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Frenemies: Coalitions and Conflict in the Dictator's Inner Circle

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Abstract

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Maintaining the small coalition of powerful individuals necessary to support her regime is not without risks for a dictator. One of the fundamental tensions of authoritarian governance is how a dictator interacts with these elites: while giving them power, riches, and access is fundamental to maintaining their loyalty, these resources can easily be turned against you. Given the inherent threat that her elite allies pose, how can a dictator consolidate her own power? Can a dictator undermine the strength of the elites within her own ruling coalition? Across the papers of this dissertation, I explore the potential strategies that a ruler can undertake to strengthen her own position relative to the elites in her ruling coalition.

First, I consider purges: a leader's forcible removal of a high-ranking party, government, or military member from office. When faced with elite rivals with varying levels of power, which member of her coalition will the dictator target for a purge? I formalize the dictator's attempt to seize power from a regime insider with a dynamic and novel adaptation of a classical flexible contest framework. The conclusions I derive show that elite coalitions can lead to more purge attempts instead of deterring them and that both powerful and relatively weak dictators will initiate conflict with their inner circle of elites, an important implication for empirical studies of purges and the personalization of power in dictatorships. I then expand the dictator's strategy set to include simultaneous power sharing and purges. In my theoretical model, a dictator faces a multi-stage process of conflict and consolidation, eliminating elites from his ruling coalition to win more power for himself. The dictator can offer rewards to other members of the elite in return for their support, but empowering a rival directly affects the dictator's ability to purge said rival in the future. Lastly, I investigate the use of the masses as a tool to undermine the strength of local elites. With a multi-stage Rubinstein alternating offer bargaining program, I show how a central ruler can use investment in distribution to the masses to reduce the bargaining power of local elites while still using them to ensure the support of the district. I compare multiple institutional arrangements, showing that unit proliferation only benefits the dictator if she does not need unanimous support and, instead, can credibly threaten to exclude some local elites from power-sharing. These varying institutions also yield different implications for where the dictator will invest in his popularity to strengthen his position vis-a-vis the local elite.

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Chapter 1

Introduction

> The wish to acquire more is admittedly a very natural and common thing; and when men succeed in this they are always praised rather than condemned.
>
> *Niccolò Machiavelli, The Prince*

Rulers do not come to power alone, nor can they maintain a regime without a network of allies that staff government offices, protect the regime from external threats, and maintain the power of the regime (Bueno De Mesquita et al. 2003; Geddes, Wright, and Frantz 2014; Myerson 2008; Svolik 2009). While the coalition of supporters that bring a leader to power and maintain her rule vary across types of regimes,[1] the threat that these "allies" pose to a leader's power is common to all: if support is withdrawn or members of the coalition remove the ruler from her position, she cannot maintain power (Bueno De Mesquita et al. 2003). This concern is most acute in authoritarian regimes[2] where the institutions that

[1] In democratic regimes, supporters may include voters and party members; in autocratic regimes, supporters include the military, regime party, economic elites, and other powerful persons in the state

[2] I use autocratic, authoritarian, and dictatorships interchangeably. I follow the procedural definition of

regularize interactions between supporters and the ruler do not adequately constrain the executive (Meng 2019) and a threat of violence underlies every conflict (Svolik 2012). The elites that make up the ruling coalition of an authoritarian regime may vary: whether the "launching organization" is the military or a civilian party organization (Geddes, Wright, and Frantz 2014), establishing a regime takes many powerful individuals working together both to control the mechanisms of the state and to exclude potential challengers.

Maintaining a small coalition of powerful individuals to support her regime is not without risks for the dictator. Over 60% of authoritarian rulers who left office by non-constitutional means were removed by a coup d'etat in which government insiders–members of the military, security forces, or government–forcibly removed the dictator from office (Svolik 2009). One of the fundamental tensions of authoritarian governance is how a dictator interacts with these elites: while giving them power, riches, and access is fundamental to maintaining their loyalty, these resources can easily be turned against you (Svolik 2012, 2013). Recent studies of authoritarian politics have built on previous considerations of the power-sharing problem (Gandhi 2008; Magaloni 2008; Myerson 2008) and coup risk (Little 2017; Singh 2014) by recognizing the relationship between them (Meng 2019; Svolik 2012). Given the inherent threat that her elite allies pose, how can a dictator consolidate her own power? Can a dictator undermine the strength of the elites within her own ruling coalition?

Across the papers of this dissertation, I explore the potential strategies that a ruler can undertake to strengthen her own position relative to the elites in her ruling coalition. First, I consider purges: a leader's forcible removal of a high-ranking party, government, or military member from office.[3] I then expand the dictator's strategy set to include simultaneous power sharing and purges. Specifically, I explore how a dictator will share power with an individual

dictatorship as regimes the fail to elect their executive or legislature in free, competitive elections. The relevant aspect of authoritarianism is that "elites cannot rely on an independent authority to enforce their agreements, and violence is the ultimate arbiter of political conflicts" (Svolik 2012, 20).

[3] This could also include the arrest, imprisonment, or execution of these individuals

member of her regime in order to get his support against a colleague, forming a temporary coalition in order to purge an elite despite the risks that sharing power entail. Lastly, I investigate the use of the masses as a tool to undermine the strength of local elites. When local elites serve as an intermediary between the dictator and her citizens, she can undermine their strength by investing directly in her popularity among the masses. These strategies are costly: important members of the government, military, or party do not want to be sidelined and can penalize the dictator for such attempts. Thus I give particular attention to conditions under which the risks and costs that undermining elites pose to the dictator are overwhelming and induce her to maintain a status quo.

Furthermore, I incorporate heterogeneity into the ruling coalition. While many models of the interactions between a dictator and her elite allies utilize unitary actor assumptions (Meng 2019; Svolik 2009) or homogeneous atoms (Little 2017), I build on recent work specifically recognizing the heterogeneity of ruling elites in terms of power, type, and information (Acemoglu, Egorov, and Sonin 2008; Kosterina 2017; Luo and Rozenas 2019). Uniformly undermining all the elites in her ruling coalition at once could be disastrously destabilizing for the regime (Luo and Rozenas 2019) not only could it foment a widely supported countercoup (Boix and Svolik 2013; Svolik 2012) but could remove all competent personnel from their positions of governance (Egorov and Sonin 2011; Woldense 2018). While a mass single purge event is not unprecedented (see Saddam Hussein's mass arrests in the Revolutionary Command Council and Ba'ath Party on July 22, 1979 (Coughlin 2005)), the historical record suggests that most dictators consolidating power move against one or a couple elites at a time (Keller and Wang; Khlevniuk 2008; Teiwes 1993). If a dictator seeks to undermine her elite allies but can only attack one at a time to minimize the instability that conflict in the inner circle begets, who should she target? If she is forming a temporary coalition with other members of the elite against her target, who should she ally with? Introducing elite heterogeneity into my models of elite-dictator interaction allow this dissertation to speak to these questions that have yet to be fully developed in the existing literature.

The consequences of these machinations are consequential not only for the dictator's tenure but for all who live under dictatorial regimes. Dictators who are successful in sidelining the powerful elites around them–who are the only de facto constraints they face (Meng 2019)–are the least likely to democratize and the most likely to face exile, imprisonment, or death if they are removed from power (relative to a natural death or surviving out of office) (Geddes, Wright, and Frantz 2014). Furthermore personalist dictators are more likely to initiate international conflicts (and lose them) (Weeks 2014). The makeup of the ruling coalition after the dictator has undermined, expelled, and eliminated her way to consolidated power has policy and personnel consequences. Who stands by the side of the ruler at the top of the regime affects who staffs the bureaucracy, security services, and military (Carter and Hassan 2021; Hassan 2017; Woldense 2018) as well as how public goods are distributed across the state (Kramon and Posner 2016). As successive rulers are often selected from the cabinet, politburo, junta, royal family, or whatever form the inner circle takes, the identities of those who survive the dictator's process of consolidating power are important for the future of the regime (Castaneda 2000; Fontana 1987; Zhou 2020).

1.1 Overview of the Dissertation

My first paper, "Who to Target? Strategic Elite Elimination in the Contest for Power," addresses the dictator's use of purges to consolidate power for herself. A dictator's attempt to purge members of her inner circle leads to uncertain conflict with her elites that could end in the dictators' ouster. When faced with elite rivals with varying levels of power, which member of her coalition will the dictator target for a purge? This choice is especially complex when the benefit that the dictator receives from purging a powerful member of her coalition is inversely related to the probability that she succeeds in the purge. My results show that when the dictator is weak relative to the elites and the difference between the two elites that she faces is low, the dictator targets the weaker elite. However, as the difference between the elites increases, the weaker elite is a less lucrative target and the dictator targets the stronger elite. If the dictator is weak relative to the uncertainty of the potential conflict, she will not initiate conflict and instead maintain her status quo power instead of initiating a risky conflict. I show that both strong and weak dictators will initiate purges under different conditions of uncertainty: a weak dictator will attempt to eliminate rivals under uncertain conflict conditions while a strong dictator will avoid conflict and protect her advantage under similar conditions. The implications of this result for empirical research on purges and the personalization of power are critical: without accounting for the uncertainty of the ensuing dictator-elite conflict, observed purges cannot be used as an indicator of dictatorial strength, a common practice in empirical literature.

By not assuming that elites will necessarily coalesce against a power-hungry dictator and, instead, are subject to a similar risk-reward tradeoff when considering the uncertainty of attempting to oust the dictator, I show the conditions under which the formation of an elite coalition leads to more consolidation conflict than there would otherwise be, rather than deterring the dictator from violating the power-sharing status quo. This is because defeating multiple elites at once offers the dictator even greater power than eliminating one. This model further contributes to formal theoretical work on contests: my novel adaptation of the framework allows the contest participants to be chosen endogenously and the benefits from winning the contest to be inversely related to the likelihood of winning.

In my second paper, "Sowing Seeds of Destruction? Empowering Elite Rivals under Contested Dictatorship," I explore the incentives of both the dictator and elites to form coalitions under the shadow of intra-regime conflict. Why would an elite join a coalition with a dictator who is likely to target him for elimination in the future? Why would a dictator empower a rival she plans on attempting to eliminate in the future? In order to recruit elites to support her in purging another elite, the dictator must offer riches, positions of power, and policy control to an elite ally in exchange for joining an uncertain conflict. Giving these rewards directly affects the dictator's likelihood of defeating this elite in a future conflict. I show that the dictator will ally with her most powerful rival, giving him even more power in exchange for the elite's support, in order to succeed in her initial, uncertain purge attempt. The elite will accept the empowerment and support the dictator in the initial conflict even when he anticipates being targeted next because the benefits he receives will help him withstand the future purge. Unlike existing explanations of dictator-elite coalition and conflict behavior that emphasize the shortsightedness or low-information of the actors, by introducing a dynamic conflict environment in which the dictator has multiple elites in her path to consolidation as well as uncertainty over conflict outcomes, I show that even with complete information about the relative power of herself and her rivals, the dictator and her rival will ally temporarily against another elite despite the anticipation of future conflict.

My third paper, "Local Elites Drive a Hard Bargain: Strategic Investment in Mass Distribution under Electoral Autocracy," expands beyond the inner circle to a dictator's relations with local elites. Because of their advantage in generating support for the dictator, local elites can extract high concessions from her in exchange for their delivery of their districts' support. I utilize a multi-stage Rubinstein alternating offer bargaining program in which the dictator will invest more in mass politics to make herself popular among voters to counteract the elites' strong local position in individual districts. This study contributes to the formal bargaining literature by exploring the diminishing returns to outside option investment across institutional contexts: while investment in the electoral outside option increases the dictator's bargaining position, it simultaneously decreases the surplus that utilizing a local intermediary provides. By comparing multiple institutional arrangements (how many districts the dictator needs to win), I show that unit proliferation–the creation of more districts and therefore more bargaining partners–only benefits the dictator if she does not need unanimous support and, instead, can credibly threaten to exclude some local elites from power-sharing. These varying institutions also yield different implications for where the dictator will invest in her popularity to strengthen her position vis-a-vis the local elite. For example, when the dictator needs the support of two of two districts, she will invest in the district in which she is less popular; when she only needs the support of one of two districts, she will invest in the district in which she is more popular.

Each of these papers contributes to not only the literature on authoritarian elite politics, but advances formal theoretical models of contests in which the relevant choice is opponent rather than effort as well as bargaining across a variety of institutions with an endogenous outside option. Beyond the academic literature, the coalitions and conflicts that occur at the top of an authoritarian regime have life and death consequences for not only the members of the ruling coalition but the millions of citizens that live there. A dictator's successful use of these strategies to undermine, expel, and eliminate her elite "allies" will yield an unconstrained ruler with undisputed discretion over policy and personnel (Gandhi and Sumner 2020; Geddes, Wright, and Frantz 2018). These personalist authoritarian regimes have been steadily increasing in number since the end of World War II, rivaling party-ruled regimes as the most common type of dictatorships (Geddes, Wright, and Frantz 2014). By carefully examining the processes by which a dictator can consolidate such unprecedented power, this dissertation furthers our understanding of an ever increasing number of regimes and the individuals that lead them.

Chapter 2

Who to Target? Strategic Elite Elimination in the Contest for Power

When a dictator moves to consolidate his own power vis-a-vis the elite allies in his inner circle, who should he target for a purge? By introducing power heterogeneity into the elites of the ruling coalition, I formalize the dictator's attempt to seize power from a regime insider with a dynamic and novel adaptation of a classical flexible contest framework. The degree to which the elites vary in their initial power endowments affects the dictator's propensity to initiate conflict as well as his preference for targeting one elite or a coalition of elites for a purge. The conclusions I derive show that elite coalitions can lead to more purge attempts instead of deterring them and that both powerful and relatively weak dictators will initiate conflict with their inner circle of elites, an important implication for empirical studies of purges and the personalization of power in dictatorships.

Personalist authoritarian regimes, in which one individual exercises discretion over policy and personnel, have been steadily increasing in number since the end of World War II, rivaling party-ruled regimes as the most common type of dictatorships (Geddes, Wright, and Frantz 2014). The dictators who head these regimes built their position of preeminence by consolidating their own power and influence relative to other contenders for leadership (Gandhi and Sumner 2020). One way that an autocratic leader can solidify his position as the dictator is through purges–forcible removals– of elites in his ruling coalition (Geddes, Wright, and Frantz 2018; Goldring 2020; Svolik 2009). While the removal of elites from the upper echelons of power could occur for a variety of reasons such as ideological disagreement, elite malfeasance, or a demonstration of personnel power (Lu and Lorentzen 2016; Montagnes and Wolton 2019; Woldense 2018), the focus of this study is consolidation purges in which the dictator removes or eliminates members of his inner circle in order to take more power for himself (Svolik 2009; Geddes, Wright, and Frantz 2018).

The elites that form the ruling coalition at the top of an authoritarian regime are distinct from one another in a variety of ways. The formal positions of power they hold as party, cabinet, junta, or politburo members entail different levels of policy influence and access to resources (Arriola, DeVaro, and Meng 2017; Meng 2019). Influence and patronage could be tied to an elite's ethnic, clan, or religious identity (Coughlin 2005; Hornsby 2013; Van Dam 1996), network of connections with other elites through education or previous positions (Lu and Lorentzen 2016), or military command (Barros 2002). When faced with a heterogeneous ruling coalition of elites, which types of elites will be purged in the dictator's movement toward consolidation? In other words, who will a power-hungry dictator target?

The makeup of the dictators ruling coalition, in particular who remains to govern after a purge, has downstream implications for both the inner circle and the regime at large. The members of the ruling coalition that continue in power after a consolidation attempt may be more or less ambitious and threatening to the dictator (Kosterina 2017) or have an unstable balance of power with the leader (Acemoglu, Egorov, and Sonin 2008; Svolik 2009). For bureaucrats, lower-level security officials, and citizens, the identity of the members of the inner circle have consequences for personnel (Carter and Hassan 2021; Hassan 2017; Woldense 2018) as well as the distribution of public goods and services (Kramon and Posner 2016). Furthermore, with ever increasing numbers of dictators consolidating power at the expense of their elite partner (Geddes, Wright, and Frantz 2014), understanding the process by which this unfolds generates important insights into regimes that are frequently closed, secretive, and the least likely to democratize (Barros 2016; Geddes, Wright, and Frantz 2014).

A ruling coalition with which to share power and govern effectively is necessary to help a regime come to power (Bueno De Mesquita et al. 2003; Haber 2006), but is the greatest threat to a dictator's tenure (Svolik 2012). Violating an agreement to share power with his elites by attempting to remove one of them in a consolidation attempt destabilizes a regime and will be met with resistance from his coalition (Svolik 2009). Thus when choosing the path of personalization and consolidation through elite eliminations, the dictator must consider the tradeoff between his desire for power and the instability that purges introduce (Luo and Rozenas 2019). The potential targets of his purge create a similar tradeoff: a more powerful, influential member of the ruling coalition would yield the dictator a high reward when removed, but attempting to purge them is risky. Violating the power-sharing agreement could cause elites to retaliate in a coup, the success of which is subject to uncertainty (Meng 2019; Svolik 2009; Singh 2014).

I model a dictator facing two elite actors with differing amounts of power in a dynamic setting. In the first round, the dictator must decide whether to initiate a purge and which elite, who vary by the amounts of power they possess, to target. If an elite avoids being targeted, he, in turn, must decide whether to form a defensive coalition against the dictator or join the dictator against his elite colleague. Conflicts between the dictator and his allies are subject to contest-style uncertainty: while having a power advantage over your opponent make you much more likely to defeat him, your success is not guaranteed. The most powerful winner of the conflict in the first round will usurp the power of the defeated side, and anyone left standing will move to a second round in which they must make similar conflict decisions. This builds on Acemoglu, Egorov, and Sonin 2008 non-democratic coalition model in which the relative power of coalition members determines who is removed from the coalition by incorporating that relative power and stochastic noise into an uncertain conflict over that removal. How the dictator resolves the risk-reward tradeoff of targeting a member of his ruling coalition for a purge yields different expectations about who he will target (or if he moves to consolidate at all) depending on both his relative power as well as the uncertainty of the ensuing conflict. The results show how the dictator's preferred target changes with his own power advantage, the difference between the possible elite targets, and the uncertainty of the conflict. When the dictator is weak relative to the elites and the difference between the elites is low, the dictator targets the weaker elite. As the difference between the elites increases, the weaker elite is a less lucrative target and the dictator targets the stronger elite. If the dictator is weak relative to the uncertainty of the potential conflict, he will not initiate conflict and instead maintain his status quo power instead of initiating a risky conflict.

In addition to the dictator's targeting decision, this model of dictatorial consolidation generates predictions about elite coalition behavior in response to a purge attempt. I build on similar models of dictator-elite conflict in which the elites overthrow the dictator to punish him for violating the power sharing agreement (Boix and Svolik 2013; Meng 2019; Svolik 2009) but do not assume that the elites, who I model as individual actors, will always coalesce against the dictator. Instead, the elites makes a similar risk-reward calculation in weighing the likelihood that they can successfully overthrow the dictator with the benefits that ousting him will yield. I show the conditions under which the formation of an elite coalition leads to more consolidation conflict than there would otherwise be, not deterring the dictator from violating the power-sharing status quo, because defeating multiple elites at once offers the dictator even greater power than eliminating one. The non-monotonic relationship between the dictator's power relative to his elites and his moves to consolidate power through elite purges has implications for the burgeoning empirical literature on personalism, consolidation, and purges in authoritarian regimes (Geddes, Wright, and Frantz 2018; Goldring 2020; Keller and Wang; Sudduth 2017b).

2.1 Power Consolidation and Conflict

Rulers come to power with the support of allies, often formally grouped into a party, council, royal family, or military junta (Gandhi 2008; Geddes, Wright, and Frantz 2018; Luo and Rozenas 2019). While some agreement to share power among the members of the ruling coalition may have been agreed upon (Bueno De Mesquita et al. 2003) or even formalized through institutions (Boix and Svolik 2013; Myerson 2008), "institutions do not eliminate the ruler's primal instinct to accumulate power" (Luo and Rozenas 2019, p. 1). Consider an authoritarian regime that has already formed: there is a leader, or dictator, and some set of elite regime members with various positions, holdings, and factional ties. While the dictator individually has more power than any single elite (which is why he is the dictator), this advantage may be very small. One of the methods by which the dictator can accumulate more power for himself is by breaking the status quo power-sharing agreement and eliminate the members of his own coalition, removing, expelling, imprisoning, exiling, or executing them and thereby taking their power for himself (Geddes, Wright, and Frantz 2018; Svolik 2009).

2.1.1 The Rewards of Elite Purges

Removing high-ranking members of the ruling coalition could offer the dictator a variety of benefits. While some purges could be conducted to remove corrupt or truly treacherous officials (Lu and Lorentzen 2016) or as a public display of arbitrary power over personnel (Woldense 2018), many elite eliminations are used to consolidate power directly from the elites to the dictator (Gandhi and Sumner 2020; Geddes, Wright, and Frantz 2018; Svolik 2009). An elite in the dictator's inner circle often holds formal positions in the government, military, and/or party and, with the executive powers over personnel and appointment that are *de jure* or *de facto* held by the leader, the dictator has control over what happens to those offices. Removing the elite can open up the position for a protégé, family member, or loyalist, increasing the dictator's influence and inspiring loyalty among the promoted. For example, in the Soviet Union, by eliminating Beria and other close associates of Stalin after the latter's death, Nikita Khruschev was able to promote his protégés Kirichenko, Brezhnev, Zhukov, and Furtseva to positions of power; they were then able to support Khruschev against a coup attempt by other members of the elite in June of 1957 (Taubman 2003).

Alternatively, the dictator can leave prominent positions open, maintaining the post-purge balance of power in the ruling coalition with *de facto* control over the domains of the open offices. Félix Houphouët-Boigny, founding president of Côte d'Ivoire, maintained vacancies in his cabinet including such prominent offices as the minister of defense and the vice presidency (Meng 2019). A dictator could even keep such titles and offices for himself, taking full control over the material resources, policy control, and prestige that the position incurs. In Mali, Moussa Traoré took on the role of general secretary of the Democratic Union of the Malian People as well as minister of defense and security in addition to his position as president. Saddam Hussein accumulated positions as the chairman of the Revolutionary Command Council, regional secretary of the Ba'ath Party, general secretary of the National

Command of the Arab Socialist Ba'ath Party, and eventually Prime Minister all in addition to being president of Iraq. Collecting governmental offices, either by installing lackeys or directly controlling them himself, puts greater control over policy, personnel, and resources in the hands of the dictator and has been used as an empirical measure of the consolidation of power (Gandhi and Sumner 2020).

How much the dictator benefits from an elite elimination depends on **who** is being eliminated: their position in the government, party, or military, the resources that once flowed to them, etc. A higher level position such as a head of party or government or minister of a major department like defense, finance, or interior, will yield the dictator much more control when he installs his loyal protégé (or himself) into that office. Targeting such a powerful member of the ruling coalition for removal, in violation of an agreement to share the power of the regime, is not without risk of potentially grave consequences.

2.1.2 The Risks of Elite Purges

The uncertainty inherent in a conflict between dictator and elites where institutions might exist but don't constrain the way they do in democracies (Gandhi 2008; Meng 2019; Svolik 2012) and violence underlies every contest (Svolik 2012) has been widely recognized, but mostly from the perspective of elites. Elites may be uncertain about the dictator's relative strength (Meng 2019), hidden actions that the dictator could be taking (Boix and Svolik 2013), what signals other elites receive (Luo and Rozenas 2019), or simply whether a coup could succeed (Little 2017; Singh 2014). The dictator, however, can be equally uncertain about whether his attempt at a purge will succeed. Even with full information about the actions and intentions of the members of the inner circle, an attempt at an elite removal could fail.

In Malaysia, for example, the dismissal and imprisonment of Deputy Prime Minister Anwar Ibrahim sparked the *Reformasi* movement of mass demonstrations against the sitting government. After several regime party politicians lost seats to opposition candidates in 1999, the Prime Minister responsible for initially ousting Anwar, Mahathir Mohamad, resigned in 2003.[1] In the Soviet Union, the formal removal of elites from their official positions required a vote from the Politburo, the members of which were not always beholden to the leader. Early in Stalin's tenure,

> members of the Politburo conducted themselves independently... forming diverse and unexpected (given subsequent events) tactical coalitions... The votes were evenly divided on whether or not Trotsky and Zinoviev should be immediately expelled from the Central Committee... On 20 June 1927, a bare majority voted to expel Trotsky and Zinoviev, but only after Stalin demanded that his vote be counted in absentia and Kalinin joined those in favor of immediate expulsion (Khlevniuk 2008, p. 3)

Even a dictator like Stalin, who is often considered a pinnacle of consolidated power and highly effective at purging his elite comrades (as well as mass party and military members), was "forced...to act cautiously...and keep an eye on the mood of his comrades-in-arms" (Khlevniuk 2008, p. 5).

Much in the way that coups are more likely to succeed if the target of the coup is relatively weak and the coupers are numerous and powerful (Little 2017; Singh 2014), I argue that the dictator is more likely to succeed in his attempt to purge a member of the ruling coalition when he (and any other elites who join him in a coalition) is more powerful than his opponent. Sergei Syrtsov, a relatively young and recently promoted member of the Central Committee, was easier to remove than the chairman of the Council of People's Commissars (and thus the head of government), Aleksei Rykov, whose replacement "dragged on for sometime... which leaves room for speculation that Stalin was wavering, weighing the advantages and

[1] Mahathir resumed power under a new party label in 2018.

disadvantages before making up his mind" (Khlevniuk 2008, p. 34). Strong targets can be strong opponents, as Burkina Faso's president Jean-Baptiste Ouedraogo found when he attempted to remove Thomas Sankara from his post as prime minister and was immediately overthrown in a coup. In addition to their relative power, the conflict between a dictator and his purge target is subject to the "fundamental and irreducible uncertainty" of conflict that results from stochastic processes (Bas and Schub 2017). The critical substantive feature is that some features of conflict between the dictator and powerful elites, whether from a reactionary mass protest, elite coup, or lack of control over personnel who carry out arrests or violence, makes the outcome of the conflict uncertain to both the dictator and his elite rivals. Thus while eliminating a member of the ruling coalition with great de jure and de facto power can offer the dictator great rewards if he succeeds in his purge, the resistance of this powerful target to removal is an important consideration. The greater the target, the greater risk the dictator will fail to remove him and instead face ouster himself.

2.1.3 The Path to Personalist Power

When he seeks more power for himself at the expense of his elite allies, who will the dictator target for a purge? How he resolves the tradeoff between the greater power that he could achieve from a bigger, stronger target and the greater uncertainty of defeating such a target determines the dictator's targeting decision. When the potential elite targets are very similar to one another, the dictator will target a weaker opponent because the greater likelihood of successfully purging him is sufficient to outweigh the lower power he can take. In general, a stronger dictator will attack a more powerful opponent, whether that is the elite with higher power or a coalition of elites who coalesce against the dictator and can all be removed. This relationship between the power advantage of the dictator and the power of his target, however, is conditioned by the uncertainty of the resulting conflict between them. In a highly uncertain environment where the dictator is *ex ante* unsure whether his purge attempt will give him the power of the eliminated elite or result in his own ouster, even an extremely powerful dictator would rather avoid a risky conflict and maintain the status quo arrangement with his elites, resisting the temptation to attempt any consolidation at the elites' expense.

How will the elites react to their leader's attempt at removing them? While I assume that the purge target will immediately launch a coup against the power-hungry dictator, I do not assume other elites will necessarily join the effort. In an elite-dictator conflict, other members of the ruling coalition have the option to join their peer against the dictator, join the dictator in ousting the elite, or stay on the sidelines of the conflict. By allowing non-targeted elites to join the conflict only when it is in their best interest to do so and not because of any previous commitment to their colleagues, I am able to further explore how the risk-reward tradeoff of conflict in the upper echelons of power affects coalition behavior from the elite's perspective. Not only will the elites not always coalesce against the dictator's consolidation attempt as some previous models assume, elite coalitions will lead to more conflict as defeating a coalition offers greater rewards despite the higher risk the dictator faces. Further, the elites' ability to form coalitions has heterogeneous effects on the elites themselves: the lower-powered elite is worse off as the higher-powered elite joins him in a coalition and alters the dictator's targeting decision. It is the anticipation of the elite's coalition behavior and the risk-reward tradeoff that conflict at the top of the regime induces that determines when the dictator consolidates power vis-a-vis elites and who he targets in the purge.

2.2 Modeling Elite-Dictator Conflict

Dictators desire to simultaneously amass more power for themselves and avoid the instability that elite-dictator conflict may bring to the regime. Elite elimination is an effective way to achieve personalist power, but there are potentially drastic consequences: a counter-coup or mass elite defection could lead to the dictator's ouster. My theory of elite-dictator conflict has two (related) stages: targeting and coalition formation. Considering the potential for his ouster or the possibility of making the regime more vulnerable to outside opposition, the dictator must first consider whether initiating any intra-regime conflict is worth the consequences. If he deems conflict beneficial, in expectation, for his personal power, the dictator must then decide who to target for elimination.

These modeling assumptions yield a few substantive scope conditions on the applicability of this model to the inner workings of autocratic regimes. First, the dictator, as the conflict "agenda-setter," must be able to target an elite for a purge. This could include introducing articles of expulsion to a legislature, ordering police to make an arrest, or prompting a paramilitary to attempt assassination. What the model does not assume is that the dictator must be such a powerful strongman that he can unilaterally remove and elite from the position in the inner circle and usurp their power. Indeed, by incorporating uncertainty over the outcome of the conflict, even a strongly advantaged dictator in my model will not be certain that his purge attempt will succeed. Second, in order to focus on the dictator's targeting of his allies, preemptive coups are outside the scope of the model. The elites only respond in a coup when the dictator violates the power-sharing agreement by initiating a purge (Meng 2019; Svolik 2009). Relatedly, there is no first-mover advantage: when the dictator and elites are in conflict, only their relative power and the conflict uncertainty determine the outcome, not an element of surprise which would make the dictator's initiation relative to a possible elite-initiated coup more relevant.[2] Lastly, I assume that there is no credible sharing of the spoils of conflict. In practice, this means that all of the benefits of removing a member of the inner circle–whether it is an elite or the dictator himself who

[2] Without a first-mover advantage, in the parameter space where the dictator does not initiate conflict but the elites may want to coup, in equilibrium the dictator would simply be indifferent between initiating conflict and allowing the elites to initiate it.

loses–flows to the most powerful member of the coalition that removed him.[3]

I formalize the heterogeneous levels of power of the dictator and two elite regime members, the uncertainty of the interested parties over the conflict outcomes, and the targeting mechanism below. The variation in targeting and elite coalition formation depends not on coordination-promoting institutions or non-credible commitments, but the self-interest of heterogeneously powerful members of the ruling coalition.

2.2.1 Model Setup

Three players, a dictator (D), an elite with a high initial endowment of power (H), and an elite with a lower initial endowment of power (L), together form a regime. Each player's type, $\tau_i > 0$, is his power endowment. I use the following notation to measure the power disparity among the players: H's initial endowment of power is fixed at H ($\tau_H \equiv H > 0$); the dictator's endowment is $d > 0$ larger than this ($\tau_D \equiv H + d$) and L's endowment is $w \in (0, H)$ less ($\tau_L \equiv H - w$). The dictator has the most power initially (which is why he is the dictator) and the high elite has more initial power than the low elite. Thus d (and $d + w$) can be interpreted as the dictator's initial power advantage over the other two elites while w denotes the relative difference between elites H and L.

Play takes place over the course of two rounds. First, the dictator chooses which elite to target for elimination or does not initiate conflict, ending the round. If he chose a target, the non-targeted elite can choose to join a coalition with the dictator, the targeted elite, or remain out of the conflict. The dictator and target(s) then participate in a contest where the probability that each side wins is the difference between their relative power plus mean-zero noise; e.g. participant (or coalition) i wins the conflict if $\tau_i \geq \tau_j + \epsilon_t$ where $\epsilon_t \sim U[-a, a]$ independent of the round.[4] Thus the probability that participant i wins is $F_\epsilon(\tau_i - \tau_j)$. While

[3] This assumption is relaxed in Cuttner 2021 in a focus on sharing power with rivals.
[4] This distribution is symmetric around 0 and the likelihood of a draw of noise is equal between the bounds. The parameter of interest regarding the uncertainty of the conflict is a: as a increases, the spread of the noise

the amount of uncertainty the potential conflict parties are subject to can vary, I assume all conflicts are uncertain; in particular, I assume that a is sufficiently large such that no individual or coalition can win a conflict with certainty.[5]

The power of the loser(s) of the conflict is transferred to the most powerful winner. Note that the winner cannot split the winnings with a coalition member because any *ex ante* commitment to split gains upon winning would be violated when the time to share power came. While both exogenous and endogenous sharing rules are commonplace in contests that are nested or allow for alliances (Konrad 2009), such agreements may have credibility problems in authoritarian regimes (Gehlbach, Sonin, and Svolik 2016; Svolik 2012). Thus I focus on the baseline case of complete lack of enforceability. This assumption is relaxed in Cuttner 2021, where the dictator and elite are able to share the spoils of coalescing against another elite.[6]

The first round ends and whichever remaining player has the most power now becomes the dictator and can choose to initiate conflict with the other remaining member of the regime (if there is a second member). If there is only one player remaining in round two, the game ends. If conflict is initiated, it occurs as previously described, with a new, independent draw of ϵ. The power of the loser(s) is transferred to the winner and the game ends.

Sequence of Play

$t = 1$

- The dictator, D chooses to target H, to target L, or not to target anyone. If no target is selected, the round ends.

distribution increases in both directions (maintaining symmetry around 0). The more advantaged party is less certain of their victory as a increases. See footnote 16 for a discussion of how distributional assumptions affect (and do not affect) the results.

[5] Technically, $a > H + d + w$.

[6] The general relationships between the dictator's power advantage, conflict uncertainty, and conflict initiation remain when sharing is an option. When conflict does occur, however, the dictator will only utilize the sequential targeting strategy instead of allowing an elite coalition to form and targeting them both.

• The non-targeted elite chooses whether to participate in the conflict on the side of the dictator, the target, or not participate.

• The dictator (or coalition) and target(s) participate in a contest where participant(s) i win if $\tau_i \geq \tau_j + \epsilon_1$ where $\epsilon_t \sim U[-a, a]$

• The loser(s)'s power is transferred to the strongest winner; whichever remaining player now has the most power is now the dictator. If only one player remains, the game ends.

$t = 2$

• The dictator chooses to target one of the remaining players, or not to target anyone. If no target is selected, the game ends.

• If there is a non-targeted elite, he chooses whether to participate in the conflict on the side of the dictator, the target, or not participate.

• The dictator (or coalition) and target(s) participate in a contest where participant(s) i win if $\tau_i \geq \tau_j + \epsilon_2$ where $\epsilon_t \sim U[-a, a]$

• The loser's power is transferred to the winner and the game ends.

Payoffs

All members of the ruling group derive utility from their endowments of power at the end of the game. These *ex post* power endowments are a function of the dictator's target choice and the coalition choices of both elites in both rounds. The actions of each player are, in turn, a function of their initial power endowments and the conflict uncertainty.

$$u_i(H, d, w, a) = \tau_{i,t=2}$$

2.3 Targeting without Coalitions

First consider a benchmark in which coalitions are not possible: the dictator simply makes his targeting decision (H, L, or no conflict) and conflict ensues accordingly, subject to the uncertainty described above. The contest function that determines each actor's probability of winning the conflict creates a tradeoff for this dictator: he is less likely to be successful in a conflict against the higher-powered elite, H, but beating him would yield higher rewards. While he is never certain of victory, a successful purge is much more likely against his lower-powered opponent, L, though the rewards are less. The parameter w defines the difference between H and L: as w approaches 0, H and L are very similar both in the rewards from ousting them and the difficulty in defeating them; as w approaches H, L is very weak and more likely to be defeated, but the rewards from the conflict are also less enticing. While all possible conflicts are subject to uncertainty[7], how much "noise" there is in the dictator's chance of winning depends on a, the dispersion of the distribution. When a is low, the dictator's power advantage strongly affects his chances of winning; as a increases, the conflict is more uncertain even when there is a large difference in power. As a approaches infinity (in the limit), the likelihood that the dictator wins a conflict goes to $\frac{1}{2}$, even if he is extremely advantaged over his opponent.

How these concerns affect the dictator's targeting decision is easily illustrated with a single round of conflict. Consider a one-shot conflict[8] in which the dictator makes a single targeting decision: he can fight H, L, or not initiate any conflict. The characterization of his optimal targeting strategy is summarized in Lemma 1 and visualized in Figure 1. When the

[7] Note that I assume the spread of the noise distribution (a) is sufficiently large to avoid deterministic conflict outcomes (assuming ϵ is distributed uniform). Using the uniform supports the tractability, but a variety of probability distributions would maintain the tradeoff between conflict uncertainty and the gains of a high-power purge. If a continuous noise distribution is highly disperse (has a high variance), the high uncertainty makes the dictator less conflict-prone if he is advantaged and more conflict-prone if he is disadvantaged. A noise distribution that is tight around zero will reduce the risk of conflict and make the dictator's expected utility of a high-powered target for a purge greater.

[8] Which is equivalent to a second round of conflict with all three players remaining in the full version

uncertainty of the conflict (a) is relatively low, the dictator will target L, the low powered elite, when L is not too weak. The benefit for defeating L is sufficiently high (with low w) that the dictator will take the higher conflict win probability and attack his weakest opponent. When L is particularly weak, however, as can be seen in the right region of Figure 1(a), the benefit from defeating L is insufficient. Instead, the dictator will take on the greater risk of fighting his high powered opponent for the greater reward.

Lemma 1. *When the elites cannot form coalitions and there is a single round of conflict, the dictator will target L for $a \in (H + d + w, 2H - w)$, target H for $a \in (2H - w, 2H + d)$, and not initiate conflict for $a > 2H + d$*

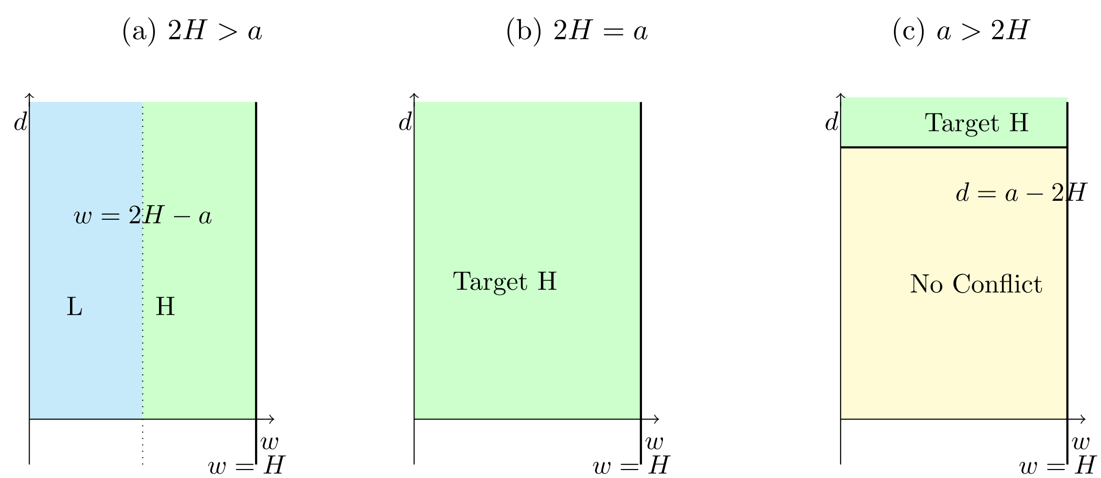


Figure 2.1: No Coalitions, Single Round of Conflict

As the possible conflicts become more uncertain, moving across the panels of Figure 1, the dictator will only target H to achieve the higher expected benefit of taking his power. When uncertainty is sufficiently high, even the prospect of taking H's power is not enough to tempt the dictator into a risky conflict. He will maintain his status quo power and not initiate any conflict, especially if his relative advantage, d, is low. As conflict uncertainty continues to grow (higher a), the point at which the dictator is advantaged enough relative to the elites that he will risk conflict gets higher.

2.3.1 Multiple Rounds of Conflict

While the above described his targeting decision in a single conflict, the dictator can use two rounds of conflict to oust the elites. In a multi-round conflict, a second round in which only two players remain is possible if there was conflict in the first round. Whoever won the first round conflict now has the most power and is the dictator; his targeting strategy when there is one opponent remaining is described in Lemma 2.

Lemma 2. *The round 2 dictator will target the remaining elite for conflict if the dictator's advantage is sufficiently high relative to the conflict uncertainty* ($d > a + w - 3H$)*, otherwise he will not initiate conflict.*

The full characterization of the dictator's two round targeting strategy takes both rounds of conflict into account: he will only receive the benefits of the full conflict if he survives both rounds, but taking one elite's power in the first round of conflict increases his power advantage in the second round. This is especially true if he fights H first: he takes H's power and uses this advantage against L, the weak opponent, defeating him with more certainty.[9] The dictator's optimal targeting strategy is described in Lemma 3 and Figure 2.

Lemma 3. *When the elites cannot form coalitions and there are two possible rounds of conflict, the dictator will target H in the first round and L in the second round for*

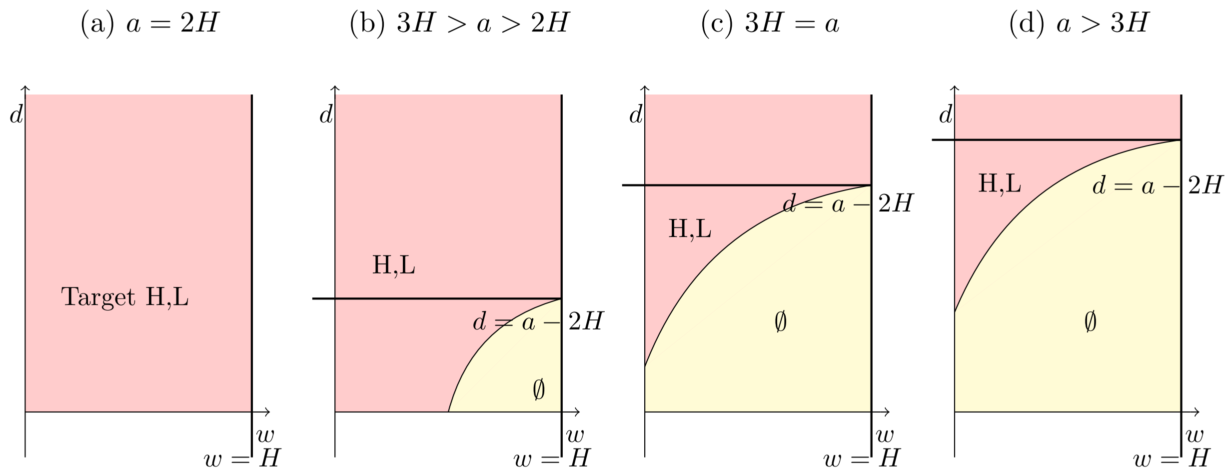
$$a \in (H + d + w, (\frac{1}{(2(3d+H+w))})((d + 3H - w)(2d + H + w) - \sqrt{(d + 3H - w)(16d^3 + (3H - w)(H + w)^2 + 16d^2(2H + w) + d(H + w)(17H + w))}))$$

and not initiate conflict for

$$a > (\frac{1}{(2(3d+H+w))})((d + 3H - w)(2d + H + w) - \sqrt{(d + 3H - w)(16d^3 + (3H - w)(H + w)^2 + 16d^2(2H + w) + d(H + w)(17H + w))})$$

[9] Though the outcome is never certain (winning with probability 1) due to restrictions on a.

Figure 2.2: Two Rounds of Conflict, No Coalitions



For low levels of uncertainty, the dictator will target H first and then, if he succeeds in his first conflict, target L in the second round. For all $d > a - 3H + w$, the expected utility from targeting two elites sequentially (H then L or L then H) is preferred to only targeting one (H or L, respectively). When the dictator is sufficiently advantaged relative to the uncertainty of the conflict, he prefers to target his stronger opponent first, then use those additional gains to take on L with an even stronger advantage. A weak dictator in an uncertain conflict environment would prefer the weak opponent first, targeting L first then using those lesser gains to stand against H. However, in the parameter space where L then H is preferred to H then L, the dictator is sufficiently weak and the conflicts sufficiently uncertain that no conflict is a dominant strategy. For more uncertain conflict environments, the level of power advantage that the dictator has and the level of overall expected benefit from beating both elites (i.e. w is not too high such that L's power is not too low) must be higher to induce him into a risky conflict.

The general trend of conflict initiation is the same whether the dictator is facing one or two rounds of potential consolidation. When uncertainty is low, the dictator will still initiate conflict; if two rounds of conflict are possible, however, he will always target H first whereas in the single round he would target L if L was not too weak to offer sufficient benefits. When conflict is more uncertain, having second round of conflict makes the dictator more conflict-prone: because the possible rewards from defeating both elites are higher than the single conflict rewards, the dictator is willing to take on the risk of conflict for more of the parameter space when beating both elites is possible. Note that this conflict behavior is limited by L's power: as L becomes weaker, the additional benefit of fighting the elites becomes too low to induce conflict and the dictator prefers to maintain his status quo power. In general, greater possible rewards from conflict will lead the dictator to take on more uncertain conflicts in order to achieve those rewards.

2.4 Coalitions and Conflict with Three Players

Now allow the members of the regime to form coalitions with one another after the dictator has made his targeting choice. Whichever elite is not targeted can join the dictator, join the targeted elite, or stay out of the conflict. We can use backwards induction, starting with $t = 2$ for Subgame Perfect Nash Equilibria. Depending on what happened in the previous round, there could be two or three players remaining. The only way that there could be three players remaining in the second round is if the dictator did not initiate conflict in the first round. Thus all three players have their initial endowments of power.[10]

2.4.1 Coalition Formation

Given their expectations about the potential outcomes of the conflict and the losses or rewards that those outcomes entail, whichever elite is not targeted by the dictator for conflict must decide whether to join the conflict and, if so, on which side. First, should a non-targeted elite join the dictator in a coalition? As credible commitment to sharing spoils is ruled out

[10] Note that round 2 with three players remaining, if considered on its own, constitutes a one-shot version of the target selection, coalition formation, and conflict among the three regime actors as all individuals have their initial power endowments and no shadow of a future conflict.

by assumption, neither elite will ever join the dictator in a coalition as there is nothing to gain.[11] For similar reasoning, the weakest member of the regime, L, will not join a coalition with the stronger elite H. Because any possible gains from the conflict would accrue to H, L is at most indifferent between joining a coalition and remaining out of the conflict to keep his status quo power. As long as the conflict is uncertain, the elites are strong enough that the potential gains of beating the dictator (which would go to H) outweigh the risk of defeat. This logic is summarized in Lemma 4.

Lemma 4.A. *L will never join a coalition with either the dictator or elite H.*

Lemma 4.B. *H will never join a coalition with the dictator, but will always join L in a coalition if L is targeted.*

Recall that this is the second round:[12] there is no shadow of future conflict driving H to fight the dictator, and he has no concern for L. The driver of coalition formation here is solely H's desire for power. He will join a coalition with L for the opportunity to defeat the dictator and take D's power upon winning, despite the risk this entails.

2.4.2 Targeting Behavior

When choosing to initiate conflict and, if so, which elite to target, the dictator must take into account the potential formation of a coalition between H and L. In particular, for the parameter space in which H is willing to join L in a coalition (which is the case for all uncertain conflict), targeting L alone is no longer an on-path option for the dictator. The

[11] Recall that the dictator, by definition, is stronger than either of the elites individually (regardless of his relative power to an elite coalition). Thus the spoils of any successful conflict would accrue to the dictator–he cannot credibly commit to sharing any of the power gained from the target with a coalition partner. The non-targeted elite will not gain anything from joining the conflict on the side of the dictator: he is, at most, indifferent between joining the conflict and staying out. If the conflict outcome is uncertain, joining the conflict would give the elite, in expectation, a non-zero probability of losing everything, with no possible gains to offset the risk. Thus neither elite would ever join a coalition with the dictator.

[12] Or a one-shot conflict.

dictator's targeting decisions that take his relative power, the conflict uncertainty, and the expected behavior of the elites into account is summarized in Proposition 1 and Figure 3.

Proposition 1.A. *If the dictator is strongly advantaged ($d > H - w$) and there is a single round of conflict, he will target H for $a \in H + d + w, 3H - w)$, target L and fight the elite coalition for $a \in (3H - w, 3H + d - w)$, and not initiate conflict for $a > 3H + d - w$.*

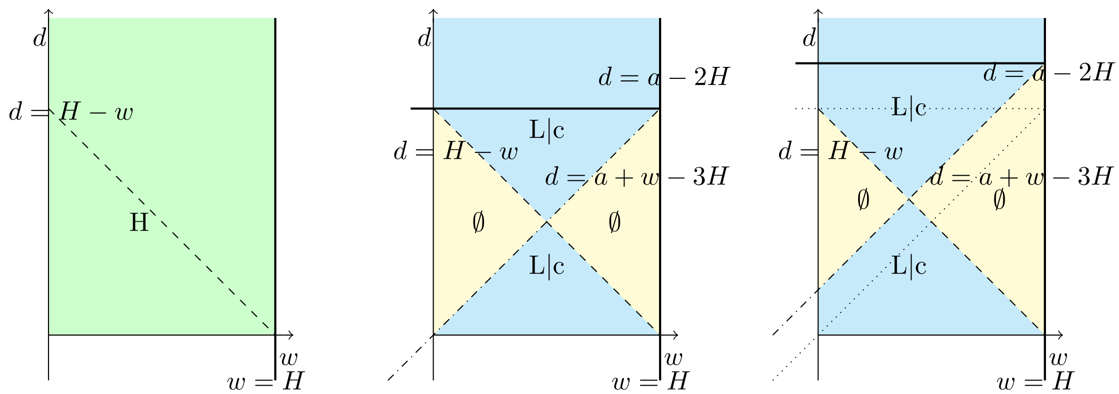
Proposition 1.B. *If the dictator is weakly advantaged ($d < H - w$) and there is a single round of conflict, he will target H for $a \in H + d + w, 3H - w)$, not initiate conflict for $a \in (3H - w, 3H + d - w)$, and target L and fight the elite coalition for $a > 3H + d - w$.*

When the uncertainty of the conflict is relatively low and the difference between the two elites, w, is relatively low, the dictator will only target H. As conflict uncertainty increases, the dictator's behavior depends on his advantage relative to the elites. If the dictator is relatively strong ($d > H - w$), he will target L in anticipation that H will join the elite coalition so that he can fight them both and take their power. Once L's power is too low to induce him into conflict, a relatively strong dictator will not initiate conflict. A weak dictator ($d < H - w$), on the other hand, benefits from greater conflict uncertainty when fighting both elites as it washes out their advantage over him with greater noise. Moving from Figure 3(b) to 3(c) shows how increasing uncertainty adjusts the parameter space. Note that the impact of conflict uncertainty on conflict behavior is nuanced here. It is not the case that a dictator facing greater uncertainty will always "exhibit prudence" and avoid conflict, as is the case in interstate war onset (Bas and Schub 2016). The relationship between the dictator's advantage over the elites and his advantage relative to the uncertainty of the conflict, when both taken into account, suggest that both extremely weak **and** extremely strong dictators will initiate conflict within their inner circle.

Note how the possibility of coalitions makes the dictator more likely to initiate conflict relative his targeting strategy in a single conflict with no coalitions. Without coalitions, a

Figure 2.3: Coalitions, Single Conflict

(a) $3H > a$ (b) $3H = a$ (c) $a > 3H$



high uncertainty environment would deter all but the dictators with very high relative from initiating conflict (Figure 1 (c)). When coalitions are possible and the dictator can fight both elites at once, he will target an elite (L) and initiate conflict for much more of the parameter space.

2.4.3 Two Players Remaining

If there are two players remaining, that implies that the dictator did initiate conflict in the first round and the third regime member either (a) did not join a coalition on either side or (b) joined a coalition and the coalition won, leaving the winning coalition members as the two remaining members of the regime, the stronger of whom is now the dictator.[13] Regardless of the identity of the dictator in the second round[14], if the dictator's advantage (which may

[13] Possible Conflicts: (1) D and H fought in round 1, now the winner (note that it does not matter whether there was a coalition or not. If L and H fought D together and won, H got all of D's power and L's power did not change. If D and L fought H together and won, D got all of H's power and L's power did not change. So regardless of what exactly the conflict was in round 1, the round 2 power distribution is $2H + d$ for the current dictator versus $H - w$ for the potential opponent.) is the dictator and can target L (or no conflict); (2) D and L fought in round 1, the winner (note that round 1 coalitions do matter for this option as the only way L can acquire D's power is through a one-on-one conflict. If H was involved, he would take D's power as the stronger coalition member and we would be in case 1.) is the dictator and can target H (or no conflict).

[14] Note that the identity of the dictator in the second round does not matter because there is no loss or decay of power upon transfer: it simply flows from the loser to the winner.

have been transferred to the new dictator) is sufficiently large, he will initiate conflict in the second round against his remaining opponent, regardless of if it is H or L who remains. If the dictator's advantage relative to uncertainty and the power of his opponents is sufficiently low, the conflict outcome is uncertain enough to deter the dictator from conflict and he will not initiate conflict, instead maintaining the power he gained from round 1.

Lemma 5. *The round 2 dictator will target the remaining elite for conflict if the dictator's advantage is sufficiently high relative to the conflict uncertainty* ($d > a + w - 3H$)*, otherwise he will not initiate conflict.*

Note that the dictator is now never interested in starting a risky, uncertain conflict as the weak dictator was in the previous subgame. Even if he began the game as a relatively weak dictator ($d < H - w$) or he began the game as an elite, by reaching the subgame with two remaining players as the dictator, he necessarily won the conflict in the first round and is now relatively advantaged in terms of power. Thus he will act as a strong dictator, preferring to maintain his status quo power than risk losing it in an uncertain conflict, similar to the advantaged dictator in the three-player subgame.

2.5 Dynamic Coalitions and Conflict

The round 1 targeting behavior must take into account both the expected round 2 behavior as well as the possible coalition formations in round 1. Note that, as above, H joining the dictator in a coalition against L is a weakly dominated strategy. This is intuitive as H has nothing to gain from siding with the dictator: if he and the dictator win together, all of the gains go to the dictator. If they lose, H loses everything. H's round 1 coalition behavior is summarized in Lemma 6(a). As in the one-shot version, H will join a coalition with L. The benefits of H joining L in a coalition and possibly defeating the dictator are no longer just the dictator's power, but also the potential for taking L's power in the second round conflict.

If H and L beat the dictator as a coalition in the first round, all of D's power is transferred to H and he will become the new dictator, targeting his former ally L when there is a sufficient dictatorial advantage. As before, L never gains from joining the conflict on either the side of H or the side of the dictator as neither party can commit to sharing the gains of conflict with him. As the weakest member of any coalition, the power of the loser will always go to the other coalition member, not to L. L's strategy is summarized in Lemma 6(b).

Lemma 6.A. *H will always join L in an elite coalition.*

Lemma 6.B. *L will never join a coalition*

2.5.1 Targeting Behavior

The dictator's full equilibrium targeting decision depends on both the expected coalition behavior of the elites as well as the round 2 conflict and coalition decisions that he anticipates. Note that a choice of no target and thus no conflict in the first round does not end the game and lead to absolutely no conflict, it simply moves all regime members with their initial power endowments to the second round, the equilibrium strategies of which are described above in the round 2 subgame. In terms of expected utilities, the dictator is indifferent between targeting one player (H) in the first round and not initiating conflict in the second and not initiating conflict in the first round and targeting H in the second.

The dictator's round 1 targeting strategy is summarized in Proposition 2 and visualized in Figure 4. The dictator must choose between targeting L and fighting the coalition, targeting H in the first round and L in the second round, only targeting H (in the first or second round), and no conflict. Note that L alone is not available as a target option because H will join a targeted L in an elite coalition.

Proposition 2.A. *If the dictator is strongly advantaged ($d > H - w$) and there are two possible rounds of conflict, he will target H in the first round and L in the second for $a \in$*

$(H + d + w, \frac{1}{2}(3H - w + \sqrt{4d^2 + (-3H + w)^2 + 4d(H + w)}))$
will target L and fight the coalition for
$a \in (\frac{1}{2}(3H - w + \sqrt{4d^2 + (-3H + w)^2 + 4d(H + w)}), 3H + d - w)$ *and will not initiate conflict for* $a > 3H + d - w$.

Proposition 2.B. *If the dictator is weakly advantaged ($d < H - w$) and there are two possible rounds of conflict, the dictator will target H in the first round and L in the second round for*

$$a \in (H + d + w, (\frac{1}{(2(3d+H+w))})((d + 3H - w)(2d + H + w) - \sqrt{(d + 3H - w)(16d^3 + (3H - w)(H + w)^2 + 16d^2(2H + w) + d(H + w)(17H + w)})))$$

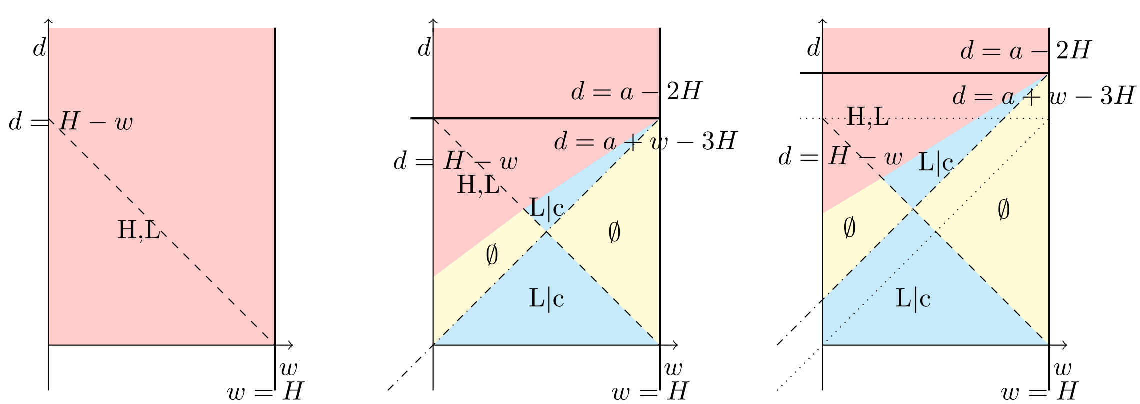
not initiate conflict for

$$a \in ((\frac{1}{(2(3d+H+w))})((d + 3H - w)(2d + H + w) - \sqrt{(d + 3H - w)(16d^3 + (3H - w)(H + w)^2 + 16d^2(2H + w) + d(H + w)(17H + w)})), 3H + d - w)$$

and target L and fight the elite coalition for $a > 3H + d - w$

Figure 2.4: Coalitions, Two Rounds Conflict

(a) $3H > a$ (b) $3H = a$ (c) $a > 3H$



Much like his multi-round targeting strategy without coalitions, targeting H and then using that additional power advantage to fight L in the second round is the dictator's equilibrium strategy when uncertainty is low. For higher levels of conflict uncertainty, the dictator's power advantage relative to the elites matters for his optimal strategy. A dictator who is weak relative to the elites ($d < H - w$) will target H then L, initiate no conflict, or target L and fight the coalition depending on his strength and the difference between the elites' power, w. For the weakest dictator (d close to 0), higher uncertainty gives him the opportunity to take on a powerful elite coalition in a conflict where the elites' advantage is counteracted by stochastic noise.[15] As his advantage increases (though he is still weak with $d < H - w$), the dictator will avoid conflict altogether, then take on his sequential targeting strategy of H then L. A strong dictator ($d > H - w$) prefers the same three strategies, but the parameter spaces in which he pursues them differ. When the benefits of conflict are low (as L's power decreases), even a relatively strong dictator will avoid initiating conflict. In an intermediate area when the dictator is of middling strength and the elites' power differential is not too great, the dictator will target L and fight the coalition.

Note that this sequential type of conflict, which occurs with and without coalitions when multiple conflicts are possible, in which elites are picked off one by one, is similar to the empirical patterns of early purges under Stalin in the Soviet Union. Further, the sequential targeting in this model, in which the high-powered elite is targeted first, is quite opposite to the "encircling" strategy suggested by Keller and Wang in which weak, peripheral members of the regime are removed first. This dictator targets H first because (a) it increases his chances of consolidating full power by the second round and (b) targeting L first could result in an elite coalition.

[15] He is actually indifferent between fighting the conflict in the first round or the second round: the gains from the conflict and probability of beating the coalition is the same regardless of the timing.

2.6 Discussion

Who should the dictator target for a purge? In general, strong dictators will go after strong targets that will yield the most rewards. Who constitutes the strongest target depends on the ability of the elites (namely H) to form a coalition against the dictator: elites together in a coalition are a more lucrative, and therefore desirable, target for a strong dictator. This drive is mitigated by uncertainty as a relatively strong dictator will maintain his status quo power instead of initiating conflict when uncertainty over the conflict is too high, especially if the low-powered elite is not bringing much reward. Weak dictators are similarly conflict prone, but not in order to preemptive strike or coup proof.[16] The power hungry dictator will utilize an uncertain conflict environment to his advantage to take on a strong elite coalition.

Most importantly for empirical work on purges and elite-dictator conflict, the relationship between the dictator's power advantage and whether any conflict will be observed is non-linear in the dictator's advantage. When conflict is uncertain, it is not the case that more elite purges are an indication of the personalist strength of the dictator. Indeed, for high levels of uncertainty, the observation of an attempt at a mass purge would indicate the *weakness* of the dictator vis-a-vis elites: he is willing to risk his small power advantage for the possibility of a huge payoff if he is able to defeat the elite coalition. A lack of conflict between the dictator and his elite allies is, likewise, not an indication of weakness. While a lack of observed conflict may indicate a relatively weak dictator in a semi-uncertain conflict environment, it may also be an extremely strong, advantaged dictator who is not willing to risk losing power in an uncertain conflict. The uncertainty of the conflict, as well as the difference in power between the possible purge targets, must be considered in order to understand the observed relationship between a dictator's power and purges.

[16] These incentives are outside the scope of the model as elites here cannot initiate conflict. However, note that even without these concerns (which we may expect will make weak dictators more likely to initiate a purge) a weak dictator is willing to initiate conflict.

2.6.1 Heterogeneity and Coalitions

While the ability of the elites to form coalitions makes the dictator more conflict-prone, particularly at higher levels of conflict uncertainty due to the higher possible benefit of removing both elites, the affect on the elites is heterogeneous. The higher powered elite's ability to join L in a coalition benefits him: he can join a conflict where he may not have before, use L's power with his own against the dictator, and reap the potential benefits if the two elites are able to oust the dictator. As the only coalition joiner in equilibrium, H will only form a coalition with L and join the conflict when, in response to (or in anticipation of, when there are two rounds of possible conflict) the dictator's targeting choice, his expected utility is greater.

This is not the case for L, who is unable to reject H as a coalition partner, and ends up being targeted because of H's joining. In the single conflict benchmark, L both benefits from and is hurt by H's ability to join him in different parts of the parameter space. When uncertainty over conflict outcomes is relatively low, the dictator switches from targeting L to targeting H because H would join L in a coalition, so L gets to avoid a conflict which he is very likely to lose.[17] In this case, L benefits from the possibility of a coalition because he avoids the conflict maintains his status quo power. For higher levels of uncertainty, however, L is always worse off. Instead of avoiding conflict, he becomes the target and H, who joins him in a coalition, reaps the benefits of the fight. Similarly when two rounds of conflict are possible, L is weakly worse off[18] from the possibility of elite coalitions. While for high levels of uncertainty being targeted in the second round might be preferred for L as it gives him the opportunity to win all of the power of the regime, this will not occur in equilibrium.[19]

[17] In this parameter space of $a < 2H + d - w$, L would prefer not being targeted to fighting the dictator; but note that for higher uncertainty, when the dictator's advantage is overcome by noise, L would like to be targeted for the chance to win the dictator's power. In equilibrium, however, he will not be.

[18] Either indifferent or strictly worse, depending on the parameter space

[19] L prefers the being targeted in the second round to no conflict when $a > 3H + d - w$, but the dictator's targeting strategy H, L is not on path in that parameter space.

Instead, there is now incentive for the dictator to target L, knowing the coalition will form and he will fight both elites, whereas before there was no conflict. These differential effects of coalitions formation on different elites depends on who gains the most from winning a conflict as a coalition. Even if H and L split the gains of the dictator's power if they beat him, that is not always sufficient compensation for L when, all else equal, he would not have been targeted without the coalition.[20] The inner workings of elite coalitions formed against a dictator's purge attempt warrant greater study and the inclusion of heterogeneity in elite power is an important aspect of understanding the different incentives and rewards of coalition behavior.

2.7 Conclusion

In an environment in which decisions are made, and disagreements are settled, under the shadow of extreme violence, even the simplest of personnel and staffing decisions can become uncertain conflicts (Svolik 2012). This uncertainty is exacerbated when elite "allies," with independent bases of power, wealth, and arms, are the regime officials being demoted or eliminated. By introducing heterogeneity in power to the ruling coalition, I have shown how the dictator will target his strongest possible opponent, whether the individual or an elite coalition, when he has a strong power advantage. While conflict uncertainty makes a strong dictator more conflict-averse, greater uncertainty induces a weak dictator into more conflict. Even without informational issues in which the dictator can hide his intent to personalize power, the balance of power among the dictator and among his allies can explain when the dictator will initiate conflict, who will be targeted for elimination, and the type of conflict observed. The differences in elite power yield differences in elite incentives to join one another in coalitions against the dictator. While a more powerful elite uses coalitions to his benefit to

[20] This is true for some parameter space.

join the conflict against the dictator, a lower-powered elite is often hurt by the possibility of a coalition because the dictator's anticipation of the coalition makes the weaker elite a more attractive target. The process by which power is consolidated into the hands of one leader not only has life and death consequences for the elites being targeted, but repercussions for the makeup of the ruling coalition, with downstream effects on lower personnel, policy implementation, and the distribution of public goods and services to citizens.

2.8 Formal Appendix

No Coalitions Possible

Round 2: Three Players

Because there is no coalition stage, the only action is the dictator's targeting decision

ϵ is distributed uniform, so

$$F_\epsilon(x) = \begin{cases} 0 & x < -a \\ \frac{x+a}{2a} & x \in [-a, a) \\ 1 & x \geq a \end{cases}$$

Assume $a > H + d + w$ such that all conflicts are uncertain. Thus the probability that the dictator wins against H is $\frac{d+a}{2a}$ and the probability the dictator wins against L is $\frac{d+w+a}{2a}$

$U_D(H) = (\frac{d+a}{2a})(2H + d) + (1 - \frac{d+a}{2a})(0)$

$U_D(L) = (\frac{d+w+a}{2a})(2H + d - w) + (1 - (\frac{d+w+a}{2a}))(0)$

$U_D(\emptyset) = H + d$

H versus no conflict

$(\frac{d+a}{2a})(2H + d) > H + d$

$2H + d > a$ if this holds, the dictator prefers targeting H relative to no conflict (equivalently $H > \frac{a-d}{2}$)

L versus no conflict

$(\frac{d+w+a}{2a})(2H + d - w) > H + d$

$2H + d - w > a$ if this holds, the dictator prefers targeting L relative to no conflict (equivalently, $H > \frac{a+w-d}{2}$)

H versus L

$(\frac{d+a}{2a})(2H+d) > (\frac{d+w+a}{2a})(2H+d-w)$

$a > 2H-w$ if this holds, the dictator prefers targeting H relative to targeting L (equivalently $H < \frac{a+w}{2}$)

Round 2: Two Players

If there are two players remaining, there was a conflict in the first round between D and H or D and L.

If the first round conflict was between D and H, the winner (and second round dictator) now has a power endowment of $2H+d$. Should he target L?

$\frac{H+d+w+a}{2a}(3H+d-w) > 2H+d$

$3H+d-w > a$ if this holds, the second round dictator should target L relative to no conflict (equivalently $H > \frac{a+w-d}{3}$)

If the first round conflict was between D and L, the winner (and second round dictator) now has a power endowment of $2H+d-w$. Should he target H?

$\frac{H+d-w+a}{2a}(3H+d-w) > 2H+d-w$

$3H+d-w > a$ if this holds, the second round dictator should target H relative to no conflict (equivalently $H > \frac{a+w-d}{3}$)

Round 1 Targeting

Possible paths for two round targeting:

$U_D(\emptyset, H) = \frac{d+a}{2a}(2H+d)$

$U_D(\emptyset, L) = \frac{d+w+a}{2a}(2H+d-w)$

$U_D(H, L) = (\frac{d+a}{2a})(\frac{H+d+w+a}{2a})(3H+d-w)$

$U_D(L, H) = (\frac{d+w+a}{2a})(\frac{H+d-w+a}{2a})(3H+d-w)$

$U_D(H,\emptyset) = \frac{d+a}{2a}(2H+d)$

$U_D(L,\emptyset) = \frac{d+w+a}{2a}(2H+d-w)$

$U_D(\emptyset,\emptyset) = H+d$

H,L versus L,H

$(\frac{d+a}{2a})(\frac{H+d+w+a}{2a})(3H+d-w) > (\frac{d+w+a}{2a})(\frac{H+d-w+a}{2a})(3H+d-w)$

$a > H-d-w$ if this holds, H,L is preferred to targeting L, H (equivalently $H < a+d+w$)

H,L versus H alone (note $H,\emptyset$ and $\emptyset,H$ are the same expected utilities)

$(\frac{d+a}{2a})(\frac{H+d+w+a}{2a})(3H+d-w) > \frac{d+a}{2a}(2H+d)$

$3H+d-w > a$ if this holds, targeting H and L is preferred to H alone (equivalently, $H > \frac{a+w-d}{3}$)

L,H versus L alone (note $L,\emptyset$ and $\emptyset,L$ are the same expected utilities)

$(\frac{d+w+a}{2a})(\frac{H+d-w+a}{2a})(3H+d-w) > \frac{d+w+a}{2a}(2H+d-w)$

$3H+d-w > a$ if this holds, targeting L and H is preferred to L alone (equivalently, $H > \frac{a+w-d}{3}$)

H,L versus L alone

$(\frac{d+a}{2a})(\frac{H+d+w+a}{2a})(3H+d-w) > (\frac{d+w+a}{2a})(2H+d-w)$

$\frac{2a(a+d)(a+d+H+w)}{(a+d+w)} > \frac{2H+d-w}{3H+d-w}$

As $3H+d-w > 2H+d-w$, the right side is less than 1

$\frac{2a(a+d)(a+d+H+w)}{(a+d+w)}$

H,L is preferred if $2a(a+d)(a+d+H+w) > (a+d+w)$ which holds for sufficiently high a, d

(see round two for comparisons of H and L alone and relative to no conflict)

Where does $H = \frac{a+w-d}{3}$ fall relative to the other cutpoints?

$\frac{a+w-d}{3} < \frac{a+w-d}{2}$

Is $\frac{a+w-d}{3} > \frac{a-d}{2}$?

$2(a+w-d) > 3(a-d)$

$d + 2w > a$

This may be possible, but by assumption $a > H + d + w$

$d + 2w > a > H + d + w$

$w > H$ this is a contradiction by definition so it must be the case that $\frac{a-d}{2} > \frac{a+w-d}{3}$

Coalitions Possible

Round 2: Three Players

Coalition Formation

Lemma 4.A. *L will never join a coalition with either the dictator or elite H.*

Lemma 4.B. *H will never join a coalition with the dictator, but will always join L in a coalition if L is targeted.*

For L, not joining a coalition weakly dominates joining either D or H. H will not join a coalition with D. If $d > H - w$, for every vector d, H, w, a, there exists a unique threshold $a_{Coal}(d, H, w) > 0$ such that H is indifferent between joining a coalition with L or staying out of the conflict and joining L against D is preferred for all $a > a_{Coal}$. If $d < H - w$, H will always join a coalition with L.

Assume H is targeted, what does L do?

$$E\left[u_L(j\,\text{oin}\,D)\right] = P(\text{ coalition wins })(H-w) + P(H \text{ wins })(0) = F_\epsilon(H+d-w)(H-w)$$

$$E\left[u_L(\neg \text{ join })\right] = H - w$$

$$E\left[u_L(\text{ join })\right] = P(D \text{ win })(0) + P(D \text{ lose })(H-w) = \left(1 - F_\epsilon(d+w-H)\right)(H-w)$$

L join iff $F_\epsilon(H+d-w)(H-w) > H-w$. This will never occur as F_ϵ is a proper CDF.L would, at most, be indifferent between joining the dictator and staying out of the conflict. L will join H in a coalition if and only if $\left(1-F_\epsilon(d+w-H)\right)(H-w) > H-w$, which is never the case. Even if the probability that the dictator wins goes to 0, L is at most indifferent.

Assume L is targeted. What should H do? Recall the CDF of the Uniform distribution, with symmetric bounds $[-a, a]$, is

$$F_\epsilon(x) = \begin{cases} 0 & x < -a \\ \frac{x+a}{2a} & x \in [-a, a) \\ 1 & x \geq a \end{cases}$$

$$E\left[u_H(joinL)\right] = P(D \text{ wins })(0) + P(D \text{ loses })(2H+d) = \left(1 - F_\epsilon(d+w-H)\right)(2H+d)$$

$$E\left[u_H(\neg \text{ join })\right] = H$$

$$E\left[u_H(\text{ join } D)\right] = P(\text{ coalition wins })(H) + P(L \text{ wins })(0) = F_\epsilon(H+d+w)(H)$$

H join iff $F_\epsilon(H+d+w)(H) > H$. This will never occur as F_ϵ is a proper CDF. H would, at most, be indifferent between joining the dictator and staying out of the conflict.

H should join the coalition with L if $(1 - F_\epsilon(d+w-H))(2H+d) > H$

If $a \geq d+w-H > 0, F_\epsilon(d+w-H) = \frac{d+w-H+a}{2a}$

Join coalition iff $1 - \frac{d+w-H+a}{2a} \geq \frac{H}{2H+d}$

$a \geq d+w+H+\frac{2H(w-H)}{d}$. Denote the point at which H is indifferent between joining the conflict in a coalition with L and staying out as $a_{Coal} = d+w+H+\frac{2H(w-H)}{d}$. For all

$a > d + w + H + \frac{2H(w-H)}{d}$ H will join the coalition with L against the dictator. Note that $a_{Coal} < H + d + w$, the lower bound on a such that all conflicts will be uncertain. Thus H will always join a coalition with L in this parameter space.

If $d < H - w$

If $a \geq H - d - w, F_\epsilon(d + w - H) = \frac{d+w-H+a}{2a}$

Join coalition iff $1 - \frac{d+w-H+a}{2a} \geq \frac{H}{2H+d}$ $a \geq d + w + H + \frac{2H(w-H)}{d}$. Denote the point at which H is indifferent between joining the conflict in a coalition with L and staying out as $a_{Coal} = d + w + H + \frac{2H(w-H)}{d}$. For all $a > d + w + H + \frac{2H(w-H)}{d}$, H will join the coalition with L against the dictator. (note that a_{Coal} is the same as above). As above, H will always join a coalition with L in this parameter space.

Targeting Behavior

Proposition 1.A. *If the dictator is strongly advantaged ($d > H - w$) and there is a single round of conflict, he will target H for $a \in H + d + w, 3H - w)$, target L and fight the elite coalition for $a \in (3H - w, 3H + d - w)$, and not initiate conflict for $a > 3H + d - w$.*

$d > H - w$

$F_\epsilon(d + w - H) = \frac{d+w-H+a}{2a}, F_\epsilon(d + w) = \frac{d+w+a}{2a}, F_\epsilon(d) = \frac{d+a}{2a}$.

The noise distribution is disperse enough that there are no certain conflict outcomes. Even against his weakest opponent, the dictator is uncertain about whether he will win the conflict. Note as $a \to \infty$, all of the win probabilities go to $\frac{1}{2}$. For sufficiently high a, the dictator will prefer the status quo and will not initiate conflict.

Crossing points: $E[u_D(\emptyset)] > E[u_D(L|\neg c)]$ if $a > 2H + d - w$

$E[u_D(\emptyset)] > E[u_D(H)]$ if $a > 2H + d$

$E[u_D(\emptyset)] > E[u_D(L|c)]$ if $a > 3H + d - w$

Note that $3H + d - w > 2H + d > 2H + d - w$, thus if the coalition forms, fighting the coalition will be the preferred conflict for a around $3H + d - w$, then no conflict is preferred for all $a > 3H + d - w$

The coalition will form across the entire range. For $a < 3H + d - w$, the dictator is choosing between targeting H alone or L knowing that the coalition will form (recall from above that for $a > 3H + d - w$, the dictator does not initiate conflict). The dictator is indifferent between targeting H and fighting the coalition at $a = 3H - w$.
If $H - w < d < 2(H - w)$, $H + d + w < 3H - w$, then for $a \in (H + d + w, 3H - w)$ the dictator targets H; for $a \in (3H - w, 3H + d - w)$, the dictator targets L and the coalition forms; for $a > 3H + d - w$, the dictator does not initiate conflict. If $d > 2(H - w)$, $3H - w$ is out of the relevant range of uncertainty. For $a \in (H + d + w, 3H + d - w)$, the dictator targets L and fights the coalition. For $a > 3H + d - w$, the dictator does not initiate conflict.

$d < H - w$

$E\left[u_D(H)\right] = F_\epsilon(d)(2H + d)$

$E\left[u_D(L)\right] = F_\epsilon(d + w - H)(3H + d - w)$

$E\left[u_D(\emptyset)\right] = H + d$

H alone is preferred to no target if $a < 2H + d$

H alone is preferred to fighting the coalition if $a < 3H - w$

$L(coal)$ is preferred to no target if $a > 3H + d - w$

Thus targeting H is preferred for $a \in (H + d + w, 2H + d)$, no target is preferred for $a \in [2H + d, 3H + d - w)$ targeting L and fighting the coalition is preferred for $a > 3H + d - w$

Round 2: Two Players Remaining

Lemma 5. *The round 2 dictator will target the remaining elite for conflict if the dictator's advantage is sufficiently high relative to the conflict uncertainty ($d > a + w - 3H$), otherwise he will not initiate conflict.*

Case 1: Dictator (whether identity was D or H in rd 1) has 2 $2H + d$. Denote round 2 dictator as D'

$E\left[u_{D'}(\emptyset)\right] = 2H + d$

$E\left[u_{D'}(L)\right] = F_{\epsilon}(H + d + w)(3H + d - w)$ as $a > H + d + w$

Target L if $F_{\epsilon}(H + d + w)(3H + d - w) > 2H + d$

$a < 3H + d - w$

Note that $3H + d - w > H + d + w$. Thus given this history, D' will target L in round 2 for all $a < 3H + d - w$

in terms of d, target if $d > a + w - 3H$

Case 2: Dictator has $2H + d - w$. Denote round 2 dictator as D''

$E\left[u_{D''}(\emptyset)\right] = 2H + d - w$

$E\left[u_{D''}(H)\right] = F_{\epsilon}(H + d - w)(3H + d - w)$ as $a > H + d - w$

Target H if $F_{\epsilon}(H + d - w)(3H + d - w) > 2H + d - w$

$a < 3H + d - w$

Note that $3H + d - w > H + d - w$. Thus given this history, D'' will target H in round 2 for all $a < 3H + d - w$

in terms of d, target if $d > a + w - 3H$

Round 1

Coalition Behavior

Lemma 6.A. *H will always join L in an elite coalition.*

Denote the expected utility of H not joining any coalition in round 1, staying out of the conflict, as $E[u_H(\neg join_1)]$. If H does not join, the winner of the conflict between D and L will choose to target H or not in accordance with the round 2 strategies defined above, which depend on the relative uncertainty (a) of the conflict.

Denote the expected utility of H joining a coalition with the dictator in round 1 as $E[u_H(join_{1D})]$. Recall that if the dictator and H together defeat L, all of the gains will go to the dictator and he will retain his position. He will target the remaining player, H, in accordance with the above strategies.

Denote the expected utility of H joining L in a coalition against the dictator in round 1 as $E[u_H(join_{1L})]$. If the elite coalition defeats the dictator, all of the gains will go to H and he will target L in the second round in accordance with the above strategies.

$a \in [H + d + w, 3H + d - w)$

$E\left[u_H\left(\neg \text{ join }_1\right)\right] = \left(1 - F_\epsilon(H + d - w)\right)(3H + d - w)$

$E\left[u_H\left(join_{1D}\right) = F_\epsilon(H + d + w)\left(1 - F_\epsilon(H + d - w)(3H + d - w)\right.$

$E\left[u_H\left(join_{1L}\right)\right] = \left(1 - F_\epsilon(d + w - H)\right) F_\epsilon(H + d + w)(3H + d - w)$

Note joining D is dominated by not joining

$E\left[u_H\left(join_{1L}\right)\right] - E\left[u_H\left(\neg join_1\right)\right] = 0$

Solve for a at the indifference point

$$= \frac{(2H + d - w)(3H + d - w) \pm \sqrt{(3H + d - w)^2\left(4Hd + 5H^2 - 4dw - 4Hw\right)}}{3H + d - w}$$

$= 2H + d - w \pm \frac{\sqrt{(3H+d-w)^2(4Hd+5H^2-4dw-4Hw)}}{3H+d-w}$

recall the bounds of this case are $H + d + w$ and $3H + d - w$

Where do the roots fall relative to the case bounds?

let $\eta = \frac{\sqrt{(3H+d-w)^2(4Hd+5H^2-4dw-4Hw)}}{3H+d-w}$

Is $2H + d - w - \eta < H + d + w$? (compare lower bound and lower root)

$H - 2w - \eta < 0$? We know that $-2w \in (0, -2H)$ by definition of w, so whether or not the inequality holds depends on the relative size of H and η i.e. if $\eta > H$, the expression is always less than 0 and the lower root is less than the lower bound.

$\sqrt{(3H + d - w)^2 (4Hd + 5H^2 - 4dw - 4Hw)} > H(3H + d - w)$

$(4Hd + 5H^2 - 4dw - 4Hw) > H^2$ (square both sides and divide by $(3H + d - w)^2$) $Hd+H^2-dw - Hw > 0$

$H(H + d) - w(H + d) > 0$ this is always true as $H > w$ by definition

Therefore $\eta > H$, therefore the lower root is less than the lower bound.

Compare upper root and upper bound:

$2H + d - w + \eta > 3H + d - w$

this holds as $\eta > H$

Check first partials at roots to show increasing/decreasing

$\frac{\partial}{\partial a} E\left[u_H(\neg \text{ join })\right] - E\left[u_H(\text{ join }_L)\right] = -\frac{(3H+d-w)((H+d+w)(d+w-H)+a(w-d-2H))}{2a^3}$

Note if $d < H - w, d + w - H$ and $w - 2H - d$ are both negative, making the entire first derivative positive for all a. Further note that when $d < H - w$, the lower root discussed above does not exist. The expression of utility difference crosses zero only once, at $2H + d - w + \eta$ and is increasing everywhere. Therefore it the expression is less than zero for all $a < 3H + d - w < 2H + d - w + \eta$ and greater than zero for all $a > 2H + d - w + \eta$. Thus H prefers to join L in this range if $d < H - w$

If $d > H - w$, both roots exist and the expression of utility difference is non-monotonic. Show that the first derivative is negative at the lower root and positive at the upper root Note the sign of $\frac{\partial}{\partial a}E\left[u_H(\neg \text{ join })\right]-E\left[u_H\left(join_L\right)\right]=-\frac{(3H+d-w)((H+d+w)(d+w-H)+a(w-d-2H))}{2a^3}$depends– on the sign of $(H+d+w)(d+w-H)+a(w-d-2H)$ as $3H+d-w$ is always positive and a is always positive by definition.

sign$(H+d+w)(d+w-H)+a(w-d-2H)$ evaluated at the upper and lower roots of $E\left[u_H(\neg \text{ join })\right]-E\left[u_H\left(join_L\right)\right]=0$

Upper root: $d^2-H^2+2dw+w^2+(2H+d-w+\eta)(w-d-2H)>0$ $-d(4H-4w+\eta)+\eta(w-2H)+H(4w-5H)>0$

As $H>w$ and $d,\eta>0$ by definition, this is a contradiction. Therefore $-d(4H-4w+\eta)+\eta(w-2H)+H(4w-5H)<0$. Therefore $\frac{\partial}{\partial a}E\left[u_H(\neg join)\right]-E\left[u_H\left(j\,\text{oin}_L\right)\right]>0$. The expression is increasing at the upper root.

Lower root: $d^2-H^2+2dw+w^2+(2H+d-w-\eta)(w-d-2H)>0$ (recall $d>H-w$) $\eta(2H+d-w)>H^2+4(H+d)(H-w)$ while we know from above $\eta>H$, the actual magnitude matters for signing the derivative here. So plug in the actual value of $\eta=\frac{\sqrt{(3H+d-w)^2(4Hd+5H^2-4dw-4Hw)}}{3H+d-w}$ $(2H+d-w)\sqrt{(3H+d-w)^2\left(4Hd+5H^2-4dw-4Hw\right)}>\left(H^2+4(H+d)(H-w)\right)(3H+d-w)$

Square both sides:

$(2H+d-w)^2(3H+d-w)^2\left(4Hd+5H^2-4dw-4Hw\right)>\left(H^2+4(H+d)(H-w)\right)^2(3H+d-w)^2$

$(2H+d-w)^2>4Hd+5H^2-4dw-4Hw$

$d^2+w^2+2dw-H^2>0$

$(H+d+w)(d+w-H)>0$

Note this holds as $d>H-w$. Thus $(H+d+w)(d+w-H)+(2H+d-w-\eta)(w-d-2H)>0$ so the derivative evaluated at the lower root is negative.

As $E\left[u_H(\neg \text{ join })\right]-E\left[u_H\left(\text{join}_L\right)\right]$ is decreasing at the lower root and increasing at the upper root, the expression is negative between the two roots. As the case bounds are within the roots as shown above, the expression is negative for the full range of a in this case. Thus

H prefers to join L in a coalition for this case.

$a > 3H + d - w$

$E\left[u_H\left(\neg join_1\right)\right] = H$ as there will be no round 2 conflict

$E\left[u_H\left(join_{1D}\right) = F_\epsilon(H + d + w)(H)\right.$

$E\left[u_H\left(join_{1L}\right)\right] = \left(1 - F_\epsilon(d + w - H)\right)(2H + d)$

Note don't join dominates joining D as F is a proper CDF

Join L is preferred to don't join iff $\left(1 - F_\epsilon(d + w - H)\right)(2H + d) > H$

$a > \frac{(2H+d)(d+w-H)}{d}$

Note $d + w - H > 0$ if $d > H - w$, so if strong coalition, join L for whole range

Is $\frac{(2H+d)(d+w-H)}{d} < 3H + d - w$?

$2w(H + d) < 2H(H + d)$ true by definition

Therefore H will join L across this entire range of a.

Lemma 6.B. *L will never join a coalition*

Denote the expected utility of L not joining any coalition in round 1, staying out of the conflict, as $E[u_L(\neg join_1)]$. If L does not join, the winner of the conflict between D and H will choose to target L or not in accordance with the round 2 strategies defined above, which depend on the relative uncertainty (a) of the conflict.

Denote the expected utility of L joining a coalition with the dictator in round 1 as $E[u_L(join_{1D})]$. Recall that if the dictator and L together defeat H, all of the gains will go to the dictator and he will retain his position. He will target the remaining player, L, in accordance with the above strategies.

Denote the expected utility of L joining H in a coalition against the dictator in round 1 as $E[u_L(join_{1H})]$. If the elite coalition defeats the dictator, all of the gains will go to H and he will target L in the second round in accordance with the above strategies.

$a \in [H + d + w, 3H + d - w)$

$E\left[u_L\left(\neg join_1\right)\right] = \left(1 - F_\epsilon(H + d + w)\right)\left(3H + d - w\right)$

$E\left[u_L\left(join_{1D}\right) = F_\epsilon(H + d - w)\left(1 - F_\epsilon(H + d + w)\right)\left(3H + d - w\right)$

$E\left[u_L\left(join_{1H}\right)\right] = \left(1 - F_\epsilon(d + w - H)\right)\left(1 - F_\epsilon(H + d + w)\right)\left(3H + d - w\right)$

Note that not joining dominates joining either coalition as F_ϵ is a proper CDF

$a > 3H + d - w$

$E\left[u_L\left(\neg join_1\right)\right] = H - w$ as there will be no round 2 conflict

$E[[u_L\left(join_{1D}\right)] = F_\epsilon(H + d - w)(H - w)$ as there will be no round 2 conflict.

$E\left[u_L\left(join_{1H}\right)\right] = \left(1 - F_\epsilon(d + w - H)\right)\left(H - w\right)$ as there will be no round 2 conflict

Not joining dominates either coalition as F_ϵ is a proper CDF.

Targeting Behavior

Proposition 2.A. *If the dictator is strongly advantaged ($d > H - w$) and there are two possible rounds of conflict, he will target H in the first round and L in the second for* $a \in (H + d + w, \frac{1}{2}(3H - w + \sqrt{4d^2 + (-3H + w)^2 + 4d(H + w)}))$
will target L and fight the coalition for
$a \in (\frac{1}{2}(3H - w + \sqrt{4d^2 + (-3H + w)^2 + 4d(H + w)}), 3H + d - w)$ *and will not initiate conflict for* $a > 3H + d - w$.

Proposition 2.B. *If the dictator is weakly advantaged ($d < H - w$) and there are two possible rounds of conflict, the dictator will target H in the first round and L in the second round for*

$$a \in (H + d + w, (\frac{1}{(2(3d+H+w))})((d + 3H - w)(2d + H + w) - \sqrt{(d + 3H - w)(16d^3 + (3H - w)(H + w)^2 + 16d^2(2H + w) + d(H + w)(17H + w))}))$$

not initiate conflict for

$$a \in ((\frac{1}{(2(3d+H+w))})((d+3H-w)(2d+H+w)-\sqrt{(d+3H-w)(16d^3+(3H-w)(H+w)^2+16d^2(2H+w)+d(H+w)(17H+w))}),3H+d-w)$$

and target L and fight the elite coalition for $a > 3H + d - w$

$a \in [H + d + w, 2H + d)$ assume $d < H - w$

$E[u_D(H_1)] = F_\epsilon(d)F_\epsilon(H + d + w)(3H + d - w)$

$E[u_D(L_1)] = F_\epsilon(d + w - H)(3H + d - w)$

$E[u_D(\emptyset)] = F_\epsilon(d)(2H + d)$

H alone is preferred to no target for all $a < 3H + d - w$, which is true in this range.

$E[u_D(H_1)] - E[u_D(L_1)] = \frac{(3H+d-w)\left(-a^2+a(3H-w)+d(H+d+w)\right)}{4a^2}$

$\frac{\partial}{\partial a}E[u_D(H_1)] - E[u_D(L_1)] = \frac{-(3H+d-w)\left(2d^2+a(3H-w)+2d(H+w)\right)}{4a^3}$ which is always negative by definition

of H, w, d, a. Thus the expression is monotonically decreasing everywhere. Check whether it crosses zero in the relevant range:

$\lim_{a \to H+d+w} E[u_D(H_1)] - E[u_D(L_1)] = \frac{(H-w)(3H+d-w)}{2(H+d+w)}$ which is always positive

$\lim_{a \to 2H+d} E[u_D(H_1)] - E[u_D(L_1)] = \frac{H(H-w)(3H+d-w)}{2(2H+d)^2}$ which is always positive.

Therefore targeting H alone is preferred to L and the subsequent coalition in this range.

$a \in [H + d + w, 3H + d - w)$ assume $d > H - w$

$E[u_D(H_1)] = F_\epsilon(d)F_\epsilon(H + d + w)(3H + d - w)$

$E[u_D(L_1)] = F_\epsilon(d + w - H)(3H + d - w)$

$E[u_D(\emptyset)] = F_\epsilon(d + w - H)(3H + d - w)$

Compare H and the other option (target L and fight coalition now, or wait until next round and fight coalition).

$E[u_D(H_1)] - E[u_D(L_1)] = \frac{(3H+d-w)\left(-a^2+a(3H-w)+d(H+d+w)\right)}{4a^2}$

$\frac{\partial}{\partial a} E\left[u_D\left(H_1\right)\right]-E\left[u_D\left(L_1\right)\right]=\frac{-(3H+d-w)\left(2d^2+a(3H-w)+2d(H+w)\right)}{4a^3}$ which is always negative by definition of H, w, d, a. Thus the expression is monotonically decreasing everywhere. Check whether it crosses zero in the relevant range:

$\lim_{a\to H+d+w} E\left[u_D\left(H_1\right)\right]-E\left[u_D\left(L_1\right)\right]=\frac{(H-w)(3H+d-w)}{2(H+d+w)}$ which is always positive

$\lim_{a\to 3H+d-w} E\left[u_D\left(H_1\right)\right]-E\left[u_D\left(L_1\right)\right]=\frac{d(w-H)}{2(3H+d-w)}$ which is always negative by definition of $H>w$.

By the intermediate value theorem, there exists an $a^{**}\in(H+d+w, 3H+d-w)$ at which the dictator is indifferent all his options. For $a<a^{**}$, targeting H alone is preferred. For $a>a^{**}$, the dictator is indifferent between targeting L and fighting the coalition in this round or choosing no target in round 1 and fighting the coalition in round 2.

In terms of d,

$\lim_{d\to a-H-w} E\left[u_D\left(H_1\right)\right]-E\left[u_D\left(L_1\right)\right]=\frac{(a+2H-2w)(H-w)}{2a}$ which is always positive

$\lim_{d\to a+w-3H} E\left[u_D\left(H_1\right)\right]-E\left[u_D\left(L_1\right)\right]=-\frac{(H-w)(a-3H+w)}{2a}$ which is always negative

By the intermediate value theorem, there exists a $d^{**}\in(a+w-3H, a-H-w)$ at which the dictator is indifferent all his options. For $d>d^{**}$ targeting H alone is preferred. For $d<d^{**}$, the dictator is indifferent between targeting L and fighting the coalition in this round or choosing no target in round 1 and fighting the coalition in round 2.

$a\in[2H+d, 3H+d-w)$ and $d<H-w$

$E\left[u_D\left(H_1\right)\right]=F_\epsilon(d)F_\epsilon(H+d+w)(3H+d-w)$

$E\left[u_D\left(L_1\right)\right]=F_\epsilon(d+w-H)(3H+d-w)$

$E\left[u_D(\emptyset)\right]=H+d$ as there would be no conflict in round 2

No target is preferred to L if $a<3H+d-w$, which is true.

Compare No target and targeting H: no target preferred if $E\left[u_D(\emptyset)\right]-E\left[u_D\left(H_1\right)\right]>0$

$\frac{\partial}{\partial a} H+d-\left(F_\epsilon(d)F_\epsilon(H+d+w)(3H+d-w)\right)=\frac{(3H+d-w)(2d(H+d+w)+a(2d+H+w))}{4a^3}$ which is always positive and therefore monotone.

$\lim_{a \to 2H+d} E[u_D(\emptyset)] - E[u_D(H_1)] = -\frac{(H+d)(H-w)(H+d+w)}{2(2H+d)^2}$ which is always negative.

$\lim_{a \to 3H+d-w} E[u_D(\emptyset)] - E[u_D(H_1)] = \frac{d(H-w)}{2(3H+d-w)}$ which is always positive.

By the intermediate value theorem, there exists an $\tilde{a} \in (2H+d, 3H+d-w)$ such that for all $a > \tilde{a}$ no target is preferred to targeting H while H is preferred for $a < \tilde{a}$

In terms of d,

$\lim_{d \to a-2H} E[u_D(\emptyset)] - E[u_D(H_1)] = -\frac{(a-H)(H-w)(a-H+w)}{2a^2}$ which is always negative.

$\lim_{d \to a+w-3H} E[u_D(\emptyset)] - E[u_D(H_1)] = \frac{(H-w)(a+w-3H)}{2a}$ which is always positive.

By the intermediate value theorem, there exists an $\tilde{d} \in (a+w-3H, a-2H)$ such that for all $d < \tilde{d}$ no target is preferred to targeting H while H is preferred for $d < \tilde{d}$

$a > 3H + d - w$

$E[u_D(H_1)] = F_\epsilon(d)(2H+d)$ as there will be no conflict in round 2

$E[u_D(L_1)] = F_\epsilon(d+w-H)(3H+d-w)$

$E[u_D(\emptyset)] = F_\epsilon(d+w-H)(3H+d-w)$ if $d < H - w$

$E[u_D(\emptyset)] = H + d$ if $d > H - w$

If the dictator is weak relative to the coalition, the dictator is indifferent between fighting L now (as the coalition will form) and not initiating conflict now and instead fighting the coalition in round 2

Targeting H is preferred $a < 3H - w$, however $3H - w < 3H + d - w$, therefore targeting H is dominated. Either no target or targeting L and fighting to coalition is preferred in this range, the dictator is indifferent between them.

If the dictator is strong relative to the coalition ($d > H - w$), no target is preferred in this range and there will be no second round conflict.

Chapter 3

Sowing Seeds of Destruction? Empowering Elite Rivals under Contested Dictatorship

Given the inherent risk elites pose to a dictator, why would a dictator dole out policy control, riches, and military command to his elite rivals? While historical accounts focus on a dictator's underestimation of his rival's strength, these explanations fail to consider the perspective of the elite rival who must choose to support the dictator. In my theoretical model, a dictator faces a multi-stage process of conflict and consolidation, eliminating elites from his ruling coalition to win more power for himself. The dictator can offer rewards to other members of the elite in return for their support, but empowering a rival directly affects the dictator's ability to purge said rival in the future. By introducing heterogeneity into the regime elite in a dynamic setting of uncertain conflict, I show the conditions underwhich both the dictator and his future opponent will be willing to work together temporarily despite the shadow of future conflict.

In the Soviet Union, when Joseph Stalin decided to move against his former allies in the Politburo, Lev Kamenev and Grigory Zinoviev, he rewarded his newest ally, Nikolai Bukharin, with a promotion as General Secretary of the Comintern Executive Committee in 1926. Bukharin's intellectualism and ideological rigor, as well as his popularity in the Party, helped legitimize the elimination of such powerful founding members of the Soviet system (Cohen 1980). In Uganda, when Milton Obote found himself at odds with the entire Bugandan region because of his deposal of Mutesa of Buganda, he empowered Idi Amin, then deputy commander of the army, to take charge of military operations (Ingham 1994, 103). With a deadlocked and ineffectual Reichstag, German President Paul von Hindenburg decided to appoint Adolf Hitler to the position of Chancellor as "...a declaration of a state of emergency was, remarkable though it now seems, seen as more worrying than a cabinet led by Hitler" (Kershaw 2014, 252-253). An agreement among Hindenburg and political rivals Hitler and conservative politician Franz von Papen, was reached: Hitler would take the post of Chancellor, Papen would become Vice Chancellor, and Hindenburg would remain Reich President and supreme commander of the armed forces. Why would each of these leaders, threatened by a powerful and ambitious rival, empower that rival with positions of influence over policy or the military? Why would the elites enter into alliances with leaders trying to consolidate power by eliminating rivals?

These questions are particularly puzzling considering how disastrously these temporary alliances ended for the parties involved. After supporting Stalin in his earlier conflicts, Bukharin, too, was eventually expelled from the party in 1929, arrested in 1937, and executed in 1938. In Uganda, while President Obote was out of the country in January 1971, Amin led a coup, beginning a reign of terror that would last eight years. Similary, in Germany, Hitler outmaneuvered his patrons, murdering many of Papen's associates and placing the Vice Chancellor himself under house arrest in the "Night of the Long Knives" (Kershaw 2014, 312). Upon Hindenburg's death, Hitler automatically became supreme commander of the armed forces (Kershaw 2014, 317). In each of these cases, both the leader and elite rival were jockeying for power, yet both decided to form an alliance. These alliances went badly in the end as, ultimately, the goals of both members was always power.

When a dictator comes to power, the greatest threat to his tenure is members of his own regime (Meng 2019; Myerson 2008; Svolik 2012). Eliminating the elites that make up his regime and consolidating more power for himself could not only protect the dictator from possible elite rebellion, but allow the dictator to implement his preferred policies, extract more resources from the state, and promote his own proteges. Successful consolidation does not occur without conflict, however: dictators purge elites (Keller and Wang; Sudduth 2017b) and elites resist to protect themselves (Goldring 2020; Luo and Rozenas 2019; Sudduth 2017a). Dictators who successfully marginalize elites become leaders of "consolidated" or "personalist" style regimes (Gandhi and Sumner 2020; Geddes, Wright, and Frantz 2014; Svolik 2009). If a power-hungry dictator wants to get rid of the members of his inner circle, why strengthen them with riches, positions, and prestige?

Sharing power[1] with the elites around him–whether through governmental or military positions, access to rents, or policy influence– has oft been thought to reduce conflict as the elites are less likely to coup (Svolik 2012; Meng 2019). While this maybe be true in some contexts, constrained peace is not the only situation in which we should observe the dictator working together with members of his inner circle. As the above examples show, a dictator on his conflict-laden path to consolidation can ally temporarily with elites in order to improve his chances of removing other threats to his power. When the elimination of elite rivals is subject to uncertainty, rewarding a coalition member in order to entice him to the dictator's side will help the dictator remove other elites in his way.

I model a muti-round contest in which a dictator can target a member of his regime

[1] I follow Arriola, DeVaro, and Meng 2017 and Meng 2019 view that access to material resources and prestige through governmental positions are the basis of power for the dictator and regime elites.

for a purge then make an offer to the non-targeted elite to share the spoils. While working together makes his side more likely to win the current conflict, rewarding his elite ally enough to entice him into the uncertain conflict directly reduces the dictator's chances of successfully eliminating that former ally in future consolidation attempts. Furthermore, the elite "ally" is willing to work with the power-hungry dictator, allowing the dictator to further grow the power that will be used against him in a future purge, for the right price. This model, in unpacking the process by which a dictator consolidates power, offers an alternative view of purges and elite eliminations (Keller and Wang; Sudduth 2017b). While previous studies similarly recognize the risks that elite purges entail and the potential benefits to the dictator of eliminating surrounding elites, the potential for elite alliances with the dictator against their colleagues has yet to be explored. Such strategic elite action in which coalitions can occur with the dictator, instead of just among elites, are understudied in this burgeoning field and are necessary for explaining the elite rivals who succeeded in overthrowing leaders (Hitler and Amin) and those who do not (Bukharin). I argue that when a dictator attempts to consolidate power vis-a-vis elites in an uncertain conflict environment, temporary coalitions with his future opponents are a necessary risk to further his own agenda.

3.1 Coalitions and Consolidation

A dictator hoping to extend his tenure and govern with limited constraints seeks to establish a consolidated regime (Geddes, Wright, and Frantz 2014, 2018). "Established" regimes are contrasted with those in which leaders are constrained by their elite allies that populate the upper echelons of the regime, often in military or party institutions (Svolik 2009). Regimes and leaders identified as unconstrained and "personalist" have lower failure rates than military regimes and military leaders (Geddes, Wright, and Frantz 2014).[2] In addition to po-

[2] While "consolidated," "established," and "personalist" are frequently used in different ways in the literature, the concepts all focus on a leader that is unconstrained by elites.

tentially lengthening his survival, an established leader benefits from the lack of constraint in other ways. More power vis-a-vis elites in the military or regime party allows a dictator to implement his preferred policies or promote his preferred personnel at will (Gandhi and Sumner 2020; Meng). By eliminating elites within their own ruling coalition, a dictator can simultaneously increase his own power and policy control and deter challengers. Dictators can use their control over personnel and budgets to limit the power of potential threats. Such personalized "despotic power"[3] is often achieved by fully removing rivals from office, instead promoting personal loyalists to positions of power or even taking those positions for yourself. In Malaysia, for example, after removing his main rival Anwar Ibrahim from office, Mahathir Mohamad added to his already powerful portfolio of prime minister and home minister the posts of finance minister and, effectively, deputy prime minister (Slater 2003).

After weighing the potential consequences of attempting to marginalize powerful elite allies against the benefits of amassing personalist power, the dictator must decide how best to undertake this strategy. While removing or eliminating all rivals and potential threats in the elite population might be the best option in terms of power consolidation, budget and personnel constraints will often prevent the dictator from taking such extreme measures. If the dictator's first priority is to retain office, as the loss of office could mean imprisonment, exile, or death, elite marginalization is a potentially risky endeavor (Svolik 2012). Whether or not a dictator, even a powerful one relative to his rivals, is successfully able to remove an elite target from office is influenced by a variety of factors outside the direct relationship between the leader and his "ally." Involvement in wars or international financial crises could suddenly shock the regime's access to financial and military resources. Domestically, intra-regime conflict outcomes further depend on people and whether they follow orders: uncertainty may stem from doubt over which side of the conflict the rank-and-file military or masses will support. General uncertainty in elite-dictator conflict outcomes means that

[3] Which Slater 2003 describes as "the power to decide" (82).

dictators often do not know in advance whether their attempted purge will succeed, even with full information about who their rival is and how powerful they are (see previous chapter for a discussion of a dictator's purge targeting under symmetric uncertainty).

How can a dictator counter such uncertainty and achieve consolidation? Bringing elite allies into a coalition against a rival will help the dictator achieve the overwhelming advantage needed to eliminate an elite with minimal risk. As Obote needed support in countering the growing threat of Bugandans in his conflict with Mutesa of Buganda, promoting and empowering Amin, a potential rival, was the best way for him to maintain power initially (Ingham 1994). Similarly, Stalin was not confident in his ability to remove the influential Kamenev and Zinoviev on his own: he took the time to build a coalition first to ensure their elimination spurred minimal backlash (Cohen 1980). Here the dictator and his elite ally are working together and sharing power (even temporarily) not in order to avoid conflict (Meng 2019), but in order to engage in conflict. By enticing one elite to support him against the other, the dictator is implementing a conventional "divide and rule" strategy (Acemoglu, Verdier, and Robinson 2004; Bates 2014; Luo and Rozenas 2019); however in this scenario it is not for the purposes of coup-proofing or preventing elites from becoming too threatening (Greitens 2016), but in order to achieve his own ends of consolidation.[4]

When faced with a heterogeneous elite where members of the ruling coalition have their own networks, skills, power-bases, and ideologies, the question is not only when to entice an elite into joining the dictator against his colleague, but who? This is particularly important when elite elimination attempts are subject to uncertainty: while a more prominent and powerful elite could be more difficult to successfully remove from his position, a dictator who succeeds in doing so will receive greater benefits. I build on a burgeoning literature that incorporates elite heterogeneity into models of elite-dictator conflict and cooperation

[4] While allowing the elites to engage in an elite-initiated conflict against the dictator (coup) would let the dictator potentially make offers to elites in order to avoid conflict, this is outside the scope of the current model. The dictator entices an elite to his side in the conflict against a rival even without such complications.

(Acemoglu, Egorov, and Sonin 2008; Kosterina 2017) in order to assess which elite (strong or weak) will be the target and which will be the dictator's ally.

Having a coalition of powerful elites against a rival makes him more likely to succeed against his current opponent, but a dictator that is liberal with rewards is bargaining his future rivalries for his current conflict. Empowering an elite coalition member with greater access to military and financial resources, personnel, and policy influence will make him all the more difficult to purge later. However, if the dictator is unable to purge his opponent now, there will be no future conflict to be concerned about. His desire to consolidate now means the dictator may be willing to empower a rival, despite the effect on his future conflict success. Empowering a rival elite, especially under the shadow of future intra-regime conflict, may be a risk for dictators. On the path to personalization of power vis-a-vis elites, however, it is a risk that many dictators seem to be willing to take.

Elite Support for Dictatorial Consolidation

From the elite's perspective, why would an elite support a dictator on his path towards consolidation? Support does not come freely: elites who support the dictator against other elite threats are rewarded handsomely.[5] If the dictator's offer is sufficient to make up for the risk of entering an uncertain conflict and possibly losing everything, a power-hungry elite will join such a coalition despite how much it will benefit the dictator. When the elite anticipates further conflict as the dictator continues to consolidate, we might expect him to avoid joining the dictator and easing his initial consolidation attempt. Why help your future attacker get more power? By joining the dictator in his conflict against another elite, he not only bolsters himself with the power he gets from winning the first round conflict, he is reducing the dictator's advantage against him. If he will be attacked in the future anyway,

[5] Note that rewards here are a one-time promotion to a governmental position or financial gain, not a commitment to future loyalty and rewards upon which the dictator may have an incentive to renege.

an elite will be better off splitting the gains of conflict now with the dictator to increase his own power and reduce the dictator's (relative to if the dictator got to keep all the gains himself), supporting his own chances of survival.

Incorporating power heterogeneity into this coalition and conflict model model is equally important from the elite's perspective: his identity not only determines whether he will be the dictator's target under certain conditions, but also how much he stands to gain from a successful elite elimination. Much like the dictator's consideration when determining whether to consolidate power through a purge, the elite joining the conflict takes into account the tradeoff between the reward from removing their colleague and the risk of being unsuccessful in the conflict. How much power their potential coalition partner, or potential enemy, brings to bear is a vital aspect of his decision to join the conflict and, if so, on which side.

I depart from existing explanations of elite-dictator coalitions in two important ways. First, by not tying the dictator's hands with a commitment to stop consolidating, power-sharing does not prevent future dictator-elite conflict (Meng 2019; Myerson 2008). Indeed, the threat of future conflict with the dictator will make the elite more likely to accept power from the dictator as such a transfer will make him more advantaged in the future conflict. Second, while conflict outcomes are uncertain (though a function of the relative power of the involved individuals), the dictator and elites all have full information about each others' power and intentions. Many historical explanations of elite-dictator coalitions and conflict between the same individuals suggest that they had incomplete information about each others' ambitions or their relative power and ability to eliminate the other (Ingham 1994; Kershaw 2014). In this line of reasoning, for a dictator to willingly strengthen his rival he must underestimate the rival's strength. But for the rival to be willing to support the dictator in his path of consolidation and help him achieve even more power, the rival must overestimate his own strength. Even in low-information environments, such divergent prior beliefs would be insupportable.[6]

While the opacity of authoritarian politics can lead to misperception over rivals' strengths, incomplete information is not the only explanation. I argue that a dictator, fully informed of the power and threat that his rival presents, is still willing to empower the elite in order to ensure his victory in the early stages of consolidation against other elites. Further, my theory simultaneously explains an elite's willingness to join a coalition with a dictator with whom he will be in conflict in the future. I show that under certain distributions of power among the ruling coalition, both the dictator and his future opponent are willing to work together *temporarily*, for the right price, despite the shadow of future conflict. It is not the case that Stalin was able to manipulate and "trick" Bukharin in a way that Obote was unable to do with Amin: both dictators chose to empower elites to support them against other elite problems (Kamenev and Zinoviev or Mutesa of Buganda, respectively) despite the uncertainty it created for their future consolidation.

3.1.1 Model Setup

Three players, a dictator (D), an elite with a high initial endowment of power (H), and an elite with a lower initial endowment of power (L), together form a regime. Each player's type,τ_i, is his power endowment. H's initial endowment of power is fixed at H ($\tau_H \equiv H$); the dictator's endowment is $d > 0$ greater than H's ($\tau_D \equiv H + d$) and L's endowment is $w \in (0, H)$ less than H's ($\tau_L \equiv H - w$). The dictator has the most power initially (which is why he is the dictator) and the high elite has more initial power than the low elite. Thus d can be interpreted as the dictator's initial power advantage over the other two elites while

[6] This is not to say that incomplete information regarding individuals' strength and conflict actions is impossible. Indeed, in some cases a lack of information may lead to conflict. My model shows that incomplete information is not the only explanation for conflict and may be more applicable in regimes where there are institutions with the purpose of avoiding such information-induced conflicts (Boix and Svolik 2013) and yet conflicts are still observed (see Luo and Rozenas 2019 for a summary of elite-dictator conflicts despite institutions).

w denotes the relative difference between elites H and L.

Play takes place over the course of two rounds. First, the dictator chooses which elite to target for elimination or does not initiate conflict, ending the round. If a target is chosen, the dictator can make a take-it-or-leave-it offer, $x \in [0, \bar{x}]$, his post-conflict budget constraint, to the non-targeted elite to share the spoils of the conflict (conditional on winning) with that elite in exchange for the elite's support in a coalition against the target.[7] The dictator's budget, $\bar{x}$, is the total amount of power he would have if he won the conflict: the combined power of the dictator and his target. The non-targeted elite can choose to accept the dictator's offer and join a coalition with the dictator, join the targeted elite, or remain out of the conflict. The dictator (or coalition) and target(s) then participate in a contest where the probability that each side wins is the difference between their relative power plus mean-zero noise; e.g. participant (or coalition) i wins the conflict if $\tau_i \geq \tau_j + \epsilon_t$ where $\epsilon_t \sim U[-a, a]$ independent of the round. Thus the probability that participant or coalition i wins is $F_\epsilon(\tau_i - \tau_j)$. While the likelihood of winning is based on the power differential between the opponents, I assume all conflicts are uncertain.[8] If a coalition formed, whichever side won the conflict splits the power of the loser(s) according to the agreed upon division. If no coalition formed, all of the loser's power is transferred to the winner.

The first round ends, and whichever remaining player has the most power now becomes the dictator and can choose to initiate conflict with the other remaining member of the regime (if there is a second member–if there is only one player remaining in round two, the game ends). If three players remain, the dictator may make a coalition offer as above. If conflict is initiated, it occurs as previously described, with a new, independent draw of ϵ. The power of the loser(s) is transferred to the winner(s), potentially shared among a coalition if they win together.

[7] I analyze an alternative bargaining protocol in the appendix for robustness.
[8] In particular, a is sufficiently large.

Sequence of Play

$t = 1$

- The dictator, D chooses to target H, target L, or no target. If no target is selected, the round ends.
- The dictator, D, chooses an offer $x \in [0, \bar{x}_1]$ to make to the non-targeted elite. $\bar{x}_1$ is the dictator's post-conflict budget constraint in the first round, which depends on the selected first round target.
- The non-targeted elite chooses whether to participate in the conflict on the side of the dictator, the target, or not participate.
- The dictator (or coalition) and target(s) participate in a contest where participant(s) i win if $\tau_i \geq \tau_j + \epsilon_1$ where $\epsilon_t \sim U[-a, a]$
- Power is transferred from the loser(s) to the winner(s), split among a winning coalition if an offer greater than 0 was made and accepted. Whichever remaining player now has the most power is the dictator. If only one player remains, the game ends.

$t = 2$

- The dictator chooses to target one of the remaining players, or no target. If no target is selected, the game ends.
- If three players remain, the dictator chooses an offer $x \in [0, \bar{x}_2]$ to make to the non-targeted elite.
- If there is a non-targeted elite, he chooses whether to participate in the conflict on the side of the dictator, the target, or not participate.

- The dictator (or coalition) and target(s) participate in a contest where participant(s) i win if $\tau_i \geq \tau_j + \epsilon_2$ where $\epsilon_t \sim U[-a, a]$
- If the dictator (or dictator's coalition) wins, the loser(s)'s power is transferred to the winner(s), split according to the accepted offer if a coalition formed. The game ends.

Payoffs

All members of the ruling group derive utility from their endowments of power at the end of the game, which are a function of the dictator's target choice(s), offer(s), and the coalition decisions of the elites across both rounds. These actions are, in turn, a function of each individual's relative power and the uncertainty of the conflict environment.

$$u_i(H, d, w, a) = \tau_{i,t=2}$$

This formalization of elite-dictator conflict and coalition formation yield a few substantive scope conditions. First, the dictator, as the conflict "agenda-setter," must be able to target an elite for a purge. This could include introducing articles of expulsion to a legislature, ordering police to make an arrest, or prompting a paramilitary to attempt assassination. I do not assume is that the dictator must be such a powerful strongman that he can unilaterally remove and elite from the position in the inner circle and usurp their power with certainty. Second, the dictator commits to his course of conflict before making an offer to the non-targeted elite. If the elite rejects his spoil-sharing coalition offer, the dictator will still engage in conflict with the purge target. This is similar to a public accusation of treason or corruption or a movement to arrest the target before any attempts at coalition formation.[9] Lastly, in order to focus on the dictator's conflict-initiation and power-sharing with elite rivals, elite-initiated conflicts (i.e. coups) are not possible. If the dictator does not initiate

[9] The alternative approach is discussed in the appendix

a consolidation purge of an elite, there will not be elite-dictator conflict.[10]

3.1.2 Results

The game is solved using backwards induction for sub-game perfect Nash equilibria. Note a second round in which all three players remain is equivalent to a one-shot version of the elimination game in which there is no shadow of future conflict.

Three Remaining Players

The last action of the game, and therefore the first to be addressed, is the non-targeted elite's decision to join a coalition or stay out of the conflict. Due to the uncertain nature of all conflicts, if no offer is made to him, either elite would prefer to stay out of the conflict and maintain his status quo power rather than risk a conflict for no additional benefit. An offer of power that the dictator makes to the non-targeted elite must be high enough to compensate the elite for the risk he is taking on by joining the conflict; I term this minimum power needed to induce an elite to join the conflict the *join condition*, stated in Result 1. As the elites are heterogeneous, they each have their own thresholds that are a function of both their status quo power (what they would keep if they stayed out of the conflict), and the probability that their coalition with the dictator would successfully defeat the targeted elite. The dictator, however, is not necessarily willing to share sufficient power to get an elite to join him. Instead, the dictator has a maximal offer, termed the *offer condition*, he is willing to make to the non-targeted elite; if he had to pay any more than this maximum, he would rather just fight alone (see Result 2).

Result 1. *If L is targeted, H will join the dictator if D offers $x_H \geq \underline{x_H} \equiv \frac{H(1-F_\epsilon(H+d+w))}{F_\epsilon(H+d+w)}$,*

[10] Obviously this is a simplification of the world, but the focus of this paper is on who the dictator targets and coalesces with and the response of the elites to this behavior in terms of accepting the dictator's inducements, not all observed elite-dictator conflict.

otherwise he will not join the conflict. If H is targeted, L will join the dictator if D offers $x_L \geq \underline{x_L} \equiv \frac{(H-w)(1-F_\epsilon(H+d-w))}{F_\epsilon(H+d-w)}$ *otherwise he will not join the conflict.*

Result 2. *If L is targeted, the maximum offer D is willing to offer H is* $\overline{x_H} \equiv \frac{(2H+d-w)(F_\epsilon(H+d+w)-F_\epsilon(d+w))}{F_\epsilon(H+d+w)}$. *If H is targeted, the maximum offer D is willing to offer L is* $\overline{x_L} \equiv \frac{(2H+d)(F_\epsilon(H+d-w)-F_\epsilon(d))}{F_\epsilon(H+d-w)}$.

Given the expected coalition decisions and minimal offers needed to induce such coalitions, will the dictator initiate conflict? If so, which elite will he target? High conflict uncertainty benefits the target and mitigates the dictator's power advantage. Even when a coalition member would increase his chance of victory, an uncertain conflict environment means that the increase in win probability is not large enough to counteract the decrease in benefits from the offered power transfer. The dictator prefers maintaining his power and avoiding risky conflict, therefore he chooses no target. If the dictator's advantage is great enough relative to the conflict uncertainty, however, the dictator is willing to choose a target and make a positive offer of power sharing to the non-targeted elite. Under this level of uncertainty, both coalitions are available: the dictator could come to a power-sharing agreement with either H or L. Additionally, both of these target and coalition options are strictly preferred to no conflict. In choosing between targets and the resulting coalition, the dictator chooses to target L and make a sufficient coalition offer to H.

Proposition 3. *If the dictator's advantage relative to the elites and uncertainty is sufficiently high* ($d > a+w-3H$)*, D will target L and make H a sufficient offer* $\underline{x_H}$ *such that H joins the dictator's coalition. If the dictator's relative advantage is sufficiently low* ($d < a+w-3H$)*, the dictator will choose no target and no conflict will occur.*

Because H is more powerful, the dictator is maximizing his win probability by joining forces with H ($F_\epsilon(H+d+w) > F_\epsilon(H+d-w)$). Despite a lower benefit from conflict in terms of elite elimination (defeating L gives the dictator less power than defeating H), H's minimal sufficient power-sharing offer is lower than L's (because of his higher likelihood of winning) so the dictator does not need to transfer as much. Note that the dictator is empowering his most serious rival, the most powerful elite in the regime, in order to support his attack on the least powerful member of the regime. At this point, however, there is no shadow of future conflict that the dictator must guard against: after they defeat L, D and H share power without issue. This changes below, when future conflict alters the incentives of both the dictator and his coalition partner.

This general result–that the dictator will coalesce with the higher-powered elite against the lower-powered elite for a large portion of the parameter space– is robust to an alternative bargaining protocol in which the dictator does not commit to a purge target. In this alternating offer Rubinstein bargaining version, the dictator can bargain with both elites and thus use the threat of coalescing with the *other* member of the regime to fight the elite in the negotiation. This alternative protocol and how the results mirror Proposition 1 are discussed in the Appendix.

Two Remaining Players

If there are two players remaining in the second round, there must have been a first round conflict that eliminated one of the members of the regime.[11] Regardless of who the second round dictator is, regardless of how much power was transferred after the first round, regardless of the identity of the remaining regime member, the dictator will target the remaining regime member for conflict if uncertainty is sufficiently low relative to the dictator's advantage. Note that the dictator's advantage is no longer necessarily D's advantage: the power d

[11] There are four ways for there to be exactly two players remaining: (1) H and D fought alone (or L joined and received no transfer); (2) L and D fought alone (or H joined and received no transfer); (3) a coalition of D and L fought H, so now D and L remain with some power transferred to L; (4) a coalition of D and H fought L, leaving D and H with some power transferred to H. If a sufficient offer was made and a coalition formed in the first round and won, some power was transferred to the elite member of the coalition according to the bargain.

now belongs to whoever won the first round conflict and is now the second round dictator. If the conflict environment is too uncertain, the dictator (whoever he is) will maintain the power he gained from the first round rather than initiate a second round conflict.

Proposition 4. *If the dictator's advantage relative to the elites and uncertainty is sufficiently high ($d > a+w-3H$), the dictator will target the remaining regime member. If the dictator's relative advantage is sufficiently low ($d < a+w-3H$), the dictator will choose no target and no conflict will occur.*

First Round Offers and Targets

The first round of conflict must take into account the potential for conflict in the second round: there is a shadow of future conflict affecting the players' decisions to join coalitions, make offers, and choose targets. Sharing power with an elite is now sharing power with a future rival, if conflict will occur in the second round. Based on the above result, we know that if uncertainty is particularly high, no conflict will occur in the second round regardless of which players remain. With no anticipated second round conflict, the elites' decision to join the dictator is the same as the one-shot version: if the dictator was to make him a sufficient offer, the non-targeted elite would join his coalition. However, in this parameter range, the dictator is not willing to make such an offer; furthermore, the dictator is not even willing to initiate a one-on-one conflict with either elite.

Proposition 5. *If the dictator's advantage relative to uncertainty is sufficiently low ($d < a+w-3H$), there will be no conflict. The first round join conditions for the elites are the same as three players in round two ($\underline{x_H}, \underline{x_L}$), but the dictator will not initiate any conflict.*

If the dictator's advantage is sufficiently greater than the conflict uncertainty, there will be conflict in the second round, the anticipation of which must be taken into account in both the targeting and offer decisions of the dictator as well as the coalition decision of the elite. Recall that any offers and power transfers made in the first round directly affect the conflict success probabilities in the second round. By joining the dictator's coalition in the first round, the non-targeted elite is taking on the risk of not only the first round conflict, but the second round conflict in which he, if his coalition won in the first round, will definitely be a participant. However, the potential benefits of such conflict involvement are extreme: instead of keeping his own power plus whatever transfer the dictator offered him as was the case in the one-shot version, the elite now has the opportunity to win all the combined power of the regime.

Result 3. *If the dictator's relative advantage is sufficiently high* ($d > a + w - 3H$)*: if* L *is targeted,* H *will join the dictator if* D *offers* $x_H \geq \underline{x'_H} \equiv \frac{(a-H-d-w)(a-H-d+w)}{2(H+d+w+a)}$*, otherwise he will not join the conflict. If* H *is targeted,* L *will join the dictator if* D *offers* $x_L \geq \underline{x'_L} \equiv \frac{(a-H-d-w)(a-H-d+w)}{2(a+H+d-w)}$*, otherwise he will not join the conflict. In this parameter range,* $\underline{x_H} > \underline{x'_H}$ *and* $\underline{x_L} > \underline{x'_L}$*.*

As before, the elites are not willing to join the dictator and take on the risk of a first round conflict without some compensation. If they do not join him, they will still be targeted for conflict in the second round and the full regime's power is still winnable. Why, then, would a non-targeted elite join a first round conflict on the side of the dictator knowing he will have to fight a second round conflict as well? While the ultimate prize is the same regardless of whether he fights both rounds or stays out until being targeted in the second round, the probability that he wins is not the same. By accepting a power transfer and taking on some of the risk of the first conflict, the non-targeted elite is not only increasing his power in the second round conflict but **decreasing the dictator's**. If he stays on the sidelines, whoever wins the first round will have a large power advantage over him because they would have the combined power of both the dictator and the first target. Taking a power transfer doubly reduces this power differential as it both increases the non-target's power by x_i and decreases the dictator's power by x_i. The elite's chance of winning the second round increases enough to not only make joining the dictator worth the risk, but to make the minimal sufficient offer lower than his join condition in a one-time conflict.

The dictator's decision is two-fold: he must choose who to target (if anyone) as well as whether to make the non-targeted elite their minimal sufficient offer. While the dictator does reduce his chances of winning the second round conflict by making a transfer to the non-targeted elite (who is his future opponent), no second round conflict would occur (for him) if he does not win the first. The dictator is willing to make a minimal sufficient offer to the non-targeted elite in order to increase his chances of winning the first round conflict despite the mitigation of his second round advantage. Note from above that $\underline{x'_H} < \underline{x'_L}$: the minimal sufficient offer to get H to join the dictator's coalition is less than the minimal sufficient offer to get L to join his coalition. As H is already closer to the dictator in power than L, he does not need as high of a transfer to agree to join the conflict. Thus while D's initial power advantage over L is greater, the higher transfer that must be made to L reduces this advantage to the point that the dictator is perfectly indifferent between initially targeting H and making L an offer and initially targeting L and making H an offer. While a coalition between H and D is stronger in the first round, D's probability of winning the second round is lower, so the total expected utility is the same as an initial coalition between L and D.

Proposition 6. *Assume the dictator's relative advantage is sufficiently large ($d > a + w - 3H$)). Offering a minimal sufficient transfer to induce a coalition ($\underline{x'_H}, \underline{x'_L}$) dominates targeting an elite and failing to form a coalition. When uncertainty is sufficiently low, targeting an elite and making a minimal sufficient offer dominates no conflict initiation.*

While making an offer and a coalition is preferred to fighting alone, it is not necessarily preferred to initiating no conflict whatsoever. As uncertainty approaches the upper case bound, no target is preferred. On the lower end of uncertainty, however, the dictator prefers to initiate conflict with one of the elites, forming a coalition with the other, rather than no conflict. Recall, however, that he is indifferent between his target and coalition options. Joining forces with the most powerful elite regime member is no longer a dominant strategy as it was in the one-shot version.

3.2 Conclusions

Why would a dictator, limited by the elites around him, empower a rival with whom he will fight in the future? Why would an elite form a coalition with a dictator attempting to consolidate power, easing his path to consolidation? While historians have pointed to the underestimation of the rival's strength or overestimation of the rival's loyalty as possible explanations, my theoretical model explains both the dictator's empowerment of a rival and the rival's willingness to support a dictator on a path of consolidation. What does drive the willingness of a dictator and his rival to work together is the general uncertainty that the dictator faces in his path to consolidation. Sudden changes in finances, mass sentiment, or arbitrary difficulties preventing arrest and detention can make an attempt at elite elimination fail. The dictator, however, can mitigate these uncertainties when he builds a coalition of overwhelming power. Thus the dictator is willing to take a risk and empower a rival in order to increase his chances of consolidation now, knowing full well that his future conflict with said rival will be affected. The elite rival, aware that he will have to face the dictator in a conflict regardless, would rather take the additional power being offered, making the future conflict more even between the two parties.

The path to dictatorial consolidation of power is littered with risks, but the potential rewards of being an unchallenged ruler, in control of policy, personnel, and resources, may be worth the conflict. For dictators with multiple elite rivals to remove, forming transitory coalitions with rivals can be the best strategy for risk mitigation. As was the case with Stalin's alliance with Bukharin, working with an elite rival was needed to successfully eliminate other rivals; in the end, such dictators are still able to achieve high levels of personalization, removing those rivals who they had previously empowered. Elite empowerment is not without risk, however, as Obote found with his disastrous reliance on Idi Amin.

By formalizing a dynamic consolidation process with global conflict uncertainty, I have furthered our understanding of elite-dictator relations with both coalitions and conflict. Institutions that reduce informational asymmetries among the dictator and elites does not necessarily imply stable, long-lasting power-sharing agreements. Even with complete information about the relative strengths of potential opponents, conflicts and coalitions of convenience can still emerge and a dictator may still successfully personalize power. The underlying balance of power and global conflict uncertainty, not only asymmetrical information about the strength of each actor, can explain not only the seemingly bizarre behavior of elite coalition behavior against their own colleagues during periods of intra-regime conflict, but the dictator's willingness to dole out offices, money, *dachas*, and other goods to his supporters only to seize them upon the next round of treason accusations.

3.3 Appendix: Alternative Bargaining Protocol

Is it reasonable that the dictator can make a take it or leave it offer to an elite? Is it reasonable to assume that the dictator is committed to a conflict target regardless of whether he induces the support of the other elite? While the mechanic of divide and rule (targeting one, coalition with the other) occurs in the above model, allowing the dictator to play the elites off of one another with the threat of being the purge target in the same round captures the spirit of divide and rule.

Model Setup

Three players, a dictator (D), an elite with a high initial endowment of power (H), and an elite with a lower initial endowment of power (L), together form a regime. Each player's type,τ_i, is his power endowment. H's initial endowment of power is fixed at H ($\tau_H \equiv H$); the dictator's endowment is $d > 0$ greater than H's ($\tau_D \equiv H + d$) and L's endowment is $w \in (0, H)$ less than H's ($\tau_L \equiv H - w$). The dictator has the most power initially (which is why he is the dictator) and the high elite has more initial power than the low elite. Thus d can be interpreted as the dictator's initial power advantage over the other two elites while w denotes the relative difference between elites H and L.

The dictator chooses which elite to bargain with first. He makes the first elite an offer $x_i \in [0, 1]$ to share the spoils of conflict, the power of elite $\neg i$, where the elite i keeps $x_i(\tau_{\neg i})$ and the dictator keeps $(1 - x_i)(\tau_{\neg i})$. If elite i accepts, they together fight elite $\neg i$ in a contest where the probability that each side wins is the difference between their relative power plus mean-zero noise; i.e. coalition dictator and i wins the conflict if $\tau_D + \tau_i \geq \tau_{\neg i} + \epsilon_t$ where $\epsilon_t \sim U[-a, a]$ independent of the round. If no agreement is reached (bargaining breaks down), the dictator will then bargain with the other elite, $\neg i$, making an offer of $x_{\neg i} \in [0, 1]$ of sharing the spoils of conflict against elite i. If they agree, the contest occurs as described above with the dictator and his coalition partner $\neg i$ against elite i. If this second bargain does not reach an agreement, the dictator has the choice to fight either elite on his own (using the same contest function with noise) or not initiate conflict.

I utilize an alternating offer Rubinstein bargaining protocol with risk of breakdown. Whenever an offer of division is rejected, with probability $\delta \in [0.1]$ the other party makes a counteroffer or bargaining breaks down and the dictator must stand for a mass election without elite support in the district with probability $1 - \delta$. If the dictator and elite fail to reach a successful bargain (or bargaining breaks down before they are able to come to an agreement), the dictator moves to the next elite. When no elites remain, the dictator chooses whether to initiate a one-on-one conflict, keeping all the spoils of the elite elimination for himself if he succeeds, or chooses no target and does not initiate conflict. Nature then draws ϵ, the conflict occurs (if there is one), and payoffs are distributed.

Results

The relevant equilibrium is subgame perfect Nash. The dictator and each elite utilize stationary strategies. The dictator proposes x proportion of the conflict spoils to the elite, keeping $1 - x$ for himself, every period and accepts the elite's proposal if and only if $y \geq y'$. The elite proposes y proportion of the conflict spoils to the dictator, keeping $1 - y$ for himself, every period and accepts the dictator's proposal if and only if $x \geq x'$. The outside options (what the expected utilities of the players are if bargaining fails) are determined by what the dictator will do at the end of the game if no coalitions form as well as the other elite's expected behavior. In equilibrium, an agreement is reached between the first elite and the dictator who then fight the other elite. By choosing the bargaining order, the dictator is choosing who to coalesce with and who to target.[12]

[12] But more indirectly than in the previous model. Indeed it is the off-path option of targeting/coalescing with the other player that affects the split of spoils.

The full equilibrium characterization is in the formal appendix (it is defined by the optimal offer and counter offer for the dictator and elite in each individual bargain, bargaining order subgames, and the dictator's outside option (lone targeting decision) and is therefore very long formally). The dictator has a different targeting/coalition strategy depending on his advantage relative to the elites and conflict uncertainty that defines his true outside option (unilateral conflict). When the dictator is relatively weak ($a \in (H+d+w, 2H-w)$), if he was going it alone he would target L, the weaker elite. In this case, he prefers to bargain L first, using the threat of joining with H against him, inducing him into a coalition against H. A dictator of middling strength relative to uncertainty ($a \in (2H-w, 2H+d)$) would target H for a one-on-one conflict. He prefers to bargain with H first, using the threat of fighting him with L, inducing him into a coalition against L. A dictator in a highly uncertain conflict environment ($a > 2H+d$) would avoid conflict if he had to go it alone. When he can play the elites off one another, he bargains with L first and uses this coalition against H. How the dictator's equilibrium strategy changes as uncertainty increases can be seen in Figure 3.1.

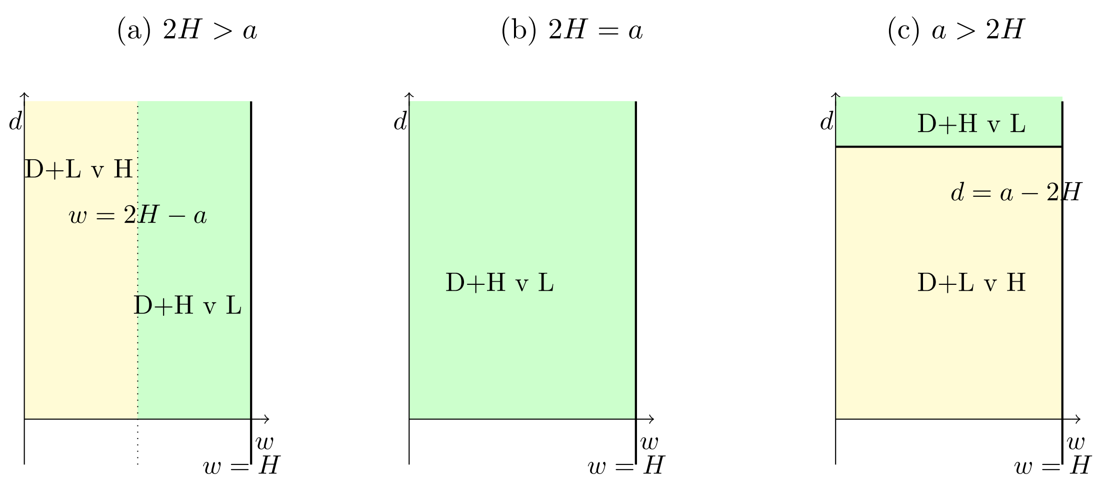


Figure 3.1: Single Conflict with Bargaining for Coalition

Comparing Bargaining Protocols

These two approaches (take it or leave it offer with conflict commitment versus bargaining) have different potential drawbacks and benefits. In the bargaining version, the dictator is able to play the elites off of one another because he has the option of bargaining with the other person– he has not committed to a target yet. For most of the parameter space (outside of the tails of extremely low uncertainty and high uncertainty), the dictator's coalition and targeting behavior is the same as the main model: he makes an offer to, and forms a coalition with, H and they fight L.

This bargaining protocol, in which the dictator can credibly threaten to not only target the elite to whom he is making an offer but fight the elite with a coalition including the third member of the regime, clearly benefits the dictator. Because the likelihood of loss is higher if the dictator's offer is refused is significantly higher than it is with the one-shot take it or leave it offer discussed in the main text, the dictator can induce a coalition across the full domain of uncertainty. Thus in the parameter space where, in the main model, we would expect the dictator to not initiate any conflict in order to protect his status quo power, instead the dictator will create a coalition with the low-powered elite, L, against H.

While this protocol is more generous to the dictator in his ability to play the elites off of one another as potential targets, limiting the options of the elites to only accepting or rejecting the dictator's office may not be fair. Not only does this protocol preclude elites from joining one another in the coalition, it also does not allow them to stay outside the conflict as, on path, they will either be an aggressor or a target. With this dichotomous choice– aggressor or target— it is not surprising that each elite would accept the dictator's offer to join his coalition for some compensation. When the dictator is initiating a purge, why would an elite join him in a coalition? This bargaining protocol shows that the elite will join to avoid being a target in addition to whatever spoils the dictator shares. In the main model, however, the elite is willing to join the dictator without the additional inducement of being a potential target of the purge. Instead, it is only the offer to share the victim's power that induces the elite to join a coalition. It is a harder case to induce the elite into a coalition. As the objective is to consider the perspectives of the dictator empowering a rival and the elite supporting a power-hungry dictator, the bargaining protocol that is more conservative in inducing the elite coalition best supports this objective.

3.4 Formal Appendix A: Main Model

Round 2: Three Players Remaining

Result 1. *If L is targeted, H will join the dictator if D offers* $x_H \geq \underline{x_H} \equiv \frac{H(1-F_\epsilon(H+d+w))}{F_\epsilon(H+d+w)}$, *otherwise he will not join the conflict. If H is targeted, L will join the dictator if D offers* $x_L \geq \underline{x_L} \equiv \frac{(H-w)(1-F_\epsilon(H+d-w))}{F_\epsilon(H+d-w)}$ *otherwise he will not join the conflict.*

Result 2. *If L is targeted, the maximum offer D is willing to offer H is* $\overline{x_H} \equiv \frac{(2H+d-w)(F_\epsilon(H+d+w)-F_\epsilon(d+w))}{F_\epsilon(H+d+w)}$. *If H is targeted, the maximum offer D is willing to offer L is* $\overline{x_L} \equiv \frac{(2H+d)(F_\epsilon(H+d-w)-F_\epsilon(d))}{F_\epsilon(H+d-w)}$.

Proof. Assume the dictator is strong ($d > H - w$) and all possible conflicts are uncertain ($a > H + d + w$).

The dictator targets L and makes an offer to H : $x_H \in [0, 2H + d - w]$

H can join a coalition with the dictator, the target L, or stay out of the conflict.

$U_H(joinD) = F_\epsilon(H + d + w)\,(H + x_H)$

$U_H(joinL) = (1 - F_\epsilon(d + w - H))\,(H)$

$U_H(\neg join) = H$

Note not joining strictly dominates joining L due to assumed uncertainty

Join D if $F_\epsilon(H + d + w)\,(H + x_H) > H$

$x_H > \frac{H(1-F_\epsilon(H+d+w))}{F_\epsilon(H+d+w)}$ denote this minimal offer H is willing to accept as $\underline{x_H}$

BR_H: join D if $x_H \geq \underline{x_H}$, else $\neg$join

Given this response function, what offer will the dictator make?

$U_D\left(x_H\right) = F_\epsilon(H+d+w)\left(2H+d-w-x_H\right)$ if $x_H > \underline{x}_H$ (the offer is sufficient)

$U_D\left(x_H\right) = F_\epsilon(d+w)(2H+d-w)$ if $x_H < \underline{x}_H$(the offer is insufficient)

The dictator prefers to make a sufficient offer if $F_\epsilon(H+d+w)\left(2H+d-w-x_H\right) > F_\epsilon(d+w)(2H+d-w)$

$x_H < \frac{(2H+d-w)(F_c(H+d+w)-F_\epsilon(d+w))}{F_\epsilon(H+d+w)}$ denote this maximal offer the dictator is willing to make as $\overline{x_H}$

Lemma 7. *Given the dictator's maximal willingness to pay is greater than the elite's minimal offer to join, the dictator will offer the elite's minimum sufficient offer.*

Proof. Assume $\overline{x_H} \geq x_H$. Recall H will join the dictator's coalition for all offers $x_H \geq \underline{x_H}$. Would the dictator ever make an offer greater than $\underline{x_H}$?

$U_D\left(\underline{x_H}\right) = F_\epsilon(H+d+w)\left(2H+d-w-\underline{x_H}\right)$

$U_D\left(\underline{x_H}+\eta\right) = F_\epsilon(H+d+w)\left(2H+d-w-\left(\underline{x_H}+\eta\right)\right)$ where $\eta > 0$

$F_\epsilon(H+d+w)\left(2H+d-w-\underline{x_H}\right) > F_\epsilon(H+d+w)\left(2H+d-w-\left(\underline{x_H}+\eta\right)\right)$

$2H+d-w-\underline{x_H} > 2H+d-w-\left(\underline{x_H}+\eta\right)$

$0 > -\eta$ this is true by definition.

Therefore if the dictator is willing to make a sufficient offer, he will only ever make the minimal sufficient offer to get H to join him. □

BR_D : offer $\underline{x_H}$ if $\overline{x_H} \geq \underline{x_H}$, else indifferent among all $x_H < \overline{x_H}$

Using $\epsilon \sim U[-a,a], \overline{x_H} \geq \underline{x_H}$ if $a \leq 3H+2d$

L targeted subgame: If $a \in (H + d + w, 3H + 2d]$, D offer $\underline{x_H}$, H join D.

If $a > 3H + 2d$, D offer any $x_H < \underline{x_H}$, H doesn't join the conflict.

The dictator targets H and makes an offer to L$x_L \in [0, 2H + d]$

L can join a coalition with the dictator, join with the targeted H, or stay out of the conflict.

$U_L(\text{join}D) = F_\epsilon(H + d - w)\,(H - w + x_L)$

$U_L(\text{join}H) = (1 - F_\epsilon(d + w - H))\,(H - w)$

$U_L(\neg\text{join}) = H - w$

Note not joining dominates joining a coalition with H under assumed uncertainty.

Join D if $F_\epsilon(H + d - w)\,(H - w + x_L) > H - w$

$x_L > \frac{(H-w)(1-F_c(H+d-w))}{F_c(H+d-w)}$ denote this minimal offer L is willing to accept to join the dictator as $\underline{x_L}$

BR_L: join D if $x_L \geq \underline{x_L}$ else $\neg$ join

Given this expected response function, what will the dictator offer?

$U_D\,(x_L) = F_\epsilon(H + d - w)\,(2H + d - x_L)$ if $x_L > \underline{x_L}$ (the offer is sufficient)

$U_D\,(x_L) = F_\epsilon(d)(2H + d)$ if $x_L < \underline{x_L}$(the offer is insufficient)

Make a sufficient offer if $F_\epsilon(H + d - w)\,(2H + d - x_L)$ if $x_L > \underline{x_L} > F_\epsilon(d)(2H + d)$

$x_L < \frac{(2H+d)(F_c(H+d-w)-F_c(d))}{F_c(H+d-w)}$ denote this maximal offer the dictator is willing to make as $\overline{x_L}$

Lemma 8. *Given the dictator's maximal willingness to pay is greater than the elite's minimal offer to join, the dictator will offer the elite's minimum sufficient offer.*

Proof. Assume $\overline{x_L} \geq \underline{x_L}$. Recall L will join the dictator's coalition for all offers $x_L \geq \underline{x_L}$. Would the dictator ever make an offer greater than $\underline{x_L}$?

$U_D\left(\underline{x_L}\right) = F_\epsilon(H + d - w)\left(2H + d - \underline{x_L}\right)$

$U_D\left(\underline{x_L}+\eta\right) = F_\epsilon(H+d-w)\left(2H+d-\left(\underline{x_L}+\eta\right)\right)$ where $\eta > 0$

$F_\epsilon(H+d-w)\left(2H+d-\underline{x_L}\right) > F_\epsilon(H+d-w)\left(2H+d-\left(\underline{x_L}+\eta\right)\right)$

$2H+d-\underline{x_L} > 2H+d-\left(\underline{x_L}+\eta\right)$

$0 > -\eta$ this is true by definition.

Therefore if the dictator is willing to make a sufficient offer, he will only ever make the minimal sufficient offer to get L to join him. □

BR_D : offer $\underline{x_L}$ if $\overline{x_L} \geq \underline{x_L}$, else indifferent among all $x_L < \underline{x_L}$

Using $\epsilon \sim U[-a,a], \overline{x_L} \geq \underline{x_L}$ if $a < 3H+2d-w$

H targeted subgame: if $a \in (H+d+w, 3H+2d-w]$, D offers $\underline{x_L}$, L joins D.

If $a > 3H+2d-w$, D offers any $x_L < \underline{x_L}$, L $\neg$ join. □

Targeting Behavior

Proposition 3. *If the dictator's advantage relative to the elites and uncertainty is sufficiently high ($d > a+w-3H$), D will target L and make H a sufficient offer $\underline{x_H}$ such that H joins the dictator's coalition. If the dictator's relative advantage is sufficiently low ($d < a+w-3H$), the dictator will choose no target and no conflict will occur.*

Proof. $a \in (H+d+w, 3H+2d-w)$ both coalitions available

$U_D(\emptyset) = H+d$

$U_D(H) = F_\epsilon(H+d-w)\left(2H+d-\underline{x_L}\right)$ make sufficient minimal offer to L, L joins D

$U_D(L) = F_\epsilon(H+d+w)\left(2H+d-w-\underline{x_H}\right)$ make sufficient minimal offer to L, H joins D

Target H vs No Target

$F_\epsilon(H+d-w)\left(2H+d-\underline{x_L}\right) > H+d$

$F_\epsilon(H+d-w)(3H+d-w) > 2H+d-w$

Using $\epsilon \sim U[-a,a]$:

$a < 3H + d - w$ if this holds, targeting H and offering $\underline{x_L}$ is preferred to no target.

Target L vs No Target

$F_\epsilon(H + d + w)\left(2H + d - w - \underline{x_H}\right) > H + d$

$F_\epsilon(H + d + w)(3H + d - w) > 2H + d$

Using $\epsilon \sim U[-a,a]$

$a < 3H + d - w$ if this holds, targeting L and offering $\underline{x_H}$ is preferred to no target.

Target H vs Target L

Targeting H preferred to Targeting L if:

$F_\epsilon(H + d - w)\left(2H + d - \underline{x_L}\right) > F_\epsilon(H + d + w)\left(2H + d - w - \underline{x_H}\right)$

$F_\epsilon(H + d - w)(3H + d - w) - (H - w) > F_\epsilon(H + d + w)(3H + d - w) - H$

Using $\epsilon \sim U[-a,a]$:

$a > 3H + d - w$ if this holds, H is preferred to targeting L. So L is a preferred target when $a < 3H + d - w$

Equilibrium: if $a \in (H + d + w, 3H + d - w)$, Target L, offer $\underline{x_H}$ to H, H joins D.

If $a \in (3H + d - w, 3H + 2d - w)$, the dictator chooses no target, no conflict occurs

$a \in (3H + 2d - w, 3H + 2d)$ Only a coalition with H is possible, the dictator and L cannot come to a power-sharing agreement.

$U_D(\emptyset) = H + d$

$U_D(H) = F_\epsilon(d)(2H + d)$L will not join D

$U_D(L) = F_\epsilon(H + d + w)\left(2H + d - w - \underline{x_H}\right)$ make sufficient minimal offer to H, H joins D

Target H vs No target

Target H if $F_\epsilon(d)(2H + d) > H + d$

$a < 2H + d$ If this holds, targeting H is preferred to no target. However, this does not hold in this range of a therefore no target is preferred.

Target L vs No Target

$F_\epsilon(H + d + w)\left(2H + d - w - \underline{x_H}\right) > H + d$

$F_\epsilon(H + d + w)(3H + d - w) > 2H + d$

Using $\epsilon \sim U[-a, a]$

$a < 3H + d - w$ if this holds, targeting L and offering x_H is preferred to no target.This does not hold here, therefore no target no conflict.

Equilibrium: for $a \in (3H + 2d - w, 3H + 2d)$, the dictator will choose no target and no conflict will occur.

$a > 3H + 2d$ no coalitions would form.

$U_D(\emptyset) = H + d$

$U_D(H) = F_\epsilon(d)(2H + d)$ L will not join D

$U_D(L) = F_\epsilon(d + w)(2H + d - w)$ H will not join D

Target H vs No target

Target H if $F_\epsilon(d)(2H + d) > H + d$

$a < 2H + d$ If this holds, targeting H is preferred to no target. However, this does not hold in this range of a therefore no target is preferred.

Target L vs No Target

Target L if $F_\epsilon(d + w)(2H + d - w) > H + d$

$a < 2H + d - w$ if this holds, target L is preferred to no target. This does not hold in this range of a, therefore no target is preferred.

Equilibrium: for $a > 3H + 2d$, the dictator will choose no target and no conflict will occur.

□

Round 2: Two Players Remaining

Proposition 4. *If the dictator's advantage relative to the elites and uncertainty is sufficiently high ($d > a+w-3H$), the dictator will target the remaining regime member. If the dictator's relative advantage is sufficiently low ($d < a + w - 3H$), the dictator will choose no target and no conflict will occur.*

Proof. There are four ways for there to be exactly two players remaining: (1) H and D fought alone (or L joined H and received no transfer); (2) L and D fought alone (or H joined L and received no transfer); (3) a coalition of D and L fought H so now D and L remain with some power transferred to L; (4) a coalition of D and H fought L, leaving D and H with some power transferred to H.

(1) D or H is the dictator with power $2H + d$, option to target L who has power $H - w$

Target L if $F_\epsilon(H + d + w)(3H + d - w) > 2H + d$

Using Uniform, target L if $a < 3H + d - w$ else no conflict

(2) D or L is the dictator with power $2H + d - w$, option to target H who has power H

Target H if $F_\epsilon(H + d - w)(3H + d - w) > 2H + d - w$

Using Uniform, target H if $a < 3H + d - w$ else no conflict

(3) A coalition between D and L fought against H in the first round, some power $x_L \in [0, 2H + d]$ was transferred to L. Theoretically, the transfer could have been so large that L's second round power is greater than the original dictator's new power, making L the new

dictator. Therefore there are two subcases.

(3a) D is still the dictator $(2H + d - x_L > H - w + x_L)$

Target L if $F_\epsilon (H + d + w - 2x_L)(3H + d - w) > 2H + d - x_L$

Using uniform target L if $a < 3H + d - w$ else no conflict

(3b) L is now the dictator and can target D $((2H + d - x_L < H - w + x_L)$

Target D if $F_\epsilon (2x_L - H - d - w)(3H + d - w) > H - w + x_L$

Using uniform, target D if $a < 3H + d - w$ else no conflict

(4) A coalition between D and H fought against L in the first round, some power $x_H \in [0, 2H + d - w]$ was transferred to H. Theoretically, the transfer could have been so large that H's second round power is greater than the original dictator's new power, making H the new dictator. Therefore there are two subcases.

(4a) D is still the dictator $(2H + d - w - x_H > H + x_H)$

Target H if $F_\epsilon (H + d - w - 2x_H)(3H + d - w) > 2H + d - w - x_H$

Using uniform, target H if $a < 3H + d - w$ else no conflict

(4b) H is now the dictator and can target D $(2H + d - w - x_H < H + x_H)$

Target D if $F_\epsilon (2x_H - H - d + w)(3H + d - w) > H + x_H$

Using uniform target D if $a < 3H + d - w$ else no conflict.

□

Round 1

Proposition 5. *If the dictator's advantage relative to uncertainty is sufficiently low ($d < a + w - 3H$), there will be no conflict. The first round join conditions for the elites are the same as three players in round two ($\underline{x_H}, \underline{x_L}$), but the dictator will not initiate any conflict.*

Result 3. *If the dictator's relative advantage is sufficiently high ($d > a + w - 3H$): if L* *is targeted, H will join the dictator if D offers* $x_H \geq \underline{x'_H} \equiv \frac{(a-H-d-w)(a-H-d+w)}{2(H+d+w+a)}$, *otherwise he will not join the conflict. If H is targeted, L will join the dictator if D offers* $x_L \geq \underline{x'_L} \equiv \frac{(a-H-d-w)(a-H-d+w)}{2(a+H+d-w)}$, *otherwise he will not join the conflict. In this parameter range,* $\underline{x_H} > \underline{x'_H}$ *and* $\underline{x_L} > \underline{x'_L}$.

Proof. Assume L is targeted.

$a < 3H + d - w$

• D makes H a low or insufficient offer

$U_H(joinD) = F_\epsilon(H+d+w)\,(1 - F_\epsilon(H + d - w))\,(3H + d - w)$ targeted in the second round

$U_H(\text{join } L) = (1 - F_\epsilon(d + w - H))\,(1 - F_\epsilon(H + d - w))\,(3H + d - w)$

$U_H(\neg \text{ join }) = (1 - F_\epsilon(H + d - w))\,(3H + d - w),$ H is targeted in round 2

Don't join dominates both coalition options. BR_H is don't join.

• D makes H a sufficient offer

$U_H(joinD) = F_\epsilon(H+d+w)\,(1 - F_\epsilon\,(H + d - w - 2x'_H))\,(3H+d-w)$ targeted in the second round

$U_H(joinL) = (1 - F_\epsilon(d + w - H))\,(1 - F_\epsilon(H + d - w))\,(3H + d - w)$

$U_H(\neg\text{join}) = (1 - F_\epsilon(H + d - w))\,(3H + d - w)$H is targeted in round 2

Note not joining dominates joining L

Join D if $F_\epsilon(H+d+w)\,(1 - F_\epsilon\,(H + d - w - 2x'_H))\,(3H+d-w) > (1 - F_\epsilon(H + d - w))\,(3H+d - w)$

$x'_H > \frac{(a-H-d-w)(a-H-d+w)}{2(H+d+w+a)}$ denote this minimal offer that H is willing to accept as $\underline{x'_H}$. If the dictator's offer meets this constraint, H will join him in a coalition in the first round. because the join threshold is the same and the ultimate expected utilities of the dictator and H are the same regardless of who becomes the dictator, it doesn't matter whether the transfer is large enough for H to become the dictator

BR_H join D if $x'_H > \underline{x'_H}$ else don't join

• D makes H a high offer, so high that if they win H will become the dictator

$U_H(\text{join}D) = F_\epsilon(H+d+w)F_\epsilon\left(2x'_H - H - d + w\right)(3H+d-w)$ targets remaining player in the second round

$U_H(\text{join}L) = (1 - F_\epsilon(d+w-H))\,(1 - F_\epsilon(H+d-w))\,(3H+d-w)$

$U_H(\neg\text{join}) = (1 - F_\epsilon(H+d-w))\,(3H+d-w)$H is targeted in round 2

Note not joining dominates joining L

Join D if $F_\epsilon(H+d+w)F_\epsilon\left(2x'_H - H - d + w\right)(3H+d-w) > (1 - F_\epsilon(H+d-w))\,(3H+d-w)$

$x'_H > \frac{(a-H-d-w)(a-H-d+w)}{2(H+d+w+a)}$ denote this minimal offer that H is willing to accept as $\underline{x'_H}$. If the dictator's offer meets this constraint, H will join him in a coalition in the first round. Note this is the same constraint as above... whether or not H becomes the dictator in the second round will ultimately depend on a.

$a > 3H + d - w$

• D makes a low or sufficient offer

$U_H(joinD) = F_\epsilon(H+d+w)(H)$

$U_H(joinL) = (1 - F_\epsilon(d+w-H))\,(H)$

$U_H(\neg\text{join}) = H$

Not joining dominates both coalition options. $BR_H\neg$ join

• D makes H a sufficiently high offer

$U_H(joinD) = F_\epsilon(H+d+w)\,(H + x_H)$

$U_H(joinL) = (1 - F_\epsilon(d+w-H))\,(H)$

$U_H(\neg\text{join}) = H$

Note not joining dominates joining L Same as third round: join D if $x_H > \frac{H(1-F_\epsilon(H+d+w))}{F_\epsilon(H+d+w)}$ denoted $\underline{x}_H$

BR_H join D if $x_H > \underline{x_H}$ else don't join $\square$

Proof. Assume H is targeted.

$a < 3H + d - w$

• D makes L a low or insufficient offer

$U_L(joinD) = F_\epsilon(H + d - w)\,(1 - F_\epsilon(H + d + w))\,(3H + d - w)$

$U_L(joinH) = (1 - (F_\epsilon(d + w - H))\,(1 - F_\epsilon(H + d + w))\,(3H + d - w)$

$U_L(\neg\text{join}) = (1 - F_\epsilon(H + d + w))\,(3H + d - w)$ Note not joining either coalition is dominant strategy BR_L : don't join

• D makes L a sufficient offer but not high enough to make L the second round dictator

$U_L(joinD) = F_\epsilon(H + d - w)\,(1 - F_\epsilon\,(H + d + w - 2x'_L))\,(3H + d - w)$

$U_L(joinH) = (1 - (F_\epsilon(d + w - H))\,(1 - F_\epsilon(H + d + w))\,(3H + d - w)$

$U_L(\neg\text{join}\ ) = (1 - F_\epsilon(H + d + w))\,(3H + d - w)$

Note not joining dominates joining H

Join D if $F_\epsilon(H+d-w)\,(1 - F_\epsilon\,(H + d + w - 2x'_L))\,(3H+d-w) > (1 - F_\epsilon(H + d + w))\,(3H+d - w)$

$x'_L > \frac{(a-d-H-w)(a-d-H+w)}{2(a+d+H-w)}$ denote this minimal offer L is willing to accept as $\underline{x'_L}$

BR_L : if $x'_L > \underline{x'_L}$ join D, else don't join

confirm that x'_L will not make L the dictator... same as above, the threshold is the same and the expected utilities of L and D will be the same, so whether or not L becomes the dictator doesn't matter

• D makes L a high enough offer to make L the second round dictator

$U_L(joinD) = F_\epsilon(H + d - w)F_\epsilon\,(2x'_L - H - d - w)\,(3H + d - w)$

$U_L(joinH) = (1 - (F_\epsilon(d + w - H))\,(1 - F_\epsilon(H + d + w))\,(3H + d - w)$

$U_L(\neg join) = (1 - F_\epsilon(H + d + w))\,(3H + d - w)$

Note not joining dominates joining H

Join D if $F_\epsilon(H{+}d{-}w)F_\epsilon\,(2x'_L - H - d - w)\,(3H{+}d{-}w) > (1 - F_\epsilon(H + d + w))\,(3H{+}d{-}w)$

$x'_L > \frac{(a-d-H-w)(a-d-H+w)}{2(a+d+H-w)}$ denote this minimal offer L is willing to accept as $\underline{x'_L}$ note this is the same as the previous constraint

BR_L: if $x'_L > \underline{x'_L}$ join D, else don't join

$a > 3H + d - w$

- D makes L a low or insufficient offer

$U_L(joinD) = F_\epsilon(H + d - w)(H - w)$

$U_L(joinH) = (1 - F_\epsilon(d + w - H))\,(H - w)$

$U_L(\neg\text{join }) = H - w$

don't join dominates both coalition options as conflicts are uncertain

- D makes L a sufficient offer

$U_L(\text{ join } D) = F_\epsilon(H + d - w)\,(H - w + x_L)$

$U_L(\text{ join } H) = (1 - F_\epsilon(d + w - H))\,(H - w)$

$U_L(\neg\text{join}) = H - w$

Note not joining dominates joining H

Join D if $F_\epsilon(H + d - w)\,(H - w + x_L) > H - w$

$x_L > \frac{(H-w)(1-F_\epsilon(H+d-w))}{F_\epsilon(H+d-w)}$ denote this minimal offer L will accept as $\underline{x_L}$ note this is the same as the third round constraint.

BR_L : join D if $x_L > \underline{x}_L$ else don't join □

Proposition 6. *Assume the dictator's relative advantage is sufficiently large ($d > a + w - 3H$)). Offering a minimal sufficient transfer to induce a coalition $(\underline{x'_H}, \underline{x'_L})$ dominates targeting an elite and failing to form a coalition. When uncertainty is sufficiently low, targeting* *an elite and making a minimal sufficient offer dominates no conflict initiation.*

Proof. Targeting Decision in Rd 1 given best responses and expected round 2 behavior

if $a < 3H + d - w$ round 2 conflict will occur

(1) Target L make H low or no offer $U_D(L) = F_\epsilon(d + w)F_\epsilon(H + d - w)(3H + d - w)$ H does not join, if dictator wins target H rd 2

(2) target L make H a sufficient offer

$U_D\left(L, \underline{x'_H}\right) = F_\epsilon(H+d+w)F_\epsilon\left(H + d - w - 2x'_H\right)(3H + d - w)$H joins D, D targets H rd 2

(3) target L make H a huge offer so H will be 2nd round dictator

$U_D\left(L, \underline{x'_H}\right) = F_\epsilon(H+d+w)\left(1 - F_\epsilon\left(2x'_H + w - H - d\right)\right)(3H+d-w)$, H joins D, H becomes rd2 dictator and targets D. note that this is equivalent to option (2) using the Uniform distribution as the minimal offers are the same

(4) target H make L low or no offer

$U_D(H) = F_\epsilon(d)F_\epsilon(H + d + w)(3H + d - w)$L does not join, if dictator wins target L rd 2

(5) target H make L sufficient offer

$U_D\left(H, \underline{x'_L}\right) = F_\epsilon(H + d - w)F_\epsilon\left(H + d + w - 2x'_L\right)(3H + d - w)$L joins D, D targets L rd 2

(6) target H make L a huge offer so L will be second round dictator

$U_D\left(H, x'_L\right) = F_\epsilon(H+d-w)\left(1 - F_\epsilon\left(2x'_L - w - H - d\right)\right)(3H+d-w)$, L joins D, L becomes rd 2 dictator and targets D. note this is equivalent to (5) using the uniform as the minimum offers are the same

(7) no round 1 conflict

$U_D(\emptyset) = F_\epsilon(H+d+w)(2H+d-w-\underline{x_H})$ L is targeted in round 2, D offers $\underline{x_H}$ and H joins D

Compare $(L, 0)$ and $(H, 0)$

$F_\epsilon(d+w)F_\epsilon(H+d-w)(3H+d-w) > F_\epsilon(d)F_\epsilon(H+d+w)(3H+d-w)$

using Uniform $Hw - w^2 > aw + dw$

$H-d-w > a$ this does not hold as $a > H+d+w$ to ensure all conflicts uncertain. Therefore $(H,0)$ dominates $(L,0)$.

Compare $\left(H, \underline{x'_L}\right)$ and $\left(L, \underline{x'_H}\right)$

$F_\epsilon(H+d-w)F_\epsilon\left(H+d+w-2\underline{x'_L}\right)(3H+d-w) > F_\epsilon(H+d+w)F_\epsilon\left(H+d-w-2\underline{x'_H}\right)(3H+d-w)$

Using uniform and substituting in the offers

$(H+d+w+a)(H+d-w+a)-(a-d-H-w)(a-d-H+w) > (H+d+w+a)(H+d-w+a)-(a-H-d-w)(a-H-d+w)$

indifferent

Compare $\left(L, \underline{x'_H}\right)$ and $(H,0)$

$F_\epsilon(H+d+w)F_\epsilon\left(H+d-w-2\underline{x'_H}\right)(3H+d-w) > F_\epsilon(d)F_\epsilon(H+d+w)(3H+d-w)$

$F_\epsilon\left(H+d-w-2\underline{x'_H}\right) > F_\epsilon(d)$

$H-w > 2\underline{x'_H}$

$H-w > \frac{(a-H-d-w)(a-H-d+w)}{(H+d+w+a)}$

Note $\lim\limits_{a\to H+d+w} H-w-\dfrac{(a-H-d-w)(a-H-d+w)}{(H+d+w+a)} = \dfrac{d(H-w)}{2H+d} > 0$

and $\lim\limits_{a\to 3H+d-w} H-w-\dfrac{(a-H-d-w)(a-H-d+w)}{(H+d+w+a)} = H-w > 0$

Check that $(L, x'_H)-(H,0)$ does not cross zero for $a \in (H+d+w, 3H+d-w)$

Roots: $\frac{1}{2}(3H+2d-w \pm \sqrt{8Hd+9H^2-8dw-6Hw+w^2})$

$\frac{1}{2}(3H+2d-w-\sqrt{8Hd+9H^2-8dw-6Hw+w^2}) < H+d+w$ the lower root is less than the lower case bound of uncertainty

$\frac{1}{2}(3H+2d-w+\sqrt{8Hd+9H^2-8dw-6Hw+w^2}) > 3H+d-w$ the upper root is greater than the upper case bound

Thus the relative utility function does not cross zero inside this case space, thus $(L, \underline{x'_H}) > (H, 0)$for all $a \in (H + d + w, 3H + d - w)$

So the dictator is indifferent between each of the target/offers, but a target with an offer dominates targeting without making an offer and forming a coalition in this range of uncertainty. How does this compare to no first round target?

Compare (L, x'_H) and $(\emptyset)$

Target an elite and make a coalition offer is preferred to no target in the first round if:

$F_\epsilon(H + d + w)F_\epsilon(H + d - w - 2x'_H)(3H + d - w) - F_\epsilon(H + d + w)(2H + d - w - x_H) > 0$

Substituting in minimal offers and using Uniform,

$\frac{2a(3H+d-w)(2H+2d-w)-(3H+d-w)(H+d-w)(H+d+w)+a^2(3w-5H-3d)}{4a^2}$

$\lim_{a \to 3H+d-w} \frac{2a(3H + d - w)(2H + 2d - w) - (3H + d - w)(H + d - w)(H + d + w) + a^2(3w - 5H - 3d)}{4a^2} = \frac{H(w - H)}{3H + d - w} < 0$ by definition of H and w. Thus as a approaches its upper bound in this range, no target is preferred to an elite target and coalition

$\lim_{a \to H+d+w} \frac{2a(3H + d - w)(2H + 2d - w) - (3H + d - w)(H + d - w)(H + d + w) + a^2(3w - 5H - 3d)}{4a^2} = \frac{(H - w)(H + d - w)}{H + d + w} > 0$ by definition of H and w. Thus as a approaches its lower bound in this range, though conflict outcomes are still uncertain, targeting an elite and making a sufficient offer to form a coalition is preferred to no target. There exists a range of parameters for which the dictator prefers initiating conflict and forming an elite coalition.

$a > 3H + d - w$ no conflict round 2

No target: $U_D(\emptyset) = H + d$

Target L, no or low offer to $H : F_\epsilon(d + w)(2H + d - w)$

Target L, sufficient offer to H : $F_\epsilon(H + d + w)\left(2H + d - w - \underline{x_H}\right)$

Target H, low or no offer: $F_\epsilon(d)(2H + d - w)(2H + d - w - x_H)$

Target H, sufficient offer to L : $F_\epsilon(A + d - w)\big(2H + d - \underline{x_L}\big)$

No target preferred to target L with offer ($a > 3H + d - w$), No target preferred to with offer, No target preferred to target L no offer ($a > 2H + d - w$), No target preferred to target H no offer ($a > 2H + d$), no target preferred to H with offer ($a > 3H + d - w$)

no target

□

3.5 Formal Appendix B: Rubinstein Bargaining

Set up: the dictator chooses the bargaining order (H first or L first)

the dictator bargains with first elite with alternating offer protocol with δ probability that bargaining continues; if they reach an agreement, they form coalition and fight the other elite, sharing power post-conflict according to the bargain

If they fail to reach an agreement, the dictator bargains with the next elite to form a coalition against the first, sharing power according to the agreement

If they fail to reach an agreement, the dictator can fight either elite alone or not start a conflict and maintain his power

All conflicts are subject to uniform noise contest

General Bargaining SPNE

Denote the dictator's outside option Ω_D and the elite's outside option Ω_E. Regardless of the history, after any rejection bargaining continues with common probability $\delta \in (0, 1)$. The dictator makes the first offer.

Let m_E and M_E be the infimum and supremum of equilibrium payoffs to the elite when he is the proposer. Let m_D and M_D be the infimum and supremum of equilibrium payoffs to the dictator when he is the proposer. The following inequalities hold:

1. $m_E \geq 1 - (\delta M_D + (1-\delta)\Omega_D)$

2. $M_E \leq 1 - (\delta m_D + (1-\delta)\Omega_D)$

3. $m_D \geq 1 - (\delta M_E + (1-\delta)\Omega_E)$

4. $M_D \leq 1 - (\delta m_E + (1-\delta)\Omega_E)$

In equilibrium, the dictator must accept an offer x where $x = (\delta M_D + (1-\delta)\Omega_D)$ as that is the most that he could get from refusing (inequality 1). It follows that the elite cannot get less than w where $w = 1 - (\delta M_D + (1-\delta)\Omega_D)$ because he can get a guaranteed w by making it his opening demand.

Similarly, in equilibrium, the dictator must get at least y for each $y = (\delta m_D + (1-\delta)\Omega_D)$ because y is guaranteed if the dictator rejects the elite's opening proposal, so the elite can get at most $1 - y$ (inequality 2).

When the dictator is the proposer, the elite must accept an offer x' where $x' = (\delta M_E + (1-\delta)\Omega_E)$, the most he could get from refusing. Thus the dictator cannot get less than $1-(\delta M_E+(1-\delta)\Omega_E)$, which he is guaranteed if he makes x' his opening proposal (inequality 3)

The elite must get at least y' for each $y' = (\delta m_E + (1-\delta)\Omega_E)$ as that is guaranteed if the elite rejects the dictator's proposal. Thus the dictator can get at most $1 - y'$ (inequality 4).

Rearranging these inequalities, we see that

$M_E \leq 1 - \delta m_D - (1-\delta)\Omega_D$

$m_E \geq 1 - \delta M_D - (1-\delta)\Omega_D$

so if $m_D = M_D$, then $m_E = M_E$

Further,

$m_D \geq 1 - \delta M_E - (1-\delta)\Omega_E$

$M_D \leq 1 - \delta m_E - (1-\delta)\Omega_E$

So if $m_E = M_E$ then $m_D = M_D$

but how do we know that this is necessarily the case?

Proof by contradiction (to show that $m_D = M_D$)

Assume $m_D < M_D$

From above, we know that $M_D \leq 1 - \delta m_E - (1-\delta)\Omega_E$ and $m_D \geq 1 - \delta M_E - (1-\delta)\Omega_E$

$m_D - M_D \geq 1 - \delta M_E - (1-\delta)\Omega_E - 1 + \delta m_E + (1-\delta)\Omega_E$ subtracting the lesser from the greater maintains the inequality

$m_D - M_D \geq \delta(m_E - M_E)$

Further,

$M_E \leq 1 - \delta m_D - (1-\delta)\Omega_D$ and $m_E \geq 1 - \delta M_D - (1-\delta)\Omega_D$

Thus $m_E - M_E \geq 1 - \delta M_D - (1-\delta)\Omega_D - (1 - \delta m_D - (1-\delta)\Omega_D)$ to maintain the inequality

$m_E - M_E \geq \delta(m_D - M_D)$

$\frac{m_E - M_E}{\delta} \geq m_D - M_D$

Combining the above,

$\frac{m_E - M_E}{\delta} \geq m_D - M_D \geq \delta(m_E - M_E)$

By hypothesis, $m_D - M_D < 0$ and as $\delta \in (0,1)$ by definition, $m_E - M_E < 0$

$\frac{m_E - M_E}{\delta} \geq \delta(m_E - M_E)$

$m_E - M_E \geq \delta^2(m_E - M_E)$

This is a contradiction as $\delta \in (0,1)$ and $m_E - M_E < 0$ (multiplying $m_E - M_E$ by a positive number less than one will make it less negative and therefore larger)

Therefore it must be that $m_D \geq M_D$

$m_D > M_D$ also does not hold as the infimum cannot be greater than the supremum by definition

If $m_D \not< M_D$ and $m_D \not> M_D$, is must be that $m_D = M_D$ and, from above, this implies that

$m_E = M_E$

Therefore $m_D = M_D$ and $m_E = M_E$. The subgame perfect equilibrium must be unique.

Single Conflict

First, cases based on the dictator's final outside option: if bargaining with both elites fails, what will the dictator do?

if $a \in (H+d+w, 2H-w)$, target L alone

if $a \in (2H-w, 2H+d)$ target H alone

if $a > 2H+d$ no conflict

Case 1 $a \in (H+d+w, 2H-w)$**, outside option is target L alone**

If the dictator fails to reach an agreement with either elite, he will target L alone and his expected utility is $F_\epsilon(d+w)(2H+d-w)$

Subgame: Bargain with H first, L second

second bargain with L: dictator proposes x_L to keep $1-x_L$; elite proposes y_L to keep $1-y_L$; these are proportions of the benefit from defeating the other elite H

$U_D(accept) = F_\epsilon(H+d-w)(H+d+Hy_L)$

$U_D(reject) = \delta(F_\epsilon(H+d-w)(H+d+H(1-x_L))) + (1-\delta)(F_\epsilon(d+w)(2H+d-w))$

$U_L(accept) = F_\epsilon(H+d-w)(H-w+Hx_L)$

$U_L(reject) = \delta(F_\epsilon(H+d-w)(H-w+H(1-y_L))) + (1-\delta)((1-F_\epsilon(d+w))(2H+d-w))$

$x = (2*a*(\delta*(((H+d-w+a)/(2*a))*(H-w+H*(1-y))) + (1-\delta)*(1-(d+w+a)/(2*a))*(2*H+d-w) - (((H-w)(a+d+H-w))/(2a))))/(H(a+d+H-w))$

$y = ((\delta * (((H + d - w + a)/(2 * a)) * (H + d + H * (1 - x))) + (1 - \delta) * ((d + w + a)/(2 * a)) * (2 * H + d - w) - ((d + H)(a + d + H - w))/(2a)) * 2 * a)/(H(a + d + H - w))$

$x_L^* = \frac{(-d^2 - 3dH - H^2 + dw + (H(d+2H) - (d+4H)w + w^2)\delta + a(d+H+w\delta))}{(H(a+d+H-w)(1+\delta))}$

$y_L^* = \frac{(-1+\delta)(H^2(1-2\delta) + Hw(-3+\delta) - 4dH\delta - (d-w)(w+d\delta) + a(-H+w+d\delta))}{(1+H(-a+d+H-w)\delta^2)}$

Leave CDFs as CDFs

$x_L^* = \frac{2H+d-w+F_\epsilon(H+d-w)(\delta(2H+d)+w-H) - F_\epsilon(d+w)(1+\delta)(2H+d-w)}{H(1+\delta)F_\epsilon(H+d-w)}$

$y_L^* = \frac{F_\epsilon(d+w)(1+\delta)(2H+d-w) - \delta(2H+d-w) + F_\epsilon(H+d-w)(\delta(2H-w)-H-d)}{H(1+\delta)F_\epsilon(H+d-w)}$

$U_D = F_\epsilon(H + d - w)(H + d + H(1 - x_L^*)$

$= \frac{F_\epsilon(d+w)(2H+d-w)(1+\delta) + F_\epsilon(H+d-w)(3H+d-w) - (2H+d-w)}{1+\delta)}$

$U_L = F_\epsilon(H + d - w)(H - w + Hx_L^*)$

$= \frac{F_\epsilon(H+d-w)\delta(3H+d-w) + (2H+d-w) - F_\epsilon(d+w)(2H+d-w)(1-\delta)}{1+\delta}$

First Bargain with H: dictator proposes x_H to keep $1 - x_H$; elite proposes y_H to keep $1 - y_H$; now the outside options take the dictator's coalition with L into account

$U_D(accept) = F_\epsilon(H + d + w)(H + d + (H - w)y_H)$

$U_D(reject) = \delta F_\epsilon(H+d+w)(H+d+(H-w)(1-x_H)) + (1-\delta)(\frac{F_\epsilon(d+w)(2H+d-w)(1+\delta)+F_\epsilon(H+d-w)(3H+d-w)-(2H+d-}{1+\delta)}$

$U_H(accept) = F_\epsilon(H + d + w)(H + (H - w)x_H)$

$U_H(reject) = \delta F_\epsilon(H+d+w)(H+(H-w)(1-y_H)) + (1-\delta)(1-F_\epsilon(H+d-w))(3H+d-w))$

(note if H doesn't come to an agreement with the dictator, he will fight D and L together with a chance to beat them both)

$y_H^* = \frac{F_\epsilon(H+d-w)(3H+d-w)(1+\delta+\delta^*) + F_\epsilon(d+w)(1+\delta)(2H+d-w) - F_\epsilon(H+d+w)(1+\delta)(H+d+\delta(w-2H)) - (2H+d-w) - \delta(3H+d-w) - \delta^2(}{F_\epsilon(H+d+w)(H-w)(1+\delta)^2}$

$x_H^* = \frac{F_\epsilon(H+d+w)(\delta(H+d-w)+\delta^2(2H+d-w)-H) + (3H+d-w) + \delta(5H+2d-2w) - F_\epsilon(H+d-w)(3H+d-w)(1+2\delta) - F_\epsilon(d+w)\delta(1+\delta)(2H+d}{F_\epsilon(H+d+w)(H-w)(1+\delta)^2}$

$U_D = F_\epsilon(H + d + w)(H + d + (H - w)(1 - x_H^*))$

$= \frac{F_\epsilon(H+d+w)(1+\delta)(3H+d-w) - (3H+d-w) - \delta(5H+2d-2w) + F_\epsilon(d+w)(2H+d-w)\delta(1+\delta) + F_\epsilon(H+d-w)(3H+d-w)(1+2\delta)}{(1+\delta)^2}$

$U_H = F_\epsilon(H + d + w)(H + (H - w)x_H^*)$

$= \frac{F_\epsilon(H+d+w)(3H+d-w)\delta(1+\delta) - F_\epsilon(H+d-w)(3H+d-w)(1+2\delta) - F_\epsilon(d+w)(2H+d-w)\delta(1+\delta) + (3H+d-w) + \delta(5H+2d-2w)}{(1+\delta)^2}$

Subgame: Bargain with L first, H second

Backwards induction start with H bargain (outside option is still target L alone)

$U_D(accept) = F_\epsilon(H+d+w)(H+d+(H-w)y_H)$

$U_D(reject) = \delta(F_\epsilon(H+d+w)(H+d+(H-w)(1-x_H))) + (1-\delta)(F_\epsilon(d+w)(2H+d-w))$

$U_H(accept) = F_\epsilon(H+d+w)(H+(H-w)x_H)$

$U_H(reject) = \delta(F_\epsilon(H+d+w)(H+(H-w)(1-y_H))) + (1-\delta)(H)$

if they don't reach an agreement, dictator fights L and H keeps his power

$y_H^* = \frac{F_\epsilon(\delta(2H-w)-d-H))+F_\epsilon(d+w)(2H+d-w)-\delta H}{F_\epsilon(H+d+w)(H-w)(1+\delta)}$

$x_H^* = \frac{F_\epsilon(H+d+w)(\delta(2H+d-w)-H)-F_\epsilon(d+w)(\delta(2H+d-w))+H}{F_\epsilon(H+d+w)(H-w)(1+\delta)}$

$U_D = F_\epsilon(H+d+w)(H+d+(H-w)(1-x_H^*))$

$= \frac{F_\epsilon(H+d+w)(3H+d-w)-H+F_\epsilon(d+w)(2H+d-w)}{1+\delta}$

$U_H = F_\epsilon(H+d+w)(H+(H-w)x_H^*)$

$= \frac{F_\epsilon(H+d+w)(\delta(3H+d-w))-F_\epsilon(d+w)\delta(2H+d-w)+H}{1+\delta}$

First bargain with L

$U_D(accept) = F_\epsilon(H+d-w)(H+d+H(y_L))$

$U_D(reject) = \delta F_\epsilon(H+d-w)(H+d+H(1-x_L))+(1-\delta)(\frac{F_\epsilon(H+d+w)(3H+d-w)-H+F_\epsilon(d+w)(2H+d-w)}{1+\delta})$

$U_L(accept) = F_\epsilon(H+d-w)(H-w+H(x_L))$

$U_L(reject) = \delta F_\epsilon(H+d-w)(H-w+H(1-y_L))+(1-\delta)((1-F_\epsilon(H+d+w))(3H+d-w))$

$y_L^* = \frac{F_\epsilon(3H+d-w)(1+\delta+\delta^2)+F_\epsilon(H+d-w)(\delta(H-d-w)+\delta^2(2H-w)-d-H)+F_\epsilon(d+w)(\delta(2H+d-w))-H-\delta(3H+d-w)-\delta^2(3H+d-w)}{F_\epsilon(H+d-w)H(1+\delta)^2}$

$x_L^* = \frac{F_\epsilon(H+d-w)(\delta(H+d+w)+\delta^2(2H+d)-H+w)+(3H+d-w)+\delta(4H+d-w)-F_\epsilon(H+d+w)(3H+d-w)(1+2\delta)-F_\epsilon(d+w)(\delta^2(2H+d-w))}{F_\epsilon(H+d-w)H(1+\delta)^2}$

$U_D = F_\epsilon(H+d-w)(H+d+H(1-x_L^*))$

$= \frac{F_\epsilon(3H+d-w+2\delta(3H+d-w))+F_\epsilon(H+d-w)(1+\delta)(3H+d-w)+F_\epsilon(d+w)\delta^2(2H+d-w)-(3H+d-w)-\delta(4H+d-w)}{(1+\delta)^2}$

$U_L = F_\epsilon(H+d-w)(H-w+H(x_L^*))$

$= \frac{F_\epsilon(H+d-w)(\delta(3H+d-w)+\delta^2(3H+d-w))+(3H+d-w)+\delta(4H+d-w)-F_\epsilon(H+d+w)(3H+d-w)(1+2\delta)-F_\epsilon(d+w)\delta^2(2H+d-w)}{(1+\delta)^2}$

Which bargaining order would the dictator choose? (which coalition member does he choose)?

Compare

$U_D(L,H) = \frac{F_\epsilon(3H+d-w+2\delta(3H+d-w))+F_\epsilon(H+d-w)(1+\delta)(3H+d-w)+F_\epsilon(d+w)\delta^2(2H+d-w)-(3H+d-w)-\delta(4H+d-w)}{(1+\delta)^2}$

$U_D(H,L) = \frac{F_\epsilon(H+d+w)(1+\delta)(3H+d-w)-(3H+d-w)-\delta(5H+2d-2w)+F_\epsilon(d+w)(2H+d-w)\delta(1+\delta)+F_\epsilon(H+d-w)(3H+d-w)(1+2\delta)}{(1+\delta)^2}$

$U_D(L,H) - U_D(H,L) =$

$\frac{F_\epsilon(H+d+w)\delta(3H+d-w)+F_\epsilon(H+d-w)(3H+d-w)(1+\delta)-F_\epsilon(d+w)\delta(2H+d-w)-\delta(3H+d-w)-(2H+d-w)}{(1+\delta)^2}$

Substituting in the uniform win probabilities, the dictator prefers bargaining with L first in this range of a ($a \in H+d+w, 2H-w$). The dictator will make a coalition with L and they will both fight H

Case 2: $a \in (2H-w, 2H+d)$ **H alone is the outside option**

If the dictator fails to reach an agreement with either elite, he will target H alone and his expected utility is $F_\epsilon(d)(2H+d)$

Subgame: Bargain with H first, L second

second bargain with L: dictator proposes x_L to keep $1-x_L$; elite proposes y_L to keep $1-y_L$; these are proportions of the benefit from defeating the other elite H

$U_D(accept) = F_\epsilon(H+d-w)(H+d+Hy_L)$

$U_D(reject) = \delta(F_\epsilon(H+d-w)(H+d+H(1-x_L))) + (1-\delta)(F_\epsilon(d)(2H+d))$

$U_L(accept) = F_\epsilon(H+d-w)(H-w+Hx_L)$

$U_L(reject) = \delta(F_\epsilon(H+d-w)(H-w+H(1-y_L))) + (1-\delta)(H-w)$ If they don't reach an agreement, dictator fights H and L stays out of the conflict

$x_L^* = \frac{F_\epsilon(H+d-w)(\delta(2H+d)-H+w)-F_\epsilon(d)\delta(2H+d)+H-w}{F_\epsilon(H+d-w)H(1+\delta)}$

$y_L^* = \frac{F_\epsilon(H+d-w)(\delta(2H-w)-d-H)+F_\epsilon(d)(2H+d)-\delta(H-w)}{F_\epsilon(H+d-w)H(1+\delta)}$

$U_D = \frac{F_\epsilon(H+d-w)(3H+d-w)-(H-w)+F_\epsilon(d)\delta(2H+d)}{1+\delta}$

$U_L = \frac{F_\epsilon(H+d-w)\delta(3H+d-w)+(H-w)-F_\epsilon(d)\delta(2H+d)}{1+\delta}$

first bargain with H: dictator proposes x_H to keep $1 - x_H$; elite proposes y_H to keep $1 - y_H$; these are proportions of the benefit from defeating the other elite L

$$U_D(accept) = F_\epsilon(H + d + w)(H + d + (H - w)y_H)$$

$$U_D(reject) = \delta F_\epsilon(H+d+w)(H+d+(H-w)(1-x_H))+(1-\delta)(\tfrac{F_\epsilon(H+d-w)(3H+d-w)-(H-w)+F_\epsilon(d)\delta(2H+d)}{1+\delta})$$

$$U_H(accept) = F_\epsilon(H + d + w)(H + (H - w)x_H)$$

$$U_H(reject) = \delta F_\epsilon(H+d+w)(H+(H-w)(1-y_H))+(1-\delta)(1-F_\epsilon(H+d-w))(3H+d-w))$$

(note if H doesn't come to an agreement with the dictator, he will fight D and L together with a chance to beat them both)

$$x_H^* = \tfrac{F_\epsilon(H+d+w)(1+\delta)(\delta(2H+d-w)-H)-F_\epsilon(H+d-w)(3H+d-w)(1+2\delta)-F_\epsilon(d)\delta^2(2H+d)+(3H+d-w)+\delta(4H+d-2w)}{F_\epsilon(H+d+w)(H-w)(1+\delta)^2}$$

$$y_H^* = \tfrac{F_\epsilon(H+d+w)(\delta(H-d-w)+\delta^2(2H-w)-d-H)+F_\epsilon(H+d-w)(3H+d-w)(1+\delta+\delta^2)+F_\epsilon(d)\delta(2H+d)-H+w-\delta(3H+d-w)-\delta^2(3H+d-w}{F_\epsilon(H+d+w)(H-w)(1+\delta)^2}$$

$$U_D = F_\epsilon(H + d + w)(H + d + (H - w)(1 - x_H^*))$$

$$= \tfrac{F_\epsilon(H+d+w)(1+\delta)(3H+d-w)+F_\epsilon(H+d-w)(3H+d-w)(1+2\delta)+F_\epsilon(d)\delta^2(2H+d)-(3H+d-w)-\delta(4H-d-2w)}{(1+\delta)^2}$$

$$U_H = F_\epsilon(H + d + w)(H + (H - w)(x_H^*))$$

$$= \tfrac{F_\epsilon(H+d+w)(3H+d-w)\delta(1+\delta)-F_\epsilon(H+d-w)(3H+d-w)(1+2\delta)-F_\epsilon(d)\delta^2(2H+d)+3H+d-w+\delta(4H+d-2w)}{(1+\delta)^2}$$

Subgame: Bargain with L first, then H

Second bargain with H: dictator proposes x_H to keep $1-x_H$; elite proposes y_H to keep $1-y_H$; these are proportions of the benefit from defeating the other elite L. If they do not reach an agreement, he will target H alone and his expected utility is $F_\epsilon(d)(2H + d)$

$$U_D(accept) = F_\epsilon(H + d + w)(H + d + (H - w)y_H)$$

$$U_D(reject) = \delta F_\epsilon(H + d + w)(H + d + (H - w)(1 - x_H)) + (1 - \delta)F_\epsilon(d)(2H + d)$$

$$U_H(accept) = F_\epsilon(H + d + w)(H + (H - w)x_H)$$

$U_H(reject) = \delta F_\epsilon(H + d + w)(H + (H - w)(1 - y_H)) + (1 - \delta)(1 - F_\epsilon(d))(2H + d))$ if they don't reach an agreement, the dictator will fight H alone

$$y^*_H = \frac{F_\epsilon(H+d+w)(\delta(2H-w)-d-H)+F_\epsilon(d)(1+\delta)(2H+d)-\delta(2H+d)}{F_\epsilon(H+d+w)(H-w)(1+\delta)}$$

$$x^*_H = \frac{F_\epsilon(H+d+w)(\delta(2H+d-w)-H)-F_\epsilon(d)(2H+d)(1+\delta)+2H+d}{F_\epsilon(H+d+w)(H-w)(1+\delta)}$$

$$U_D = F_\epsilon(H+d+w)(H+d+(H-w)(1-x^*_H))$$

$$= \frac{F_\epsilon(H+d+w)(3H+d-w)+F_\epsilon(d)(1+\delta)(2H+d)-(2H+d)}{1+\delta}$$

$$U_H = F_\epsilon(H+d+w)(H+(H-w)(x^*_H))$$

$$= \frac{F_\epsilon(H+d+w)\delta(3H+d-w)-F_\epsilon(d)(1+\delta)(2H+d)+2H+d}{1+\delta}$$

first bargain with L: dictator proposes x_L to keep $1-x_L$; elite proposes y_L to keep $1-y_L$; these are proportions of the benefit from defeating the other elite H; if they don't reach an agreement, the dictator will work with H and fight L

$$U_D(accept) = F_\epsilon(H+d-w)(H+d+Hy_L)$$

$$U_D(reject) = \delta(F_\epsilon(H+d-w)(H+d+H(1-x_L)))+(1-\delta)\left(\frac{F_\epsilon(H+d+w)(3H+d-w)+F_\epsilon(d)(1+\delta)(2H+d)-(2H+d)}{1+\delta}\right)$$

$$U_L(accept) = F_\epsilon(H+d-w)(H-w+Hx_L)$$

$$U_L(reject) = \delta(F_\epsilon(H+d-w)(H-w+H(1-y_L)))+(1-\delta)(1-F_\epsilon(H+d+w))(3H+d-w)$$

$$y^*_L = \frac{F_\epsilon(H+d+w)(1+\delta+\delta^2)(3H+d-w)+F_\epsilon(H+d-w)(\delta^2(2H-w)+\delta(H-d-w)-d-H)+F_\epsilon(d)(1+\delta)(2H+d)+d-2H-\delta(3H+d-w)-\delta^2(3H+}{F_\epsilon(H+d-w)H(1+\delta)^2}$$

$$x^*_L = \frac{F_\epsilon(H+d-w)(\delta(H+d+w)+\delta^2(2H+d)-H+w)-F_\epsilon(H+d+w)(3H+d-w)(1+2\delta)-F_\epsilon(d)\delta(1+\delta)(2H+d)+3H+d-w+\delta(5H+2d-w)}{F_\epsilon(H+d-w)H(1+\delta)^2}$$

$$U_D = F_\epsilon(H+d-w)(H+d+H(1-x^*_L)$$

$$= \frac{F_\epsilon(H+d+w)(3H+d-w)(1+2\delta)+F_\epsilon(H+d-w)(3H+d-w)(1+\delta)+F_\epsilon(d)\delta(1+\delta)(2H+d)-(3H+d-w)-\delta(5H+2d-w)}{(1+\delta)^2}$$

$$U_L = F_\epsilon(H+d-w)(H-w+Hx^*_L)$$

$$= \frac{F_\epsilon(H+d-w)\delta(1+\delta)(3H+d-w)-F_\epsilon(H+d+w)(3H+d-w)(1+2\delta)-F_\epsilon(d)\delta(1+\delta)(2H+d)+3H+d-w+\delta(5H+2d-w)}{(1+\delta)^2}$$

Who does the dictator want to bargain with first?

$U_D(L,H) - U_D(H,L) =$
$\frac{F_\epsilon(H+d+w)(3H+d-w)(\delta^2-\delta-1)-F_\epsilon(H+d-w)(3H+d-w)(2+3\delta)-F_\epsilon(d)(2H+d)(1+2\delta)+(3H+d-w)(2+3\delta)}{(1+\delta)^2}$

Substituting in the uniform win probabilities, this is negative in this subgame range of a ($a \in (2H-w, 2H+d)$). So the dictator prefers to bargain with H first then L. In equilibrium, the dictator forms a coalition with H and the fight L together.

Case 3: $a > 2H + d$ no conflict is dictator's outside option

If the dictator fails to reach an agreement with either elite, he will target not target either elite and there will be no conflict

Subgame: Bargain with H first, L second

second bargain with L: dictator proposes x_L to keep $1 - x_L$; elite proposes y_L to keep $1 - y_L$; these are proportions of the benefit from defeating the other elite H

$U_D(accept) = F_\epsilon(H + d - w)(H + d + Hy_L)$

$U_D(reject) = \delta(F_\epsilon(H + d - w)(H + d + H(1 - x_L))) + (1 - \delta)(H + d)$ no conflict, he keeps his status quo power

$U_L(accept) = F_\epsilon(H + d - w)(H - w + Hx_L)$

$U_L(reject) = \delta(F_\epsilon(H + d - w)(H - w + H(1 - y_L))) + (1 - \delta)(H - w)$ If they don't reach an agreement, no conflict keep status quo power

$y_L^* = \frac{F_\epsilon(H+d-w)(\delta(2H-w)-d-H)-\delta(H-w)+H+d}{F_\epsilon(H+d-w)H(1+\delta)}$

$x_L^* = \frac{F_\epsilon(H+d-w)(\delta(2H+d)-H+w)-\delta(H-d)+H-w}{F_\epsilon(H+d-w)H(1+\delta)}$

$U_D = F_\epsilon(H + d - w)(H + d + H(1 - x_L^*)$

$= \frac{F_\epsilon(H+d-w)(3H+d-w)+\delta(H+d)-(H-w)}{1+\delta}$

$U_L = F_\epsilon(H + d - w)(H - w + Hx_L^*)$

$= \frac{F_\epsilon(H+d-w)\delta(3H+d-w)-\delta(H+d)+H-w}{1+\delta}$

first bargain with H: dictator proposes x_H to keep $1 - x_H$; elite proposes y_H to keep $1 - y_H$; these are proportions of the benefit from defeating the other elite L

$U_D(accept) = F_\epsilon(H + d + w)(H + d + (H - w)y_H)$

$U_D(reject) = \delta F_\epsilon(H+d+w)(H+d+(H-w)(1-x_H))+(1-\delta)(\frac{F_\epsilon(H+d-w)(3H+d-w)+\delta(H+d)-(H-w)}{1+\delta})$

$U_H(accept) = F_\epsilon(H + d + w)(H + (H - w)x_H)$

$U_H(reject) = \delta F_\epsilon(H+d+w)(H+(H-w)(1-y_H))+(1-\delta)(1-F_\epsilon(H+d-w))(3H+d-w))$

(note if H doesn't come to an agreement with the dictator, he will fight D and L together with a chance to beat them both)

$y_H^* = \frac{F_\epsilon(H+d+w)(\delta^2(2H-w)+\delta(H-d-w)-d-H)+F_\epsilon(H+d-w)(1+\delta+\delta^2)(3H+d-w)-\delta^2(3H+d-w)-\delta(2H-w)-(H-w)}{F_\epsilon(H+d+w)(H-w)(1+\delta)^2}$

$x_H^* = \frac{F_\epsilon(H+d+w)(\delta^2(2H+d-w)+\delta(H+d-w)-H)-F_\epsilon(H+d-w)(3H+d-w)(1+2\delta)-\delta^2(H+d)+\delta(4H+d-2w)+3H+d-w}{F_\epsilon(H+d+w)(H-w)(1+\delta)^2}$

$U_D = F_\epsilon(H + d + w)(H + d + (H - w)(1 - x_H^*))$

$= \frac{F_\epsilon(H+d+w)(1+\delta)+F_\epsilon(H+d-w)(3H+d-w)(1+2\delta)+\delta^2(H+d)-\delta(4H+d-w)-(3H+d-w)}{(1+\delta)^2}$

$U_H = F_\epsilon(H + d + w)(H + (H - w)x_H^*)$

$= \frac{F_\epsilon(H+d+w)(3H+d-w)\delta(1+\delta)-F_\epsilon(H+d-w)(3H+d-w)(1+2\delta)-\delta^2(H+d)+\delta(4H+d-2w)+3H+d-w}{(1+\delta)^2}$

Subgame: bargain with L first, H second

Second bargain with H: dictator proposes x_H to keep $1-x_H$; elite proposes y_H to keep $1-y_H$; these are proportions of the benefit from defeating the other elite L. If they do not reach an agreement, no conflict

$U_D(accept) = F_\epsilon(H + d + w)(H + d + (H - w)y_H)$

$U_D(reject) = \delta F_\epsilon(H + d + w)(H + d + (H - w)(1 - x_H)) + (1 - \delta)(H + d)$

$U_H(accept) = F_\epsilon(H + d + w)(H + (H - w)x_H)$

$U_H(reject) = \delta F_\epsilon(H + d + w)(H + (H - w)(1 - y_H)) + (1 - \delta)(H)$ if H doesn't come to an agreement with the dictator, no conflict

$$y_L^* = \frac{F_\epsilon(H+d+w)(\delta(2H-w)-d-H)-\delta H+H+d}{F_\epsilon(H+d+w)(H-w)(1+\delta)}$$
$$x_L^* = \frac{F_\epsilon(H+d+w)(\delta(2H+d-w)-H)-\delta(H+d)+H}{F_\epsilon(H+d+w)(H-w)(1+\delta)}$$

$$U_D = F_\epsilon(H+d+w)(H+d+(H-w)(1-x_H^*)$$
$$= \frac{F_\epsilon(H+d+w)(3H+d-w)+\delta(H+d)-H}{1+\delta}$$
$$U_H = F_\epsilon(H+d+w)(H+(H-w)x_H^*)$$
$$= \frac{F_\epsilon(H+d+w)\delta(3H+d-w)-\delta(H+d)+H}{1+\delta}$$

first bargain with L: dictator proposes x_L to keep $1-x_L$; elite proposes y_L to keep $1-y_L$; these are proportions of the benefit from defeating the other elite H; if they don't reach an agreement, the dictator will work with H and fight L

$$U_D(accept) = F_\epsilon(H+d-w)(H+d+Hy_L)$$
$$U_D(reject) = \delta(F_\epsilon(H+d-w)(H+d+H(1-x_L))) + (1-\delta)(\frac{F_\epsilon(H+d+w)(3H+d-w)+\delta(H+d)-H}{1+\delta})$$

$$U_L(accept) = F_\epsilon(H+d-w)(H-w+Hx_L)$$
$$U_L(reject) = \delta(F_\epsilon(H+d-w)(H-w+H(1-y_L))) + (1-\delta)(1-F_\epsilon(H+d+w))(3H+d-w)$$

$$y_L^* = \frac{F_\epsilon(H+d+w)(3H+d-w)(1+\delta+\delta^2)+F_\epsilon(H+d-w)(\delta^2(2H-w)+\delta(H-d-w)-H-d)-\delta^2(3H+d-w)-\delta(2H-w)-H}{F_\epsilon(H+d-w)H(1+\delta)^2}$$
$$x_L^* = \frac{F_\epsilon(H+d-w)(\delta^2(2H+d)+\delta(H+d+w)-H+w)-F_\epsilon(H+d+w)(3H+d-w)(1+2\delta)-\delta^2(H+d)+\delta(4H+d-w)+3H+d-w}{F_\epsilon(H+d-w)H(1+\delta)^2}$$

$$U_D = F_\epsilon(H+d-w)(H+d+H(1-x_L^*))$$
$$= \frac{F_\epsilon(H+d+w)(3H+d-w)(1+2\delta)+F_\epsilon(H+d-w)(3H+d-w)(1+\delta)\delta^2(H+d)-\delta(4H+d-w)-(3H+d-w)}{(1+\delta)^2}$$
$$U_L = F_\epsilon(H+d-w)(H-w+Hx_L^*)$$

$$= \frac{F_\epsilon(H+d-w)(3H+d-w)\delta(1+\delta)-F_\epsilon(H+d-w)(3H+d-w)(1+2\delta)-\delta^2(H+d)+\delta(4H+d-w)+(3H+d-w)}{(1+\delta)^2}$$

Which order does he prefer in this subgame?

$U_D(L,H) - U_D(H,L) = \frac{(F_\epsilon(H+d+w)-F_\epsilon(H+d-w)))((3H+d-w)+w)\delta}{(1+\delta)^2}$

This is positive (or 0) because of non-decreasing proper CDF. Therefore if the dictator's post-bargain preference is for no conflict (uncertainty is sufficiently high), he prefers to choose the bargaining order L, then H. He comes to an agreement with L and they both fight H.

$[x_L^* = \frac{F_\epsilon(H+d-w)(\delta^2(2H+d)+\delta(H+d+w)-H+w)-F_\epsilon(H+d+w)(3H+d-w)(1+2\delta)-\delta^2(H+d)+\delta(4H+d-w)+3H+d-w}{F_\epsilon(H+d-w)H(1+\delta)^2}]$

Chapter 4

Local Elites Drive a Hard Bargain: Strategic Investment in Mass Distribution under Electoral Autocracy

Electoral regimes need some combination of the sincere votes of citizens and votes bought by local brokers or fraudulently cast to maintain power. Locally embedded elites' informational advantage as well as traditional social structures of patronage and clientelism give local leaders a strong bargaining position vis-a-vis a central government seeking the area's support. By circumventing local elites and appealing directly to the masses, a ruler can generate popular support in a geographic area without having to pay off a high-priced patron. With a multi-stage Rubinstein alternating offer bargaining program, I show how a central ruler can use investment in distribution to the masses to reduce the bargaining power of local elites while still using them to ensure the support of the district. I compare multiple institutional arrangements, showing that unit proliferation–the creation of more districts and therefore

more bargaining partners–only benefits the dictator if he does not need unanimous support and, instead, can credibly threaten to exclude some local elites from power-sharing. These varying institutions also yield different implications for where the dictator will invest in his popularity to strengthen his position vis-a-vis the local elite. When the dictator needs unanimous support, he will invest in the district in which he is less popular; when he only needs majority support, he will invest in the district in which he is more popular.

Leaders of electoral regimes need votes to achieve and maintain power. In electoral autocracies and developing democracies, this can be achieved with sincere popularity among voters, fraud, or some combination thereof. Local elites[1] are advantaged, relative to the central government[2], in garnering the electoral support of the citizens in their district. In some cases, these local elites are powerful patrons and traditional leaders whose clients will follow the political directives of the local leader due to existing relationships or expectations of reward (Koter 2016). In other cases, local brokers simply have an informational advantage of local embeddedness: even without a hierarchical structure, they can recognize which voters in their district to target with vote buying and more easily monitor and punish defectors (Frye, Reuter, and Szakonyi 2019; Hidalgo and Nichter 2016; Larreguy, Marshall, and Querubin 2016; Stokes et al. 2013). As local elites are thus advantaged in delivering their districts' support to the central leader, they are an invaluable component to the regime's maintenance of power.

When local support is needed to achieve and maintain power, the dictator could attempt to win local support by coopting these powerful local elites. In many authoritarian states and developing democracies in Eurasia, Africa, and Latin America, clientelistic structures permeate politics (Hale 2014; Koter 2016; Migdal 1988). In situations when the local areas are organized into clientelist pyramids emanating from a single powerful patron (Hale 2014), the dictator can use the structure to his advantage by only buying off the top elite. If there is a strong local elite who can whip votes, ensure effective policy implementation in the district, and generally support the needs of the regime at the local level, the dictator can simply buy off this local elite in order to bring the entire locality into his fold. Similarly a high-ranking broker with local relationships and information can be an effective "purchase" for the dictator: by coopting this single elite or local machine, the dictator can ensure his

[1] I use local notable, elite, chief, and local leader interchangeably.
[2] I use central government, leader, ruler, and dictator interchangeably.

victory in the district through vote buying or fraud. How exactly elites are included into the coalition in a way that ties their hands to supporting the regime can be varied: positions in government (Blaydes 2010), commitment to a dominant party (Reuter 2017), or just more transfers than a challenger can offer(Groseclose and Snyder Jr 1996; Bueno De Mesquita et al. 2003). While needing to buy off only one person at the top of the district "pyramid" may be most efficient, the bargaining position of a local leader that has a tight grip on her district is strong and may drive up the "price" that she can charge the dictator in exchange for her support.

For a dictator trying to extract resources for himself, reliance on local power brokers can be problematic. How can the dictator win and maintain electoral support without being held hostage by powerful local elites? By going around the local elites and appealing directly to the masses, a ruler can generate popular support in a geographic area without having to pay off a high-priced patron. While electoral autocracies are not often recognized for their public goods distribution, mass coalition building through distributive politics is common across the developing world (Stokes et al. 2013). Turning out the vote with subsidies, transfers, and other benefits to mobilize supporters (Kasara 2007) is common. Dictators may in fact pursue public policies that benefit large portions of the population such as land reform (Albertus 2015) or more targeted benefits that can only be accessed by citizens in certain localities, ethnic groups, or industries. By using public goods and transfers to voters, the dictator can directly increase his own sincere popularity among voters, reducing the need for local elites to whip votes and commit fraud on his behalf.

I show how a central ruler can use costly distribution to the masses to reduce the bargaining power of local elites. Investment in the infrastructure of mass politics allows a dictator to build a cheaper coalition of local elites because of the indirect effect of the dictator's popularity with citizens on the elites' bargaining strength, regardless of whether the dictator actually bypasses the local elites. My results show that while investment in mass politics supports the dictator's bargaining position relative to the local elite, it reduces the mutual surplus (i.e. efficiency relative to no elite) that using the elite intermediary provides. Thus there is a limit to how much costly investment in public goods the dictator will make when coopting the local leader is an option. Furthermore, the institutional environs of the central-local bargain drastically affects the dictator's popularity investment and resulting elite bargain. In a multi-district environment, the dictator's ability to play elites off of each other to create a minimal winning coalition further reduces the bargaining power of local elites.

This study has implications for not only the power of the central government vis-a-vis local strongmen but the distribution of public and nonprogrammatic goods. Graft and corruption, the representation of certain districts in the central government, and other benefits that the local elites can demand from the center directly depend on the dictator's ability to build local support among voters and the institutional rules that define central-local relations. Additionally, a dictator's use of investment in mass politics to increase his bargaining power while simultaneously coopting local elites to deliver their district's support (albeit at a lower price) may explain the outsized margins of voter fraud in areas where the dictator is already popular (Rundlett and Svolik 2016).

4.1 Building and Maintaining Local Support

The strength, legitimacy, and authority of local elites varies substantially across states and even within countries. Politicians can mobilize voters "...by indirectly working through electoral intermediaries: local leaders who command moral authority, control resources and can influence the electoral behavior of their dependents" (Koter 2016, 17). These local elites may derive their authority over voters through a religious order, such as Marabouts in Senegal (Koter 2016), economic dominance in an area, as is the case of oligarchs in Eurasia (Hale 2014) and prominent employers in Russia and Venezuela (Frye, Reuter, and Szakonyi

2019), or through official governmental position as a mayor of a large city, governor, or provincial administrator(Barkan and Chege 1989; Bueno De Mesquita et al. 2003; Reuter 2017). Regardless of how they derive their power, local elites can wield their influence over voters in their district to help or hurt the electoral chances of the central ruler. Even without the religious or moral authority to ensure the compliance of their voters, local leaders have advantages over the central government in ensuring citizens follow their directions.

Local elites have information, monitoring, and punishment power that allow for micro-targeting of goodies, favors, jobs, and other benefits (or punishments) within a district. This allows them to condition benefits for individuals in their geographic domain on the actual support the individual gives, whether through party membership, voting, or tacitly supporting regime policies (Stokes et al. 2013). The relationships that a local leader has with the voters themselves or the network of brokers and subordinates that they utilize to whip votes are invaluable and often irreplaceable as the ability to monitor their behavior and only reward those who follow the directives of the patron is paramount to the transaction (Larreguy, Marshall, and Querubin 2016; Stokes et al. 2013). Whether the electoral outcomes are a result of individual vote-buying or its variants (Gans-Morse, Mazzuca, and Nichter 2014), fraud at the polling place or aggregation level (Beber and Scacco 2012), true moral or religious persuasion, or voters' heuristic belief that voting according to their patron's wishes will yield more preferable policies or development spending from the center (Carter and Hassan 2021; Koter 2016), the local elites' ability to deliver their locality makes them an important partner for the central leader to court.

By using these intermediaries, however, the regime is giving both resources and the control of their distribution to local powerholders. In addition to all the agency problems that this entails (will the local powerbroker actually deliver the votes/support? are they serving the interested of the principal?), a local elite that becomes an independent power base can be problematic for the dictator's tenure. While it is highly unlikely that a local notable will lead a coup, the defection of one powerful chief (and therefore all of her clients in her district) to a challenger could be disastrous for the incumbent. The dictator might need to overpay (i.e. pay significantly more than the elite distributes to his clients) the local elites in his coalition, particularly those in large or pivotal districts, to keep them loyal. Without a sufficient number of districts to win a national-level election, an incumbent ruler will lose power. Recognizing their importance to the dictator's tenure, local elites can make high demands of the central government.

How can the central government avoid being held hostage by strong local leaders who demand much in exchange for their base of voters? If there is not currently an elite intermediary, it is difficult to simply set one up. A regime lackey will not have the support and trust of locals. An effective intermediary needs to be locally embedded, respected, and powerful. If people like this already exist in the community, their bargaining power to be coopted by the regime makes them very expensive (Garfias and Sellars 2018; Gerring et al. 2011). According to Koter, "as much as it might be tempting for political actors to manufacture intermediaries where they were hitherto lacking, such efforts are likely to fail. Strong relations between local leaders and their followers are not built overnight" (Koter 2016, 39).

If he cannot replace a local leader at the peak of a district hierarchy with his loyal lackey, the dictator's best option is to go around the local hierarchy and appeal directly to voters. The use of public goods and "pork" targeted at voters in different electoral districts has been widely studied in the literature (Golden and Min 2013; Cox and McCubbins 1986; Dixit and Londregan 1996). Unlike the distribution that the local elite could undertake to buy (or legally persuade) voters, mass redistribution is inefficient in its lack of targeting of the most persuadable voters on an individual level and monitoring their resulting votes. Micro-targeting benefits to supporters or individuals who need a transfer in order to support the incumbent is not feasible without local intermediaries.

Infrastructure projects, subsidies for particular industries, or special treatment for certain ethnic groups are all ways that a dictator can build support among certain mass groups without a local elite intermediary. While these policies may have some level of excludability (i.e. you can only access the subsidy if you farm a particular crop), they are not excludable at the micro-level. There could be many cocoa farmers, for example, that may not actually deliver their votes or support to the regime but receive the subsidy anyway (Kasara 2007). Without a local intermediary to monitor which individuals received benefits and failed to give their support, the center cannot punish individuals who do not support the regime. Public goods at a district level instead boost the overall popularity of the central government who provides such benefits, but does not necessarily persuade all the affected voters to vote for the incumbent.

What the extant work on distributive politics fails to consider, however, is how the general distribution of public goods and more popular policies will affect the relationship between the dictator and local elite. Increasing his popularity among voters does not remove the local power structure from the local elite, it simply increases the dictator's likelihood of winning *without* the support of the local elite. I argue that the incumbent can use mass politics to increase his popularity with voters as a bargaining tactic: by making himself more likely to win his election without elite support, the dictator reduces the local elites' advantage in their districts and increases his own bargaining position. This means that the dictator can still coopt the local elite and utilize their influence and political machine to deliver the district, just at a lower price than if he had not increased his mass popularity. As I show below, this effect has a limit: as the dictator invests more in the district, the efficiency surplus that using the elite intermediary supplies is reduced. Thus the dictator will make a costly investment in mass politics to improve his bargaining position vis-a-vis the local elite, still utilize the local elite's machine at a reduced price of cooptation, and keep the investment sufficiently low that the technology of a local intermediary continues to yield a surplus. I compare multiple institutional environments to see how the number of potential bargaining partners (and later heterogeneous districts) affect the dictator's investment decisions.

As alluded to by the above discussion, there are some conditions that limit where I expect this theory to apply. First, there must be local elites who can effectively deliver their geographic area electorally (or at least having their support must make the dictator significantly more likely to win than he would without them), though how they generate the votes in their individual district does not matter. I focus on electoral autocratic regimes and developing/unconsolidated democracies because it is more likely that the local power broker can truly deliver the district (possibly with fraud) in these cases. Second, there must be some policies or goods that the central government can provide to voters that the local elite cannot appreciably interfere with. If the local elite was in charge of the distribution of such a good, she could intercept the dictator's attempt to popularize himself with voters and (1) claim credit for the distribution and increase her own hold on the district, minimizing the effect the good has on the dictator's popularity, or (2) refuse to distribute the good altogether and prevent the dictator from altering his electoral chances and, therefore, his bargaining position.

4.2 Model

Consider a regime subdivided into districts $\mathcal{I} = \{J, K, L...\}$, which each have a local elite chief and a continuum of masses. In order to stay in power, the dictator must win the support of a sufficient number of districts.[3] If the dictator is successful in winning at least the minimum number of districts, he will achieve a regime benefit R. To win districts, the dictator can appeal directly to the masses of the district and sidestep the local elite or meet the demands of the local elite and recruit her support in controlling the district. While the expression of support in a district could take a variety of forms (including simply paying

[3] I will address requiring one of one, two of two, and one of two in the main text. Three district configurations are addressed solely in the appendix.

taxes and acquiescing to the dictator's policies without unrest), I will model the district's support for the dictator (with or without the local elite as an ally) as an election. If the dictator utilizes a local elite intermediary (meeting her cooptation demand), he will win the support of the district with certainty. Without the elite, the dictator may still win the district, but is subject to the uncertainty of the election (see below). The dictator will bargain with each elite individually with no reconsideration: once an agreement on the split of his regime benefit between himself and local elite has been reached, it will occur. If there is more than one district, nature chooses the order by which elites bargain with the dictator.

The dictator can improve his chances in the district election by investing in costly mass redistribution. In particular, he can use costly distribution to the masses to make himself more popular without having to go through the elite intermediary. The dictator derives utility from the regime benefit R, less any cooptation cost from utilizing elite intermediaries ($E_{\mathcal{I}}$) and cost of investment in mass redistribution ($c(\nu)$): he has no preference over ideological positioning or mass redistribution for ideological reasons. If he fails to win a sufficient number of districts, with or without elite support, the dictator's reversion utility is 0.

$$U_D(\nu, \sum \text{Bargain}_{\mathcal{I}}) = \begin{cases} 1 - \sum E_{\mathcal{I}} - c(\nu) & \text{Win Sufficient Districts} \\ 0 & \text{Lose} \end{cases}$$

Local elites derive utility from the amount of the regime benefit that they can extract from the dictator in exchange for delivering their district. If the dictator wins the district without making a deal with the local elite, the elite has a reversion utility of 0. I assume that the local elite does not incur any additional cost for capturing their district on the dictator's behalf: she has sufficient local power and knowledge to use the resources she receives from the dictator to convert the district efficiently. The elite only receives her promised cooptation price if the dictator is ultimately successful; because she is receiving a portion of the regime benefit, the local elite will get nothing if the dictator does not win enough districts to succeed.[4]

$$U_{E_{\mathcal{I}}}(\text{Bargain}_{\mathcal{I}}) = \begin{cases} E_{\mathcal{I}} & \text{Dictator Wins Sufficient Districts} \\ 0 & \text{Lose} \end{cases}$$

The Election

The dictator (D) faces a non-strategic challenger (C) in an election in every district in $\mathcal{I}$. The winner of a sufficient number of districts receives the regime benefit R, which I normalize to 1. The continuums of voters in each district are identical across districts. Each district is characterized by a representative voter, z, who evaluates the dictator relative to the challenger based on the dictator's valence, which can be positive, where the dictator is more popular than the challenger, or negative, where the challenger is more popular.

$$U_z = \begin{cases} 0 + V & \text{Vote D} \\ 0 & \text{Vote C} \end{cases}$$

V denotes the valence of the dictator relative to the challenger for individual voter z; it is distributed uniformly on $[-\beta + \alpha, \beta + \alpha]$ where $\beta > 0$. While β determines the dispersion of V, the mean of the distribution is determined by α which can be positive or negative. This is the natural valence the voter has for the dictator prior to any investment. If $\alpha > 0$, the dictator is idiosyncratically more popular than the challenger; if $\alpha < 0$, the dictator is at a disadvantage; if $\alpha = 0$ the voter is neutral. Voter z will vote for the dictator D if and only if $V \geq 0$. I assume that the dictator's minimum *ex ante* probability of winning is non-zero: while his likelihood of winning a district may be extremely small, it is always positive.

If the dictator invests in mass redistribution, he moves his valence distribution by $\nu > 0$

[4] I assume the local elite's fate in her own district is not tied to that of the dictator. She does not incur a cost for "backing the wrong horse" if she delivers the district and the dictator ultimately loses. Any power and benefits she receives for being the district chief is unaltered by the dictator's election or lack thereof.

in all districts.[5] Specifically, $V_I \sim U[-\beta+\alpha+\nu, \beta+\alpha+\nu]$ where $\beta, \nu > 0$. The probability that the dictator wins is still $1 - F_{V_I}(0)$. The dictator can choose a level of investment $\nu \in [0, \infty)$ and pays cost $c(\nu)$ where $c'(\nu) > 0$, $c'' > 0$, and $c(0) = 0$. The dictator's decision to invest occurs prior to any cooptation negotiations with the local elite.

Note that as valence becomes more uncertain, the dictator's expectation of winning goes to $\frac{1}{2}$ regardless of the positive idiosyncratic lean of the voter, α. The *ex ante* probability that the dictator wins the district is increasing in $\alpha > 0$ as he is more popular as well as in ν, the investment he makes in appealing to the masses.

4.2.1 Bargaining Protocol and Sequence of Play

I utilize an alternating offer Rubinstein bargaining protocol with risk of breakdown. Within a bargaining period, the dictator and local elite bargain over the division of the benefit the dictator achieves when the elite supports him, which is normalized to 1. Whenever an offer of division is rejected, with probability $\delta \in [0, 1]$ the other party makes a counteroffer or bargaining breaks down and the dictator must stand for a mass election without elite support in the district with probability $1-\delta$. If the dictator and elite fail to reach a successful bargain (or bargaining breaks down before they are able to come to an agreement), the dictator moves to the next elite in the nature-determined sequence or, if no districts remain, the election proceeds as described above: nature draws V from the appropriate distribution (F_V or F_{V_I} if investment was made) and the citizens cast their votes for the dictator or challenger. If the dictator wins the district without making an agreement with the elite, he keeps the regime benefit (1) all to himself. If the dictator and elite fail to reach an agreement, the excluded elite gets a reversion utility of 0. If they do reach an agreement, each party receives the proportion of the regime benefit that they agreed to.[6]

[5] This assumption (that the dictator can make an investment across all districts) is relaxed later, allowing the dictator to target specific districts for valence increases.

[6] See formal appendix for general Rubinstein SPNE existence and uniqueness for this bargaining protocol.

The game proceeds as follows. Nature chooses the order of elites with whom the dictator will bargain (if there is more than one district). The dictator then chooses a level of investment, ν in mass politics, at convex cost $c(\nu)$. The elite in the first bargaining position proposes a split of the regime benefit between herself and the dictator, which the dictator can accept or reject. Upon rejection, with probability δ the dictator can make a counter proposal that the elite may accept or reject, and with probability $1 - \delta$ bargaining breaks down and the dictator moves to the next bargaining position. If a proposed split is accepted (by either party), the dictator moves to the next bargaining position until the districts are exhausted. Once the dictator has bargained with the local elite in every district, regardless of whether an agreement was reached or bargaining broke down, the election is held and payoffs are distributed.

4.3 Results

The relevant equilibria are subgame perfect Nash. The dictator and each elite utilize stationary strategies. Every time it is her turn to propose, the elite proposes x to the dictator, keeping $1 - x$ for herself, every period and accepts the dictator's proposal if and only if $y \geq y'$. The dictator proposes y to the elite, keeping $1 - y$ for himself, every period and accepts the elite's proposal if and only if $x \geq x'$.

The demand the elite makes of the dictator depends on whether the dictator has invested in mass politics as well as the dictator's incentives to reject her offers, which is a function of both his electoral prospects and the number of elites he has to bargain with.[7] Every time they fail to reach an agreement, the negotiations between the dictator and elite can breakdown and the election occurs without the elite support. This "outside option" of an unsupported election still yields the dictator a positive expected utility: he could win the

[7] All proofs are in the appendix.

election without the elite. The elite's utility if bargaining breaks down, however, is zero. While the elite is privileged in making the first offer, the dictator's non-zero outside option in case of breakdown gives him a stronger bargaining position.

How the dictator and elites negotiate is subject to both the electoral environment (how popular the dictator is and how uncertain his electoral prospects are), but also the *institutional environment* in which these bargains are taking place. In particular, how many districts does the dictator need to win the regime benefit? Even without the elite intermediaries, different configurations would yield the dictator different expected utilities: for example, the dictator's probability of winning two of two districts is less than his probability of winning one of two. Furthermore, how many districts the dictator needs to win determines whether there are "extra" districts. Not only is winning one of two districts more likely than two of two with the mass election, a one of two configuration also makes one of the elite intermediaries superfluous. The dictator doesn't *need* his first bargaining partner, he can simply reject a bad offer and move on to the second elite as winning just the second district is sufficient to win. To explore how both his electoral outside option and his negotiations with elites change across different institutional environments, I present a single district baseline, two of two, one of two, and two of three homogeneous districts. I then relax the homogeneity assumption to show how different levels of popularity and uncertainty drive different investments in public goods in his districts.

4.3.1 One District

Consider the simplest case in which there is only one district in the state. In order to achieve the regime benefit, the dictator must win the single district. He can stand for an election without the support of the elite and be subject to the uncertainty this entails, or coopt the district's local elite to win this district with certainty. Without an elite intermediary to bargain with, the dictator will achieve the regime benefit, normalized to 1, with the probability that he wins the district in the election, $\gamma = \frac{\alpha+\beta}{2\beta}$. Note that his expected utility is always increasing in his popularity in the district, but the effect of uncertainty depends on his popularity: when he is popular, his expected utility decreases in the uncertainty of the district; when he is unpopular, the dictator benefits from uncertainty in the district. Denote this electoral chance (the probability he wins the election without the support of the local elite) as $\Omega_{1,1}$ and the dictator's chances of winning with the elite support as Ξ which I have assumed is 1 as a coopted local elite will deliver her district with certainty. The marginal benefit of utilizing the local elite is $\Xi - \Omega_{1,1}$. I term this difference in utilizing the local elite relative to standing for election alone as the *mutual surplus*. From bargaining with the elite, the dictator will retain a portion $\lambda \in [0, 1]$ of this surplus. In the case of a single district, $\lambda_1 = \frac{\delta}{1+\delta}$.

The offer made (and accepted) is the proportion of the regime benefit that the dictator keeps from succeeding in the election with the support of the local elite. The remainder of the regime benefit is transferred to the elite as the price of her support. Note that the dictator never actually makes a counteroffer in equilibrium: he accepts the elite's initial offer, but that offer from the elite is taking the potential counteroffer into account.

Lemma 9. *When the dictator must win one of one districts, for any investment $\nu \geq 0$, the local elite makes him an offer of $\Omega_{1,1} + \frac{\delta}{1+\delta}(\Xi - \Omega_{1,1})$, which he accepts. If he were to make a counteroffer, the dictator would offer the local elite $\frac{\delta(1+\gamma)}{(1+\delta)(1-\gamma)}(\Xi - \Omega_{1,1})$.*

I term the equilibrium portion that the dictator takes as the *dictator's share*, which is a direct reflection of his bargaining power. As discussed above, the dictator is coming to this bargain in a privileged position: if the parties fail to reach an agreement, he maintains his electoral outside option. Because the probability that he wins the district without the local elite is non-zero, the dictator will never accept an offer from the elite that is less than his positive outside option of going to the mass election alone. Thus the dictator will always receive at minimum his electoral outside option, $\Omega_{1,1}$, from the bargain.

The mutual surplus of coopting the elite intermediary–the difference between going it alone and winning with certainty, $\Xi - \Omega_{1,1}$–is then split between the elite and the dictator according to $\lambda_{1,1}$. In the extreme case of $\delta = 1$, bargaining would continue infinitely and the parties would share this surplus 50:50.[8] For $\delta < 1$, the portion of the surplus that the elite will take in equilibrium is larger than the dictator's portion.

The dictator's share is increasing in α, his *ex ante* popularity in the district. The dictator's popularity in the district directly affects his bargaining position with the local elite and thus his equilibrium share: as his popularity increases, his outside option, $\Omega_{1,1}$ increases. Note, however, that the mutual surplus that the parties share, $\Xi - \Omega_{1,1}$, is *decreasing* in α. As the dictator is more likely to win his election without elite support, the benefit that the elite provides is lessened. Thus while the dictator's outside option grows in his *ex ante* popularity, the surplus decreases. For the single district case, the effect of the outside option dominates the decreasing surplus: the dictator's overall utility is increasing in his popularity.

The dictator has the option to actively increase his outside option of his unsupported election probability by investing in mass politics, but at a cost. For any $\nu > 0$, the voter's valence evaluation of the dictator has a positive shift, increasing his chances of being elected without elite support. He must pay a convex cost of $c(\nu)$ for this investment. Recall that the dictator makes his investment decision prior to bargaining with any elites and will thus pay the cost of mass redistribution regardless of whether he stands for election unsupported or not.

Assumption 1. *The dictator's optimal investment will not yield a certain electoral outcome:* $max\{\frac{\alpha+\beta+2\beta\delta}{2\beta^2(1+\delta)^2-1}, \frac{\beta-\alpha}{1+\beta^2(1+\delta)^2}\} < \beta - \alpha$

To simplify the analysis of the dictator's optimal investment in mass politics, I assume that the costs of investment are sufficient that he will never make an investment so large

[8] See Nash bargaining solution.

that winning the election on his own is guaranteed. The formal threshold is derived from the dictator's maximal optimal investment and the boundary of the uniform distribution. What his maximal optimal investment is depends on parameter magnitudes. A more complete treatment of the dictator's investment in boundary cases where this assumption is violated (and the probability of winning without investment is zero) is in the appendix.

Proposition 7. *When the dictator must win one of one districts and anticipates the equilibrium elite offer of* $\Omega_{1,1} + \frac{\delta}{1+\delta}(\Xi - \Omega_{1,1})$, *he makes optimal investment* $\nu^*_{1,1} = c'^{-1}(\frac{1}{2\beta(1+\delta)})$

Corollary 1. *When the dictator must win one of one districts, anticipates the equilibrium elite offer of* $\Omega_{1,1} + \frac{\delta}{1+\delta}(\Xi - \Omega_{1,1})$, *and* $c(\nu) = \frac{\nu^2}{2}$, *he makes optimal investment* $\nu^*_{1,1} = \frac{1}{2\beta(1+\delta)}$.

Note that, in equilibrium, the dictator is still coopting the elite, not actually utilizing the election. The investment in mass politics is an instrument by which the dictator reduces the bargaining power of the elite by making the his outside option more attractive, thereby increasing his utility. By increasing his mass election probability, the dictator makes a more lucrative deal with the local elite. The dictator's optimal investment is decreasing in the uncertainty of his reelection, β. Higher levels of electoral uncertainty reduce the impact of investment in mass politics on the dictator's outside option and, thus, the bargain he can strike with the local elite.

How does the technology of an elite intermediary affect the dictator's optimal investment? If the elite intermediary was not an option, as would be the case without strong local party machines or chiefs that are able to deliver the district, the dictator would make an optimal investment of $\nu^*1, 1(Mass) = c'^{-1}(\frac{1}{2\beta})$.[9] This optimal investment simply balances the direct increase in election probability that the dictator receives from mass investment with its cost, $c(\nu)$. Note that $\nu^*_{1,1}(M) > \nu^*_{1,1}(Elite)$: the optimal investment that the dictator makes without the use of an elite intermediary is greater than the optimal investment he makes

[9] With the convex cost functional form assumption of $c(\nu) = \frac{\nu^2}{2}$, this is $\nu^*_{1,1}(Mass) = \frac{1}{2\beta}$.

when an elite bargain is available. Because investment is used instrumentally under the elite bargaining technology, it's effect on the dictator's utility is mediated by the bargain. Recall that though $\Omega_{1,1}$ is increasing in the dictator's investment in mass redistribution, the mutual surplus of $\Xi - \Omega_{1,1}$ is *decreasing* in the investment. Thus the increase in the dictator's share that greater investment brings is mitigated by its negative effect on the surplus that the dictator and local elite share. When the dictator invests a lot in his popularity, he is no longer gaining as much from utilizing the elite intermediary.

4.3.2 Two Districts: Two of Two

A single district for the dictator to win and achieve the regime benefit is a baseline for the model: most states have multiple geographic areas over which to rule. While control over all districts might maximize tax revenue (Cederman and Girardin 2010; Garfias and Sellars 2018), a regime may be comfortable in power without the support of every region.

First consider a two-district state where the dictator needs the support of both districts in order to achieve the regime benefit. The dictator still has the option of investing in mass politics, which increases his likelihood of mass election uniformly in all districts. As described above, nature chooses which elite the dictator bargains with first and, after that bargain is concluded either with an agreement or breakdown, the dictator can then bargain with the second elite before the election occurs. Without bargaining, the likelihood that the dictator wins both districts through the election, and thus his expected utility for the no-elite no-investment world, is $\Omega_{2,2} = (\Omega_{1,1})^2 = \gamma^2$. As above, this is increasing in his popularity among voters and decreasing in electoral uncertainty if he is popular, increasing in electoral uncertainty if he is unpopular.

If the dictator and the elite in the first bargaining position fail to reach an agreement, the dictator's bargain with the elite in the second position is scaled by $\Omega_{1,1}$, the probability he wins one district alone. This is because even if they reach an agreement, the dictator will only get the regime benefit to split with the local elite if he wins the other district, which occurs with probability $\Omega_{1,1}$. The maximum mutual surplus that the dictator and second position elite are bargaining over is $1 * \Omega_{1,1}$. If the dictator and second elite also fail to reach an agreement, the dictator's true electoral outside option is $(\Omega_{1,1})^2$, the probability he wins both districts alone. Thus the bargain that the dictator and second elite will come to is $(\Omega_{1,1})^2 + \dot{\lambda}(\Omega_{1,1} - (\Omega_{1,1})^2) = \Omega_{1,1}(\Omega_{1,1} + \dot{\lambda}(\Xi - \Omega_{1,1}))$. As we know the dictator will make a deal with the first elite, this is off path.

If the dictator and first elite do reach an agreement, the dictator's bargain with the second elite is scaled by x_1, the amount of the regime benefit the dictator is left with after the agreement with the first elite. The maximum mutual surplus over which the two parties bargain is x_1. If the dictator and second elite fail to reach an agreement in this case, the outside option is what the dictator gets if he and the second elite fail to reach an agreement: $\Omega_{1,1}x_1$. Thus the bargain they come to is $\Omega_{1,1}x_1 + \lambda(x_1 - \Omega_{1,1}x_1) = x_1(\Omega_{1,1} + \lambda(\Xi - \Omega_{1,1}))$. Therefore in equilibrium we know that the dictator will keep $(\Omega_{1,1} + \lambda(\Xi - \Omega_{1,1}))$ portion of whatever is left after his bargain with the first elite. The first elite will leave him with $(\Omega_{1,1} + \lambda(\Xi - \Omega_{1,1}))$ between the first and second round of bargaining, so the dictator's utility at the end of both bargains after coopting both districts is $(\Omega_{1,1} + \lambda(\Xi - \Omega_{1,1}))^2$.

Lemma 10. *When the dictator must win two of two districts, for any investment $\nu \geq 0$, each homogeneous elite makes him an offer of $\Omega_{1,1} + \lambda_{1,1}(\Xi - \Omega_{1,1})$, which he accepts. If he were to make a counteroffer, the dictator would offer each elite $\frac{\delta(1+\gamma)}{(1+\delta)(1-\gamma)}(\Xi - \Omega_{1,1})$.*

Note that the individual bargains that the dictator conducts with each elite precisely follow his bargains in the one of one institutional arrangement. As he needs both elites and successfully coopting only one does not guarantee him the regime benefit (though it does affect his probability of winning it), each individual bargain is the same despite the difference in his general outside option. Both elites offer $\Omega_{1,1} + \lambda_{1,1}(\Xi - \Omega_{1,1})$, so the dictator's utility is

$(\Omega_{1,1}+\lambda_{1,1}(\Xi-\Omega_{1,1}))^2-c(\nu)$. Using his true outside option in this institutional configuration, the dictator's utility is $\Omega_{2,2}+\lambda_{2,2}(\Xi-\Omega_{2,2})-c(\nu)$ where $\lambda_{2,2}\equiv\frac{\delta^2}{(1+\delta)^2}+\frac{2\delta\gamma}{(1+\delta)^2(1+\gamma)}$ (which is equivalent to $(\lambda_{1,1})^2+\frac{2\delta\gamma}{(1+\delta)^2(1+\gamma)}$). As the dictator's outside option (winning with an election only) is lower when he has to win two districts instead of one, the surplus that he shares with the elite is greater. The share of the surplus that the dictator keeps, $\lambda_{2,2}$, is less than his single district share $\lambda_{1,1}$ as the elite are in a strong bargaining position with the surplus that they offer and the dictator is splitting the regime benefit with not just one but both elites.[10]

Prior to bargaining with the elites, the dictator can make his investment decision. For now, I assume that a single investment ν at cost $c(\nu)=\frac{\nu^2}{2}$ affects all districts uniformly: the dictator's popularity increases by ν in both districts.

Proposition 8. *When the dictator must two of two districts, anticipates the equilibrium elite offers of* $(\Omega_{1,1}+\lambda_{1,1}(\Xi-\Omega_{1,1}),\Omega_{1,1}+\lambda_{1,1}(\Xi-\Omega_{1,1}))$*, he makes optimal investment which satisfies the first order condition* $\frac{\alpha+\beta+2\beta\delta+\nu}{2\beta^2(1+\delta)^2}-c'(\nu)=0$.

Corollary 2. *When the dictator must win two of two districts, anticipates the equilibrium elite offers of* $(\Omega_{1,1}+\lambda_{1,1}(\Xi-\Omega_{1,1}),\Omega_{1,1}+\lambda_{1,1}(\Xi-\Omega_{1,1}))$*, and* $c(\nu)=\frac{\nu^2}{2}$*, he makes optimal investment*

$$\nu^*_{2,2}=\begin{cases}0 & \beta\leq\sqrt{\frac{1}{2}}\\ \frac{\alpha+\beta+2\beta\delta}{2\beta^2(1+\delta)^2-1} & \beta>\sqrt{\frac{1}{2}}\end{cases}$$

When the uncertainty of the election gets extremely low (β is extremely low), the dictator's utility gets infinitely large. Any investment in this region of the parameter space does materially change his utility, he only incurs the cost of putting resources into mass distribution. Thus only when there is some uncertainty in his electoral outside option will the

[10] An **individual** elite's portion of the surplus is larger in the single district setting than two of two ($(1-\lambda_{1,1})>\frac{(1-\lambda_{2,2})}{2}$).

dictator invest in his popularity among citizens. Similar to the single district state, optimal investment is increasing in the dictator's popularity and decreasing in electoral uncertainty when the dictator is popular. If he is sufficiently unpopular, investment is increasing in uncertainty.[11]

4.3.3 Two Districts: One of Two

Consider a state with two districts of which the dictator only needs one to succeed. Even without the elite intermediaries, the dictator's baseline outside option of winning the election is now $\Omega_{1,2} = 2\gamma - \gamma^2$.[12] As above, the dictator's likelihood of winning sufficient districts is increasing in his popularity and decreasing in the uncertainty of the election if he is popular. Note that without the elites, the dictator is unambiguously better off when he needs to only win one of two instead of two of two districts as his likelihood of winning sufficient districts is higher. In addition to the increase in his electoral outside option, only needing one of two elites supports the dictator's bargaining position. Because he only needs one district, if the dictator achieves an agreement with the first elite, the second is completely superfluous: no offer of sharing will entice the dictator to accept. The second elite will be left with nothing and the dictator will only share the regime benefit with the first elite. Despite her exclusion from the regime benefit, the second elite still matters a great deal: the bargain that the dictator strikes with the elite in the first bargaining position is materially different from the one-district bargain.

Lemma 11. *In equilibrium, the elite in the second bargaining position will never receive a portion of the regime benefit greater than 0.*

[11] Specifically if $\alpha < -\frac{1+2\beta^2}{4\beta}$

[12] Recall γ is the generic probability the dictator wins a single district. With the micro-voting model, $\gamma = \frac{\alpha+\beta}{2\beta}$.

Lemma 12. *When the dictator must win one of two districts, for any investment* $\nu \geq 0$*, the local elite in the first bargaining position makes him an offer of* $\Omega_{1,2} + \lambda_{1,2}(\Xi - \Omega_{1,2})$*, which he accepts; if he were to make a counteroffer, the dictator would offer the first elite* $\frac{\delta+\delta\gamma(2-\gamma)}{(1+\delta)(\gamma-1)^2}(\Xi - \Omega_{1,2})$*. The local elite in the second bargaining position makes the dictator an offer of 1, which he accepts; if he were to make a counteroffer, the dictator would offer the second elite 0.*

As discussed above, having a second district to win but only needing one changes the probability that the dictator is successful in a mass election, his outside electoral option $\Omega_{1,2}$. The probability of winning one of two districts is always greater than or equal to the probability of winning one of one or one of two.[13] While the dictator's electoral outside option is stronger, this also reduces the surplus from using the elite intermediaries. Further, having an additional potential bargaining partner increases the share of the regime benefit that the dictator can keep. If the dictator rejects the first elite's offer and their negotiations breakdown, he still has the opportunity to bargain with the second elite rather than being forced into an immediate election. Thus the elite has to give up more of the mutual surplus $(\Xi - \Omega_{1,2})$ in order to get the dictator to agree to his terms; if he were to demand as much as the local elite in the one-district or two of two version, the dictator would simply move on to the second elite and leave the first with nothing. Note that the elites (the districts they represent) are homogeneous: the only difference between them is the nature-chosen bargaining position that determines whether they share in the regime benefit or are left with nothing.

This change in institutional environment–where the dictator now has a back-up bargaining partner–can be seen in the difference between $\lambda_{1,1}$, the portion of the mutual surplus the dictator got with one district, and $\lambda_{1,2}$ (which is $\frac{\delta(2+\delta)}{(1+\delta)^2}$), the portion of the surplus he gets with needing only one of two. Note that $\lambda_{1,2} > \lambda_{1,1}$ for all $\delta > 0$: if there is any opportunity

[13] Assuming, as I do here, that the individual district probabilities are independent and the same.

to bargain, the dictator keeps more of the mutual surplus when he has a second, superfluous district.[14] Having a superfluous district, while beneficial for the dictator's overall utility, complicates his investment decision.

Proposition 9. *When the dictator must win one of two districts and anticipates the equilibrium elite offers of* $(\Omega_{1,2} + \frac{\delta(2+\delta)}{(1+\delta)^2}(\Xi - \Omega_{1,2}), 1)$*, he makes optimal investment which satisfies the first order condition* $\frac{\beta-\alpha-\nu}{2\beta^2(1+\delta)^2} - c'(\nu) = 0$.

Corollary 3. *When the dictator must win one of two districts, anticipates the equilibrium elite offers of* $(\Omega_{1,2}+\frac{\delta(2+\delta)}{(1+\delta)^2}(\Xi-\Omega_{1,2}), 1)$ *and his convex cost function* $c(\nu)$ *takes the functional form* $\frac{\nu^2}{2}$*, he makes optimal investment* $\nu^*_{1,2} = \frac{\beta-\alpha}{1+\beta^2(1+\delta)^2}$.

Utilizing this simple functional form for costs, the dictator's optimal investment balances the increase in his electoral outside option with the negative effect that mass investment has on the mutual surplus as well as its outright cost. The optimal investment in this case is decreasing in α, the dictator's *ex ante* popularity. The dictator is strongly advantaged by both having a superfluous back-up bargaining partner as well as the increased outside option simply from needing one of two districts; additional investment in his outside option via mass redistribution is less helpful. The dictator's equilibrium investment further depends on the uncertainty of the election: if he is sufficiently popular,[15] investment is increasing in the uncertainty of the electoral environment. At the upper end of his utility function (high α) the marginal returns to increasing his popularity are extremely small (flat slope), so an increase in β extends the domain and makes investment more lucrative. In other words, when the dictator is extremely popular he will already invest very little, but an increase in the uncertainty of the electoral environment gives him room to invest more in his popularity.

While the movement from needing one of one to two of two changed the dictator's outside option and share of the regime benefit he kept, it did not materially change his incentives

[14] $\lambda'_{1,2} > \lambda_{2,2}$ as well

[15] $\alpha > \frac{\beta^2(1+\delta)^2-1}{2\beta(1+\delta)^2}$

to invest in mass politics. In this institutional set up, however, the dictator no longer uses investment to simply increase his popularity (investing more when he is already popular). Instead, the dictator invests more when he is *unpopular*: investment is decreasing in his popularity, α. As the one of two institutional arrangement is so beneficial to the dictator in terms of both his outside option and the outcome of his bargain, he does not need to maximize his popularity but instead minimize his unpopularity.

4.3.4 The Effect of Institutions: Homogeneous Baseline

Despite using exactly the same bargaining protocol and homogeneous elites, the institutional rules of how many districts the dictator needs to win the regime benefit have drastic consequences for both the amount of the regime benefit the elites can extract and the dictator's optimal investment. Increasing the dictator's electoral outside option–his chances of achieving the regime benefit without local elite support– both increases the baseline demand he can make, but also reduces the surplus that using an elite intermediary generates. Having a superfluous district, however, unambiguously increases the dictator's utility through strengthening his bargaining position. Because he still has an opportunity to coopt the second elite and win the regime benefit, the dictator's position vis-a-vis the first elite he bargains with is much more advantaged. These simultaneous effects of altering the dictator's bargaining position relative to the elite and altering both the electoral outside option and surplus that the elite intermediary technology generates demonstrates the complexity of unit proliferation from the dictator's perspective. Where he to choose his own institutional situation, having to win one of two is preferred. However, general unit proliferation (the creation of more districts) does not necessarily imply that he will still only need a bare majority to win.

The varied institutional configurations also yields different predictions regarding the dictator's optimal investment in mass redistribution to increase his electoral popularity. Recall that the dictator will invest the most in mass redistribution when there are no elite intermediaries to bargain with: if he must stand for election without elite support, he will invest more in mass redistribution as the effect of his investment on his expected utility is direct. Investment with elite intermediaries is still beneficial to the dictator: increases his outside option improves his bargaining position, but because he does not actually stand for election unsupported, the effect is indirect. Which institutional configuration yields the greatest investment in mass politics? Which optimal investment is greater depends on the dictator's *ex ante* popularity, α. Holding the other parameters (β, δ) constant, if the dictator is unpopular such that $\alpha < -\frac{1+2\beta^2\delta(1+\delta)}{2\beta(1+\delta)}$, the dictator's optimal investment under one of two districts is greater than his optimal investment when he must win two of two. If he is not this unpopular ($\alpha > -\frac{1+2\beta^2\delta(1+\delta)}{2\beta(1+\delta)}$), his optimal investment when he needs to win two of two is greater. Recall that the dictator wants to invest more in his popularity when he is already popular when he needs to win two of two districts: his optimal investment is increasing in his popularity as the marginal benefit of increasing his popularity is higher when he is already popular. When he needs to win one of two districts, however, optimal investment highest when the dictator is unpopular: the marginal benefit of increasing his popularity is highest when α is extremely low.

Note that these relationships between the institutional environment, the dictator's electoral outside option, and the dictator's optimal investment are not particular to these particular institutions. When the dictator needs to win two of three districts and maintains that superfluous bargaining partner, his optimal investment mimics his optimal investment under one of two.[16] See the appendix for a full treatment and discussion of the electoral outside option, equilibrium offers, and optimal investment when the dictator needs to win two of three districts.

[16] In general his optimal investment is decreasing in his popularity except for the extremes of α where the slope of his electoral option is very flat.

4.3.5 Heterogeneous Districts

A multi-district state may incorporate districts which differ in a variety of ways including geography, distance to the capital, demography, and, most importantly, valence for the dictator. I consider a two district state in which the districts differ in three possible ways: (1) the dictator's ex ante popularity, (2) the uncertainty of the dictator's election without the support of the local elite, or (3) both popularity and uncertainty. There are two possible institutional configurations for a two-district state: either the dictator must win both districts to stay in office or he only needs one of two. The dictator now makes investment specific to a district. He can invest in both districts, one or the other, or neither, but he must make all investments prior to bargaining with the elites. As before, investment is costly; I now further assume a complementarity in costs between districts, χ. Investing in a single district involves the cost of building infrastructure, bureaucracy, and moving resources to voters in the district; investing in two districts requires those bureaucrats, builders, and central government administrators to split their attention between multiple locations. Whether the dictator wants to invest in both districts (and how much) is influenced by how much added difficulty there is in working in two different districts simultaneously. As described above, nature chooses which elite the dictator bargains with first and, after that bargain is concluded either with an agreement or breakdown, the dictator can then bargain with the second elite before the election occurs.

Two of Two Districts

Consider a two-district state in which the dictator must win both in order to achieve the regime benefit. Let each district have distinct valence distributions for the dictator: $V_1 \sim U[-\beta_1 + \alpha_1 + \nu_1, \beta_1 + \alpha_1 + \nu_1]$ and $V_2 \sim U[-\beta_2 + \alpha_2 + \nu_2, \beta_2 + \alpha_2 + \nu_2]$.[17] Let $\hat{\omega}_i$ be the dictator's electoral outside option in each individual district, the independent probabilities

[17] This includes the possibility that $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$ for completeness. See appendix.

that the dictator wins each district, such that $\hat{\omega}_1 = \gamma_1$ and $\hat{\omega}_2 = \gamma_2$ for district 1 and district 2, respectively. As he must win both, the dictator's true electoral outside option of winning both districts without elite support is $\hat{\Omega}_{2,2} \equiv \gamma_1\gamma_2$.

Lemma 13. *When the dictator must win two of two districts, for any investment vector* $(\nu_1, \nu_2) \geq 0$*, each local elite* $i \in \{1, 2\}$ *makes him an offer of* $\hat{\omega}_i + \lambda_{1,1}(\Xi - \hat{\omega}_i)$*, which he accepts. If he were to make a counteroffer, the dictator would offer each local elite* $\frac{\delta(1+\hat{\omega}_i)}{1+\delta}$*.*

Note that which bargaining position the elites are in does not matter: they each get a share of the regime benefit according to the characteristics of their district, not the bargaining order.[18] As above, the dictator does not actually make a counteroffer: the initial equilibrium offer from the each elite is immediately accepted. As γ_i is a function of the dictator's ex ante popularity in each district, his share is increasing in his ex ante popularity in each district. After bargaining with each elite, the dictator has $\hat{\Omega}_{2,2} + \frac{\delta(\delta+\gamma_1+\gamma_2)-\delta\gamma_1\gamma_2(2+\delta)}{(1+\delta)^3(1-\gamma_1\gamma_2)}(\Xi - \hat{\Omega}_{2,2})$.

How does investment in mass politics change when there are two heterogeneous districts? To parameterize the cost of the dictator's investment in each district, I utilize the specific functional form of $c(\nu_1, \nu_2) = \frac{\nu_1^2}{2} + \frac{\nu_2^2}{2} - \chi\nu_1\nu_2$ where $\chi \geq 0$ indicates a flexible complementarity in costs (i.e. costs are not purely independent of each other). If this complementarity is sufficiently low (χ is sufficiently small) the dictator will invest in both districts simultaneously; if the costs are too high, the dictator will only invest in one district.

Lemma 14. *Anticipating the elite equilibrium offers, the dictator will always make some investment in mass politics.*

Proposition 10. *If* $\chi < \hat{\chi}_{2,2} = 1 + \frac{1}{4\beta_1\beta_2(1+\delta)^2}$*, the dictator invests*

$$\hat{\nu}_1^* = -\frac{\alpha_1 + \beta_1 + 2\beta_1\delta + 4\beta_1\beta_2(1+\delta)^2(\alpha_2 + \beta_2 + 2\beta_2\delta) - 4\beta_1\beta_2(1+\delta)^2(\alpha_1 + \beta_1 + 2\beta_1\delta)\chi}{1 + 8\beta_1\beta_2(1+\delta)^2(-\chi + 2\beta_1\beta_2(1+\delta)^2(-1+\chi^2))}$$

[18] See the homogeneous two of two section above for a more extensive discussion of the sequential bargain.

$$\hat{\nu}_2^* = -\frac{\alpha_2 + \beta_2 + 2\beta_2\delta + 4\beta_1\beta_2(1+\delta)^2(\alpha_1 + \beta_1 + 2\beta_1\delta) - 4\beta_1\beta_2(1+\delta)^2(\alpha_2 + \beta_2 + 2\beta_2\delta)\chi}{1 + 8\beta_1\beta_2(1+\delta)^2(-\chi + 2\beta_1\beta_2(1+\delta)^2(-1+\chi^2))}$$

in district 1 and district 2, respectively. If $\chi \geq \hat{\chi}_{2,2}$*, the dictator will invest either* $\hat{\nu}_1 = \frac{\alpha_2+\beta_2+2\beta_2\delta}{4\beta_1\beta_2(1+\delta)^2}$ *or* $\hat{\nu}_2 = \frac{\alpha_1+\beta_1+2\beta_1\delta}{4\beta_1\beta_2(1+\delta)^2}$*: if and only if* $\alpha_2 - \alpha_1 > (\beta_1 - \beta_2)(1+2\delta)$*, invest in* $\hat{\nu}_1$ *in district 1, else invest* $\hat{\nu}_2$ *in district 2.*

Under weak conditions on the parameters such that the dictator faces a strict probability of winning or losing (i.e. not 0 or 1, see appendix), the dictator will always make some investment in mass politics to support his bargaining position vis-a-vis the local elites. When the costs of investing in mass politics in one district affect the costs of investing in the other, the dictator may not always invest in both districts despite the fact that he needs both to succeed. If the cost complementarity (χ) is sufficiently low, however, the dictator will invest in both districts. The total investment that the dictator makes in mass redistribution, $\hat{\nu}_1^* + \hat{\nu}_2^* = \frac{\alpha_1+\alpha_2+(1+2\delta)(\beta_1+\beta_2)}{4\beta_1\beta_2(1+\delta)^2(1+\chi)-1}$, is increasing the dictator's popularity in each district. While investment across heterogeneous districts is not directly comparable to investment in homogeneous districts,[19] the relationship between the dictator's investment and popularity is maintained: when the dictator must win two of two districts, whether homogeneous or heterogeneous, his investment is increasing in his popularity in the districts. Higher uncertainty (β) in the district reduces the dictator's investment there as uncertainty reduces the effectiveness of his investment.

The specific investments in each district, however, differ. In particular, investment in district one decreases in the dictator's popularity in district one and increases in the dictator's popularity in district two. Similarly investment in district two is decreasing in the dictator's popularity there and increasing in his popularity in the other district. He puts more of his resources where is he less popular, but his overall investment is still increasing in his

[19] Under homogeneous districts, a single investment with its cost $c(\nu)$ affects multiple districts simultaneously. When investment is targeted to heterogeneous districts, not only is there a complementarity in costs across the districts, χ, each individual investment costs $c(\nu_i)$.

popularity in each district. This division of resources becomes clear when we consider the dictator's investment in a single district. When the frictions of working in both districts simultaneously generate too much costs (χ is high), the dictator must choose which single district to invest in. Note that which district he invests in is not a function of the bargaining order of each district. Instead, only his own potential electoral win probability in each district affects his investment decision. When he must win both districts, the dictator will prefer to invest in the district in which he is less popular and more certain i.e. his investment will be more effective. Because these parameters are independent (a district can be more uncertain but he can be ex ante more popular), the magnitudes of the difference between the two districts matter when the dictator is simultaneously more popular and more uncertain in one district over the other. As can be seen from Proposition 4, the difference in the electoral uncertainties of the districts is weighted more than the difference in popularities. If district one's election is more uncertain, the dictator would have to be very unpopular in district one (relative to district two) to induce him to invest there. Note that some investment will always occur; while a high complementarity in costs might reduce his investment, zero investment in either district is always a dominated strategy for the dictator.

One of Two Districts

The other possible institutional arrangement in a state with two heterogeneous districts is that the dictator need only win one of the two to win the regime benefit. As in the homogeneous district case, because he only needs one district, if the dictator achieves an agreement with the first elite, the second is completely superfluous: no offer of sharing will entice the dictator to accept. The second elite will be left with nothing and the dictator will only share the regime benefit with the first elite.

With an "extra" district, the dictator's bargaining position is affected in two ways. First, having a back up elite to bargain with if his negotiation with the first elite breaks down increases the dictator's share of the regime benefit that he can keep from the first elite. Second, the dictator's true electoral outside option–his probability of achieving the regime benefit without the support of either elite–increases as he only needs one of two. In particular, his electoral outside option is $\hat{\Omega}_{1,2} \equiv \gamma_1 + \gamma_2 - \gamma_1\gamma_2$. Both of these forces benefit the dictator.

Lemma 15. *In equilibrium, the elite in the second bargaining position will never receive a portion of the regime benefit greater than 0.*

Lemma 16. *When the dictator must win one of two districts, for any investment vector* $(\nu_1, \nu_2) \geq 0$*; the local elite in the first bargaining position makes him an offer of* $\hat{\Omega}_{1,2} + \lambda_{1,2}(\Xi - \hat{\Omega}_{1,2})$*, which he accepts. If he were to make a counteroffer, the dictator would offer this elite* $\frac{\delta(1+2\delta+\hat{\Omega}_{1,2})}{(1+\delta)^2(1-\gamma_1)(1-\gamma_2)}(\Xi - \hat{\Omega}_{1,2})$*. The local elite in the second bargaining position makes the dictator an offer of 1, which he accepts; if he were to make a counteroffer, the dictator would offer the second elite 0.*

Despite the exclusion of the superfluous elite in the share of the regime benefit, the existence of this elite still affects the bargain that the dictator can strike with his first local elite partner. Note that even if the second district in the bargaining order is is better for the dictator in terms of outside win probability ($\gamma_2 > \gamma_1$), the dictator will prefer to come to an agreement with the first elite as the superfluous bargaining partner will always make him better off. Further note that even if the dictator could choose the bargaining order (instead of nature), he will be indifferent between either bargaining order as his expected utility is the same and he has no preference over which local elite shares more of the regime benefit. The portion of the surplus that the dictator keeps, $\frac{\delta(2+\delta)}{(1+\delta)^2}$, is the same as the portion of the surplus he kept under one of two homogeneous districts ($\lambda_{1,2}$).

Lemma 17. *Anticipating the elite equilibrium offers, the dictator will always make some investment in mass politics.*

Proposition 11. *If* $\chi < \hat{\chi}_{1,2} = 1 - \frac{1}{4\beta_1\beta_2(1+\delta)^2}$*, the dictator invests*

$$\hat{\nu}_1^* = \frac{\beta_1 + 4\beta_1\beta_2(1+\delta)^2(\alpha_2 - \beta_2 + \chi\beta_1) + \alpha_1(-1 - 4\beta_1\beta_2\chi(1+\delta)^2)}{1 + 8\beta_1\beta_2(1+\delta)^2(\chi + 2\beta_1\beta_2(1+\delta)^2(\chi^2 - 1))}$$

$$\hat{\nu}_2^* = \frac{\beta_2 + 4\beta_1\beta_2(1+\delta)^2(\alpha_1 - \beta_1 + \chi\beta_2) + \alpha_2(-1 - 4\beta_1\beta_2(1+\delta)^2\chi)}{1 + 8\beta_1\beta_2(1+\delta)^2(\chi + 2\beta_1\beta_2(1+\delta)^2(\chi^2 - 1))}$$

in district 1 and district 2, respectively. If $\chi \geq \hat{\chi}_{1,2}$*, the dictator will invest either* $\hat{\nu}_1 = \frac{\beta_2 - \alpha_2}{4\beta_1\beta_2(1+\delta)^2}$ *or* $\hat{\nu}_2 = \frac{\beta_1 - \alpha_1}{4\beta_1\beta_2(1+\delta)^2}$*: if and only if* $\alpha_1 - \alpha_2 > \beta_1 - \beta_2$ *invest* $\hat{\nu}_1$ *in district 1, else invest* $\hat{\nu}_2$ *in district 2.*

As above, under weak conditions on his win probability, the dictator will always make some non-zero investment in mass politics. I utilize the same functional form for convex costs as above: $c(\nu_1, \nu_2) = \frac{\nu_1^2}{2} + \frac{\nu_2^2}{2} - \chi\nu_1\nu_2$ where $\chi \geq 0$ indicates a flexible complementarity in costs (i.e. costs are not purely independent of each other). As in the two of two institutional configuration, if the cost complementarity (χ) is sufficiently low, the dictator will invest in both districts. The total investment that the dictator makes, $\hat{\nu}_1^* + \hat{\nu}_2^* = \frac{\beta_1 - \alpha_1 + \beta_2 - \alpha_2}{1 + 4\beta_1\beta_2(1+\delta)^2(1+\chi)}$, which is decreasing the dictator's popularity in each district. This parallels the dictator's investment in one of two homogeneous districts, which also was decreasing in his popularity, α. Similar to the two of two institution, higher uncertainty (β) in the district reduces the dictator's investment there as uncertainty reduces the effectiveness of his investment.

Each targeted piece of this overall investment, however, has the opposite relationship with the dictator's popularity. Investment in district one (two) decreases in the dictator's ex ante popularity in district two (one) but increases in his popularity in district one (two). If costs are too high, the dictator will choose a single district to invest in. The order in which he will bargain with the districts does not affect his investment choice, only his potential to win each district without the elite support matters. The dictator prefers to invest in the district in which he is already more popular and in which his chances are more certain. As these are independent, he could be both popular and have high electoral uncertainty in a single district. Where he invests in this case depends on his relative popularity and uncertainty between the districts as can be seen in Proposition 5.

Note that which district the dictator invests in (or how his two district investment amount changes) differs under the two institutional configurations described here. When the dictator must win both districts (two of two), investment in mass politics is complementary, particularly if χ is low. Investment in district 1 is increasing in investment in district 2 and vice versa. When he must choose which district in which to invest, the dictator will invest in the district in which he is less popular. Because he needs both districts to win, he will use his resources in his weakest district.

By contrast when only one of two districts is needed to win, investment in districts is substitutive: as he invests more in district 1, he will invest less in district 2 and vice versa. Both districts affect his expected utility through the bargain (and internal outside option), so even investment in the district that he does not end up coopting helps the dictator's position. If costs are so high that he must choose which district to invest in, the one of two institutional configuration will lead him to invest in the district in which he is more popular. Instead of investing in his weakest district, he invests in his strongest district to make it as easy to win as possible *even if this is not the district that he ends up coopting through the elite bargain.* His bargain with the elites takes both district win probabilities into account, thus he will invest more where he is popular regardless of which elite he ends up coopting.

The thresholds in the cost complementarity, χ, at which the dictator moves from investing in both districts to just one also depends on the institutional configuration. The range of parameters for which he prefers to invest in both districts is greater when he must win two of two than one of two. Relative to his optimal investment scheme if there were no intermediaries (and he still needs one of two districts), the parameter range for which he invests in both districts is larger when he bargains with the elites. Investing in the superfluous district is more attractive when the elite intermediaries are available because investment not only increases his win probability but also increases his bargaining position. Note this is not to say that he invests more when there are elite intermediaries (he does not), but that he will invest in both districts, instead of just one, more frequently.

4.4 Discussion

In summary, the series of alternating-offer bargains with local elites under a variety of institutional configurations has generated predictions regarding where the leader will invest in his popularity among citizens: districts in which he is already popular or districts in which he is unpopular. While using mass politics to improve his own bargaining position vis-a-vis local elites benefits the dictator, the use of this strategy is limited not only by the outright costs of redistribution but the decreasing returns to popularity when elite intermediaries are available. The surplus in utility that using a local elite intermediary who can deliver their area with certainty provides is diminished as the dictator's electoral chances improve.

In what cases might a dictator utilize this investment strategy to undermine the local elites? As discussed above, when the institutional arrangement improves his bargaining position, the dictator may invest less in public goods.[20] How can the dictator move to a more beneficial institutional arrangement in which he has superfluous bargaining partners and can therefore keep more of the regime benefit? While institutional choice is outside the scope of the current model, it is clear that while the dictator has an obvious preference for a one of two arrangement, the local elites do not. Elites that are powerful enough to control their districts can cause a lot of problems for a dictator attempting to change the institutions of the state, whether that be through unit proliferation, centralization, or changes in electoral

[20] This is not universally true: when he is unpopular, optimal investment is greater in the one of two configuration. Equilibrium two of two investment is greater than one of two for a majority of the parameter space.

rules for higher offices.

This is precisely what happened in Kenya under Daniel Arap Moi in the 1980s. He wanted to reduce the power of provincial leaders, so first adjusted to a populist message to shore up his own popularity among citizens. He then implemented an institutional reform that devolved the power of provincial leaders to a lower level of government (districts) that were easier for him to control and limited the bargaining position of any one province/district (more bargaining partners). In 1983, Moi unilaterally instituted a massive institutional change across Kenya called “District Focus” (Barkan and Chege 1989). Prior to Moi’s ascension to the presidency,“Kenyatta [the previous president] governed via a stable clientelist system composed of regional and district-level leaders, who were permitted to establish their own political identities and local bases of power, provided they supported the President” (Barkan and Chege 1989, 437). When the Kikuyu officers staged an unsuccessful coup in 1982, Moi was no longer tolerant of the strong Kikuyu provincial and district leaders, many of whom had been appointed by Kenyatta (Barkan and Chege 1989; Hornsby 2013). Instead, “Moi sought to bypass such leaders and their organizations to create directly his own personal following in the countryside” (Barkan and Chege 1989, 437). The District Focus, which was announced in October 1982 and implemented in 1983, was advertised as a focus on rural development and more efficient local governance. Indeed, “Moi’s populist mode of governance... had its intended effect of circumventing the influence of the most senior politicians of the Kenyatta era, especially those from the Central Province” (Barkan and Chege 1989, 437-438). Despite the purported focus on equitable development, making the lower level district as the primary operational unit (rather than larger provinces) allowed Moi to “reduce the power of P.C.s [Provincial Commissioners]” (Barkan and Chege 1989, 440).

What this example demonstrates is how devolution and unit proliferation has already been recognized as a strategy by which a dictator can strengthen himself relative to local elites. This model has demonstrated the two mechanisms by which this is the case: (1) the change in the dictator's likelihood of maintaining his regime without the use of the elites and (2) the bargaining strength of the dictator when there are superfluous bargaining partners to play off one another. Furthermore, in order to undermine the resistance of local leaders, who would likely oppose such a large institutional change that would disadvantage them, the dictator invests in his own popularity among citizens. By adopting populist policies and increasing his own "personal following," Moi supported the survival of his regime against the possible defections and difficulties that the local elites he subverted.

4.5 Conclusion

Under what conditions can an elected autocrat undermine the power of local elites while still utilizing their embedded capacity on behalf of his regime? By extending a local-center bargaining framework to multiple heterogeneous districts and varying institutional rules, I show that while investment in his own popularity among citizens can improve his bargaining position, the positive effect that this has on his overall utility is diminishing. The greater benefit that using a local elite intermediary provides to the leader– the ability to monitor, reward, and punish at a targeted, micro level because of her local embeddedness– is mitigated as the dictator becomes so popular he no longer needs the aforementioned strategies of generating support. Furthermore, where he invests in his electoral chances is a function of how difficult it is to redistribute to multiple districts simultaneously, his popularity and electoral uncertainty in each district, as well as the institutional arrangement he faces (unanimous or majority support).

The importance of the number of districts and electoral rules in determining the dictator's investment and utility have important implications for unit proliferation– the creation of more subnational governments. More units do not necessarily benefit the dictator if he still needs unanimous support. Local elites' abilities to deliver their districts to the dictator (or not if they are not paid sufficiently) implies a level of local corruption that merits further study. Does the use of central mass politics undermine local leader's ability to deliver? If the individuals in their districts receive everything they want from the central government, how can a local elite still control them? This consideration could further depress a dictator's investment in centrally-distributed public goods if it has further deleterious consequences for local power that, at the end of the day, he still wants to use for his own benefit. Lastly, the simultaneous use of investment in mass politics and local intermediaries to deliver a district's support could explain vote-buying and fraudulent electoral practices even when the dictator is popular (Rundlett and Svolik 2016). Over the top returns– where the leader wins by such a large margin it is obviously irregular– could include sincere votes that he won through mass redistribution and true popularity among citizens as well as local brokers and patrons earning their share of the regime's power.

4.6 Formal Appendix

Micro Probabilistic Voting Model

The dictator (D) faces a challenger (C) in an election in some district J. The district has a continuum of voters with unit mass. The winner of district J receives benefit R which I normalize to 1.

Representative voter z cares only about their idiosyncratic preference for the dictator relative to the challenger. The utility of voter z is

$$U_z = \begin{cases} 0 + V & \text{Vote D} \\ 0 & \text{Vote C} \end{cases}$$

V denotes the valence of the dictator relative to the challenger for voter z; it is distributed uniformly on $[-\beta + \alpha, \beta + \alpha]$ where $\beta > 0$ and is uncorrelated with a voter's ideology. While β determines the dispersion of V, the mean of the distribution is determined by α which can be positive or negative. This is natural valence the district has for the dictator prior to any investment. If $\alpha > 0$, the dictator is idiosyncratically more popular than the challenger in district J; if $\alpha < 0$, the dictator is at a disadvantage; if $\alpha = 0$ the district is neutral.

Voter z will vote for the dictator D iff $V \geq 0$

If the voter has a positive valence for the dictator, she will vote for the dictator. The probability that $V \geq 0$ is $1 - F_V(0)$. Using the uniform, the probability the valence is sufficient for the dictator to win the election is $\frac{\alpha+\beta}{2\beta}$

Note that $\lim_{\beta \to \infty} \frac{\alpha + \beta}{2\beta} = \frac{1}{2}$ as valence becomes more uncertain, the dictator's expectation of winning goes to $\frac{1}{2}$ regardless of the positive idiosyncratic lean of the district, α. In general, the *ex ante* probability that the dictator wins the district is increasing in $\alpha > 0$ as he is more popular.

If the dictator invests in mass redistribution in district J, he moves his valence distribution by $\nu > 0$. Specifically, $V_I \sim U[-\beta + \alpha + \nu, \beta + \alpha + \nu]$ where $\beta, \nu > 0$. The probability that the dictator wins is still $1 - F_{V_I}(0)$ which using the uniform is $\frac{\beta+\alpha+\nu}{2\beta}$.

General Bargaining SPNE

Denote the dictator's outside option $\Omega \in (0, 1)$. Regardless of the history, after any rejection bargaining continues with common probability $\delta \in (0, 1)$. The size of the regime benefit over which the two parties are bargaining is 1, but note this benefit can be scaled by any constant. The elite makes the first offer.

Let m_E and M_E be the infimum and supremum of equilibrium payoffs to the elite in the game. Let m_D and M_D be the infimum and supremum of equilibrium payoffs to the dictator when he is the proposer. The following inequalities hold:

1. $m_E \geq 1 - (\delta M_D + (1 - \delta)\Omega)$

2. $1 - M_E \geq (\delta m_D + (1 - \delta)\Omega)$

3. $m_D \geq 1 - \delta M_E$

4. $1 - M_D \geq \delta m_E$

In equilibrium, the dictator must accept an offer x where $x = (\delta M_D + (1 - \delta)\Omega)$ as that is the most that he could get from refusing (inequality 1). It follows that the elite cannot get less than w where $w = 1 - (\delta M_D + (1 - \delta)\Omega)$ because she can get a guaranteed w by making it her opening demand.

Similarly, in equilibrium, the dictator must get at least y for each $y = (\delta m_D + (1 - \delta)\Omega)$ because y is guaranteed if the dictator rejects the elite's opening proposal, so the elite can get at most $1 - y$ (inequality 2).

When the dictator is the proposer, the elite must accept an offer x' where $x' = \delta M_E$, the most she could get from refusing (note the elite's outside option is 0). Thus the dictator cannot get less than $1 - \delta M_E$, which he is guaranteed if he makes x' his opening proposal (inequality 3)

The elite must get at least y' for each $y' = \delta m_E$ as that is guaranteed if the elite rejects the dictator's proposal. Thus the dictator can get at most $1 - y'$ (inequality 4).

Rearranging these inequalities, we see that

$M_E \leq 1 - \delta m_D - (1 - \delta)\Omega$

$m_E \geq 1 - \delta M_D - (1 - \delta)\Omega$

so if $m_D = M_D$, then $m_E = M_E$

Further,

$M_D \leq 1 - \delta m_E$

$m_D \geq 1 - \delta M_E$

So if $m_E = M_E$ then $m_D = M_D$

but how do we know that this is necessarily the case?

Proof by contradiction (to show that $m_D = M_D$)

Assume $m_D < M_D$

From above, we know that $M_D \leq 1 - \delta m_E$ and $m_D \geq 1 - \delta M_E$

$m_D - M_D \geq 1 - \delta M_E - 1 + \delta m_E$ subtracting the lesser from the greater maintains the inequality

$m_D - M_D \geq \delta(m_E - M_E)$

Further,

$M_E \leq 1 - \delta m_D - (1 - \delta)\Omega$ and $m_E \geq 1 - \delta M_D - (1 - \delta)\Omega$

Thus $m_E - M_E \geq 1 - \delta M_D - (1 - \delta)\Omega - (1 - \delta m_D - (1 - \delta)\Omega)$ to maintain the inequality

$m_E - M_E \geq \delta(m_D - M_D)$

$\frac{m_E - M_E}{\delta} \geq m_D - M_D$

Combining the above,

$\frac{m_E - M_E}{\delta} \geq m_D - M_D \geq \delta(m_E - M_E)$

By hypothesis, $m_D - M_D < 0$ and as $\delta \in (0,1)$ by definition, $m_E - M_E < 0$

$\frac{m_E - M_E}{\delta} \geq \delta(m_E - M_E)$

$m_E - M_E \geq \delta^2(m_E - M_E)$

This is a contradiction as $\delta \in (0,1)$ and $m_E - M_E < 0$ (multiplying $m_E - M_E$ by a positive number less than one will make it less negative and therefore larger)

Therefore it must be that $m_D \geq M_D$

The infimum cannot be greater than the supremum by definition

If $m_D \not< M_D$ and $m_D \not> M_D$, is must be that $m_D = M_D$ and, from above, this implies that $m_E = M_E$

Therefore $m_D = M_D$ and $m_E = M_E$. The subgame perfect equilibrium must be unique.

One District Bargaining

Lemma 9. *When the dictator must win one of one districts, for any investment* $\nu \geq 0$*, the local elite makes him an offer of* $\Omega_{1,1} + \frac{\delta}{1+\delta}(\Xi - \Omega_{1,1})$*, which he accepts. If he were to make a counteroffer, the dictator would offer the local elite* $\frac{\delta(1+\gamma)}{(1+\delta)(1-\gamma)}(\Xi - \Omega_{1,1})$.

Denote the dictator's outside option as Ω. In the single district case, the dictator's outside option is the probability he wins district J. $\Omega_1 = \frac{\alpha+\beta}{2\beta}$

Stationary strategies: The elite proposes x to the dictator, keeping $1-x$ for herself, every period and accepts the dictator's proposal if and only if $y \geq y'$. The dictator proposes y to the elite, keeping $1-y$ for himself, every period and accepts the elite's proposal if and only if $x \geq x'$

Continuation Values (no investment):

$U_D(Accept|x \geq x') = x'$

$U_D(Reject|x \geq x') = \delta(1-y') + (1-\delta)(\Omega_1)$

$U_E(Accept|y \geq y') = y'$

$U_E(Reject|y \geq y') = \delta(1-x') + (1-\delta)0$

Set x such that the dictator is indifferent between accepting and rejecting.

$x' = \delta(1-y') + (1-\delta)(\Omega_1)$

Similarly set y' such that the elite is indifferent between accepting and rejecting.

$y' = \delta(1-x') + (1-\delta)0$

Plug and solve

$x^* = \frac{\Omega+\delta}{1+\delta} = \frac{\alpha+\beta+2\beta\delta}{2\beta(1+\delta)}$

$y^* = \frac{\delta(1+\Omega)}{1+\delta} = \frac{\delta(\beta-\alpha)}{2\beta(1+\delta)}$

Utilities:

$U_E(x^*) = 1 - \frac{\Omega+\delta}{1+\delta} = \frac{\beta-\alpha}{2\beta(1+\delta)}$

$U_D(\neg Invest, Accept\ x^*) = \frac{\Omega+\delta}{1+\delta} = \frac{\alpha+\beta+2\beta\delta}{2\beta(1+\delta)}$

Algebraically adjust to outside option + share of surplus:

$U_D(\neg Invest, Accept\ x^*) = \frac{\Omega+\delta}{1+\delta} = \Omega + \frac{1}{1+\delta}(\Omega + \delta - \Omega(1+\delta))$

$U_D(\neg Invest, Accept\ x^*) = \Omega + \frac{\delta}{1+\delta}(1-\Omega)$

Continuation Values (with investment):

$U_D(Accept|x \geq x') = x' - c(\nu)$

$U_D(Reject|x \geq x') = \delta(1-y') + (1-\delta)(\Omega_I) = \delta(1-y') + (1-\delta)(\frac{\alpha+\beta+\nu}{2\beta}) - c(\nu)$

$U_E(Accept|y \geq y') = y'$

$U_E(Reject|y \geq y') = \delta(1-x') + (1-\delta)0$

$x_I^* = \frac{\Omega_I+\delta}{1+\delta} = \frac{\frac{\alpha+\beta+\nu}{2\beta}+\delta}{1+\delta} = \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)}$

$y_I^* = \frac{\delta(1+\Omega_I)}{1+\delta} = \frac{\delta(\beta-\alpha-\nu)}{2\beta(1+\delta)}$

Utilities:

$U_E(x^*) = \frac{\beta-\alpha-\nu}{2\beta(1+\delta)}$

$U_D(Invest, Accept\ x^*) = \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - c(\nu)$

$U_D(Invest, Accept\ x^*) = \Omega_I + \frac{\delta}{1+\delta}(1-\Omega_I) - c(\nu)$

Note investment solely affects the dictator's outside option of the mass election.

Comparative Statics

The dictator's share is $\frac{\alpha+\beta+2\beta\delta}{2\beta(1+\delta)}$

This is increasing in α, the change in β depends on the sign of α. If α is positive, decreasing in β. If α is negative, increasing in β

The elite's share is $\frac{\beta-\alpha}{2\beta(1+\delta)}$

This is decreasing in α, the change in β depends on the sign of α. If α is positive, increasing in β. If α is negative, decreasing in β

Investment and Surplus

Proposition 8. *When the dictator must two of two districts, anticipates the equilibrium elite offers of* $(\Omega_{1,1}+\lambda_{1,1}(\Xi-\Omega_{1,1}), \Omega_{1,1}+\lambda_{1,1}(\Xi-\Omega_{1,1}))$*, he makes optimal investment which satisfies the first order condition* $\frac{\alpha+\beta+2\beta\delta+\nu}{2\beta^2(1+\delta)^2} - c'(\nu) = 0$.

While I have focused on the continuous probability of winning the district, the Uniform assumption requires the following cases:

$$F_V(0) = \begin{cases} 0 & \beta < \alpha + \nu \\ \frac{\beta-\alpha-\nu}{2\beta} & -\beta + \alpha + \nu \leq 0 \leq \beta + \alpha + \nu \\ 1 & 0 > \beta + \alpha + \nu \end{cases}$$

Therefore

$$P(\text{win one district}) = \begin{cases} 1 & \beta < \alpha + \nu \\ \frac{\beta+\alpha+\nu}{2\beta} & -\beta + \alpha + \nu \leq 0 \leq \beta + \alpha + \nu \\ 0 & 0 > \beta + \alpha + \nu \end{cases}$$

If $\beta < \alpha + \nu$, the dictator will win the mass election with certainty and the elite will make the only acceptable offer of $x^* = 1$. The dictator's investment problem would then be $\max_\nu 1 - c(\nu)$ and the dictator's optimal investment is 0: there is no benefit to investing as it is impossible to improve his outside option above 0, so the dictator will make no investment.

If $-\beta + \alpha + \nu \leq 0 \leq \beta + \alpha + \nu$

$\max_\nu \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - c(\nu)$

confirm concavity: $\frac{\partial^2}{\partial \nu^2} \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - c(\nu) = 0 - c''$ where $c'' > 0$ by assumption. Thus the objective function is concave.

FOC: $\frac{1}{2\beta(1+\delta)} - c'(\nu) = 0$ set ν such that this holds for optimal investment

Denote the optimal investment when the dictator must win one of one districts to achieve the regime benefit ν_1^*

$\nu_1^* = c'^{-1}(\frac{1}{2\beta(1+\delta)})$

In this internal case, it could be that a large enough investment will move the dictator into a certain victory in the district i.e. there is a $\overline{\nu} < \nu^*$ such that $-\beta + \alpha < 0 < -\beta + \alpha + \overline{\nu}$

If the dictator makes investment $\underline{\nu}$, his expected offer from the elite is 1 and his expected overall utility will be $1 - c(\overline{\nu})$ where $1 - c(\overline{\nu}) > \frac{\alpha+\beta+2\beta\delta+\nu^*}{2\beta(1+\delta)} - c(\nu^*)$ as $c(\overline{\nu}) < c(\nu^*)$ by definition and $\frac{\alpha+\beta+2\beta\delta+\nu^*}{2\beta(1+\delta)} < 1$. Thus the dictator will make investment $\overline{\nu}$ to maximize his outside option to an outside win probability of 1 and keep the full regime benefit less the cost of investment.

If $\beta + \alpha + \nu < 0$, the dictator will lose the mass election. It could be the case that sufficient investment will move him from a win probability of 0 to a positive district win probability. Let $\underline{\nu}$ be the minimal level of investment the dictator needs to make such that $\beta + \alpha + \underline{\nu} \geq 0 > \beta + \alpha$. Consider cases:

(1) The dictator's optimal investment ν^* is greater than $\underline{\nu}$.

The dictator maximizes $\max_\nu \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - c(\nu)\ s.t.\ \nu \geq \underline{\nu}$

As $\nu^* = c'^{-1}(\frac{1}{2\beta(1+\delta)}) \geq \underline{\nu}$, the constraint is satisfied and the dictator will make his optimal investment and increase his probability of winning the district alone from 0 to a positive probability.

(2) It could be the case that $\nu^* < \underline{\nu}$. If the dictator makes investment $\underline{\nu}$, he will increase his probability of winning from 0 to something minimally positive, but at cost $c(\underline{\nu})$

If he does not make the investment, his outside option is 0, so he expects the elite will offer him $\frac{\delta}{1+\delta}$, which he will accept. If he invests, $\underline{\nu}$, he expects the elite will offer him$\frac{\alpha+\beta+2\beta\delta+\underline{\nu}}{2\beta(1+\delta)} - c(\underline{\nu})$. Should he invest?

$\frac{\alpha+\beta+2\beta\delta+\underline{\nu}}{2\beta(1+\delta)} - c(\underline{\nu}) - \frac{\delta}{1+\delta} \geq 0$

$\frac{\alpha+\beta+\underline{\nu}}{2\beta+2\beta\delta} - c(\underline{\nu}) \geq 0$

$\frac{\alpha+\beta+\underline{\nu}}{2\beta+2\beta\delta} \geq c(\underline{\nu})$

$\frac{\alpha+\beta+\underline{\nu}}{2\beta} \geq c(\underline{\nu})(1+\delta)$

The boost in his outside option that the dictator receives from investing a sufficient amount to generate a positive win probability in the district must outweigh the cost of investment (scaled).

Investment with convex cost functional form: $c(\nu) = \frac{\nu^2}{2}$

$U_D = \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - c(\nu)$

$U_D = \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - \frac{\nu^2}{2}$

$\max_\nu \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - \frac{\nu^2}{2}$

$\nu^* = \frac{1}{2\beta(1+\delta)}$

Decreasing in β, decreasing in δ

Surplus

Mutual surplus $\equiv E[U_D(intermediary)] + E[U_E(intermediary)] - (E[U_D(\neg intermediary)] + E[U_E(\neg intermediary)])$

$= 1 - (\Omega + 0)$

The total benefit of using the elite intermediary is 1 which will be divided according to the equilibrium bargain. The total benefit if the dictator does not use the elite is $\frac{\alpha+\beta}{2\beta}$. We could describe this as being divided 1, 0 between the dictator and elite, respectively. So the mutual surplus of going through the elite is $1 - \frac{\alpha+\beta}{2\beta} = \frac{\beta-\alpha}{2\beta}$. This mutual surplus is decreasing in α (if the dictator is ex ante popular, going without the elite approaches the using the elite as α increases). How the mutual surplus changes in β depends on α. If α is positive, the mutual surplus is increasing in β. If α is negative, the mutual surplus is decreasing in β.

Without investment ($\nu = 0$), the change in the dictator's share in β depends on the dictator's *ex ante* popularity: if the dictator is initially popular such that $\alpha > 0$, the dictator's share is decreasing in β. Increased uncertainty counteracts the dictator's popularity, reducing his outside option and worsening his bargaining position. If the dictator is *ex ante* unpopular, however, his share is increasing in β. Increasing uncertainty makes an unpopular dictator more likely to win, increasing his outside option and bargaining position.

With investment, the optimal investment is in itself a function of β

Share with investment: $\frac{\alpha+\beta+2\beta\delta+\nu^*(\beta)}{2\beta(1+\delta)} = \frac{\alpha+\beta+2\beta\delta}{2\beta(1+\delta)} + \frac{\nu^*(\beta)}{2\beta(1+\delta)}$
$\frac{\partial}{\partial\beta}\frac{\alpha+\beta+2\beta\delta}{2\beta(1+\delta)} + \frac{\nu^*(\beta)}{2\beta(1+\delta)} = -\frac{\alpha}{2\beta^2(1+\delta)} + \nu^{*'}(\beta)(\frac{1}{2\beta(1+\delta)}) + -\frac{1}{2\beta^2(1+\delta)}\nu^*(\beta)$
I know that $v^* = c^{'-1}(\frac{1}{2\beta(1+\delta)})$ is decreasing in β so $\nu^{*'}$ is negative (and ν^* is positive unless it is 0).
So if α is positive, $\frac{\partial share}{\partial\beta}$ is definitely negative. But if α is negative, the magnitudes determine the sign which would require a functional form for c to determine.
$\frac{-(\alpha+\nu^*(\beta))}{2\beta^2(1+\delta)} + \frac{\nu^{*'}(\beta)}{2\beta(1+\delta)}$ the second term is negative, the sign of the first term depends on the relative magnitude of α and $\nu^*(\beta)$

Investment Comparison Without Bargaining Technology

Let ν_E^* denotes the dictator's optimal investment when he uses an elite intermediary and ν_M^* is the dictator's optimal investment when he goes directly to the masses without elite cooptation.
Elite Bargain Objective Function: $\max_\nu \frac{\alpha+\beta+2\beta\delta+\nu}{2\beta(1+\delta)} - c(\nu)$
No Elite Intermediary Objective Function: $\max_\nu \frac{\alpha+\beta+\nu}{2\beta} - c(\nu)$

As c' is invertible, $\nu_E^* = c^{'-1}(\frac{1}{2\beta(1+\delta)})$ and $\nu_M^* = c^{'-1}(\frac{1}{2\beta})$. Note that $\nu_M^* > \nu_E^*$: the dictator's optimal level of investment if there is not elite intermediary is greater than the optimal level of investment with the elite intermediary.

Extreme Bargaining Protocols

Dictator makes the elite a take-it-or-leave-it offer (dictator extremely advantaged)
Sequence of play:
The dictator makes an investment decision of $\nu \in [0, 1]$ at convex cost $c(\nu)$
The dictator makes a TIOLI offer of e to the elite
The elite accepts the offer, receiving e, or rejects, leaving her with her reservation utility of

0

If the dictator coopted the elite, he wins the election with certainty; without the elite he runs for election in the district unsupported according to the micro-voting model (win with probability $\frac{\alpha+\beta+\nu}{2\beta}$. If the dictator wins the district, he keeps regime benefit $R = 1$ less any offer he made to the elite

The dictator will make the elite an offer of $e = 0$ as the elite will be indifferent between accepting and rejecting the offer (accept on indifference). As the dictator will win the district with certainty without having to transfer anything to the elite, his investment optimization problem is $\max_{\nu} 1 - c(\nu)$ and will thus make 0 investment in mass politics.

Elite makes the dictator a take-it-or-leave-it offer (elite extremely advantaged)

Sequence of play:

The dictator makes an investment decision of $\nu \in [0, 1]$ at convex cost $c(\nu)$

The elite makes the dictator a TIOLI offer of d, if the dictator accepts her offer, she will keep $1 - d$

The dictator accepts the offer, receiving d with certainty, or rejects and stands for the election unsupported

If the dictator coopted the elite, he wins the district with certainty; without the elite he runs for election in the district unsupported according to the micro-voting model (win with probability $\frac{\alpha+\beta+\nu}{2\beta}$. If the dictator wins the district, he keeps regime benefit $R = 1$ less any deal made with the elite

The dictator's expected utility is $d - c(\nu)$ if he accepts the elite's proposal and $\frac{\alpha+\beta+\nu}{2\beta}(1) - c(\nu)$ if he rejects the elite's proposal. To maximize her utility, the elite will make the dictator a minimal offer that makes her indifferent between accepting and rejecting: $d = \frac{\alpha+\beta+\nu}{2\beta}$, which the dictator accepts. The dictator's investment optimization problem is thus $\max_{\nu} \frac{\alpha+\beta+\nu}{2\beta} - c(\nu)$. The dictator's optimal investment here is $\nu^* = c'^{-1}(\frac{1}{2\beta})$, which is equivalent to ν^*_M

Under more extreme bargaining protocols, the dictator's optimal investment is less than or equal to the investment he makes without the elite bargaining technology. In particular, if the dictator is in the extreme privileged bargaining position and can make a take-it-or-leave it offer to the local elite, the elite will support him with an offer of 0 as she will accept on indifference. Thus the dictator will make an investment of 0, give the elite 0, and still win the regime benefit with certainty. In the other extreme, if the elite can make a take-it-or-leave-it offer to the dictator, the minimal offer dictator will accept is that which makes him indifferent between using the elite's support or standing for election alone $(\frac{\alpha+\beta+\nu}{2\beta})$. In this case, the dictator's optimal investment is $c'^{-1}(\frac{1}{2\beta})$, which is the same as ν^*_M. Thus the optimal investment the dictator will make under the Rubinstein alternating offer bargaining protocol is between the optimal investments he would make if he were strongly advantaged (0) or disadvantaged (ν^*_M) by the bargaining protocol.

Two Districts

There are two homogeneous districts. The value of the regime is normalized to 1.

One of Two

First, consider the case in which the dictator must win at least one district to receive this benefit (need at least one of two).

Nature chooses which elite bargains with the dictator first, then bargaining occurs as described above. After the dictator completes his bargain with the first elite (either through an agreement or breakdown), the dictator bargains with the second elite following the same protocol. Both the probability of bargaining breakdown and the probability the dictator wins each district is the same for both elites/districts. After the dictator completes his bargain with the second elite (either through an agreement or breakdown), the elections are held in both districts.

$P(winJ) = \frac{\alpha+\beta}{2\beta}$ (from micro model)

$P(win\ at\ least\ one) = \Omega_2' = 2(\frac{\alpha+\beta}{2\beta}) - (\frac{\alpha+\beta}{2\beta})^2$ (win one or both districts)

Second Bargaining Position

Consider the subgame in which the first elite has already been coopted. Because the dictator has already made an agreement with the first elite, they are now bargaining over the dictator's remaining portion. Let x_1 be the portion of the benefit that the dictator is left with after the first bargain. The second elite will propose x_2x_1, dividing x_1 into two portions: x_2 for the dictator, $1 - x_2$ for himself. Similarly, the dictator proposes a split of $(1 - y_2)x_1$ for himself and y_2x_1 for the second elite.

This yields continuation values:

$U_D(acceptx_2) = x_2x_1$

$U_D(reject) = \delta(1 - y_2)x_1 + (1 - \delta)x_1$

$x_2 = 1 - \delta y_2$

$U_E(accepty_2) = y_2x_1$

$U_E(reject) = \delta(1 - x_2)x_1 + (1 - \delta)0$

$y_2 = \delta(1 - x_2)$

plug and solve

$x_2^* = 1$ the dictator gets everything, as expected

$y_2^* = 0$ the elite gets nothing, as expected

Because the dictator has already successfully coopted the first elite, the second district is superfluous. There is no offer of sharing that the dictator would accept.

Consider the subgame in which the first elite has not been coopted.

$U_D(accept) = x_2$

$U_D(reject) = \delta(1-y_2) + (1-\delta)\Omega_2' = \delta(1-y_2) + (1-\delta)(2(\frac{\alpha+\beta}{2\beta}) - (\frac{\alpha+\beta}{2\beta})^2)$ [the probability that the dictator wins one or both districts]

$U_E(accept) = y_2$

$U_E(reject) = \delta(1-x_2) + (1-\delta)(0)$

$x_2^* = \frac{\Omega_2'+\delta}{1+\delta} = \frac{2(\frac{\alpha+\beta}{2\beta})-(\frac{\alpha+\beta}{2\beta})^2+\delta}{1+\delta} = \frac{(\frac{\alpha+\beta}{\beta})-(\frac{\alpha+\beta}{2\beta})^2+\delta}{1+\delta}$

$y_2^* = \frac{\delta(1+\Omega_2')}{1+\delta} = \frac{(\alpha-\beta)^2\delta}{4\beta^2(1+\delta)}$

Utilities: $U_E(x_2^*) = \frac{(\alpha-\beta)^2}{4\beta^2(1+\delta)}$

$U_D(Accept) = \frac{(\frac{\alpha+\beta}{\beta})-(\frac{\alpha+\beta}{2\beta})^2+\delta}{1+\delta}$

First Bargaining Position

If the dictator coopts the first elite with successful bargaining, he gets x_1. If bargaining breaks down and elite 1 is not coopted, the dictator will bargain with the second elite and we know that bargain will yield $\Omega_2' = \frac{\Omega_2'+\delta}{1+\delta} = \frac{(\frac{\alpha+\beta}{\beta})-(\frac{\alpha+\beta}{2\beta})^2+\delta}{1+\delta}$.

$U_D(accept) = x_1$

$U_D(reject) = \delta(1-y_1) + (1-\delta)\Omega_2' = \delta(1-y_1) + (1-\delta)(\frac{(\frac{\alpha+\beta}{\beta})-(\frac{\alpha+\beta}{2\beta})^2+\delta}{1+\delta})$

$U_E(accept) = y_1$

$U_E(reject) = \delta(1-x_1) + (1-\delta)(0)$

$x_1^* = \frac{\Omega_2'+\delta}{1+\delta} = \frac{(\frac{\Omega_2'+\delta}{1+\delta})+\delta}{1+\delta} = \frac{(\alpha+\beta+2\beta\delta)(\beta(3+2\delta)-\alpha)}{4\beta^2(1+\delta)^2}$ [with investment $\frac{(\alpha+\beta+2\beta\delta+\nu)(\beta(3+2\delta)-\alpha-\nu)}{4\beta^2(1+\delta)^2}$]

$y_1^* = \frac{\delta(1+\Omega_2')}{1+\delta} = \frac{\delta(\alpha-\beta)^2}{4\beta^2(1+\delta)^2}$

Utilities: $U_D(acceptx_1^*) = \frac{(\alpha+\beta+2\beta\delta)(\beta(3+2\delta)-\alpha)}{4\beta^2(1+\delta)^2}$

$U_E(x_1^*) = \frac{(\alpha-\beta)^2}{4\beta^2(1+\delta)^2}$

Substitute in Ω_2':

With two districts (need one of two), $U_D(x_1^*, x_2^*) = \Omega_2' + \frac{\delta(2+\delta)}{(1+\delta)^2}(1 - \Omega_2')$ where Ω_2' is the probability the dictator wins one or both districts without using an elite intermediary.

the dictator accepts the elite's first offer, x_1^*. The elite in the second bargaining position is indifferent between making an offer of 1 (where the dictator gets everything and the elite gets nothing) and making unacceptable offers until bargaining breaks down (she gets zero either way). The dictator's overall utility is $\frac{(\alpha+\beta+2\beta\delta)(\beta(3+2\delta)-\alpha)}{4\beta^2(1+\delta)^2}$. For the elites, their utility depends on the bargaining position nature selects for them. Let the probability that an elite is chose for the first position be ρ. Both elites' expected utility is then $E[U_E(x_1^*, x_2^*)] = \rho(\frac{(\alpha-\beta)^2}{4\beta^2(1+\delta)^2}) + (1-\rho)(0)$. As the probability of going first is symmetric for both elites, $\rho = \frac{1}{2}$

Comparative Statics: $\frac{\partial}{\partial\alpha}U_D(acceptx_1^*) = \frac{(\alpha+\beta+2\beta\delta)(\beta(3+2\delta)-\alpha)}{4\beta^2(1+\delta)^2} = \frac{\beta-\alpha}{2\beta^2(1+\delta)^2}$ which is positive. The dictator's share (and equilibrium utility) are increasing in his ex ante popularity.

$\frac{\partial}{\partial\beta}U_D(acceptx_1^*) = \frac{(\alpha+\beta+2\beta\delta)(\beta(3+2\delta)-\alpha)}{4\beta^2(1+\delta)^2} = \frac{\alpha(\alpha-\beta)}{2\beta^3(1+\delta)^2}$ the sign of which depends on α (if $\alpha > 0$, decreasing in β, if $\alpha < 0$ increasing in β)

$\frac{\partial}{\partial\alpha}U_E(x_1^*) = \frac{(\alpha-\beta)^2}{4\beta^2(1+\delta)^2} = \frac{\alpha-\beta}{2\beta^2(1+\delta)^2}$ which is negative, decreasing in the dictator's popularity

$\frac{\partial}{\partial\beta}U_E(x_1^*) = \frac{(\alpha-\beta)^2}{4\beta^2(1+\delta)^2} = -\frac{\alpha(\alpha-\beta)}{2\beta^3(1+\delta)^2}$ the sign of which depends on α.

Mutual Surplus $= 1 - (\frac{\alpha+\beta}{\beta} - (\frac{\alpha+\beta}{2\beta})^2)$ note this is different than the one district mutual surplus because now if the dictator bypasses the elite he has two possible districts of which he must win one.

$1 - (\frac{\alpha+\beta}{\beta} - (\frac{\alpha+\beta}{2\beta})^2) = \frac{(\alpha-\beta)^2}{4\beta^2}$

The mutual surplus is decreasing in α and how it changes in β depends on the sign of α (if positive, increasing in β, if negative decreasing in β)

Compare utilities with one district versus two

Elites: One district elite - two districts elite $= \frac{\beta-\alpha}{2\beta(1+\delta)} - (\rho(\frac{(\alpha-\beta)^2}{4\beta^2(1+\delta)^2}) + (1-\rho)0) = \frac{(\beta-\alpha)(\alpha+\beta(3+4\delta))}{8\beta^2(1+\delta)^2}$

Dictator: Two districts - one district $= \frac{(\alpha+\beta+2\beta\delta)(\beta(3+2\delta)-\alpha)}{4\beta^2(1+\delta)^2} - \frac{\alpha+\beta+2\beta\delta}{2\beta(1+\delta)} = \frac{(\beta-\alpha)(\alpha+\beta+2\beta\delta)}{4\beta^2(1+\delta)^2}$

literally transfers some gain to the dictator recall $\beta > \alpha$ for a proper probability)

Investment

Note the probability of winning one of two districts is non-monotonic in investment. For investment greater than 0, the probability of winning one of two districts is non-monotonic in α. Thus the optimal investment depends on α

Equilibrium investment with two districts:

$\max_\nu \frac{(\alpha+\beta+2\beta\delta+\nu)(\beta(3+2\delta)-\alpha-\nu)}{4\beta^2(1+\delta)^2} - c(\nu)$

FOC is $\frac{\beta-\alpha-\nu}{2\beta^2(1+\delta)^2} - c'(\nu) = 0$

Note optimal investment is decreasing in α

Compare to investment when dictator needs one of one districts:

One district FOC: $\frac{1}{2\beta(1+\delta)} - c'(\nu) = 0$

Assume $c(\nu) = \frac{\nu^2}{2}$ such that $c'(\nu) = \nu$

$\nu^*_{2D} = \frac{\beta-\alpha}{1+2\beta^2(1+\delta)^2}$

Comparative Statics:

$\frac{\partial\nu^*_{2D}}{\partial\alpha} = \frac{-1}{1+2\beta^2(1+\delta)^2}$ investment is decreasing in the dictator's popularity

$\frac{\partial\nu^*_{2D}}{\partial\beta} = \frac{1+2(2\alpha-\beta)\beta(1+\delta)^2}{(1+2\beta^2(1+\delta)^2)^2}$

This is not always negative. if α is sufficiently high, this is positive. In particular, if $\alpha > \frac{\beta}{2} - \frac{1}{4\beta(1+\delta)^2}$, the derivative is positive and investment is increasing in district uncertainty.

$\nu^*_{1D} = \frac{1}{2\beta(1+\delta)}$

$\nu^*_{1D} - \nu^*_{2D} = \frac{1+2\alpha\beta(1+\delta)+2\beta^2\delta(1+\delta)}{2\beta(1+\delta)(1+2\beta^2(1+\delta)^2)}$ which is positive if $\alpha > \frac{-1}{2\beta(1+\delta)} - \beta\delta$ (this is obviously satisfied for all positive α but also some negative ones depending on other parameter magnitudes)

When this holds, the equilibrium level of investment when there is only one district is greater than when there are two districts (but the dictator only needs one) with this particular functional form of convex costs.

My interpretation of this is that having two districts to play off each other increases the dictator's bargaining power so much that he doesn't need to affect his own outside option with investment quite as much. In the one-district bargain, the only way to affect his bargaining power is through investment; with an additional outside option of another district, the dictator can already keep much more of the regime benefit without investment. Note that the equilibrium investment with two districts is still positive (decreasing in α, increasing in β if $\alpha > \frac{\beta}{2} - \frac{1}{4\beta(1+\delta)^2}$)

Two Districts Win Two of Two

Consider the case of two districts in which the dictator must win both in order to achieve the regime benefit: if he loses either or both districts, he will not get the benefit. For simplicity, denote the probability the dictator wins one district as γ and assume the two districts are homogeneous and independent. The probability that he wins both districts without elite support is $\gamma^2 = (\frac{\alpha+\beta+\nu}{2\beta})^2$.

Second Bargaining Position

First, consider the history where the previous elite has not been coopted.

$U_D(accept\ x_2) = x_2 P(win) = x_2\gamma$

$U_D(reject) = \delta(1-y_2)\gamma + (1-\delta)\gamma^2$

Note the dictator only gets the regime benefit to share with the elite if he wins the election in the second district

$x_2 = \delta(1 - y_2) + (1 - \delta)\gamma$

$U_E(accept\ y_2) = y_2\gamma$

$U_E(reject) = \delta(1 - x_2)\gamma + (1 - \delta)0$

Note the elite will only have access to a portion of the regime benefit if the dictator wins the election in the second district

$y_2 = \delta(1 - x_2)$

Plug and solve:

$x_2^* = \frac{\delta+\gamma}{1+\delta}$

$y_2^* = \frac{\delta(1+\gamma)}{1+\delta}$

Note that the parties' expected utilities are still dependent on winning the other district:

$U_E(x_2^*) = (1 - \frac{\delta+\gamma}{1+\delta})\gamma$

$U_D(accept) = (\frac{\delta+\gamma}{1+\delta})\gamma$

Second, consider the history where the elite in the first bargaining position has already been coopted, leaving the dictator with remainder x_1 with which to bargain.

$U_D(accept\ x_2) = x_2 x_1$

$U_D(reject) = \delta(1 - y_2)x_1 + (1 - \delta)x_1\gamma$

$x_2 = \delta(1 - y_2) + (1 - \delta)\gamma$

$U_E(accept\ y_2) = y_2 x_1$

$U_E(reject) = \delta(1 - x_2)x_1 + (1 - \delta)0$

$y_2 = \delta(1 - x_2)$

Plug and solve:

$x_2^* = \frac{\delta+\gamma}{1+\delta}$

$y_2^* = \frac{\delta(1+\gamma)}{1+\delta}$

Note that the parties' expected utilities are still dependent on the bargain that the dictator made with the first elite:

$U_E(x_2^*) = (1 - \frac{\delta+\gamma}{1+\delta})x_1$

$U_D(accept) = (\frac{\delta+\gamma}{1+\delta})x_1$

First Bargaining Position The dictator anticipates making the aforementioned bargain with the elite in the second position.

$U_D(accept\ x_1) = x_2 x_1$

$U_D(reject) = \delta(1 - y_1)x_2 + (1 - \delta)x_2\gamma$

$x_1 = \delta(1 - y_1) + (1 - \delta)\gamma$

$U_E(accept\ y_1) = y_1$

$U_E(reject) = \delta(1 - x_1) + (1 - \delta)0$

$y_1 = \delta(1 - x_1)$

Plug and solve:

$x_1^* = \frac{\delta+\gamma}{1+\delta}$

$y_1^* = \frac{\delta(1+\gamma)}{1+\delta}$

$U_E(x_1^*) = (1 - \frac{\delta+\gamma}{1+\delta})$

$U_D(accept) = (\frac{\delta+\gamma}{1+\delta})x_2 = (\frac{\delta+\gamma}{1+\delta})^2$

Outside option form:

If the dictator and the elite in the first bargaining position fail to reach an agreement, the dictator's bargain with the elite in the second position is scaled by Ω_1, the probability he wins one district alone. This is because even if they reach an agreement, the dictator will only get the regime benefit to split with the local elite if he wins the other district, which occurs with probability Ω_1. The maximum mutual surplus that the dictator and second position elite are bargaining over is $1 * \Omega_1$. If the dictator and second elite also fail to reach an agreement, the dictator's true electoral outside option is $(\Omega_1)^2$, the probability he wins both districts alone. Thus the bargain that the dictator and second elite will come to is $(\Omega_1)^2 + \dot{\lambda}(\Omega_1 - (\Omega_1)^2) = \Omega_1(\Omega_1 + \dot{\lambda}(1 - \Omega_1))$. As we know the dictator will make a deal with the first elite, this is off path.

If the dictator and first elite do reach an agreement, the dictator's bargain with the second elite is scaled by x_1, the amount of the regime benefit the dictator is left with after the agreement with the first elite. The maximum mutual surplus over which the two parties bargain is x_1. If the dictator and second elite fail to reach an agreement in this case, the outside option is what the dictator gets if he and the second elite fail to reach an agreement: $\Omega_1 x_1$. Thus the bargain they come to is $\Omega_1 x_1 + \lambda(x_1 - \Omega_1 x_1) = x_1(\Omega_1 + \lambda(1 - \Omega_1))$. Therefore in equilibrium we know that the dictator will keep $(\Omega_1 + \lambda(1 - \Omega_1))$ portion of whatever is left after his bargain with the first elite. The first elite will leave him with $(\Omega_1 + \lambda(1 - \Omega_1))$ between the first and second round of bargaining, so the dictator's utility at the end of both bargains after coopting both districts is $(\Omega_1 + \lambda(1 - \Omega_1))^2$.

Investment

Objective function: $\max_\nu (\frac{\delta + (\frac{\alpha+\beta+\nu}{2\beta})}{1+\delta})^2 - c(\nu)$

FOC: $\frac{\alpha+\beta+2\beta\delta+\nu}{2\beta^2(1+\delta)^2} - c'(\nu) = 0$

Assume $c(\nu) = \frac{\nu^2}{2}$ such that $c'(\nu) = \nu$.

FOC: $\frac{\alpha+\beta+2\beta\delta+\nu}{2\beta^2(1+\delta)^2} = \nu$

$\nu^* = \frac{\alpha+\beta+2\beta\delta}{2\beta^2(1+\delta)^2-1}$ whether or not this is positive depends on the magnitude of β. If β is sufficiently small, the dictator should invest 0 as investment cannot be negative.

$$\nu^* = \begin{cases} 0 & \beta \leq \sqrt{\frac{1}{2}} \\ \frac{\alpha+\beta+2\beta\delta}{2\beta^2(1+\delta)^2-1} & \beta > \sqrt{\frac{1}{2}} \end{cases} \tag{4.1}$$

As β gets very low, the dictator's utility gets infinitely large (in the limit as $\beta \to 0$, the dictator's utility approaches ∞). Thus any investment here will just incur costs, so the dictator does not want to invest.

$\frac{\partial \nu^*}{\partial \alpha} = \frac{1}{-1+2\beta^2(1+\delta)^2}$ increasing in α

$\frac{\partial \nu^*}{\partial \beta} = -\frac{1+2\delta+2\beta(1+\delta)^2(2\alpha+\beta+2\beta\delta)}{(1-2\beta^2(1+\delta)^2)^2}$ negative: always decreasing in dictator's popularity

Three Districts

For simplicity, assume the districts all have a homogeneous win probability γ. The dictator needs at least two of the three districts to receive the benefit normalized to 1. With the micro-voting model, we know that $\gamma = \frac{\alpha+\beta}{2\beta}$ or, with investment, $\gamma_I = \frac{\alpha+\beta+\nu}{2\beta}$

Without any elites, the dictator will win the benefit with probability $\gamma^3 + (1-\gamma)3\gamma^2$

Third Bargaining Position

Consider the subgame where no prior elites have been coopted.

$U_D(accept) = \gamma(2-\gamma)x_3$

$U_D(reject) = \delta(\gamma(2-\gamma))(1-y_3) + (1-\delta)(3\gamma^2 - 2\gamma^3)$

$U_E(accept) = \gamma(2-\gamma)y_3$

$U_E(reject) = \delta(\gamma(2-\gamma)(1-x_3)) + (1-\delta)0$

$x_3 = \delta(1-y_3) + \frac{(1-\delta)(3\gamma^2-2\gamma^3)}{2\gamma-\gamma^2}$

$y_3 = \delta(1-x_3)$

$x_3^* = \frac{2\gamma^2+\gamma(\delta-3)-2\delta}{(\gamma-2)(1+\delta)}$

$y_3^* = \frac{2(\gamma-1)^2\delta}{(2-\gamma)(1+\delta)}$

Utilities: $U_D(accept|win\ other\ district) = \frac{2\gamma^2+\gamma(\delta-3)-2\delta}{(\gamma-2)(1+\delta)}$

$U_D(accept|\neg win\ other\ district) = 0$

$U_D(accept) = U_D(accept|win\ other\ district)P(win) + U_D(accept|\neg win\ other\ district)(1 - P(win))$

$U_D(accept) = (\frac{2\gamma^2+\gamma(\delta-3)-2\delta}{(\gamma-2)(1+\delta)})(2\gamma-\gamma^2)+(1-(2\gamma-\gamma^2))(0) = -\frac{\gamma(2\gamma^2+\gamma(\delta-3)-2\delta)}{1+\delta} = \frac{\gamma(2\delta+\gamma(3-\delta)-2\gamma^2)}{1+\delta}$

$U_E(x_3^*|win\ other\ district) = -\frac{2(\gamma-1)^2}{(\gamma-2)(1+\delta)} = \frac{2(\gamma-1)^2}{(2-\gamma)(1+\delta)}$

$U_E(x_3^*|\neg win\ other\ district) = 0$

$U_E(x_3^*) = U_E(x_3^*|win\ other\ district)P(win) + U_E(x_3^*|\neg win\ other\ district)(1 - P(win))$

$U_E(x_3^*) = \frac{2(\gamma-1)^2}{(2-\gamma)(1+\delta)}(2\gamma - \gamma^2) + (1 - (2\gamma - \gamma^2))(0) = \frac{2\gamma(\gamma-1)^2}{1+\delta}$

Check: sum to $1(2\gamma - \gamma^2)$

One prior elite coopted: dictator has x_2 or x_1 (I will use x_2 for simplicity) to bargain with but will not have it with certainty if he doesn't coopt this elite...

$U_D(accept) = x_3x_2$

$U_D(reject) = \delta(1 - y_3)x_2 + (1 - \delta)[x_2(2\gamma - \gamma^2) + (1 - (2\gamma - \gamma^2))(0)]$

$U_E(accept) = y_3x_2$

$U_E(reject) = \delta(1 - x_3)x_2 + (1 - \delta)0$

$y_3 = \delta(1 - x_3)$

$x_3 = \delta(1 - y_3) + (1 - \delta)(2\gamma - \gamma^2)$

$x_3^* = \frac{2\gamma-\gamma^2+\delta}{1+\delta}$

$y_3^* = \frac{(\gamma-1)^2\delta}{1+\delta}$

$U_D(accept) = \frac{2\gamma-\gamma^2+\delta}{1+\delta}x_2$

$U_E(x_3^*) = \frac{(\gamma-1)^2}{1+\delta}x_2$

Check: sum to x_2

Two prior elites coopted: dictator has $x_2 x_1$ with certainty regardless of his bargain with the third elite

$U_d(accept) = x_3 x_2 x_1$

$U_D(reject) = \delta(1 - y_3)x_2 x_1 + (1 - \delta)x_2 x_1$

$U_E(accept) = y_3 x_2 x_1$

$U_E(reject) = \delta(1 - x_3)x_2 x_1 + 0$

$x_3^* = 1$, $y_3^* = 0$ as the third elite is superfluous

$U_D(accept) = x_2 x_1$

$U_E(x_3^*) = 0$

Second Bargaining Position

One prior elite coopted:

If the dictator makes a successful bargain with elite 2, he will be left with $x_1 x_2 x_3$ where $x_3 = 1$

If the dictator does not make a successful bargain with elite 2, he will make a deal with the third elite and end up with $x_1 x_3$ where $x_3 = \frac{2\gamma - \gamma^2 + \delta}{1+\delta}$ so his utility will be $\frac{2\gamma - \gamma^2 + \delta}{1+\delta} x_1$

$U_D(Accept) = x_1 x_2(1)$

$U_D(Reject) = \delta(1 - y_2)x_1 + (1 - \delta)(\frac{2\gamma - \gamma^2 + \delta}{1+\delta})x_1$

$U_E(Accept) = y_2 x_1$

$U_E(Reject) = \delta(1 - x_2)x_1 + (1 - \delta)(0)$

$x_2^* = \frac{2\gamma - \gamma^2 + \delta(2+\delta)}{(1+\delta)^2}$

$y_2^* = \frac{(\gamma-1)^2\delta}{(1+\delta)^2}$

$U_D(Accept) = \frac{2\gamma - \gamma^2 + \delta(2+\delta)}{(1+\delta)^2} x_1$

$U_E(x_2^*) = \frac{(\gamma-1)^2}{(1+\delta)^2} x_1$

Check: sum to x_1

No prior elite coopted:

If the dictator makes a successful bargain with elite 2, he will also bargain with elite 3 and get $\frac{2\gamma-\gamma^2+\delta}{1+\delta}x_2$

If the dictator fails to make a successful bargain with elite 2, he will bargain with elite 3 and get $\frac{\gamma(2\delta+\gamma(3-\delta)-2\gamma^2)}{1+\delta}$ (the expected utility of bargaining with the third elite taking into account the uncertainty of winning another district without elite support)

$U_D(Accept) = \frac{2\gamma-\gamma^2+\delta}{1+\delta}x_2$

$U_D(reject) = \delta(1-y_2)\frac{2\gamma-\gamma^2+\delta}{1+\delta} + (1-\delta)(\frac{\gamma(2\delta+\gamma(3-\delta)-2\gamma^2)}{1+\delta})$

$U_E(Accept) = y_2$

$U_E(Reject) = \delta(1-x_2) + (1-\delta)0$

$x_2 = \delta(1-y_2) + \frac{\gamma(2\gamma^2+\gamma(\delta-3)-2\delta)(\delta-1)}{2\gamma-\gamma^2+\delta}$

$y_2 = \delta(1-x_2)$

$x_2^* = \frac{-2\gamma^3+\gamma^2(3-2\delta)+4\gamma\delta+\delta^2}{(1+\delta)(2\gamma-\gamma^2+\delta)}$

$y_2^* = \frac{\delta(\gamma-1)^2(2\gamma+\delta)}{(1+\delta)(2\gamma-\gamma^2+\delta)}$

$U_D(Accept) = \frac{2\gamma-\gamma^2+\delta}{1+\delta} * \frac{-2\gamma^3+\gamma^2(3-2\delta)+4\gamma\delta+\delta^2}{(1+\delta)(2\gamma-\gamma^2+\delta)} = -\frac{(\gamma(2\gamma-3)-\delta)(\gamma+\delta)}{(1+\delta)^2} = \frac{(\gamma(3-2\gamma)+\delta)(\gamma+\delta)}{(1+\delta)^2}$

$U_E(x_2^*) = 1 - \frac{-2\gamma^3+\gamma^2(3-2\delta)+4\gamma\delta+\delta^2}{(1+\delta)(2\gamma-\gamma^2+\delta)} = \frac{(\gamma-1)^2(2\gamma+\delta)}{(1+\delta)(2\gamma-\gamma^2+\delta)}$

Check: $U_D + U_{E2} + U_{E3}$ sums to 1

First Bargaining Position

If the dictator makes a successful bargain with elite 1, he will then bargain with the later elites and get $x_1x_2x_3$ where $x_3 = 1$ and $x_2 = \frac{2\gamma-\gamma^2+\delta(2+\delta)}{(1+\delta)^2}$ so his utility will be $\frac{2\gamma-\gamma^2+\delta(2+\delta)}{(1+\delta)^2}x_1$

If the dictator does not complete a successful bargain with elite 1, he will still coopt elites 2 and 3 and get $\frac{(\gamma(3-2\gamma)+\delta)(\gamma+\delta)}{(1+\delta)^2}$

$U_D(accept) = \frac{2\gamma-\gamma^2+\delta(2+\delta)}{(1+\delta)^2}x_1$

$U_D(reject) = \delta(1-y_1)\frac{2\gamma-\gamma^2+\delta(2+\delta)}{(1+\delta)^2} + (1-\delta)\frac{(\gamma(3-2\gamma)+\delta)(\gamma+\delta)}{(1+\delta)^2}$

$U_E(accept) = y_1$

$U_E(reject) = \delta(1 - x_1) + (1 - \delta)(0)$

$x_1 = \delta(1 - y_1) + (1 - \delta)\frac{(\gamma(3-2\gamma)+\delta)(\gamma+\delta)}{2\gamma-\gamma^2+\delta(2+\delta)}$

$x_1^* = \frac{\delta^2-\delta(\gamma-3)+\gamma(2-2\gamma)}{(1+\delta)(2+\delta-\gamma)}$

$y_1^* = \frac{\delta(\gamma-1)^2(2\gamma+2\delta+1)}{(1+\delta)^3}$

$U_D(Accept) = \frac{\delta^2-\delta(\gamma-3)+\gamma(2-2\gamma)}{(1+\delta)(2+\delta-\gamma)} * \frac{2\gamma-\gamma^2+\delta(2+\delta)}{(1+\delta)^2} = \frac{(3+\delta-2\gamma)(\delta+\gamma)^2}{(1+\delta)^3}$

$U_E(x_1^*) = \frac{2(\gamma-1)^2}{(1+\delta)(2+\delta-\gamma)}$

Check: $U_D + U_{E1} + U_{E2}$ sums to 1

Equilibrium

The dictator will complete successful bargains with elites in the first and second bargaining positions. He will reject any offer less than 1 from the third elite, the elite is indifferent between offering 1 and continuing the bargain as he will get 0 regardless.

$U_D(acceptx_1^*, acceptx_2^*) = \frac{(3-2\gamma+\delta)(\gamma+\delta)^2}{(1+\delta)^3}$

$\frac{\partial}{\partial\gamma}U_D(acceptx_1^*, acceptx_2^*) = \frac{6(1-\gamma)(\gamma+\delta)}{(1+\delta)^3}$ positive, increasing in γ

$\frac{\partial}{\partial\delta}U_D(acceptx_1^*, acceptx_2^*) = \frac{6(\gamma-1)^2(\gamma+\delta)}{(1+\delta)^4}$ positive, increasing in δ

Plug in micro probability $\gamma = \frac{\alpha+\beta}{2\beta}$

$U_D(acceptx_1^*, acceptx_2^*) = \frac{(3-2(\frac{\alpha+\beta}{2\beta})+\delta)(\frac{\alpha+\beta}{2\beta}+\delta)^2}{(1+\delta)^3} = \frac{(\alpha+\beta+2\beta\delta)^2(\beta(2+\delta)-\alpha)}{4\beta^3(1+\delta)^3}$

$\frac{\partial}{\partial\alpha}U_D(acceptx_1^*, acceptx_2^*) = \frac{3(\beta-\alpha)(\alpha+\beta+2\beta\delta)}{4\beta^3(1+\delta)^3}$ positive; increasing in α the dictator's ex ante popularity

$\frac{\partial}{\partial\beta}U_D(acceptx_1^*, acceptx_2^*) = \frac{3\alpha(\alpha-\beta)(\alpha+\beta+2\beta\delta)}{4\beta^4(1+\delta)^3}$ sign depends on the sign of α. If $\alpha > 0$, decreasing in β. If $\alpha < 0$, increasing in β

$U_{E1}(x_1^*) = \frac{2(\gamma-1)^2}{(2-\gamma+\delta)(1+\delta)}$

$\frac{\partial}{\partial\gamma}U_{E1}(x_1^*) = \frac{2(1-\gamma)(\gamma-3-2\delta)}{(1+\delta)(2-\gamma+\delta)^2}$

$\gamma - 3 - 2\delta > 0$ this will never hold, thus the entire derivative is negative. decreasing in γ

$\frac{\partial}{\partial\delta} U_{E1}(x_1^*) = \frac{2(\gamma-1)^2(\gamma-3-2\delta)}{(1+\delta)^2(2-\gamma+\delta)^2}$ negative, decreasing in δ

Plug in micro probability $\gamma = \frac{\alpha+\beta}{2\beta}$

$U_{E1}(x_1^*) = \frac{2(\gamma-1)^2}{(2-\gamma+\delta)(1+\delta)} = \frac{2((\frac{\alpha+\beta}{2\beta})-1)^2}{(2-\frac{\alpha+\beta}{2\beta}+\delta)(1+\delta)} = \frac{(\alpha-\beta)^2}{\beta(1+\delta)(\beta(3+2\delta)-\alpha)}$

$\frac{\partial}{\partial\alpha} U_{E1}(x_1^*) = \frac{(\beta-\alpha)(\beta(5+4\delta)-\alpha)}{\beta(1+\delta)(\alpha-\beta(3+2\delta))^2}$ this is positive (increasing in α)

$\frac{\partial}{\partial\beta} U_{E1}(x_1^*) = \frac{\alpha(\alpha-\beta)(\alpha-\beta(5+4\delta))}{\beta^2(1+\delta)(\alpha-\beta(3+2\delta))^2}$ sign depends on α. If α is positive, increasing in β. If α is negative, decreasing in β.

$$U_{E2}(x_2^*) = \frac{(\gamma-1)^2}{(1+\delta)^2} x_1 = \frac{(\gamma-1)^2}{(1+\delta)^2} * \frac{-2\gamma^2+\gamma(3-\delta)+\delta(3+\delta)}{(1+\delta)(2-\gamma+\delta)} = \frac{(\gamma-1)^2(3-2\gamma+\delta)(\gamma+\delta)}{(2-\gamma+\delta)(1+\delta)^3}$$

$\frac{\partial}{\partial\gamma} U_{E2}(x_2^*) = \frac{2(\gamma-1)(-3+3\gamma^3+4\delta+5\delta^2+\delta^3-3\gamma^2(4+\delta)+\gamma(13+2\delta-2\delta^2))}{(1+\delta)^3(2-\gamma+\delta)^2}$

this is not easy to sign, but from pictures the utility function looks nonmonotonic in γ if δ is sufficiently low. When δ gets close to .5 or higher, E2's utility is decreasing in γ

$\frac{\partial}{\partial\delta} U_{E2}(x_2^*) = -\frac{(\gamma-1)^2(-3+3\gamma^3+4\delta+5\delta^2+\delta^3-3\gamma^2(4+\delta)+\gamma(13+2\delta-2\delta^2))}{(1+\delta)^4(2-\gamma+\delta)^2}$

this is not easy to sign, but from numerical simulation the utility function looks nonmonotonic in δ when γ is sufficiently low, then decreasing in δ when γ is around .3 or greater.

Plug in micro probability $\gamma = \frac{\alpha+\beta}{2\beta}$

$U_{E2}(x_2^*) = \frac{((\frac{\alpha+\beta}{2\beta})-1)^2(3-2\frac{\alpha+\beta}{2\beta}+\delta)(\frac{\alpha+\beta}{2\beta}+\delta)}{(2-\frac{\alpha+\beta}{2\beta}+\delta)(1+\delta)^3} = \frac{(\alpha-\beta)^2(\alpha+\beta+2\beta\delta)(\beta(2+\delta)-\alpha)}{4\beta^3(1+\delta)^3(\beta(3+2\delta)-\alpha)}$

$\frac{\partial}{\partial\alpha}$ it is negative. decreasing in α

$\frac{\partial}{\partial\beta}$ depends on α as above. If $\alpha > 0$ increasing in β; if $\alpha < 0$ decreasing in β

If I simplify and just look at $(1 - x_2^*)$, the elite's portion of x_1, it is easier to work with. $\frac{\partial}{\partial\alpha}(1 - x_2^*) = \frac{\alpha-\beta}{2\beta^2(1+\delta)^2}$ which is negative, decreasing in α. But x_1^* the dictator's share of the first bargain is increasing in α

$$U_{E3}(x_3^*) = 0$$

$$U_{E1} - U_{E2} = \frac{(\gamma-1)^2(2+2\gamma^2+\gamma(\delta-3)+\delta+\delta^2)}{(2-\gamma+\delta)(1+\delta)^3}$$

from numerical simulation, this quantity is always positive and always decreasing in δ, always decreasing in γ

$U_{E1} - U_{E2} = \frac{(\alpha-\beta)^2(\alpha^2+\alpha\beta(\delta-1)+\beta^2(2+3\delta+2\delta^2))}{4\beta^3(1+\delta)^3(\beta(3+2\delta)-\alpha)}$

Always positive; this is decreasing in α; change in β depends on α: if α is positive, increasing in β; if α negative, decreasing in β

Three District Investment

Using the micro-founded model with uniform valence, the dictator's ex ante probability of winning a district without elite support is $\frac{\beta+\alpha}{2\beta}$ where $\beta > 0$ and $\alpha > 0$ indicates the dictator is idiosyncratically popular ($\alpha < 0$ indicates the dictator is idiosyncratically unpopular)

If the dictator invests in popular policies (redistribution), his valence distribution is increased by a factor of $\nu \in [0, \infty)$ in all districts $\sim U[-\beta+\alpha+\nu, \beta+\alpha+\nu]$. His probability of winning any one district is $\frac{\beta+\alpha+\nu}{2\beta}$

Investment is costly. The cost of investment $c(\nu)$ is a function of the amount of investment ν, such that $c'(\nu) > 0$, $c'' > 0$, and $c(0) = 0$.

The dictator's utility from the three-district (must win 2) rubinstein alternating-offers bargain: $\frac{(3-2\gamma+\delta)(\gamma+\delta)^2}{(1+\delta)^3}$

Substitute in the micro-founded win probability: $\frac{(3-2(\frac{\beta+\alpha+\nu}{2\beta})+\delta)((\frac{\beta+\alpha+\nu}{2\beta})+\delta)^2}{(1+\delta)^3} - c(\nu)$

Confirm concavity: $\frac{\partial^2}{\partial\nu^2}\frac{(3-2(\frac{\beta+\alpha+\nu}{2\beta})+\delta)((\frac{\beta+\alpha+\nu}{2\beta})+\delta)^2}{(1+\delta)^3} - c(\nu) = -\frac{3(\alpha+\beta\delta+\nu)}{2\beta^3(1+\delta)^3} - c''(\nu)$

Recall c is convex by definition, so $-c$ is concave

The first term is negative iff $\alpha + \beta\delta + \nu > 0$. By definition, β, δ, ν are all positive. If the dictator is idiosyncratically unpopular, α may be negative. It must hold that $\beta\delta+\nu > -\alpha$ for the dictator's objective function to be strictly concave. If α is extremely large and negative relative to the other parameters, the dictator is so unpopular that any investment in his popular support will not make a difference in his likelihood of winning. In the limit where

$\delta \to 1$, this requirement that $\beta + \nu \geq -\alpha$ ensures that the dictator's probability of winning any individual district is greater than or equal to 0 (and thus a proper probability).

Assume the dictator is sufficiently popular.

$$\max_\nu \frac{(3-2(\frac{\beta+\alpha+\nu}{2\beta})+\delta)((\frac{\beta+\alpha+\nu}{2\beta})+\delta)^2}{(1+\delta)^3} - c(\nu)$$

$$\frac{\partial}{\partial\nu}\frac{(3-2(\frac{\beta+\alpha+\nu}{2\beta})+\delta)((\frac{\beta+\alpha+\nu}{2\beta})+\delta)^2}{(1+\delta)^3} - c(\nu) = \frac{3(\beta-\alpha-\nu)(\alpha+\beta+2\beta\delta+\nu)}{4\beta^3(1+\delta)^3} - c'(\nu) = 0 \text{ (FOC)}$$

Let's assume a functional form for the convex cost. Let $c(\nu) = \frac{\nu^2}{2}$ so $c'(\nu) = \nu$ and $c''(\nu) = 1$.

FOC: $\frac{3(\beta-\alpha-\nu)(\alpha+\beta+2\beta\delta+\nu)}{4\beta^3(1+\delta)^3} - \nu = 0$

$\nu^* = -\frac{1}{3}\beta^3(1+\delta)^3(2 + \frac{3\alpha}{(\beta^3(1+\delta)^3)} + \frac{3\delta}{(\beta^2(1+\delta)^3)} - \sqrt{4 + \frac{(9+12\beta(1+\delta)(\alpha+\beta\delta))}{(\beta^4(1+\delta)^4)}})$

$\frac{\partial\nu^*}{\partial\alpha} = \frac{2}{\sqrt{4+\frac{(9+12\beta(1+\delta)(\alpha+\beta\delta))}{(\beta^4(1+\delta)^4)}}} - 1$ this could be positive or negative depending on parameter magnitudes. Specifically positive if $\alpha < \frac{-3-4\beta^2\delta-4\beta^2\delta^2}{4\beta(1+\delta)}$; if α is sufficiently low, investment is increasing in the dictator's popularity in that district. Else investment is decreasing in the dictator's popularity.

$\frac{\partial\nu^*}{\partial\beta} = -\delta - 2\beta^2(1+\delta)^3 - \frac{(2(3+\beta(1+\delta)(3\alpha+2\beta\delta)))}{(\beta^2(1+\delta)\sqrt{4+\frac{(9+12\beta(1+\delta)(\alpha+\beta\delta))}{(\beta^4(1+\delta)^4)}})} + \beta^2(1+\delta)^3\sqrt{4 + \frac{(9+12\beta(1+\delta)(\alpha+\beta\delta))}{(\beta^4(1+\delta)^4)}}$ can be positive or negative depending on magnitudes. Positive for extreme values (high and low) of α, negative in the middle. Specifically investment is decreasing in β if $\alpha \in \frac{1}{6}(-4\beta\delta + \frac{-1-2\delta}{\beta(1+\delta)^3} - \frac{2}{\beta+\beta\delta} - \sqrt{\frac{(3+6\delta+4\delta^2)(\delta+2\beta^2(1+\delta)^3)^2}{\beta^2(1+\delta)^6}}), \frac{1}{6}(-4\beta\delta + \frac{-1-2\delta}{\beta(1+\delta)^3} - \frac{2}{\beta+\beta\delta} + \sqrt{\frac{(3+6\delta+4\delta^2)(\delta+2\beta^2(1+\delta)^3)^2}{\beta^2(1+\delta)^6}}))$

Surplus

Social Surplus $= 1 - P(win\neg elite) = 1-(3\gamma^2-2\gamma^3) = 1-(3(\frac{\alpha+\beta}{2\beta})^2 - 2(\frac{\alpha+\beta}{2\beta})^3) = \frac{(\alpha-\beta)^2(\alpha+2\beta)}{4\beta^3}$

$\frac{\partial}{\partial\alpha} = \frac{3(\alpha-\beta)(\alpha+\beta)}{4\beta^3}$ the surplus is decreasing in α

$\frac{\partial}{\partial\beta} = \frac{3(\alpha\beta^2-\alpha^3)}{4\beta^4}$ if α is positive, increasing in β. If α is negative, non-monotonic in β

The above social surplus is the difference between a coalition with no elites and a coalition with sufficient elites (2 or more) to win with certainty. Theoretically (off path) the dictator could coopt fewer elites (1) and still generate a surplus as having one district with certainty reduces the overall uncertainty the dictator faces. In this case, the surplus is the difference between a 1-elite coalition and a 0-elite coalition: $2\gamma - \gamma^2 - (3\gamma^2 - 2\gamma^3) = 2\gamma(\gamma-1)^2 = 2\frac{\alpha+\beta}{2\beta}(\frac{\alpha+\beta}{2\beta} - 1)^2 = \frac{(\alpha-\beta)^2(\alpha+\beta)}{4\beta^3}$

Obviously the surplus of going from no elites to sufficient elites to win with certainty is greater than the surplus of going from none to one: while having one elite in the coalition makes the dictator more likely to succeed, achieving the regime benefit is still uncertain.

I used another definition of surplus above that I want to try again here: just from the dictator's perspective, the surplus is the difference in his expected utility from using the elite intermediary option relative to no elites (off path).

$U_D(elites) = \frac{(3-2\gamma+\delta)(\gamma+\delta)^2}{(1+\delta)^3}$

$U_D(\neg elites) = 3\gamma^2 - 2\gamma^3$

Dictator surplus $= \frac{(3-2\gamma+\delta)(\gamma+\delta)^2}{(1+\delta)^3} - (3\gamma^2 - 2\gamma^3) = \frac{\delta(\gamma-1)^2(6\gamma+\delta^2(1+2\gamma)+\delta(3+6\gamma))}{(1+\delta)^3}$

positive (obviously) and less than the social surplus as the dictator's expected utility from using the elite intermediaries is less as it excludes the portion that the elites take

This quantity is decreasing in α, change in β depends on sign of α. If α is negative, decreasing in β; if α is positive, increasing in β

Two of Three Districts Discussion

Consider a state with three homogeneous districts of which the dictator needs the support of at least two in order to remain in power and receive the regime benefit, normalized to

1. If he does not coopt any of the elites, the dictator wins each district via mass support with probability from the micro voting model, $\gamma = \frac{\alpha+\beta}{2\beta}$. Thus his overall electoral outside option $\Omega_3 = \gamma^3 + 3\gamma^2(1-\gamma)$. This electoral outside option is increasing in the dictator's popularity and decreasing in electoral uncertainty when the dictator is popular, increasing in the electoral uncertainty when the dictator is unpopular. The dictator's outside option when he must win two of three districts to achieve the regime benefit is greater than his likelihood of winning two of two when he goes it alone in the election. However, his electoral outside option for two of three is worse than one of two ($\Omega_3 < \Omega_2'$). While, like one of two, there is a superfluous elite, needing to win two districts to keep the regime does depress his outside option. Coopting two elites will make the dictator win with certainty; adding the third elite does not increase his benefit. While there may be other reasons a dictator wants an oversized coalition or super-majority support (see Groseclose and Snyder 1996, Magaloni 2006), these specific incentives are outside the scope of the current model.

Lemma 18. *In equilibrium, the elite in the third bargaining position will never receive a portion of the regime benefit greater than 0.*

Lemma 19. *Let the probability the dictator wins each district equal γ. When the dictator must win two of three districts, for any investment $\nu \geq 0$, the dictator coopts all three elites; the first elite takes $\frac{2}{(1+\delta)(2+\delta-\gamma)(1+2\gamma)}(\Xi-\Omega_3)$, the second elite takes $\frac{(3+\delta-2\gamma)(\delta+\gamma)}{(1+\delta)^3(2+\delta-\gamma)(1+2\gamma)}(\Xi-\Omega_3)$, the third elite takes 0. The dictator is left with $\Omega_3 + \lambda_3(\Xi - \Omega_3)$. If he were to make counter offers to each elite, they would be $(\frac{\delta(\gamma-1)^2(2\gamma+2\delta+1)}{(1+\delta)^3}, (\frac{(\gamma-1)^2\delta}{(1+\delta)^2})x_1, 0)$*

The portion of the surplus that utilizing the elites instead of using the election alone that the dictator keeps, λ_3, is $(\frac{\delta^2(3+\delta)}{(1+\delta)^3} + \frac{6\delta\gamma}{(1+\delta)^3(1+2\gamma)})$. As one elite is superfluous and the dictator can use this to play the elites off of one another to strengthen his bargaining position, the portion of the surplus that he keeps in this institutional arrangement is greater than his portion when he needed two of two districts ($\lambda_3 > \lambda_2$). However, this portion is less than what he keeps in the one of two institutional configuration ($\lambda'_2 > \lambda_3$).

Which bargaining position is best for an elite? Obviously being in the third position leaves the elite worst off as she will maintain a utility of 0. The first bargaining position is always best for the elite, but the difference between the utilities of being in the first or second positions depends on the other parameters. The first position's advantage over the second decreases as the dictator is more popular. When the dictator is *ex ante* popular, the first position advantage is increasing in the uncertainty of the mass election; when the dictator is unpopular, the first position advantage is decreasing in the uncertainty of the mass election.

Proposition 12. *When the dictator must win two of three districts and anticipates the equilibrium elite offers of* $(\frac{\delta^2+\delta(3-\gamma)+2\gamma(1-\gamma)}{(1+\delta)(2+\delta-\gamma)}, \frac{(\delta+\gamma)(\delta^2+\delta(3-\gamma)+2\gamma(1-\gamma))}{(1+\delta)^3}, 1)$ *he makes optimal investment which satisfies the first order condition* $\frac{3(\beta-\alpha-\nu)(\alpha+\beta+2\beta\delta+\nu)}{4\beta^3(1+\delta)^3} - c'(\nu) = 0$.

Corollary 4. *When the dictator must win two of three districts, anticipates the equilibrium elite offers of* $(\frac{\delta^2+\delta(3-\gamma)+2\gamma(1-\gamma)}{(1+\delta)(2+\delta-\gamma)}, \frac{(\delta+\gamma)(\delta^2+\delta(3-\gamma)+2\gamma(1-\gamma))}{(1+\delta)^3}, 1)$*, he makes optimal investment* $\nu^* = -\frac{1}{3}\beta^3(1+\delta)^3(2+\frac{3\alpha}{(\beta^3(1+\delta)^3)}+\frac{3\delta}{(\beta^2(1+\delta)^3)}-\sqrt{4+\frac{(9+12\beta(1+\delta)(\alpha+\beta\delta))}{(\beta^4(1+\delta)^4)}})$

Much like investment in the one of two district configuration, investment is decreasing in the dictator's district popularity α unless α is sufficiently low. For extreme values of α (high and low), investment in increasing in β; for middling β, investment is decreasing in β. These comparative statics are similar to the one of two institutional configuration in the dictator's incentive to invest more when he is unpopular and the sensitivity of investment to electoral uncertainty depends on the dictator's popularity.

Takeaways Homogeneous Districts

One of One

Outside option: $\frac{\alpha+\beta}{2\beta}$

$\frac{\partial}{\partial\alpha} = \frac{1}{2\beta}$

$\frac{\partial}{\partial\beta} = -\frac{\alpha}{2\beta^2}$ sign depends on α: positive if α is negative. In general, increasing β makes the slope flatter, but also extends the domain of α. So when α is negative, the upward shift of the utility curve when β increases overwhelms the flattening slope

Dictator's utility through bargaining (no investment): $\frac{\alpha+\beta+2\beta\delta}{2\beta(1+\delta)}$ this is the same as no bargaining if $\delta = 0$. Slope (still linear) is lower (flatter) as δ increases

Optimal investment without bargaining: $\frac{1}{2\beta}$

Optimal investment with bargaining: $\frac{1}{2\beta(1+\delta)}$

both decreasing in β

The difference between the dictator's utility (with optimal investment) when bargaining with the elites relative to no elites is the greatest when β is low.

Two of Two

Outside Option: $(\frac{\alpha+\beta}{2\beta})^2$

$\frac{\partial}{\partial\alpha} = \frac{\alpha+\beta}{2\beta^2}$ this is always positive

$\frac{\partial}{\partial\beta} = -\frac{\alpha(\alpha+\beta)}{2\beta^3}$ positive if α is negative, negative if α is positive. Same as above: the slope of the dictator's utility flattens as β increases, but when α is negative the changing domain is the overwhelming effect

Dictator's utility through bargaining (no investment): $\frac{(\delta+\frac{\alpha+\beta}{2\beta})^2}{(1+\delta)^2}$

Optimal investment without bargaining: $\frac{\alpha+\beta}{2\beta^2-1}$ investment is always increasing in α, but change in β depends on parameter magnitudes. Increasing in β if $\alpha < -\frac{1+2\beta^2}{4\beta}$, else decreasing in β

Optimal investment with bargaining: $\frac{\alpha+\beta+2\beta\delta}{2\beta^2(1+\delta)^2-1}$ similarly always increasing in α (for investment greater than 0). Increasing in β if α is sufficiently low, specifically if $\alpha < -\frac{(1+2\delta)(1+2\beta^2(1+\delta)^2))}{4\beta(1+\delta)^2}$ else decreasing in β

This follows similar logic as above: the slope of investment (in α) is flattening as β increases, but for very low α the change in domain overwhelms this effect and increasing β has a positive effect on investment

One of Two

Outside Option: $2(\frac{\alpha+\beta}{2\beta}) - (\frac{\alpha+\beta}{2\beta})^2 = \frac{(3\beta-\alpha)(\alpha+\beta)}{4\beta^2}$

As β increases, the outside option curve (in α) gets flatter. For high α, the curve is already very flat so the changing domain in increasing β dominates

Dictator's utility through bargaining (no investment): $\frac{(\alpha+\beta+2\beta\delta)(\beta(3+2\delta)-\alpha)}{4\beta^2(1+\delta)^2}$

Optimal Investment (no bargaining): $\frac{\beta-\alpha}{1+2\beta^2}$

decreasing in α, decreasing in β unless α is sufficiently high. In particular, if $\alpha > \frac{2\beta^2-1}{4\beta}$, increasing in β

Optimal Investment (with bargaining): $\frac{\beta-\alpha}{1+2\beta^2(1+\delta)^2}$

decreasing in α, β partial depends on relative parameters. Positive if α is sufficiently large, specifically if $\alpha > \frac{\beta^2(1+\delta)^2-1}{2\beta(1+\delta)^2}$.

Two of Three

Outside Option: $\frac{(2\beta-\alpha)(\alpha+\beta)^2}{4\beta^3}$

Dictator's utility through bargaining (no investment): $\frac{(\alpha+\beta+2\beta\delta)^2(\beta(2+\delta)-\alpha)}{4\beta^3(1+\delta)^3}$

Optimal investment (no bargaining): $\frac{1}{3}(-3\alpha - 2\beta^3 + \beta\sqrt{9 + 12\alpha\beta + 4\beta^4})$

Optimal Investment (with bargaining): $-\frac{1}{3}\beta^3(1+\delta)^3(2+\frac{3\alpha}{(\beta^3(1+\delta)^3)}+\frac{3\delta}{(\beta^2(1+\delta)^3)}-\sqrt{4+\frac{(9+12\beta(1+\delta)(\alpha+\beta\delta))}{(\beta^4(1+\delta)^4)}})$

depends on parameters; increasing in α if α is sufficiently low, else decreasing in α. Increasing in β for low and high α, decreasing in β for middling α (but if δ is closer to 1, just increasing for high α else decreasing)

Two Heterogeneous Districts

There are three possible ways to define district heterogeneity in terms of the dictator's electoral chances[21]:

- same dispersion, different mean st $V_1 \sim U[-\beta+\alpha_1, \beta+\alpha_1]$ and $V_2 \sim U[-\beta+\alpha_2, \beta+\alpha_2]$
- same mean, difference dispersion st $V_1 \sim U[-\beta_1+\alpha, \beta_1+\alpha]$ and $V_2 \sim U[-\beta_2+\alpha, \beta_2+\alpha]$
- different mean and dispersion st $V_1 \sim U[-\beta_1+\alpha_1, \beta_1+\alpha_1]$ and $V_2 \sim U[-\beta_2+\alpha_2, \beta_2+\alpha_2]$

I will utilize generic probabilities γ_1 and γ_2 as placeholders.

One of Two

Without elite support, the dictator's probability of winning at least one of the two districts and achieving the regime benefit is $\gamma_1 + \gamma_2 - \gamma_1\gamma_2$.

When the dictator must win one of two districts, the bargain follows as described above, but now his outside electoral option when bargaining with the second elite and not coopting the first is $\gamma_1 + \gamma_2 - \gamma_1\gamma_2$.

The dictator's expected utility of this bargaining game is $U_D(x_1^*, x_2^*) = \frac{2\delta+\delta^2+\gamma_1+\gamma_2-\gamma_1\gamma_2}{(1+\delta)^2}$ Note that even if the second district in the bargaining order is is better for the dictator in terms of outside win probability ($\gamma_2 > \gamma_1$), the dictator will prefer to come to an agreement with the first elite as the superfluous bargaining partner will always make him better off. Further note that even if the dictator could choose the bargaining order (instead of nature), he will be indifferent between either bargaining order as his expected utility is the same and he has no preference over which local elite shares more of the regime benefit.

[21] there also could be heterogeneity in investment costs or something like that, but I am going to focus on differences in win probability

For investment, we need to specify the heterogeneous win probabilities. I will use the most heterogeneous where $V_1 \sim U[-\beta_1 + \alpha_1, \beta_1 + \alpha_1]$ and $V_2 \sim U[-\beta_2 + \alpha_2, \beta_2 + \alpha_2]$. The dictator's objective function is

$$\max_{\nu_1,\nu_2} \frac{2\delta + \delta^2 + \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1} + \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2} - \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1} * \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}}{(1+\delta)^2} - c(\nu_1, \nu_2)$$

$$\frac{\partial}{\partial \nu_1} = \frac{-\alpha_2+\beta_2-\nu_2}{4\beta_1\beta_2(1+\delta)^2} - c'(\nu_1, \nu_2)$$

$$\frac{\partial}{\partial \nu_2} = \frac{-\alpha_1+\beta_1-\nu_1}{4\beta_1\beta_2(1+\delta)^2} - c'(\nu_1, \nu_2)$$

Assume $c(\nu_1, \nu_2) = \frac{\nu_1^2}{2} + \frac{\nu_2^2}{2} + \chi\nu_1\nu_2$ where $\chi \geq 0$ indicates the complementarity of costs from investing in each district. $\frac{\partial}{\partial \nu_1} c(\nu_1, \nu_2) = \nu_1 + \chi\nu_2$ and $\frac{\partial}{\partial \nu_2} c(\nu_1, \nu_2) = \nu_2 + \chi\nu_1$

First Order Conditions:

$$\frac{\partial}{\partial \nu_1} = \frac{-\alpha_2+\beta_2-\nu_2}{4\beta_1\beta_2(1+\delta)^2} - (\nu_1 + \chi\nu_2) = 0$$

$$\frac{\partial}{\partial \nu_2} = \frac{-\alpha_1+\beta_1-\nu_1}{4\beta_1\beta_2(1+\delta)^2} - (\nu_2 + \chi\nu_1) = 0$$

Second Partials:

$$\frac{\partial^2}{\partial \nu_1^2} = -1$$

$$\frac{\partial^2}{\partial \nu_2^2} = -1$$

$$\frac{\partial^2}{\partial \nu_1 \nu_2} = -\chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2}$$

Hessian: $\begin{bmatrix} -1 & -\chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2} \\ -\chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2} & -1 \end{bmatrix}$

$$\begin{bmatrix} -1 & -\chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2} \\ -\chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2} & -1 \end{bmatrix} - \lambda I = \begin{bmatrix} -1-\lambda & -\chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2} \\ -\chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2} & -1-\lambda \end{bmatrix}$$

Characteristic polynomial: $(-1-\lambda)^2 - (-\chi - \frac{1}{4\beta_1\beta_2(1+\delta)^2})^2 = 0$

$$\lambda = -1 \pm (\chi + \frac{1}{4\beta_1\beta_2(1+\delta)^2})$$

If eigenvalues are all non-negative, positive semi-definite. If eigenvalues are all non-positive, negative semi-definite.

$-1-\chi-\frac{1}{4\beta_1\beta_2(1+\delta)^2}>0$ is not possible by definition of χ, so the hessian is not positive semi-definite.

$-1-\chi-\frac{1}{4\beta_1\beta_2(1+\delta)^2}<0$ always holds by definition of χ

Check $-1+\chi+\frac{1}{4\beta_1\beta_2(1+\delta)^2}<0$

$\chi<1-\frac{1}{4\beta_1\beta_2(1+\delta)^2}$ if this holds, the hessian is negative semi-definite

To be negative definite, it must be the case that the determinant of the negative semi-definite matrix is not zero.

$(-1)^2-(-\chi-\frac{1}{4\beta_1\beta_2(1+\delta)^2})^2\neq 0$

This would be violated if $\chi=1-\frac{1}{4\beta_1\beta_2(1+\delta)^2}$ which is already ruled out by the eigenvalue condition on χ, therefore the hessian is negative definite and the solution $(\nu_1^*=\frac{-\alpha_2+\beta_2-\nu_2}{4\beta_1\beta_2(1+\delta)^2}-\chi\nu_2, \nu_2^*=\frac{-\alpha_1+\beta_1-\nu_1}{4\beta_1\beta_2(1+\delta)^2}-\chi\nu_1)$ is a local maximum if $\chi<1-\frac{1}{4\beta_1\beta_2(1+\delta)^2}$

CLOSED FORM

$\nu_1^*=\frac{-\alpha_2+\beta_2-\nu_2}{4\beta_1\beta_2(1+\delta)^2}-\chi\nu_2, \nu_2^*=\frac{-\alpha_1+\beta_1-\nu_1}{4\beta_1\beta_2(1+\delta)^2}-\chi\nu_1$

For all the following comparative statics, assume that $\beta_1\beta_2>\frac{1}{4(1+\delta)^2}$, the districts are sufficiently uncertain. Further, $\chi\in[0,1-\frac{1}{4\beta_1\beta_2(1+\delta)^2})$ so investing in both districts is optimal.

$\nu_1^*=\frac{\beta_1+4\beta_1\beta_2(1+\delta)^2(\alpha_2-\beta_2+\chi\beta_1)+\alpha_1(-1-4\beta_1\beta_2\chi(1+\delta)^2)}{1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$

Comparative Statics:

$\frac{\partial\nu_1^*}{\partial\alpha_1}=\frac{-1-4\beta_1\beta_2(1+\delta)^2\chi}{1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$ numerator and denominator are negative, so derivative is positive. Investment in district 1 is increasing in the dictator's ex ante popularity in district 1.

$\frac{\partial\nu_1^*}{\partial\alpha_2}=\frac{4\beta_1\beta_2(1+\delta)^2}{1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$ negative. Investment in district 1 is decreasing in the dictator's ex ante popularity in district 2.

$$\frac{\partial \nu_1^*}{\partial \beta_1} =$$

$$\frac{(1+4\beta_2(1+\delta)^2(\alpha_2+(\alpha_1+2\beta_1)\chi-16\alpha_2\beta_2^2\beta_1^2(1+\delta)^4(-1+\chi^2)+16\beta_2^3\beta_1^2(1+\delta)^4(-1+\chi^2)+16\alpha_1\beta_2^2\beta_1^2(1+\delta)^4\chi(-1+\chi^2)+\beta_2(-1+4\beta_1(1+\delta)^2(-2\alpha_1+\beta_1+(}{(1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(-1+\chi^2)))^2}$$

Limits summary:

$\beta_2 > \beta_1$

$\alpha_1 >> \alpha_2$ derivative negative

$\alpha_1 > \alpha_2$ negative

$\alpha_1 < \alpha_2$ negative

$\alpha_1 << \alpha_2$ positive

$\beta_1 > \beta_2$

$\alpha_1 >> \alpha_2$ derivative negative

$\alpha_1 > \alpha_2$ negative

$\alpha_1 < \alpha_2$ negative

$\alpha_1 << \alpha_2$ positive

the derivative is either decreasing in χ or non-monotonic in χ depending on parameter magnitudes

As χ approaches its maximum (in the limit), $\frac{\partial \nu_1^*}{\partial \beta_1}$ is negative if $\alpha_1 - \alpha_2 > \beta_1 - \beta_2$

$$\frac{\partial \nu_1^*}{\partial \beta_2} =$$

$$\frac{(4\beta_1(1+\delta)^2(\alpha_2+(\alpha_1-\beta_1)\chi-16\alpha_2\beta_1^2\beta_2^2(1+\delta)^4(-1+\chi^2)+8\beta_1\beta_2^2(1+\delta)^2\chi(-1+2(\alpha_1-\beta_1)\beta_1(1+\delta)^2(-1+\chi^2))+2\beta_2(-1+4(\alpha_1-\beta_1)\beta_1(1+\delta)^2(-1+\chi^2)))}{(1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(-1+\chi^2)))^2}$$

Limits summary:

$\beta_2 > \beta_1$

$\alpha_1 >> \alpha_2$ negative

$\alpha_1 > \alpha_2$ positive

$\alpha_1 < \alpha_2$ positive

$\alpha_1 << \alpha_2$ positive

$\beta_1 > \beta_2$

$\alpha_1 >> \alpha_2$ negative

$\alpha_1 > \alpha_2$ negative

$\alpha_1 < \alpha_2$ positive

$\alpha_1 << \alpha_2$ positive

Generally increasing in χ but non-monotonic in χ for certain magnitudes.

As χ approaches its max, derivative is negative if $\alpha_1 - \alpha_2 > \beta_1 - \beta_2$

For ν_2^*:

$\nu_1^* = \frac{-\alpha_2+\beta_2-\nu_2}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_2, \nu_2^* = \frac{-\alpha_1+\beta_1-\nu_1}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_1$

$\nu_2^* = \frac{\beta_2+4\beta_1\beta_2(1+\delta)^2(\alpha_1-\beta_1+\chi\beta_2)+\alpha_2(-1-4\beta_1\beta_2(1+\delta)^2\chi)}{1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$

$\frac{\partial\nu_2^*}{\partial\alpha_2} = \frac{-1-4\beta_1\beta_2(1+\delta)^2\chi}{(1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(-1+\chi^2))}$ numerator and denominator are negative so derivative is positive. Investment in district 2 is increasing in the dictator's ex ante popularity in district 2.

$\frac{\partial\nu_2^*}{\partial\alpha_1} = \frac{4\beta_1\beta_2(1+\delta)^2}{1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$ negative. Investment in district 2 is decreasing in the dictator's ex ante popularity in district 1.

$\frac{\partial\nu_2^*}{\partial\beta_2} =$

$\frac{(1+4\beta_1(1+\delta)^2(\alpha_1+(\alpha_2+2\beta_2)\chi-16\alpha_1\beta_1^2\beta_2^2(1+\delta)^4(-1+\chi^2)+16\beta_1^3\beta_2^2(1+\delta)^4(-1+\chi^2)+16\alpha_2\beta_1^2\beta_2^2(1+\delta)^4\chi(-1+\chi^2)+\beta_1(-1+4\beta_2(1+\delta)^2(-2\alpha_2+\beta_2+(}{(1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(-1+\chi^2)))^2}$

Limits summary:

$\beta_2 > \beta_1$

$\alpha_1 >> \alpha_2$ positive

$\alpha_1 > \alpha_2$ positive

$\alpha_1 < \alpha_2$ negative

$\alpha_1 << \alpha_2$ negative

$\beta_1 > \beta_2$

$\alpha_1 >> \alpha_2$ positive

$\alpha_1 > \alpha_2$ negative

$\alpha_1 < \alpha_2$ negative

$\alpha_1 << \alpha_2$ negative

$$\frac{\partial \nu_2^*}{\partial \beta_1} = \frac{(4\beta_2(1+\delta)^2(\alpha_1+(\alpha_2-\beta_2)\chi-16\alpha_1\beta_1^2\beta_2^2(1+\delta)^4(-1+\chi^2)+8\beta_1^2\beta_2(1+\delta)^2\chi(-1+2(\alpha_2-\beta_2)\beta_2(1+\delta)^2(-1+\chi^2))+2\beta_1(-1+4(\alpha_2-\beta_2)\beta_2(1+\delta)^2(-1+\chi^2)))}{(1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(-1+\chi^2)))^2}$$

Limits summary:

$\beta_2 > \beta_1$

$\alpha_1 >> \alpha_2$ positive

$\alpha_1 > \alpha_2$ positive

$\alpha_1 < \alpha_2$ positive

$\alpha_1 << \alpha_2$ negative

$\beta_1 > \beta_2$

$\alpha_1 >> \alpha_2$ positive

$\alpha_1 > \alpha_2$ positive

$\alpha_1 < \alpha_2$ negative

$\alpha_1 << \alpha_2$ negative

In general, investment is increasing in the dictator's popularity in the district (invest more where you are ahead). How district uncertainty affects investment depends on the relative parameters: increasing uncertainty in the district decreases investment in that district unless the dictator is severely unpopular in the district (relative to the other district). When the dictator is very behind, district uncertainty actually benefits him, so his investment in increasing in the uncertainty of the district only in that case.

if χ is too big....

Options (possible corner solutions): $(\nu_1, 0), (0, \nu_2), (0, 0)$

$$\frac{2\delta+\delta^2+\frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}+\frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}-\frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}*\frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}}{(1+\delta)^2} - \frac{\nu_1^2}{2} - \frac{\nu_2^2}{2} - \chi\nu_1\nu_2$$

$(\nu_1, 0)$: $\frac{2\delta+\delta^2+\frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}+\frac{\alpha_2+\beta_2}{2\beta_2}-\frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}*\frac{\alpha_2+\beta_2}{2\beta_2}}{(1+\delta)^2} - \frac{\nu_1^2}{2}$

maximizing wrt ν_1 yields FOC $\frac{-\alpha_2+\beta_2-4\beta_1\beta_2(1+\delta)^2\nu_1}{4\beta_1\beta_2(1+\delta)^2} = 0$

$\hat{\nu_1} = \frac{\beta_2-\alpha_2}{4\beta_1\beta_2(1+\delta)^2}$

$U_D(\hat{\nu_1}, 0) = \frac{2\delta+\delta^2+\frac{\alpha_1+\beta_1+(\frac{\beta_2-\alpha_2}{4\beta_1\beta_2(1+\delta)^2})}{2\beta_1}+\frac{\alpha_2+\beta_2}{2\beta_2}-\frac{\alpha_1+\beta_1+(\frac{\beta_2-\alpha_2}{4\beta_1\beta_2(1+\delta)^2})}{2\beta_1}*\frac{\alpha_2+\beta_2}{2\beta_2}}{(1+\delta)^2} - \frac{(\frac{\beta_2-\alpha_2}{4\beta_1\beta_2(1+\delta)^2})^2}{2} =$
$1 + \frac{(\alpha_2-\beta_2)(\alpha_2-\beta_2(1+8(\alpha_1-\beta_1)\beta_1(1+\delta)^2))}{32\beta_1^2\beta_2^2(1+\delta)^4}$

$(0, \nu_2)$: $\frac{2\delta+\delta^2+\frac{\alpha_1+\beta_1}{2\beta_1}+\frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}-\frac{\alpha_1+\beta_1}{2\beta_1}*\frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}}{(1+\delta)^2} - \frac{\nu_2^2}{2}$

maximizing wrt ν_2 yields FOC $\frac{-\alpha_1+\beta_1-4\beta_1\beta_2(1+\delta)^2\nu_2}{4\beta_1\beta_2(1+\delta)^2}$

$\hat{\nu_2} = \frac{\beta_1-\alpha_1}{4\beta_1\beta_2(1+\delta)^2}$

$U_D(0, \hat{\nu_2}) = \frac{2\delta+\delta^2+\frac{\alpha_1+\beta_1}{2\beta_1}+\frac{\alpha_2+\beta_2+(\frac{\beta_1-\alpha_1}{4\beta_1\beta_2(1+\delta)^2})}{2\beta_2}-\frac{\alpha_1+\beta_1}{2\beta_1}*\frac{\alpha_2+\beta_2+(\frac{\beta_1-\alpha_1}{4\beta_1\beta_2(1+\delta)^2})}{2\beta_2}}{(1+\delta)^2} - \frac{(\frac{\beta_1-\alpha_1}{4\beta_1\beta_2(1+\delta)^2})^2}{2}$
$= 1 + \frac{(\alpha_1-\beta_1)(\alpha_1-\beta_1(1+8(\alpha_2-\beta_2)\beta_2(1+\delta)^2))}{32\beta_1^2\beta_2^2(1+\delta)^4}$

$(0, 0)$: $\frac{2\delta+\delta^2+\frac{\alpha_1+\beta_1}{2\beta_1}+\frac{\alpha_2+\beta_2}{2\beta_2}-\frac{\alpha_1+\beta_1}{2\beta_1}*\frac{\alpha_2+\beta_2}{2\beta_2}}{(1+\delta)^2}$

$U_D(0, 0) = 1 + \frac{(\alpha_1-\beta_1)(-\alpha_2+\beta_2)}{4\beta_1\beta_2(1+\delta)^2}$

$(0, 0)$ is dominated by investment in a single district, so just compare only investing in district one versus only investing in district two:

Compare $U_D(\hat{\nu_1}, 0), U_D(0, \hat{\nu_2})$:

$U_D(\hat{\nu_1}, 0) - U_D(0, \hat{\nu_2}) = \frac{-\alpha_1^2+\alpha_2^2+2\alpha_1\beta_1-\beta_1^2+\beta_2^2-2\alpha_2\beta_2}{32\beta_1^2\beta_2^2(1+\delta)^4}$

This quantity is positive (and thus investing in district 1 is preferred) iff $(\beta_1 - \alpha_1 + \beta_2 - \alpha_2)(\alpha_1 - \beta_1 + \beta_2 - \alpha_2)$. Because $\beta > \alpha$ by assumption for a proper probability, district 1 is preferred iff $\beta_2 - \alpha_2 > \beta_1 - \alpha_1$

$\alpha_1 - \alpha_2 > \beta_1 - \beta_2$

Note that if the uncertainty in each district is the same or district 2 is more uncertain, being more popular in district 1 guarantees the dictator will invest there. Even when he is not ahead in district 1, the dictator may invest there depending on the relative uncertainties (i.e. if district 2 is very uncertain)

General takeaway:

If the complementarity of costs is sufficiently low, the dictator will invest in both districts. If the complementarity of costs is not sufficiently low, the dictator will invest in one district (no investment is dominated). He will invest in the less uncertain district in which his investment will make a greater impact (higher mean ex ante popularity, lower dispersion).

Comparative statics for individual district investment (when χ is too high)

$\hat{\nu_1} = \frac{\beta_2-\alpha_2}{4\beta_1\beta_2(1+\delta)^2}$

$\frac{\partial\hat{\nu_1}}{\partial\alpha_2} = \frac{-1}{4\beta_1\beta_2(1+\delta)^2} < 0$ investment is decreasing in the dictator's popularity in the other district

$\frac{\partial\hat{\nu_1}}{\partial\beta_2} = \frac{\alpha_2}{4\beta_1\beta_2^2(1+\delta)^2}$ sign depends on whether α_2 is positive or negative. If α_2 is positive, investment in district 1 is increasing in the uncertainty of district 2. If α_2 is negative (in which case uncertainty in district 2 benefits the dictator), investment in district 1 is decreasing in the uncertainty of district 2.

$\frac{\partial\hat{\nu_1}}{\partial\beta_1} = \frac{\alpha_2-\beta_2}{4\beta_1^2\beta_2(1+\delta)^2} < 0$ investment decreasing in the dispersion of the district

$\hat{\nu_2} = \frac{\beta_1 - \alpha_1}{4\beta_1\beta_2(1+\delta)^2}$

$\frac{\partial \hat{\nu_2}}{\partial \alpha_1} = \frac{-1}{4\beta_1\beta_2(1+\delta)^2} < 0$ investment is decreasing in the dictator's popularity in the other district

$\frac{\partial \hat{\nu_2}}{\partial \beta_2} = \frac{\alpha_1 - \beta_1}{4\beta_1\beta_2^2(1+\delta)^2} < 0$ investment decreasing in the dispersion of the district

$\frac{\partial \hat{\nu_2}}{\partial \beta_1} = \frac{\alpha_1}{4\beta_1^2\beta_2(1+\delta)^2}$ sign depends on whether α_1 is positive or negative

One of Two with no Elites

Comparison of investment with and without bargaining protocol

One of two with no bargaining: Without elite support, the dictator's probability of winning at least one of the two districts and achieving the regime benefit is $\gamma_1 + \gamma_2 - \gamma_1\gamma_2$.

$\frac{\alpha_1+\beta_1+\nu_1}{2\beta_1} + \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2} - \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1} * \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}$

What is optimal investment without bargaining?

$\max_{\nu_1,\nu_2} \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1} + \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2} - \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1} * \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2} - c(\nu_1, \nu_2)$

$\max_{\nu_1,\nu_2} \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1} + \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2} - \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1} * \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2} - \frac{\nu_1^2}{2} - \frac{\nu_2^2}{2} - \chi\nu_1\nu_2$

$\frac{\partial}{\partial \nu_1} = \frac{1}{2\beta_1} - \frac{\alpha_2+\beta_2\nu_2}{4\beta_1\beta_2} - \chi\nu_2 - \nu_1$

$\nu_1^* = \frac{1}{2\beta_1} - \frac{\alpha_2+\beta_2\nu_2}{4\beta_1\beta_2} - \chi\nu_2$

$\frac{\partial}{\partial \nu_2} = \frac{1}{2\beta_2} - \frac{\alpha_1+\beta_1\nu_1}{4\beta_1\beta_2} - \chi\nu_1 - \nu_2$

$\nu_2^* = \frac{1}{2\beta_2} - \frac{\alpha_1+\beta_1\nu_1}{4\beta_1\beta_2} - \chi\nu_1$

Confirm concavity/interior conditions

$\frac{\partial}{\partial \nu_1^2} = -1$

$\frac{\partial}{\partial \nu_2^2} = -1$

$\frac{\partial}{\partial \nu_1\nu_2} = -\frac{1}{4\beta_1\beta_2} - \chi$

Hessian: $\begin{bmatrix} -1 & -\frac{1}{4\beta_1\beta_2} - \chi \\ -\frac{1}{4\beta_1\beta_2} - \chi & -1 \end{bmatrix}$

Determinant: $(-1)^2 - (-\frac{1}{4\beta_1\beta_2} - \chi)^2$ which is positive if $\chi < 1 - \frac{1}{4\beta_1\beta_2}$

Assume $\chi \in [0, 1 - \frac{1}{4\beta_1\beta_2})$ If this holds, interior solution (invest in both districts) is optimal.

Closed Form

$\nu_1^* = \frac{\beta_1+4\beta_1\beta_2(\alpha_2-\beta_2+\beta_1\chi)-\alpha_1(1+4\beta_1\beta_2\chi)}{(1+4\beta_1\beta_2(\chi-1))(1+4\beta_1\beta_2(1+\chi))}$

$\nu_2^* = \frac{\beta_2+4\beta_1\beta_2(\alpha_1-\beta_1+\beta_2\chi)-\alpha_2(1+4\beta_1\beta_2\chi)}{(1+4\beta_1\beta_2(\chi-1))(1+4\beta_1\beta_2(\chi+1))}$

Basic Comparative Statics

ν_1^* is increasing in α_1, the dictator's popularity in district 1

ν_1^* is decreasing in α_2, the dictator's popularity in the other district

ν_2^* is increasing in α_2, the dictator's popularity in district 2

ν_2^* is decreasing in α_1, the dictator's popularity in the other district

Corner Solutions: $(\nu_1, 0), (0, \nu_2), (0, 0)$

$(\nu_1, 0)$

$\max_{\nu_1} \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1} + \frac{\alpha_2+\beta_2}{2\beta_2} - \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1} * \frac{\alpha_2+\beta_2}{2\beta_2} - \frac{\nu_1^2}{2} - \chi\nu_1 * 0$

$\nu_1^* = \frac{\beta_2-\alpha_2}{4\beta_1\beta_2}$

$U_D(\nu_1^*, 0) = \frac{\alpha_2^2-2\alpha_2(1+4(\alpha_1-\beta_1)\beta_1)\beta_2+(1+8\beta_1(\alpha_1+3\beta_1))\beta_2^2}{32\beta_1^2\beta_2^2}$

$(0, \nu_2)$

$\max_{\nu_2} \frac{\alpha_1+\beta_1}{2\beta_1} + \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2} - \frac{\alpha_1+\beta_1}{2\beta_1} * \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2} - \frac{\nu_2^2}{2} - \chi * 0 * \nu_2$

$\nu_2^* = \frac{\beta_1-\alpha_1}{4\beta_1\beta_2}$

$U_D(0, \nu_2^*) = \frac{\alpha_1^2-2\alpha_1\beta_1(1+4\alpha_2\beta_2-4\beta_2^2)+\beta_1^2(1+8\alpha_2\beta_2+24\beta_2^2)}{32\beta_2^2\beta_1^2}$

$(0, 0)$

$U_D = \frac{\alpha_1+\beta_1}{2\beta_1} + \frac{\alpha_2+\beta_2}{2\beta_2} - \frac{\alpha_1+\beta_1}{2\beta_1} * \frac{\alpha_2+\beta_2}{2\beta_2}$
$= \frac{\alpha_1(\beta_2-\alpha_2)+\beta_1(\alpha_2+3\beta_2)}{4\beta_1\beta_2}$

$(0, 0)$ is dominated

Whether $(\nu_1^*, 0)$ or $(0, \nu_2^*)$ is preferred depends on relative parameter magnitudes.
$U_D(\nu_1^*, 0) - U_D(0, \nu_2^*) = \frac{(\alpha_1-\alpha_2+\beta_2-\beta_1)(\beta_1-\alpha_1+\beta_2-\alpha_2)}{32\beta_1^2\beta_2^2}$
$\beta > \alpha$ within district by proper probability. So investing in district 1 is preferred if $(\alpha_1 - \alpha_2 + \beta_2 - \beta_1) > 0$
Invest in district 1 if $\frac{\alpha_1-\alpha_2}{\beta_1-\beta_2} > 1$ which is the same condition as with the elite intermediary

The point of this is to compare investment/utility with and without the bargaining protocol.
Assume χ is sufficiently low so the dictator invests in both districts
Investment in district 1 with bargaining: $\nu_1^* = \frac{\beta_1+4\beta_1\beta_2(1+\delta)^2(\alpha_2-\beta_2+\chi\beta_1)+\alpha_1(-1-4\beta_1\beta_2\chi(1+\delta)^2)}{1+8\beta_1\beta_2(1+\delta)^2(\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$
Investment in district 1 without bargaining: $\nu_1^* = \frac{\beta_1+4\beta_1\beta_2(\alpha_2-\beta_2+\beta_1\chi)-\alpha_1(1+4\beta_1\beta_2\chi)}{(1+4\beta_1\beta_2(\chi-1))(1+4\beta_1\beta_2(1+\chi))}$
For $\delta > 0$, investment is higher without bargaining

Note that the χ (cost complementarity) at which the dictator no longer wants to invest in both districts differs with and without elite intermediaries. The range of χs for which the dictator invests in both districts is larger when there are elite intermediaries to bargain with.

Investing in the superfluous district (since he only needs one of two) is more attractive in the elite bargaining case because he is not just increasing his probability of winning but increasing his bargaining position.

Two of Two

Without elite support, the dictator's probability of winning both districts and achieving the regime benefit is $\gamma_1\gamma_2$. The bargain follows as described above, but now the dictator's true electoral outside option is $\gamma_1\gamma_2$ and if he fails to coopt the first elite, his expected utility is scaled by the probability that he wins the first district through unsupported electoral means as he needs both to get the benefit.

The dictator's objective function is

$\max_{\nu_1,\nu_2}(\frac{\delta+\gamma_2}{1+\delta})(\frac{\delta+\gamma_1}{1+\delta}) - c(\nu_1,\nu_2)$

Substituting the heterogeneous district win probabilities:

$\max_{\nu_1,\nu_2}(\frac{\delta+\frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}}{1+\delta})(\frac{\delta+\frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}}{1+\delta}) - c(\nu_1,\nu_2)$

Assume $c(\nu_1,\nu_2) = \frac{\nu_1^2}{2} + \frac{\nu_2^2}{2} + \chi\nu_1\nu_2$ where $\chi \geq 0$ indicates the complementarity of costs from investing in each district. $\frac{\partial}{\partial\nu_1}c(\nu_1,\nu_2) = \nu_1 + \chi\nu_2$ and $\frac{\partial}{\partial\nu_2}c(\nu_1,\nu_2) = \nu_2 + \chi\nu_1$

First Order Conditions:

$\frac{\partial}{\partial\nu_1} = \frac{\alpha_2+\beta_2+2\beta_2\delta+\nu_2}{4\beta_1\beta_2(1+\delta)^2} - \nu_1 - \chi\nu_2 = 0$

$\frac{\partial}{\partial\nu_2} = \frac{\alpha_1+\beta_1+2\beta_1\delta+\nu_1}{4\beta_1\beta_2(1+\delta)^2} - \nu_2 - \chi\nu_1 = 0$

Second Partials:

$\frac{\partial^2}{\partial\nu_1^2} = -1$

$\frac{\partial^2}{\partial\nu_2^2} = -1$

$\frac{\partial^2}{\partial\nu_1\nu_2} = \frac{1}{4\beta_1\beta_2(1+\delta)^2} - \chi$

Hessian: $\begin{bmatrix} -1 & \frac{1}{4\beta_1\beta_2(1+\delta)^2} - \chi \\ \frac{1}{4\beta_1\beta_2(1+\delta)^2} - \chi & -1 \end{bmatrix}$

Determinant: $(-1)^2 - (\frac{1}{4\beta_1\beta_2(1+\delta)^2} - \chi)^2$ which is positive if $\chi < 1 + \frac{1}{4\beta_1\beta_2(1+\delta)^2}$

As $-1 < 0$ and the determinant of the Hessian is positive if χ is sufficiently low, $\nu_1^* = \frac{\alpha_2+\beta_2+2\beta_2\delta+\nu_2}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_2, \nu_2^* = \frac{\alpha_1+\beta_1+2\beta_1\delta+\nu_1}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_1)$ is a local maximum

Note this cutoff in χ is higher than the cutoff when he needed to win one of two. There is a larger range of cost complementarities for which investing in both districts is optimal.

Closed form:

$\nu_1^* = \frac{\alpha_2+\beta_2+2\beta_2\delta+\nu_2}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_2, \nu_2^* = \frac{\alpha_1+\beta_1+2\beta_1\delta+\nu_1}{4\beta_1\beta_2(1+\delta)^2} - \chi\nu_1)$

$\nu_1^* = -\frac{\alpha_1+\beta_1+2\beta_1\delta+4\beta_1\beta_2(1+\delta)^2(\alpha_2+\beta_2+2\beta_2\delta)-4\beta_1\beta_2(1+\delta)^2(\alpha_1+\beta_1+2\beta_1\delta)\chi}{1+8\beta_1\beta_2(1+\delta)^2(-\chi+2\beta_1\beta_2(1+\delta)^2(-1+\chi^2))}$

$\frac{\partial\nu_1^*}{\partial\alpha_1} = -\frac{1-4\beta_1\beta_2(1+\delta)^2\chi}{1+8\beta_1\beta_2(1+\delta)^2(-\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$ mostly negative (maybe positive for super low βs and low χ

$\frac{\partial\nu_1^*}{\partial\alpha_2} = -\frac{4\beta_1\beta_2(1+\delta)^2}{1+8\beta_1\beta_2(1+\delta)^2(-\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$ mostly positive (maybe negative for super low βs but they probably violate my conditions

$\frac{\partial\nu_1^*}{\partial\beta_1} = \frac{((-1-2\delta-4\alpha_2\beta_2(1+\delta)^2-4\beta_2^2(1+\delta)^2(1+2\delta)+4\beta_2\chi(1+\delta)^2(\alpha_1+2\beta_1+4\beta_1\delta))(1+8\beta_1\beta_2(-\chi(1+\delta)^2+2\beta_1\beta_2(-1+\chi^2)(1+\delta)^4))-8\beta_2(-\chi(1+\delta)^2-}{(1+8\beta_1\beta_2(-\chi(1+\delta)^2+2\beta_1\beta_2(-1+\chi^2)(1+\delta)}$

whether this is positive or negative depends on relative parameter sizes (like one of two)

$\frac{\partial\nu_1^*}{\partial\beta_2} = \frac{(4\beta_1(1+\delta)^2(-\chi(\alpha_1+\beta_1+2\beta_1\delta)+\alpha_2(-1+16\beta_1^2\beta_2^2(-1+\chi^2)(1+\delta)^4)-8\beta_1\beta_2^2\chi(1+\delta)^2(-1-2\delta+2\beta_1(-1+\chi^2)(1+\delta)^2(\alpha_1+\beta_1+2\beta_1\delta))+2\beta_2(-1-}{(1+8\beta_1\beta_2(-\chi(1+\delta)^2+2\beta_1\beta_2(-1+\chi^2)(1+\delta)^4))^2}$

this is positive or negative depends on relative parameter sizes (like one of two) but mostly positive

Closed form

$\nu_2^* = \frac{\alpha_2(-1+4\beta_1\beta_2\chi(1+\delta)^2)+\beta_2(-1-2\delta-4\beta_1(1+\delta)^2(\alpha_1+(\beta_1-\beta_2\chi)(1+2\delta)))}{1+8\beta_1\beta_2(1+\delta)^2(-\chi+2\beta_1\beta_2(1+\delta)^2(-1+\chi^2))}$

$\frac{\partial\nu_2^*}{\partial\alpha_2} = \frac{-1+4\beta_1\beta_2(1+\delta)^2\chi}{1+8\beta_1\beta_2(1+\delta)^2(-\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$ negative

$\frac{\partial\nu_2^*}{\partial\alpha_1} = -\frac{4\beta_1\beta_2(1+\delta)^2}{1+8\beta_1\beta_2(1+\delta)^2(-\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$ positive

$\frac{\partial\nu_2^*}{\partial\beta_2} = \frac{(-1-2\delta+4\beta_1(1+\delta)^2(-16\alpha_2\beta_1^2\beta_2^2\chi(-1+\chi^2)(1+\delta)^4+16\beta_1^3\beta_2^2(-1+\chi^2)(1+\delta)^4(1+2\delta)+\chi(-\alpha_2+2\beta_2+4\beta_2\delta)+\alpha_1(-1+16\beta_1^2\beta_2^2(-1+\chi^2)(1+\delta)^4)}{1+8\beta_1\beta_2(1+\delta)^2(-\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$

whether this is positive or negative depends on relative parameter sizes (like one of two)

$\frac{\partial\nu_2^*}{\partial\beta_1} = \frac{(4\beta_2(1+\delta)^2(-\chi(\alpha_2+\beta_2+2\beta_2\delta)+\alpha_1(-1+16\beta_1^2\beta_2^2(-1+\chi^2)(1+\delta])^4)-8\beta_1^2\beta_2\chi(1+\delta)^2(-1-2\delta+2\beta_2(-1+\chi^2)(1+\delta)^2(\alpha_2+\beta_2+2\beta_2\delta))+2\beta_1(-1-}{1+8\beta_1\beta_2(1+\delta)^2(-\chi+2\beta_1\beta_2(1+\delta)^2(\chi^2-1))}$

whether this is positive or negative depends on relative parameter sizes (like one of two)

What if χ is too big?

Possible corner solutions: $(\nu_1, 0), (0, \nu_2), (0, 0)$

$(\nu_1, 0) : (\frac{\delta + \frac{\alpha_2+\beta_2}{2\beta_2}}{1+\delta})(\frac{\delta + \frac{\alpha_1+\beta_1+\nu_1}{2\beta_1}}{1+\delta}) - \frac{\nu_1^2}{2}$

maximize wrt ν_1 yields FOC: $\frac{\alpha_2+\beta_2+2\beta_2\delta}{4\beta_1\beta_2(1+\delta)^2} - \nu_1 = 0$

$\hat{\nu_1} = \frac{\alpha_2+\beta_2+2\beta_2\delta}{4\beta_1\beta_2(1+\delta)^2}$

$U_D(\hat{\nu_1}, 0) = \frac{(\alpha_2+\beta_2+2\beta_2\delta)(\alpha_2+\beta_2+2\beta_2\delta+8\beta_1\beta_2(1+\delta)^2(\alpha_1+\beta_1+2\beta_1\delta))}{32\beta_1^2\beta_2^2(1+\delta)^4}$

$(0, \nu_2) : (\frac{\delta + \frac{\alpha_2+\beta_2+\nu_2}{2\beta_2}}{1+\delta})(\frac{\delta + \frac{\alpha_1+\beta_1}{2\beta_1}}{1+\delta}) - \frac{\nu_2^2}{2}$

maximize wrt ν_2 yields FOC: $\frac{\alpha_1+\beta_1+2\beta_1\delta}{4\beta_1\beta_2(1+\delta)^2} - \nu_2 = 0$

$\hat{\nu_2} = \frac{\alpha_1+\beta_1+2\beta_1\delta}{4\beta_1\beta_2(1+\delta)^2}$

$U_D(0, \hat{\nu_2}) = \frac{(\alpha_1+\beta_1+2\beta_1\delta)(\alpha_1+\beta_1+2\beta_1\delta+8\beta_1\beta_2(1+\delta)^2(\alpha_2+\beta_2+2\beta_2\delta))}{32\beta_1^2\beta_2^2(1+\delta)^4}$

$(0, 0) : (\frac{\delta + \frac{\alpha_2+\beta_2}{2\beta_2}}{1+\delta})(\frac{\delta + \frac{\alpha_1+\beta_1}{2\beta_1}}{1+\delta})$

$U_D(0, 0) = \frac{(\alpha_1+\beta_1+2\beta_1\delta)(\alpha_2+\beta_2+2\beta_2\delta)}{4\beta_1\beta_2(1+\delta)^2}$

$(0, 0)$ is dominated by investment in a single district, so just compare investment in district one versus investment in district two:

$U_D(\hat{\nu_1}, 0) - U_D(0, \hat{\nu_2}) =$

$\frac{(\alpha_2+\beta_2+2\beta_2\delta)(\alpha_2+\beta_2+2\beta_2\delta+8\beta_1\beta_2(1+\delta)^2(\alpha_1+\beta_1+2\beta_1\delta))}{32\beta_1^2\beta_2^2(1+\delta)^4} - \frac{(\alpha_1+\beta_1+2\beta_1\delta)(\alpha_1+\beta_1+2\beta_1\delta+8\beta_1\beta_2(1+\delta)^2(\alpha_2+\beta_2+2\beta_2\delta))}{32\beta_1^2\beta_2^2(1+\delta)^4}$

positive if $-(\alpha_1 + \alpha_2 + \beta_1 + \beta_2 + 2\delta(\beta_1 + \beta_2))(\alpha_1 - \alpha_2 + (\beta_1 - \beta_2)(1 + 2\delta)) > 0$

The first term is always positive

Invest in district 1 if and only if

$\alpha_2 - \alpha_1 > (\beta_1 - \beta_2)(1 + 2\delta)$

Comparative statics for individual district investment (χ too high)

$\hat{\nu_1} = \frac{\alpha_2+\beta_2+2\beta_2\delta}{4\beta_1\beta_2(1+\delta)^2}$

$\frac{\partial}{\partial\alpha_2} = \frac{1}{4\beta_1\beta_2(1+\delta)^2}$ positive: increasing in popularity in OTHER district

$\frac{\partial}{\partial\beta_2} = \frac{-\alpha_2}{4\beta_1\beta_2^2(1+\delta)^2}$ sign depends on sign of α_2

$\frac{\partial}{\partial\beta_1} = -\frac{\alpha_2+\beta_2+2\beta_2\delta}{4\beta_1\beta_2^2(1+\delta)^2}$ negative: decreasing in uncertainty dispersion in this district

$\hat{\nu_2} = \frac{\alpha_1+\beta_1+2\beta_1\delta}{4\beta_1\beta_2(1+\delta)^2}$

$\frac{\partial}{\partial\alpha_1} = \frac{1}{4\beta_1\beta_2(1+\delta)^2}$positive: increasing in popularity in OTHER district

$\frac{\partial}{\partial\beta_2} = -\frac{\alpha_1+\beta_1+2\beta_1\delta}{4\beta_1^2\beta_2(1+\delta)^2}$ negative: decreasing in uncertainty dispersion in this district

$\frac{\partial}{\partial\beta_1} = \frac{-\alpha_1}{4\beta_1^2\beta_2(1+\delta)^2}$ sign depends on sign of α_1

Takeaways

When the dictator needs to win one of two districts, investments in his electoral outside options across the two districts are substitutes. The optimal investment level for one district is decreasing in the investment level for the other. When the dictator must win both districts in order to receive the regime benefit, investments in his electoral outside options across the districts are complements. The optimal investment level for one district is increasing in the investment level for the other.

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