Move your Math
A Pedagogical Study of the Interdisciplinary Research between Math and Movement in Higher Education

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An abstract of
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Abstract

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Cross-disciplinary studies have been a recent focus on improving education qualities. Combining different subjects invites more mathematical students to explore areas they are less familiar with, and therefore when disciplines collide, the process can create new sparks for each subject. At Emory, Math & Movement as a directed study has been active since 2019. Led by Pr. Lori Teague from Emory Dance and Movement Studies Program and Dr. Manuela Manetta from Emory Math Department, students upgrade their knowledge in each field by being challenged to combine them to achieve an embodied understanding of math concepts. and to convey them to someone else in an effective way. This paper focuses on a directed research study offered in the Fall of 2022, designed, and guided by Flora Zhang, the author of this thesis. This work endeavors to show the procedure of the research in steps and intends to draw more attention to the topic on the Emory campus, and beyond by presenting the motivation, analyzing the effects of two specific activities throughout six lessons across three math topics (in Linear Algebra, Calculus, and Differential Equations), and projecting possible future applications.

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Acknowledgments

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The research of the Math of Movement has accompanied me through my Emory experience and invited me to the world of math that I was once a stranger to and gave me a chance to come back to the area of dance that I was familiar with. The interdisciplinary study of the two areas has dearly combined the two subjects that will believably influence my life, and I am more than excited to witness it influencing even more in the near future.

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1 Introduction

At Emory, since the Spring of 2019, math professor Manuela Manetta and dance professor Lori Teague have engaged students in a novel approach to mathematics. Through many different courses and activities, "Mathematics through Movement" has brought on board different groups of students, some with solid math backgrounds, some experienced dancers, and students who would not claim to be in either category. By involving more students year after year and exposing the slowly maturing idea to diverse audiences, it has been possible to witness the value of learning through embodiment. by experimenting with a broader group of people around the Emory campus.

As an Applied Math major student, the author was fortunate to be introduced to the program at the beginning of the junior year, exploring both interests in math education and dance in a directed research course, since then the author has had the opportunity to be a learner, a teaching assistant, and a "teacher". While changing roles, the idea of combining math and movement to explain abstract higher education math concepts became a different reality, especially when leading a directed study course independently. This experience helped the author identify ways to embed the methods into higher education, introduce diverse approaches, and establish pedagogical practices for varied learning styles that would be efficient.

In the trend of machine learning and rapid technology revolutions, the underlying logic explained by math has become increasingly important. While the subject reveals its significance in modern society, the basic math requirement in American universities did not immediately reflect this relevance. Since math is "notoriously" hard, students who are convinced they are "bad at math" are not often able to change this label in the traditional lecture hall. Calculus, Linear Algebra, and

Differential Equations are courses that continue to be either the requirement for graduation or a prerequisite for higher-level science and math courses and rarely offer a student an authentic opportunity to learn. Students are too easily discouraged or overwhelmed by the complexity of the material to realize how important it is to build on foundational knowledge and apply that knowledge in more advanced courses.

Adding movement to a mathematics class makes them more relaxed, self-aware, and less self-conscious, which invites the body to live in a state more conducive to learning. This state of body and mind (somatic) provides students with a new angle to perceive both subjects, encouraging them to collaborate and develop different tools that allow them to reflect and understand abstract concepts. Traditional movement classes, with warm-up and repetitive practices, are intended to make students feel comfortable, reduce the stress that comes from the possibility of making mistakes, and direct students' attention to their feelings as the catalyst for expression. Furthermore, a movement class typically allocates time to collaborate, rehearse, and eventually present one's idea in public. Including the above-mentioned practices in a math learning process will facilitate students' understanding of abstract concepts while engaging them, according to a student-centered pedagogy.

Through different lenses (student, observer, assistant, and instructor), the work focuses on the process of designing, delivering, and documenting the consequent reflections of the directed research, pointing out the activities and teaching strategies that were effective for the group of students we worked with. It offers examples of mathematical concepts and the related proposed movement activities, discusses some possible changes to the course, and offers some prompts for future studies.

2 Motivation

Undergraduate students, often entering college to pursue their dream major, are encouraged to explore the variety of subjects Emory has to offer with an open mind. The basic requirements of graduation, including humanities, arts, social science, science, and math, are meant to form well-rounded students and allow them to experience a positive mindset.

Current math education mostly focuses on giving firm lectures and building solid math skills; however, students usually care more about getting correct answers on exams rather than understanding certain concepts and how they are applied, or how to approach a new math concept. While these effects are often the result of antiquated teaching methods, changes are needed to ameliorate the anxiety of achieving accuracy. Math educators should switch gears to focus on understanding concepts. This may allow students time to perceive the scientific learning process differently, build their systematic thinking of math from the ground up, and free the mind from stress so that a student can practice critical thinking skills in math which may help them in their area of study.

To many people, math has its convoluted equation that acts as a stand-off sign posted at the front gate of the math world, listed as an uninviting "welcome" sign. This entry point needs to be accessible to a general audience – at least at the basic level – to appreciate its full potential. More ambassadors who show (can demonstrate) how math can be perceived outside of class are needed. An accessible approach could change people's general attitude toward the subject. Inviting students to share their ups and downs, while navigating their personal or scientific academic growth will help them be more resilient. Dance educators deal with the nervousness and discomfort of stepping on stage and the potential possibility of making mistakes. Math educators lack the process of telling students how to deal with such fear when facing a math problem.

In this fast-developing world of technologies, thousands of educators, parents, and even children themselves started to realize the importance of learning computer languages. While education that combines machine learning is the trend, math is still the basic logic under everything. That is, if students cannot see how math contributes to such a fast development of technologies, and they are just finishing tasks in a designed classroom, then they have no chance of being creative and surviving in the actual world.

To think in words and to think in actions seem to be two opposite poles. Claiming one may be better than the other is not useful; therefore we are rather inclined to see how the combination of the two can communicate things more precisely and even arouse more discussions.

In the dance community, people are used to expressing emotions related to their work and they ponder how the audience would view their ideas. Therefore, they learn to deal with the nervousness and discomfort of being in public. While bearing the possibility of making mistakes, dancers are trained more intentionally to accept failure, and welcome comments for improvements, to eventually turn their fears into performing ideas. Comparatively, in traditional lecture halls, math classes seem to focus on quickly understanding, explaining, and calculating precisely. Precision should not always be the main focal point in a math class, Instead, understanding and seeing the connections among different concepts is more important. Perhaps in the past, it sounded appealing if a student was able to perform impressive calculations. However, in the era of computers, smartphones, and the internet, mere computation skills are not even useful anymore. It becomes clear that learning and understanding abstract math concepts is the key. Keeping the calculation precision is certainly valuable., but this study argues that being able to discuss math problems or applications needs an equal amount of attention. Offering time to ask questions, reducing expectations, and worrying less about grades may be more enraging in the learning process. Incorporating movements in the learning experience intends to address the “math anxiousness” provoked by the fear of “being wrong”, and consequently will embolden students to speak up in class. Aware of their feelings and emotions, students recognize that unknowns naturally bring anxiousness to the picture, however, excitement and potential fulfillment are also following. This realization may help transform their attitude about making mistakes into a new opportunity to learn, and a chance to step out of their old system of knowledge. This new attitude may make students more creative when facing challenges and more excited to find out the root of the cause.

Even though there is potential in learning math from moving, not all students are relaxed enough even with the guidance of an experienced dance teacher. People hold a fear to dance in front of others. This self-consciousness is like the one math must deal with. In this study, especially those participants who define themselves as science students found it harder to let go of their limbs and focus on how they felt without noticing and wondering how others may view them. Several semesters of observing and adjusting the method have pinpointed that group project activities in which students are invited to work in pairs help build a tighter community, especially when students from opposite poles of self-identification work together. This setting allows for group improvement in a safe space of sharing, where students feel free to point out things to work on and are able to empathize from a perspective one may once not ignore. Likewise, the feelings that come from solving math problems, or the fear of performing movements in front of others do not vanish through a few courses in the dance studio. This methodology is meant to provide a different access point to the sciences and surprise students with many possibilities of moving to explore any subject.

And there is no doubt that the combination of math and movement has to build an inclusive environment for finding those possibilities.

Math and movement share more than nervousness, of course. Dancing and math require effort. Both subjects appreciate repetitions, practicing, and making minor changes after reflections. Practicing repetitively builds confidence in expressing movements or solving familiar problems. While most students, stepping into college, forgot how they felt rewarded when they learned addition and multiplications; Similarly, some basic dance steps and warmups may remind them how basic matters and how making a minor progression is worth noticing. And vice versa, most math concepts, breaking down from the top, may be able to break some stereotypes of rocket science and motivate the students to explore more, to feel the excitement of learning something again. Movement and math also share the keyword of "pattern" in the range of undergraduate studying. Dance demonstrates patterns in a professional dancing piece or just pedestrian movements. Movement analysis searches for patterns to decode the choreographic work and uses patterns to teach basic movement concepts: releasing weight, creating order using a form, creating a line with energy while relating to space, and expressing rhythm and dynamics. Similarly, math formulae are meant to summarize the patterns using mathematical languages and shed light on the unchanging in the stream of a fast-changing world.

In summary, intuitively, math and movement are two very different worlds, but they share the underlying ideas which the study built on and endeavored to promote.

This study is an attempt to connect more roads to the same destination. Often, students only know a set way of approaching a problem or even memorizing instead of learning. Showing the connections between movement and math can help students find more ways to the same solution while inspiring them to understand the logic behind the questions instead of simply wanting to ace a test or finish a homework assignment.

College-level math concepts are deeply rooted in one another. It is believable that students will find out how they need prior knowledge as the reliable bedrock for later learning to be natural. Soon, learning to connect each move in the dance studio, explaining them step by step, and waiting for the “Aha” moments to emerge through repetitions and varied perspectives will be a natural process for participants in math and movement study.

To clarify, this study is not intended to find a creative way to substitute math classes. The designed movement activities are meant to support students while dealing with abstract concepts.

When engaging in movement, group work is essential to the class's success; it is not often the case in traditional math lectures. Of course, on one hand, there are solo dancers, and on the other hand, some math results are possible through the effort of groups of people across the globe. However, at a very basic level, we do not expect two students in Calculus to cooperate during exams to solve an integral, and there is no final project in the usual dance course to create their moves in five minutes. These very natural and basic structures of the two subjects differ and even contrast, but that is an exciting aspect of mathematics through movement: the subjects complement each other and help us be acquainted with an interdisciplinary perspective.

Admittedly, there is rarely a shared vocabulary bank for movements. Though groups of dancers who have been working on a certain genre of dancing may recognize “battement” as a straight leg kick, it still needs more explanations in where the leg is directed, what kind of effort to apply, and what the chosen speed is when approaching the kick. While Laban Movement analysis aids in the process of decoding, students are encouraged to find more ways of describing the concepts to their peers. This procedure centered the students and encouraged more active thinking such that students not only need to show and tell what their movements mean but they also are required to understand the movement concepts fully to lead the conversation and answer the questions from the instructor as well as their peers.

With all traits shared by the two fields and the complementary differences between the two subjects from above, the combination? aims to add a new aspect to math studying. With multiple groups, the fall study was able to test and optimize through several semesters of teaching. More detailed designs of the courses, the aims of each activity, and a possible future for the study will be expanded on in the next chapters.

3 STEAM – When STEM Meets the Arts

Many resources are available on the connections between math and dance. However, papers are devoted to PreK – 12 activities and case studies. In higher education, the math-required courses sometimes stop students' interest in STEM-related subjects. However, as the world is growingly focusing on technology and electronic-related evolutions, this study believes interdisciplinary approaches align well with the college education goal of raising responsible, farsighted talents alongside compassion. Technology, rather than using it, the harder part appears to understand and use it well. Compassion is needed as the basis for understanding technology and is needed for an even bigger crowd. Therefore, the research process of Math and Movement inspired the idea of STEAM: Adding an "Art" perspective to the traditional STEM point of view, to encourage students to grasp onto concepts through an artistic approach – movement, and to invite participants to sympathize with other perspectives.

The inclusion of movement has its foundation in the study of neuroscience. Students were exposed to more approaches to problem-solving and managed their anxiety better with the help of "breath control, mindful physical exertion" [1]. Guiding the students to search for better ways to approach math problems, the study observed that the students found the process of math learning more accessible. The students were able to make personalized analyses of their quantitative learning habits and open their minds to the non-rigid approach. Building their system of digesting the problems, with the add-on of movement, brings in a new element that aids students' cognitive learning. [2]

The interdisciplinary model blends math and dance to build a comprehensive learning experience. Combining Diene's theory of learning mathematics and Gardner's theory of Multiple Intelligence and educational neuroscience[3], Laban's educational form of dance was incorporated to optimize the teaching/learning method. Research has shown that to enhance neurobiological systems (cognition, emotions, immune, circulatory, perceptual motor), bodily-kinesthetic arts are a necessity to be put into classrooms.

Dr.Chan and Dr.Stern [4] also started to study the effectiveness of adding movement elements to college math classrooms. They addressed that the current course deliveries are causing more and more discomforts and avoidances toward quantitative literacy requirements. Therefore, not only does a math and movement class need to get the classroom moving, but it also needs constant reflections on students' feelings, making them more conscious about the changes in the learning process, and challenging the possible walls they built up toward STEM learning or adding new facade to the system they own. They experimented with a 90-minute workshop at Weber State University and were excited to witness the concentration and the energy students were able to offer.

With Professor Lori Teague, the study prompted the group first with warm-ups in movements, her guidance that leads students into the atmosphere of a dance class ensured the choreographic activities displayed enormous potential in energizing mathematical cognitions. With introductions of materials in small pieces and conscious reflections, the courses were structured in order of dance permutations guided by careful wording of questions. The course evaluator collected qualitative and quantitative data from the class and did one-on-one interviews for assessment and impact on students' perception of mathematical concepts after a whole semester immersed in this novel experience.

Through the process of implementing and adjusting, more benefits of combining the two subjects were revealed, as mentioned in previous studies, related to cognitive learning. Stern drew "Three Pillars" from the class which bolstered the movement's role of aid in knowing math concepts [5]. Pillar One referred to the overlap of choreographic patterns and mathematical patterns; Pillar Two acknowledged the artistic process through physical explorations is a paradigm for mathematical inquiry; Pillar Three emphasized body awareness builds self-awareness and being aware of oneself assisted learners to develop their learning strategies. In conclusion, Stern reassured the tradition of fostering creativity within dance education and looked ahead to its capability of eliciting the ability of problem-solving, which fits the value of math learning in courses like Calculus and Linear Algebra.

The idea of STEAM continued to be the center of the directed study's intention and influenced the design of the course: what are the main concepts to include? As basic requirements for graduation, major or minor prerequisites, and helpful bedrock to follow modern technology developments, the courses of Calculus I, Calculus II, Linear Algebra, and Differential Equations were reviewed and taught in three sections. Laban/Bartenieff's method of movement analysis was slowly infiltrated during every warmup to expand students' ways of moving and expressing the concepts, orienting them to observe with accessibility to more varied word banks.[1]

[1] Dance Preliminaries: Laban/Bartenieff movement analysis is the original work of Rudolf Laban. It is a method and language for describing, visualizing, interpreting, and documenting human movement. Labanotation is a notation system that aids the documentation and description of movements. Among the system of explaining and notating the movements using Laban's theory and Labanotation, the directed study especially focused on the five organizing themes of Body, Effort, Shape, Space, and Flow to help students to navigate ways of moving and collaborating.[6]

4 Mathematics through Movement

4.1 Embodied Learning at Emory University

Before guiding the directed research, multiple experiences related to Math & Movement at Emory and beyond have solidified the connection between the two subjects and made some explorations in teaching the subject in this way.

Mathematics through Movement began as a collaborative experiment between math professor Manuela Manetta and dance professor Lori Teague in the Spring of 2019. They taught a one-credit Sidecar course called *Dancing Dynamical Systems*. Their methodologies continued to develop within a Directed Study called *Mathematics through Movement* during the Fall of 2019, thanks to the Funds for Innovative Teaching (FIT grant) awarded by the Center for Faculty Development and Excellence at Emory University. This directed study was conducted online during the Spring of 2021 and moved back to in-person in the Fall of 2021. In the Spring semester of 2022, professors co-taught a Freshman Seminar (MATH 190 or DANC 190) called *Move your Math*. Additionally, in the same semester, they were part of the LINC initiative (Learning Through Inclusive Collaboration. Funded by the Howard Hughes Medical Institute), combining Dance Literacy (DANC 240) and Partial Differential Equations (MATH 351).

Below is a summary of the author's experiences with courses and initiatives before the individual project.

Fall 2021 has brought the Math through Movement experience back to in-person. Meeting once a week allowed the group of students to practice basic Laban's movement analysis theories in a dance studio and sufficiently cooperate. Together, the group familiarized themselves with effort combination phrases and was able to embody the concepts and guided revisits to the logic behind the theories. Some later course activities were inspired by individual presentations and group discussions from week to week.

4.1.1 Freshman Seminar: Move your Math – MATH/DANC 190

First-year students at Emory are required to take the so-called freshman seminar. Seminars are offered based on the instructors' interests or research. Move your Math (MATH/DANC 190) was offered in Spring 2022.

Working to observe and give background information in math, the role of a teaching assistant views the Math & Movement experience from a new angle. The group of participants, compared to the previous group, was made up of all freshman students. Furthermore, after a semester, the group was shown to be more polarized in the interest between math and movement, that is, most students are found to be very intrigued in one area while rather rusty in the other. Admittedly, the learning experience was influenced by the reason that engaged this certain group. The Freshman Seminar Move your Math met twice a week, as a requirement for Emory freshman students, the group that has not experienced much at Emory needed more time in math concepts and had more barriers in ideating their group projects. However, numerous takeaways from a semester with first-year students and the related observations allow for ameliorating anxieties toward math studying.

Reflections were made mostly on the teaching process rather than on class materials. Additional engaging reflections in each class could be included by asking questions about how participants felt, and more interactions and comments on each other's work may accelerate the willingness to cooperate. While most topics are developed starting from a short math lecture, students are also encouraged to connect the warm-up routine to what they have learned before college and develop more student-centered discussions from there.

4.1.2 Learning through Inclusive Collaboration

LINC, Learning Through Inclusive Collaboration, connected a required course for applied math majors, Partial Differential Equation (MATH 351), and a required course for dance, Dance Literacy majors (DANC 240). The collaboration between the two courses invited students from a high-level math course to step onto a performance stage and welcomed the dancers to sit in a math lecture but collaboratively design the studying process using movements. The group was polarized between the two subjects, but because of how the experience acted as an add-on to each course in its traditional form, students were able to put in more. The differences between Freshman Seminar and LINC experience guided the study to notice the diverse effect this combination has on higher-level math courses and how that engaged the students' effort in explaining the concepts with moves. Many students from the math course admitted it was their first class in a dance room while dancers admitted their first visit to the Math and Science Building. The curiosity and the feeling of the unknown boosted the efficiency of combining the two subjects, and the wave topic discussed in both classes was able to check understanding while guiding participants to think out of the box. Exchanging the traditional environment for each course, certain invisible walls in students' minds were also broken or even brought down, and they were stimulated to think differently in the area they thought they were familiar with.

4.1.3 The National Dance Education Organization Conference

The National Dance Education Organization held its conference in Atlanta in October 2022. A poster presentation positioned the idea of Math & Movement in the area of dance education from a student perspective for the first time. In the process of guiding the directed research study in the same semester, the experience of bringing the study beyond the campus environment has pushed the presenter to come to a temporary conclusion, summarize the pros and cons of the studies so far, and project the future that builds from the study.

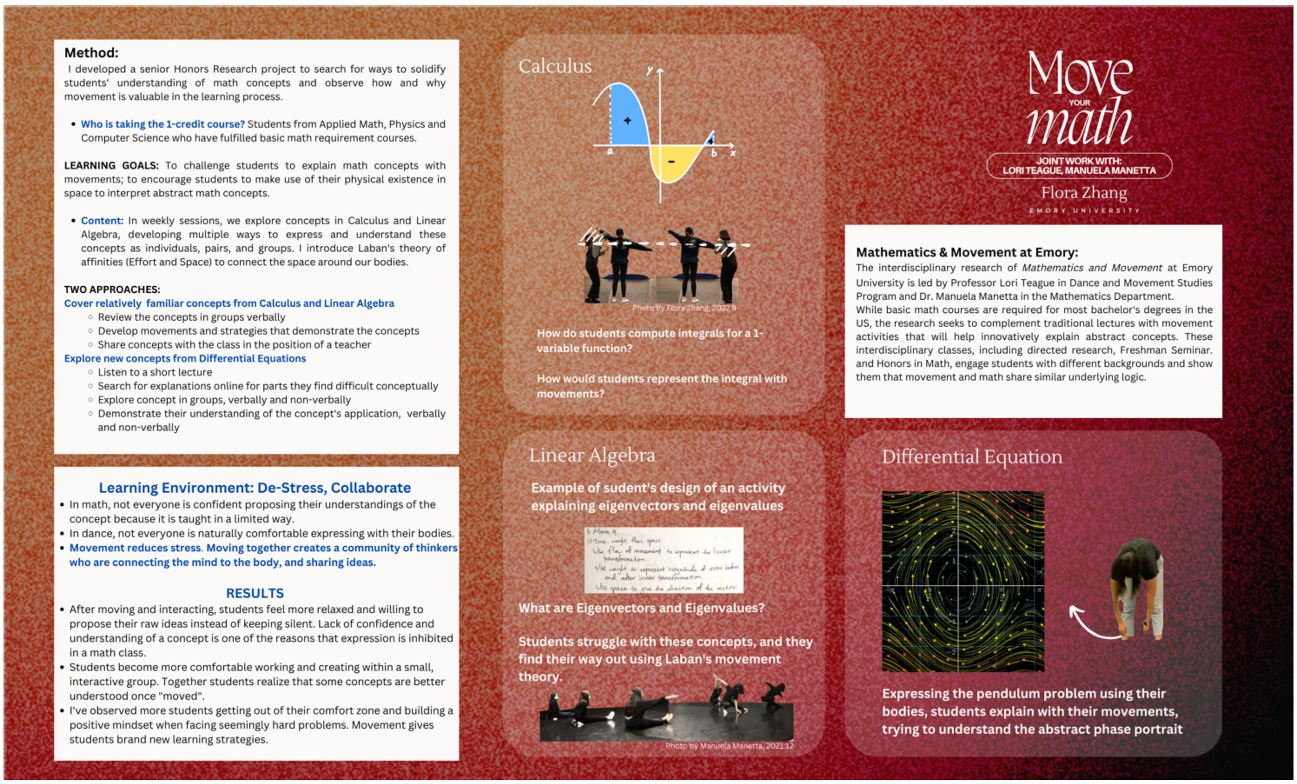


Figure 1: *Poster presentation material developed from the experience at NDEO*

4.1.4 Directed Study Spring 2023

A more diverse group that includes students from all years and majors was built. While the framework of math concepts remains, more diverse approaches were applied, and one of the classes from the individual research was revisited. The goal was to clarify what allowed more engagement in the activity and consequent discussion, despite the topic (differential equation for a pendulum) was not the most intuitive among those proposed. Starting with fewer people having experience with differential equations, this retake aroused great discussions and students were observed explaining to each other the behavior of the physical system using movements without specific requirements. The latter half of the discussion was inspired by questioning how the movement of swaying the upper body related to the fixed point and the moving process on a pendulum and how to understand the phase graph of the angular speed vs angle plane. Students completely took over the discussion and were separated into three groups, each examining the problem from the point of view they felt curious about.

Revisiting the “pendulum activity” was a success: it was student-centered and encouraged a deep understanding of abstract graphs. Specific course activities will be mentioned in the next chapter.

5 Move your Math: Directed Research

Figure 2: *Poster calling for participants in the student-led directed research*

5.1 Course Activities

The following tables summarize the structure of the course.

Session I: Calculus		
Topics	Reveal of concepts: Function, dependent/independent variables, linear/non-linear relationship.	Review of last week's work and continue to review concepts: second derivative, concavity. Reflection: What do we want to do with the help of movements? Think about meanings and what took the longest to understand.
Activities	Move in a group to express: a single variable, a linear function with multiple variables, and a non-linear function. Express limits within one's own body and then in a group, collect ways to embody the same concept. Think about the core-distal and cross-lateral relationship in one's body, how limbs come near, and how bodies fold and expand. Imagine then exercise on moving the first and second derivatives— a guide to thinking about meaning, premises, explanations, and applications.	Warm up with boxing techniques to build rapport among students Refine the trigonometry activity from the previous session Derivatives and Integrals—what was hard, and how do the two relate and differ?

Session II: Linear Algebra		
Topics	Concepts review: vectors and vector spaces, matrix---review verbally and physically now that students adjust to "moving the concepts" Concepts reviews: basis.	Eigenvectors and eigenvalues: mathematics part was explained, including calculations (the process of calculation helped this group of students specifically, but it may not help everyone).
Activities		This class is a simulation of a "normal" learning experience: lecture, material

	Embody the concepts and refine the movements to include more aspects of the concepts. In group activities, question and explore the role of each member. Guide the participants to think not only in making a shape, but also express the concept in motion.	watching, homework, with movement activities imprinted. Invite the students to be more active and involved and engage in the class by re-experiencing the learning process of an "old" concept while reflecting on what was difficult for them back then.

Session III: Differential Equations		
Topics	Review of concepts: verbally, and recall the previous sessions (derivative, rate of change, vectors, directions) and move the concepts to warm up the Math & Movement part of the brain.	Find the characters: what are the three characteristics you want others to describe you (potentially finding the best "ability").
Activities	Warm up for a rolling, head-tail relationship. Body-swaying problem (pendulum problem): Find the knowns, set up the problem, and explore practical solutions. Two groups each experimenting with different angle theta and different length of string l—find the fixed points, and stability. Think about the relationship between the swaying movement and the more abstract graph of angle and angular velocity.	Warm up as a group for holding hands and standing up or kneeling. Activity 1 with two groups, two islands: one shape "normal" one pointy, results as pointy one with 1 out, "normal" one in peace. Activity 2: allow changes in characters, go into bigger groups: 1st round without competing hints, people tend to be peaceful; 2nd round grouped, more competitive, and a winner group (2:3 left) within 3 minutes of limited time; 3rd round again competitive and allow strategies, again one group won (1:3). Draw the population time graph and understand the population A vs population B graph.

5.2 Student Engagement

Students' gradual adaptation to "moving" some abstract concepts reveals as harder materials are involved. When challenged with "try to interpret this concept with movement", not only are students more active and ready with more thoughts, but also more willing to share their ideas in groups.

As the "instructor", the design of the course was able to revolute from a stubborn structure of "learn from the board and then on the dance floor" to more "learn by dancing on the floor".

Admittedly, the first section on calculus was not as informative on math concepts. For the group of students is made up of junior and senior students in math-related majors and minors (Applied Math, Physics, CS). However, it was able to build some relationships among the group of students and get the students, which include no dance majors, more used to moving around in the space of the dance studio.

Figure 3: *Prof. Teague guiding students through warm-ups about balance in pairs, the pairs also switch so they work as a group on the topic of balance in math*

The second section was not as helpful in teaching harder concepts. Furthermore, the teaching structure, which was intended to reflect the normal pattern of learning, did not distribute the in-class time in the best way. Following the process of listening to lectures, finding materials online, finishing homework, and explaining in groups, the intention was not reflected the most through the material of Linear Algebra. Being basic material, the group of students was not particularly drawn to lecturing and video watching. The intention of student-centered learning was not able to be realized eventually.

In the third section, math materials were more challenging for the group, and some of the students were seeing the concepts for the first time. As they felt challenged, as most students admitted, it was more natural for them to focus on understanding the unknown concepts. Their willingness to acquire knowledge actively engaged them in activities that could help them explain the topic to their peers. The resulting collaboration was an unquestionable success. At this stage of the semester, students were more familiar with each other, and the team activities witnessed more interactions and less reliance on the instructor. This sparked the students to reflect on their past learning experiences and mentioned more related concepts, making them excited in discovering how the new concept related to the past while searching for patterns within.

While combining concepts in dance literacy, students are challenged to explain the movements as well. Movement courses, unfortunately not a basic requirement mount to mathematics from K-12, students do not usually share a vocabulary bank in generalizing various movements. However, this allows them to articulate and think through movements step by step and reflect on their intentions with each part of the movement actively. By questioning how certain moves relate to the concepts and ways to be more explicit in movements, the students are encouraged to think from a learner's perspective and empathize with others with diverse backgrounds which helps them to see the concepts from more angles, therefore receiving a better understanding.

5.3 Successful Activities… or Not!

5.3.1 Linear Algebra - Session 2

In the second lesson on Linear Algebra, the course design aimed to simulate the learning process of listening to lectures, working on practice problems, finding extra materials, and finally finishing homework and adding the process of moving the math concepts before writing the exercises and solving the homework problems.

The course first revisited the concepts of eigenvalues and eigenvectors as a traditional math lecture. From observing the equation of $Ax = \lambda x$, the students were asked to explain the linear transformation from their understanding without reviewing. Clarifying that a linear transformation is a mapping between two vector spaces that follows addition and scalar multiplication operations. The discussion focused on the meaning of eigenvectors and eigenvalues, the two concepts the group responded to as being "unfamiliar", "hard to understand when first learning," or "unable to grasp its whole meaning." An eigenvector of a linear transformation is a nonzero vector that changes by scalar multiplication when the transformation applies, referring to the x in the equation above. While an eigenvalue is a scalar value that is multiplied by the eigenvector when the linear transformation applies, the λ in the equation above.

Figure 4: *Students expressing vector additions as a group while discussing how to improve their movements to be more precise in what they want to interpret*

After the brief lecture, the students were organized into groups and asked to explain to each other how to prove the following two facts.

1. The eigenvalues of a triangular matrix are on its diagonal.
2. If $v_1, v_2, \dots, v_r$ are eigenvectors that correspond to distinct eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_r$ of an $n \times n$ matrix A, then the set $\{v_1, v_2, \dots, v_r\}$ is linearly independent. That is, in the set of vectors, there exist no nontrivial linear combinations of the vectors that equal the zero vector.

The process is not required to implement movements, however, one group started to describe the matrices and vectors using their body, which encouraged other groups to do the same.

Then, the class was shown a video, further explaining the two ideas using clear and colored animations. This is to mimic how students may get confused after listening to the lecture and look for materials online that endeavor to clarify the concept from a new perspective. However, the five minutes of video watching under the dim light at 7:30 pm in the middle of the week put most students to rest. Though the students were interested in using more movements as the group got familiar and the students are more comfortable orienting themselves around the dance studio, the video-watching process might need to move before class as a preview.

The lecture arrived at answering the questions of “Why would we need it?” and “How would we get it?” And the video watched aimed to address the “What is usually hard in the concepts to grasp as a student?” and “How to visualize the abstract ideas outside of the textbook?” However as mentioned above, the pure design of delivery of this part impeded the process of answering the two questions above, as a result, provide insufficient motivation for students to participate in the movement activities following.

The activities were based on students’ acknowledgments of the difficulties they had in reviewing the concepts, and the ideas inspired by the video to identify the shared barrier of understanding. Activity 1 checked the general understanding by asking the students to implement movements in explaining the concepts when first meeting them. Activity 2 asked students to move in a group to address the hardships they have in understanding the concepts.

After the class, students were asked to reflect on how they showed the concepts of “vectors”, “linear transformations”, “eigenvectors”, and “eigenvalues”. They were also invited to think about how the activities changed or deepened their understanding, and how they would like to do the movements differently than in class to make the process more helpful. The students were familiar with describing their movements using Laban language such as

> *"We used the three planes (wheel, wall, and table) to represent the three dimensions and showed vectors in R3 with our arms."*

The combination of the two subjects expanded the vocabulary one has in describing the movements and concepts and helped the students understand the abstract concepts to a new extent.

> *"We expressed the two bases of the vector space using arms and hands, and then we performed the head-to-tail vector addition to show possible linear combinations."*

And a new perspective of understanding arose

> *"I did not really see the matrix in matrix multiplication as an agent that shapes all kinds of vectors in the same way. Now it makes a lot of sense."*

However, the reflections also addressed how the participants wanted to improve the class. A lot of participants mentioned the length of the video and suggested it better be shorter or watched at a different time during class. Someone suggested adding music to the movement activities to make the atmosphere more active. Considering the difficulties of the topics, it was more advised to start by testing the understanding of students instead of lecturing again on the topic most participants were familiar with. One also mentioned redoing the activity by explaining $Ax = b$ in groups instead of pairs, such that

> *"We could let three people be A's columns, and one person be the vector x, who decides how the columns' magnitude will change."*

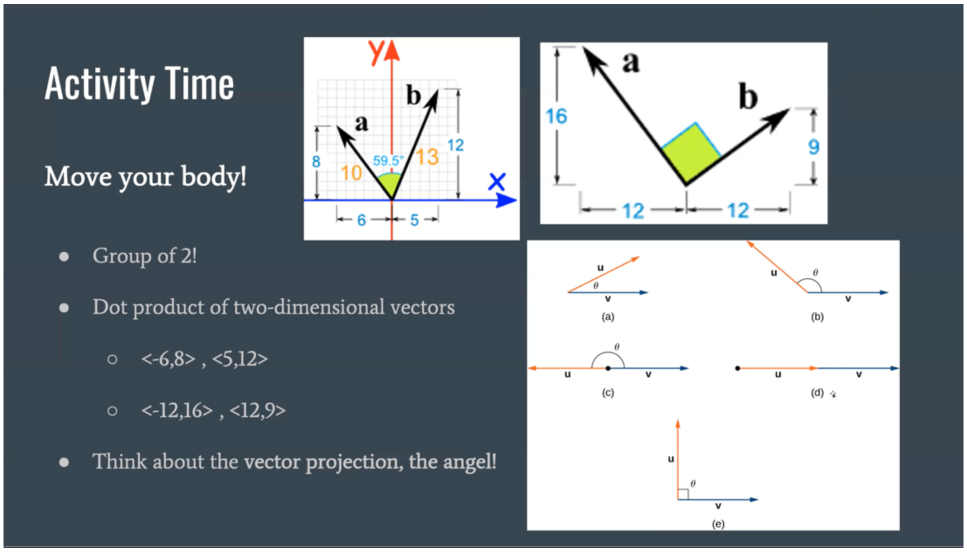


Figure 5: *Student's final presentation on the topic of vectors and projections of vectors, they are implementing more questions for their peers to reflect on their feelings*

Glad to see students initiate more activities themselves, thinking even more from a teaching perspective, though the class needed improvements in arrangements and design of activities, this meeting in the middle of the semester was a good reflecting point. Students were seen to be more adapted to the interdisciplinary thinking process, engaged in meetings with more student-centered mindsets, and actively reimagine ways to improve the learning process.

5.3.2 Differential Equation – Session 1

In the first session of differential equations, the goal was to construct the differential equation explaining the pendulum problem. That is, the equation expresses the relationship between the velocity and the position of the heavy object hanging on the non-elastic string. A graph relating the angle the string creates with the vertical line to the angular velocity, the vector explaining the time rate at which an object rotates.

The lesson was one of the most effective ones despite the topic not being as familiar to the group as the previous two sections. The students were challenged and therefore more curious about learning and understanding the concepts. They also had fewer premises due to a lack of interaction with the topic. Although the instructor needed to circumvent several parts of calculations and only briefly touched on some prerequisite knowledge in constructing the equation, the explicit motions of the pendulum manifested a clear image for students to build their interpretations. This, for the author, was the key to the success of this lesson.

Figure 6: *Students revisiting how they thought about math problems related to motions in a plane, the student running to the left is expressing the behavior if she is on a phase portrait that is first attracted to the origin, then move toward the infinite along the axis*

The class started with a warm-up movement, inviting the students to explore the head-tail relationship by rolling down and up in their bodies. The students were asked to reflect on how they felt the most relaxed when bending down from their head to their spine, eventually to bending their knees. They were also questioned about where the movement was initiated when rolling back up, and which body part exerted the most power for them to stand up straight. Starting from movements, the rolling from head to tail changed to side to side, giving students more time to explore freely how starting from a different height would change the swaying of the bodies, but all results were similar at the bottom, same as the posture when rolling down to the bottom.

After warming up, the participants are engaged by thinking about how to describe the relationship between the position of one's head and its velocity using a differential equation. The lecture on constructing the equation was not to get the exact equation to solve but intended to realize what variables to include and how the differential relationship among them allows us to result in an equation to approximate the movement. The construction also catches the participants on the same page, relating to previous topics on Calculus and Linear Algebra, and introduce the motivation of differential equation, quickly giving the example of how the area helped mathematician in modeling changes in the real world.

The group was then separated into two, each exploring how starting from different heights changes the swaying process, as well as where is the "most stable point" to stay such that forces exerted on the body would not change one's posture. In groups of four, it was obvious to both groups that the lowest point the body can reach by bending over is the most stable -- in the sense that any "small" force exerted on the body would slightly change the position to eventually go back to the "minimum". Another stationary point was standing straight up; however, students discussed and realize the position was easier to be changed comparing to the previous point (unstable equilibrium).

The instructor then introduced the concept of fixed points and asked students to identify the fixed points through the "body-swaying" movement. Finding this question rather intuitive to answer, students are then asked to understand the following graph on the relationship between angle theta and angular speed omega.

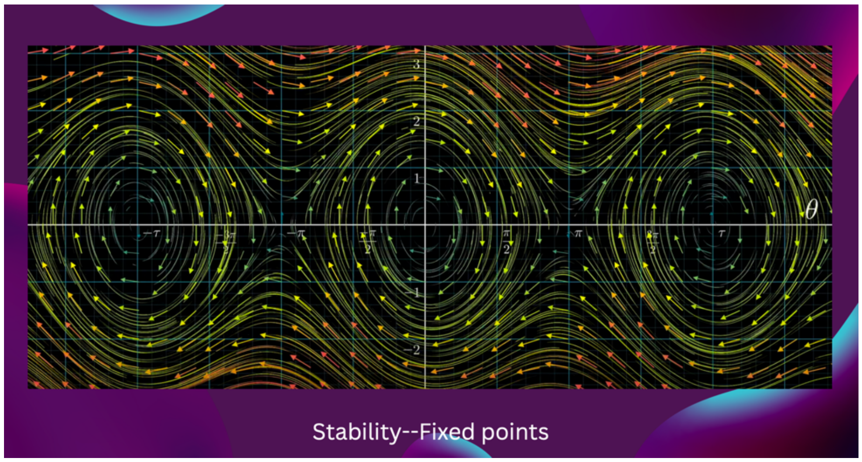


Figure 7: *Graph on pendulum problem usually abstract for students to understand in lectures, students were encouraged to start from any point on the graph and explain how their movements were equivalent to that in the graph*

In a differential equation class, understanding the abstract graph above is usually time-consuming and difficult to clarify using visual aids and verbal explanations. However, in the meeting on this topic, students were intrigued by understanding the graph and were seen to discuss each part, raise questions, and answer them within groups.

Starting with the question of identifying where the two fixed points are on the graph, students added their questions to explain what the positive and negative axis meant, and how the circular motion suggested by the arrows on the graph could be embodied.

This session successfully reached most of the lesson goals, and participants had the most impressive experience when evaluating their entire semester. When asked the reason for using differential equations, participants responded

> *"Because we don't always know the exact expression concerning some variables and a dependent variable, but we know some relations between them. Thus, we can set up differential equations based on the given information to calculate the exact expression."*

Building to the final explanation of the graph, participants also commented on how including movements helped them look at the equations that they already learned to solve numerically, differently.

> *"Here it helps me to visualize the function, for example, some specific patterns. Also, we could know how controlling one independent variable will influence the whole event."*

Considering the success of the meeting, the instructor revisits the meeting with the group of students in Spring 2023. Students' background is more heterogeneous, and only a few students have already taken courses in differential equations. They were asked not to construct the equations but to focus on finding the stable and unstable fixed points, and explaining different pathways shown in the same graph. The group was quickly engaged and progressed to explaining the graphs to each other. Some gathered in front of the screen, observing different paths, and finding which ones refer to starting at fixed points; some connect their movements and the graph by drawing on the board, breaking the graph on the screen down step by step, reconstructing a portrait of the movement on their own; some explained to each other only using movements, asking questions like "What would happen if I start from a higher point?" to help their peers understand. The students put themselves into the position of a learner and a teacher, learning through explaining, denying their premises, and interpreting from a new perspective.

The two meetings and the activities were considered successful as they combined math and movement naturally and achieved the goal of student-centered pedagogy by proposing questions and inviting students to work on explanations in group activities.

6 Conclusions and Future Work

We witnessed how students become more adapted to explaining scientific concepts using movements, working in a group easily, and embedding their characteristic moves or gestures without realizing it, especially when trying to convey their ideas to others.

As mentioned in Motivation, this work did not intend to find a way to substitute traditional lectures, but to incorporate embodiment into the mathematical learning process, as a “lab”, and invite students to think out of the box. There are various ways researchers are trying to contribute to innovative teaching techniques, based especially on the role the instructor should have in class (i.e., flipped classroom/ active learning), but rarely are their discussions about the well-roundness of the educators, that is, show that a mathematician is also a mover. As all movements can be described using equations, we have hope that mathematical reasoning can be supported by body movements. Overall, the students involved were impressed by the interdisciplinarity of this project and seemed to agree that novel approaches can change attitudes toward mathematics.

Different from most students who are doing honor thesis, I assume, I did not start college knowing my passion is on this side. Neither did I decide to show my talent in this area through honor research. However, I was able to feel welcomed and intrigued in this world of math with the help of passionate professors and cooperative peers.

The author wants to encourage females, people with an artistic mind, people who were denied the possibility to access mathematical reasoning because of negative prior experiences, and those who peeked but feel intimidated to challenge themselves again.

This study has got a chance to briefly be introduced to dance educators (as mentioned), and it will be exciting to announce a presentation to a larger crowd after more practices with more groups of diverse components will have taken place.

References

[1] R. Laban and L. Ullmann, “The mastery of movement.,” 1971.

[2] P. Evangelopoulou, *A case study on Maths Dance: The impact of integrating dance and movement in maths teaching and learning in preschool and primary school settings*. diva-portal.org, 2014.

[3] M. Z. Altan, “Intelligence Reframed: Multiple Intelligences for the 21st Century.: Howard Gardner.,” pp. 204–205, 2001.

[4] E. Stern and J. Chan, “Putting Your Best Foot Forward: Movement and Mathematics in College.,” *Proceedings of Bridges 2016: Mathematics, Music, Art, Architecture, Education, Culture*, p. 641, 2016.

[5] E. Stern and R. Bachman, “Pattern play: the case for dance in college mathematics and beyond,” *Journal of Dance Education*, vol. 21, no. 3, pp. 158–167, Jul. 2021, doi: 10.1080/15290824.2021.1939357.

[6] C. Wahl, *Laban/bartenieff movement studies: contemporary applications*. Human Kinetics, 2019.

Appendix:

Questionnaires:

1. Remember we approach Math and Movement in two ways: Start with the math concept and try to express it in movement or start with the training and then go to the math behind it. Which one do you think works better for you?
2. What is your expectation every time before coming to the class?
3. What is the usual reason you are confused in class?
4. Has there been an Aha moment during the class?
5. How do you think the handouts help? Do they get you to move the math?
6. Do you think the idea of Math and Movement will help you in the future in math and movement learning?
7. Do you think the two disciplines are helping each other? Why? Why not?
8. During the final project presentation, what do you consider the most difficult/unnatural?
9. What did you notice about yourself as a learner?
10. What did you notice about yourself as a collaborator?

Actions: Write about your experiences last semester and this semester

Fall 2021:

- First impression: I could not think of anything else movements can do except make shapes
- Quickly into warm-up

- How everyone shared generously and was willing to build on others' ideas, everyone else opened their minds and hearts to new ideas. We are all trying to make it work
- Effective group work (which I especially appreciate during the freshman seminar)
- Personally, when I am learning something new, I would think of how to make connections to movement so that it is clearer to show, and so that I may make sure my knowledge of the concept was solid and thorough.
- We may sometimes lean too much to the dance side, which has more obvious connections with physics than math
- It may be on the free ride of teaching concepts to someone else
- When entering the freshman seminar, I realized how the process of connecting two disciplines is not natural, I was used to it. But this gave me the chance of observing how the process grew naturally for others
- It can be hard to practice group work to meet expectations since the students' math understanding may differ. However, some aha moments appear during the process of group work which helps us as observers make sure they were understanding and helped

Assessment questions for Freshman Seminar:

- **Past evaluation**
- Remember we approach Math and Movement in two ways: Start with the math concept and try to express it in movement or start with the training and then go to the math behind it. Which one do you think works better for you?
- What is your expectation every time before coming to the class?
- What is the usual reason you are confused in class?

- Has there been an Aha moment during the class?
- How do you think the handouts help? Are they helpful? Do you get to move?
- How do you feel when moving with other students in class?
- **Future**
- Do you think the idea of Math and Movement will help you in the future in math learning?
- Do you think the idea of Math and Movement will help you in the future in movement learning?
- Do you think the two disciplines are helping each other? Why? Why not?
- **Final Project**
- During the final project presentation, what do you consider the most difficult/unnatural?

Math Preliminaries

- The course design was structured into three sections, each closely checking and practicing the explanations of math concepts. The concepts were excerpted from three main courses related to math and science majors: Calculus, Linear Algebra, and Differential Equations. Among them, Calculus is usually the requirement of finishing college degrees; Linear Algebra is often a prerequisite course for higher-level sciences; and Differential Equation classes usually set the foundation for higher-level math and science courses from undergraduate to beyond.
- While the student group of this leading research course was familiar with the first two sections, only one-third of them have taken courses related to Differential Equations, and half of them claimed to be not as familiar with the concepts. This is not assuming the

students were able to recall all concepts from the top of their heads even with the first two sections, over half of the students have raised their hands admitting they did not understand certain topics during the first time learning them at least once.

- Therefore, following math preliminaries are offered to show materials given in class, and help identify the hardships students had in understanding.

- ## Calculus

- Function: a function from one set to another assigns each element from one to exactly one element in another.
- Independent and Dependent variables: Independent variables are the controlled inputs of functions while the dependent variables are outputs of functions that are usually being measured.
- Linear Function: a linear function has a graph as a straight line, in form of y=ax + b.
- Limit: a limit is the value of a dependent variable, the output of a function, as the independent variable approaches a value.
- Derivative: a derivative measures the rate of change of a function concerning its independent variable.
- Concavity: a function is convex if the line segment of any two points on the graph is above the function; a function is concave if it is not convex.
- Integration: an integral is the reverse calculation of a derivative. It is the collected sum of a continuous function or piece of a function, usually used to calculate areas and volumes.

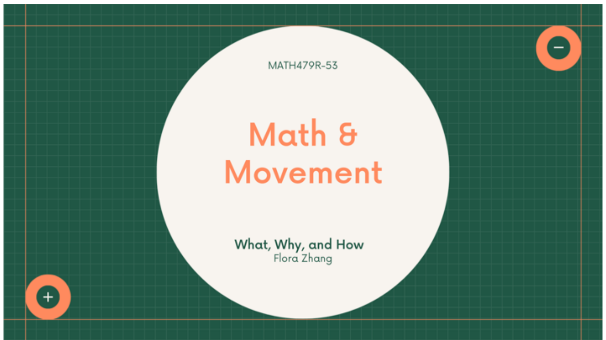
- ## Linear Algebra

- Vector: a quantity that has both magnitude and direction.
- Scalar: a quantity with only magnitude.

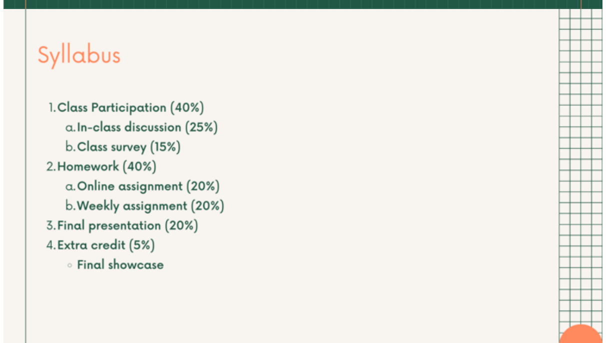
- Vector Space: a vector or linear space is a set of vectors that can be added and multiplied by scalars.
- Matrix: a rectangular table of numbers arranged in rows and columns, representing a mathematical object made of several vectors.
- Basis: a set of vectors in a vector space that can be written as a unique finite linear combination of elements from the set.
- Linear Transformation: a linear transformation is a mapping between two vector spaces that follows addition and scalar multiplication operations.
- Eigenvector: an eigenvector of a linear transformation is a nonzero vector that changes by scalar multiplication when the transformation applies.
- Eigenvalue: an eigenvalue is a scalar value that is multiplied by the eigenvector when the linear transformation applies.

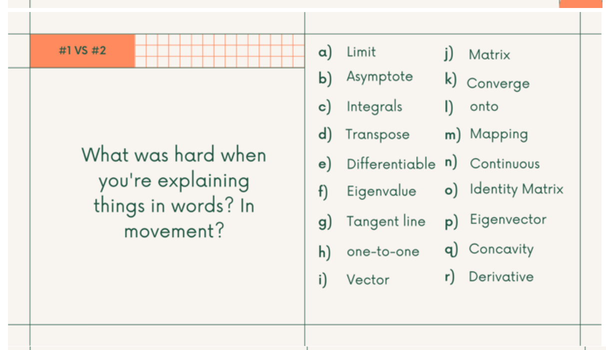
- ## Differential Equations

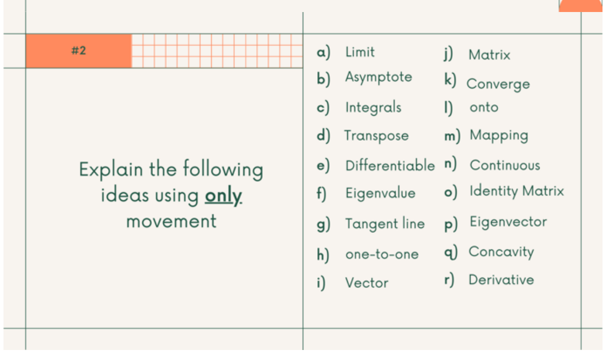
- Differential Equation: a differential equation contains single or multiple terms including the derivative of one dependent variable concerning other variables.
- Predator and Prey model: a predator and prey model observes the change in population between two groups that each represent predator and prey. The result usually shows an increase in one population cause a decrease in the other and vice versa because they are interdependent.

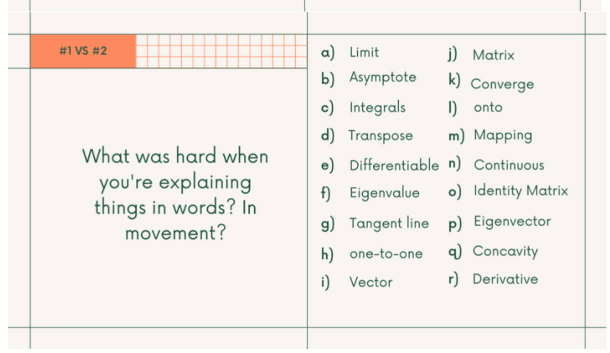

MATH479R-53
Math &
Movement
What, Why, and How
Flora Zhang

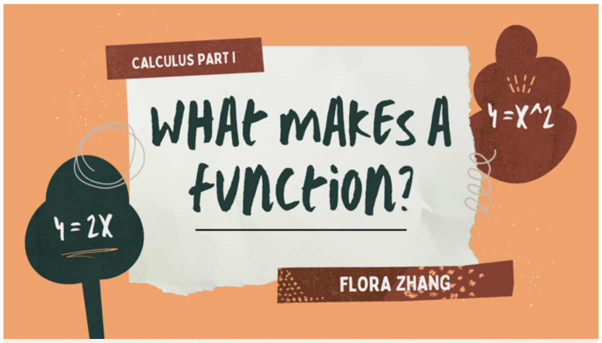

Syllabus
Flora Zhang lzha382@emory.edu
Course schedule: 18:00~19:15 Tuesday, Rich Studio
(two continuous week followed by an online assignment week)
Office Hour: upon request with TA via email
With the aims listed, you are not required to have strong backgrounds in either math or movement to be fully engaged in the course.
While moving, please note that videos will be taken to do further analysis.

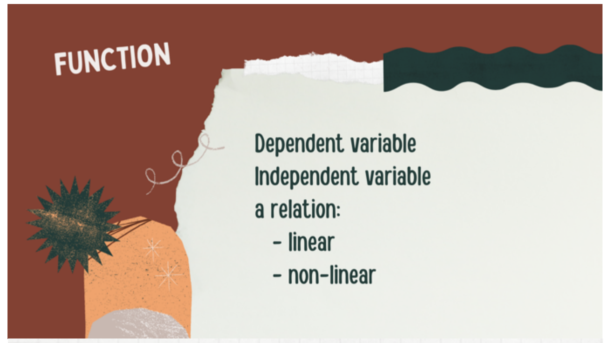

Syllabus
1. Class Participation (40%)
a. In-class discussion (25%)
b. Class survey (15%)
2. Homework (40%)
a. Online assignment (20%)
b. Weekly assignment (20%)
3. Final presentation (20%)
4. Extra credit (5%)
Final showcase

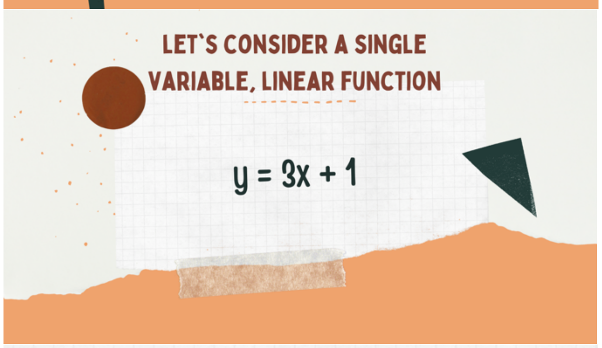

#1 VS #2
What was hard when you're explaining things in words? In movement?
a) Limit
b) Asymptote
c) Integrals
d) Transpose
e) Differentiable
f) Eigenvalue
g) Tangent line
h) one-to-one
i) Vector
j) Matrix
k) Converge
l) onto
m) Mapping
n) Continuous
o) Identity Matrix
p) Eigenvector
q) Concavity
r) Derivative

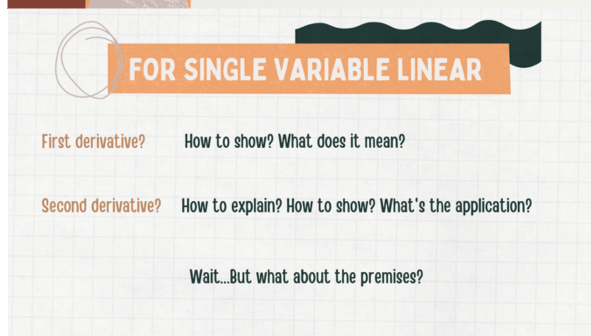

#2
Explain the following ideas using only movement
a) Limit
b) Asymptote
c) Integrals
d) Transpose
e) Differentiable
f) Eigenvalue
g) Tangent line
h) one-to-one
i) Vector
j) Matrix
k) Converge
l) onto
m) Mapping
n) Continuous
o) Identity Matrix
p) Eigenvector
q) Concavity
r) Derivative

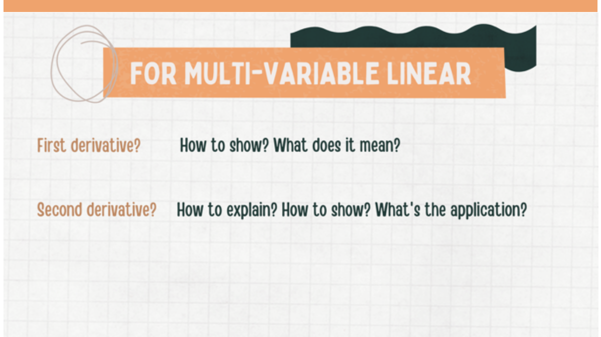

#1 VS #2
What was hard when you're explaining things in words? In movement?
a) Limit
b) Asymptote
c) Integrals
d) Transpose
e) Differentiable
f) Eigenvalue
g) Tangent line
h) one-to-one
i) Vector
j) Matrix
k) Converge
l) onto
m) Mapping
n) Continuous
o) Identity Matrix
p) Eigenvector
q) Concavity
r) Derivative

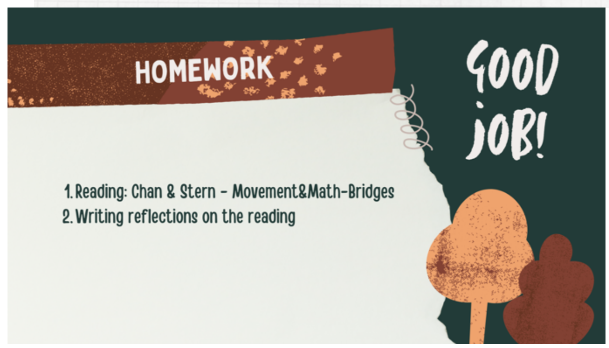

CALCULUS PART I
WHAT MAKES A FUNCTION?
FLORA ZHANG
y = 2X
y = X^2

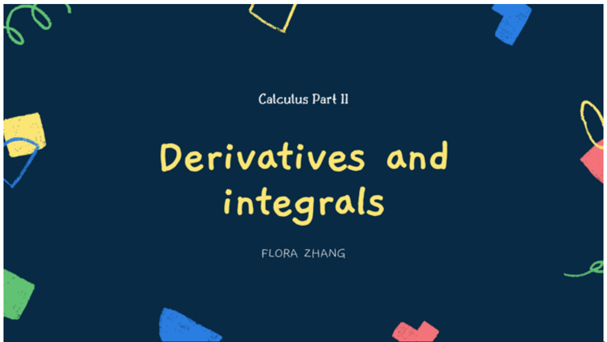

FUNCTION
Dependent variable
Independent variable
a relation:
- linear
- non-linear

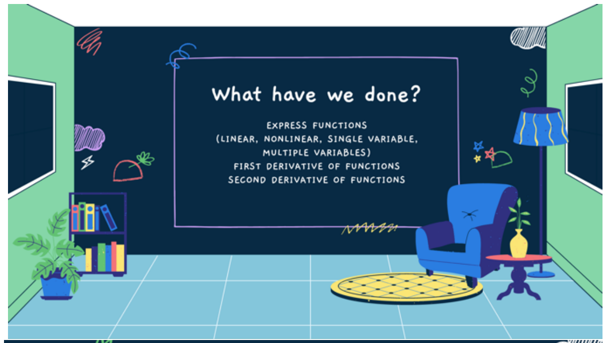

LET'S CONSIDER A SINGLE VARIABLE, LINEAR FUNCTION
y = 3x + 1


FOR SINGLE VARIABLE LINEAR
First derivative?
How to show? What does it mean?
Second derivative?
How to explain? How to show? What's the application?
Wait...But what about the premises?

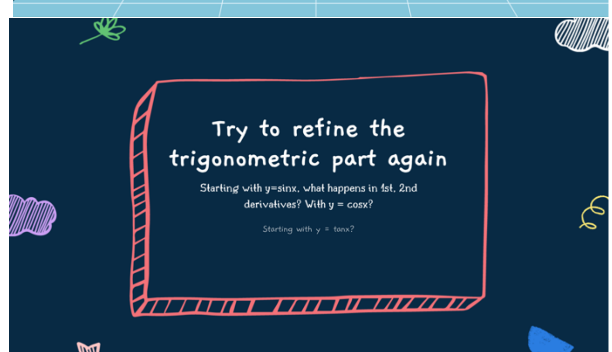

FOR MULTI-VARIABLE LINEAR
First derivative?
How to show? What does it mean?
Second derivative?
How to explain? How to show? What's the application?

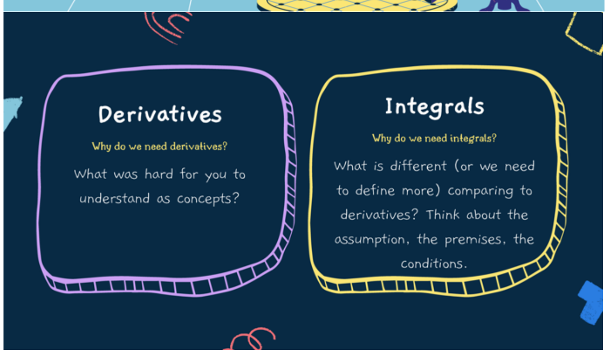

HOMEWORK
1. Reading: Chan & Stern - Movement&Math-Bridges
2. Writing reflections on the reading
GOOD JOB!


Calculus Part II
Derivatives and integrals
FLORA ZHANG

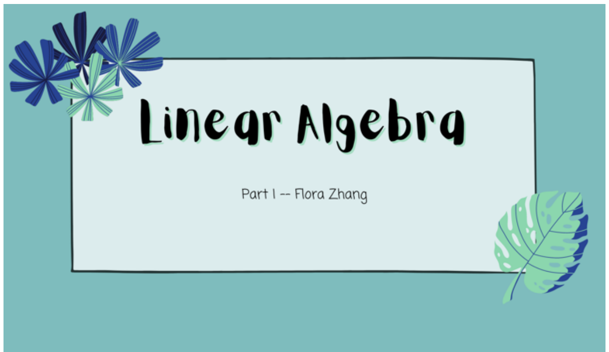

What have we done?
EXPRESS FUNCTIONS
(LINEAR, NONLINEAR, SINGLE VARIABLE, MULTIPLE VARIABLES)
FIRST DERIVATIVE OF FUNCTIONS
SECOND DERIVATIVE OF FUNCTIONS

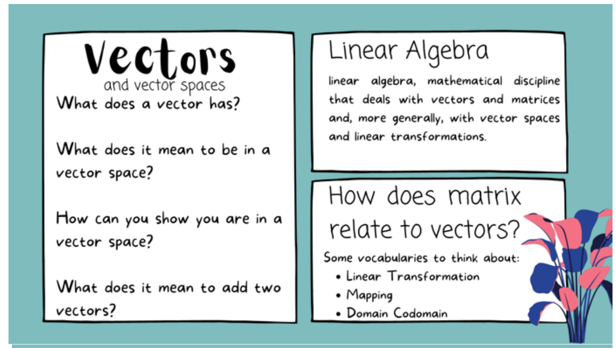

What do we want to do with the help of movement?
THINK ABOUT MEANINGS AND WHAT TOOK YOU THE LONGEST TO UNDERSTAND.

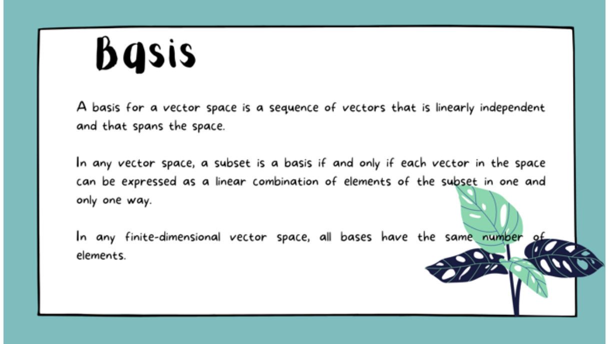

Try to refine the trigonometric part again
Starting with y=sinx, what happens in 1st, 2nd derivatives? With y = cosx?
Starting with y = tanx?

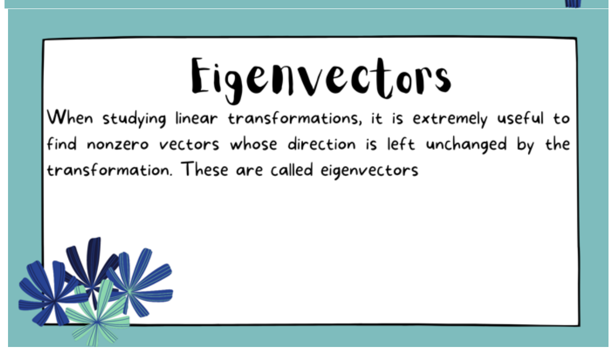

Derivatives
Why do we need derivatives?
What was hard for you to understand as concepts?
Integrals
Why do we need integrals?
What is different (or we need to define more) comparing to derivatives? Think about the assumption, the premises, the conditions.

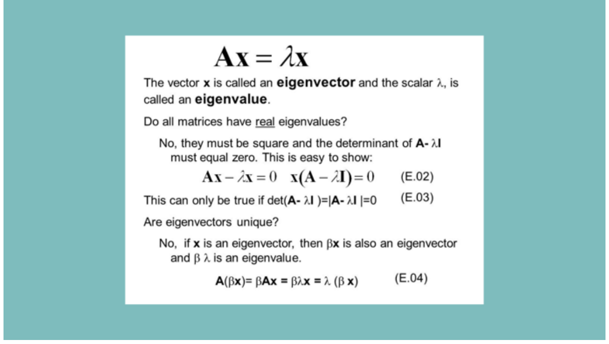

Reflecting on what we've done, how do they connect with the things we read/watched?
End of section 1
END OF SECTION REFLECTION
1 page double space
CHOOSE A CONCEPT IN CALCULUS, DESIGN A SET OF MOVEMENT IN THE ROLE OF TEACHING
Partial derivative, limits,
(Find something you considered hard)
1 page double space
VIDEO WATCHING
https://math.gatech.edu/news/linear-algebra-musical-interview-aren-russ

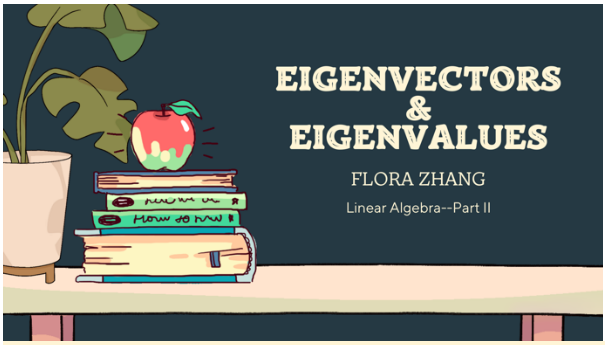

Linear Algebra
Part I -- Flora Zhang

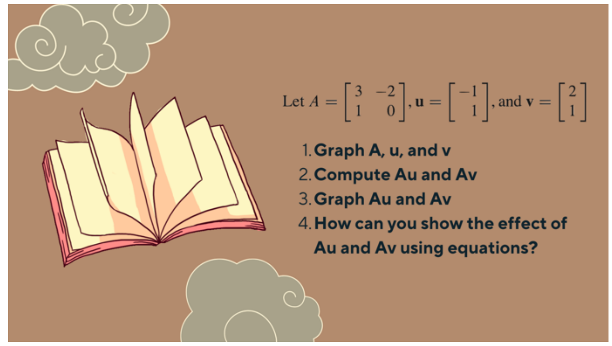

Vectors
and vector spaces
What does a vector has?
What does it mean to be in a vector space?
How can you show you are in a vector space?
What does it mean to add two vectors?
Linear Algebra
linear algebra, mathematical discipline that deals with vectors and matrices and, more generally, with vector spaces and linear transformations.
How does matrix relate to vectors?
Some vocabularies to think about:
• Linear Transformation
• Mapping
• Domain Codomain

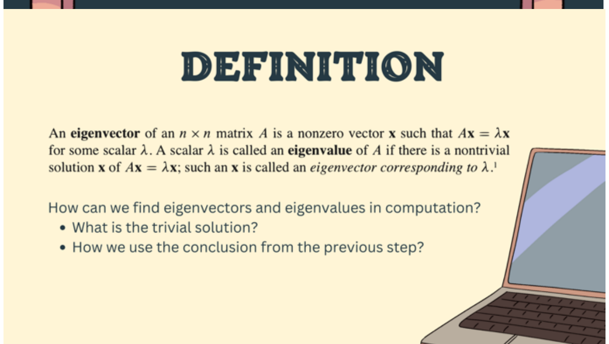

Basis
A basis for a vector space is a sequence of vectors that is linearly independent and that spans the space.
In any vector space, a subset is a basis if and only if each vector in the space can be expressed as a linear combination of elements of the subset in one and only one way.
In any finite-dimensional vector space, all bases have the same number of elements.

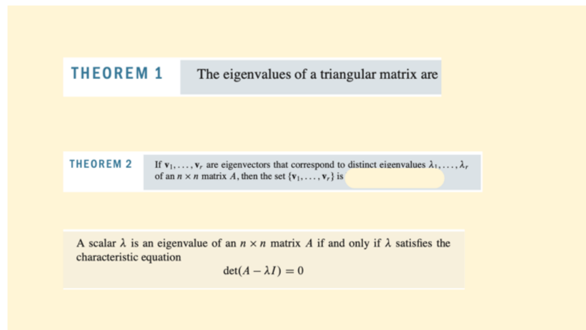

Eigenvectors
When studying linear transformations, it is extremely useful to find nonzero vectors whose direction is left unchanged by the transformation. These are called eigenvectors

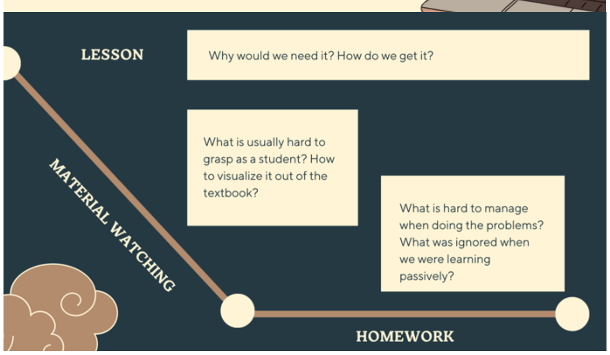

$\mathbf{Ax} = \lambda\mathbf{x}$
The vector **x** is called an **eigenvector** and the scalar λ, is called an **eigenvalue**.
Do all matrices have real eigenvalues?
No, they must be square and the determinant of **A**- λ**I** must equal zero. This is easy to show:
$\mathbf{Ax} - \lambda\mathbf{x} = 0 \quad \mathbf{x}(\mathbf{A} - \lambda\mathbf{I}) = 0$ (E.02)
This can only be true if det(**A**- λ**I**)=|**A**- λ**I** |=0 (E.03)
Are eigenvectors unique?
No, if **x** is an eigenvector, then β**x** is also an eigenvector and β λ is an eigenvalue.
A(β**x**)= β**Ax** = βλ**x** = λ (β **x**) (E.04)

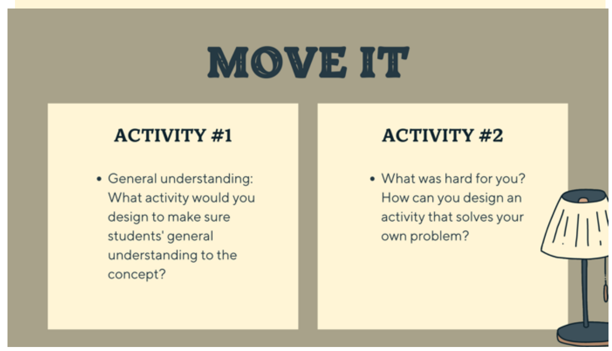

EIGENVECTORS
&
EIGENVALUES
FLORA ZHANG
Linear Algebra--Part II

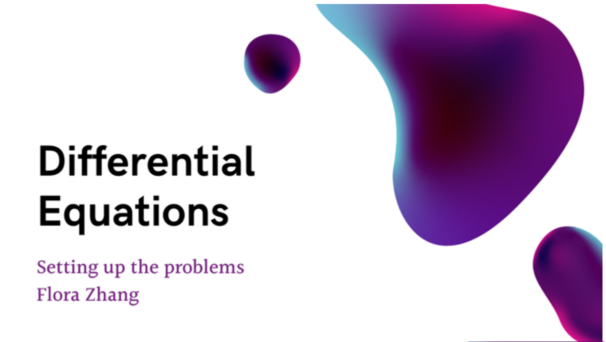

Let $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, and $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$
1. Graph A, u, and v
2. Compute Au and Av
3. Graph Au and Av
4. How can you show the effect of Au and Av using equations?

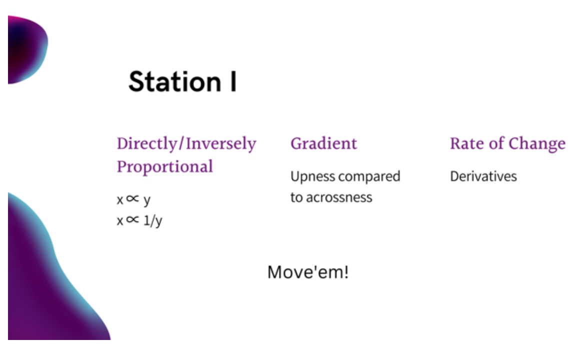

DEFINITION
An **eigenvector** of an $n \times n$ matrix A is a nonzero vector $\mathbf{x}$ such that $A\mathbf{x} = \lambda\mathbf{x}$ for some scalar λ. A scalar λ is called an **eigenvalue** of A if there is a nontrivial solution $\mathbf{x}$ of $A\mathbf{x} = \lambda\mathbf{x}$; such an $\mathbf{x}$ is called an *eigenvector corresponding to* λ.[1]
How can we find eigenvectors and eigenvalues in computation?
• What is the trivial solution?
• How we use the conclusion from the previous step?

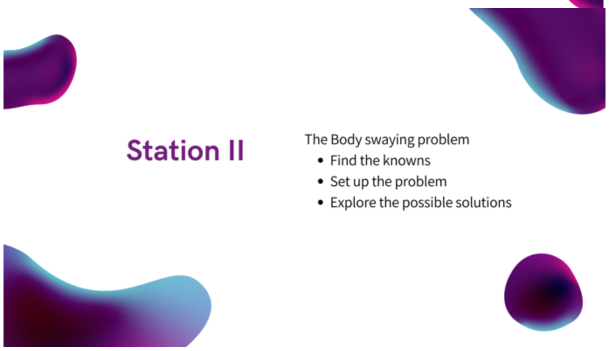

THEOREM 1
The eigenvalues of a triangular matrix are
THEOREM 2
If $\mathbf{v}_1, \ldots, \mathbf{v}_r$ are eigenvectors that correspond to distinct eigenvalues $\lambda_1, \ldots, \lambda_r$ of an $n \times n$ matrix A, then the set $\{\mathbf{v}_1, \ldots, \mathbf{v}_r\}$ is
A scalar λ is an eigenvalue of an $n \times n$ matrix A if and only if λ satisfies the characteristic equation
$\det(A - \lambda I) = 0$

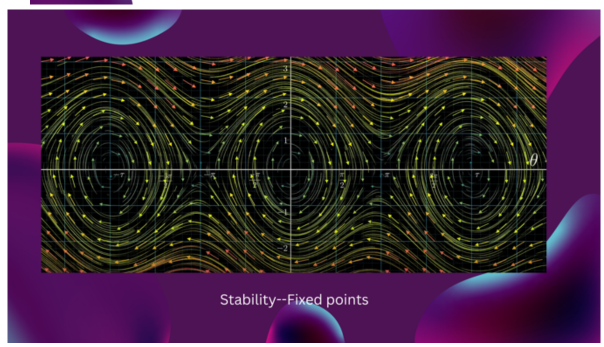

LESSON
Why would we need it? How do we get it?
What is usually hard to grasp as a student? How to visualize it out of the textbook?
MATERIAL WATCHING
What is hard to manage when doing the problems? What was ignored when we were learning passively?
HOMEWORK

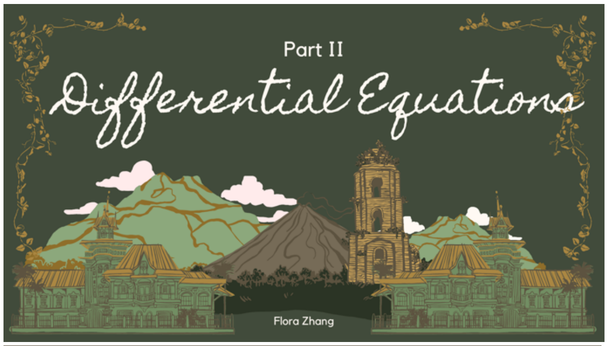

MOVE IT
ACTIVITY #1
• General understanding: What activity would you design to make sure students' general understanding to the concept?
ACTIVITY #2
• What was hard for you? How can you design an activity that solves your own problem?

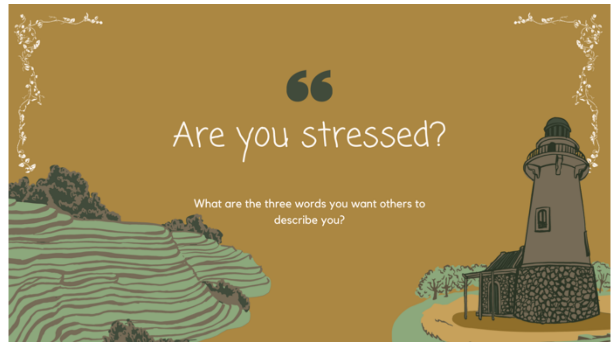

Differential Equations
Setting up the problems
Flora Zhang


Station I
Directly/Inversely Proportional
x ∝ y
x ∝ 1/y
Gradient
Upness compared to acrossness
Rate of Change
Derivatives
Move'em!


Station II
The Body swaying problem
• Find the knowns
• Set up the problem
• Explore the possible solutions

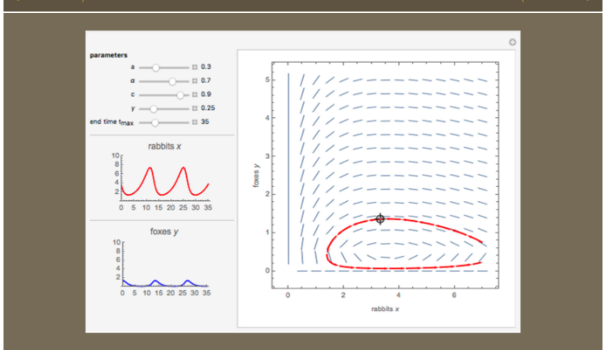

Stability--Fixed points

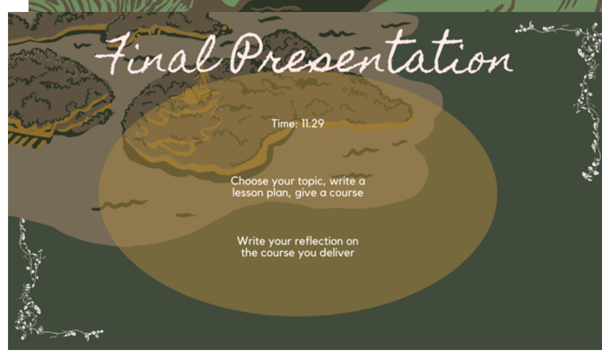

Part II
Differential Equations
Flora Zhang


Are you stressed?
What are the three words you want others to describe you?


Share
• List your three characteristics
• Express them using movement
• Choose your favorite


WAIT...WHAT HAPPENED?
What happened in smaller group?
In bigger group?
Recall the graphs we have from last session, draw it.
Reflect on our "experiment today, how does the graph look in terms of "population"?


parameters
rabbits x
foxes y


Final Presentation
Time: 11.29
Choose your topic, write a lesson plan, give a course
Write your reflection on the course you deliver