1(x\in[\underline{x},\overline{x})) = qEU_1(Peace) + (1-q)EU_1(Multilateral)$$

Consider an offer $x < \underline{x}$, in the sustained equilibrium it will be always rejected, hence it is weakly dominated by $x < \underline{x}$. Even when player 2 plays the strategy that is not sustained, following the similar proof in proposition 2,

$$EU_1(x < \underline{x}) = qEU_1(Bilateral) + (1-q)EU_1(Multilateral)$$
$$EU_1(x = \underline{x}) = qEU_1(Peace) + (1-q)EU_1(Multilateral)$$

$EU_1(x < \underline{x}) < EU_1(x = \underline{x})$ when

$$EU_2(Bilateral) < EU_2(Peace)$$

The proof of $EU_2(Bilateral) < EU_2(Peace)$ is shown in proposition 2. Such inequality will sustain unless the value of c_2 is extremely high. The only difference is that change the A in proposition 2 to $\underline{\theta}$.

Hence, by elimination of the dominated strategy, player 1 will offer $\underline{x}$ or $\overline{x}$ where $\overline{x} > \underline{x}$ in the first stage.

QED

Proposition 9. Assume the likelihood of the asset is likely to be a high type $q < q^*$. When the cost of war for player 1 is high and the prior belief suggests the asset is likely to be a high value type, there is a PBE that player 1 will offer $\overline{x}$, player 2 plays a pooling strategy of accept regardless of the type of the asset, resulting peace. When the cost of war for player 1 is low and the prior belief suggests the asset is likely to be a high value type, player 1 will offer $\underline{x}$, player 2 plays a pooling strategy of reject regardless of the type of the asset, and player 3 mobilize resulting a multilateral war. Furthermore, in such PBE, when $r'p' < \overline{x}^2$, player 1 will risk war when $q < \frac{-(p'-\overline{x}^2)-\overline{\theta}(r'p'-\underline{x}^2)-c_1}{(\underline{\theta}-\overline{\theta})(r'p'-\overline{x}^2)}$. When $r'p' > \overline{x}^2$, player 1 will risk war when $q > \frac{-(p'-\overline{x}^2)-\overline{\theta}(r'p'-\underline{x}^2)-c_1}{(\underline{\theta}-\overline{\theta})(r'p'-\overline{x}^2)}$.

Proof:

From Proposition 8, we derived the sustained strategy for player 2 in stage 2 when $q < q^*$. When player 1 offers $\overline{x}$, player 2 will play a pooling strategy where choose to accept regardless of θ. When player 2 offers $\underline{x}$, player 2 will play a pooling strategy where choose to reject regardless of θ. Therefore,

$$EU_1(x = \overline{x}) = q(-(\hat{x}_1 - \overline{x})^2 - (\hat{x}_1 - \overline{x})^2\underline{\theta}) + (1-q)(-(\hat{x}_1 - \overline{x})^2 - (\hat{x}_1 - \overline{x})^2\overline{\theta})$$
$$EU_1(x = \underline{x}) = q(-p' - c_1 - r'p'\underline{\theta}) + (1-q)(-p' - c_1 - r'p'\overline{\theta})$$

Suppose $EU_1(x = \overline{x}) > EU_1(x = \underline{x})$,

Simplify, get

$$c_1 > -p' - r'p'(q\underline{\theta} + \overline{\theta} - q\overline{\theta}) + \overline{x}^2(q\underline{\theta} + \overline{\theta} - q\overline{\theta} + 1)$$

Hence, player 1 will offer $\overline{x}$ to pacify player 2 when the cost of war is high, namely, when

$$c_1 < -p' - r'p'(q\underline{\theta} + \overline{\theta} - q\overline{\theta}) + \overline{x}^2(q\underline{\theta} + \overline{\theta} - q\overline{\theta} + 1)$$

Player 1 will offer $\underline{x}$ and risk war.

Reorder the inequality, we get

$$q(\underline{\theta} - \overline{\theta})(r'p' - \overline{x}^2) > -(p' - \overline{x}^2) - \overline{\theta}(r'p' - \underline{x}^2) - c_1$$

If $r'p' < \overline{x}^2$, player 1 will risk war when

$$q < \frac{-(p' - \overline{x}^2) - \overline{\theta}(r'p' - \underline{x}^2) - c_1}{(\underline{\theta} - \overline{\theta})(r'p' - \overline{x}^2)} \equiv q^*$$

If $r'p' > \overline{x}^2$, player 1 will risk war when

$$q > \frac{-(p' - \overline{x}^2) - \overline{\theta}(r'p' - \underline{x}^2) - c_1}{(\underline{\theta} - \overline{\theta})(r'p' - \overline{x}^2)} \equiv q^*$$

QED