Appendix A: Equilibrium Graphs and Comparative Statics

Equilibrium Analysis and Comparative Statics

Computation of Equilibrium Optimal Value of q*

In[•]:=
```
Solve[(q) * (1 / (1 + δ)) + (1 - q) * ((δ ^ 2 / (1 + δ)) - p * B) == (1 / (1 + δ)) - c - p * B, q]
```

Out[•]= $$\left\{\left\{q \to \frac{-c + \frac{1}{1+\delta} - \frac{\delta^2}{1+\delta}}{B\,p + \frac{1}{1+\delta} - \frac{\delta^2}{1+\delta}}\right\}\right\}$$

In[•]:= $$\text{Simplify}\left[\frac{-c + \frac{1}{1+\delta} - \frac{\delta^2}{1+\delta}}{B\,p + \frac{1}{1+\delta} - \frac{\delta^2}{1+\delta}}\right]$$

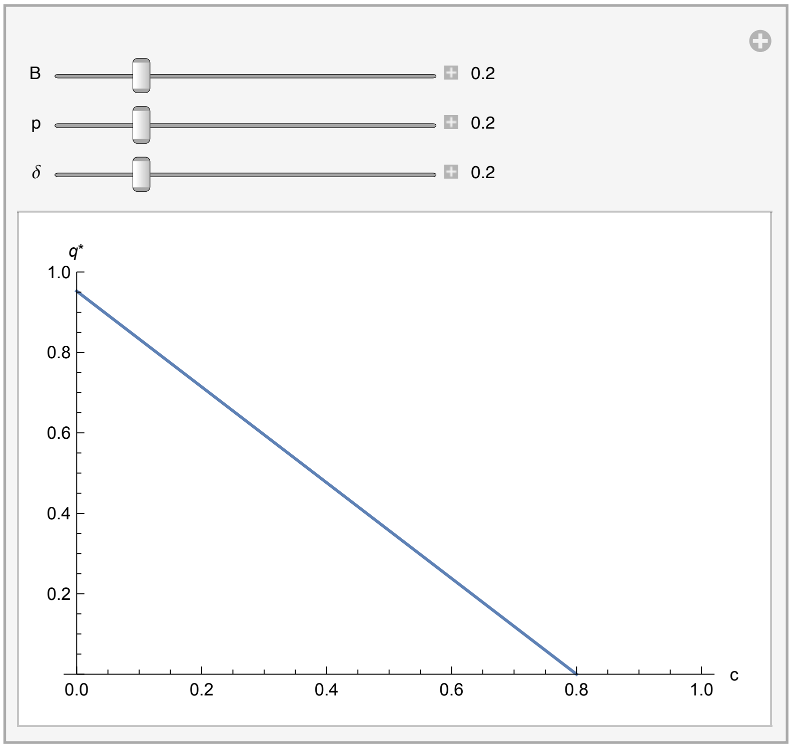
Out[•]= $$\frac{-1 + c + \delta}{-1 - B\,p + \delta}$$

This is the equilibrium value of q. Below q* there is no rejection of offer or conflict as State 1 will make the generous demand of x_g and at q* and above it, rejection of offer is observed with probability 1-q and terrorism is observed with probability p.

Graph q* against c, vary B, p, and δ

In[∘]:=
```
Manipulate[Plot[(-1 + c + δ)/(-1 - B p + δ), {c, 0, 1}, PlotRange → {0, 1}, AxesLabel → {"c", "q*"}],
 {B, 0, 1, Appearance → "Labeled"}, {p, 0, 1, Appearance → "Labeled"},
 {δ, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

Out[∘]=



As c increases, q* decreases at a linear rate with all other parameters held constant. The y intercept decreases for higher values of B and p. The x intercept is lower for higher values of δ

Relationship between q* and c

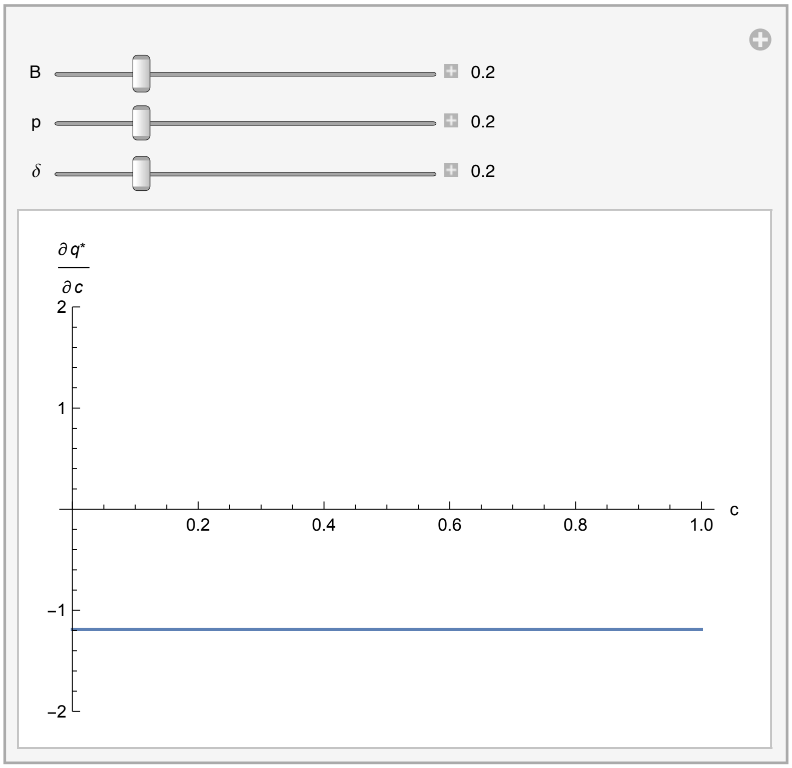
In[∘]:=
```
D[(-1 + c + δ)/(-1 - B p + δ), c]
```

Out[∘]=

$$\frac{1}{-1 - B\,p + \delta}$$

In[∘]:=
```
Manipulate[Plot[1/(-1 - B p + δ), {c, 0, 1}, PlotRange → 2, AxesLabel → {"c", "∂q*/∂c"}],
 {B, 0, 1, Appearance → "Labeled"}, {p, 0, 1, Appearance → "Labeled"},
 {δ, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

Out[∘]=

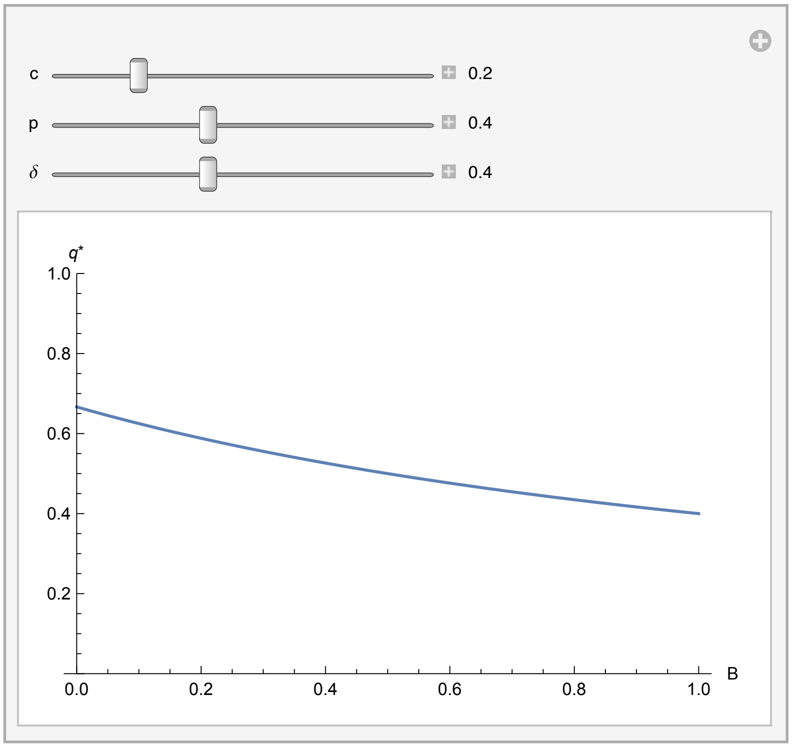


This shows that the rate of change of q* with respect to c is negative and linear.

Graph q * against B, vary c, p, and δ

In[•]:=
```
Manipulate[Plot[(-1 + c + δ)/(-1 - B p + δ), {B, 0, 1}, PlotRange → {0, 1}, AxesLabel → {"B", "q*"}],
  {c, 0, 1, Appearance → "Labeled"}, {p, 0, 1, Appearance → "Labeled"},
  {δ, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

Out[•]=



As B increases, q* decreases at a non-linear rate with all other parameters held constant. The y intercept decreases for higher values of c and δ. The graph becomes linear and parallel to the x axis when p=0, since B and p are expressed as a product in the equilibrium expression and becomes non-linear and decreases for higher values of p

Relationship between q* and B

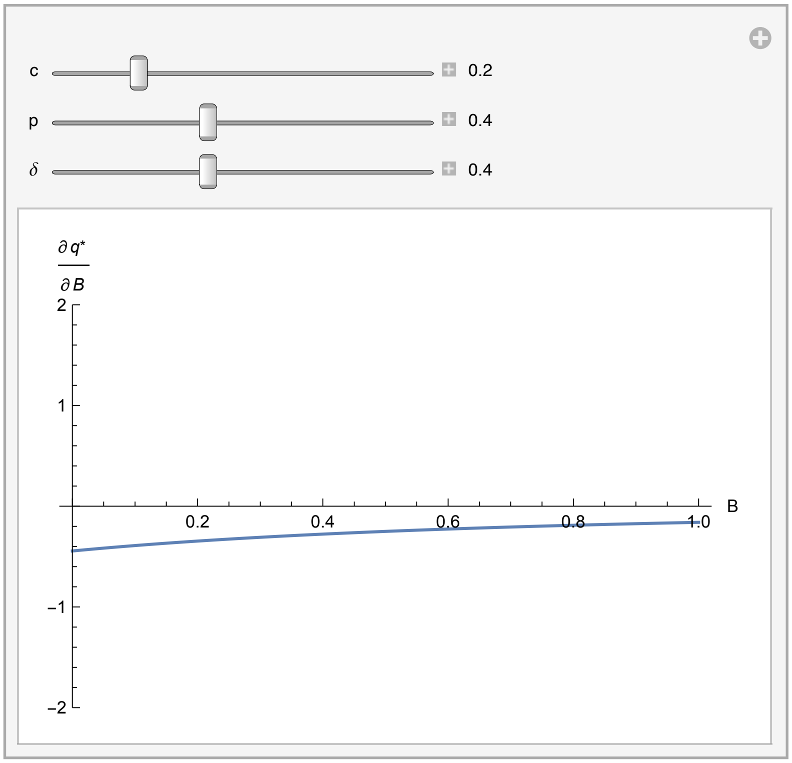
In[•]:=
```
D[(-1 + c + δ)/(-1 - B p + δ), B]
```

Out[•]=

$$\frac{p\,(-1 + c + \delta)}{(-1 - B\,p + \delta)^2}$$

In[∘]:=
```
Manipulate[Plot[p (-1 + c + δ)/(-1 - B p + δ)^2, {B, 0, 1}, PlotRange → 2, AxesLabel → {"B", "∂q*/∂B"}],
 {c, 0, 1, Appearance → "Labeled"}, {p, 0, 1, Appearance → "Labeled"},
 {δ, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

Out[∘]=

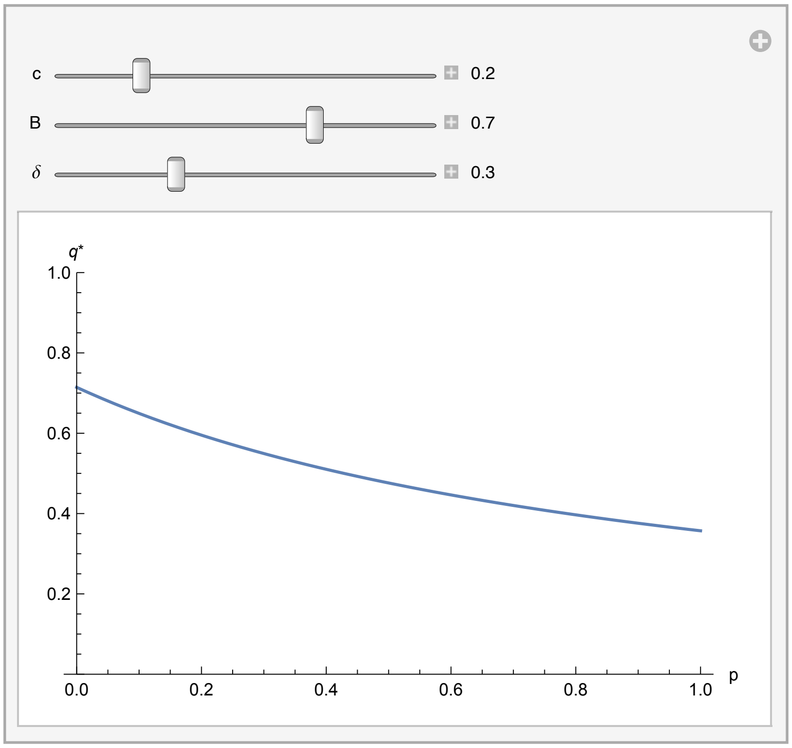


This shows that the rate of change of q* with respect to B is negative and non-linear (increasing).

Graph q*against p, vary c, B, and δ

In[∘]:=
```
Manipulate[Plot[(-1 + c + δ)/(-1 - B p + δ), {p, 0, 1}, PlotRange → {0, 1}, AxesLabel → {"p", "q*"}],
 {c, 0, 1, Appearance → "Labeled"}, {B, 0, 1, Appearance → "Labeled"},
 {δ, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

Out[∘]=



As p increases, q* decreases at a non-linear rate with all other parameters held constant. The y intercept decreases for higher values of c and δ. The graph becomes linear and parallel to the x axis when B=0, since B and p are expressed as a product in the equilibrium expression, and becomes non-linear and decreases for higher values of B

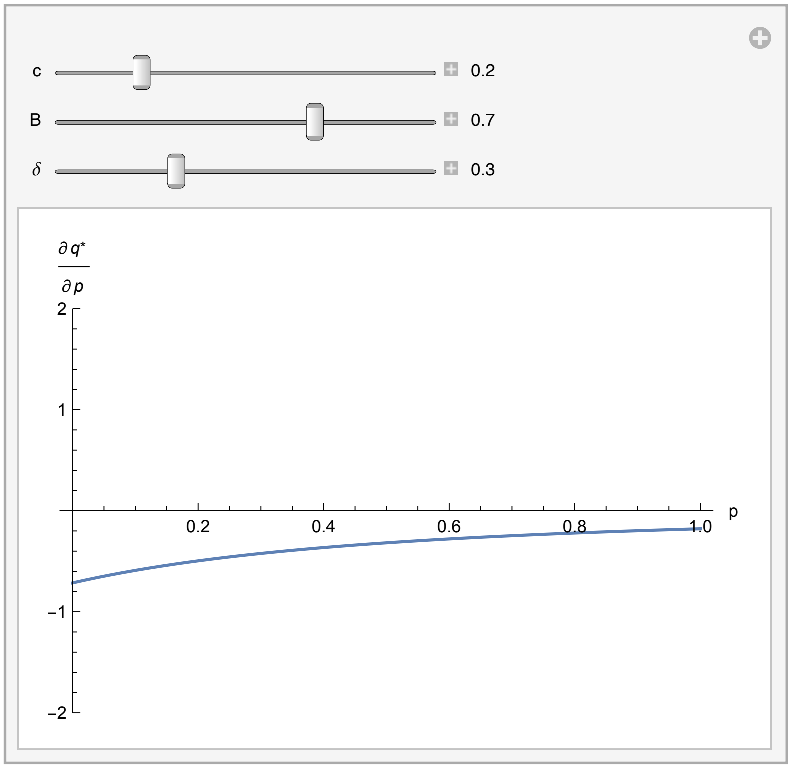
Relationship between q* and p

In[∘]:= $D\left[\frac{-1+c+\delta}{-1-B\,p+\delta},\ p\right]$

Out[∘]= $$\frac{B\,(-1+c+\delta)}{(-1-B\,p+\delta)^2}$$

In[•]:=
```
Manipulate[Plot[B (-1 + c + δ)/(-1 - B p + δ)^2, {p, 0, 1}, PlotRange → 2, AxesLabel → {"p", "∂q*/∂p"}],
 {c, 0, 1, Appearance → "Labeled"}, {B, 0, 1, Appearance → "Labeled"},
 {δ, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

Out[•]=

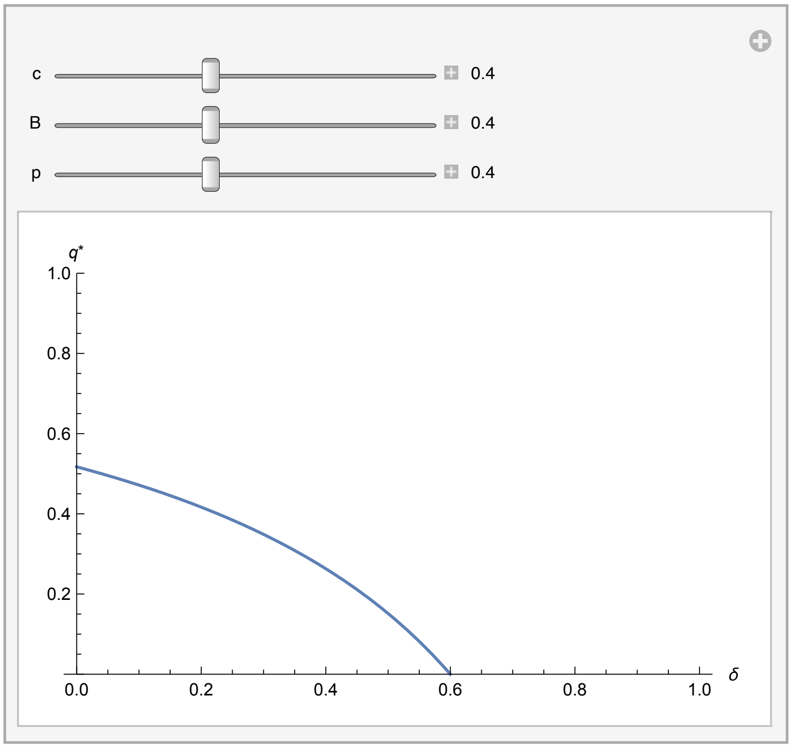


This shows that the rate of change of q* with respect to p is negative and non-linear (increasing).

Graph q*against δ, vary c, B, and p

In[◦]:=
```
Manipulate[Plot[(-1 + c + δ)/(-1 - B p + δ), {δ, 0, 1}, PlotRange → {0, 1}, AxesLabel → {"δ", "q*"}],
 {c, 0, 1, Appearance → "Labeled"}, {B, 0, 1, Appearance → "Labeled"},
 {p, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

Out[◦]=



As δ increases, q* decreases at a non-linear rate with all other parameters held constant. The y intercept and x intercept is lower for higher values of c. The y-intercept is lower for higher values of B and p, causing the curvature of the graph to become lesser as well.

Relationship between q* and δ

In[◦]:=
$$D\left[\frac{-1+c+\delta}{-1-B\,p+\delta},\ \delta\right]$$

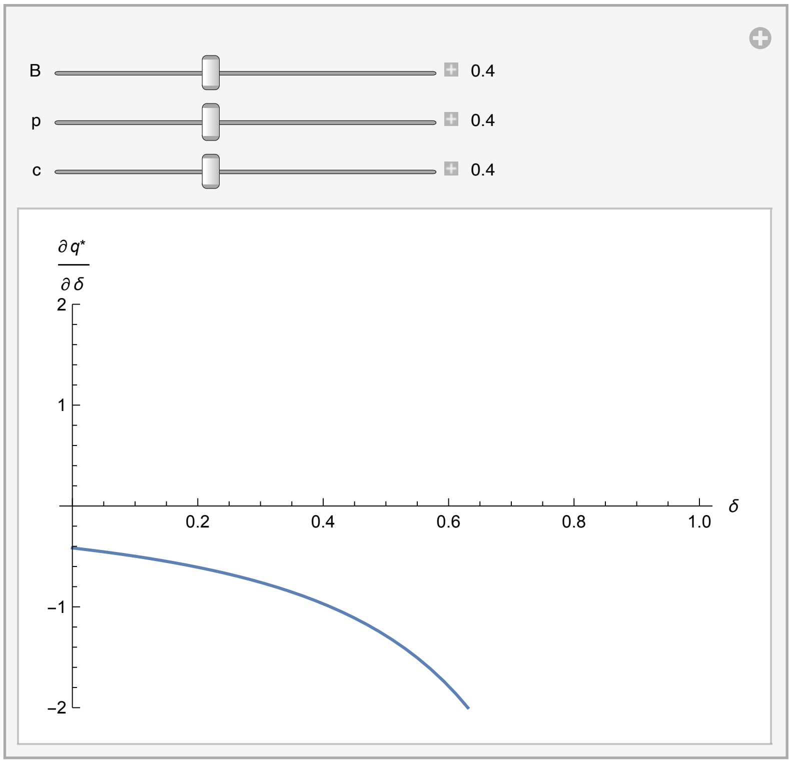
$$-\frac{-1+c+\delta}{(-1-B\,p+\delta)^2}+\frac{1}{-1-B\,p+\delta}$$

In[◦]:=
$$\text{Simplify}\left[-\frac{-1+c+\delta}{(-1-B\,p+\delta)^2}+\frac{1}{-1-B\,p+\delta}\right]$$

Out[◦]=
$$-\frac{c+B\,p}{(1+B\,p-\delta)^2}$$

In[∘]:=
```
Manipulate[Plot[-(c + B p)/(1 + B p - δ)^2, {δ, 0, 1}, PlotRange → 2, AxesLabel → {"δ", "∂q*/∂δ"}],
 {B, 0, 1, Appearance → "Labeled"}, {p, 0, 1, Appearance → "Labeled"},
 {c, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

Out[∘]=



This shows that the rate of change of q* with respect to δ is negative and non-linear.

Computation of Equilibrium Optimal Value of c*

In[∘]:=
```
Solve[(q) * (1 / (1 + δ)) + (1 - q) * ((δ^2 / (1 + δ)) - p * B) == (1 / (1 + δ)) - c - p * B, c]
```

Out[∘]= $\{\{c \to 1 - q - B\,p\,q - \delta + q\,\delta\}\}$

In[∘]:=
```
Simplify[1 - q - B p q - δ + q δ]
```

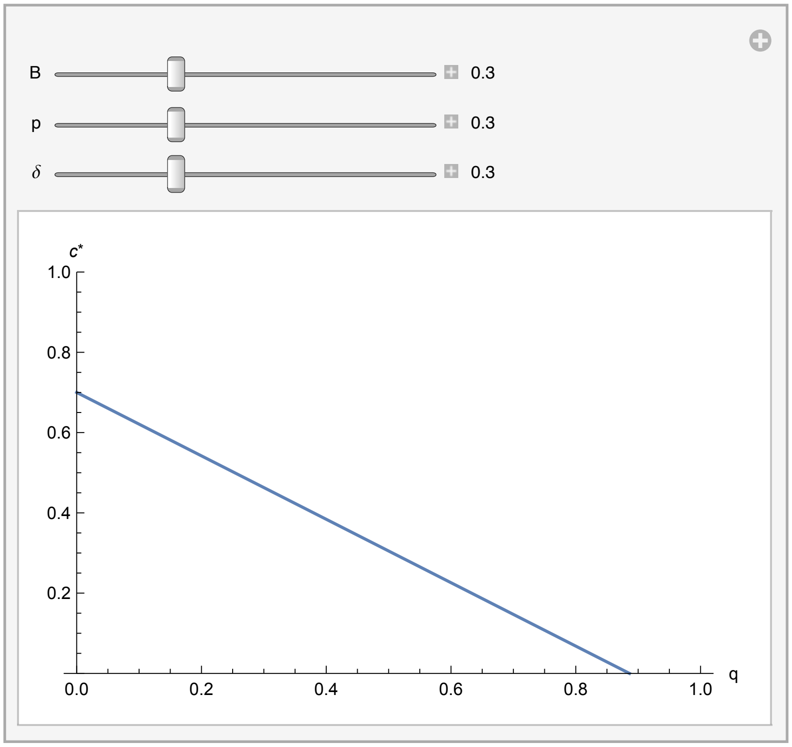
Out[∘]= $1 - \delta + q\,(-1 - B\,p + \delta)$

This is the equilibrium value of c. Below c* there is no rejection of offer or conflict as State 1 will make the generous demand of x_g and at c* and above it, rejection of offer is observed with probability 1-q and terrorism is observed with probability p.

Graph c* against q, vary p, B, and δ

In[•]:=
```
Manipulate[
 Plot[1 - δ + q (-1 - B p + δ) , {q, 0, 1}, PlotRange → {0, 1}, AxesLabel → {"q", "c*"}],
 {B, 0, 1, Appearance → "Labeled"}, {p, 0, 1, Appearance → "Labeled"},
 {δ, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

Out[•]=



As q increases, c* decreases at a linear rate with all other parameters held constant. The y intercept decreases for higher values of δ . The x intercept is lower for higher values of B and p

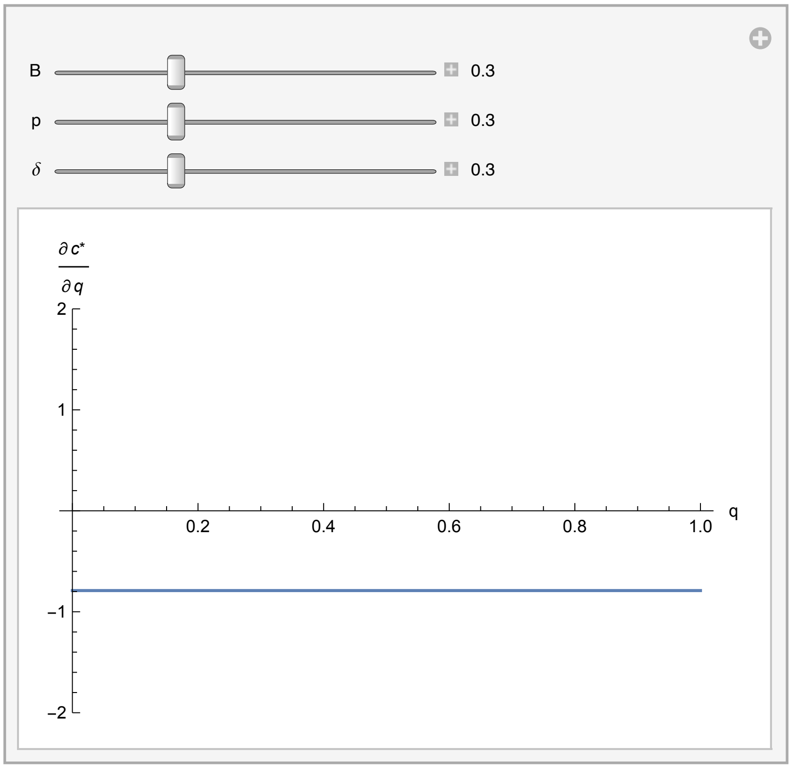
Relationship between c* and q

In[•]:=
```
D[1 - δ + q (-1 - B p + δ), q]
```

Out[•]= $-1 - B\,p + \delta$

In[∘]:=
```
Manipulate[Plot[-1 - B p + δ , {q, 0, 1}, PlotRange → 2, AxesLabel → {"q", "∂c*/∂q"}],
  {B, 0, 1, Appearance → "Labeled"}, {p, 0, 1, Appearance → "Labeled"},
  {δ, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

Out[∘]=

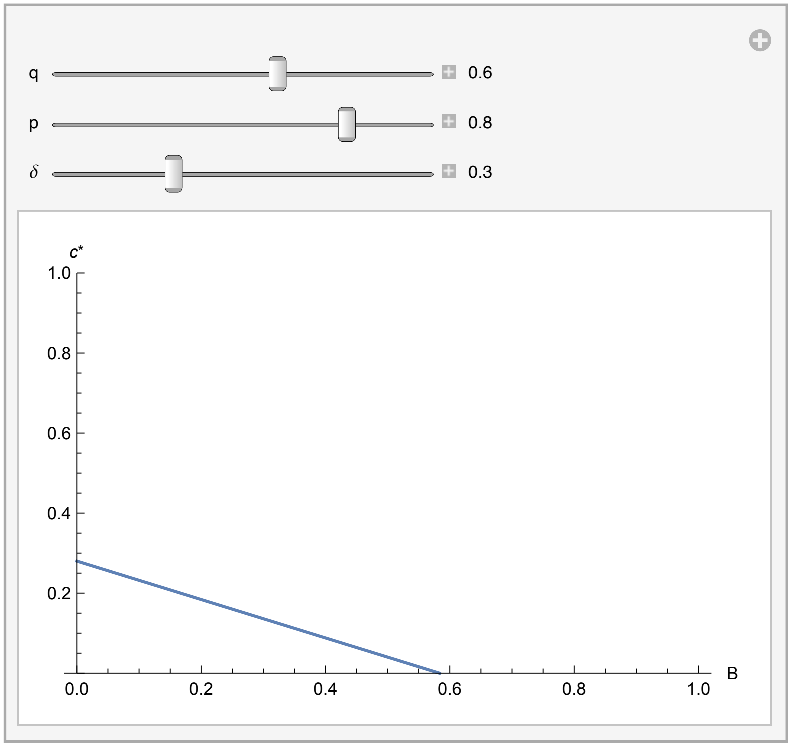


This shows that the rate of change of c* with respect to q is negative and linear.

Graph c* against B, vary q, p, and δ

In[•]:=
```
Manipulate[
 Plot[1 - δ + q (-1 - B p + δ) , {B, 0, 1}, PlotRange → {0, 1}, AxesLabel → {"B", "c*"}],
 {q, 0, 1, Appearance → "Labeled"}, {p, 0, 1, Appearance → "Labeled"},
 {δ, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

Out[•]=



As B increases, c* decreases at a linear rate with all other parameters held constant. The x and y intercept decreases for higher values of q and δ. The x intercept is lower for higher values of p

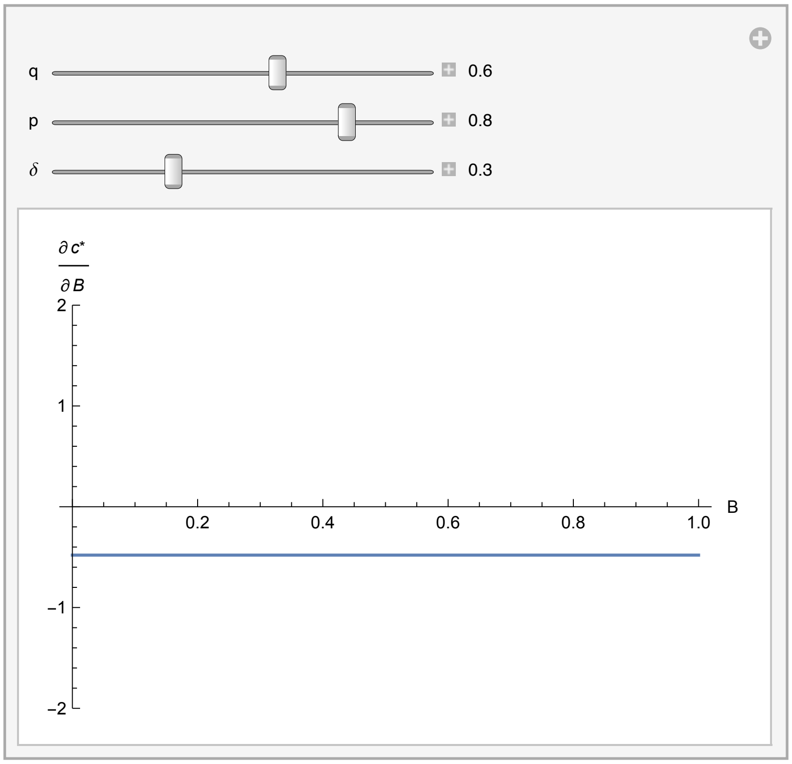
Relationship between c* and B

In[•]:=
```
D[1 - δ + q (-1 - B p + δ), B]
```

Out[•]= $-p\,q$

```
Manipulate[Plot[-p q , {B, 0, 1}, PlotRange → 2, AxesLabel → {"B", "∂c*/∂B"}],
 {q, 0, 1, Appearance → "Labeled"}, {p, 0, 1, Appearance → "Labeled"},
 {δ, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

Out[ ]=

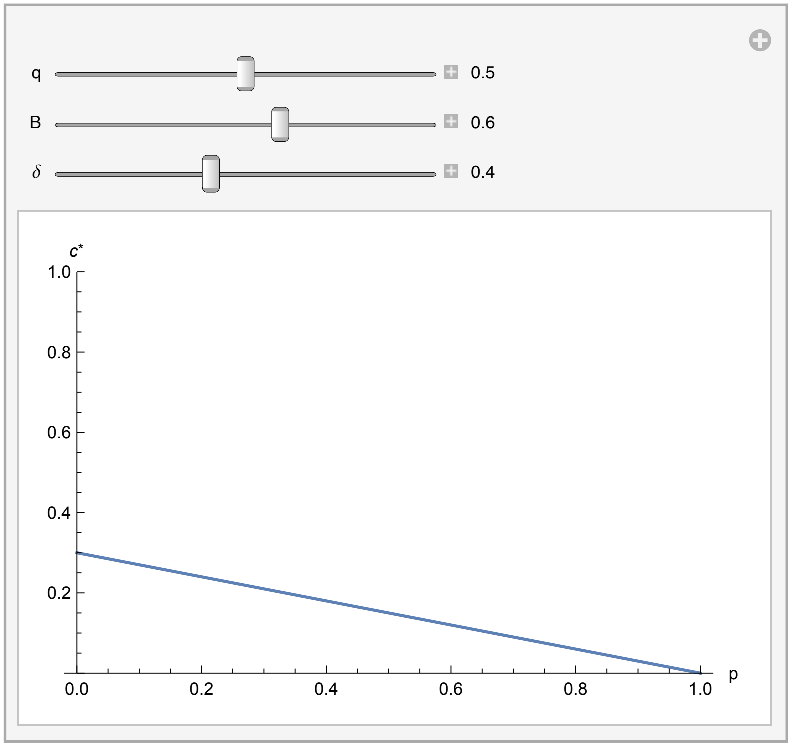


This shows that the rate of change of c* with respect to B is negative and linear.

Graph c* against p, vary q, B, and δ

In[•]:=
```
Manipulate[
 Plot[1 - δ + q (-1 - B p + δ) , {p, 0, 1}, PlotRange → {0, 1}, AxesLabel → {"p", "c*"}],
 {q, 0, 1, Appearance → "Labeled"}, {B, 0, 1, Appearance → "Labeled"},
 {δ, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

Out[•]=



As p increases, c* decreases at a linear rate with all other parameters held constant. The x and y intercept decreases for higher values of q and δ. The x intercept is lower for higher values of B

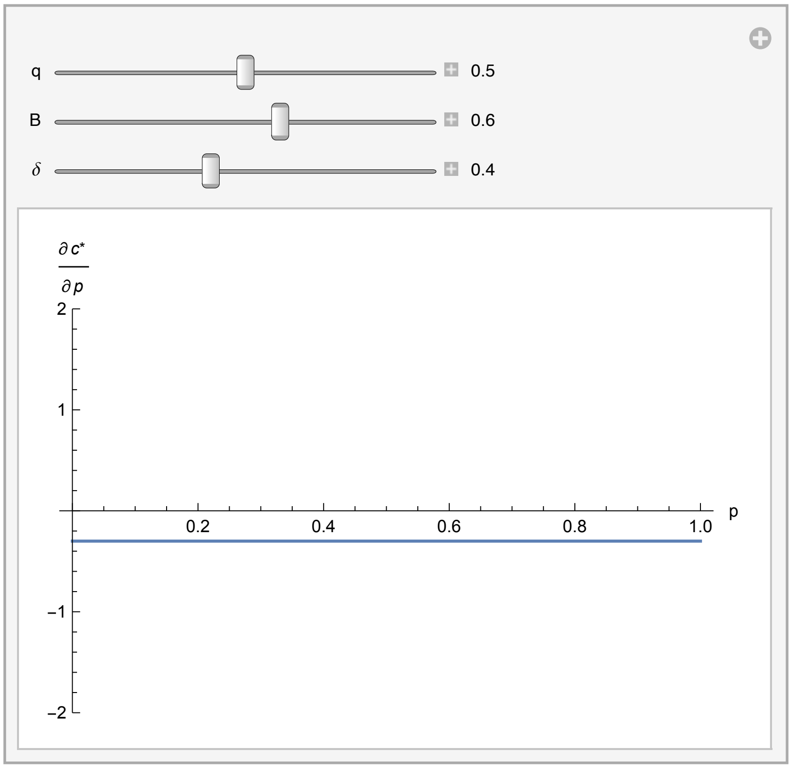
Relationship between c* and p

In[•]:=
```
D[1 - δ + q (-1 - B p + δ), p]
```

Out[•]= $-B\,q$

```
Manipulate[Plot[-B q , {p, 0, 1}, PlotRange → 2, AxesLabel → {"p", "∂c*/∂p"}],
 {q, 0, 1, Appearance → "Labeled"}, {B, 0, 1, Appearance → "Labeled"},
 {δ, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

Out[ ]=

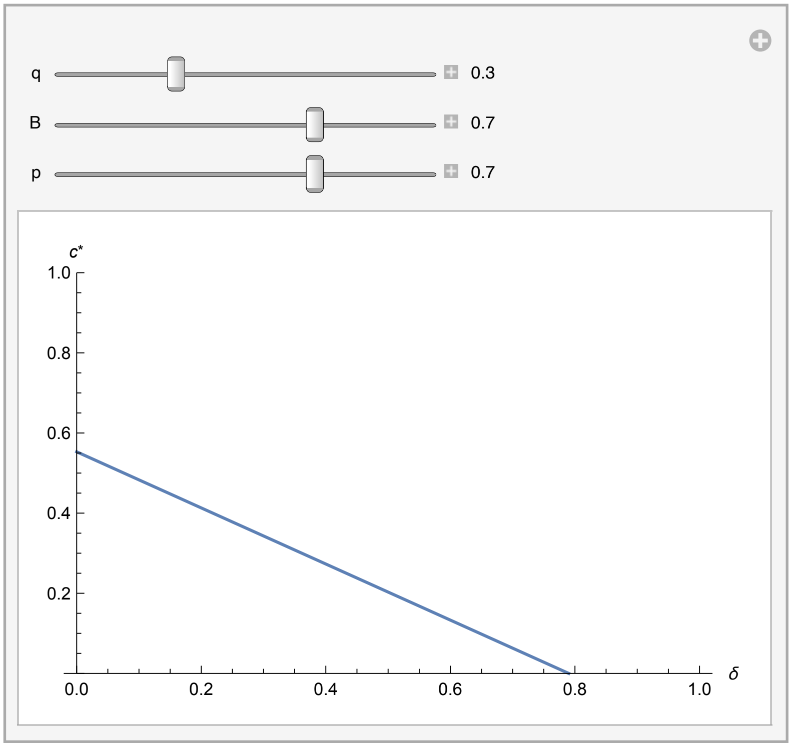


This shows that the rate of change of c* with respect to p is negative and linear.

Graph c* against δ, vary q, B, and p

In[•]:=
```
Manipulate[
 Plot[1 - δ + q (-1 - B p + δ) , {δ, 0, 1}, PlotRange → {0, 1}, AxesLabel → {"δ", "c*"}],
 {q, 0, 1, Appearance → "Labeled"}, {B, 0, 1, Appearance → "Labeled"},
 {p, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

Out[•]=



As δ increases, c* decreases at a linear rate with all other parameters held constant. The x and y intercept decreases for higher values of q, B, and p . This change is very sensitive to change in q but not as sensitive for change in B and p.

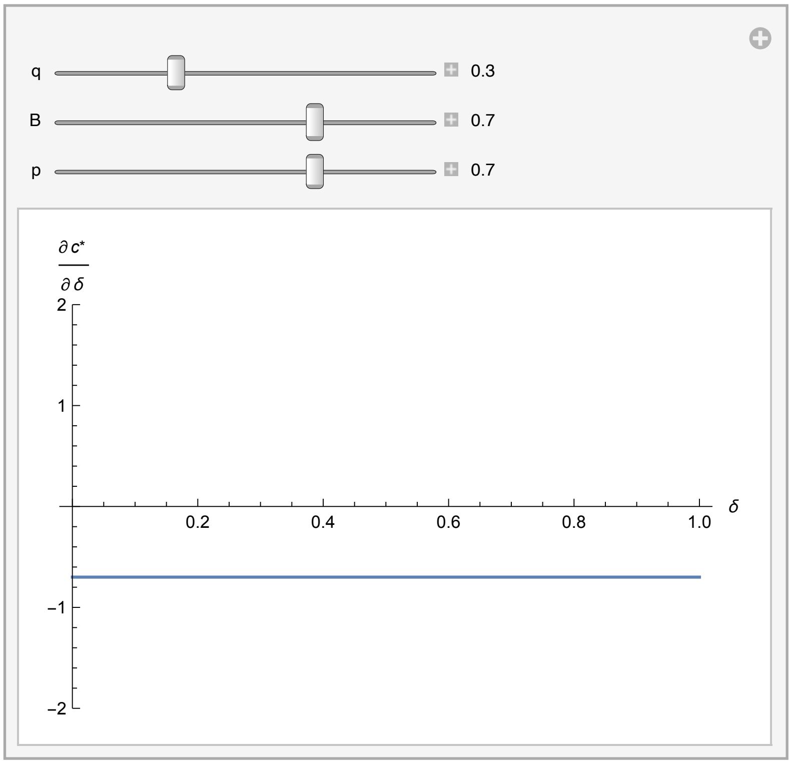
Relationship between c* and δ

In[•]:=
```
D[1 - δ + q (-1 - B p + δ), δ]
```

Out[•]= $-1 + q$

```
Manipulate[Plot[-1 + q , {δ, 0, 1}, PlotRange → 2, AxesLabel → {"δ", "∂c*/∂δ"}],
 {q, 0, 1, Appearance → "Labeled"}, {B, 0, 1, Appearance → "Labeled"},
 {p, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

Out[∘]=



This shows that the rate of change of c* with respect to δ is negative and linear.

Computations of Equilibrium Optimal values of B*, p*, and δ*

Equilibrium for B*

In[∘]:=
```
Solve[(q) * (1 / (1 + δ)) + (1 - q) * ((δ^2 / (1 + δ)) - p * B) == (1 / (1 + δ)) - c - p * B, B]
```

Out[∘]= $\left\{\left\{B \to \frac{1 - c - q - \delta + q\,\delta}{p\,q}\right\}\right\}$

Equilibrium for p*

In[∘]:=
```
Solve[(q) * (1 / (1 + δ)) + (1 - q) * ((δ^2 / (1 + δ)) - p * B) == (1 / (1 + δ)) - c - p * B, p]
```

Out[∘]= $\left\{\left\{p \to \frac{1 - c - q - \delta + q\,\delta}{B\,q}\right\}\right\}$

Equilibrium for δ*

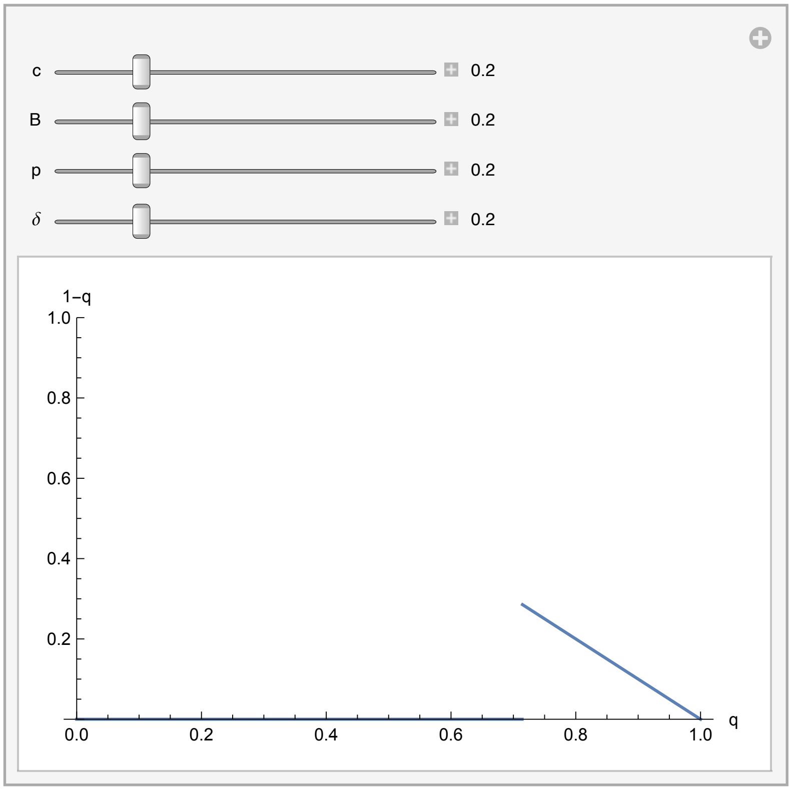
In[∘]:=
```
Solve[(q) * (1 / (1 + δ)) + (1 - q) * ((δ^2 / (1 + δ)) - p * B) == (1 / (1 + δ)) - c - p * B, δ]
```

$\left\{\left\{\delta \to \frac{-1 + c + q + B\,p\,q}{-1 + q}\right\}\right\}$

Graph Probability of Demand Rejection against q

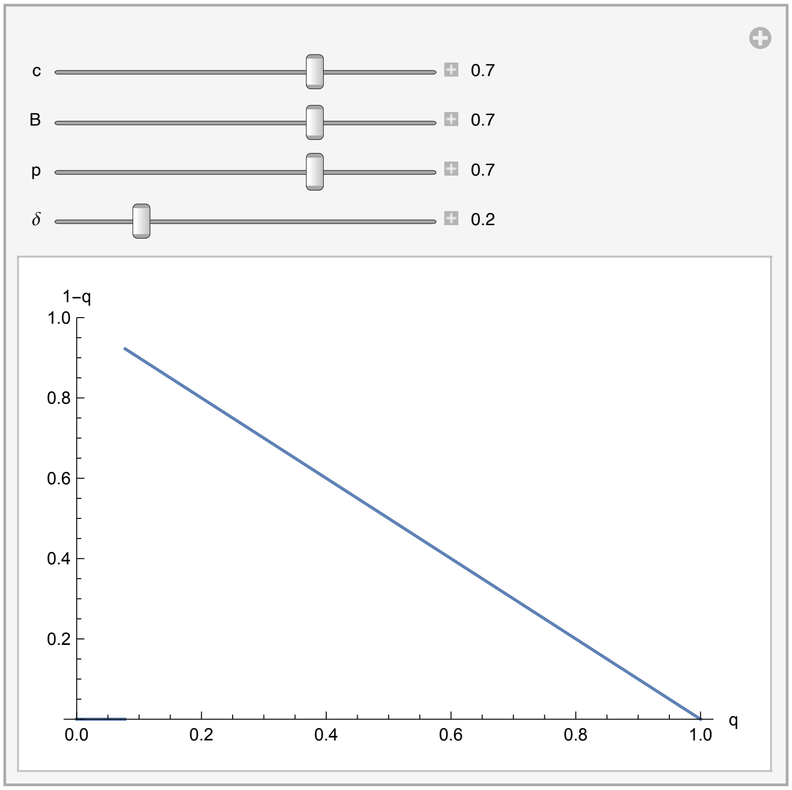
In[∘]:=
```
Manipulate[Plot[Piecewise[{{0, q < (-1 + c + δ)/(-1 - B p + δ)}, {1 - q, q > (-1 + c + δ)/(-1 - B p + δ)}}], {q, 0, 1},
  PlotRange → {0, 1}, AxesLabel → {"q", "1-q"}], {c, 0, 1, Appearance → "Labeled"},
 {B, 0, 1, Appearance → "Labeled"}, {p, 0, 1, Appearance → "Labeled"},
 {δ, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

Out[∘]=

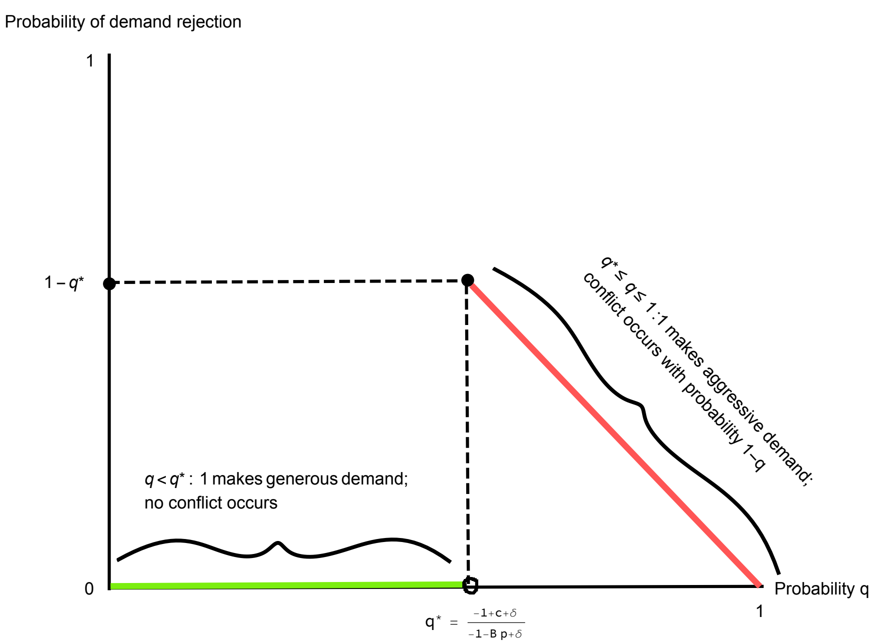


This shows that the equilibrium q* is higher for lower values of c, B, p, and δ.

Out[ ]=



This shows that the equilibrium q* is lower for higher values of c, B, p, but a low δ . If δ is higher, the piecewise function disappears and only the decreasing linear graph remains.

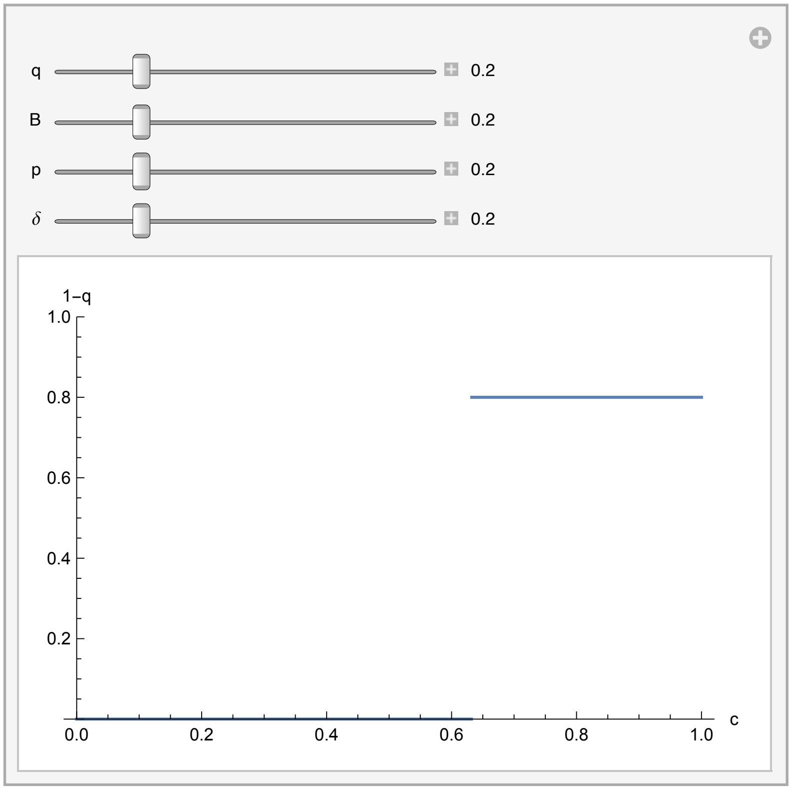


This is an illustration of the graphs computed above.

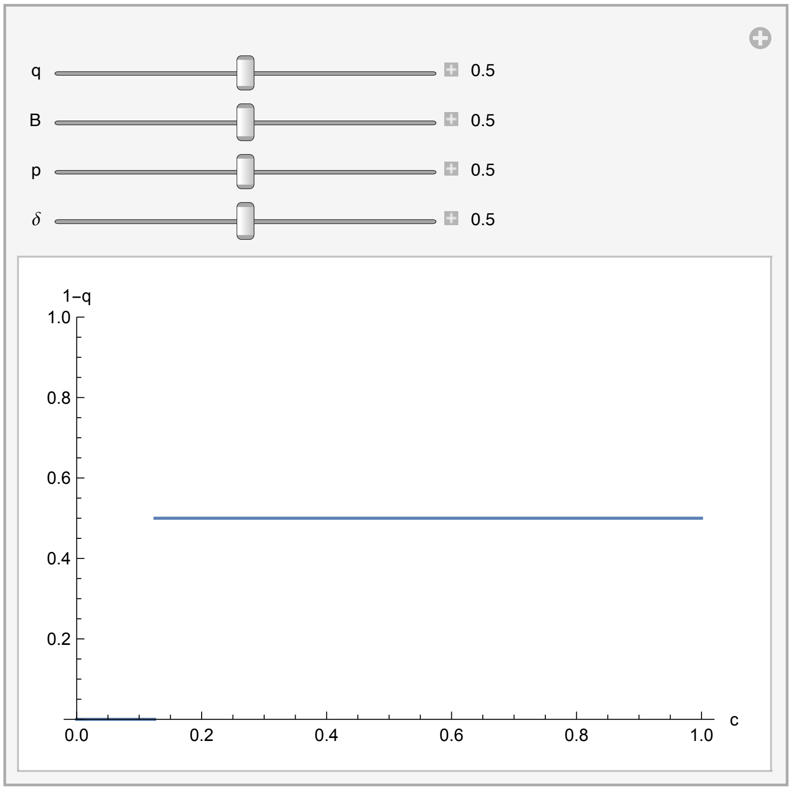
Graph Probability of Demand Rejection against c

In[∘]:=
```
Manipulate[
 Plot[Piecewise[{{0, c < 1 - δ + q (-1 - B p + δ)}, {1 - q, c > 1 - δ + q (-1 - B p + δ)}}],
  {c, 0, 1}, PlotRange → {1, 0}, AxesLabel → {"c", "1-q"}],
 {q, 0, 1, Appearance → "Labeled"}, {B, 0, 1, Appearance → "Labeled"},
 {p, 0, 1, Appearance → "Labeled"},
 {δ, 0, 1, Appearance → "Labeled"}, AutorunSequencing → All]
```

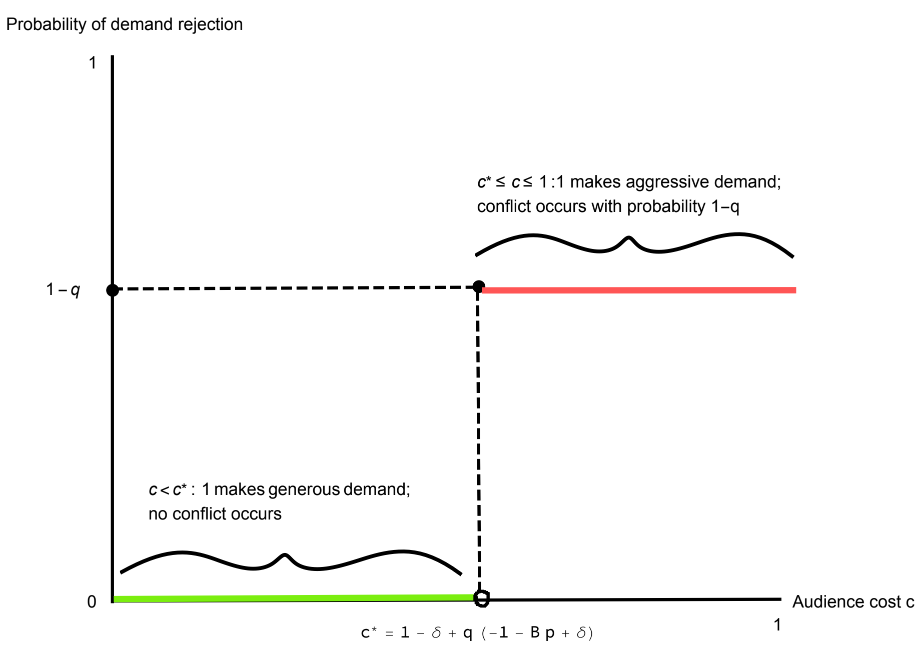
Out[∘]=



This shows that the equilibrium c* is higher for lower values of c, B, p, and δ and the probability of rejection is constant after the equilibrium at 1-q.



This shows that the equilibrium q* is lower for medium values of c, B, p, and δ. If the values are higher, the piecewise function disappears and only the constant linear graph remains.

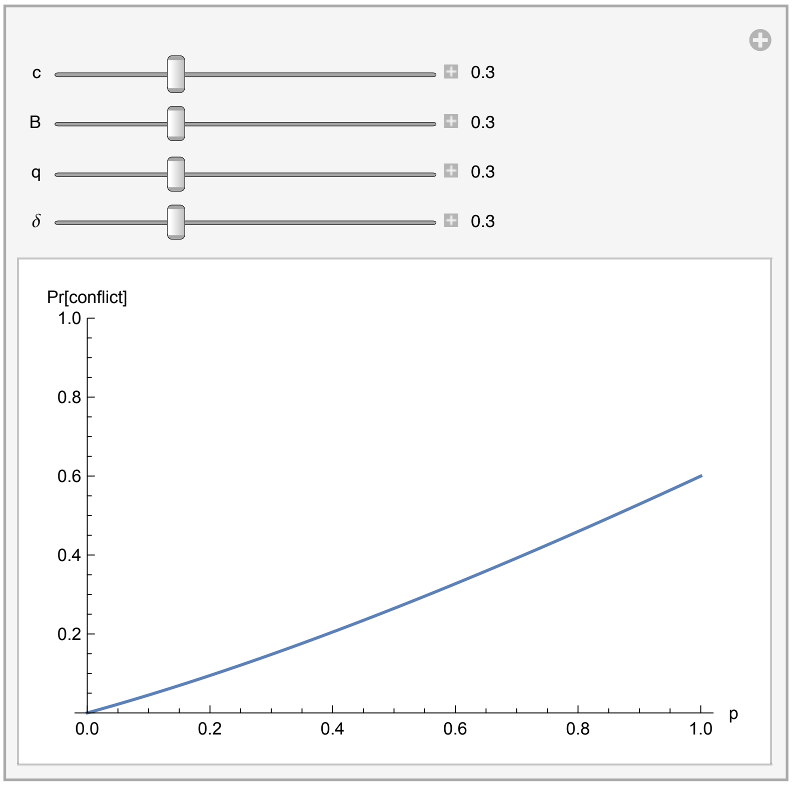


This is an illustration of the graphs computed above.

Probability of Conflict (Reject and Terrorist Attack) = (1-q*)(p) against p, vary q, c, B, δ

In[∘]:=
```
Manipulate[Plot[{(1 - (-1 + c + δ)/(-1 - B p + δ)) * p}, {p, 0, 1},
  PlotRange → {0, 1}, AxesLabel → {"p", "Pr[conflict]"}],
 {c, 0, 1, Appearance → "Labeled"}, {B, 0, 1, Appearance → "Labeled"},
 {q, 0, 1, Appearance → "Labeled"}, {δ, 0, 1, Appearance → "Labeled"}]
```

Out[∘]=



Probability of conflict i.e. demand rejection and terrorist attack is p(1-q). For both lower and higher values of the parameters, as p increases, the probability of conflict also increases. This is not trivial because p is directly proportional to p(1-q).