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Stock Market Bubbles: Effects on Fixed Investment and Financial Market

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Doctor of Philosophy

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M.A., Emory University, 2007

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An abstract of

A dissertation submitted to the Faculty of the Graduate School of Emory University

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Abstract

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Previous studies on stock market bubbles have developed theoretical models showing that the stock market bubble is a determinant of the stock price, and the presence of bubbles is also supported by empirical evidence. Based on these results, this dissertation further explores the effects of stock market bubbles on fixed investment and financial market.

The first research question focuses on testing the hypothesis that stock returns are more sensitive to investor sentiment during stock market crashes than during stock market booms. The empirical results confirm that sentiment betas are asymmetric across stock market cycles.

The next research question aims to examine the dynamic effects of stock market misvaluation on firm fixed investment. We apply a Bayesian vector autoregression (BVAR) model to calculate the impulse response function of investment to misvaluation shock. And we find that investment responds 47%-55% at maximum annually to one standard deviation of misvaluation.

Finally, we address why stock return volatility is typically higher after the stock market falls than after it rises (referred as asymmetric volatility). By maximizing the likelihood functions of dividends and stock reruns from a quadratic generalized autoregressive conditional heteroscedasticity (QGARCH) model, we decompose this asymmetric volatility into the volatility feedback effect due to dividend news, and the bubble effect explained by bubble news.

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It has been an extraordinary journey since I joined Emory economics Ph.D. program in 2003. I have taken away a lot of treasures from my Ph.D. education, such as solid background in economics and finance, more importantly, the ability of critical and creative thinking, and the capability of conducting research independently. I believe these precious abilities will well prepare me for my future career development. As I approach to the finishing line of this amazing journey, I would like to take this opportunity to express my acknowledgement to those who helped me to get to this point.

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Chapter 1: INTRODUCTION

Stock prices frequently deviate from the fundamentals--the summations of discounted prospective dividends. Some studies argue these deviations are due to the inappropriate specifications of dividends and discount. However, Shiller (2003) and LeRoy (2004) find the deviations do not disappear no matter how dividends and discount rates are specified. Therefore, this phenomenon is inconsistent with the efficient market theory, which argues that stock prices are determined by the fundamentals. The inability of fundamental-based models to fully capture stock price movements has inspired the search for factors beyond the fundamentals. Stock market bubbles, which are the differences between stock market prices and their fundamentals, have been of great interest in recent asset pricing research. Previous studies on stock market bubbles have developed theoretical models showing that the stock market bubble is a determinant of the stock price, and the presence of bubbles is also supported by empirical evidence. Based on these results, this dissertation further explores the roles that bubbles play in the economy. More specifically, the focus of this dissertation is to examine the effects of stock market bubbles on fixed investment and financial market.

The rest of this dissertation is composed of three chapters, chapter 2 to chapter 4. The second chapter “Does Investor Sentiment Have a Larger Effect on Bear Markets or Bull Markets?” tests the hypothesis that stock returns are more sensitive to investor sentiment during stock market crashes than during stock market booms, which is confirmed by the empirical results. This hypothesis is developed based on two lines of studies. First, it is motivated by the theoretical and empirical evidence that investor sentiment is a determinant of stock prices. The second motivation is loss aversion, a feature of prospect theory developed by Kahneman and Tversky (1979). This loss aversion shows that investors are more sensitive to losses than gains, indicating that investors are more risk averse during stock market crashes. Therefore, it may be reasonable to conjecture that investor sentiment risk is compensated more during stock market crashes than booms.

To test this hypothesis, the two-stage least squares (2SLS) method is applied to estimate the coefficient of stock returns on investor sentiment for each individual stock, which is referred to as sentiment beta since it captures the sensitivity of stock returns to investor sentiment. The empirical results show that the magnitudes of sentiment betas are larger during stock market crashes than stock market booms, which is consistent with the hypothesis. These results are robust regardless of how stock market cycles are identified and how investor sentiment is measured.

The third chapter "Fixed Investment and the Stock Market: Evidence from BVAR Models (With Yan Liu)" is motivated by the observation that more stocks are issued and more money is invested in firm fixed investment as stock prices go higher. This brings up an interesting question: Does the stock market matter in firm fixed investment decision making or is it only a sideshow? This chapter examines the effects of the stock market on fixed investment. Previous studies on this issue have conducted regression analysis, providing insights into how fixed investment responds to misvaluation contemporaneously. However, how persistent the effect is and when the peak response occurs could not be answered by regression methods. To capture this dynamic effect, this essay applies Bayesian vector autoregression (BVAR) models to calculate the impulse response of investment to misvaluation shocks. As a result, this BVAR method keeps track of the complete responses of investment to misvaluation.

The empirical results under different priors show that: 1) investment responds 47%-55% at maximum annually to one standard deviation of misvaluation; 2) the peak response occurs in the first year and starts to diminish afterwards; and 3) the effect lasts persistently for at least 5 years at a relatively high level. A key implication of these results is that timing plays an important role in linear regression of investment on misevaluation.

Based on the empirical results of this paper, it is better to include current and 2 lagged misvaluations in the linear regression to capture the dynamic effect of misvaluation on investment.

Similarly, chapter four “Stock Market Bubbles, Fundamentals, and Volatility Asymmetry” is inspired by the stylized fact that stock return volatility is typically higher after the stock market falls than after it rises, referred to as volatility asymmetry. This indicates that today’s stock returns are negatively correlated with tomorrow’s volatility. This chapter studies what drive the negative relationship between stock returns and their volatility.

This Previous literature has attributed this asymmetry to the leverage effect and volatility feedback effect. While the leverage effect argues that this asymmetry is due to the change of leverage ratio, and the volatility feedback effect shows that it is the results of dividend news, empirical results find both effects cannot fully account for this asymmetric volatility. Inspired by the evidence that the stock price is composed of both fundamental and bubble, this paper introduces stock bubble news into the volatility feedback effect model developed by Campbell and Hentschel (1992), shedding some light on the asymmetry of volatility by providing a bubble based explanation.

The model in this paper decomposes stock returns into three parts: expected stock returns, the volatility feedback effect which captures the effect of dividend news on returns, and the bubble effect defined as the effect of bubble news on returns.

Based on this decomposition, the correlation between stock returns and volatility becomes a function of dividend news, bubble news, and their respective volatility. Estimated using the maximum likelihood method, the empirical results show that 1) the larger the news is, the more negative the correlations are for both the volatility feedback effect and bubble effect; 2) the correlations due to the volatility feedback effect are much smaller than those due to the bubble effect, which account for more than 90% of the total effect on average; 3) when both dividend news and bubble news are present, the bubble effect dominates the volatility feedback effect because the bubble news is much larger than dividend news; and 4) despite the relatively small magnitude of the volatility feedback effect, it has a very significant impact on the correlations accounting for about 20% during stock market crashes.

Chapter 2: Does Investor Sentiment Have a Larger Effect on Bear Markets or Bull Markets?

2.1 Introduction

There is mounting psychological evidence indicating that investors are subject to systematic judgment and decision bias. Under this bias, investors could be either optimistic or pessimistic. Investor sentiment, which measures investor moods, has been taken into account as a determinant of asset prices in theoretical models, and examined empirically in asset pricing. The studies in investor sentiment have increased our understanding of some anomalies that are inconsistent with fully rational models of asset pricing. For example, Lee, Shleifer and Thaler (1991) show that investment sentiment could explain the size effect, why small cap stock returns are higher than the returns of large cap stocks. Barberis, Shleifer and Vishny (1998) find investor sentiment causes the underreaction of stock prices to earning announcements and overreaction to a series of good or bad news. Baker and Wurgler (2006) prove that investor sentiment could be used to predict stock returns. Baker and Wurgler (2006) and Glushkov (2006) find that the effects of sentiment on stock returns vary with firm characteristics. Specifically, companies that are younger, smaller, more volatile, unprofitable, non-dividend paying, distressed or having analogous characteristics are more sensitive to investor sentiment than other companies, indicating asymmetric sensitivity of stock returns to investor sentiment. Instead examining the asymmetry across stocks, this paper studies the asymmetric effect of sentiment on stock returns across time, aiming to find whether there is significant difference in the sensitivity of stock returns to investor sentiment during stock market booms and crashes.

To investigate whether there exists asymmetric effect of investor sentiment on stock returns in stock market cycles, the hypothesis developed in this paper postulates that stock returns are more sensitive to investor sentiment during stock market crashes than booms. This hypothesis is developed based on two lines of studies. One is motivated by the theoretical and empirical evidence that investor sentiment is priced in stock prices found by De Long, Shleifer, Summers and Waldmann (1990) (hereafter DSSW) and Lee, Shleifer and Thaler (1991) among others. The other is motivated by loss aversion, a feature of prospect theory developed by Kahneman and Tversky (1979). This loss aversion shows that investors are more sensitive to losses than gains, indicating investors are more risk averse during stock market crashes. Therefore, it maybe reasonable to conjecture that investor sentiment risk is compensated more during crashes than booms.

Before testing this hypothesis, two problems need to be solved. One is to identify stock market cycles. Based on previous studies on stock market cycles identification, we use two methods to define stock market booms and crashes. The first is developed by Pagan and Sossounov (2003), which labels the stock market into two categories, bull or bear market. The second method combines the studies of Mishkin and White (2002) and Bordo and Wheelock (2006). This method divides the stock market into three categories: stock market booms, stock market crashes and neutral markets, which are neither stock market booms nor crashes.

The other challenge is to measure investor sentiment using some proxies, since sentiment is not straightforward to measure. Based on previous studies, we choose two sentiment proxies to serve as sentiment indicators. The first is closed-end-fund discount (CEFD), and the other is a composite sentiment indicator. Closed-end-fund issues a fixed number of shares, and the only way investors can liquidate their positions is to trade on the stock market. It has been found that CEFD, the difference between the net asset value of closed-end-fund and the market price, widens in bear markets and becomes narrower in bull markets, thus, making it a popular proxy for investor sentiment. To extract common information from other sentiment proxies, the composite sentiment is constructed using the first principle component of six sentiment proxies: closed-end-fund discount, market turnover, volume of the initial public offerings (IPO), the first-day return on IPO, the dividend premium, and the equity share over total new shares.

In the empirical part, we first use the two-stage least squares (2SLS) regression to estimate the coefficient on investor sentiment for each individual stock by regressing stock returns on sentiment during stock market booms and crashes. These coefficients on sentiment are defined as sentiment beta, which capture the sensitivity of stock returns to sentiment. Next, these individual sentiment betas are sorted into ten deciles for all sentiment betas, positive and negative sentiment betas respectively. The empirical results show that the magnitudes of sentiment betas are larger during stock market crashes than stock market booms. Therefore, the hypothesis that stock returns are more sensitive to sentiment in bear markets than in bull markets is confirmed.

To make sure this asymmetry is robust, we then apply the same model to Russell small-cap RUI and large-cap index RUT daily data, and choose the CBOE (Chicago Board Option Exchange) implied volatility index VIX as a sentiment proxy. Similar to the individual monthly stock returns, both daily RUT and RUI returns respond to investor sentiment proxy VIX asymmetrically. Another robust check is to examine whether similar asymmetry exists across the other risk factors as shown in the multifactor model. The empirical results are mixed, with higher sensitivity of stock returns to SML (the difference between the return on portfolio of small stocks and the return on a portfolio of large stocks) across all the deciles during stock market crashes than booms, and with some exceptions to market returns and HML (the difference between the return on portfolio of high-book-to-market stocks and the return on a portfolio of low-book-to-market stocks HML) in some deciles.

The rest of the paper is organized as follows. Section 2 presents the hypothesis that stock returns are more sensitive to investor sentiment during stock market crashes than during stock market booms. Section 3 uses two different identification methods to categorize the stock market into two groups, bull or bear markets, or three groups, stock market booms, stock market crashes and neutral markets. Section 4 describes two different ways to proxy investor sentiment. The data description and estimation method are provided in section 5, and section 6 presents the empirical results. In section 7, we check whether the asymmetry found for individual monthly data is robust for index daily data. The asymmetry across other risk factors is also examined. Section 8 concludes the paper.

2.2 The hypothesis

The investigation of this paper is motivated by two lines of research. One is the theoretical reasoning and empirical evidence on the role investor sentiment plays in stock returns. De Long, Shleifer, Summers and Waldmann (1990) develop a theoretical model in which stock prices are affected by irrational noise traders, defined as traders with erroneous stochastic beliefs. Noise traders trade on noise, namely some pseudo signals they get from technical analysis and economic media, and irrationally believe this noise carry information about the future price of stock. Thus, noise trader risk or investor sentiment is defined as the subject expectation about stock returns not warranted by fundamentals. The theoretical model developed by DSSW shows that it is costly and risky for arbitragers to bet against noise traders because of the unpredictability of investors' stochastic sentiment. As a result, stock prices could deviate from fundamentals significantly, and noise traders could survive and earn higher return than arbitrageurs by bearing more sentiment risk. Shleifer and Vishny (1997) provide further theoretical evidence that arbitrage is limited from the perspective of the agent problem faced by institutional arbitrageurs, which indirectly supports DSSW's reasoning. Therefore, arbitrages could be limited when arbitrageurs are risk-averse, leveraged and run the risk of losing fund under management in case of poor performance. This costly and risky arbitrage has been taken into account in theoretical asset pricing models, where sentiment risk is priced into stock prices.

Consistent with this definition in DSSW model, in empirical studies, investor sentiment is often defined as a measure of investor public mood which could not be explained by fundamentals.The role of investor sentiment in the stock market has also been explored empirically. Lee, Shleifer and Thaler (1991) find that both closed-end funds and small stocks tend to be held by individual investors, and that investor sentiment affects discounts on closed-end funds in the same way that it affects small firm returns. Specifically, the discounts on closed-end funds narrow when small stocks do well. Thus, they argue closed-end-fund discounts are a measure of the sentiment of individual investors. In addition, the size effect is due to that small cap stocks expose more to noise trader risk than large cap stocks. Neal and Wheatley (1998) also examine the ability of individual investor sentiment to predict returns. They find that discounts on closed-end funds predict the difference between small and large firm returns, as well as net mutual fund redemptions. Using GARCH in mean model, Lee, Jiang and Indro (2002) estimate the effect of noise trader risk on both conditional volatility and returns. Their empirical results show that excess returns are positively correlated with contemporaneous sentiment, and changes in sentiment lead to revision in volatility and future returns. Further, Brown and Cliff (2005) find that a direct survey measure of investor sentiment predicts market returns over the next 1-3 years, which provides support that sentiment affects asset valuation.

The other motivation is based on loss aversion, an important feature of Kahneman and Tversky's (1979) prospect theory. This theory uses experimental evidence to argue that people are more sensitive to losses than to gains. This is characterized by the shape of the value function, which is sharply kinked at the reference point and steeper for loss than for gains by a factor of about 2-2.5. Some later research extends this loss aversion developed in one-shot gambles to intertemporal framework by conducting experiments on how people evaluate sequences of gambles. Thaler and Johnson (1990) find that the degree of loss aversion depends on prior gains and losses. Specifically, a loss followed by prior gains is less painful than usual since it is cushioned by the earlier gains. However, a loss that comes after other losses is more painful than usual. Investors become more sensitive to additional loss after being burned by the previous ones.

Loss aversion is closely related to dual beta literature, which examines whether market betas differ in up-market with positive market excess returns and down-market where excess market returns are negative. Kim and Zumwalt (1979) find that the statistically significant differences between up-market and down-market betas exhibited by more securities do not occur by chance, which implies that investors do require a premium for taking on downside variation and do pay a premium for upside variation. Similarly, Howton and Peterson (1998) find that there is a significantly positive (negative) relationship between cross-sectional stock return and up market (down market) beta for the US market. Supportive evidence of dual beta is also found in other markets. Isakov (1999) finds significant different relationship between stock return and beta exists in the Swiss stock market over the period 1973 to 1991. Faff (2001) finds strong support for the dual-beta CAPM model for Australian stock market. Tang and Shum (2004) identify a similar pattern for Singapore market. And Chio and Tian (2005) claim that there exists only a significantly negative relationship between realized return and beta in the down markets; the relationship in the up markets is flat in New Zealand stock market.

Applying similar methodology in dual beta model, we regress 10 size portfolio stock returns on market excess returns for up-market and down-market respectively, and the results show that portfolio returns are mostly more sensitive to excess market returns in down-markets than in up-markets, which is consistent with Kim and Zumwalt (1979). The different responses of stock returns to market returns in up and down markets implies that stock returns may also respond to investment in different ways during stock market booms and crashes.

Although the hypothesis test of asymmetric sensitivity betas is similar to the dual beta models in terms that betas differ in different markets, the definitions of markets are very different. In dual beta models, up-market is defined as the excess returns are positive, and down-market is the market where the market return is lower than the risk free rate. However, in this paper, we are testing whether sentiment betas are different during stock market crashes and booms, which capture stock market cycles. By these definitions, there is up-market during stock market crashes, and down-market during stock market booms. Therefore, asymmetric sentiment beta test is different from dual beta models.

The most important characteristics that distinguish stock market booms from stock market crashes is that investors on average enjoy juicy profits in bull markets and suffer substantial losses in bear markets. Investors usually could make consistent money for about 2-5 years during each stock market boom, making them less risk averse. On the other hand, continuous losses are often observed during stock market crashes, which last 1-1.5 years on average. These losses are more unbearable on a continuous basis in bear market. As a consequence, investors are observed more risk averse during stock market crashes, and rational investors are less willing to run the risk of betting sentiment. Therefore, returns are expected to be disproportionately sensitive to investor sentiment during crashes than booms. Based on that sentiment risk is priced and that investors are more sensitive to losses over gains, we develop a hypothesis regarding the asymmetric effect of sentiment on stock returns. This hypothesis postulates that returns are more sensitive to sentiment during stock market crashes than stock market booms, or the magnitudes of coefficients on sentiment during market crashes are larger than those during market booms.

2.3 Specifications of stock market booms and crashes

To capture how stock returns respond to investor sentiment during stock market cycles, we first need to find a way to identify the stock cycles. Although there is no precise definition of stock market booms and crashes, previous research has imposed some stylized facts as filters to identify stock market cycles. Since there is no consensus of the definitions of stock market booms and crashes, we use two different specification methods that have been studied by previous literature. One is developed by Pagan and Sossounov (2003), which divides stock markets into two categories, bull market and bear market. The other method categorizes stock market into three groups, stock market boom, stock market crashes and neither booms or crashes based on the research of Mishkin and White (2002) and Bordo and Wheelock (2006).

Inspired by the business cycle recognition algorithm developed by Bry and Boschan (1971), Pagan and Sossounov (2003) adapt this algorithm for use in the stock market. The algorithm modified in Pagan and Sossounov (2003) works as follows: 1) Local peaks and troughs are identified as stock returns higher or lower than those on 8 months (window size) either side, and turning points are chosen as the highest of the multiple peaks or the lowest of the multiple troughs in one phase period (peak to trough or vise versa) whose minimal length is 4 months; 2) the complete cycle (peak to peak or trough to trough) must span at least 16 months; and 3) the minimal phase length constraint is ignored when the stock price falls by 20% in a single month.

Pagan and Sossounov (2003) perform the algorithm on monthly data for the equivalent of the S&P 500 for the USA over the years 1865/1-1997/5 and list the post-war US stock market cycles. Gonzalez, Powell, Shi and Wilson (2005) also apply the Bry and Boschan (1971) algorithm and present the peaks and troughs for USA stock price index running from January 1800 to September 2000, whose results are exactly the same as Pagan and Sossounov (2003) over the post-war period. Specifically, both identify 11 booms and crashes from 1966 to 2000, and the booms and crashes are continuous in the sense that each time period is either a stock market boom or crash.

Unlike the Bry and Boschan (1971) algorithm, describing stock markets as either bull markets or bear markets, another way is to categorize stock markets into three types, stock market booms, stock market crashes, and neutral markets based on the criteria used in Mishkin and White (2002) and Bordo and Wheelock (2006). Mishkin and White (2002) use the Dow Jones index from 1903 to 1940 and shift to the S&P 500 around 1946 when it is first reported. They look at stock price declines over windows of 1 day, 5 days, 1 month, 3 months and 1 year, and sort the percentage changes for each window and identify 15 largest declines which are over 20 percent to define stock market crashes in the 20th century. According to this specification, 5 episodes of crashes occur after 1966 including Nov. 1968-June 1970, Jan. 1973-Dec. 1974, Aug. 1987-Dec. 1987, Oct. 1989-Oct. 1990, and Aug. 2000-Dec. 2001. Bordo and Wheelock (2006) classify booms as all periods of at least three years from trough to peak with an average annual rate of increase in the real stock price index of at least 10 percent and a few episodes of exceptional real stock appreciation that were shorter than three years are also included as booms. Based on this definition, there are 5 stock market booms in the United States from the early 1920s onwards, which are Oct. 1923-Sept. 1929, Mar. 1935-Feb. 1937, Sept. 1953-Apr. 1956, June 1962-Jan. 1966, July 1984- Aug. 1987, and Apr. 1994-Aug. 2000. Table 2.1 lists all the stock market booms and crashes defined using these methods over the period from1966 to 2000.

Table 2.2 presents the summary statistics of stock market booms and crashes. Comparisons of booms and crashes, regardless of their identification methods, show that stock market booms on average last about 26-57 months, much longer than stock market crashes whose average duration is about 11-15 months. In terms of the comparison of stock market booms specified using different methods, the monthly return, cumulative return and duration are 3.1%, 177.74% and 57.5 months respectively for booms identified by Bordo and Wheelock (2006), which are larger than 2.22%, 52.18% and 26.91 months in booms specified by Pagan and Sossounov (2003). During the crashes identified by Pagan and Sossounov (2003), investors on average lose 2.69% per month and the total loss is 25.2% for each crash, which lasts 11.73 months. The returns are even worse in the crashes defined by Mishkin and White (2002) in terms of longer duration and higher total loss for each crash. The comparisons between these two identification methods show that stock market booms identified by Bordo and Wheelock (2006) and stock market crashes specified by

Mishkin and White (2002) on average last longer and investors enjoy more gains during booms and suffer more losses during crashes compared to the stock market cycles defined by Pagan and Sossounov (2003). Since the characteristics of stock market cycles are more distinguished using the criteria of Mishkin and White (2002) and Bordo and Wheelock (2006), we conjecture that the difference between stock returns sensitivity of sentiment during stock market crashes and booms is larger than that using Pagan and Sossounov (2003) method.

2.4 Sentiment measures

Since investor sentiment is not straightforward to measure, some proxies for sentiment are needed to capture whether investors are optimistic or pessimistic about the stock market in general. Some previous research has used direct investor surveys, indirect measures which are observed in stock trading or composite index to investigate the effect of sentiment on stock market. So far, there are two popular sentiment surveys. The first one is Investor Intelligence Index surveyed on the outlook of over 100 market newsletter writers conducted by Investor Intelligence of New Rochelle (see Lee, Jiang, and Indro (2002) and Brown and Cliff (2005) among others), and it is available from 1963 on a biweekly basis and weekly basis from 1969. The other is a survey targeted towards individuals conducted by the American Association of Individual Investors (Brown and Cliff, 2004), which started from 1987 on a weekly basis.

The indirect sentiment measures are observed via investor trading activities. The sentiment can be captured by all kinds of indicators, including indicators extracted from funds activities such as closed-end-fund discount (CEFD) and mutual fund flow; indicators observed in firms equity offering market such as return of first-day initial public offerings, volume of initial public offerings and equity issues over total new issues; option implied volatility interpreted from option market; trading volume, insider trading and dividend premium observed from stock market; and some other proxies not discussed in this paper. We describe some commonly used sentiment indicators as follows.

The most popular proxy for sentiment is closed-end-fund discount (CEFD). Closed-end-fund issues a fixed number of shares that are traded on the stock market. However, unlike an open-end fund, which investors can redeem with the funds to liquidate their holding, closed-end-fund investors can only liquidate their positions by trading on the stock market. Since closed-end funds are primarily held by retail investors, the difference between the net asset value of closed-end-fund and the market price should be able to serve as a sentiment indicator of individual investors. If the market price of a closed-end-fund is lower than its net asset value, the difference is referred as closed-end-fund discount. The closed-end-fund discount is often observed when investors are pessimistic. However, if the market price of a closed-end-fund is higher than its net asset value, the difference becomes a premium. This phenomena is often observed in bull markets. Therefore, the closed-end-fund discount is a good proxy for investor sentiment.

Baker and Stein (2004) find market liquidity can capture investor sentiment. Specifically, under short-sales constraint, high liquidity is a sign that the sentiment of these irrational investors is positive. Therefore, market turnover (TURN), the ratio of trading volume to the number of shares listed on the New York Stock Exchange, is a simple indicator of sentiment.

Investor sentiment is also reflected in firms' initial public offerings (IPO).

Average first-day returns (RIPO) and the number of IPO (NIPO) have been observed to be highly correlated with investor sentiment. Therefore, high RIPO and NIPO are interpreted as a symptom of investors being enthusiastic about the market, and low IPO returns and volume as a signal of investors being bearish.

The dividend premium (PDND), the log difference between the average market-to-book-value ratios of dividend payers and nonpayers, also contains information about investor sentiment. When investors experience high sentiment, they value dividend non-paying firms, which in general are young, high growing firms, more than the dividend paying firms such as large, profitable with weaker growth opportunities. This produces relatively high demand, thus higher stock prices for dividend nonpayers than dividend payers, thus lowers the dividend premiums. The higher the investor sentiment is, the lower the dividend premiums become. Baker and Wurgler (2004) suggest that dividend premium can serve as another proxy for sentiment.

The equity issue over total new issues is defined as the gross equity issuance divided by the summation of equity and long-term debt issuance. It is well known that firms are more likely to issue equity at high prices and repurchase it at low prices to exploit the mispricing in the stock market. Baker and Wurgler (2002) find the market timing of equity issue has a persistent effect on firms' capital structure. Specifically, high sentiment is correlated with high ration of equity issue to total new issue and vice versa.

To combine these various sentiment indicators and extract common features of the proxies, Brown and Cliff (2004) apply the principle component analysis, which effectively decreases data dimensions to one by doing first principle component or two by using the second principle component analysis. Similar to Brown and Cliff

(2004), Baker and Wurgler (2006) construct a monthly composite sentiment change index over the period January 1966 to December 2005 using the first principle component of the six indirect sentiment proxies, including close-end-fund discounts, New York Stock Exchange turnover, IPO volume, first-day average returns on IPO, the dividend premium and the equity issue over the total issue. Equation 2.1 and eqn. 2.2 specify investor sentiment SENT and sentiment change $\Delta SENT$.

$$SENT = -0.23CEFD + 0.23TURN + 0.24NIPO + 0.29RIPO - 0.32PDND + 0.28\Delta S \quad (2.1)$$

$$\Delta SENT = -0.17\Delta CEFD + 0.32\Delta TURN + 0.17\Delta NIPO + 0.41\Delta RIPO - 0.49\Delta PDND - 0.28\Delta S \quad (2.2)$$

In this paper, we choose two sentiment indicators to test our hypothesis that stock returns are more sensitive to investor sentiment during stock market crashes than during booms. One is closed-end-fund discount CEFD, the other is the composite sentiment indicator used in Baker and Wurgler (2006). Figure 1 plots these two sentiment indicators together. First, this figure shows that the closed-end-fund discounts are negatively correlated with the composite sentiment index, with higher discounts during low sentiment periods, and close-end-fund premium during high sentiment. Moreover, these sentiment indictors line up with the anecdotal stock market booms and crashes identified in section 3.

Specifically, during the stock market crash over the period of November 1968 to June 1970, while the stock market dropped 30.6%, the composite sentiment started its downward trend and the close-end-fund premiums shrank. Then the stock market lost 45.7% value during January 1973 to December 1974, one of the longest and largest stock market collapses, SENT continued its downward trend and stayed negative while CEFD kept on increasing, resulting in widened discounts. Next, the stock market witnessed a high sentiment period due to the high-tech and biotech booms from July 1984 to August 1987, when the stock price gained about 120%. This high-tech bubble started to burst in August and did not end until December 1987. During this period, the stock market dropped 26.8% while SENT was low and CEFD remained a local high. Following this collapse was another one, which lasted from October 1989 to October 1990, and by then the market had fallen by 28%, and SENT remained negative while CEFD positive. After two crashes, the market enjoyed another uptrend ride starting from April 1994 and ending in August 2000, brought on by the internet boom. The SENT was high and developed in the rising tunnel, while CEFD was relatively low, and the market return was about 237% during this period. The last identified crash was seen from August 2000 to December 2001. Again the market gave up its value by 22.9%, and both SENT and CEFD reversed their trends. Similar to the analysis of Mishkin and White (2002) stock market crash and Bordo and Wheelock (2006) boom classifications, the behaviors of SENT and CEFD also line up with Pagan and Sossounov (2003) bull and bear market identifications.

Table 2.3 reports the summary statistics of the sentiment indicators CEFD, SENT and $\Delta SENT$ for stock market booms and crashes respectively over the period of January 1966 to December 2005. Pagan and Sossounov (2003) bull and bear market identifications show that SENT on average increased by 0.07 monthly during booms and dropped 0.15 during crashes. CEFD is positively skewed during booms and the median is 9.93, smaller than 10.48 during crashes, which is negatively skewed. The fact that both the mean and median of SENT are negative during booms and positive during crashes does not mean that SENT fails to capture the sentiment. In fact, it reflects that sentiment is on the upward trend in bull markets and downward trend in bear markets. Similarly, applying Mishkin and White (2002) stock market crash and Bordo and Wheelock (2006) boom specification, SENT drops 0.18 monthly when markets are bearish and goes up 0.03 while markets are bullish. SENT is 0.49

during crashes, still larger than 0.3 during booms. However, we see that the median of SENT in bull market is 0.26, much larger than 0.02 in bear markets. In sum, most of the statistics of sentiment indicators again show evidence that they capture the stock market public mood.

Finally, we investigate the relationship between investor sentiment and consumer confidence. The University of Michigan Consumer Sentiment Index is a consumer confidence index published monthly by the University of Michigan. This index provides a near time assessment of consumer attitudes on the business climate, personal finance, and spending, and it has been used to judge the level of optimism/pessimism in the consumer's mind. Since consumer spending and investment are affected by consumer confidence, the index of consumer confidence (ICS) has implications which can influence stock market. Therefore, ICS should be closely related to investor sentiment, which measures whether investors are optimistic or pessimistic.

To examine the relationship between investor sentiment and consumer confidence, we plot monthly CEFD and ICS for the period from 1978 to 2005 (ICS monthly data is available from 1978) in Figure 2. It shows that CEFD and ICS have very similar historical pattern. When investors are optimistic, we observe higher ICS. Similarly, lower CEFD corresponds to lower ICS. Since ICS and CEFD are closely linked to each other, in this paper, we only use stock market indicator CEFD and SENT to measure investor sentiment.

2.5. Data and estimation methods

2.5.1 Data

The data used in this paper is monthly stock returns, sentiment change indicators $\Delta SENT$ and CEFD, and Fama-French three factors. Monthly stock returns are collected from the close price colleted from CRSP. Sentiment indicators $\Delta SENT$ and CEFD are available at Jeffery Wurgler's website www.stern.nyu.edu/~jwurgler. Fama-French three factors constitutes the excess return on market portfolio R_M-R_f, where R_f is the risk free rate defined as the one-month treasury bill rate, the difference between the return on portfolio of small stocks and the return on a portfolio of large stocks SML, and the difference between the return on portfolio of high-book-to-market stocks and the return on a portfolio of low-book-to-market stocks HML. These factors are available at French's data library http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. Due to the data availability of $\Delta SENT$, the empirical analysis is conducted at monthly frequency over the period of January 1966 to December 2005. The total observations are 480 months for sentiment indicators and Fama-French factors. However, due to individual stock monthly price varies with their going public data, the time-series regressions vary with the data length of individual stock price availability.

2.5.2 Estimation methods

In order to capture how stock returns respond to sentiment during stock market booms and crashes, dummy variables are added to differentiate the coefficients on sentiment. Since Pagan and Sossounov (2003) divide stock markets into two categories, either bull or bear markets, we use D_{crash} to differentiate these two categories, with D_{crash}=1 in bear markets and 0 in bull markets and the regression model is specified as follows.

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_1(R_{M,t} - R_{f,t}) + \beta_2 SMB_t + \beta_3 HML_t \\ + \beta_4 \Delta SENT_t + \beta_5 \Delta SENT_t * D_{crash} + e_{i,t} \tag{2.3}$$

Combining Mishkin and White (2002) stock market crash and Bordo and Wheelock (2006) booms identifications, stock market can be divided into booms, crashes and neutral markets, the rest of time which is neither bullish nor bearish. Therefore, two dummy variables are needed in this method, with D_{boom}=1 indicating stock market booms, D_{crash}=1 crashes, and D_{boom}=0 and D_{crash}=0 denoting neutral market. And the regression equation is

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_1(R_{M,t} - R_{f,t}) + \beta_2 SMB_t + \beta_3 HML_t \\ + \beta_4 \Delta SENT_t + \beta_5 \Delta SENT_t * D_{boom} + \beta_6 \Delta SENT_t * D_{crash} + e_{i,t} \tag{2.4}$$

Based on the specifications of the regression equation and the definitions of the dummy variables, we define β_4 in equation (2.3) and the summation of β_4 and β_5 in equation (2.4) as β_{boom} , which captures how stock returns respond to sentiment during stock market booms. Correspondingly, β_5 in equation (2.3) and the summation of β_5 and β_6 in equation (2.4) are defined as β_{crash}, which describes the sensitivity of stock returns to sentiment during stock market crashes.

One econometric problem encountered in these regressions is endogeniety due to endogenous dummy variables. More specifically, the dummy variable D_{crash} in equation (2.3) and the dummies D_{boom} and D_{crash} in equation (2.4) are correlated with the right hand dependent variable individual stock returns, resulting in the endogeniety problem in these regressions. In particular, this endogeniety is generated by the ways stock market booms and crashes are defined. These identification methods developed by Pagan and Sossounov (2003), Mishkin and White (2002) and Bordo and Wheelock (2006) all use market returns, which is highly correlated with individual stock returns, as a critical criteria to distinguish stock market booms and crashes. Therefore, the right hand dependent variable individual stock returns are also used to explain the left hand stock market booms and crashes dummy variables, indicating the existence of endogeniety in the regressions.

One way to solve this problem is to use the two-stage least squares (2SLS) method to estimate the coefficients. Specifically, we first use two lagged interaction dummies as the instrumental variables (IV) to compute the IV interaction dummy estimators. Then these IV interaction dummy estimators combined with Fama-French factors and sentiment indicators are used as the regressors in the least squares regression. The empirical results of sentiment beta during booms and crashes are discussed in the next section.

2.6 Empirical results

In order to examine whether stock returns respond differently to investor sentiment during stock market booms and crashes, we first sort these sentiment beta during booms and crashes into 10 deciles respectively and calculate the percentage differences between them. Table 2.4 presents the sentiment betas using the CEFD indicator. The boom sentiment beta, crash sentiment beta and percentage change between them using Pagan and Sossounov (2003) method are reported in column 2 to column 4. And those using the definitions of Mishkin and White (2002) and Bordo and Wheelock (2006) are reported in the last 4 columns in panel A. In this panel, we can first observe that the magnitudes of β_{boom} are all smaller than those of β_{crash} in every decile except the 6th decile for both stock market cycle identification methods. Since the deciles displaying larger absolute value of β_{boom} show up in the middle of the ten deciles, we conjecture that it may be caused by positive and negative betas canceling each other out when calculating mean beta for this deciles. To check whether this is the case, we further divide all betas into positive beta and negative beta, and then sort them into ten deciles, which are reported in panel B and panel C respectively. The results in panel B and panel C show that the absolute values of β_{crash} are all larger than those of β_{boom} in every decile, which confirm the above conjecture.

Second, we compare the percentage change between β_{crash} and β_{boom} using two different bull and bear market identification methods. The changes in column 4 using Pagan and Sossounov (2003) method are larger than those in column 8 using Mishkin and White (2002) and Bordo and Wheelock (2006) method. This is consistent with the fact that the criteria for bull and bear markets used by Mishkin and White (2002) and Bordo and Wheelock (2006) are stricter than those of Pagan and Sossounov (2003). Therefore, the percentage change between β_{crash} and β_{boom} are more dramatic in the former method than in the latter one.

Similar to table 2.4, table 2.5 reports the betas using the composite sentiment indicator SENT. These results indicate that stock returns are more sensitive to investor sentiment during stock market crashes than during stock market booms is robust to the way investor sentiment is defined.

Corresponding to table 2.4 which uses sentiment indicator CEFD, figure 2.3 plots all the betas in panel a, positive betas in panel b, and negative betas in panel c respectively using Pagan and Sossounov (2003) method. Those using the methods of Mishkin and White (2002) and Bordo and Wheelock (2006) are plotted in Figure 2.5

respectively. And corresponding to table 2.5, which uses the SENT as sentiment indicator, figures 2.4 and 2.6 plot the betas using two and three categories for stock market identification respectively.

To further investigate whether these sentiment betas in bull markets are statistically different from those in bear markets, we use the t test. The results are reported in table 2.6. The t statistics of the null hypothesis that sentiment beta during stock market crashes are the same as those during stock market booms is rejected at the 1 percent level for every decile for all the betas. Similar t statistics are obtained for positive betas and negative betas. Therefore, these differences in sensitivity of stock returns to investor sentiment are statistically significant.

2.7 Robustness check

2.7.1 Daily index return sentiment asymmetry

The empirical results show that individual stock monthly returns are more sensitive to investor sentiment during stock market crashes than stock market booms. In this section, we examine whether this asymmetric response to sentiment holds for index daily stock returns RUI and RUT during stock market cycles. RUI, the ticker symbol for Russell 1000 Index, is the large-cap index of the top 1,000 stocks in the Russell 3000 Index, and RUT, Russell 2000 Index, is the small-cap index of the bottom 2,000 stocks in the Russell 3000 index.

Since sentiment proxy SENT and CEFD is not available on daily frequency, we choose VIX, the Chicago Board Options Exchange Volatility Index, as daily investor sentiment proxy. VIX is a popular measure of the implied volatility of S&P index options. Referred to by some as the fear index, VIX represents one measure of the market's expectation of volatility over the next 30 day period. Since investors are more likely to panic and cut losses during bear markets, which leads to big stock price drop and high volatility, VIX is higher in bear markets than in bull markets. Figure 2.7 plots the VIX and S&P 500 index together from 1990 to 2008. From the plot, we observe higher VIX during 1990 and 2000 crashes, and much lower ones during the 1994-2000 stock market booms. Thus, it seems that VIX can capture investor daily sentiment. Due to the availability of VIX, we regress equation (2.2) for the Russell index daily return during the period1990 to 2001. The specifications of stock market cycles are the same as in equation (2.2).

The empirical results are plotted in figure 2.8, which illustrate the sentiment betas for RUT and RUI during bear and bull markets. Similar to the individual monthly stock returns, both index daily returns RUT and RUI respond to investor sentiment proxy VIX asymmetrically, where returns are more sensitive to sentiment during stock market crashes than booms. Therefore, the conclusion of the asymmetric responses of stock returns to investor sentiments is robust, it holds for both individual monthly stock returns and index daily returns.

2.7.2 Portfolio returns sentiment asymmetry

In this section, we first examine the sensitivity of ten size portfolio returns to investor sentiment. Since stock size is closely correlated with its stock return volatility, the smaller the size is, the higher return volatility is observed. For instance, the standard deviation of S&P 500 large cap, middle cap and small size cap are 0.99, 1.04 and 1.17 respectively. Therefore, the results based on size portfolio returns should give similar ones based on volatility portfolio returns in Baker and Wurgler (2007). We regress ten portfolio returns (decile 1 is the largest size and decile 10 is the smallest size portfolio) on market excess return and investor sentiment SENT, and the sentiment betas are plotted in figure 9. This figure shows that from decile 3 to decile 10 the smaller the size of the portfolio, the higher sentiment beta is. Thus, stocks of low capitalization are likely to be more sensitive to investor sentiment. This is consistent with what is found in Baker and Wurgler (2007).

To investigate whether portfolio returns have asymmetric sensitivity to sentiment during stock market booms and crashes, we regress portfolio returns on market excess return, SENT and stock market cycle dummies. The sentiment betas from this regression are reported in figure 2.10, where higher sentiment betas are observed during stock market crashes than booms across every size portfolio. These results confirm that stock returns have higher sensitivity to investor sentiment in bear markets than bull markets.

2.7.3 Asymmetric response check for other risk factors

In equation (2.2) the multifactor stock return model, besides investor sentiment, excess market returns MKT, the return difference between small and big companies SMB, the return difference between high and low book value companies all are used as risk factors to explain the stock returns. Based on loss aversion, investors are more risk averse during stock market crashes than booms; we may suspect that stock returns may respond asymmetrically to MKT, SMB and HML as well. To examine whether there is asymmetry across all the betas, we regress the following

model:

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_1 (R_{M,t} - R_{f,t}) + \beta_2 (R_{M,t} - R_{f,t}) * D_{boom} + \beta_3 (R_{M,t} - R_{f,t}) * D_{crash}$$
$$\beta_4 SMB_t + \beta_5 SMB_t * D_{boom} + \beta_6 SMB_t * D_{crash} + \beta_7 HML_t + \beta_8 HML_t * D_{boom} \quad (2.5)$$
$$+ \beta_9 HML_t * D_{crash} + \beta_4 \Delta SENT_t + \beta_5 \Delta SENT_t * D_{boom} + \beta_6 \Delta SENT_t * D_{crash} + e_{i,t}$$

Figure 2.11 plots all of the betas of the risk factors during stock market cycles. As expected, stock returns are more sensitive to SML across all ten deciles, and shows similar pattern in the 4th decile to excess market return and HML. However, the pattern is reversed in the 6th decile in both betas of market returns and HML, where betas are observed to be larger in bull market than in bear market. One possible explanation is that investors are more sensitive to both sentiment and SML risk factors than to market returns and HML.

2.8 Conclusion

Motivated by the research that sentiment risk is priced in stock price as a consequence of limit arbitrage and loss aversion, this paper tests the hypothesis that sentiment beta are asymmetric in the stock market cycles: stock returns are more sensitive to sentiment during stock market crashes than during stock market booms. Based on two different methods of identifying the stock market cycles and two ways of measuring sentiment, we first regress individual stock returns on sentiment to obtain sentiment betas during stock market booms and crashes. Sorting these sentiments into ten deciles shows that the magnitudes of sentiment betas during stock market crashes are larger than those during stock market booms and these differences are statistically significant. Therefore, stock returns have larger sensitivity to investor sentiment in bull markets than in bear markets.

Empirical evidence that investor sentiment is priced and that stock returns respond to sentiment differently in stock market cycles suggests some directions for future research: building different sentiment trading strategies during stock market booms and crashes; examining the asymmetric effects of sentiment on stock returns across firms' characteristics and time; differentiating the empirical link between sentiment premium and the limit of arbitrages in both bull markets and bear markets. Much remains to be exploited in terms of the asymmetric sensitivity of stock returns to sentiment. By doing this, we can substantially improve our understanding of the role that sentiment plays on stock returns when investors are either overconfident or pessimistic.

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TABLE 2.1: STOCK MARKET BOOMS AND CRASHES SPECIFICATION

Pagan and Sossounov (2003)					
Crashes	Duration	Cumulative Return	Booms	Duration	Cumulative Return
Jan 1966–Sep 1966	9	-21.15%	Sep 1966–Sep 1967	13	26.26%
Sep 1967–Feb 1968	6	-9.24%	Feb 1968–Nov 1968	10	22.00%
Nov 1968–Jun 1970	20	-39.80%	Jun 1970–Apr 1971	11	40.26%
Apr 1971–Nov 1971	8	-11.12%	Nov 1971–Dec 1972	14	25.20%
Dec 1972–Sep 1974	22	-62.04%	Sep 1974–Dec 1976	16	33.60%
Dec 1976–Feb 1978	15	-21.90%	Feb 1978–Nov 1980	34	53.04%
Nov 1980–Jul 1982	21	-27.30%	Jul 1982–Jun 1983	12	51.12%
Jun 1983–May 1984	12	-11.64%	May 1984–Aug 1987	40	84.80%
Aug 1987–Nov 1987	4	-43.64%	Nov 1987–May 1990	31	48.98%
May 1990–Oct 1990	6	-19.98%	Oct 1990–Jan 1994	40	49.20%
Jan 1994–Jun 1994	6	-9.42%	Jun 1994–Sep 2000	75	139.50%

Mishkin and White (2002)			Bordo and Wheelock (2006)		
Crashes	Duration	Cumulative Return	Booms	Duration	Cumulative Return
Nov. 1968-June 1970	20	-30.60%	July 1984- Aug. 1987	38	118.90%
Jan. 1973-Dec. 1974	24	-45.70%	Apr. 1994-Aug. 2000	77	236.58%
Aug. 1987-Dec. 1987	5	-26.80%			
Oct. 1989-Oct. 1990	13	-28%			
Aug. 2000-Dec. 2001	16	-22.90%			

Sources: Pagan and Sossounov (2003), Mishkin and White (2002) and Bordo and Wheelock (2006)

TABLE 2.2: SUMMARY STATISTICS OF STOCK MARKET BOOMS AND CRASHES

		Booms		Crashes	
		Pagan et al (2003)	Bordo et al (2006)	Pagan et al (2003)	Mishkin et al (2002)
Monthly Return	Mean	2.22%	3.10%	-2.69%	-2.48%
	SD	0.92%	0.04%	2.82%	1.64%
	Min	1.23%	3.07%	-10.91%	-5.36%
	Max	4.26%	3.13%	-0.97%	-1.43%
Cumulative Return	Mean	52.18%	177.74%	-25.20%	-30.80%
	SD	33.92%	83.21%	16.90%	8.78%
	Min	22.00%	118.90%	-62.04%	-45.70%
	Max	139.50%	236.58%	-9.24%	-22.90%
Phase Duration (months)	Mean	26.91	57.50	11.73	15.60
	SD	19.91	27.58	6.71	7.23
	Min	10	38	4	5
	Max	75	77	22	24

TABLE 2.3: SUMMARY STATISTICS OF INVESTOR SENTIMENT

Pagan et al (2003)

Statistics	Full Period			Stock Market Booms			Stock Market Crashes		
	SENT	CEFD	$\Delta SENT$	SENT	CEFD	$\Delta SENT$	SENT	CEFD	$\Delta SENT$
N	480	486	480	326	332	326	154	154	154
Mean	-0.0002	9.6229	0.0000	-0.1445	10.1900	0.0710	0.3052	8.4003	-0.1501
Median	-0.095	10.065	0.02	-0.125	9.93	0.08	0.14	10.475	-0.115
Min	-2.36	-10.91	-4.55	-2.36	-6.63	-4.55	-2.12	-10.91	-2.67
Max	3.49	25.28	3.51	2.02	25.28	3.51	3.49	21.96	3.02
S.D.	1.0000	7.3267	1.0001	0.7476	6.7624	0.9917	1.3435	8.3085	1.0044
Skewness	0.5402	-0.1699	-0.0390	-0.3075	0.3358	-0.1902	0.3308	-0.6116	0.2812
Kurtosis	3.6780	2.7082	4.6319	3.1424	2.3517	5.1831	2.1958	2.3337	3.8986

Bordo et al (2006) and Mishkin et al (2002)

Statistics	Neutral Stock Market			Stock Market Booms			Stock Market Crashes		
	SENT	CEFD	$\Delta SENT$	SENT	CEFD	$\Delta SENT$	SENT	CEFD	$\Delta SENT$
N	287	287	287	115	121	115	79	79	79
Mean	-0.2562	10.8776	0.0381	0.3043	8.6464	0.0337	0.4852	6.5827	-0.1891
Median	-0.33	10.88	0.08	0.26	9.68	-0.09	0.02	9.17	-0.2
Min	-2.36	-6.63	-3.11	-1.4	0.17	-4.55	-1.5	-10.91	-2.67
Max	2.92	25.28	3.17	1.72	19.69	3.51	3.49	24.24	3.02
S.D.	0.9489	7.4473	0.9583	0.4643	4.0208	1.0322	1.3811	9.4317	1.0841
Skewness	0.4436	0.0185	-0.1209	0.0783	-0.1926	-0.1369	0.4924	-0.3388	0.3960
Kurtosis	3.2967	1.8669	3.9622	4.0914	3.0113	6.7501	1.9885	1.9834	3.8598

TABLE 2.4: SENTIMENT BETAS USING CEFD AS SENTIMENT INDICATOR

Panel A: all beta sorting							
Deciles	β_{boom}	β_{crash}	%change	β_{boom}	β	β_{crash}	%change
1	-0.98 (0.44)	-2.42 (1.254)	147%	-1.05 (0.052)	-2.30 (0.687)	-2.37 (1.337)	125%
2	-0.38 (0.066)	-0.76 (0.144)	100%	-0.44 (0.082)	-0.66 (0.484)	-0.90 (0.176)	103%
3	-0.20 (0.033)	-0.36 (0.068)	80%	-0.26 (0.024)	-0.29 (0.026)	-0.47 (0.089)	78%
4	-0.10 (0.019)	-0.17 (0.032)	70%	-0.15 (0.017)	-0.13 (0.183)	-0.25 (0.053)	61%
5	-0.04 (0.013)	-0.07 (0.021)	60%	-0.08 (0.014)	-0.06 (0.015)	-0.11 (0.036)	41%
6	0.002 (0.012)	0.006 (0.019)	-118%	-0.02 (0.018)	-0.01 (0.007)	-0.01 (0.023)	-68%
7	0.06 (0.019)	0.08 (0.024)	20%	0.06 (0.016)	0.05 (0.021)	0.11 (0.014)	74%
8	0.16 (0.03)	0.21 (0.055)	34%	0.17 (0.023)	0.15 (0.029)	0.26 (0.013)	50%
9	0.34 (0.068)	0.55 (0.126)	62%	0.38 (0.041)	0.42 (0.058)	0.59 (0.010)	55%
10	0.99 (0.48)	1.89 (0.914)	91%	1.03 (0.671)	1.84 (0.031)	1.70 (0.009)	64%
Panel B: positive beta sorting							
Deciles	β_{boom}	β_{crash}	%change	β_{boom}	β	β_{crash}	%change
1	0.01 (0.006)	0.02 (0.009)	42%	0.02 (0.021)	0.01 (0.054)	0.02 (0.003)	50%
2	0.04 (0.008)	0.05 (0.009)	28%	0.05 (0.033)	0.04 (0.064)	0.07 (0.013)	45%
3	0.07 (0.009)	0.09 (0.012)	26%	0.09 (0.084)	0.07 (0.077)	0.13 (0.015)	41%
4	0.11 (0.011)	0.14 (0.017)	29%	0.14 (0.023)	0.11 (0.011)	0.19 (0.012)	37%
5	0.16 (0.013)	0.21 (0.024)	37%	0.20 (0.013)	0.17 (0.014)	0.27 (0.012)	37%
6	0.22 (0.019)	0.32 (0.038)	48%	0.27 (0.014)	0.26 (0.021)	0.38 (0.019)	39%
7	0.31 (0.028)	0.50 (0.058)	62%	0.37 (0.149)	0.41 (0.029)	0.55 (0.023)	47%
8	0.44 (0.041)	0.77 (0.083)	73%	0.52 (0.130)	0.64 (0.046)	0.79 (0.025)	53%
9	0.67 (0.086)	1.25 (0.166)	87%	0.76 (0.115)	1.06 (0.097)	1.23 (0.041)	62%
10	1.42 (0.493)	2.77 (1.046)	96%	1.47 (0.911)	2.98 (0.468)	2.37 (0.115)	61%
Panel C: negative beta sorting							
Deciles	β_{boom}	β_{crash}	%change	β_{boom}	β	β_{crash}	%change
1	-1.28 (0.453)	-3.27 (1.337)	155%	-1.32 (0.615)	-3.10 (0.404)	-3.01 (0.962)	129%
2	-0.61 (0.071)	-1.33 (0.176)	119%	-0.66 (0.052)	-1.14 (0.070)	-1.48 (0.253)	125%
3	-0.41 (0.041)	-0.81 (0.089)	100%	-0.45 (0.035)	-0.68 (0.042)	-0.96 (0.241)	111%

4	-0.29 (0.026)	-0.53 (0.053)	85%	-0.33 (0.012)	-0.43 (0.026)	-0.66 (0.142)	96%
5	-0.20 (0.018)	-0.36 (0.036)	77%	-0.25 (0.021)	-0.27 (0.018)	-0.46 (0.032)	85%
6	-0.14 (0.014)	-0.24 (0.023)	69%	-0.18 (0.012)	-0.17 (0.014)	-0.33 (0.022)	76%
7	-0.09 (0.009)	-0.16 (0.015)	64%	-0.13 (0.001)	-0.11 (0.009)	-0.23 (0.011)	72%
8	-0.06 (0.008)	-0.10 (0.013)	58%	-0.09 (0.011)	-0.07 (0.007)	-0.15 (0.012)	66%
9	-0.04 (0.006)	-0.05 (0.010)	46%	-0.05 “(0.017)	-0.04 (0.006)	-0.08 (0.023)	58%
10	-0.01 (0.007)	-0.02 (0.009)	37%	-0.02 (0.0150	-0.01 (0.006)	-0.03 (0.012)	46%

TABLE 2.5: SENTIMENT BETAS USING SENT AS SENTIMENT INDICATOR

Panel A: all beta sorting							
Deciles	β_{boom}	β_{crash}	%change	β_{boom}	β	β_{crash}	%change
1	-31.28	-38.13	22%	-27.16	-51.04	-52.29	93%
	(2.682)	(4.411)		(3.981)	(2.798)	(5.982)	
2	-9.36	-16.17	73%	-10.02	-15.19	-22.60	126%
	(0.484)	(2.497)		(2.435)	(1.876)	(3.421)	
3	-5.00	-9.69	94%	-5.49	-7.22	-13.26	141%
	(0.497)	(1.707)		(1.210)	(0.947)	(1.891)	
4	-2.28	-5.50	141%	-2.64	-3.41	-7.24	174%
	(0.583)	(1.413)		(0.874)	(0.657)	(1.094)	
5	-0.17	-2.32	1231%	-0.38	-0.91	-2.89	662%
	(0.152)	(1.374)		(0.086)	(0.201)	(0.981)	
6	1.67	0.44	-74%	1.64	1.37	0.69	-58%
	(0.298)	(0.121)		(0.431)	(0.299)	(0.152)	
7	4.04	3.55	-12%	3.76	3.98	4.83	29%
	(0.589)	(0.436)		(0.984)	(0.597)	(0.873)	
8	7.24	7.64	6%	6.75	8.49	10.38	54%
	(2.192)	(1.604)		(1.092)	(1.368)	(2.051)	
9	12.43	14.14	14%	11.87	18.49	18.75	58%
	(3.189)	(2.305)		(2.074)	(2.421)	(3.050)	
10	33.69	36.55	8%	30.97	65.45	47.89	55%
	(3.941)	(4.128)		(4.451)	(3.186)	(4.872)	
Panel B: positive beta sorting							
Deciles	β_{boom}	β_{crash}	%change	β_{boom}	β	β_{crash}	%change
1	0.48	0.63	30%	0.54	0.58	0.85	57%
	(0.088)	(0.052)		(0.043)	(0.084)	(0.785)	
2	1.45	2.02	39%	1.61	1.76	2.66	66%
	(0.086)	(0.651)		(0.152)	(0.126)	(0.022)	
3	2.56	3.54	38%	2.66	2.95	4.73	78%
	(0.233)	(0.741)		(0.2430	(0.084)	(0.461)	
4	3.95	5.22	32%	3.84	4.58	7.07	84%
	(0.358)	(0.747)		(0.351)	(0.432)	(0.820)	
5	5.55	7.29	31%	5.26	6.73	9.77	86%
	(0.633)	(1.010)		(0.455)	(1.012)	(0.971)	
6	7.43	9.72	31%	7.12	9.68	13.18	85%
	(0.961)	(1.069)		(0.869)	(1.962)	(1.912)	
7	9.83	12.83	31%	9.44	14.34	17.15	82%
	(0.988)	(1.096)		(0.904)	(1.996)	(1.652)	
8	13.35	17.21	29%	13.05	21.79	22.38	72%
	(1.922)	(1.777)		(1.0920	(2.076)	(2.032)	
9	19.42	24.35	25%	18.36	35.24	31.40	71%
	(2.147)	(2.392)		(1.452)	(3.017)	(3.011)	
10	43.92	49.61	13%	41.75	94.17	67.51	62%
	(3.146)	(5.225)		(3.426)	(7.083)	(6.054)	
Panel C: positive beta sorting							
Deciles	β_{boom}	β_{crash}	%change	β_{boom}	β	β_{crash}	%change
1	-45.48	-48.37	6%	-38.95	-74.02	-67.36	73%
	(5.421)	(4.948)		(3.644)	(6.963)	(6.972)	
2	-17.23	-24.76	44%	-17.17	-29.16	-34.44	101%
	(1.412)	(2.634)		(1.982)	(2.532)	(3.650)	
3	-11.72	-17.46	49%	-12.15	-18.54	-24.49	102%
	(1.053)	(1.421)		(1.1220	(1.922)	(1.982)	
4	-8.55	-13.14	54%	-9.09	-12.53	-18.53	104%
	(0.972)	(1.325)		(0.844)	(1.125)	(1.654)	
5	-6.47	-10.10	56%	-6.86	-8.78	-13.98	104%
	(0.678)	(1.054)		(0.769)	(0.941)	(1.431)	

6	-4.89 (0.861)	-7.69 (0.891)	57%	-5.23 (0.652)	-6.17 (0.651)	-10.31 (0.964)	97%
7	-3.57 (0.467)	-5.57 (0.632)	56%	-3.78 (0.407)	-4.33 (0.396)	-7.41 (0.851)	96%
8	-2.40 (0.198)	-3.74 (0.401)	56%	-2.61 (0.234)	-2.88 (0.302)	-4.97 (0.522)	91%
9	-1.39 (0.145)	-2.21 (0.231)	59%	-1.52 (0.145)	-1.66 (0.168)	-2.78 (0.321)	83%
10	-0.44 (0.049)	-0.75 (0.068)	68%	-0.47 (0.037)	-0.56 (0.068)	-0.90	94%

TABLE 2.6: T STATISTICS OF EQUAL SENTIMENT BETA TESTS

	1 dummy		2 dummy	
Deciles	CEFD	SENT	CEFD	SENT
1	-31.32	-19.29	-32.43	-4.94
2	-67.01	-101.04	-71.73	-77.66
3	-63.20	-105.67	-69.11	-101.78
4	-50.17	-93.34	-58.81	-92.74
5	-23.57	-63.11	-35.42	-79.81
6	8.69	-25.18	-4.40	-44.94
7	28.83	22.32	12.99	-13.64
8	34.17	51.52	29.61	7.96
9	36.34	59.40	45.19	18.62
10	20.38	10.55	24.68	2.47

Figure 2.1: Monthly closed-end-fund discount (CEFD)
and composite sentiment SENT

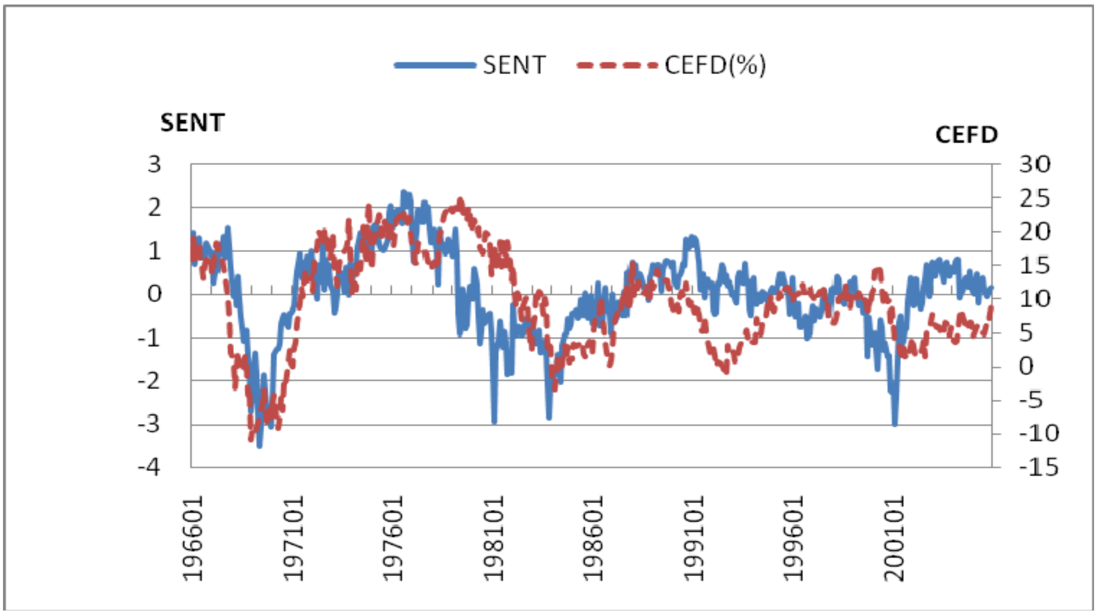


Figure 2.2: Monthly closed-end-fund discount (CEFD) and
the index of consumer sentiment (ICS)

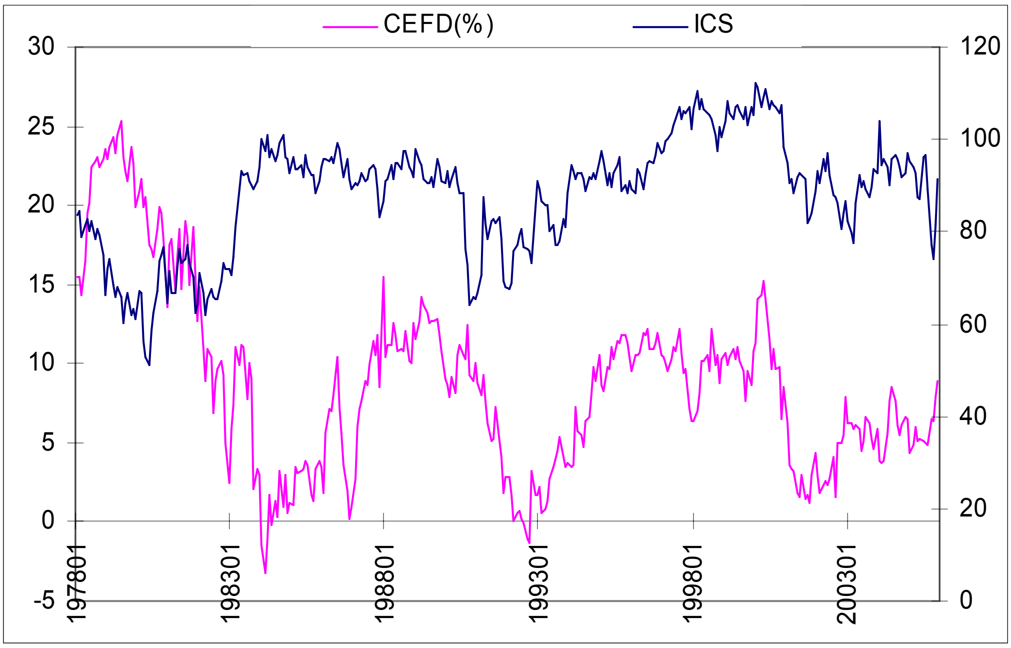
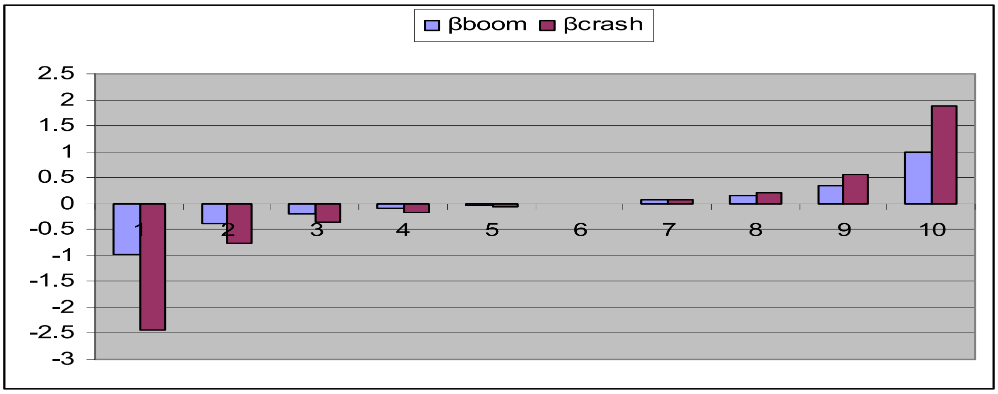
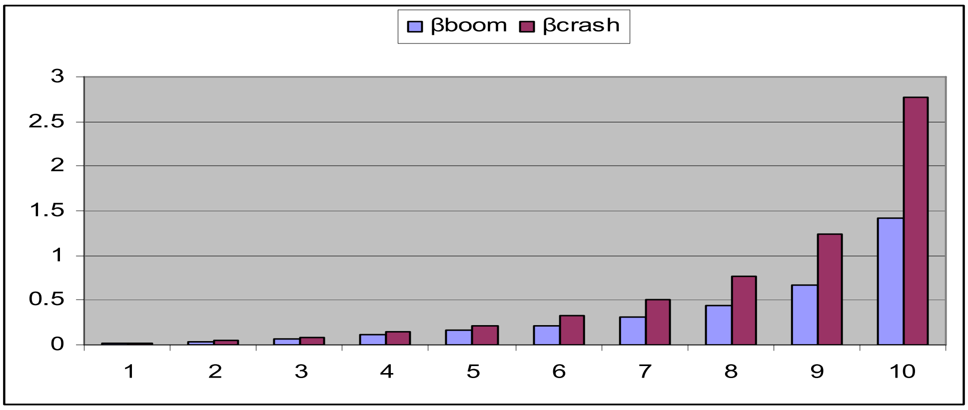


Figure 2.3: Sentiment betas using sentiment indicator CEFD

2.3.1: All betas sorting



2.3.2: Positive betas sorting



2.3.3: Negative betas sorting

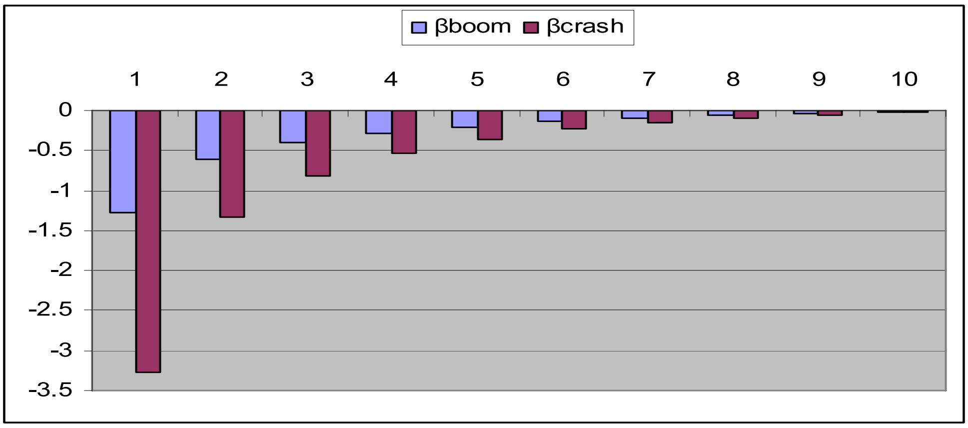
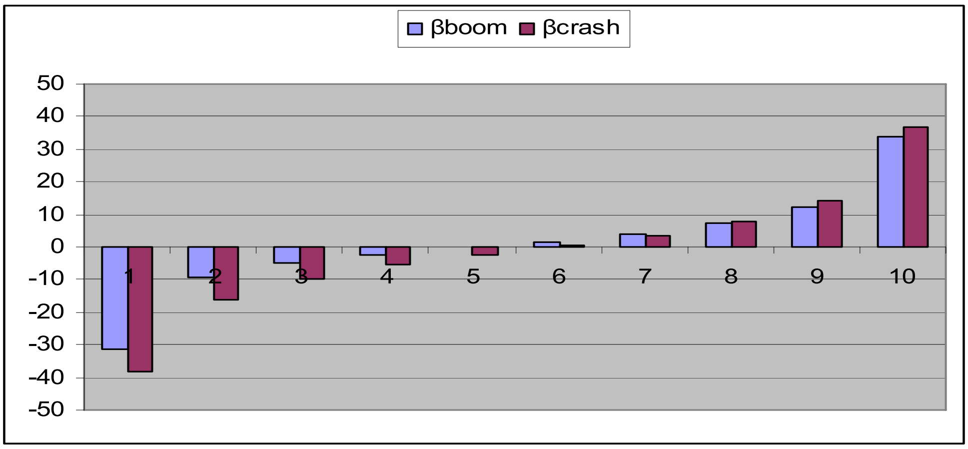
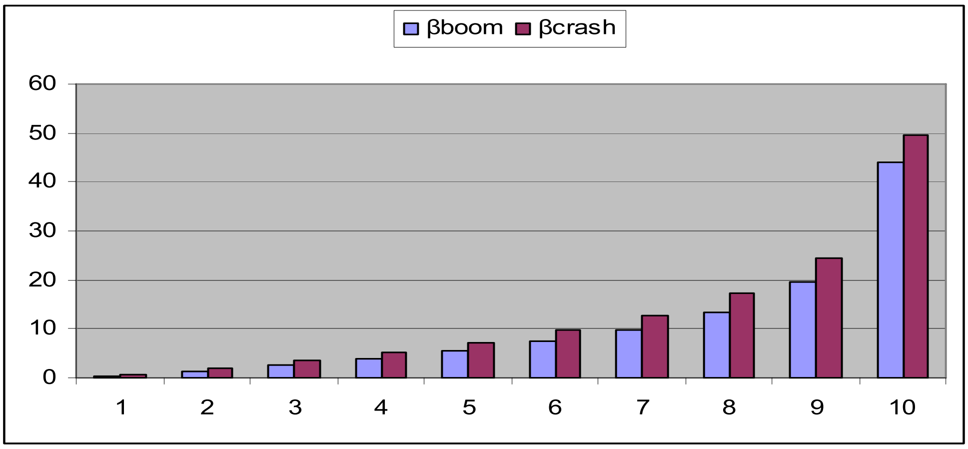


Figure 2.4: Sentiment betas using sentiment indicator SENT

2.4.1: All betas sorting



2.4.2: Positive betas sorting



2.4.3: Negative betas sorting

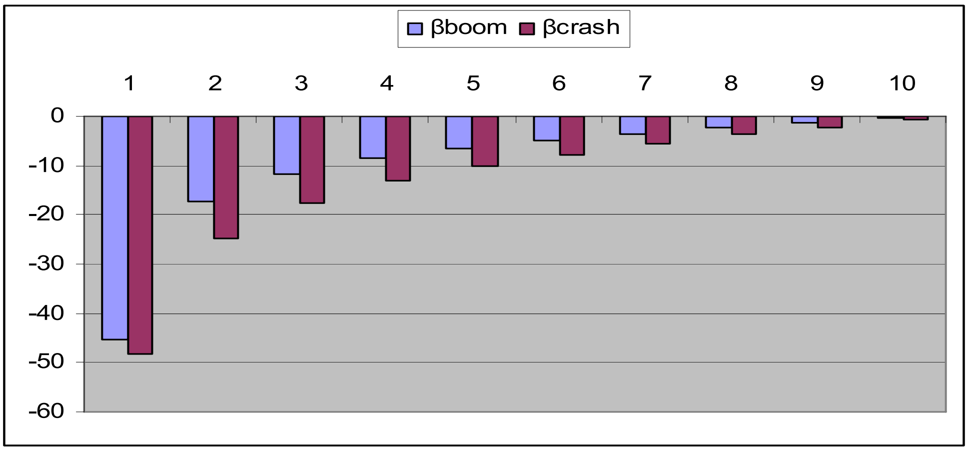
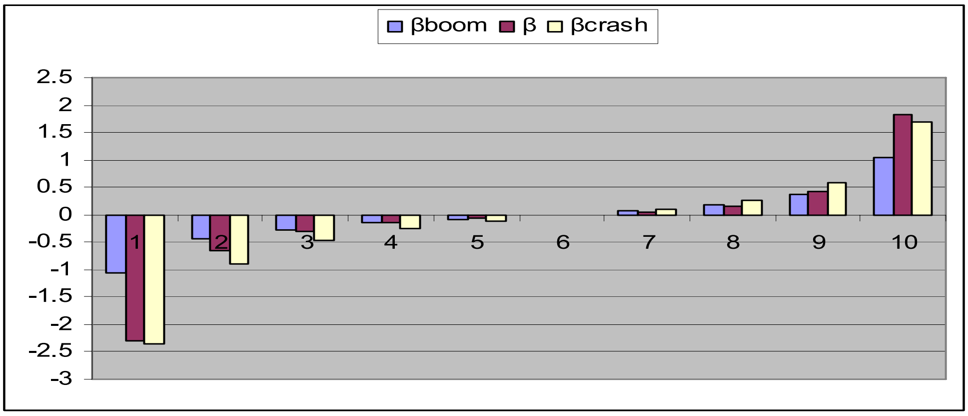
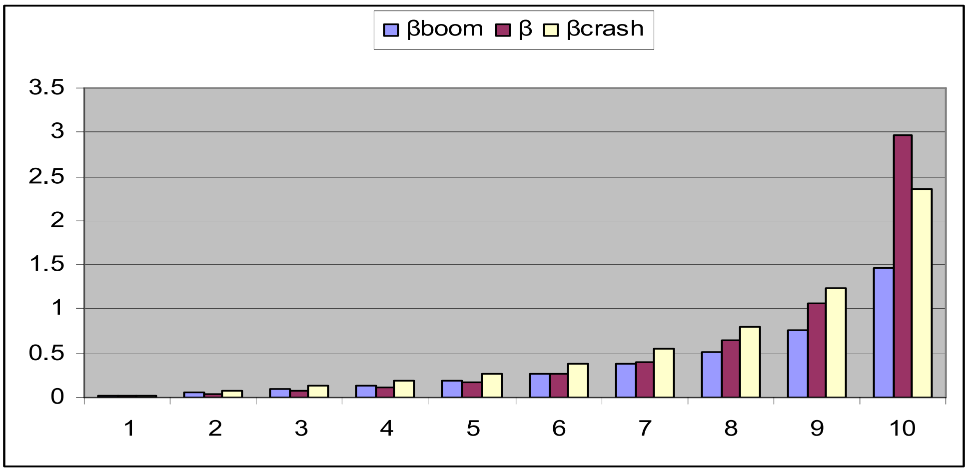


Figure 2.5: Sentiment betas using sentiment indicator CEFD

2.5.1: All betas sorting



2.5.2: Positive betas sorting



2.5.3: Negative betas sorting

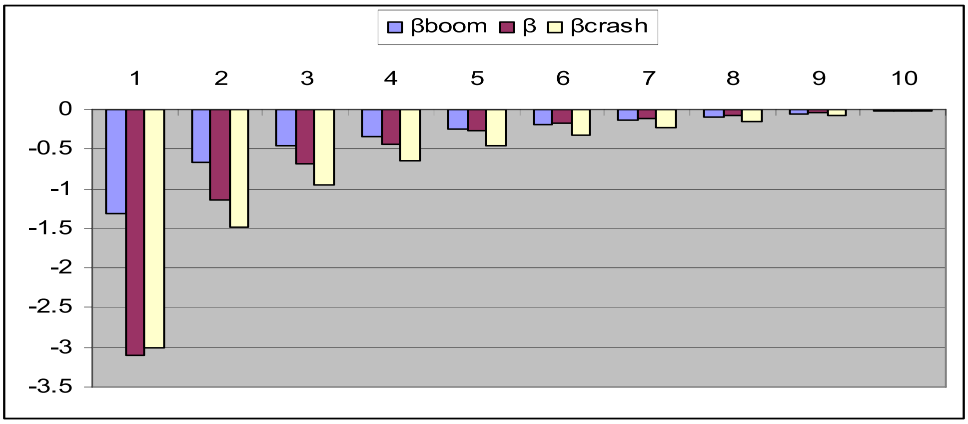
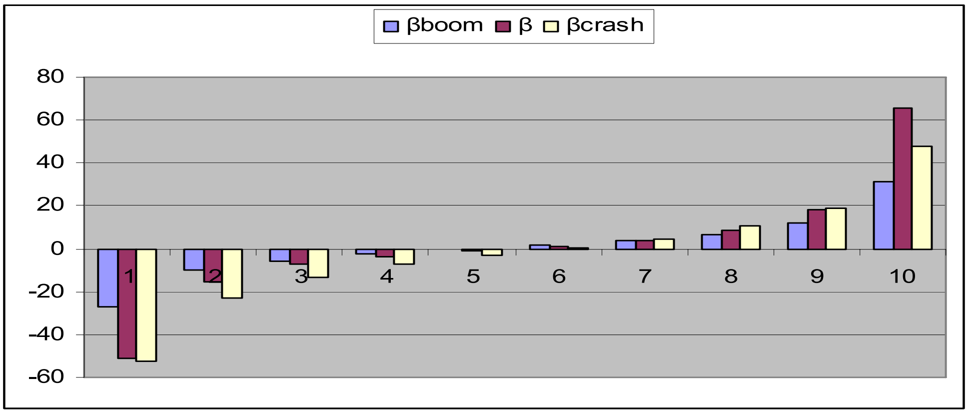
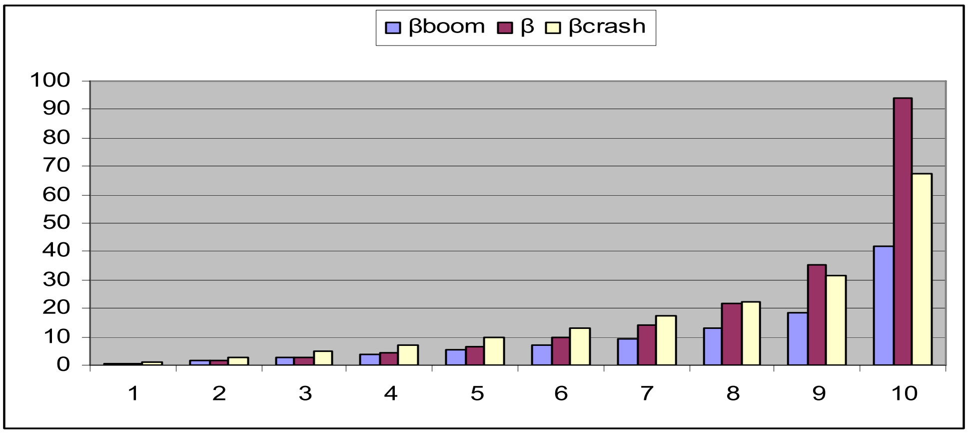


Figure 2.6: Sentiment betas using sentiment indicator SENT

2.6.1: All betas sorting



2.6.2: Positive betas sorting



2.6.3: Negative betas sorting

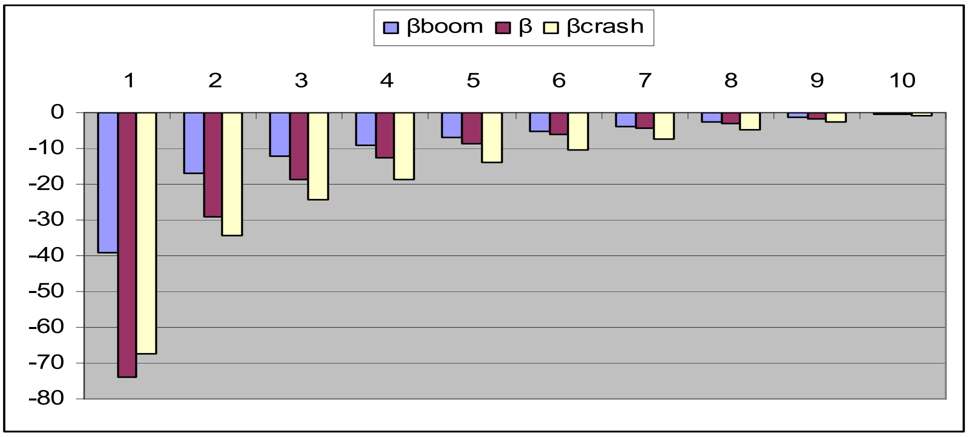
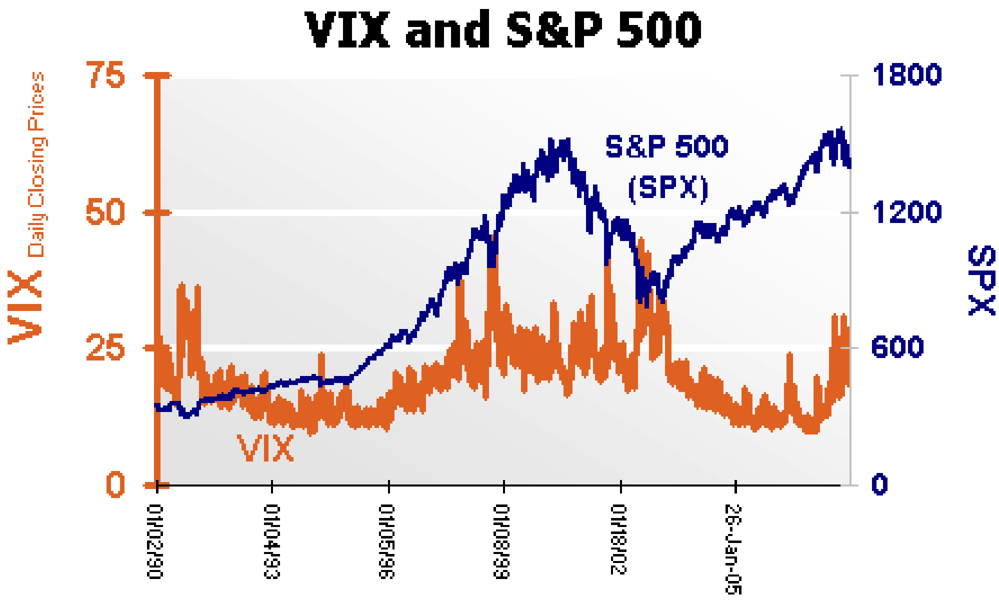
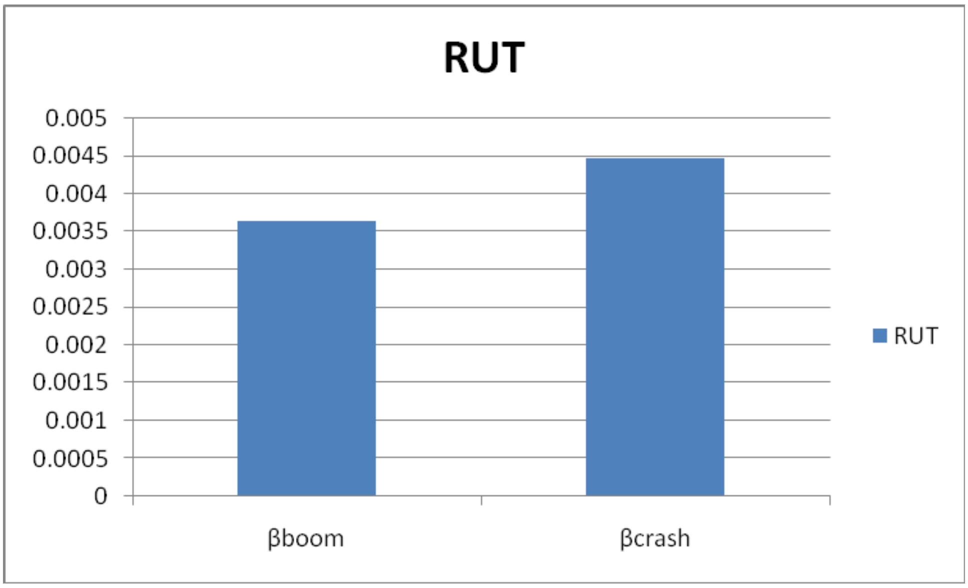


Figure2. 7: VIX and S&P 500 index



Sources: CBOE and Bloomberg. (Jan. 2, 1990 - Jan. 14, 2008).
www.cboe.com/VIX

Figure 2.8: Sentiment beta for RUT and RUI



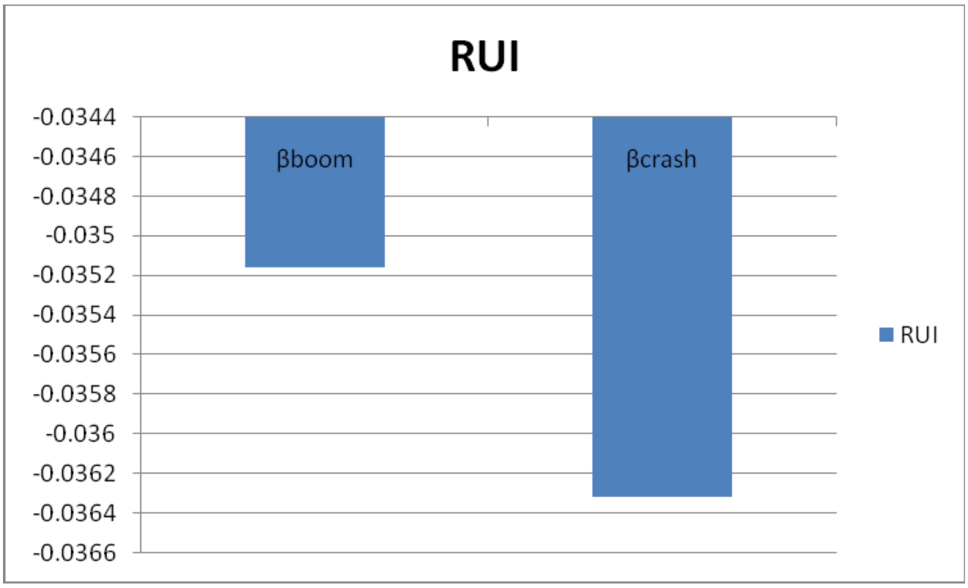


Figure 2.9: Sentiment betas of ten size portfolios

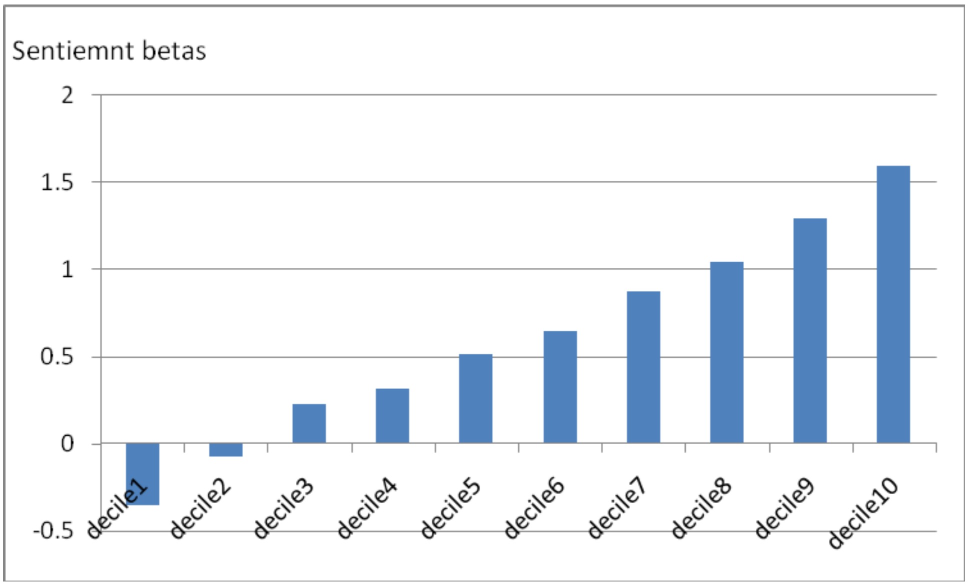
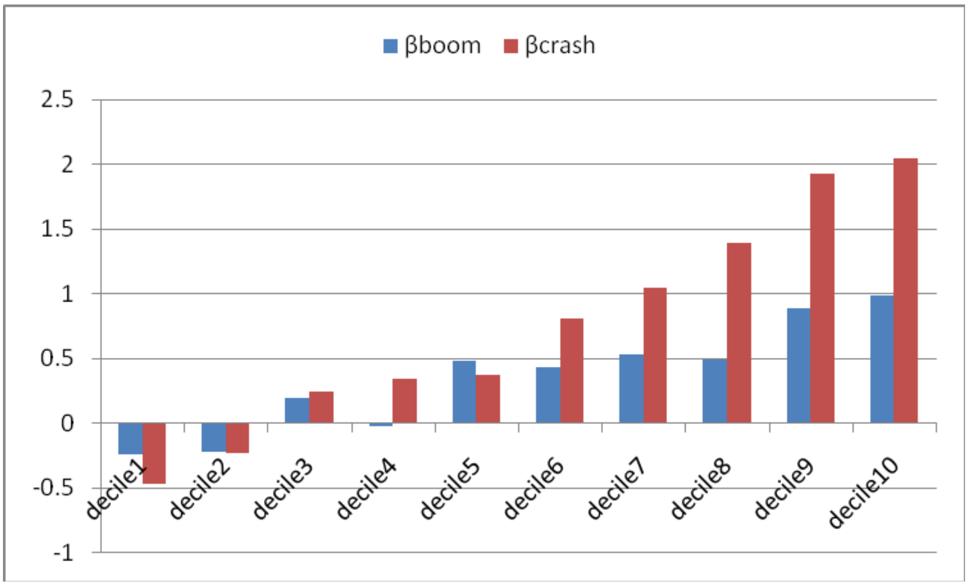


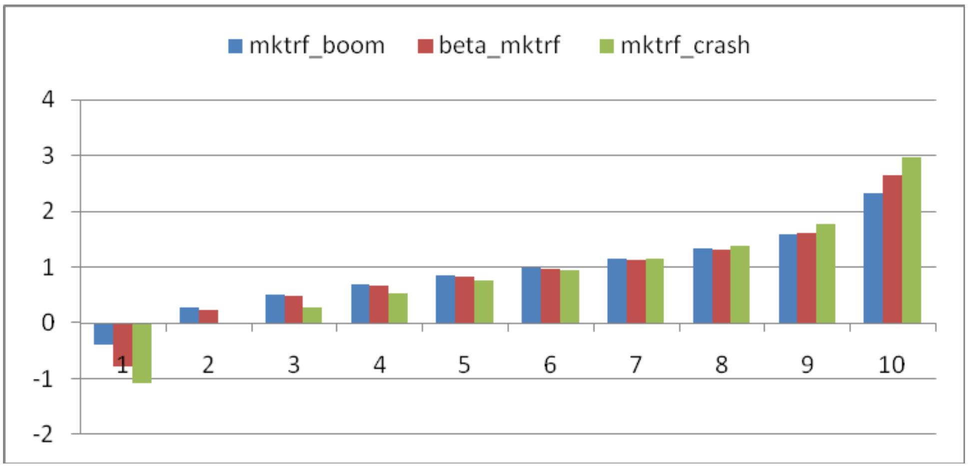
Figure 2.10: Sentiment betas of ten size portfolios across stock market cycles



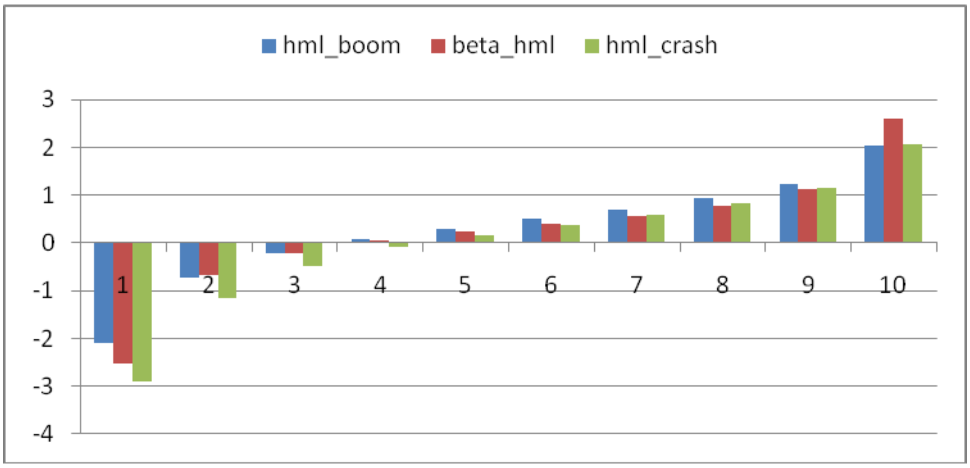
Note: decile 1 is the largest size portfolio, and decile 10 is the smallest size portfolio.

Figure 2.11: Asymmetric betas on other risk factors

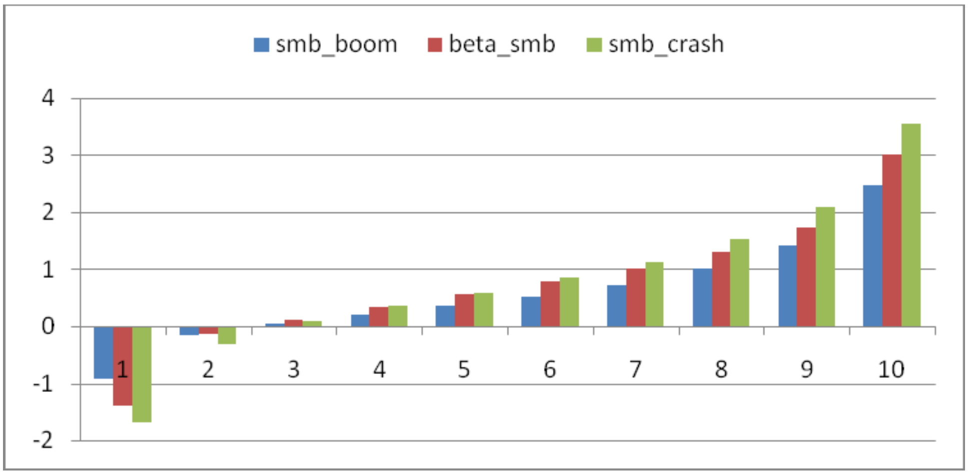
2.11.1Market return sentiment beta



2.11.2 HML Beta



2.11.3 SMB beta



Chapter 3: Fixed Investment and the Stock Market: Evidence from BVAR Models

3.1 Introduction

Does the stock market play a central role in firm investment decision making or it is just a sideshow? If investment does respond to the stock market, how big is the effect? There is no consensus view yet on these questions. Bosworth et al (1975) argue that a long-run horizon manager will not scrap investment plans in response to stock prices since it is highly volatile in short-run. In contrast, Fisher and Merton (1984) demonstrate mangers will adjust investment expenditures in response to stock price changes even in the extreme cases where managers hold their belief in certainty and the stock prices changes are inconsistent with their beliefs.

Following these mixed theories, later studies attempt to address this question by looking for empirical evidence. However the empirical results are still ambiguous. Morck, Scheilfer, and Vishny (MSV,1990) show that after controlling for the fundamentals, the explanatory power of the stock market (incremental R^2), is very small in both firm level and aggregate data. They find that a 30 percent abnormal stock return is associated with a 10 percent extra investment growth over three years for firm level data and a 10 percent rise in the lagged market return only leads to 0.8 percent increase in investment growth for aggregate data. Therefore, they conclude that the market may not be a sideshow, but nor is it very central. Blanchard, Rhee, and Summers (BRS, 1993) regress log investment on current and one lag log Tobin q and log fundamental q, expected present value of profits, and find that “an increase of 1 percent in market evaluation not matched by an increase in fundamentals leads to an increase in investment of 0.45 percent, whereas an increase in market evaluation matched by an increase in fundamentals leads to an increase in investment of 2 percent”. Replacing fundamental q with profit as a measurement of fundamentals, an increase in market valuation of 1 percent only increases investment by 0.06 percent.

Based on these evidences, they argue "market valuation appears to play a limited role, given fundamentals, in the determination of investment decisions."

Contrary to the limited effect of misvaluation on investment, there is some evidence that stock prices do matter in firm investment decision. For example, Polk and Sapienza (PS, 2003) show that a typical change (standard deviation) in one of mispricing proxies results in roughly a two to four percent change in the firm's investment capital ratio, about 10% of the sample mean of I/K (0.31). Baker, Stein, and Wurgler (BSW, 2003) find that one standard deviation shock to q alter ratio of capital expenditure to assets by 0.031 for equity dependent firms, which is quite substantial compared to either 0.06 median and 0.079 standard deviation of the investment ratio. Similarly, Chirinko and Schaller (CS, 2004) show that one standard deviation increases in misvaluation leading to a 40% to 50% increase in investment ratio I/K, indicating a quantitatively large effect on investment. Goyal and Yamada (GY, 2004) find that investment is significantly more responsive to non-fundamental or residual stock valuation around stock bubble period of late 1980's in Japan. The regression of investment on fundamental q and residual q shows that the coefficient on fundamental q is not significant even at 10% level. Gilchrist, Himmelberg, and Huberman (GHH, 2005) demonstrate the peak response of investment to one standard deviation of dispersion (0.4) is on the order of 0.1, indicating 25% dispersion elasticity of investment.

There are three factors accounting for the mixed effect of stock market on investment. One is that measures of misvaluation are different. For example, MSV (1990) use firm excess stock return alpha as market misvaluation. CS (2004) define misvaluation as the difference between market q and augmented fundamental q. GY (2004) decompose q into fundamental q and residual q. GHH (2005) use dispersion of a firm's stock analyst forecasts of its future earnings as a proxy for the dispersion of investor's belief. BRS (1993) and BSW (2003) run regression of investment on Tobin q and fundamental directly instead of measuring misvaluation. Different measures of market valuation lead to different sensitivities of investment to stock market misvaluation.

The other factor is the regression specification, including the type of dataset and regression time framework. MSV (1990) run regression on panel data for 1960-87 and aggregate data for 1935-1988 separately. BRS (1993) do regression on aggregate data from 1900-1990. PS (2003), BSW (2003) and CS (2004) run regressions on panel data from 1963-2000, 1980-1999, and 1980-2001 respectively. GHH (2005) use vector autoregression (VAR) model with panel data from 1986-2000. It seems that the effect of stock market on investment is larger since 1980's from the above results.

The third factor is the timing specification in the linear regression models. Some regressions are simultaneous, and the other studies regress investment on lag misvaluation. Understanding the dynamic effect of the stock market on investments helps to specify regression models appropriately. If investment respond misvaluation immediately and the effect of misvaluation is not persistent, then simultaneous specification is the way to go. However, if misealuation has persistent effect, lag misevaluation should be included in the regression model. This paper aims to answer these questions by capturing investment dynamics.

There are three innovations in this paper. First, unlike GHH (2005), which use annual panel data, we use manufacturing aggregate industry quarterly data to calculate the impulse response functions. Shiller (1984) and De Long et al (1990) find investor sentiment is more likely to be more pronounced in the aggregate dataset. Therefore, we investigate the investment dynamics at aggregate industry level. Due to data availability, we use times series data from 1984:1 to 2003:4 in this paper. Since the total observations for estimation is 80, which is relatively small compared to 49 coefficients needed to be estimated in a 4-variable 2-lag VAR model, VAR is subject to the overfitting problem in this case. This overfitting problem will result in inaccurate estimation. To prevent overfitting, BVAR is used to calculate impulse response functions in this paper. Since BVAR estimates the coefficients combining the priors and the data, it has been proven to have better performance of estimation than VAR.

Second, we derive full system BVAR estimators under Minnesota prior by analyzing the equations simultaneously. The estimators are more efficient than the previous estimations based on single equations, which are not efficient unless the prior variance-covariance matrices of the coefficients are identical for each equation. Instead of use arbitrary hyperparameter, we specify hyperparameters more properly by maximizing the marginal likelihood than taking arbitrary common value conjectures.

Third, we use BVAR to calculate fundamental q and define misevaluation as the difference between the market q and fundamental q. Abel and Blanchard (1986) estimate fundamental q using VAR for quarterly data from 1948:2-1979:3, the total observation is 126, which is relatively small compared to the number of coefficient needed to be estimated for calculation of fundamental q (49 coefficients totally). Overfitting is a problem here, which results in inaccurate estimation of q. In order to avoid over-parameterization, unlike Abel and Blanchard (1986), we calculate fundamental q using BVAR models.

The remainder of the paper is organized as follows. Section 2 describes how to measure misvaluation. Section 3 explains the methodology of estimation, BVAR model. Section 4 specifies the 4-variable 2-lag BVAR model. In section 5, the data and the empirical results are reported. Section 7 concludes the paper.

3.2 Measure of misvaluation

3.2.1 Market q

Market q is the ratio of market value of the firm to the replacement cost of capital. Following Lindenberg and Ross (1981), market value of the firm is the summation of the market value of the equity, preferred stock and debt. Replacement cost of the capital is the summation of the replacement cost of plant and equipment, inventory and other assets. Since replacement cost is the minimum cost needed to purchase the current productive capacity of the firm with the most modern technologies available, it is a cost adjusting both inflation and technology progress. Perfect and Wiles (PW, 1994) compare five alternative constructions of market q and find empirical results are sensitive to the method used to estimate market q. In this paper, a market q is constructed in the way, whose empirical results are proved to be robust by PW (1994). Details of calculation of market q are provided in the appendix.

3.2.2 Fundamental q

Marginal q, also referred as fundamental q, is the ratio of the market value of an additional unit of capital to its replacement cost. Since it is a forward-looking variable given current information, therefore it is not directly observable. Abel and Blanchard (1986) construct fundamental q conditional on observed fundamentals by using VAR. Following them closely, fundamental q is defined as the expected present value of marginal profit to capital in this paper:

$$q^*_t = E\left[\sum_{j=0}^{\infty}(\prod_{i=0}^{j}\beta_{t+i})M_{t+j} \mid \Omega_{t-1}\right] \tag{3.1}$$

Where β is the discount factor, M_{t+j} is the marginal profit of capital in period t+j, and Ω_{t-1} is the information set at time t-1.

Linearize q_t around the sample mean $\overline{\beta}$ and $\overline{M}$ respectively, equation (3.1) can be approximated by:

$$q^*_t = \overline{q} + \overline{M}(1-\overline{\beta})^{-1}\sum_{j=0}^{\infty}\overline{\beta}^{j}(\beta_{t+j} - \overline{\beta}) + \sum_{j=0}^{\infty}\overline{\beta}^{j+1}(M_{t+j} - \overline{M}) \tag{3.2}$$

Where $\overline{q} = \overline{M}\,\overline{\beta}(1-\overline{\beta})^{-1}$ (3.3)

Assume ex post discount factor $\widetilde{\beta}$ and M follow the following specification:

$$\widetilde{\beta}_t = b' Z_t \tag{3.4}$$

$$M_t = a' Z_t \tag{3.5}$$

Where Z_t follows AR (1) process:

$$Z_t - \overline{Z} = A(Z_{t-1} - \overline{Z}) + \varepsilon_t \tag{3.6}$$

ε_t is white noise.

Based on the above specification, fundamental q can be calculated as the following:

$$q^*_t \approx \overline{q} + \overline{M}(1-\overline{\beta})^{-1} b'(I - A\overline{\beta})^{-1} A(Z_{t-1} - \overline{Z}) + \overline{\beta} a'(I - A\overline{\beta})^{-1} A(Z_{t-1} - \overline{Z}) \tag{3.7}$$

Which factors should go into vector Z as components of information set Ω is a potential question. Abel and Blanchard (1986) use equity discount factor, debt discount factor, wage capital ratio, output capital ratio, inflation, market q and investment ratio. Chirinko and Schaller (2004) include discount rate (which is weighted average cost of capital), sales capital ratio, cost capital ratio, price of investment and output ratio, and investment to capital ratio. Gilchrist and Himmelberg (1995) compose profit to capital and sales to capital ratios. Following the above the literature, we specify vector Z as cost of debt i_t^d, cost of equity i_t^e, sales to capital ratio S_t / K_t, cost to capital ratio C_t / K_t, price inflation π, market q q_m, and investment to capital ratio I_t / K_t, that is Z_t=[$i_t^d \ i_t^e \ S_t / K_t \ \ C_t / K_t \ \ \pi \ \ q_m \ \ I_t / K_t$],and A is 7×7 matrix. We define ex post discount factor $\widetilde{\beta}$ as weighted average cost of capital. Without lost of generality, we assume constant return and perfect competition in manufacturing industry, which implies that marginal productivity of capital equal to average productivity of capital. Therefore, $a = \begin{bmatrix} 0 & 0 & 1 & 1-\partial & 0 & 0 & 0 \end{bmatrix}$, $b = \begin{bmatrix} 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$, where ∂ is the leverage ratio, ratio of long term debt to total asset.

3.2.3 Misvaluation

There are different ways to measure miavaluation. Following Chirinko and Schaller (2004), we define misvaluation as the difference between market q and fundamental q, which can be written as

$$m = q - q^* \tag{3.8}$$

The difference between q and q^* reflect not only the fads and fashions in stock market, but also asymmetric information between mangers and investors, therefore it captures the mismeasurement between stock markets and valuation based fundamentals.

3.3 The methodology: BVAR

The incentive to use BVAR is to avoid the overfitting problem in VAR. In VAR model, the total number of coefficients to be estimated is $n(np+1)$, where n is the dimension of the vector, p the length of the lag. Therefore, the coefficients needed to be estimated increase by $(2n+1)p+1$ with each additional variable and n^2 with each additional lag. Even in a simple VAR with $n=5,\ p=4$, there are 21 coefficients per equation or total 105 for VAR. The coefficients are very large compared to the number of observations that the coefficients will not only capture the stable relationship among variables, but also reflect the random features of the data when these coefficients are estimated by fitting the data (see Todd (1984)). Therefore, the parameter estimates and prediction will generally be imprecise due to the overfitting.

In order to prevent over-parameterization, model specification has to be parsimonious about not only adding explanatory variables but also the length of the lag given limited number of observations. One way to solve overfitting is to reduce the number of coefficients by excluding variables or lags, which assign zero coefficients to the excluded variables with complete certainty. However, the exclusion is so rigid that some useful information in historical data is completely ignored. Doan et al (1983) and Minnesota (1984) present an alternative solution, BVAR, to the problem. The idea behind BVAR is to impose some prior, observed from statistical regularities, on the coefficients instead of reducing them. Following Theil (1971) mixed estimation technique, the prior information can be introduced in the form of extra “dummy” observation and the coefficients are estimated by combining the data and the prior.

Since the data’s influence on the coefficients is reduced by the prior, overfitting is relieved in BVAR, which has been proved by the better performance of

BVAR in estimation and predicting. Minnesota (1985) shows that mean square error (MSE) of forecasting obtained from BVAR performs favorably in comparison with MSE from ARIMA, univariate AR, and two other models by DRI (Data Resources, Inc.) and CHASE (Chase Econometric Association, Inc.) . Roberson and Tallman (1999) also show that the forecasting of BVAR outperform that of VAR.

3.3.1 VAR framework

Consider a structure VAR (SVAR) as below

$$y_t A_0 = C + y_{t-1}A_1 + \cdots + y_{t-p}A_p + \varepsilon_t \tag{3.9}$$

Where y_t is the row vector of m variables of interest observed at time t, A_0 is Choleski decomposition of $m \times m$ dimension, C is $1 \times m$, $A_i, i = 1, 2 \cdots p$ are parameter matrices of dimension $m \times m$, ε_t has a Gaussian disturbance, namely $\varepsilon_t \sim N(0, I_{m\times m})$. Multiplying SVAR by A_0^{-1}, we obtain the reduced form VAR

$$y_t = B_0 + y_{t-1}B_1 + ... + y_{t-p}B_p + u_t = x_t B + u_t \tag{3.10}$$

Where $B_i = A_0^{-1}A_i, i = 1, 2 \cdots p$ are parameter matrices of dimension $m \times m$, $B_0 = A_0^{-1}C$ is $1 \times m$, $x_t = \begin{bmatrix} 1 & y_{t-1} & y_{t-2} & \cdots & y_{t-p} \end{bmatrix}$ is $1 \times k$ $(k = mp + 1)$, $B = \begin{bmatrix} B_0 & B_1 & \cdots & B_p \end{bmatrix}'$ is $k \times m$, u_i is $1 \times m$ and $u_t' \sim i.i.d\ N(0, \Sigma_u), \Sigma_u = (A_0 A_0')^{-1}$ is $m \times m$.

Stacking the T observations and rewrite the model in the following form:

$$Y = XB + U \tag{3.11}$$

Where Y is $T \times m$, X is $T \times k$, $U = \begin{bmatrix} u_1 & u_2 & \cdots & u_T \end{bmatrix}'$ is $T \times m$. From Bayesian perspective B is random variable matrix instead of being fixed in VAR.

3.3.2 Minnesota prior

When Minnesota (1986) first uses Bayesian vector autoregression (BVAR) to overcome overfitting problem in vector autoregression (VAR), the coefficients of BVAR are estimated equation by equation due to computational difficulty. However, the estimators are inefficient unless the prior variance-covariance matrices of the coefficients are identical for each equation.

Since computational software and computer techniques have been developed dramatically since then, the full system estimation of BVAR is feasible as long as the VAR system is not too big. Therefore, we derive the full system BVAR estimators by analyzing the equations simultaneously under the Minnesota prior.

Following Litterman (1986), most economic variables are approximately random walk, therefore the mean for the coefficient on the first own lag is set to unity and the mean of constant and the remaining coefficients are set to zero. That is the mean of B, $\overline{B} = (0\ I\ 0 \cdots 0)'$, where 0 is $m \times 1$ column vector, I $m \times m$ identity matrix. The confidence of the prior means differ with its own lag or other lags and the length of the lags, which is captured by the variances of the coefficients. In general, the standard deviation of its own lags are larger than those of other lags, reflecting the confidence of mean zero of the other lags is much tighter than mean unity of its own lag. In addition, the longer the lag is, the tighter the prior, expressing the belief that coefficient on longer lags are more likely to be zero. Specifically, the standard deviation of constant of the $i-th$ equation is specified as $\sigma_i \lambda_4$ and the standard deviation of the $ij-th$ element of the $l-th$ lag coefficient σ_{ij}^l is specified as

$$\sigma_{ij}^l = \begin{cases} \lambda_1 / l^{\lambda 3} & if\ i = j \\ \lambda_1 \lambda_2 \sigma_i / \sigma_j l^{\lambda 3} & if\ i \neq j \end{cases} \tag{3.12}$$

Where σ_i, σ_j are scale factors, making the standard deviation to be scale invariant, not affected by the unit of measurement. In practice, σ_i is set to the estimated error of the residual in an unrestricted univariante autoregression of variable i.

The interpretations of the hyperparameters λ_0 λ_1 λ_3 λ_4 are described in Table 3.1.

If B is multivariate normal distributed, Minnesota prior can be written as $B \sim MN(\overline{B}, \overline{S}, \overline{H})$, where $\overline{S}$ is specified as diagonal matrix with diagonal elements $(\sigma_i / \lambda_0)^2$, $\overline{H}$ is a diagonal matrix with the prior standard deviation of the coefficient for lag l of variable j in equation i specified as follows:

$$\sigma_{ij}^{l} = \lambda_o \lambda_1 /(l^{\lambda 3} \sigma_j) \tag{3.13}$$

The hyperparameter λ_0 controls the tightness of beliefs on contemporaneous matrix A_0.

Under Minnesota prior, $B \sim MN(\overline{B}, \overline{S}, \overline{H})$, which is equivalent to impose restriction on the VAR as:

$$\overline{B} = RB + V \qquad\qquad V \sim MN(0, \overline{S}, \overline{H}) \tag{13.4}$$

Where R is $k \times k$ identity matrix whose columns represent linear combination of the coefficients, which describes the random walk prior. Denote $\overline{b} = vec(\overline{B})$, $\beta = vec(B)$, $v = vec(V) \sim N(0, \overline{S} \otimes \overline{H})$, the vec operator transform (3.14) into

$$\begin{aligned} vec(\overline{B}) &= vec(RB) + vec(V) \\ &= (I_m \otimes R) vec(B) + vec(V) \end{aligned}$$

Therefore $\overline{b} = (I_m \otimes R)\beta + v$ (3.15)

Notice equation (14) is of the same form of reduced VAR form

$$Y = XB + U \qquad U \sim MN(0, \Sigma_u, I_T) \tag{3.16}$$

Similarly, denote $y = vec(Y)$, $u = vec(U) \sim N(0, \Sigma_u \otimes I_T)$, the vec form of (3.16) is

$$y = (I_m \otimes X)\beta + u \tag{3.17}$$

By Theil mixed estimation (1971), the prior is equivalent to add a set of k dummy observations to the data, with $\overline{B_i}$ corresponding to the observation of Y_t and R_i the explanatory variables X_t.

Combining the prior and the reduced form VAR, we get the following system

$$Y^* = X^* \beta + U^* \tag{3.18}$$

Where $Y^* = \begin{bmatrix} \bar{b} \\ y \end{bmatrix}$, $X^* = \begin{bmatrix} I_m \otimes R \\ I_m \otimes X \end{bmatrix}$, $U^* = \begin{bmatrix} v \\ u \end{bmatrix}$, $V^* = E(U^* U^{*}{}') = \begin{bmatrix} \overline{S} \otimes \overline{H} & 0 \\ 0 & \Sigma_u \otimes I_T \end{bmatrix}$

Therefore, the GLS estimator for the system $\widehat{\beta}$ is

$$\begin{aligned}
\widehat{\beta} &= (X^*{}' V^{*-1} X^*)^{-1} (X^*{}' V^{*-1} y^*) \\
&= \left\{ \begin{bmatrix} I_m \otimes R' & I_m \otimes X' \end{bmatrix} \begin{bmatrix} \overline{S} \otimes \overline{H} & 0 \\ 0 & \Sigma_u \otimes I_T \end{bmatrix}^{-1} \begin{bmatrix} I_m \otimes R \\ I_m \otimes X \end{bmatrix} \right\}^{-1} \\
&\quad \left\{ \begin{bmatrix} I_m \otimes R' & I_m \otimes X' \end{bmatrix} \begin{bmatrix} \overline{S} \otimes \overline{H} & 0 \\ 0 & \Sigma_u \otimes I_T \end{bmatrix}^{-1} \begin{bmatrix} \bar{b} \\ y \end{bmatrix} \right\} \\
&= \left[(I_m \otimes R')(\overline{S}^{-1} \otimes \overline{H}^{-1})(I_m \otimes R) + (I_m \otimes X')(\Sigma_u^{-1} \otimes I_T)(I_m \otimes X) \right]^{-1} \\
&\quad \left[(I_m \otimes R')(\overline{S}^{-1} \otimes \overline{H}^{-1})\bar{b} + (I_m \otimes X')(\Sigma_u^{-1} \otimes I_T) y \right] \qquad (3.19) \\
&= \left[(I_m \overline{S}^{-1} \otimes R' \overline{H}^{-1})(I_m \otimes R) + (I_m \Sigma_u^{-1} \otimes X' I_T)(I_m \otimes X) \right]^{-1} \\
&\quad \left[(I_m \overline{S}^{-1} \otimes R' \overline{H}^{-1})\bar{b} + (I_m \Sigma_u^{-1} \otimes X' I_T) y \right] \\
&= \left[\overline{S}^{-1} \otimes \overline{H}^{-1} + \Sigma_u^{-1} \otimes X' X \right]^{-1} \left[(\overline{S}^{-1} \otimes \overline{H}^{-1})\bar{b} + (\Sigma_u^{-1} \otimes X') y \right]
\end{aligned}$$

$$Var(\widehat{\beta}) = (X^{*}{}'V^{*-1}X^{*})^{-1} = \left[\overline{S}^{-1} \otimes \overline{H}^{-1} + \Sigma_u^{-1} \otimes X'X\right]^{-1} \quad (3.20)$$

The above GLS estimators, equation (3.19) and (3.20), are equivalent to posterior mean and variance. Under Minnesota prior, $\beta \sim N(\overline{b}, \overline{S} \otimes \overline{H})$, the prior distribution of β is

$$\mathrm{p}(\beta) \propto \left|\overline{S} \otimes \overline{H}\right|^{-1/2} \exp\left[-\frac{1}{2}(\beta - \overline{b})'(\overline{S} \otimes \overline{H})^{-1}(\beta - \overline{b})\right] \quad (3.21)$$

Since $U \sim N(0, \Sigma_u \otimes I_T)$, therefore the likelihood function of β is

$$L(\beta \mid X, y) = (2\pi)^{-mT} \left|\Sigma_u \otimes I_T\right|^{-1/2} \exp\left[-\frac{1}{2}(y - (I_m \otimes X)\beta)'(\Sigma_u \otimes I_T)^{-1}(y - (I_m \otimes X)\beta)\right] \quad (3.22)$$

Combining the above likelihood with prior distribution, the posterior distribution of β is

$$\begin{aligned} \mathrm{p}(\beta \mid X, y) &\propto \mathrm{p}(X, y \mid \beta) \times \mathrm{p}(\beta) \\ &= L(\beta \mid X, y) \times \mathrm{p}(\beta) \\ &\propto \exp\left[-\frac{1}{2}(\beta - \widehat{\beta})'(\Sigma_\beta)^{-1}(\beta - \widehat{\beta})\right] \end{aligned} \quad (3.23)$$

Therefore, the posterior distribution of β is $\beta \sim N(\widehat{\beta}, \widehat{Var(\beta)})$, where the Bayesian estimator of β are the same as GLS estimator $\widehat{\beta}$ and $Var(\widehat{\beta})$ (eqn. (3.19) and (3.20)).

3.3.3 Sims and Zha prior (SZ prior)

The specification of the Minnesota prior implies that the variance matrix of the residual is fixed and diagonal, which means the residual variance is known and the equations are independent. One way to generalize the prior is to allow for non-diagonal residual variance by introducing prior on the residual variance.

When the prior beliefs are of the Minnesota type, a number of prior distributions can be used. Kadiyala and Karlsson (1997) compare the performance of analyzing the posterior distribution and the credibility of the prior specifications among diffuse, normal-Wishart, normal-diffuse, and extended natural conjugate prior, and normal-Wishart is preferred. Under a Normal-Wishart prior, the prior distribution of coefficient is Normal $B \sim MN(\overline{B}, \Sigma, \overline{\overline{H}})$, while the prior distribution of covariance matrix Σ is inverse Wishart $\Sigma \sim \text{IW}(\overline{S}, \overline{v})$, where $\overline{v}$ is the degree of freedom, which is equal to the number of dependent variables plus one. The difference between $\overline{\overline{H}}$ and $\overline{H}$ in the Minnesota prior is that λ_2 is set to unity in $\overline{\overline{H}}$ so that all equations can be treated symmetrically. Unlike the Minnesota prior, Normal-Wishart prior can't be written as dummy observations and estimate the coefficients with the data using GLS, however we can estimate them by using posterior distribution.

Consider the reduced form VAR model

$$Y = XB + \text{E} \qquad \text{E}_t \sim i.i.d\ N(0,\Sigma)\ and\ \text{E} \sim MN(0,\Sigma,I_T) \tag{3.24}$$

The likelihood function of equation (3.24) is

$$\begin{aligned} L(B,\Sigma \mid X,Y) &= (2\pi)^{-\text{T}m/2} |\Sigma|^{-\text{T}/2} \exp\left\{-\frac{1}{2}tr\left[\Sigma^{-1}(Y-XB)'(Y-XB)\right]\right\} \\ &\propto |\Sigma|^{k/2} \exp\left\{-\frac{1}{2}tr\left[\Sigma^{-1}(B-\overline{\overline{B}})'X'X(B-\overline{\overline{B}})\right]\right\} \\ &\quad \times |\Sigma|^{-(n-k)/2} \exp\left\{-\frac{1}{2}tr\left[\Sigma^{-1}S\right]\right\} \\ &= N(\overline{\overline{B}},\Sigma \otimes (X'X)^{-1}) \times iW(S,v) \end{aligned} \tag{3.25}$$

Where the MLE of B and Σ are, respectively, $\overline{\overline{B}} = (X'X)^{-1}X'Y$ and $S = (Y - X\overline{\overline{B}})'(Y - X\overline{\overline{B}})$, $v = T - k - m - 1$ is the degree of freedom of inverse Wishart distribution.

Under the Normal-Wishart prior, $B \sim MN(\overline{B}, \Sigma, \overline{\overline{H}})$, $\Sigma \sim IW(\overline{S}, \overline{v})$, the mean and variance of B are $E(B) = \overline{B}$ and $Var(vec(B)) = (\overline{v} - m - 1)^{-1}\overline{S} \otimes \overline{\overline{H}}$, where Σ is the variance matrix of the columns of B and $\overline{H}$ the variance matrix of the rows of B. Therefore, the prior distribution of B and Σ is

$$\begin{aligned} p(B,\Sigma) \propto |\Sigma|^{-k/2} \exp\left\{-\frac{1}{2} tr\left[\Sigma^{-1}(B-\overline{B})'\overline{H}^{-1}(B-\overline{B})\right]\right\} \\ \times |\Sigma|^{-(\overline{v}+m+1)/2} \exp\left\{-\frac{1}{2} tr\left[\Sigma^{-1}\overline{S}\right]\right\} \end{aligned} \tag{3.26}$$

Combining the likelihood function, equation (3.25) and prior distribution, equation (3.26), the posterior distribution of B and Σ is

$$\begin{aligned} p(B,\Sigma \mid X,Y) &\propto p(X,Y \mid B,\Sigma) \times p(B,\Sigma) \\ &= L(B,\Sigma \mid X,Y) \times p(B,\Sigma) \\ &= |\Sigma|^{-k/2} \exp\left\{-\frac{1}{2} tr\left[\Sigma^{-1}(B-B^*)'H^{*-1}(B-B^*)\right]\right\} \\ &\times |\Sigma|^{-(v^*+m+1)/2} \exp\left\{-\frac{1}{2} tr\left[\Sigma^{-1}S^*\right]\right\} \\ &\propto N(B^*,\Sigma,H^*) \times iW(S^*,v^*) \end{aligned} \tag{3.27}$$

Where

$$\begin{aligned} H^* &= (\overline{\overline{H}}^{-1} + X'X)^{-1} \\ B^* &= H^*(\overline{\overline{H}}^{-1}\overline{B} + X'X\overline{\overline{B}}) \\ S^* &= \overline{S} + S + \overline{\overline{B}}'X'X\overline{\overline{B}} + \overline{B}\overline{\overline{H}}^{-1}\overline{B} - B^{*\prime}H^*B^* \\ v^* &= \overline{v} + T \end{aligned} \tag{3.28}$$

The marginal posterior distribution of B can be obtained by integrating out Σ of the joint posterior distribution, which is matricvariante t-distribution:

$$B \sim MT(B^*, H^{*-1}, S^*, v^*) \tag{3.29}$$

Therefore, the estimation of coefficient B and Σ have the following forms:

$$
\begin{aligned}
B^* &= (\overline{H}^{-1} + X'X)^{-1}(\overline{H}^{-1}\overline{B} + X'Y) \\
S^* &= \overline{S} + S + \overline{\overline{B}}'X'X\overline{\overline{B}} + \overline{BH}^{-1}\overline{B} - B^{*}{}'H^*B^*
\end{aligned}
\tag{3.30}
$$

Besides the Normal-Wishart prior, two additional priors on linear combinations of the coefficients are implemented by using initial dummy observations instead of specifying the prior covariance structures. The sum of coefficient prior, due to Doan et al (1984), allows unit root in the first difference of data, namely the sum of the coefficients on the lags of the dependent variable in each equation equal to one while coefficients on lags of other variables sum to zero (Robeson and Tallman 1999). This prior adds m (number of dependent variables) initial dummy observations with $\mu_5 \overline{y^i}$ of-diagonal, zero off-diagonal elements for the dependent variable, which is defined as matrix A.The dummy observations for the regressors are specified as $[0\,A \ldots A]$, where 0 is of $m \times 1$ dimension and the total number of matrix A equals to the lag of the dependent variable p. Take 2-variable, 2-lag (m=2, p=2) for example, $A = \begin{bmatrix} \mu_5 \overline{y_1} & 0 \\ 0 & \mu_5 \overline{y_2} \end{bmatrix}$, then the dummy observation becomes

$$
\begin{bmatrix} \mu_5 \overline{y_1} & 0 \\ 0 & \mu_5 \overline{y_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{21} \\ \beta_{12} & \beta_{22} \end{bmatrix} \begin{bmatrix} \mu_5 \overline{y_1} & 0 \\ 0 & \mu_5 \overline{y_2} \end{bmatrix} + \begin{bmatrix} \beta_{13} & \beta_{23} \\ \beta_{14} & \beta_{23} \end{bmatrix} \begin{bmatrix} \mu_5 \overline{y_1} & 0 \\ 0 & \mu_5 \overline{y_2} \end{bmatrix} \tag{3.31}
$$

Equivalently, $\beta_{11} + \beta_{13} = 1, \beta_{22} + \beta_{24} = 0, \beta_{12} + \beta_{14} = 1 \; and \; \beta_{21} + \beta_{23} = 0$, namely the coefficients of its own lags sum to unity while coefficients on lags of other variables sum to zero. The hyperparameter $\mu_5 \geq 0$ expresses the tightness of the prior, as $\mu_5 \to \infty$, the model tend to be unit root.

The prior of the sum of coefficient introduces m dummy observations, which express the belief that each variable is a unit root and there is no cointergration. However, it is possible that there are some long-run stable relations among the VAR variable. The second prior, developed by Sims (1992), considers the possibility of cointergration by introducing one more dummy observation with $\mu_6 \overline{y_i}$, where $i = 1,2,3...m$ for dependent variable and [1 $\mu_6 \overline{y_i} \cdots \mu_6 \overline{y_i}$], where $i = 1,2,3...m$ and the total number of $\mu_6 \overline{y_i}$ equals to the lag of the dependent variable p. Take 2-variable, 2-lag (m=2, p=2) for example, the prior is expressed as

$$\begin{bmatrix} \mu_6 \overline{y_1} & \mu_6 \overline{y_2} \end{bmatrix} = \begin{bmatrix} \beta_{10} & \beta_{20} \end{bmatrix} + \begin{bmatrix} \mu_6 \overline{y_1} & \mu_6 \overline{y_2} \end{bmatrix} \begin{bmatrix} \beta_{11} & \beta_{21} \\ \beta_{12} & \beta_{22} \end{bmatrix} + \begin{bmatrix} \mu_6 \overline{y_1} & \mu_6 \overline{y_2} \end{bmatrix} \begin{bmatrix} \beta_{13} & \beta_{23} \\ \beta_{14} & \beta_{24} \end{bmatrix} \quad (3.32)$$

Denote $Y = \begin{bmatrix} \mu_6 \overline{y_1} & \mu_6 \overline{y_2} \end{bmatrix}, C = \begin{bmatrix} \beta_{10} & \beta_{20} \end{bmatrix}, B_1 = \begin{bmatrix} \beta_{11} & \beta_{21} \\ \beta_{12} & \beta_{22} \end{bmatrix}, B_2 = \begin{bmatrix} \beta_{13} & \beta_{23} \\ \beta_{14} & \beta_{24} \end{bmatrix}$, the dummy observation after rearranging is $(I - B_1 - B_2)Y = C$, which implies there is only one stochastic trend in the VAR. The hyperparameter $\mu_6 \geq 0$, and $\mu_6 \to \infty$ implies there is only one unit root in the equations.

3.4. Model specifications

3.4.1 The model

In order to answer whether misevaluation affect real variable, we estimate the effect of misvaluation on MPK, equity issue, and investment by using BVAR. The definition of the variables see appendix B. The order of the vector is $\begin{bmatrix} MPK_t & Misvaluation_t & Issue_t & I_t / K_t \end{bmatrix}$. By the Akaike information criterion (AIC), the lag length is set equal to 2. Lower Choleski decomposition is used to identify the VAR model, which means misvaluation shock has contemporaneous effect on equity issue and investment, but not on MPK. Similar order and identification are used in GHH (2005). The specifications of the BVAR models under the Minnesota prior and Sims-Zha prior are as follows.

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} B = \begin{bmatrix} \beta_{10} & \beta_{20} & \beta_{30} & \beta_{40} \\ \beta_{11} & \beta_{21} & \beta_{31} & \beta_{41} \\ \beta_{12} & \beta_{22} & \beta_{32} & \beta_{42} \\ \beta_{13} & \beta_{23} & \beta_{33} & \beta_{43} \\ \beta_{14} & \beta_{24} & \beta_{34} & \beta_{44} \\ \beta_{15} & \beta_{25} & \beta_{35} & \beta_{45} \\ \beta_{16} & \beta_{26} & \beta_{36} & \beta_{46} \\ \beta_{17} & \beta_{27} & \beta_{38} & \beta_{47} \\ \beta_{18} & \beta_{28} & \beta_{38} & \beta_{48} \end{bmatrix} \overline{B} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\overline{H} = \begin{bmatrix} (\lambda_0\lambda_4)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (\lambda_0\lambda_1/\sigma_1)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (\lambda_0\lambda_1/\sigma_2)^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (\lambda_0\lambda_1/\sigma_3)^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (\lambda_0\lambda_1/\sigma_4)^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (\lambda_0\lambda_1/2^{\lambda 3}\sigma_1)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & (\lambda_0\lambda_1/2^{\lambda 3}\sigma_2)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & (\lambda_0\lambda_1/2^{\lambda 3}\sigma_3)^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (\lambda_0\lambda_1/2^{\lambda 3}\sigma_4)^2 \end{bmatrix}$$

$$\overline{S} = \begin{bmatrix} (\sigma_1/\lambda_0)^2 & 0 & 0 & 0 \\ 0 & (\sigma_2/\lambda_0)^2 & 0 & 0 \\ 0 & 0 & (\sigma_3/\lambda_0)^2 & 0 \\ 0 & 0 & 0 & (\sigma_4/\lambda_0)^2 \end{bmatrix}$$

$$\overline{v} = 5$$

$$Y = \begin{bmatrix} \mu_5 \overline{y_1} & 0 & 0 & 0 \\ 0 & \mu_5 \overline{y_2} & 0 & 0 \\ 0 & 0 & \mu_5 \overline{y_3} & 0 \\ 0 & 0 & 0 & \mu_5 \overline{y_4} \\ \mu_6 \overline{y_1} & \mu_6 \overline{y_2} & \mu_6 \overline{y_3} & \mu_6 \overline{y_4} \\ y_{1,3} & y_{2,3} & y_{3,3} & y_{4,3} \\ y_{1,4} & y_{2,4} & y_{3,4} & y_{4,4} \\ & \vdots & & \\ & \vdots & & \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & \mu_5 \overline{y_1} & 0 & 0 & 0 & \mu_5 \overline{y_1} & 0 & 0 & 0 \\ 0 & 0 & \mu_5 \overline{y_2} & 0 & 0 & 0 & \mu_5 \overline{y_2} & 0 & 0 \\ 0 & 0 & 0 & \mu_5 \overline{y_3} & 0 & 0 & 0 & \mu_5 \overline{y_3} & 0 \\ 0 & 0 & 0 & 0 & \mu_5 \overline{y_4} & 0 & 0 & 0 & \mu_5 \overline{y_4} \\ \mu_6 & \mu_6 \overline{y_1} & \mu_6 \overline{y_2} & \mu_6 \overline{y_3} & \mu_6 \overline{y_4} & \mu_6 \overline{y_1} & \mu_6 \overline{y_2} & \mu_6 \overline{y_3} & \mu_6 \overline{y_4} \\ 1 & y_{1,2} & y_{2,2} & y_{3,2} & y_{4,2} & y_{1,1} & y_{2,1} & y_{3,1} & y_{4,1} \\ 1 & y_{1,3} & y_{2,3} & y_{3,3} & y_{4,3} & y_{1,2} & y_{2,2} & y_{3,2} & y_{4,2} \\ & & & & \vdots & & & & \\ & & & & \vdots & & & & \end{bmatrix}$$

3.4.2 Hyperparameter specifications

The hyperparameters used in the BVAR express the belief of priors. Some previous works use common values of the hyperparemeters, which will affect the estimation accuracy, since the hyperparemeters depend on the dataset. In order to make sure the priors are proper, the hyperparameters in the priors are chosen to fit the data instead of arbitrary conjecture. Doan et al (1983) and Todd (1988) select hyperparemeters by minimizing the out-of-sample forecast error. Since measuring the fit by forecasting performance corresponds to how well a set of priors fit the data, we choose the hyperparemeters by maximizing the marginal likelihood of the data directly. The log of the marginal likelihood is

$$\ln p(X,Y) = \ln p(X,Y \mid B,\Sigma) + \ln p(B,\Sigma) - \ln p(B,\Sigma \mid X,Y) \tag{3.33}$$

Where $p(X,Y \mid B,\Sigma)$ is the likelihood function $L(B,\Sigma \mid X,Y)$, $p(B,\Sigma)$ the prior, $p(B,\Sigma \mid X,Y)$ the posterior density.

First we take an initial guess of these hyperparemeters, and then the marginal likelihood is maximized by using axial search. Each hyperparemeter is searched on the grid, and the point of the grid is picked when it optimizes the marginal likelihood while holding the other hyperparemeters constant. The procedure is repeated for each hyperparemeter until the marginal likelihood stay stable. Table 2 reports the hyperparameters estimated by maximizing the marginal likelihood under Minnesota and SZ prior respectively.

3.5 Data and empirical results

3.5.1 Data

The data is from Compustat. Due to data availability, manufacturing industrial quarterly data are used and the time period is from 1984:1 to 2003:4, therefore the total observation is 80. Since the time series is short compared to the total number of coefficients to be estimated, the overfitting problem is serious for VAR estimation. Therefore, we apply BVAR not only in the estimation of matrix A, which is needed to calculate fundamental q, but also in the estimation of the effect of misvaluation on investment. We drop the observations if either one of the data is missing. Since the number of firms Compustat industrial quarterly includes is different for each time period, therefore we use average data, which is obtained by first summing up the variable, then dividing the aggregate variable by the number of firms. We first estimate the coefficients and then obtain the impulse response under Minnesota prior and SZ prior respectively to test whether investment always respond to misvaluation under different priors. The summary statistics of these variables under SZ prior is reported after seasonal adjustment in table 3.

3.5.2 Empirical results

Both the statistics and the path figure show that fundamental q is pretty stable during the time period while market q is much volatile due to market price volatility. Therefore, the misvaluaton rises with the increase of market q and hits the peak area during 1998-2001 period, which coincides with the stock market boom during late 1990’s (refer to figure 3.1 for the path of Dow-Jones industrial index). Corresponding to the increase of the misvaluation, equity issue and investment rise during the period (see figure 3.2 for reference), which support the rough idea that equity issuance and investment respond to misvaluation.

Table 3.4 reports the estimates of the coefficients in the BVAR system under different priors. It shows that investment is positively correlated with the first lag of misvaluation, MPK and issuance. Although some of the signs of the second lag of misvaluation, MPK and issuance are negative, the sums of coefficients of the lags are still positive, which implies investments will increase if there is positive shock to misvaluation, MPK and issuance. The results are robust to different priors.

Figure 3.7 shows the impulse response functions. The effect of misvaluation shock on MPK, equity issue and investment are reported under Minnesota and SZ priors respectively. When there is a positive misvaluation shock, the stock market bubble gets bigger and firms will issue more equity and invest more. Under Minnesota prior, a 0.1 misvaluation shock results in investment increase by 0.0015 at the peak quarterly, approximately 3.2% (0.0015/0.047) of the mean of investment quarterly, or 12.8% annually. In order to compare the effect of misvaluation with other empirical results using one standard deviation of the misvaluation, we calculate the response of investment to one stand deviation misvaluation shock 0.361, which results in about 46.2% investment increases annually under the Minnesota prior.

The results under the SZ prior are similar to the results under Minnesota prior except that issue and investment response are a little bit stronger. Under SZ prior, investment increases about 15.2% corresponding to 0.1 mievaluation shock and 54.7% annually corresponding to one standard deviation of the misvaluation 0.361. The above results are consistent with the result of BSW (2003), indicating over 50% increase of investment relative to the sample median for equity dependent firms, and CS (2004), reporting 40% to 50% increase in investment ratio corresponding to one standard deviation increases in misvaluation respectively.

Both the dynamics of the impulse response functions under the Minnesota and SZ priors show that investments respond to misvaluation with some lag. If the shock hits misvaluaton at time 0, investment does not respond immediately, however investment starts to increase since then and reach the peak area in the first year. After that, the respond keeps on diminishing, but still at a relatively high level in the following 3 years. Then the effect of misvaluation on investment starts to die off. The pattern of the response shows that the effect of misvaluation on investment is quite persistent, at least keeping a relatively high level in 5 years since the shock of misvaluation hits the economy. Therefore, misvaluation is not a sideshow, its important role in investment come not only from the magnitude of the effect, but also from the persistence of the effect.

Another interesting finding from the dynamic path of the response is that timing is important when running linear regression of investment on misvaluation since the peak response occurs in the first year and starts to diminish after that. Regressions of year end investment or annual average investment on current misvaluation or one lagged misvaluation will make the coefficient of misvaluation on investment different. The response path indicates that regression of year end investment on one lagged misvaluation, which best captures the effect in terms of timing, will show larger coefficient than those of other regressions. In order to check the above speculation, we run simple regressions of investment on one lagged and four lagged miavaluation using our quarterly data, the coefficient of the misvaluation decreases from 0.014 for one lagged to 0.011 for four lagged misvaluation, which indicates that the timing of the regression may be one possible explanation for the different magnitude of the effect in the literature so far. For example, in MSV (1990), the regression of investment growth on one lag stock market return, defined as the value-weighted index cumulative dividend return, cannot best capture the response of investment to misvaluation in terms of the timing. We suspect the regression of investment growth on current value-weighted index cumulative dividend return will demonstrate a larger coefficient of misvaluation.

In order to determin whether the results are sensitive to the average data, we estimate the model by using aggregate data, summing up all the unbalanced panel data. Another alternative way to do it is to apply to balanced panel data. Due to the data availability, there are 32 firms with complete data during 1984:1 to 2003:4. The results show that misvaluation affects marginal product of capital, equity issue, and investment in the similar way where average data is applied. Therefore, the results are not affected by the type of data applied in BVAR.

3.6 Robustness check

3.6.1 IRF using VAR model

In order to demonstrate the advantages of BVAR models over VAR models for short time series data, in this section, we apply VAR model to the 80 monthly observations and compare the impulse response functions estimated from VAR model to those from BVAR model. The impulse response functions estimated from VAR model are plotted in figure 3.5. From this figure, we can see that misvaluation and MPK's responses to positive misvaluation shock are similar to those estimated using BVAR model. However, equity issue and investment drop responding to positive misevaluation shock, which is not consistent with the theory. This inconsistency is a reflection of the outfitting problem caused of applying VAR model to short time series data.

3.6.2 Forecasting error comparison between BVAR and VAR models

In order to examine the advantages of applying BVAR models to short time series data over VAR models, we forecast 12 quarters ahead MPK, misvalation, equity issue and investment using both VAR and BVAR models, and the results are reported in table 3.7 and figure 3.6. As expected, for all variables BVAR models give much more precise forecasts with much lower mean square error compared to VAR models given the mean square errors calculated using VAR model are much larger than those using BVAR model.

3.7 Conclusion

To capture the dynamic effect of the stock market on investment in short time series, we develop a BVAR model under different priors to calculate the impulse response of investment to the shock to misvaluation, defined as the difference of market q and fundamental q. Our empirical results under Minnesota and SZ priors show that investment responds 47%-55% at maximum annually to one standard deviation of misvaluation and the effect lasts persistently for at least 5 years at a relatively high level. We also find that the peak response occurs in the first year and starts to diminish afterwards. Therefore, misvaluation is no longer a sideshow, its important role in investment come not only from the magnitude of the effect, but also from the persistence of the effect.

Our work highlights the importance of timing when running linear regression of investment. Regressions of year end investment or annual average investment on current misvaluation or lagged misvaluation will make the coefficient of misvaluation on investment very different, which may be one possible explanation for the mixed story of the misvaluaion effect in the previous literature. Since the investment dynamics investigated in this paper are based on manufacturing industry average data of the United States from 1984:1 to 2003:4, the story may be different for other data sets, for example, in GHH (2005), the response of investment reaches the peak in the third year and diminishes after that. Although the specific pattern may be different, investment always responds with some lags to the misvaluation shock, therefore it is more reasonable to estimate the investment effect based on the lagged misevaluation than the current one. To capture the effect mostly, it is more appropriate to use three lags of misvaluation for annual data when estimate misvaluation effect on investment in linear regression, suggested by the results in the paper.

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Appendix A: Calculation of market q

Largely following Lindenberg and Ross (1981) and Hall (1990)

A.1 Market value of the firm

V_c = Market value of common stock

= Fiscal-year-end closing price (Compustat 14) *number of common shares outstanding (#61)

V_p = Market value of preferred stock

= Capitalized value of preferred dividends

$$= \frac{Total\ \ preferred\ dividends\ (Compustat\ \#24\ )}{Moody's\ Baa\ quarterly\ industrial\ bond\ \ yield\ /\ 4}$$

V_{sd} = Market value of short-term liabilities

= book value of current liabilities (#49)

V_{ld} = Market value of long term debt (assume debt is evenly distributed in 20 years)

$$= \sum_{i-2}^{20} \frac{Long\ term\ debt\ (\#51)/19}{Moody's\ Baa\ quarterly\ industrial\ bond\ \ yield_i\ /\ 4}$$

V = Market value of the firm

$$= V_c + V_p + V_{sd} + V_{ld}$$

A.2 Replacement cost of capital

RC_{qa} = Replacement cost of quick assets

= current asset (#40)-inventory (#38)

$RC_{p\&e}$ = Replacement cost of plant and equipment

$$= \ Net\ \ property, plant\ \&\ equipment(\#42) * \frac{GNP_t}{GNP_{t-AA}}$$

Where GNP is GNP deflator, from Department of Commerce: Bureau of Economic

Analysis

AA = Average age of property, plant & equipment

$$= \frac{Gross\ plant\ \&\ equipment(\#118) - Net\ plant\ \&\ equipment(\#42)}{Depreciation\ and\ amortization(\#5)}$$

RC_{inv} = Replacement cost of inventory

$$= \begin{matrix} Inventory(\#38) \quad if\ FIFO \\ Inventory_t(\#38) * (1+\pi_t) + (Inventory_{t+1} - Inventory_t) \quad if\ LIFO \end{matrix}$$

If the firm uses more than one method of inventory valuation (from Compustat industrial annual #59), then the inventory is the weighted summation of different method of inventory. Following Hall (1990), the weight is determined by the following:

Number of methods	Rank of LIFO	LIFO as weight
1	1	1
2	1	2/3
2	2	1/3
3	1	1/2
3	2	1/3
3	3	1/6

RC = Replacement cost of capital

$$= RC_{qa} + RC_{inv} + RC_{p\&e}$$

A.3 Market q

q_m = Market q

$$= \frac{Market\ value\ of\ the\ firm\ (V)}{\text{Re}\ placemnt\ \cos t\ of\ capital\ (RC)}$$

B. Data construction and sources

The variables used are constructed from Compustat industrial quarterly, specifically, we define the variables from 1984:1 to 2004:3 as follows.

i_t^d = Cost of debt, from Moody's Baa quarterly industrial bonds yield

i_t^e = Cost of equity

= EPS (Compustat #11)/Price-close-3rd month of quarter (#14)

∂ = Leverage ratio

= Long-term debt (#51)/Total asset (#44)

S_t / K_t = Sales to capital ratio

= Sales (#2)/Property, plant and equipment (#42)

C_t / K_t = Cost to capital ratio

$$= \frac{Cost\ of\ goods\ sold\ (\#30) + selling, general, and\ ad\min exp enses(\#1)}{Property, plant\ and\ equipment\ (\#42)}$$

π_t = Inflation

$$= \frac{PPI_t - PPI_{t-1}}{PPI_{t-1}}$$

PPI: Producer Price Index (finished goods), from Bureau of Labor Statistics

m_t = Misevaluation (difference between market q and fundamental q)

$= q_m - q_f$

MPK_t = Marginal product of capital

= S_t / K_t - C_t / K_t (Assume constant return to scale and perfect competition, $MPK_t = APK_t$)

neq_t = Equity issuance (normalized by investment)

= Sales of common & preference stock (#84) / Capital expenditures (#90)

I_t / K_t = Investment

= Capital expenditures (#90)/Property, plant and equipment (#42)

Table 3.1: DEFINITIONS OF HYPERPARAMETER

Parameter	Value	Interpretation
λ_1	>0	Overall tightness of random walk prior
λ_2	$(0\,1]$	Weight of other lags
λ_3	>0	Lag decay: rate at which prior variance shrinks with increasing lag length
λ_4	≥ 0	Scale of standard deviation around constant term

TABLE 3.2 HYPERPARAMETER SPECIFICATION

Prior	*ML*	λ_0	λ_1	λ_2	λ_3	λ_4	μ_5	μ_6
Minnesota	-815.4	0.968	18.098	0.220	3.873	6.044		
SZ	-794.46	0.699	13.068	1.000	3.825	7.731	0.998	0.989

Table 3.3: DESCRIPITIVE STATISTICS UNDER SIMS&ZHA PRIORS

Parameter	Mean	S.D.	Min	25th	50th	75th	Max
q_f	1.503	0.080	1.272	1.457	1.509	1.559	1.674
q	1.595	0.403	1.006	1.329	1.507	1.777	2.702
Mis	0.099	0.361	-0.502	-0.133	-0.026	0.249	1.195
MPK	0.080	0.017	0.030	0.069	0.077	0.091	0.124
Issue	0.230	0.124	0.070	0.150	0.202	0.278	0.783
Inves	0.047	0.008	0.032	0.041	0.046	0.052	0.078

TABLE 3.4: BVAR ESTIMATES UNDER MINNESOTA PRIOR

Variable	Mis	MPK	Issue	Inves
Mis(-1)	0.803 (0.062)	0.012 (0.002)	2.323 (0.321)	0.020 (0.0018)
MPK(-1)	1.238 (0.215)	0.452 (0.0037)	6.596 (0.592)	-0.020 (0.003)
Issue(-1)	-0.005 (0.0006)	0.000 (0.000)	0.336 (0.039)	-0.001 (0.000)
Inves(-1)	0.002 (0.00019)	0.000 (0.000)	1.025 (0.189)	0.001 (0.000)
Mis(-2)	0.103 (0.007)	0.001 (0.000)	-0.573 (0.068)	-0.006 (0.000)
MPK(-2)	-0.023 (0.002)	0.007 (0.0006)	0.153 (0.017)	-0.001 (0.0001)
Issue(-2)	0.000 (0.000)	0.001 (0.0001)	-0.214 (0.019)	0.002 (0.0002)
Inves(-2)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Costant	-0.071 (0.008)	0.040 (0.0035)	1.254 (0.187)	0.046 (0.0032)
R square	0.701	0.952	0.785	0.622

TABLE 3.5: BVAR ESTIMATES UNDER SIMS&ZHA PRIOR

Variable	Mis	MPK	Issue	Inves
Mis(-1)	0.985 (0.089)	0.016 (0.002)	2.074 (0.312)	0.007 (0.000)
MPK(-1)	0.839 (0.081)	0.496 (0.054)	-0.344 (0.042)	0.018 (0.002)
Issue(-1)	-0.015 (0.001)	0.000 (0.000)	0.242 (0.003)	0.000 (0.000)
Inves(-1)	0.348 (0.041)	-0.078 (0.008)	8.513 (0.074)	0.627 (0.056)
Mis(-2)	-0.021 (0.003)	-0.001 (0.000)	-0.291 (0.019)	-0.002 (0.000)
MPK(-2)	-0.132 (0.03)	0.034 (0.012)	0.039 (0.002)	-0.004 (0.001)
Issue(-2)	0.000 (0.000)	0.000 (0.000)	-0.025 (0.003)	0.000 (0.000)
Inves(-2)	0.198 (0.022)	-0.004 (0.000)	-0.976 (0.084)	-0.002 (0.000)
Constant	-0.041 (0.0038)	0.039 (0.004)	1.267 (0.187)	0.016 (0.002)
R square	0.749	0.9656	0.773	0.689

TABLE 3.6: VAR ESTIMATES

Estimates	Mis	MPK	Issue	Inves
Mis(-1)	1.171 (0.116)	1.806 (0.653)	-0.016 (0.004)	-0.034 (0.012)
MPK(-1)	0.026 (0.028)	-0.872 (0.157)	0.005 (0.001)	0.011 (0.003)
Issue(-1)	6.228 (3.395)	-128.423 (19.175)	0.832 (0.111)	1.794 (0.358)
Inves(-1)	-0.570 (1.330)	-16.396 (7.510)	0.038 (0.043)	0.236 (0.140)
Mis(-2)	-0.240 (0.120)	-1.127 (0.675)	0.015 (0.004)	0.041 (0.013)
MPK(-2)	0.006 (0.035)	0.535 (0.195)	-0.004 (0.001)	-0.007 (0.004)
Issue(-2)	-2.718 (3.524)	56.677 (19.900)	-0.676 (0.115)	-0.782 (0.372)
Inves(-2)	2.162 (1.370)	-14.833 (7.738)	0.032 (0.045)	0.225 (0.144)
C	-0.167 (0.175)	4.755 (0.988)	0.002 (0.006)	0.011 (0.018)
R square	0.532	0.944	0.667	0.471

TABLE 3.7: REAL DATA AND FORECASTS USING VAR AND BVAR MODELS

Date	MPK			Misvaluation		
	Observed	VAR	BVAR	Observed	VAR	BVAR
2001q1	1.292	1.422	0.765	0.061	0.472	0.127
2001q2	1.569	1.689	2.479	0.075	0.481	0.085
2001q3	2.534	2.242	2.077	0.133	0.457	0.057
2001q4	2.588	1.174	1.723	0.151	0.462	0.072
2002q1	1.966	1.427	1.831	-0.020	0.474	0.141
2002q2	1.815	1.725	1.905	-0.156	0.478	0.173
2002q3	2.769	2.204	1.879	-0.203	0.481	0.140
2002q4	2.360	1.451	1.865	-0.237	0.486	0.123
2003q1	2.663	1.521	1.873	-0.141	0.491	0.145
2003q2	2.233	1.756	1.877	-0.096	0.495	0.155
2003q3	3.049	2.153	1.877	-0.008	0.500	0.118
2003q4	3.259	1.602	1.877	-0.069	0.504	0.098

Date	Equity Issue			Investment		
	Observed	VAR	BVAR	Observed	VAR	BVAR
2001q1	0.010	0.010	0.013	0.062	0.078	0.070
2001q2	0.012	0.005	0.003	0.059	0.069	0.046
2001q3	0.011	0.006	0.009	0.036	0.055	0.053
2001q4	0.007	0.013	0.011	0.027	0.067	0.058
2002q1	0.008	0.010	0.009	0.042	0.059	0.056
2002q2	0.007	0.007	0.009	0.042	0.056	0.055
2002q3	0.005	0.008	0.009	0.035	0.050	0.056
2002q4	0.005	0.013	0.009	0.027	0.062	0.056
2003q1	0.007	0.011	0.009	0.037	0.061	0.056
2003q2	0.005	0.008	0.009	0.038	0.059	0.056
2003q3	0.003	0.008	0.009	0.033	0.053	0.056
2003q4	0.002	0.012	0.009	0.030	0.061	0.056

TABLE 3.8: THE DIEBOLD-MARIANO TEST OF THE VARIABLES

Variables	Method	MSE	MSE Difference	S(1)	p-value
MPK	VAR	.72	.1233	1.262	0.2069
	BVAR	5967			
Misvaluation	VAR	.2923	.2434	10.57	0.0000
	BVAR	.04887			
Equity issue	VAR	.0000281	7.08e-06	1.575	0.1153
	BVAR	.000021			
Investment	VAR	.0005525	.0001342	5.726	0.0000
	BVAR	.0004183			

Figure 3.1 DJAI from 1984 to 2004

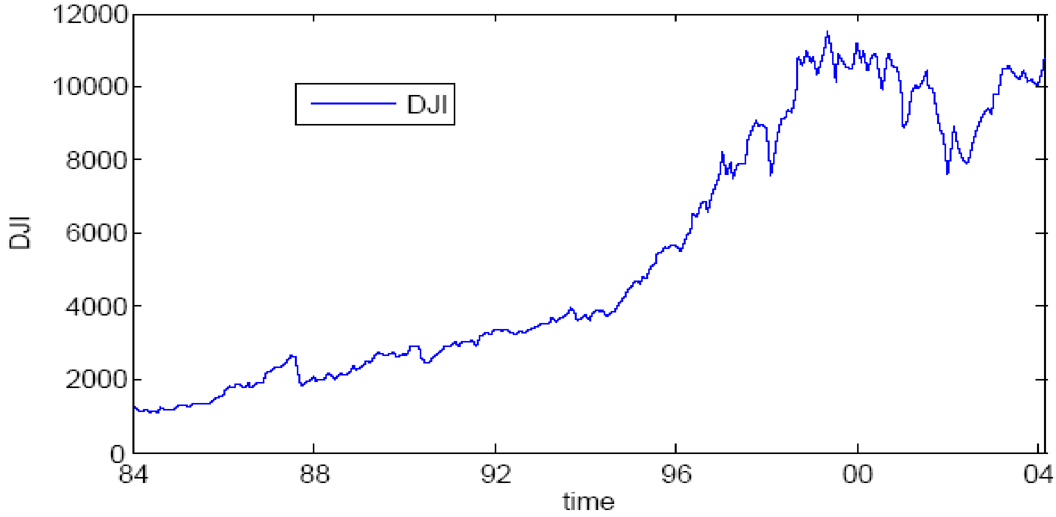


Figure 3.2 Fundamental q, market q and misvaluation

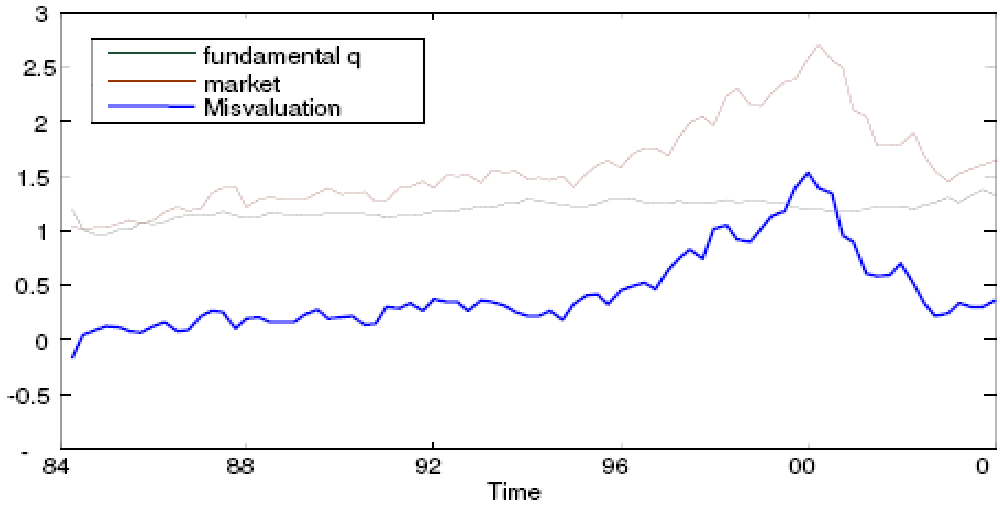


Figure 3.3 Investment

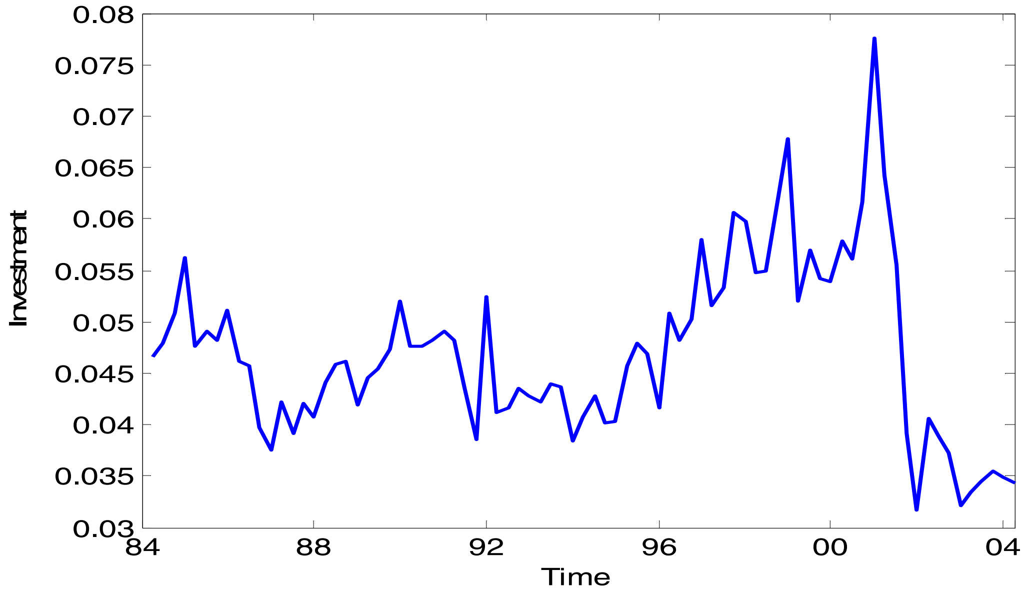


Figure 3.4 Equity issue

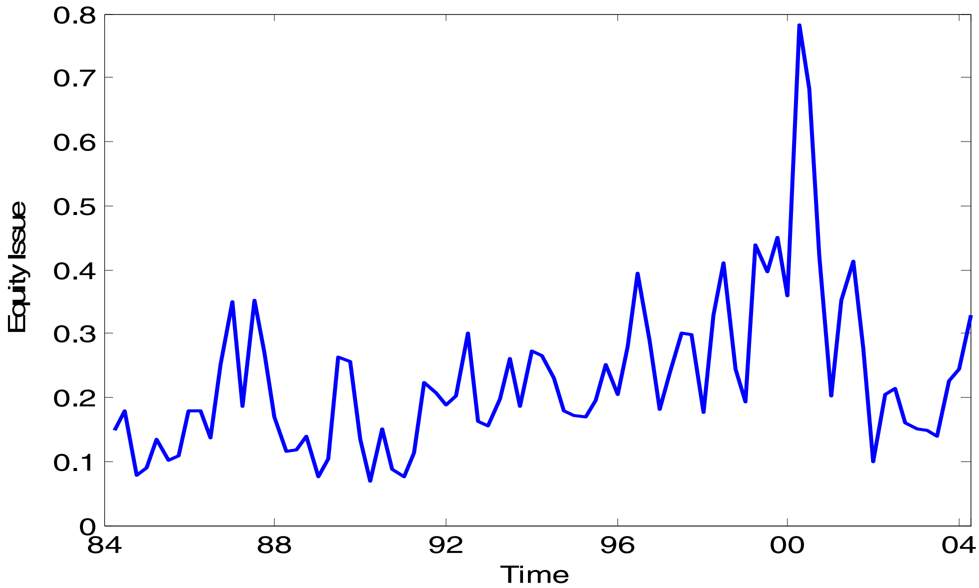


Figure 3.5: Impulse response functions using VAR model

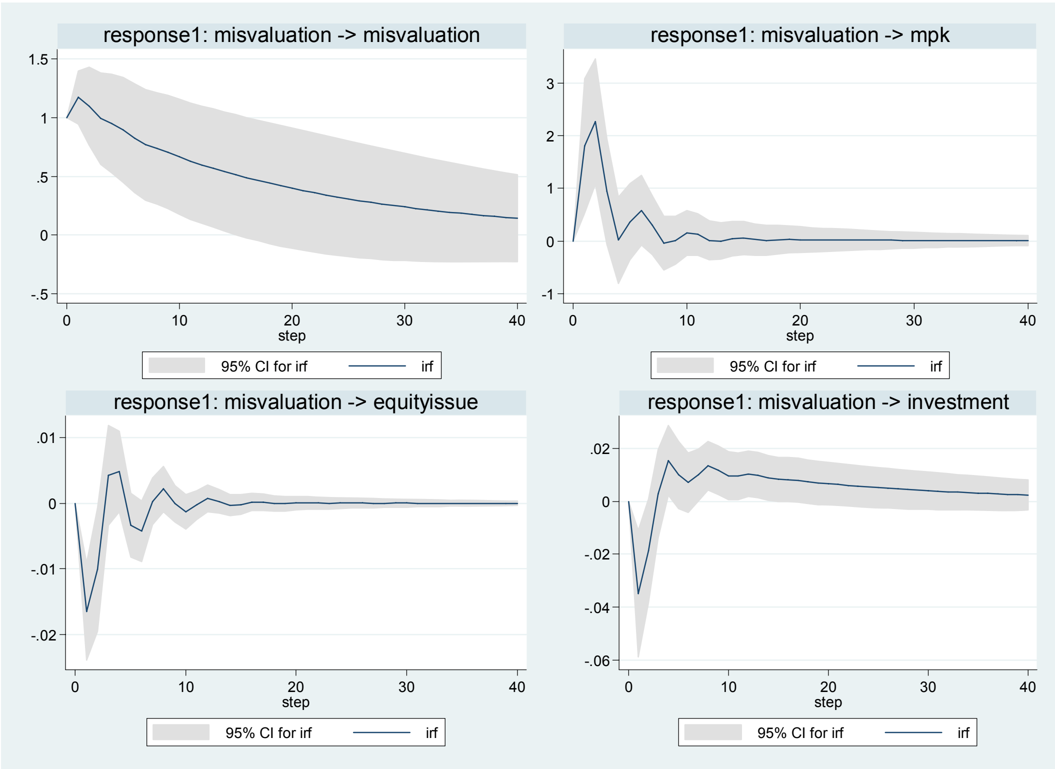
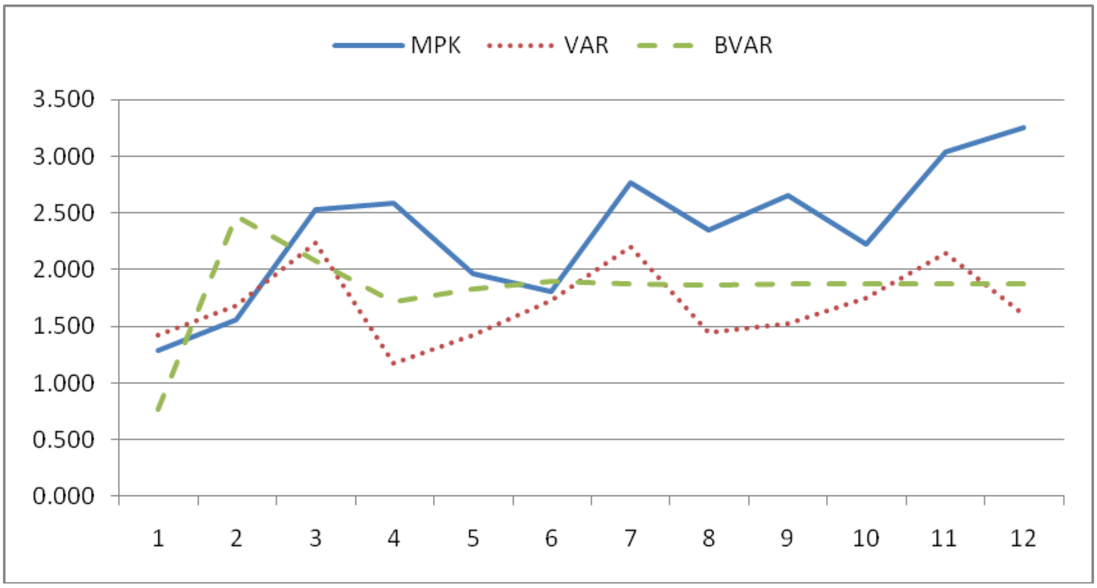
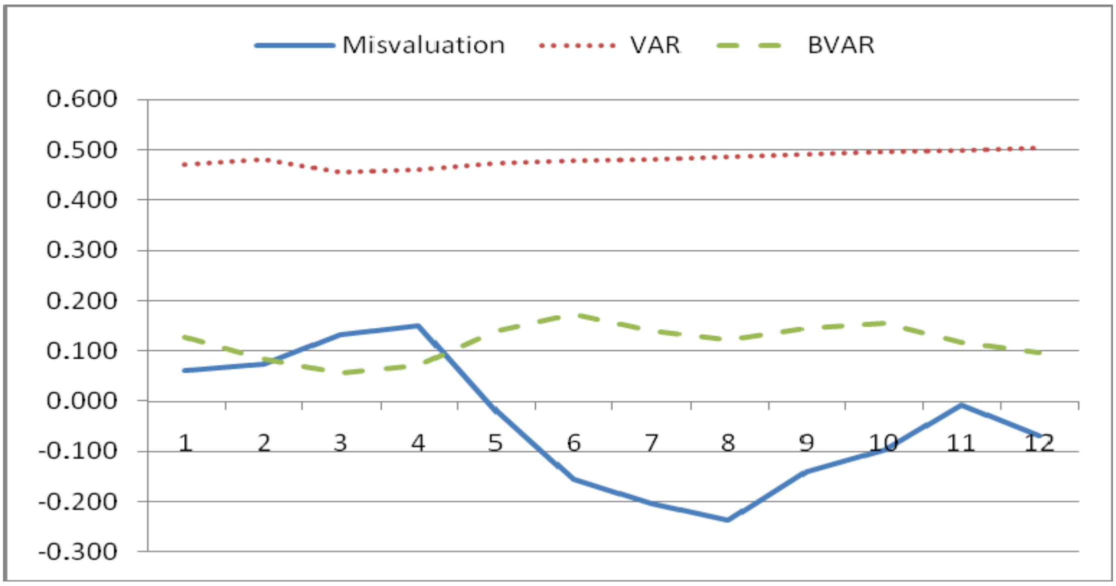
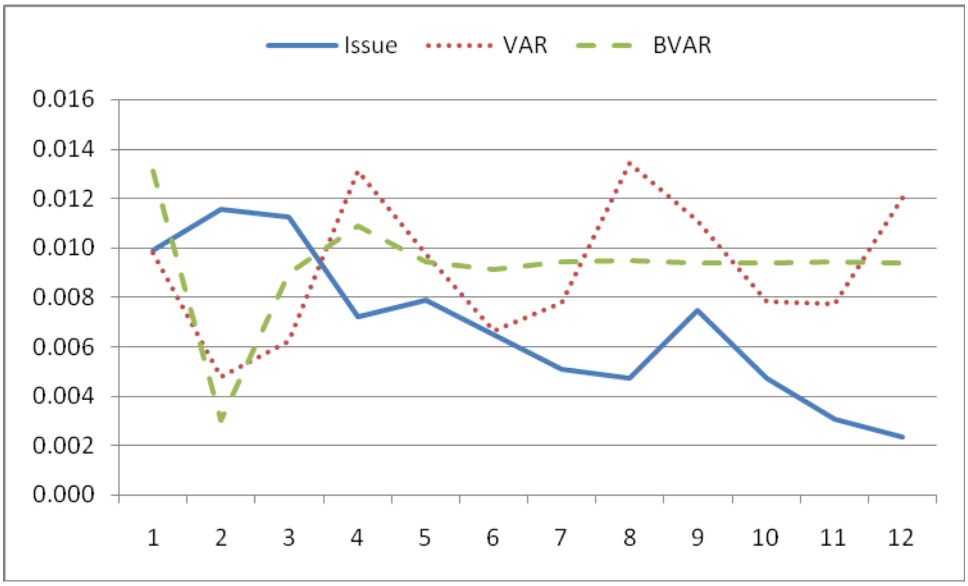


Figure 3.6: Observed data and 12 quarters- ahead forecasting using VAR and BVAR models






Issue
VAR
BVAR
0.016
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
1
2
3
4
5
6
7
8
9
10
11
12

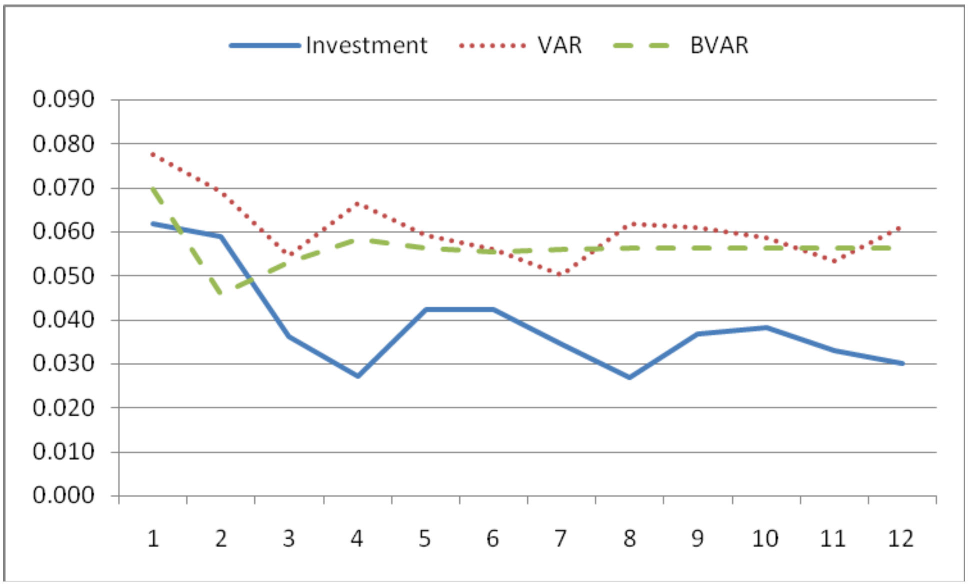
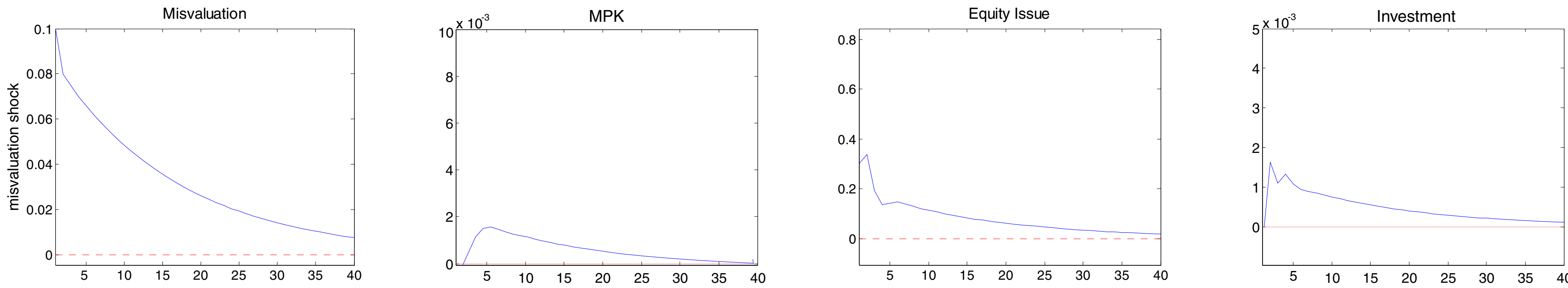
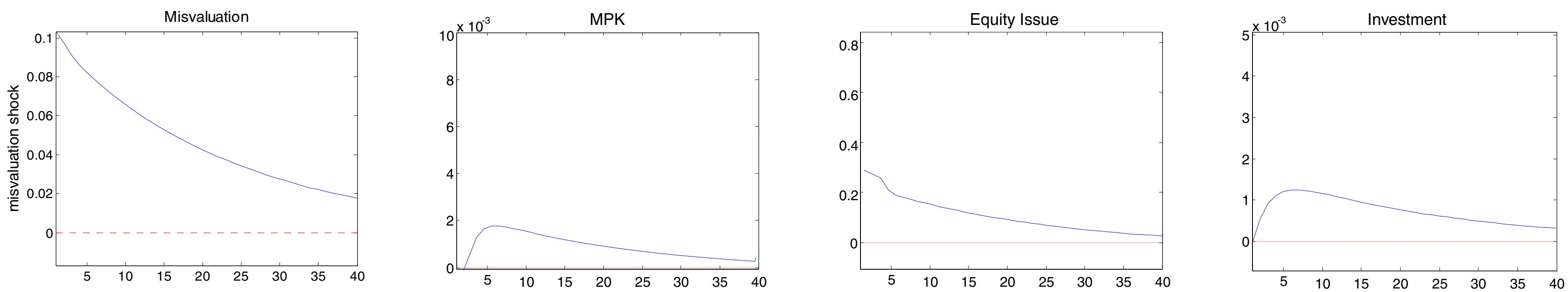

Investment
VAR
BVAR
0.090
0.080
0.070
0.060
0.050
0.040
0.030
0.020
0.010
0.000
1
2
3
4
5
6
7
8
9
10
11
12

Figure 3.7.Impulse Response Function under Minnesota and SZ priors

Minnesota prior



Sims-Zha prior



Chapter 4: Stock Market Bubbles, Fundamentals and Volatility Asymmetry

4.1 Introduction

Understanding the behavior of the variance of stock returns is of great significance for two main reasons. One is that the volatility of stock returns plays a central role in capital asset pricing and pricing of contingent claims such as options. The other is that the changes in market volatility can have important effects on firms fixed investment and consumptions. Many researches on variance have focused on the relationship between stock returns and their conditional volatility since Black (1976) points out this period's stock returns and their next period's conditional variance are typically negatively correlated. This stylized fact that volatility is typically higher after the stock market falls than after it rises is referred to as asymmetric volatility.

The previous literature has attributed the negative relation between stock returns and volatility to the leverage effect and the volatility feedback effect. The leverage effect, due to Black (1976) and Christie (1982), posits that a decrease (increase) in stock prices raises (lowers) financial leverage, which makes the stock more (less) risky, hence resulting in larger (smaller) volatility of subsequent returns. However, as showed in Schwert (1989), Bekaert and Wu (2000), pure leverage effect can only explain a relatively small part of the movements in stock volatility. Daouk and Ng (2006) finds a large portion of market level asymmetry is not explained by financial or operating leverage. More important evidence that questions the effect of the leverage effect on the correlations comes from Avramov, Chordia and Goyal (2006), which not only demonstrates that the asymmetric volatility effect is strong and robust at the daily level where leverage changes are economically small and has no impact on the correlation, but also finds asymmetric volatility for stocks with no leverage.

The volatility feedback effect, developed by Pindyck (1984), French et al (1987) and Campbell and Hentschel (1992), is based on time-varying risk premiums. The main idea of this effect is that large good news about future dividends increases future expected volatility due to volatility persistence, which results in higher required return and lowers the stock return, dampening the positive effect of dividend news. In contrast, large bad dividend news amplifies the negative effect of the news, since large bad news about future dividend increases the expected volatility and required return, which lowers the stock return. On the other hand, small news about future dividend implies lower expected volatility and required return, resulting in higher stock return. Therefore, the volatility feedback effect indicates that stock returns are negatively correlated with their future volatility. Empirically, Campbell and Hentschel (1992) find that the volatility feedback normally has little effect on returns, thus they believe the volatility is due to “other changes in expected stock returns and not to news about future dividends” (pp. 312). Further, Bekaert and Wu (2000) shows that the volatility feedback effect fails to account for the full volatility responses, which indicates there may be other factors driving the time varying risk premiums.

Nonetheless, both the leverage effect and volatility feedback effect cannot fully account for volatility asymmetry, which calls for new insights in understanding how volatility responds to returns. While there are some indications in studies by Blanchard and Watson (1982) and LeRoy (2004) that rational bubbles could cause the negative correlation between returns and volatility, the understanding of how asymmetric volatility could be a result of rational bubbles is far from complete. In this paper, I extensively study the bubble effect on the volatility asymmetry.

Rational bubbles arise when the stock price deviate from the fundamental, which is the summation of discounted prospective dividends, in response to arbitrary, self-fulfilling expectations. These arbitrary, self-fulfilling expectations are mostly driven by variables intrinsically irrelevant to fundamentals[1], therefore, they make stock prices more volatile than those that market fundamentals can account for. Similar to dividend news, large good news about future bubble increases expected future volatility and required return, which lows the stock return, weakening the positive effect of good bubble news. Large bad news of future bubble amplifies the positive effect of the news, because bad bubble news increases the expected volatility and required return, resulting in lower stock return. In contrast, small bubble news lowers expected future volatility and required return, which in turn increases stock returns. Therefore, negative correlations between returns and volatility are generated during the stochastic processes of bubble[2] growth and collapses[3]

Although there are some arguments against bubbles [Diba and Grossman (1988), Flood and Garber (1980), Hamilton and Whiteman (1985) and Hamilton (1986)], they do not necessarily prelude the existence of bubbles. The theoretical models presented in Tirole (1985) and O'Connell and Zeldes (1988) show that rational bubbles can arise in an economy with a growing number of asset holders. Empirically, Rappoport and White (1993, 1994) and West (1987) reject the no bubble hypothesis. Evans (1991) points out that bubble tests in Diba and Grossman (1988) and Hamilton and Whiteman (1985), which are based on investigating whether stock

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[1] There is no consensus yet on the specification of bubbles. Bubbles also specified as dependent on There is no consensus yet on the specification of bubbles. Bubbles also specified as dependent on dividends alone, referred as intrinsic bubble by Froot and Obstfeld (1991), or on both time and dividends, defined as fundamental-dependent bubbles by Ikeda and Shibata (1992).

[2] Previous research on bubbles has specified different forms of bubbles. Since deterministic bubble, which grows monotonically, is not plausible. Stochastic bubbles, which either grow or collapse, and periodically collapsing bubbles developed by Evans (1991) are more realistic.

[3] The large negative returns during crashes can also be explained by investors' preferences as loss aversions or disappointment aversions, which imply asymmetric aversions to gains versus losses. See Kahneman and Tversky (1979) Gul (1991), Barberis and Huang (2001), Barberis, Huang and Santos (2001), Hirshleifer (2001), Ang, Bekaert and Liu (2005) for details.

prices are more explosive or less stationary than dividends, can not detect periodically collapsing bubbles, which is demonstrated by simulations.

Despite the widely recognized existence of bubbles in stock prices, and the bubble explanation of the negative correlation between stock returns and volatility is intuitive, to my best knowledge, there is no formal model addressing how much the correlation is attributed to bubbles. In this paper, I extend the model of Campbell and Hentschel (1992) by decomposing stock prices into fundamentals and bubbles so that the excess stock return is decomposed of news about dividends, news of future stock returns and news about bubbles. Under the assumptions that both the news of dividends and bubbles follow quadratic generalized autoregressive conditional heteroscedasticity (QGARCH) models, and that the expected return on stock is a linear function of the summation of the volatility of both news of dividends and bubbles, stock returns are expressed by both news of dividends and bubbles, thus the negative correlation between returns and volatility are explained by both the volatility feedback effect and bubble effect.

The model is applied to the value-weighted CRSP S&P 500 index over the period of 1926 to 2005 at monthly frequency, and estimated by the maximum likelihood method. Empirical results show that 1) the larger the news is, the more negative the correlations are for both the volatility feedback effect and bubble effect; 2) the correlations due to the volatility feedback effect are much smaller than those due to the bubble effect, which account for more than 90% of the total effect on average;3) when both dividend news and bubble news are present, the bubble effect dominates the volatility feedback effect because of bubble news is larger than dividend news; and 4) despite the relatively small magnitude of the volatility feedback effect, it has a very significant impact on the correlations accounting for about 20%

on average during stock market crashes.

The contribution of this paper is to provide a bubble based explanation for the volatility asymmetry. Further, by decomposing the correlations into the volatility feedback effect and bubble effect, the relative contributions of dividend news and bubble news to the behaviors of the correlations are differentiated, thus the determinants of volatility asymmetry are better understood.

The remainder of the paper is organized as follows. Section 2 derives the correlation between returns and volatility based on an asymmetric model with bubbles. Section 3 describes the data and reports the empirical results estimated by the maximum likelihood method. Section 4 concludes the paper.

4.2 The asymmetric model with bubbles

4.2.1 Rational bubbles

Consider the gross rate of stock return R_t

$$R_t = \frac{P_t + D_t}{P_{t-1}} \tag{4.1}$$

Where P_t denotes the real stock price at the end of time t and D_t is the real dividend paid over time t. Under "no arbitrage" condition, $E_t R_t = 1 + r$, the conditional expected net return is equal to the constant r, and equation (4.1) can be written as first-order difference equation of P_t

$$P_t = (1+r)^{-1} E_t (P_{t+1} + D_{t+1}) \tag{4.2}$$

If the transversality condition, $\lim_{i \to \infty} (1+r)^{-i} E_t P_i = 0$, is satisfied, then the solution to equation (4.2) is the market fundamentals F_t

$$F_t = \sum_{j=1}^{\infty}(1+r)^{-j}E_t D_{t+j} \tag{4.3}$$

However, if the transversality condition does not hold, then the general solution to equation (4.2) is given by

$$P_t = F_t + B_t \tag{4.4}$$

Where B_t is referred as rational bubble, the disparity between the price and the fundamental, which satisfies

$$E_t B_{t+1} = (1+r)B_t \tag{4.5}$$

The bubble is rational in the sense that the expected return on bubbles is the same as that on stock. Let the lower-case letters denote the log form of the variables, then equation (4.5) can be written as the following form provided that bubble is normal distributed, $b_{t+1} = \log B_{t+1} \sim N(E_t b_{t+1}, \sigma^2_{b,t+1})$

$$E_t b_{t+1} = \log(1+r) + b_t - 0.5\sigma^2_{b,t+1} \tag{4.6}$$

Dropping the expectation, equation (4.6) is rewritten as

$$b_{t+1} = \log(1+r) + b_t - 0.5\sigma^2_{b,t+1} + z_{t+1} \tag{4.7}$$

Where z_{t+1} is a random variable, which satisfies $E_{t-j} z_{t+1} = 0$ for all $j \geq 0$

4.2.2 The decomposition of excess stock returns

Unlike Campbell and Shiller (1988), which derives a log-linear approximation to the log real return on stocks with absence of bubbles, we decompose price into fundamentals and bubbles, as equation (4.4) implies, and obtain the log return r_{t+1}, in the presence of bubbles:

$$r_{t+1} \equiv \log(P_{t+1} + D_{t+1}) - \log(P_t) = \log(F_{t+1} + B_{t+1} + D_{t+1}) - \log(F_t + B_t) \tag{4.8}$$

Using a first-order Taylor expansion, and using equation (4.6), equation (4.8) can be approximated by:

$$\begin{aligned} r_{t+1} \approx\ & \theta + \rho_1 f_{t+1} - (1-\lambda-\rho_2)b_{t+1} + (1-\rho_1-\rho_2)d_{t+1} - \lambda f_t \\ & - 0.5(1-\lambda)\sigma_{b,t+1}^2 + (1-\lambda)z_{t+1} \end{aligned} \tag{4.9}$$

Where the constant θ is a nonlinear function of λ, ρ_1, ρ_2. The parameters λ, ρ_1, ρ_2 are the average ratio of the fundamental to the sum of fundamental and bubble, ratio of the fundamental to the sum of fundamental, dividend and bubble, ratio of the bubble to the sum of fundamental, dividend and bubble respectively.

Rewriting equation (4.9) as a first-order difference equation about f_t, its ex ante version can be solved forward to obtain

$$\begin{aligned} f_t =\ & \frac{\delta}{1-\alpha} + \beta E_t \sum_{j=0}^{\infty} \alpha^j d_{t+1+j} - \nu E_t \sum_{j=0}^{\infty} \alpha^j b_{t+1+j} - \lambda^{-1} E_t \sum_{j=0}^{\infty} \alpha^j r_{t+1+j} \\ & - \psi E_t \sum_{j=0}^{\infty} \alpha^j \sigma_{b,t+1+j}^2 + \vartheta E_t \sum_{j=0}^{\infty} \alpha^j z_{t+1+j} + E_t \lim_{i\to\infty} \alpha^i f_{t+i} \end{aligned} \tag{4.10}$$

There are no economic models behind equation (4.10), it is simply derived by approximating an identity, and therefore, it is best thought of as a consistency condition that must be satisfied by any reasonable expectations.

Combining equation (4.9) and (4.10), we have the expression of the excess returns on stock

$$r_{t+1} - E_t r_{t+1} = \eta_{d,t+1} - \eta_{r,t+1} + \eta_{b,t+1} \tag{4.11}$$

Where $\eta_{d,t+1}, \eta_{r,t+1}, \eta_{b,t+1}$ denote news about future dividends, returns and bubbles respectively, whose specific definitions can be found in the appendix. More specifically, $\eta_{d,t+1}$ is the expectation difference of future dividends, and $\eta_{b,t+1}$ is the summation of the expectation difference of future bubbles and volatility of bubbles, thus the magnitudes of news of bubbles and volatility of bubbles are on average much larger than those of dividends. Similar to equation (4.10), there are no economic theories behind equation (4.11), and it is obtained based on approximations and can be thought as a consistency identity.

4.2.3 News about dividends and bubbles

Following Campbell and Hentschel (1992), the news about future dividends $\eta_{d,t+1}$ is assumed to follow a conditionally normal quadratic generalized autoregressive conditional heteroscedasticity (QGARCH) process[4], which permits an asymmetric response to shocks. Specifically, $\eta_{d,t+1}$ is described as QGARCH (1, 1) process as follows:

$$\eta_{d,t+1} \sim N(0, \sigma_{d,t}^2), \tag{4.12}$$

$$\sigma_{d,t}^2 = \omega_d + \partial_d (\eta_{d,t} - b_d)^2 + \beta_d \sigma_{d,t-1}^2 \tag{4.13}$$

Where parameters $\omega_d, \partial_d, b_d, \beta_d$ are all positive. The presence of b_d make the volatility of negative shocks higher than equal-size positive ones, which match the stylized facts of stock returns.

Similar to $\eta_{d,t+1}$, the asymmetric response to the news about news $\eta_{b,t+1}$ is modeled by the assumption that $\eta_{b,t+1}$ also follows QGARCH (1, 1) process, namely

$$\eta_{b,t+1} \sim N(0, \sigma_{b,t}^2), \tag{4.14}$$

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[4] See Engle (1990), Sentana (1991) for detail of QGARCH model. Also see Hentschel (1995) for a survey of the family of symmetric and asymmetric GARCH models.

$$\sigma_{b,t}^2 = \omega_b + \partial_b(\eta_{b,t} - b_b)^2 + \beta_b \sigma_{b,t-1}^2 \tag{4.15}$$

Based on the specifications of $\eta_{d,t+1}, \eta_{b,t+1}$, the third determinant of excess stock return in equation (4.13), news about future expected returns $\eta_{r,t+1}$ can be identified under the assumption that the conditional expected return $E_t r_{t+1}$ is a linear function of the summation of the volatility of $\eta_{d,t+1}, \eta_{b,t+1}$ instead of the volatility of the return itself. The empirical results of Campbell and Hentschel (1992) show the discrepancy is small.

$$E_t r_{t+1} = \mu + \gamma(E_t \eta_{d,t+1}^2 + E_t \eta_{b,t+1}^2) = \mu + \gamma(\sigma_{d,t}^2 + \sigma_{b,t}^2) \tag{4.16}$$

Where γ is the coefficient of relative risk aversion by Merton (1980).

Combining the QGARCH (1, 1) process of $\eta_{d,t+1}, \eta_{b,t+1}$ and equation (4.16), we can obtain the expected stock return $E_t r_{t+1+j}$

$$E_t r_{t+1+j} = \mu + \gamma \left[\frac{\omega_d + \partial_d b_d^2}{1-(\partial_d + \beta_d)} + \frac{\omega_b + \partial_b b_b^2}{1-(\partial_b + \beta_b)} \right] + \gamma \left[(\partial_d + \beta_d)^j \sigma_{d,t}^2 + (\partial_b + \beta_b)^j \sigma_{b,t}^2 \right] \tag{4.17}$$

Equation (4.17) implies that the summation of discounted future expected return $E_{t+1} \sum_{j=1}^{\infty} \alpha^j r_{t+1+j}$ is given by:

$$\begin{aligned} E_{t+1} \sum_{j=1}^{\infty} \alpha^j r_{t+1+j} = \alpha E_{t+1} \sum_{j=0}^{\infty} \alpha^j r_{t+2+j} &= \frac{\mu\alpha}{1-\alpha} + \frac{\gamma\alpha}{1-\alpha} \left[\frac{\omega_d + \partial_d b_d^2}{1-(\partial_d + \beta_d)} + \frac{\omega_b + \partial_b b_b^2}{1-(\partial_b + \beta_b)} \right] \\ &+ \frac{\gamma\alpha}{1-\alpha(\partial_d + \beta_d)} \sigma_{d,t+1}^2 + \frac{\gamma\alpha}{1-\alpha(\partial_b + \beta_b)} \sigma_{b,t+1}^2 \end{aligned} \tag{4.18}$$

Therefore, the news of future expected return $\eta_{r,t+1}$ is:

$$\eta_{r,t+1} = (E_{t+1} - E_t)\sum_{j=1}^{\infty}\alpha^j r_{t+1+j}$$
$$= \lambda_d(\eta_{d,t+1}^2 - \sigma_{d,t}^2 - 2b_d\eta_{d,t+1}) + \lambda_b(\eta_{b,t+1}^2 - \sigma_{b,t}^2 - 2b_b\eta_{b,t+1}) \quad (4.19)$$

$$\textit{Where } \lambda_d = \frac{\gamma\alpha\partial_d}{1-\alpha(\partial_d + \beta_d)}, \quad \lambda_b = \frac{\gamma\alpha\partial_b}{1-\alpha(\partial_b + \beta_b)}.$$

Finally, the stock return r_{t+1} can be obtained by combining equations (4.11), (14.3), (4.15), (4.16) and (4.19):

$$r_{t+1} = \mu + \gamma(\sigma_{d,t}^2 + \sigma_{b,t}^2) + \eta_{d,t+1} + \eta_{b,t+1} - \lambda_d(\eta_{d,t+1}^2 - \sigma_{d,t}^2 - 2b_d\eta_{d,t+1})$$
$$- \lambda_b(\eta_{b,t+1}^2 - \sigma_{b,t}^2 - 2b_b\eta_{b,t+1}) \quad . \quad (4.20)$$
$$= \mu + \gamma(\sigma_{d,t}^2 + \sigma_{b,t}^2) + \kappa_d\eta_{d,t+1} - \lambda_d(\eta_{d,t+1}^2 - \sigma_{d,t}^2) + \kappa_b\eta_{b,t+1} - \lambda_b(\eta_{b,t+1}^2 - \sigma_{b,t}^2)$$

Where $\kappa_d = 2\lambda_d b_d + 1$ is bigger than 1 and $\kappa_b = 2\lambda_b b_b + 1$ is negative. Equation (4.20) decomposes stock returns into three parts, expected stock returns $\mu + \gamma(\sigma_{d,t}^2 + \sigma_{b,t}^2)$, the volatility feedback effect $\kappa_d\eta_{d,t+1} - \lambda_d(\eta_{d,t+1}^2 - \sigma_{d,t}^2)$ and the bubble effect $\kappa_b\eta_{b,t+1} - \lambda_b(\eta_{b,t+1}^2 - \sigma_{b,t}^2)$. The volatility feedback effect investigates how dividend news affects the correlations between returns and volatility. Unlike the volatility feedback effect, which focuses on the fundamentals of stocks, the news of dividends, the bubble effect captures how processes of bubble growth and collapses affect the correlation between returns and volatility. In the real economy, the volatility feedback effect and bubble effect are usually both present and the sign of correlations are determined by their total effect.

4.2.4 Correlations of stock returns and volatility

As observed in equation (4.20), both news of dividends and bubbles have effect on returns, implying that both the volatility feedback effect and bubble effect contribute to the correlations between returns and volatility. Using equation (4.13), (4.15) and (4.20) gives the conditional correlation of today's stock returns and tomorrow's volatility:

$$Corr_t(r_{t+1},\sigma_{t+1}^2)=-\frac{2\left[\partial_d\sigma_{d,t}^2(\kappa_d b_d+\lambda_d\sigma_{d,t}^2)+\partial_b\sigma_{b,t}^2(\kappa_b b_b+\lambda_b\sigma_{b,t}^2)\right]}{\left\{\left[\sigma_{d,t}^2(\kappa_d^2+2\lambda_d^2\sigma_{d,t}^2)+\sigma_{b,t}^2(\kappa_b^2+2\lambda_b^2\sigma_{b,t}^2)\right]\left[2\partial_d^2\sigma_{d,t}^2(\sigma_{d,t}^2+2b_d^2)+2\partial_b^2\sigma_{b,t}^2(\sigma_{b,t}^2+2b_b^2)\right]\right\}^{1/2}} \quad (4.21)$$

The contents of equation (4.21) are interpreted as the follows. First, it shows the volatility of both dividends and bubbles has impacts on the correlations. The relative contributions of the volatility feedback effect and bubble effect to the correlations depend on the magnitudes of the news. Since dividend news s is on average much smaller than that of bubbles, the bubble effect usually contributes more to the correlation than the volatility feedback effect as long as both news of bubbles and dividends are present.

The volatility feedback effect can be the only source of the correlation if there is no bubble news, i.e. $\eta_{b,t+1}=0$, then the correlation becomes

$$Corr_t(r_{t+1},\sigma_{t+1}^2)=-\sqrt{2}(\kappa_d b_d+\lambda_d\sigma_{d,t}^2)/\left[(\kappa_d^2+2\lambda_d^2\sigma_{d,t}^2)(\sigma_{d,t}^2+2b_d^2)\right]^{1/2} \quad (4.22)$$

Equation (4.22) implies that the correlations contributed only by dividend news are negative. Another implication of equation (4.22) is the correlations approach -1 as volatility $\sigma_{d,t}^2$ increases. In other words, the larger the news of dividends is, the stronger the volatility feedback effect becomes. The correlations also approach -1 if volatility $\sigma_{d,t}^2$ is asymptotically equal to zero. This is also implied in equation (4.20), where return r_{t+1} is linear function of news $\eta_{d,t+1}$ when the volatility is very small.

If there is no news of dividends, i.e. $\eta_{d,t+1} = 0$, then the volatility feedback effect has no impacts on both the returns and their volatility, and the correlations of return and volatility are generated only by the bubble effect, which is

$$Corr_t(r_{t+1}, \sigma_{t+1}^2) = -\sqrt{2}(\kappa_b b_b + \lambda_b \sigma_{b,t}^2) / \left[(\kappa_b^2 + 2\lambda_b^2 \sigma_{b,t}^2)(\sigma_{b,t}^2 + 2b_b^2)\right]^{1/2} \quad (4.23)$$

Similar to pure volatility feedback effect, the correlations approach to -1 as volatility $\sigma_{b,t}^2$ increases, implying the larger the bubble news becomes, the stronger the bubble effect is. The correlations approach to -1 if volatility $\sigma_{b,t}^2$ is close to zero. In other words, no news of bubble i.e. $\sigma_{b,t}^2 \to 0$ will result in lower volatility and required returns, which in turn generates higher returns and very negative correlation.

In reality, neither pure volatility feedback effect nor pure bubble effect can solely account for the correlation. What is often seen in market is that the stock price responds to both news of dividends and bubbles, and that the volatility feedback effect and bubble effect interact with each other, thus the correlations are the total result of both effects.

In the next section, I estimate the correlations and those due to the volatility feedback effect and bubble effect respectively by applying the numerical maximum likelihood method to equation (4.20).

4.3 Data and empirical results

4.3.1 Data and estimation methods

Data needed to estimate this model is monthly stock returns and dividends. U.S stock returns over the period from 1926-2005 are obtained from CRSP. The stock returns are the log gross returns on the value-weighted CRSP S&P 500 index. Dividends are the log of dividends calculated from twelve-month moving sums of dividends paid on the S&P 500 index. Dividends over years 1971-2004 are from S&P Corporation and Robert Shiller's website over the period 1926-1970. Same as Campbell and Hentschel (1992), the year 1951 is chosen as a break point for two reasons. First, it corresponds to a change in interest rate regime with the Fed-Treasury Accord. The second is that it separates the Great Depression from the postwar period since previous studies have shown evidence that the behavior of volatility during the Great Depression was different from other periods. Descriptive statistics for stock returns and dividends are reported in Table 4.1.

The model is estimated by numerical maximum likelihood. There are two steps to estimate it. Both news of dividend and bubble are present in equation (4.20), so one of them needs to be specified. Since news about dividend is easier to estimate than the proxy of bubble, therefore, I estimate news about dividends by combining unit root of dividend[5] and QGARCH (1, 1) of its variance, which is system equation (4.24).

$$\begin{aligned} d_{t+1} &= \omega + d_t + \eta_{d,t+1} \\ \sigma_{d,t}^2 &= \omega_d + \partial_d(\eta_{d,t} - b_d)^2 + \beta_d \sigma_{d,t-1}^2 \end{aligned} \tag{4.24}$$

Then news about dividend and its variance are plugged in system equation (4.25) to estimate the QGARCH (1, 1) for bubbles.

$$\begin{aligned} r_{t+1} &= \mu + \gamma(\sigma_{d,t}^2 + \sigma_{b,t}^2) + \kappa_d \eta_{d,t+1} - \lambda_d(\eta_{d,t+1}^2 - \sigma_{d,t}^2) + \kappa_b \eta_{b,t+1} - \lambda_b(\eta_{b,t+1}^2 - \sigma_{b,t}^2) \\ \sigma_{b,t}^2 &= \omega_b + \partial_b(\eta_{b,t} - b_b)^2 + \beta_b \sigma_{b,t-1}^2 \end{aligned} \tag{4.25}$$

[5] Froot and Obstfeld (1991) provide several tests of the log-dividend specification in (4.22).

Since the return is a quadratic function of $\eta_{b,t+1}$, there are two different roots, one is positive and the other is negative, which satisfy equation (4.25). Therefore, the conditional density of r_{t+1}, $f_r(r_{t+1})$, is the summation over densities of both roots:

$$f_r(r_{t+1}) = \sum_{i=1}^{2} (\kappa_b - 2\lambda_b \eta_{b,t+1}^i)^{-1} f_\eta(\eta_{b,t+1}^i) \tag{4.26}$$

Note that different root gives different volatility by equation (4.24), therefore, two roots give two different time series of volatility for the same time period, which is a problem for estimation. Since it is reasonable to assume that positive (negative) news of bubbles is expected as the stock price goes up (down), the criteria is set to pick the positive root (i=1) if higher price is observed, negative (i=2) if price is lower. When one root is picked, the density of the other root is equal to zero. Therefore, the conditional log likelihood function of r_{t+1}, which is a function of normal distributed news of bubbles $\eta_{b,t+1}$, can be written as follows:

$$L(r_{t+1}) = -\log(\kappa_b - 2\lambda_b \eta_{b,t+1}^i) - 0.5\log(\sigma_{b,t}^{i\ 2}) - 0.5\eta_{b,t+1}^{i\ 2} / \sigma_{b,t}^{i\ 2} \tag{4.27}$$

Parameters that maximize the above log likelihood function give the estimators of the models, which are reported in table 4.2.

4.3.2 Empirical results

Table 4.3 and 4.4 report the monthly parameter estimates of the volatility feedback and bubble effect respectively. The results are summarized as follows. First, the values of ∂_d are 0.19 and 0.38 at monthly and quarterly levels, much lower than the corresponding values ∂_b, which are 0.39 and 0.45 respectively. This implies the quadratic effect of the QGARCH process of bubble news is stronger than that of dividend news. The values of β_d are 0.58 and 0.44 at monthly and quarterly levels, very similar to those of β_b, 0.53 and 0.45 respectively. The sums $\partial_d + \beta_d$ are 0.77 at monthly level and 0.73 at quarterly frequency, showing the volatility of dividend is quite persistent. Like dividend volatility process, bubble volatility is observed similar persistence. The sums $\partial_b + \beta_b$ are 0.91 and 0.89 at monthly and quarterly levels respectively, implying stronger persistence than dividend volatility due to larger ∂_b.

Second, the monthly and quarterly estimates of b_d are 8.77×10^{-5} and 9.08×10^{-5}, much smaller than those corresponding estimates of b_b, which are 0.014 and 0.023 respectively. As the result of the size differences between dividend news and bubble news, the difference between b_d and b_b also implies stronger QGARCH effect for bubble news than for dividend news.

The estimates of λ_d are 1.18 and 1.28 at monthly and quarterly levels, smaller than those corresponding values of λ_d, which are 6.22 and 5.05 respectively. The implication of this is the indirect effect of bubble news is stronger than that of dividend news. In contrary, the direct effect of bubble news is smaller than that of dividend news, since the values of κ_d are 1.002 at both monthly and quarterly levels, bigger than the magnitudes of corresponding values of κ_b, which are 1.1741 and 1.1686 respectively.

Since volatility feedback effect is a nested model of the model which takes into both volatility feedback effect and bubble effect into consideration, we further conducts a likelihood ratio test to investigate whether this model fits the stock return dataset significantly better than the volatility feedback model. The test statistics is 25.2, which is higher than the critical value 15.507 at 5% significant level. Therefore, the likelihood ratio test demonstrates that bubble news helps to explain the stock return data better.

4.3.3 The economic importance of the bubble effect

To better understand the roles the volatility feedback effect and bubble effect play in determining the correlations between returns and volatility, Figure 4.1-4.4 plot the correlations due to the volatility feedback effect, correlations due to the bubble effect, the total correlations and the impact of the volatility effect on correlations at monthly and quarterly levels respectively. The analyses of these plots are further explored as follows.

In order to see the relationship between the volatility feedback effect and the size of dividend news, Figure 4.1 plots the volatility feedback effect and dividends volatility together. Two characteristics can be seen from these figures. First, the correlations due to the volatility feedback effect are relatively stable, ranging between -0.094 and -0.029 at monthly frequency. Second, consistent with the predictions of the model, large negative correlations are observed when the volatility is either large or small. Large dividends volatility is either the results of stock market crashes or stock market booms.

According to Miskin and White (2003), there have been 10 main stock market crashes during 1926 to 2005 in U.S., which are crash of 1929-1933, 1937, 1940, 1946, 1962, 1970, 1973-74, 1987, 1990, and 2000. And according to Bordo and Wheelock (2006), the U.S. stock market has enjoyed six booms since 1923, which are October 1923-September 1929, March 1935-Febuary 1937, September 1953-April 1956, June 1962-Jan 1966, July 1984-August 1987 and April 1994-August 2000. Based on these results, large volatility arising during crashes 1929-1933, 1937, 1940, followed by stock market boom around 1953 and crash 1987 are observed in these plots. Also more negative correlations also occurred in the low volatility periods such as the period around and 1971.

Similar to Figure 4.1, Figure 4.2 plots the bubble effect and bubble volatility together to show how correlations respond to bubble news. Compared to the volatility feedback effect, the correlations due to the bubble effect are much more volatile than those due to volatility feedback effect, corresponding to that the volatility of bubble is more volatile than that of dividends. Also consistent with the predictions of the model, large negative correlations are observed when the bubble volatility is large, and positive correlations when there is small good bubble news. More specifically, large negative correlations correspond to large volatility arising during crashes 1929-1933, 1937, 1940, 1946, followed by stock market boom around 1953-1956, crash 1962, 1970, 1973-74, 1987 1990, stock market boom around 1997-1998 and crash around 2000 are observed in these plots. Although we cannot say all these crashes and booms were bubbles for sure, the bursting of the bubbles surely results in stock market crashes, and bubble growth leads to stock market booms. Therefore, what we see in the plots is justified by the stochastic process of bubble movements.

To differentiate the relative contribution of the volatility feedback effect and bubble effect to the correlations, Figure 4.3 shows the correlations, correlations due to pure volatility effect and correlations due to the bubble effect in the same plot. The most important feature of this figure is the correlations and correlations due to bubble effect are so similar that they seem overlap each other. In other words, the bubble effect dominates the volatility effect. The reason behind this phenomenon is that the sizes of dividend volatility is generally much smaller than those of bubbles, therefore, when the volatility feedback effect interacts with bubble effect, the bubble effect becomes dominant, which is also implied in equation (4.21). To better understand the contributions of the volatility feedback effect to the correlations, Figure 4.4 plots the correlations due to the volatility feedback effect and the difference between the correlations and those due to the bubble effect. The basic feature of these plots is the volatility feedback effect makes the correlations due to bubble effect smaller since the correlations due to the volatility feedback effect are all negative. The larger the volatility feedback effect is, the stronger the impact of the volatility feedback effect on the correlations. Although the impacts of the volatility feedback effect on the correlations are relatively small, usually lower than their own sizes, large impacts are observed during periods 1929, 1937, 1940, 1987 and late 1990s, implying that the impact of the volatility feedback effect on the correlations can be much larger than its own size during stock market crashes.

4.4 Conclusion

Studies on asymmetric volatility find neither the leverage effect nor the volatility feedback effect could fully account for this asymmetry. Based on Campbell and Hentschel (1992) which examine asymmetric volatility using only dividend news, referred to as the volatility feedback effect, I incorporate stock bubble news into this model given that stock price can be decomposed of fundamentals and bubbles. Therefore, the new model could explain volatility asymmetry using both dividend news and stock bubble news and allow differentiating the contributions of the volatility feedback effect and bubble effect due to bubble news to the asymmetry respectively.

The model is estimated using the maximum likelihood method and the empirical results find that 1) the larger the news is, the more negative the correlations are for both the volatility feedback effect and bubble effect; 2) the correlations due to the volatility feedback effect are much smaller than those due to the bubble effect, which account for more than 90% of the total effect on average;3) when both dividend news and bubble news are present, the bubble effect dominates the volatility feedback effect because of bubble news is larger than dividend news; and 4) despite the relatively small magnitude of the volatility feedback effect, it has a very significant impact on the correlations accounting for about 20% on average during stock market crashes.

The results obtained in this paper are based on several assumptions, and relaxations of these assumptions indicate some directions for future research. First, it would be interesting to examine the contribution of both of the bubble effect and dividend effect to the asymmetric volatility when the bubble new and dividend news are correlated, for instance, there exists bubbles of dividends. Second, in this paper, the residuals of the total returns are considered as bubbles. Therefore, measurement errors are included in the bubbles. Would the conclusions in this paper hold if bubbles are defined in other ways? Lastly, this paper does not take leverage effects, although the empirical result is not dramatic in explaining the negative relation, into account in the model. It would be interesting to integrate all the leverage, volatility feedback and bubble effects in exploring their roles in generating the negative correlation of return and volatility.

Appendix

A. Derivation of equation (4.9)

$$r_{t+1} \equiv \log(P_{t+1} + D_{t+1}) - \log(P_t) = \log(F_{t+1} + B_{t+1} + D_{t+1}) - \log(F_t + B_t)$$

$$V_{t+1} = F_{t+1} + B_{t+1} + D_{t+1}$$

$$\frac{V_{t+1}}{F_{t+1}} = 1 + \frac{B_{t+1}}{F_{t+1}} + \frac{D_{t+1}}{F_{t+1}}$$

$$\log(V_{t+1}/F_{t+1}) = \log\left[1 + e^{\log(B_{t+1}/F_{t+1})} + e^{\log(D_{t+1}/F_{t+1})}\right]$$

$$Let\ \log\frac{B}{F} = x,\ \log\frac{D}{F} = y,\ First-order\ Taylor\ \exp ansion\ around\ F, B, D$$

$$v_{t+1} - f_{t+1} = \log(1 + e^x + e^y) + \frac{e^x}{1 + e^x + e^y}\left[\log(B_{t+1}/F_{t+1}) - x\right] + \frac{e^y}{1 + e^x + e^y}\left[\log(D_{t+1}/F_{t+1}) - y\right]$$

$$= \log(1 + \frac{B}{F} + \frac{D}{F}) + \frac{B/F}{1 + B/F + D/F}(b_{t+1} - f_{t+1} - x) + \frac{D/F}{1 + B/F + D/F}(d_{t+1} - f_{t+1} - y)$$

$$= \log(1 + \frac{B+D}{F}) + \frac{B}{F+B+D}(b_{t+1} - f_{t+1} - x) + \frac{D}{F+B+D}(d_{t+1} - f_{t+1} - y)$$

$$= \chi + \frac{B}{F+B+D}b_{t+1} + \frac{D}{F+B+D}d_{t+1} - \frac{B+D}{F+B+D}f_{t+1}$$

$$v_{t+1} = \chi + \rho_1 f_{t+1} + \rho_2 b_{t+1} + (1 - \rho_1 - \rho_2)d_{t+1}$$

$$Where\ \chi = \log(1 + \frac{B+D}{F}) - \frac{B}{F+B+D}x - \frac{D}{F+B+D}y$$

$$= \log(1 + \frac{B+D}{F}) - \frac{B}{F+B+D}\log\frac{B}{F} - \frac{D}{F+B+D}\log\frac{D}{F}$$

Similarly

$$P_t = F_t + B_t$$

$$p_t = \lambda f_t + (1-\lambda)b_t$$

Therefore,

$$r_{t+1} \approx \kappa + \rho_1 f_{t+1} + \rho_2 b_{t+1} + (1 - \rho_1 - \rho_2)d_{t+1} - \lambda f_t - (1-\lambda)b_t$$

$$= \theta + \rho_1 f_{t+1} - (1 - \lambda - \rho_2)b_{t+1} + (1 - \rho_1 - \rho_2)d_{t+1} - \lambda f_t - 0.5(1-\lambda)\sigma^2_{b,t+1} + (1-\lambda)z_{t+1}$$

$$Where\ \theta = \kappa + (1-\lambda)\log(1+r),\ \lambda = F/P = F/(F+B)$$

$$\rho_1 = F/(P+D) = F/(F+B+D),\ \rho_2 = B/(P+D) = B/(F+B+D)$$

B. Derivation of equation (4.11)

$$f_t = \frac{\theta}{\lambda} + \frac{\rho_1}{\lambda} f_{t+1} + \frac{1-\rho_1-\rho_2}{\lambda} d_{t+1} - \frac{1-\lambda-\rho_2}{\lambda} b_{t+1} - \frac{1}{\lambda} r_{t+1} - \frac{1-\lambda}{2\lambda}\sigma^2_{b,t+1} + \frac{1-\lambda}{\lambda} z_{t+1}$$

$$= \delta + \alpha f_{t+1} + \beta d_{t+1} - \nu b_{t+1} - \lambda^{-1} r_{t+1} - \psi\sigma^2_{b,t+1} + \vartheta z_{t+1}$$

$$Where\ \alpha = \rho_1 / \lambda = (F+B)/(F+B+D) < 1$$

$$f_t = \frac{\delta}{1-\alpha} + \beta E_t \sum_{j=0}^{\infty} \alpha^j d_{t+1+j} - \nu E_t \sum_{j=0}^{\infty} \alpha^j b_{t+1+j} - \lambda^{-1} E_t \sum_{j=0}^{\infty} \alpha^j r_{t+1+j}$$

$$-\psi E_t \sum_{j=0}^{\infty} \alpha^j \sigma^2_{b,t+1+j} + \vartheta E_t \sum_{j=0}^{\infty} \alpha^j z_{t+1+j} + E_t \lim_{i\to\infty} \alpha^i f_{t+i}$$

$$r_{t+1} - E_t r_{t+1} = \rho_1\beta(E_{t+1}-E_t)\sum_{j=0}^{\infty}\alpha^j d_{t+2+j} + \rho_1\nu(E_{t+1}-E_t)\sum_{j=0}^{\infty}\alpha^j b_{t+2+j} - \rho_1\lambda^{-1}(E_{t+1}-E_t)\sum_{j=0}^{\infty}\alpha^j r_{t+2+j}$$

$$+\rho_1\psi(E_{t+1}-E_t)\sum_{j=0}^{\infty}\alpha^j \sigma^2_{b,t+2+j} + (1-\rho_1-\rho_2)(E_{t+1}-E_t)d_{t+1} + (1-\lambda-\rho_2)(E_{t+1}-E_t)b_{t+1}$$

$$+0.5(1-\lambda)(E_{t+1}-E_t)\sigma^2_{b,t+1} + (1-\lambda)(E_{t+1}-E_t)z_{t+1}$$

$$r_{t+1} - E_t r_{t+1} = \eta_{d,t+1} - \eta_{r,t+1} + \eta_{b,t+1}$$

$$Where\ \eta_{d,t+1} = \rho_1\beta(E_{t+1}-E_t)\sum_{j=0}^{\infty}\alpha^j d_{t+2+j} + (1-\rho_1-\rho_2)(E_{t+1}-E_t)d_{t+1}$$

$$\eta_{r,t+1} = \rho_1\lambda^{-1}(E_{t+1}-E_t)\sum_{j=0}^{\infty}\alpha^j r_{t+2+j} = (E_{t+1}-E_t)\sum_{j=0}^{\infty}\alpha^{j+1} r_{t+2+j} = (E_{t+1}-E_t)\sum_{j=1}^{\infty}\alpha^j r_{t+1+j}$$

$$\eta_{b,t+1} = \rho_1\nu(E_{t+1}-E_t)\sum_{j=0}^{\infty}\alpha^j b_{t+2+j} + (1-\lambda-\rho_2)(E_{t+1}-E_t)b_{t+1}$$

$$+\rho_1\psi(E_{t+1}-E_t)\sum_{j=0}^{\infty}\alpha^j \sigma^2_{b,t+2+j} + 0.5(1-\lambda)(E_{t+1}-E_t)\sigma^2_{b,t+1}$$

C. Derivation of equation (4.20)

$$E_t r_{t+1+j} = E_t E_{t+1} E_{t+2} \cdots E_{t+j-1} E_{t+j} r_{t+1+j} = E_t E_{t+1} E_{t+2} \cdots E_{t+j-1} \left[\mu + \gamma(\sigma^2_{d,t+j} + \sigma^2_{b,t+j}) \right]$$

$$= \mu + \gamma E_t E_{t+1} E_{t+2} \cdots E_{t+j-1} (\sigma^2_{d,t+j} + \sigma^2_{b,t+j})$$

$$E_{t+j-1} \sigma^2_{d,t+j} = \omega_d + \partial_d E_{t+j-1} (\eta_{d,t+j} - b_d)^2 + \beta_d \sigma^2_{d,t+j-1}$$

$$= (\omega_d + \partial_d b_d^2) + (\partial_d + \beta_d) \sigma^2_{d,t+j-1}$$

$$E_t E_{t+1} E_{t+2} \cdots E_{t+j-1} \sigma^2_{d,t+j} = (\omega_d + \partial_d b_d^2) \left[1 + (\partial_d + \beta_d) + \cdots \right] + (\partial_d + \beta_d)^j \sigma^2_{d,t}$$

$$= \frac{\omega_d + \partial_d b_d^2}{1 - (\partial_d + \beta_d)} + (\partial_d + \beta_d)^j \sigma^2_{d,t}$$

Similarly, $E_t E_{t+1} E_{t+2} \cdots E_{t+j-1} \sigma^2_{b,t+j} = (\omega_b + \partial_b b_b^2) \left[1 + (\partial_b + \beta_b) + \cdots \right] + (\partial_b + \beta_b)^j \sigma^2_{b,t}$

$$= \frac{\omega_b + \partial_b b_b^2}{1 - (\partial_b + \beta_b)} + (\partial_b + \beta_b)^j \sigma^2_{b,t}$$

$$E_t r_{t+1+j} = \mu + \gamma \left[\frac{\omega_d + \partial_d b_d^2}{1 - (\partial_d + \beta_d)} + \frac{\omega_b + \partial_b b_b^2}{1 - (\partial_b + \beta_b)} \right] + \gamma \left[(\partial_d + \beta_d)^j \sigma^2_{d,t} + (\partial_b + \beta_b)^j \sigma^2_{b,t} \right]$$

$$E_{t+1} r_{t+2+j} = \mu + \gamma \left[\frac{\omega_d + \partial_d b_d^2}{1 - (\partial_d + \beta_d)} + \frac{\omega_b + \partial_b b_b^2}{1 - (\partial_b + \beta_b)} \right] + \gamma \left[(\partial_d + \beta_d)^j \sigma^2_{d,t+1} + (\partial_b + \beta_b)^j \sigma^2_{b,t+1} \right]$$

$$E_t \sum_{j=1}^{\infty} \alpha^j r_{t+1+j} = \frac{\mu\alpha}{1-\alpha} + \frac{\gamma\alpha}{1-\alpha} \left[\frac{\omega_d + \partial_d b_d^2}{1 - (\partial_d + \beta_d)} + \frac{\omega_b + \partial_b b_b^2}{1 - (\partial_b + \beta_b)} \right]$$

$$+ \frac{\gamma\alpha(\partial_d + \beta_d)}{1 - \alpha(\partial_d + \beta_d)} \sigma^2_{d,t} + \frac{\gamma\alpha(\partial_b + \beta_b)}{1 - \alpha(\partial_b + \beta_b)} \sigma^2_{b,t}$$

$$E_{t+1} \sum_{j=1}^{\infty} \alpha^j r_{t+1+j} = \alpha E_{t+1} \sum_{j=0}^{\infty} \alpha^j r_{t+2+j} = \frac{\mu\alpha}{1-\alpha} + \frac{\gamma\alpha}{1-\alpha} \left[\frac{\omega_d + \partial_d b_d^2}{1 - (\partial_d + \beta_d)} + \frac{\omega_b + \partial_b b_b^2}{1 - (\partial_b + \beta_b)} \right]$$

$$+ \frac{\gamma\alpha}{1 - \alpha(\partial_d + \beta_d)} \sigma^2_{d,t+1} + \frac{\gamma\alpha}{1 - \alpha(\partial_b + \beta_b)} \sigma^2_{b,t+1}$$

$$\eta_{r,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \alpha^j r_{t+1+j} = \lambda_d (\eta^2_{d,t+1} - \sigma^2_{d,t} - 2 b_d \eta_{d,t+1}) + \lambda_b (\eta^2_{b,t+1} - \sigma^2_{b,t} - 2 b_b \eta_{b,t+1})$$

Where $\lambda_d = \dfrac{\gamma\alpha\partial_d}{1 - \alpha(\partial_d + \beta_d)}, \quad \lambda_b = \dfrac{\gamma\alpha\partial_b}{1 - \alpha(\partial_b + \beta_b)}$

$$r_{t+1} = \mu + \gamma(\sigma^2_{d,t} + \sigma^2_{b,t}) + \eta_{d,t+1} + \eta_{b,t+1} - \lambda_d (\eta^2_{d,t+1} - \sigma^2_{d,t} - 2 b_d \eta_{d,t+1}) - \lambda_b (\eta^2_{b,t+1} - \sigma^2_{b,t} - 2 b_b \eta_{b,t+1})$$

$$= \mu + \gamma(\sigma^2_{d,t} + \sigma^2_{b,t}) + \kappa_d \eta_{d,t+1} - \lambda_d (\eta^2_{d,t+1} - \sigma^2_{d,t}) + \kappa_b \eta_{b,t+1} - \lambda_b (\eta^2_{b,t+1} - \sigma^2_{b,t})$$

Where $\kappa_d = 2\lambda_d b_d + 1, \quad \kappa_b = 2\lambda_b b_b + 1$

D. Derivation of equation (4.21)

$$\eta_{d,t+1} \sim N(0, \sigma_{d,t}^2)$$

$$\frac{\eta_{d,t+1}}{\sigma_{d,t}} \sim N(0,1)$$

$$\frac{\eta_{d,t+1}^2}{\sigma_{d,t}^2} \sim \chi_1^2$$

$$Var_t(\frac{\eta_{d,t+1}^2}{\sigma_{d,t}^2}) = 2$$

$$Var_t(\eta_{d,t+1}^2) = 2\sigma_{d,t}^4$$

$$Similarly, Var_t(\eta_{b,t+1}^2) = 2\sigma_{b,t}^4$$

$$Since\ skewness\ of\ normal\ distribution\ is\ zero,\ E_t(\eta_{d,t+1}^3) = E_t(\eta_{b,t+1}^3) = 0$$

$$E_t(\eta_{d,t+1}^4) = E_t\left[(\eta_{d,t+1}^2)^2\right] = Var_t(\eta_{d,t+1}^2) + E_t{}^2\left[(\eta_{d,t+1}^2)\right]$$

$$= 2\sigma_{d,t}^4 + \sigma_{d,t}^4 = 3\sigma_{d,t}^4$$

$$mgf\ of\ normal\ distribution\ N(\mu, \sigma^2)$$

$$m(t) = e^{\mu t + \sigma^2 t^2}$$

$$E_t(\eta_{d,t+1}^5) = 0$$

$$E_t(\eta_{d,t+1}^6) = 15\sigma_{d,t}^6$$

$$r_{t+1} = \mu + \gamma(\sigma_{d,t}^2 + \sigma_{b,t}^2) + \kappa_d \eta_{d,t+1} + \kappa_b \eta_{b,t+1} - \lambda_d(\eta_{d,t+1}^2 - \sigma_{d,t}^2) - \lambda_b(\eta_{b,t+1}^2 - \sigma_{b,t}^2)$$

$$Assume\ \eta_{d,t+1},\ \ \eta_{b,t+1}\ are\ independent$$

$$Var_t(r_{t+1}) = \kappa_d^2\sigma_{d,t}^2 + \kappa_b^2\sigma_{b,t}^2 + \lambda_d^2 Var_t(\eta_{d,t+1}^2) + \lambda_b^2 Var_t(\eta_{b,t+1}^2)$$

$$= \kappa_d^2\sigma_{d,t}^2 + \kappa_b^2\sigma_{b,t}^2 + 2\lambda_d^2\sigma_{d,t}^4 + 2\lambda_b^2\sigma_{b,t}^4$$

$$= \sigma_{d,t}^2(\kappa_d^2 + 2\lambda_d^2\sigma_{d,t}^2) + \sigma_{b,t}^2(\kappa_b^2 + 2\lambda_b^2\sigma_{b,t}^2)$$

$$Var_t(\sigma_{t+1}^2) = Var_t(\sigma_{d,t+1}^2 + \sigma_{b,t+1}^2)$$

$$= \partial_d^2 Var_t(\eta_{d,t+1}^2) + 4\partial_d^2 b_d^2 Var_t(\eta_{d,t+1}) + \partial_b^2 Var_t(\eta_{d,t+1}^2) + 4\partial_b^2 b_b^2 Var_t(\eta_{b,t+1})$$

$$= 2\partial_d^2\sigma_{d,t}^4 + 4\partial_d^2 b_d^2\sigma_{d,t}^2 + 2\partial_b^2\sigma_{b,t}^4 + 4\partial_b^2 b_b^2\sigma_{b,t}^2$$

$$= 2\partial_d^2\sigma_{d,t}^2(\sigma_{d,t}^2 + 2b_d^2) + 2\partial_b^2\sigma_{b,t}^2(\sigma_{b,t}^2 + 2b_b^2)$$

$$Cov_t(r_{t+1},\sigma^2_{t+1}) = E_t[(\kappa_d\eta_{d,t+1} + \kappa_b\eta_{b,t+1} - \lambda_d\eta^2_{d,t+1} - \lambda_b\eta^2_{b,t+1} + \lambda_d\sigma^2_{d,t} + \lambda_b\sigma^2_{b,t})$$

$$(\partial_d\eta^2_{d,t+1} - 2\partial_d b_d\eta_{d,t+1} - \partial_d\sigma^2_{d,t} + \partial_b\eta^2_{b,t+1} - 2\partial_b b_b\eta_{b,t+1} - \partial_b\sigma^2_{b,t})]$$

$$= -2\left[\partial_d\sigma^2_{d,t}(\kappa_d b_d + \lambda_d\sigma^2_{d,t}) + \partial_b\sigma^2_{b,t}(\kappa_b b_b + \lambda_b\sigma^2_{b,t})\right]$$

$$Corr_t(r_{t+1},\sigma^2_{t+1}) = \frac{Cov_t(r_{t+1},\sigma^2_{t+1})}{\left[Var_t(r_{t+1})Var_t(\sigma^2_{t+1})\right]^{1/2}}$$

$$= -\frac{2\left[\partial_d\sigma^2_{d,t}(\kappa_d b_d + \lambda_d\sigma^2_{d,t}) + \partial_b\sigma^2_{b,t}(\kappa_b b_b + \lambda_b\sigma^2_{b,t})\right]}{\sqrt{\left[\sigma^2_{d,t}(\kappa^2_d + 2\lambda^2_d\sigma^2_{d,t}) + \sigma^2_{b,t}(\kappa^2_b + 2\lambda^2_b\sigma^2_{b,t})\right]\left[2\partial^2_d\sigma^2_{d,t}(\sigma^2_{d,t} + 2b^2_d) + 2\partial^2_b\sigma^2_{b,t}(\sigma^2_{b,t} + 2b^2_b)\right]}}$$

$$r_{t+1} = \mu + \gamma(\sigma^2_{d,t} + \sigma^2_{b,t}) + \kappa_d\eta_{d,t+1} - \lambda_d(\eta^2_{d,t+1} - \sigma^2_{d,t}) + \kappa_b\eta_{b,t+1} - \lambda_b(\eta^2_{b,t+1} - \sigma^2_{b,t})$$

$$\lambda_b\eta^2_{b,t+1} - \kappa_b\eta_{b,t+1} + r_{t+1} - \mu - (\gamma + \lambda_d)\sigma^2_{d,t} - (\gamma + \lambda_b)\sigma^2_{b,t} - \kappa_d\eta_{b,t+1} + \lambda_d\eta^2_{d,t+1} = 0$$

$$\eta^1_{b,t+1} = \frac{\kappa_b + \left\{\kappa_b{}^2 - 4\lambda_b\left[r_{t+1} - \mu - (\gamma + \lambda_d)\sigma^2_{d,t} - (\gamma + \lambda_b)\sigma^2_{b,t} - \kappa_d\eta_{d,t+1} + \lambda_d\eta^2_{d,t+1}\right]\right\}^{1/2}}{2\lambda_b}$$

$$\eta^2_{b,t+1} = \frac{\kappa_b - \left\{\kappa_b{}^2 - 4\lambda_b\left[r_{t+1} - \mu - (\gamma + \lambda_d)\sigma^2_{d,t} - (\gamma + \lambda_b)\sigma^2_{b,t} - \kappa_d\eta_{d,t+1} + \lambda_d\eta^2_{d,t+1}\right]\right\}^{1/2}}{2\lambda_b}$$

The conditional density of r_{t+1} $f_r(r_{t+1})$ *is*

$$f_r(r_{t+1}) = \sum_{i=1}^{2}(\kappa_b - 2\lambda_b\eta^i_{b,t+1})^{-1} f_\eta(\eta^i_{b,t+1})$$

$$L(r_{t+1}) = -\log(\kappa_b - 2\lambda_b\eta^i_{b,t+1}) - 0.5\log(\sigma^i_{b,t}{}^2) - 0.5\eta^i_{b,t+1}{}^2 / \sigma^i_{b,t}{}^2$$

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TABLE 4.1: SUMMARY STATISTICS OF THE DATA

Item	Frequency	No. of observations	Mean	Variance	Skewness	Kurtosis
Return	Monthly	960	0.0080864	0.0031054	-0.43223	10.985
	Quarterly	320	0.01408	0.011808	0.10812	11.209
Dividend	Monthly	960	-1.4694	1.3768	0.13977	1.7081
	Quarterly	320	-0.3674	1.3815	0.13769	1.7102

This table lists the moments of monthly and quarterly returns and dividends data the value-weighted CRSP S&P 500 index over the period 1926 to 2005.

TABLE 4.2: ESTIMATES OF THE VOLATILITY FEEDBACK EFFECT IN THE NESTED MODEL

Parameters		Monthly	
	1926-2005	1926-1951	1952-2005
$\omega \times 10^5$	9.624	10.960	22.894
	(3.621)	(5.492)	(10.951)
∂_d	0.130	0.132	0.141
	(0.024)	(0.032)	(0.032)
$b_d \times 10^2$	1.653	1.878	5.463
	(0.422)	(1.278)	(1.021)
β_d	0.855	0.822	0.126
	(0.320)	(0.036)	(0.054)
$\mu \times 10^3$	4.562	8.956	9.742
	(1.024)	(4.115)	(2.517)
γ	1.028	0.344	0.136
	(0.355)	(1.196)	(0.122)
λ_d	0.783	0.670	1.147
	(0.162)	(0.180)	(0.521)

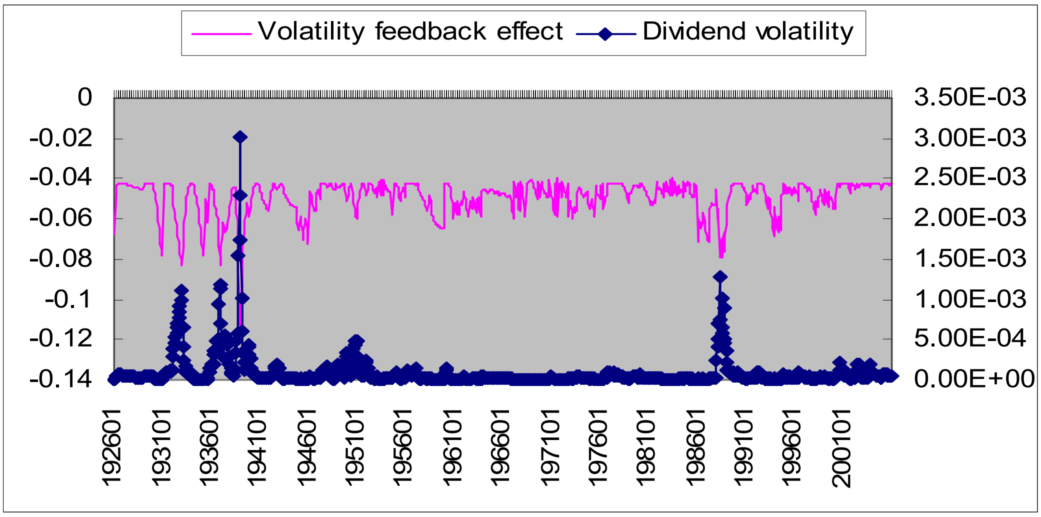
TABLE 4.3: ESTIMATES OF THE VOLATILITY FEEDBACK EFFECT

Parameters	Monthly 1926-2005	1926-1951	1952-2005
$\omega \times 10^4$	0.80517 (0.079)	4.1509 (0.384)	1.7036 (0.241)
$\omega_d \times 10^6$	1.4462 (0.167)	0.028801 (0.003)	0.62011 (0.058)
∂_d	0.19229 (0.022)	0.32367 (0.032)	0.12699 (0.011)
$b_d \times 10^5$	8.7696 (0.792)	2.7595 (0.344)	7.8723 (0.899)
β_d	0.57571 (0.024)	0.36259 (0.029)	0.78531 (0.947)
λ_d	1.1833 (0.247)	1.5616 (0.322)	2.0661 (0.158)
κ_d	1.0002 (0.132)	1.0001 (0.112)	1.0003 (0.116)

TABLE 4.4: ESTIMATES OF THE BUBBLE EFFECT

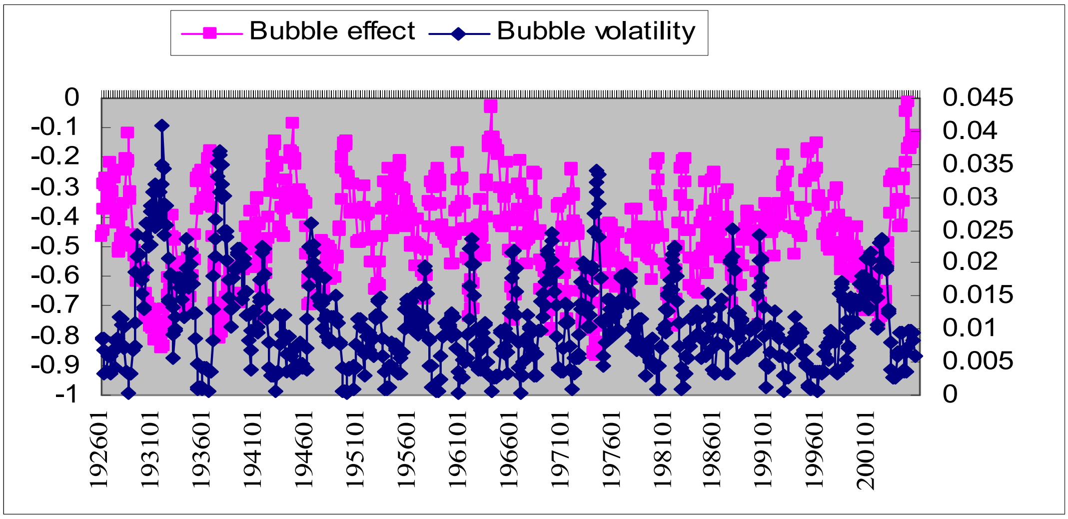
Parameters	Monthly 1926-2005	1926-1951	1952-2005
$\mu \times 10^5$	4.3259 (0.347)	1.7168 (0.241)	7.0319 (0.645)
γ	1.4524 (0.232)	1.533 (0.115)	1.4921 (0.326)
$\omega_b \times 10^5$	3.5627 (0.455)	4.9637 (0.258)	5.4824 (0.684)
∂_b	0.38513 (0.045)	0.38611 (0.047)	0.37825 (0.283)
$b_b \times 10^2$	1.399 (0.232)	1.4484 (0.189)	1.304 (0.201)
β_b	0.52898 (0.841)	0.52828 (0.435)	0.53822 (0.724)
λ_b	6.222 (0.891)	6.6048 (0.823)	6.4466 (0.924)
κ_b	1.1741 (0.212)	1.1913 (0.125)	1.1681 (0.154)

Figure 4.1: Correlations of return and volatility due to the volatility feedback effect



The left axis the monthly correlations due to the volatility feedback effect and the right axis is the monthly dividend variance.

Figure 4.2: Correlations of return and volatility due to the bubble effects



The left axis the monthly correlations due to the bubble effect and the right axis is the monthly bubble variance.

Figure 4.3: Correlations of stock return and its volatility

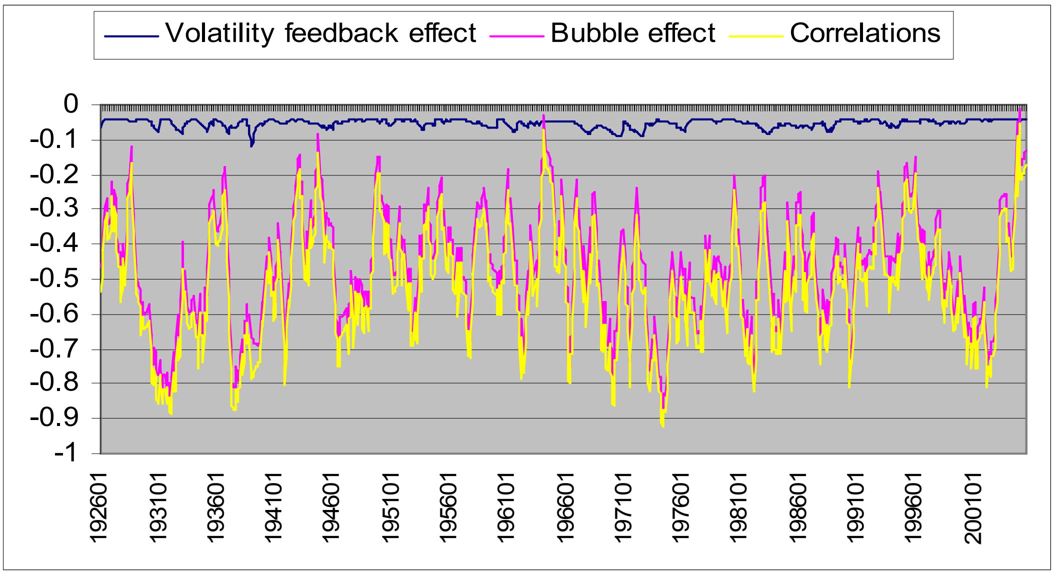
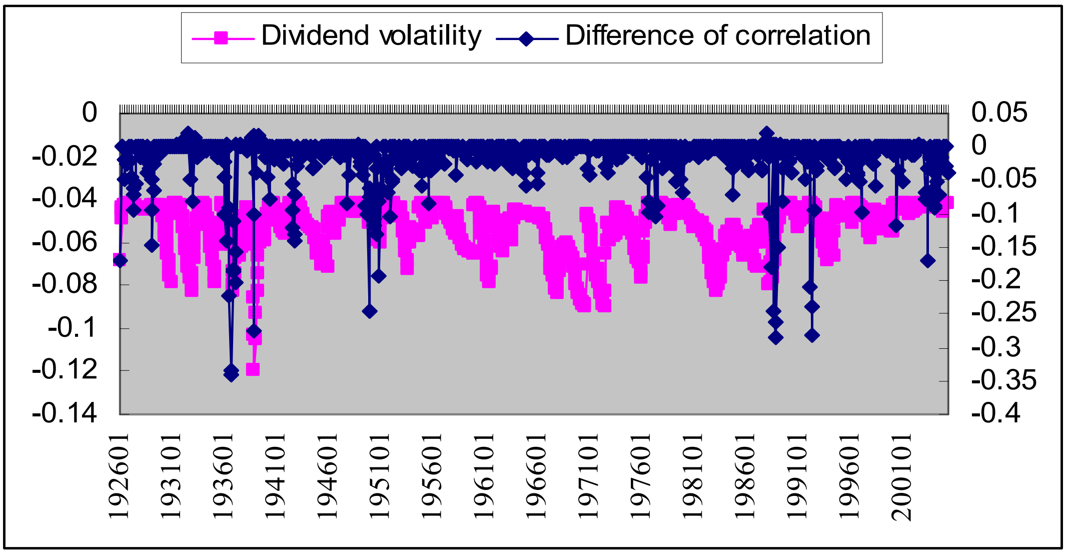


Figure 4.4: The impact of volatility feedback effect on the correlations



The left axis is the monthly correlations due to the volatility feedback effect and the right axis is the difference between the monthly correlations and monthly correlations due to the bubble effect.