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A Theoretical Analysis of Interventions Aimed at Reducing Inequality in College Admissions

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An abstract of
a thesis submitted to the Faculty of Emory College of Arts and Sciences
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Abstract

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This paper builds a mathematical model of the college admissions process to better understand the inequality present in it and to analyze the effect of government interventions. I find that, in accordance with the empirical consensus on the topic, there is substantial preference given to applicants with high level of resources available to them. The government is considered to have the objective of minimizing the role that students' resources play in their likelihood of admission. In pursuit of this end, the interventions tested are partial revelation of student resources and direct redistribution of those resources. Revelation of partial information about a student's resource level has positive effect in reducing material inequality in the admissions process. However, monotonic redistribution is shown to have no effect if the partial information is already revealed.

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1 Introduction

Perhaps no communal goal is more important for promoting long-run welfare than the provision of quality education. Educating and training citizens is vital to promoting a productive economy and to the functioning of an informed democracy. Thus, the question of who is given access to excellent education is one that must be considered seriously.

When compared to countries with developed economies, the United States has very poor rates of intergenerational elasticity of income [7]— a measure of social mobility that compares the income of parents to the income of their children when they become adults. The measure takes years to materialize and is not forward looking, but the inelasticity of intergenerational income suggests that the United States is lagging behind other countries in creating opportunities for otherwise motivated and talented people to succeed. This creates an inimical cycle of poverty as poor parents raise children that are, more often than not, doomed to poverty. A system that allows the accidental circumstances of one's birth to have nearly deterministic power over an individual is disastrous and inefficient.

Unequal access to higher education obviously hurts individuals born into financially tenuous situations, but it also hurts the economy and society at large. Since talent and aptitude are distributed across people of all wealth types, keeping poor students from quality education arbitrarily restricts the national talent pool. There are countless examples of students with exceptional potential whose talents are kept hidden from the rest of society because they were never able to receive a proper education. Ensuring equal access to education is essential to promoting economic and cultural welfare. Doing so will increase the pool of high-skilled workers who make the greatest individual contribution to society.

I examine the impact that information and redistribution on the college admissions process, where a student's admission is determined by their aptitude and resources. A single college evaluates candidates based on their performance and its corresponding expectation of aptitude. There is inequality in the baseline model, as students with higher level of resources are more likely to gain admission. I consider the government objective of ensuring the most talented students are admitted to college. In pursuit of this end, I find that revealing partial information about the resources available to a student has a positive effect, while a certain redistribution has no effect if this information has already been revealed.

The inequality I find in the baseline version of the model mirrors the consensus among social researchers — specifically, the college admissions process benefits children who had access to better resources, all else being equal. This inequality hinders economic, intellectual, and cultural production, which is a problem for the government, which I argue wants admissions to be based solely on talent. Revealing partial information about a student's resources helps address inequality by improving the quality of information that evaluators use to assess talent. Since colleges genuinely want to accept the most talented students, helping them generate more accurate assessments of potential will invariably lead to more talented students gaining admissions. However, if this information is already revealed, a redistribution of resources that does not change the order of students with respect to resource endowments has no effect on admissions. This is a consequence of the fact that the admissions process is competitive and students are evaluated only in relation to each other. If properly implemented, revealing information about the public resources available to an applicant — such as the quality of and funding to a student's high school and statistics about the health and safety of their community — can help reduce inequality in the college admissions process.

2 Literature Review

Empirical work on education inequality has found strong links between financial expenditure and positive student outcomes.

Miller (2022) [8] analyzes neighborhoods that saw sharp rises in median property value. These unanticipated appreciations left school districts — which are partially funded by local property value taxes — with more money to spend than they had before. The paper was able to establish that the resulting increase in education spending alone accounted for a tangible improvement in student outcomes. Specifically, a 10% increase in spending led to a 2.1% – 4.4% increase in graduation rate and improved student performance on standardized tests by 0.05-0.09 standard deviations. These effects may seem small but in the highly competitive world of college admissions, these increases have tangible effect on a student's candidacy.

Chetty et al. (2016) [2] studies the impact of relocation to better neighborhoods on the performance of children. In the mid-1990's the Department of Housing and Urban Development experimented with a housing voucher program called Moving to Opportunity (MTO). The effort was motivated by observations that children in poor areas have no choice but to attend schools of inferior quality. The program administered vouchers to randomly selected families to move to wealthy communities with stronger school systems. The program provides excellent data for isolating the effect of neighborhood and school quality on a child's educational performance and attainment. The study compared the outcomes of children that moved from a high-poverty area to a wealthy neighborhood through the MTO program with the outcomes of their similarly situated peers that did not receive vouchers. Controlling for parent education and income, the researchers found that — beyond a transitory regression that due to challenges of moving to a new location — children in the program were substantially better off from attending the wealthier school. The effect of the program is more intense with exposure, meaning that children who moved earlier in life were able to experience more of the benefit of the higher quality school than children who moved later in life. Specifically, the researchers found that children in the program that moved before the age of 13 are significantly more likely to attend college — and attended more selective colleges when they did — and attain substantially higher levels of income as adults than their peers that were not selected for the program.

Jackson et al. (2016) [1] researches the effects of policies intended to address education inequality that were implemented in the 1970's and 1980's. The researchers compared the outcomes of students under the reforms to those who did not benefit from them — comparing cohorts of students in areas subject to them with cohorts that did not implement the reform and with students in the same area that had graduated before the programs began. Thus, the students compared differed from the treatment either by time or place. Overall the reforms had a sizable impact on the long-run success on students. For a 10% increase in per-pupil spending over 12 years, overall education attainment increased by 0.27 years, wages later in life increased by 7.25%, and the incidence of adult poverty decreased by 3.67%. These improvements were more pronounced for low-income students. To explain this, the researchers highlight improvements in teacher-student ratios, longer school years, and increases in teacher salaries that resulted from the aid of the government programs.

Perhaps the most convincing evidence for the advantage that school quality alone confers on its students in the college admissions process comes from Berkowitz and Hoekstra (2011) [3]. The researchers were given access to administrative data from a highly selective private school. Using admissions scores, the researchers could control for intangible qualities of candidates that are used in evaluation for admissions. This was combined with information on student's GPA, scores on standardized tests, family characteristics, and more to help single out the effect of attending the private school on admission to college. The researchers found that attending the school alone allowed students to attend a school whose median SAT score is 20 points higher. That could be the difference between attending Princeton over Georgetown or NYU over Boston College.

Two policy proposals have received the bulk of attention in the literature surrounding educational equality: promoting economic community diversification and restructuring of school funding. The potential of the housing vouchers has been demonstrated through works similar to Chetty et al. (2016). Yet, another way to equalize the economic status of a community was explored in Fernandez and Rogerson (1996) [4]. Instead of giving vouchers to poor families to move to wealthy neighborhoods, the authors suggest that we apportion funds to poor communities to make them more appealing to wealthy individuals. In addition to redistributing wealth to the poor and bolstering the schools in their communities, the author found that any policies designed to attract wealthy inhabitants while not displacing current residents creates a long-run equilibrium in which schools across the country are indistinguishable from each other.

Policies that tackle school funding are clouded in a bit more controversy. Some have argued that we simply need to equalize funding for schools in all districts. However, as Kotera and Seshadri (2017) [5] points out, this is not sufficient. The economists build a model to explain social mobility under different types of education funding mechanisms. They first consider 'full state funding' which eliminates local discrepancies between school districts and gives the central body — either the state or federal government, depending on the particular policy — power to divide funds uniformly across all schools. There are no local taxes or supplementary funding to particular public schools beyond this allocation. In their simulations, the researchers find that this policy helps improve social mobility but is severely limited in its power. The primary reason for this is that kids are still dependent on their parents' human and financial capital, and are still underprivileged with respect to total resources. Further, if wealthy parents are dissatisfied with their child's schooling, they can afford to place him or her in a superior private school or hire tutors to compensate. The authors argue that it may be more effective to employ a floor-based strategy where schools are uniformly distributed in the first stage of a policy and then more money is provided to schools that teach children from lower socioeconomic class. This extra funding could work to offset the advantage that wealthy student have over poor students.

3 The Baseline Model

There is mass 1 of students, each of which will want to attend college. Their behavior prior to applying is exogenously fixed. There are two main aspects of pre-collegiate performance π: aptitude α and resources r. Each of these will be uniformly distributed on $[0, 1]$, independent of each other. The form of π is given by $\pi = \alpha r$ and is perfectly observed by the college. A single college ranks students according to this π, as well as the expectation of a student's inherent aptitude $E[\alpha|x]$ — whether or not it has been realized — through the information available to it, that is x. The function by which this college ranks students is given by

$$S = (1 - t)\pi + tE[\alpha|x], t \in (0, 1)$$

where t represents this college's preference between preparedness π and potential $E[\alpha]$. This college has capacity k and will grant admission to the k students with the highest rank, who will accept the offer of admission. Students who do not meet the cutoff will not be able to attend any college.

3.1 The Signal: π

Colleges want to accept the students that will contribute the most to the social and intellectual life on campus while they are in attendance and will achieve success that can reinforce the reputation and financial endowment of the college itself. The evaluation of candidates, though, is difficult and requires much estimation and subjective judgement on the part of evaluators. The natural abilities and attitudes of applicants combine with the resources available to them create an imperfect signal that admissions committees use to rank applications for admission. The college admissions committee can initially only observe an applicant's application materials. Most often, schools will analyze standardized test scores, letters of recommendation, student essays, resumes, and high school academic performance. Though each of these aspects reveals different things about a given applicant — academic preparedness, aptitude, community involvement, etc. — they are all signals to admissions officers of how a prospective student would perform in college and in professional pursuits thereafter. In essence, the committee wants to accept those students that best enrich their community — academically and socially — during their time in school and will be successful afterwards. These materials create a signal π that demonstrates a student's actualized skill level across many different factors.

In reality it is difficult or often impossible to determine a particular candidate's actual preparedness. Countless unobservable factors create variations in an admissions committee's estimation of an individual applicant's preparedness for college and subsequent professional life. For simplicity, we assume that a given student's π is determined via the inputs observed in the next section.

3.2 The Inputs: α, r

The admissions committee will prefer students with higher levels of π to students with lower π, ceteris paribus. However, the importance of π is that is also can serve as a signal of inherent aptitude α. This aptitude represents all aspects of individual character that help applicant excel in all the relevant dimensions. Embedded in this α are characteristics like intellectual and social intelligence, charisma, and work ethic. However, there is often a vast difference between aptitude and performance — many hard-working, intelligent students are outpaced by peers possessing less distinguishing traits.

The gap between potential and performance is filled by resources, motivation, social network, and even chance. Of these, only the resources of a student can be properly measured and are, thus, the main subject of policy interventions. Resources include the quality of a student's instruction, his or her external inspiration to achieve their potential, and the safety net needed to excel as a student and community member. These factors determine to what extent students can develop and actualize their potential. Resources and aptitude are independent of each other and are distributed uniformly across all students.

Aptitude and resources are complements. To achieve near perfect levels of preparation a student has to have elite levels of aptitude and resources. Higher levels of resources cannot make up for a lack of aptitude beyond a point and vice versa. Specifically, we assume that $\pi = \alpha r$.

3.3 The Ranking of Candidates: S

There is only one college that instructs every admitted student. In its evaluation, this college ranks every applicant. The matching of students to college is based solely on the decision of a committee, with students attending the school if they are admitted. The college will admit enough students to fill its capacity k and no more.

The ranking of each candidate is given by

$$S = (1-t)\pi + tE[\alpha|x], t \in (0,1)\,.$$

where x is the set of information available to the committee when it makes its decision.

4 Baseline Outcome

Recall that the initial performance function is given by

$$\pi = \alpha r, \text{ where } \alpha, r \sim U[0,1].$$

The joint cumulative distribution function (cdf) for this π is

$$F_\pi(z) = P(\alpha r \leq z) = \int_0^1 P\left(r \leq \frac{z}{\alpha}\right) f_r(\alpha)\, d\alpha = \int_0^z \, d\alpha + \int_z^1 \frac{z}{\alpha}\, d\alpha$$
$$= z - z\ln(z),$$

with corresponding probability density function (pdf)

$$f_\pi(z) = \tfrac{d}{dz}z - z\ln(z) = -\ln(z) \text{ for } z \in [0,1].$$

Using this, we can solve for $E[\alpha|\pi]$, which is the admissions committee's prediction of a student's aptitude α based on their application materials π, starting with the conditional distribution of performance π for a given α

$$P(\pi|\alpha) = P(\alpha r \leq \pi) = P(r \leq \tfrac{\pi}{\alpha}) = \tfrac{\pi}{\alpha} \Rightarrow f(\pi|\alpha) = \tfrac{1}{\alpha}.$$

Bayes' Theorem guarantees that $P(A|B) = \frac{P(B|A)\cdot P(A)}{P(B)}$. Here, this means that $P(\alpha|\pi) = \frac{P(\pi|\alpha)\cdot P(\alpha)}{P(\pi)}$. Since α is distributed uniformly, it has pdf $h(\alpha) = \frac{1}{1-0} = 1$ for $\alpha \in [0,1]$. The pdf for $P(\alpha|\pi)$ is found below.

$$h(\alpha|\pi) = \tfrac{f(\pi|\alpha)\cdot h(\alpha)}{f(\pi)} = \tfrac{\frac{1}{\alpha}\cdot 1}{-\ln(\pi)} = \tfrac{-1}{\alpha\ln(\pi)}$$

For a given value of π, α has lower bound at π, since its multiplicative partner r can be no higher than 1. Knowing this we can solve for $E[\alpha|\pi]$:

$$E[\alpha|\pi] = \textstyle\int_\pi^1 \alpha \cdot h(\alpha|\pi) d\alpha = \int_\pi^1 \frac{-1}{\ln(\pi)} d\alpha = \frac{\pi-1}{\ln(\pi)}.$$

Since the only information available to admissions committees is π, the choice of t in the selection of $S = (1-t)\pi + tE[\alpha]$ is irrelevant. Notice that,

$$\frac{d}{d\pi}S = \frac{d}{d\pi}\left((1-t)\pi + t\frac{\pi - 1}{\ln(\pi)}\right) = 1 - t + \frac{t(\pi\ln(\pi) - \pi + 1)}{\pi\ln^2(\pi)} > 0$$

meaning S is strictly increasing in π. Thus, students with the highest π will be admitted to college.

Figure 1 below gives depicts the admissions process in this case. The curve S_i represents the level set of S, such that the region above the curve (the number of admitted students) is equal to the college's capacity k. Notice that, as we would expect, students with high resource endowments are more likely to be admitted than student with lower level of resources.

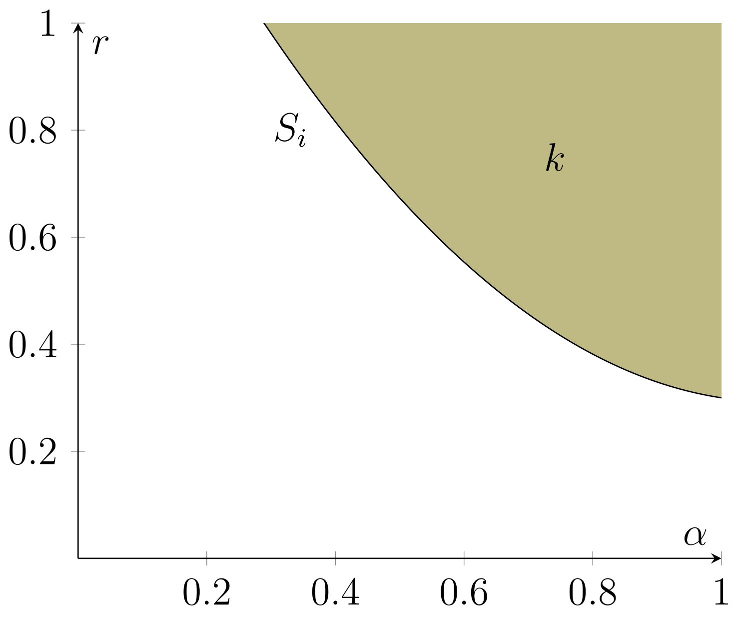

Figure 1

5 Private and Public Resources

It may be more helpful to distinguish between resources available to individual students and communal resources available to groups of students in a given community. The first we can call parental resources (or r_p), as it largely includes the financial and human capital of the parents, used to purchase additional tutoring, encourage children to do well academically and extracurricularly, and various other resources. The second set of resources will be called government resources (or r_g) and refers to more public resources in a students immediate geographic community.This group includes the student's school of attendance, the safety and well-being of their community, and various other opportunities afforded or denied the student by virtue of their geo-social status. Like the simple resource variable r, both r_p and r_g will be independent and uniformly distributed on $[0,1]$, this bifurcation has no effect on α, so it will remain uniformly distributed on $[0,1]$ as well.

5.1 Manipulated Set-Up

The pdf of the new production function is calculated as follows, starting with the cdf for $r_g r_p$

$$\Phi_{r'}(z) = P\left(r_p r_g \leq z\right) = \int_{r_g=0}^{1} P\left(r_p \leq \frac{z}{r_g}\right) f_{r_p}(r_g)\, dr_g = \int_{r_g=0}^{z} dr_g + \int_{r_g=z}^{1} \frac{z}{r_g}\, dr_g$$

$$= z - z\ln(z), \quad z \in (0,1].$$

Differentiating with respect to z yields the following pdf

$$\phi_{r'}(z) = \frac{d}{dz} z - z\ln(z) = -\ln(z), \quad z \in (0,1].$$

The cdf for π, then, is

$$F_\pi(z) = P\left(\alpha r' \leq z\right) = \int_{\alpha=0}^{1} P\left(\alpha \leq \frac{z}{\alpha}\right) \phi_{r'}(\alpha) d\alpha = -\int_{\alpha=0}^{z} \ln(\alpha) d\alpha - \int_{\alpha=z}^{1} \frac{z}{\alpha} \ln(\alpha) d\alpha$$

$$= z - z\ln(z) + \frac{z\ln^2(z)}{2},$$

with corresponding cdf

$$f_\pi(z) = \frac{d}{dz} z - \ln(z) + \frac{z\ln^2(z)}{2} = \frac{\ln^2(z)}{2}.$$

5.2 Expectation of Aptitude given π

We can now calculate $E[\alpha|\pi]$ similar to before, starting with the cdf of the conditional distribution

$$F(\pi|\alpha) = P(\alpha r' \leq \pi) = P\left(r_g r_p \leq \frac{\pi}{\alpha}\right) = \frac{\pi}{\alpha} - \frac{\pi}{\alpha} \ln\left(\frac{\pi}{\alpha}\right),$$

and its pdf

$$f(\pi|\alpha) = \frac{d}{d\pi}\frac{\pi}{\alpha} - \frac{\pi}{\alpha}\ln\left(\frac{\pi}{\alpha}\right) = \frac{-\ln(\frac{\pi}{\alpha})}{\alpha}.$$

Applying Bayes' theorem, we have the following expression for the conditional distribution of α for given π

$$h(\alpha|\pi) = \frac{f(\pi|\alpha)h(\alpha)}{f(\pi)} = \frac{\frac{-\ln(\frac{\pi}{\alpha})}{\alpha}\cdot 1}{\frac{\ln^2(\pi)}{2}} = \frac{-2\ln(\frac{\pi}{\alpha})}{\alpha\ln^2(\pi)}.$$

Using this distribution, we can solve for the admissions committee's anticipation of aptitude for given level of π

$$E[\alpha|\pi] = \int_{\pi}^{1} \alpha \cdot \frac{-2\ln(\frac{\pi}{\alpha})}{\alpha\ln^2(\pi)}d\alpha = \frac{-2}{\ln^2(\pi)}\int_{\pi}^{1} \ln(\pi) - \ln(\alpha)d\alpha = \frac{-2}{\ln^2(\pi)}(\ln(\pi) + 1 - \pi).$$

6 Government Objective

With these distributions in mind, we can now consider the objective of the government. Of the many dimensions of responsibility that governing bodies can be said to have I will focus on the government's aim to minimize the effect of wealth on college admissions. That is, the goal of the government to make sure the most talented students are admitted to college, regardless of the resource endowments of their parents and communities. In pursuit of this aim, the government can choose to perfectly reveal the level of r_g for each student to admissions committees, and can redistribute resources from r_p into r_g.

6.1 Efficiency on α

The traits that enable students to do well in school are the same ones that enhance and enable economic productivity and innovation beyond the classroom. Many economic arguments for education equality are centered on this fact. If a central planner wants to maximize total economic output and efficiency, it will make sure to educate the most talented, gifted students to produce the most efficient and innovative technologies; since aptitude is distributed irrespective of initial endowment, students from all backgrounds should have access to high quality education. Once a student attends college, he or she takes on the resources of that college. Professors replace teachers, alumni networks replace parental connections, and career centers replace professional advice from parents. To take full advantage of the exceptional resources available at the most prestigious universities, the government would prefer the admissions process be based purely on aptitude.

This goal would lead the government to prefer a resource-independent admissions process where the aptitude of admitted students is maximized. The government's primary objective is

$$\max \quad E[\alpha|A]$$

where A is the region of admitted students.

When this objective is maximized, the college will select candidates for admission in a manner consistent with Figure 2 below. Here, the college's level set S_i is a vertical line that does not change with respect to r_g or r_p and is thus completely resource-neutral.

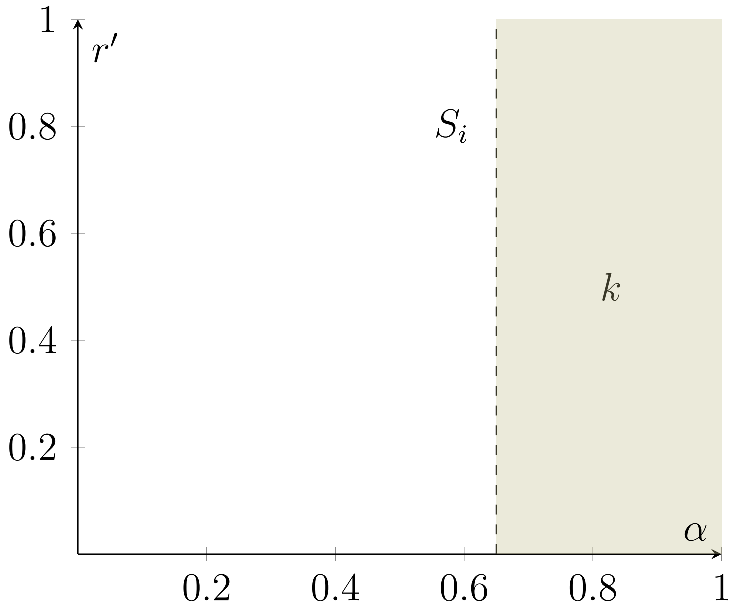

Figure 2

Since the college wants to admit the best students and — for $t \neq 0$ — will give some preference to students to students with high expectations of aptitude. The government can move closer to its primary objective, then, by making the expectation as accurate and precise to given α as possible. This is achieved through the secondary objective

$$\min_{x=\{\pi,(\pi,r_g)\}} \quad \left| E[E[\alpha|x] - \alpha] \right|.$$

6.2 Interventions

Parental resources are personal and private. Revealing them is either difficult and controversial — such as parental income — or downright impossible — as is the case for parental human capital or involvement in education. Governmental resources, on the other hand, are often direct and measurable. Metrics such as school quality and community crime statistics, among others, can be directly observed and reported. This revelation of r_g to admissions committees may enable them to make more accurate assessments of candidate's aptitudes. If the committees can make better predictions of ability, they are less likely to be biased towards students with exceptional resources as we have seen. For our purposes, r_p will remain hidden but the government has the option to reveal r_g to admissions committees. The government also has the power to redistribute the resource parameters themselves. To do this, it can tax r_p at a constant rate δ, and reapportion these funds into r_g uniformly, so that each person receives the same amount of increase in r_g. The effect that this intervention and the revelation of r_g has on the government's objective will be considered below.

7 Analysis

With these government objectives and interventions in mind, I will first examine the effect of revealing r_g and will then consider the effects of redistribution when r_g is known.

7.1 Revelation of r_g

This revelation will result in the following calculations, starting with the new conditional pdf of π given α, r_g

$$F(\pi|\alpha, r_g) = P(\pi|\alpha, r_g) = P(\alpha r_g r_p \leq \pi) = P\left(r_p \leq \frac{\pi}{\alpha r_g}\right) = \frac{\pi}{\alpha r_g}.$$

The corresponding pdf of the distribution is given by

$$f(\pi|\alpha, r_g) = \frac{d}{d\pi}\frac{\pi}{\alpha r_g} = \frac{1}{\alpha r_g}.$$

The cumulative distribution of π conditional on r_g is similar to the distribution of π given α — since α, r_g are distributed in the same manner — and is given by

$$F(\pi|r_g) = P(\alpha r_p r_g \leq \pi) = P\left(\alpha r_p \leq \frac{\pi}{r_g}\right) = \frac{\pi}{r_g} - \frac{\pi}{r_g}\ln\left(\frac{\pi}{r_g}\right),$$

with corresponding pdf

$$f(\pi|r_g) = \frac{d}{d\pi}\frac{\pi}{r_g} - \frac{\pi}{r_g}\ln\left(\frac{\pi}{r_g}\right) = \frac{-\ln(\frac{\pi}{r_g})}{r_g}.$$

The distributions of α and r_g are independent of each other, therefore $h(\alpha|r_g) = h(\alpha) = 1$. Using this, we can apply Bayes' Theorem to determine the distribution of α, π, r_g as

$$h(\alpha|\pi, r_g) = \frac{f(\pi|\alpha, r_g) \cdot h(\alpha|r_g)}{f(\pi|r_g)} = \frac{\frac{1}{\alpha r_g} \cdot 1}{\frac{-\ln(\frac{\pi}{r_g})}{r_g}} = \frac{-1}{\alpha \ln(\frac{\pi}{r_g})}.$$

The highest r_p can be is 1, meaning the lower bound for α is $\frac{\pi}{r_g}$ for given π, r_g. Our new conditional expectation is now given by

$$E[\alpha|\pi, r_g] = \int_{\alpha=\frac{\pi}{r_g}}^{1} \alpha h(\alpha|\pi, r_g) d\alpha = \int_{\alpha=\frac{\pi}{r_g}}^{1} \frac{-1}{\ln(\frac{\pi}{r_g})} d\alpha = \frac{\pi - r_g}{r_g \ln(\frac{\pi}{r_g})}.$$

Recall, the government's objective function with respect to the expectation is

$$\underset{x=\{\pi,(\pi,r_g)\}}{\text{minimize}} \quad \Big| E[E[\alpha|x] - \alpha] \Big|.$$

Regardless of whether they are revealed to admissions committees, r_g and π are both observable variables. $f(\alpha|\pi, r_g)$ gives a more accurate distribution of α than $f(\alpha|\pi)$ since r_g is also a component of π and $f(\alpha|\pi, r_g)$ accounts for the variations in r_g.

$E[y] = [E[y|x]]$ via the law of total expectation. Using this we can calculate that

$$\begin{aligned} E[E[\alpha|\pi, r_g] - \alpha] &= E[E[E[\alpha|\pi, r_g] - \alpha|\pi, r_g]] \\ &= E[E[\alpha|\pi, r_g] - E[\alpha|\pi, r_g]] \\ &= E[0] \\ &= 0. \end{aligned}$$

On the other hand, if r_g is kept hidden we have

$$\begin{aligned} E[E[\alpha|\pi] - \alpha] &= E[E[E[\alpha|\pi] - \alpha|\pi, r_g]] \\ &= E[E[\alpha|\pi] - E[\alpha|\pi, r_g]] \\ &= E[\alpha|\pi] - E[\alpha|\pi, r_g] \\ &\neq 0. \end{aligned}$$

To see the effect of revelation graphically, first consider that, for fixed r_g

1. $S(\pi) = L, S(\pi, r_g) = L'$ for some $L, L' \in [0,1]$. These are the cutoff values for admission that generate admitted students of mass k.

2. $S_\alpha(\pi), S_{r_p}(\pi), S_\alpha(\pi, r_g), S_{r_p}(\pi, r_g)$ are all continuous and differentiable, a fact that will be shown explicitly below.

3. $S_{r_p}(\pi), S_{r_p}(\pi, r_g) = 0$. Below we will see this always true, except when $\alpha r_g = t = 0$. If this is the case, though, the college cares only about π and the student will have $\pi = 0$, thus they will only be accepted if every student is admitted. For now, we can focus only on the case when at least one of these values is positive.

Thus, the implicit function guarantees that there is an implicit curve of S in (α, r_g) space. The derivative $\frac{\partial r_p}{\partial \alpha}$ is computed for both $S(\pi)$ and $S(\pi, r_g)$ below:

1. $S(\pi) = (1-t)\alpha r_p r_g + t\left(\frac{-2}{\ln^2(\alpha r_p r_g)}(\ln(\alpha r_p r_g) + 1 - \alpha r_p r_g)\right)$

$$S_\alpha(\pi) = (1-t)r_p r_g + t\left(\frac{2}{\alpha\ln(\pi)} + \frac{4}{\alpha\ln^3(\pi)} + \frac{2r_g r_p(\ln(\pi) - 2)}{\ln^3(\pi)}\right)$$

$$S_{r_p}(\pi) = (1-t)\alpha r_g + t\left(\frac{2}{r_p\ln(\pi)} + \frac{4}{r_p\ln^3(\pi)} + \frac{2\alpha r_p(\ln(\pi) - 2)}{\ln^3(\pi)}\right)$$

$$\frac{\partial r_p}{\partial \alpha} = \frac{-S_\alpha(\pi)}{S_{r_p}(\pi)} = \frac{-r_p}{\alpha}$$

2. $S(\pi, r_g) = (1-t)\alpha r_p r_g + t\left(\frac{\alpha r_g r_p - r_g}{r_g \ln\left(\frac{\alpha r_p r_g}{r_g}\right)}\right)$

$$S_\alpha(\pi, r_g) = (1-t)r_p r_g + t\left(\frac{r_p(\ln(\alpha r_p) - 1}{\ln^2(\alpha r_p)} + \frac{1}{\alpha\ln^2(\alpha r_p)}\right)$$

$$S_{r_p}(\pi, r_g) = (1-t)\alpha r_g + t\left(\frac{\alpha(\ln(\alpha r_p) - 1}{\ln^2(\alpha r_p)} + \frac{1}{r_p\ln^2(\alpha r_p)}\right)$$

$$\frac{\partial r_p}{\partial \alpha} = \frac{-S_\alpha(\pi, r_g)}{S_{r_p}(\pi, r_g)} = \frac{-r_p}{\alpha}$$

When given information about r_g, evaluators will adjust the cutoff value, effectively raising the standards for those with high values in r_g and lowering them for those with lower r_g. The effect is a shift in the level set in (α, r_g) space that represents the admissions cutoff, resulting in more students with lower r_g being admitted at the expense of students with higher r_g.

This shift can be seen in Figure 3 and Figure 4 below. Figure 3 shows the case for applicants with high r_g. The curve $S_i(\pi)$ represents the level set of S in (α, r_p) that is the cutoff for admission for this fixed r_g in the case when r_g is not revealed. The curve $S_i(\pi, r_g)$ shows the case when r_g is revealed. For high values of r_g, the standards for admission are higher and fewer students are admitted in the case when r_g is accounted for than in the baseline case — the region A is the group of students excluded with the intervention that are admitted in the baseline case.

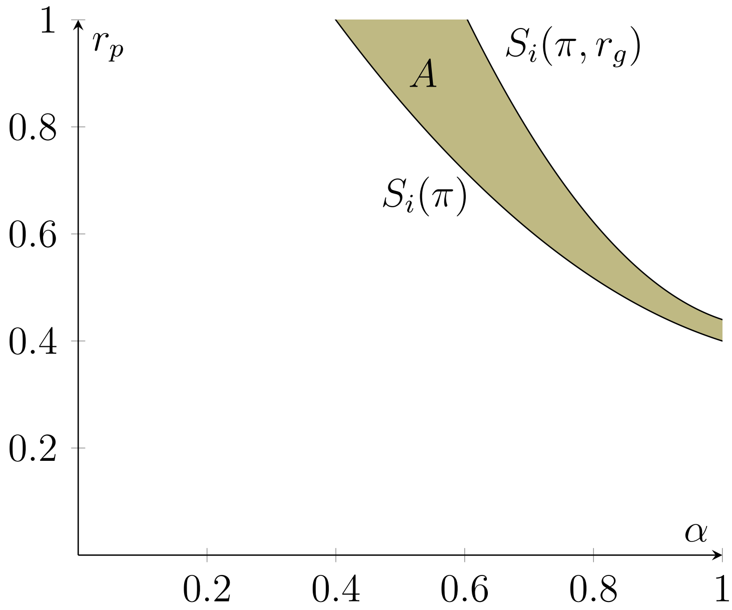

Figure 3

Figure 4 below shows the effect that revelation has on individuals with low values of r_g. The capacity of college isn't affected by the revelation and they accept fewer students with high r_g, meaning the college will accept more students with low r_g. This is shown by area B in Figure 4. This is the group of students that is accepted for this fixed, low r_g with revelation of r_g that are not admitted in the baseline case.

For some fixed value of r_g, the level sets of $S(\pi)$ and $S(\pi, r_g)$ perfectly align.

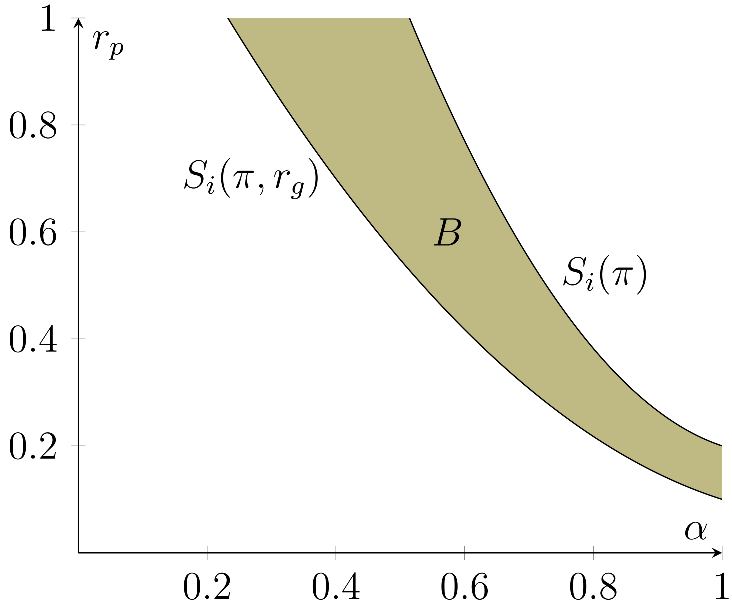

Figure 4

For all values r_g above this, the effect of revelation is the same as in Figure 3, while the effect resembles Figure 4 for any value below this r_g. Thus, the revelation of r_g results in more students with low values of r_g being accepted at the expense of some students with high r_g, which is an improvement on the government's goal of minimizing the effect of r_g in admissions.

7.2 Discussion on Redistribution

Given the analysis above, we will examine the effect of a redistribution of resources in addition to the revelation of r_g. This will adjust the parameters r_p, r_g through the taxation and apportionment of the initial endowments. A priori, adjustments made to r_g will have little impact on $E[\alpha|\pi, r_g]$, since the level of r_g is accounted for in the prediction. However, if the government taxed r_p at a constant rate δ, using funds to distribute to r_g, it might be able to reduce the stochastic noise in π generated by variations in r_p. In this case, the total tax accumulated would be

$$T = \int_0^1 \delta r_p dr_p.$$

Though r_p will remain uniformly distributed after the tax, this will constrict the distribution of r_p to $[0, (1-\delta)]$,. The revenue collected through these taxes will be redistributed into r_g by uniform amount τ, shifting the bounds of r_g to $[\tau, 1+\tau]$. This redistribution does not affect α, which will still be uniformly distributed on

$[0, 1]$.

The new statement of the signaling function is

$$\pi' = \alpha r_p' r_g' \text{ where } \alpha \sim U[0,1], r_g' \sim U[\tau, 1+\tau], r_p' \sim U[0, 1-\delta].$$

To calculate the distribution of π', we start by computing the cdf of $\alpha r_p'$ as

$$P(\alpha r_p' \leq z) = \int_0^1 P\left(r_p' \leq \frac{z}{\alpha}\right) f_\alpha(\alpha) d\alpha = \int_0^{\frac{z}{1-\delta}} d\alpha + \int_{\frac{z}{1-\delta}}^1 \frac{z}{\alpha} d\alpha$$

$$= \frac{z}{1-\delta} - z \ln\left(\frac{z}{1-\delta}\right).$$

Using this we can calculate the cumulative conditional distribution of π' given r_g'

$$F'(\pi|r_g) = P\left(\alpha r_p' \leq \frac{\pi'}{r_g'}\right) = \frac{\pi'}{r_g'(1-\delta)} - \frac{\pi'}{r_g'} \ln\left(\frac{\pi'}{r_g'(1-\delta)}\right)$$

with corresponding pdf

$$f'(\pi'|r_g') = \frac{d}{d\pi'} F'(\pi'|r_g') = \frac{1}{r_g'(1-\delta)} - \frac{1 + \ln\left(\frac{\pi'}{\alpha(1-\delta)}\right)}{\alpha.}$$

Conditioning further on α gives

$$F'(\pi'|r_g', \alpha) = P\left(r_p' \leq \frac{\pi'}{r_g' \alpha}\right) = \frac{\pi'}{(1-\delta)\alpha r_g'},$$

with pdf

$$f'(\pi'|r_g', \alpha) = \frac{d}{d\pi'} F'(\pi'|r_g', \alpha) = \frac{1}{(1-\delta)\alpha r_g'.}$$

Since α and r_g' are distributed independently, $h(\alpha|r_g) = 1$. By Bayes' Theorem the conditional distribution of α given π' is

$$h(\alpha|\pi', r_g') = \frac{f'(\pi|r_g, \alpha) \cdot h(\alpha|r_g)}{f'(\pi|r_g')} = \frac{\frac{1}{(1-\delta)\alpha r_g'}}{\frac{1}{r_g'(1-\delta)} - \frac{1+\ln\left(\frac{\pi'}{\alpha(1-\delta)}\right)}{\alpha}} = \frac{1}{(1-\delta) r_g' \left(1 + \ln\left(\frac{\pi'}{\alpha(1-\delta)}\right)\right)}.$$

This distribution allows us to calculate the expectation of α given pi', r_g,

$$E[\alpha|\pi', r_g^{'}] = \int_0^1 \alpha \cdot h(\alpha|\pi', r_g^{'}) d\alpha = \frac{\pi' - r_g^{'}(1-\delta)}{r_g^{'}(1-\delta)\ln\left(\frac{\alpha r_p^{'}}{1-\delta}\right)}.$$

Notice that, because $r_p^{'} = r_p(1-\delta)$

$$\frac{\pi' - r_g^{'}(1-\delta)}{r_g^{'}(1-\delta)\ln\left(\frac{\alpha r_p^{'}}{1-\delta}\right)} = \frac{\alpha r_p - 1}{\ln(\alpha r_p)} = \frac{\pi - r_g}{r_g \ln\left(\frac{\pi}{r_g}\right)}.$$

Thus, the redistribution of r_p into r_g has no effect on the admissions committees' prediction of aptitude for prospective students. Since this redistribution does not affect the rank of resource endowments — students have the same level of wealth relative to their peers — this won't have any effect on the colleges' evaluation of candidates. Though this will likely change the levels of π achieved by students, admissions committees can simply adjust the specific weight of t to make up for absolute differences. Thus, this redistribution has no effect when r_g is already known.

8 Limitations and Further Research

In the model, the distribution of α is independent of the distributions of resources. This reflects the assumption that aptitude is given at birth and doesn't change throughout one's life. Arguments that the resources available to a child in adolescence permanently improve aptitude — such as the work done by Lynn and Vanhanen (2006) [6]— may cast doubt on these assumptions about α. Though aptitude is not simply intelligence quotients or SAT Scores but, rather, a measure of how efficient a student is with his or her resources, it is possible that these elements, too, can be dynamic with respect to initial resources.

A more sophisticated analysis of the interventions discussed here would determine the effect of revelation and redistribution on $E[\alpha|A]$, where A is the region of students accepted into a given school. Unfortunately, the form of the model considered did not allow for a precise determination of these values or the effect that the interventions have on them. Intuitively, improving the efficiency of $E[\alpha]$ will help the government better achieve its match-making objective, given that colleges weigh the predicted aptitude of students more heavily than their actual level of preparedness. Despite this, this paper would undoubtedly be improved by a proper evaluation of the effects of the interventions on $E[\alpha|A]$.

Finally, as with any economic model, the functional form of π can be disputed. The choice to treat the variables as complements, as well as the choice of variables themselves are not incontrovertible. Factors such as student effort and community safety and well-being were included in the variables α and r_g, respectively. Other variables were omitted entirely, such as a student's ability to excel academically — sometimes, highly gifted individuals may excel in other domains but still struggle (even with strong levels of effort and resource) in academic settings. Further, I assumed r_g and r_p are distributed independently, which may not be the case — as wealthy parents are likely to move to areas with better community resources. Parsing out the implicit variables and including the omitted ones may provide a more robust production function. I argue that the model considered in this paper is an excellent start to explaining the college admissions process but am aware that improvements can be made.

9 Conclusion

The college admissions process has important implications not only for applicants seeking to gain entry into institutions of higher learning but also for their community at large. At its best, higher education provides talented, hardworking students the necessary framework for lifelong success and social contribution, regardless of the resources available to them in adolescence. As it stands the admissions process benefits wealthy students — specifically, those who are born into affluent families and communities. Students with exceptional parental and government resource can signal a higher level of aptitude to college admissions committees than those less well off, which helps affluent students achieve a higher rank than less fortunate students with higher innate ability. For many reasons, the government has incentive to reverse this trend and enact policies that help match the most talented students with the best colleges. I have examined the effect of two possible interventions. First, I argued that governments revealing the precise level of government resources help colleges generate more accurate expectations of aptitude and thus improve the ability of schools to select the most talented students. Second, I suggest that — if a student's communal resources are revealed to admissions committees — a certain redistribution of parental resources toward communal resources has no positive effect on achieving efficiency on α. If properly enacted, the revelation of information has the power to reduce the inequality and inefficiency of the college admissions process.

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