Distribution Agreement

Essays in Macroeconomic Dynamics

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Doctor of Philosophy

Economics

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James T. Laney School of Graduate Studies of Emory University
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Abstract

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This dissertation consists of three chapters in macroeconomic dynamics. In the first chapter, I provide a microfoundation for the domestic financial dollarization in many emerging economies. Dollarization is modeled as the currency denomination choice of domestic households' saving and experts' borrowing in a small economy with financial constraints and downward wage rigidity. The competitive equilibrium allocation is constrained inefficient due to a pecuniary externality and imperfect financial market. The government allows dollarization to alleviate the dilemma between achieving domestic risk-sharing and lifting nominal rigidity to implement constrained efficient allocation. This chapter describes the optimal policy choice of dollarization and state-contingent exchange rates.

The second chapter studies the intergenerational wealth transfer induced by government housing market intervention. Through a simplified over-lapping generation model, this chapter finds government can achieve optimal intergenerational transfer via housing price subsidy and purchase rationing. With a quantitative model calibrated to match aggregate and cross-sectional empirical moments, this chapter evaluates the welfare impact of the housing market reform in China on the initial old generation.

The third chapter studies the terms-of-trade management of a small open economy with defaultable external debt. It extends the consumption goods space of the sovereign default model and endogenizes the terms of trade by introducing a foreign demand. The model features the interaction of terms-of-trade movement with external borrowing and sovereign default. The paper shows competitive equilibrium is constraint inefficient. A quantitative analysis shows that the government committing to a zero tariff rate is more credible and borrows more external debt with a lower interest rate in the equilibrium than the one freely setting tariff.

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Contents

[1]This chapter is based on a joint research project with one committee member, Kaiji Chen, Ph.D.. With this dissertation committee's approval, I included my contribution as this chapter in the dissertation.

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Chapter 1

Financial Dollarization, Exchange Rate, and Macroprudential Policy

1.1 Introduction

Many emerging economies have featured financial dollarization, which means the dollar plays the role of a de facto second currency on the financial market in these economies. Some of the households' savings are denominated in dollars, and a significant share of firms' debt is in dollars at the same time. Financial dollarization is often a focus of policy debate and academic research. On the one hand, liability dollarization is perceived as a source of financial instability for emerging economies due to the financial amplification via the balance sheet effect, especially during exchange rate depreciation. A widespread view is that dollarization raises the risk of sovereign debt crises, banking crises, and other financial crises. On the other hand, dollarization is argued to facilitate international risk sharing. Deposit dollarization can also serve as an insurance arrangement for domestic savers.

This chapter focuses on providing a microfoundation for dollarization in the domestic financial system. Several key empirical features have been documented in the literature about financial dollarization in emerging economies, [6]. First, countries characterized by high levels of liability dollarization are also countries where domestic households tend to save in foreign currency. Second, in economies with higher dollarization, local currency bonds are characterized by higher excess returns over comparable dollar bonds. Lastly, countries with more counter-cyclical exchange rates have a higher level of dollarization.

Based on these empirical observations, I propose a theory for the domestic financial dollarization for emerging economies. I consider a small economy populated by households and experts. Experts produce nontradable goods, build up capital and issue financial claims subject to a collateral constraint. Households receive tradable goods endowment, earn labor income, and own financial claims issued by experts. The competitive equilibrium allocation is constrained inefficient due to a pecuniary externality and imperfect financial market. Constraint efficient allocation can be implemented with the state-contingent claim. Yet, a dilemma arises in an economy with non-state contingent debt and nominal rigidity if only local currency debt can be issued. The insurance arrangement between households and experts requires a set of state-contingent exchange rates to achieve wealth redistribution between households and experts. At the same time, exchange rates affect the experts' net-worth and the tightness of the collateral constraint. The consideration for financial stability may lead to a different set of relative prices in the presence of nominal rigidity. The theory shows that domestic dollarization may arise to ameliorate the dilemma posed on the exchange rate policy. This chapter describes the optimal policy choice of dollarization and state-contingent exchange rates.

This chapter contributes to the literature studying financial dollarization, see [6] and [9]. Unlike the existing research, this chapter provides the characterization of the constraint efficient allocation and its implementation via dollarization. This chapter also contributes to the literature studying the private currency denomination choice of the private contracts, see [20], [7], and [11]. While these papers abstract away from the government optimal policies, this chapter stressed the interaction between government policy and the private currency denomination decision. This chapter is also related to the literature studying the interaction between the currency choice of external debt and government policy, see [24], [14], and [12]. The main difference is that this chapter focuses on domestic risk sharing while abstracting away from the external debt.

1.2 Model

I consider a small economy populated by households and experts. Experts produce nontradable goods, build up capital and issue financial claims. Households receive tradable goods endowment, earn labor income from experts, and own financial claims issued by experts. We begin by describing the details of the optimization problem of households, experts, and the government with nominal terms and nominal non-state contingent financial claims. Then, we describe a corresponding real economy with real state-contingent claims.

1.2.1 The Nominal Economy with Non-state Contingent Claims

Households Households are identical and atomic. They have Epstein-Zin preferences over consuming tradable and nontradable goods described by the recursive utility function

$$V_{h,t} = \left\{(1-\beta_h)c_{h,t}^{1-\rho} + \beta_h \left[\mathbb{E}_t(V_{h,t+1}^{1-\sigma})\right]^{\frac{1-\rho}{1-\sigma}}\right\}^{\frac{1}{1-\rho}}, \tag{1.1}$$

where $V_{h,t}$ denotes households utility in period t and c_t denotes consumption. Households have a discount factor, β_h, the elasticity of intertemporal substitution, ρ, and risk aversion, σ. The consumption is a composite of tradable and nontradable consumption aggregated by a constant elasticity of substitution technology of the form

$$c_{h,t} = A(c_{h,t}^T, c_{h,t}^N) = \left[a(c_{h,t}^T)^{1-\frac{1}{\xi}} + (1-a)(c_{h,t}^N)^{1-\frac{1}{\xi}}\right]^{\frac{\xi}{\xi-1}}, \tag{1.2}$$

where $c_{h,t}^T$ and $c_{h,t}^N$ denote tradable and nontradable consumption and ξ is the elasticity of substitution between tradable and nontradable goods.

Households have access to a one-period, non-state contingent bond issued by experts. The bond can be denominated in foreign or local currency. We use b_t^f and B_t^l to denote the unit of the bond denominated in foreign and local currency in period t and the prices of bonds are q_t^f and q_t^l. The sequential budget constraint of the households is

$$P_t^T c_{h,t}^T + P_t^N c_{h,t}^N + E_t q_t^f b_{t+1}^f + q_t^l B_{t+1}^l = P_t^T y_t^T + W_t h_t + E_t b_t^f + B_t^l + T_{h,t}, \tag{1.3}$$

where P_t^T and P_t^N denotes the nominal price of tradable and nontradable goods, E_t is the nominal exchange rate (the price of foreign currency in terms of local currency), y_t^T denotes tradable goods endowment owned by households, W_t is the nominal wage, and $h_t \leq \bar{h}$ is hours worked, T_t is the lump-sum transfer from the government. The tradable endowment y_t^T is stochastic and governed by a finite state Markov process, $y_t^T = y(s_t)$, where s_t is the state of the process at time t.

We assume that the price of tradabel goods in foreign currency is constant at the level of one. Thus, the nominal price of tradable goods in local currency is $P_t^T = E_t$. Let $p_t := \frac{P_t^N}{P_t^T}$ be the relative price of nontradable goods, $w_t := \frac{W_t}{P_t^T}$ be the real wage, and $\epsilon_t := \frac{E_t}{E_{t-1}}$ be the depreciation rate of the local currency. An optimal allocation of the household's problem satisfies the first-order conditions

$$p_t = \frac{A_2(c_{h,t}^T, c_{h,t}^N)}{A_1(c_{h,t}^T, c_{h,t}^N)}, \tag{1.4}$$

$$\lambda_t = (1-\beta^h) A_1(c_{h,t}^T, c_{h,t}^N) \left(\frac{V_{h,t}}{A(c_{h,t}^T, c_{h,t}^N)} \right)^{\rho}, \tag{1.5}$$

$$q_t^f \lambda_t = \beta_h V_{h,t}^{\rho} \left(\mathbb{E}_t V_{h,t+1}^{1-\sigma} \right)^{\frac{\sigma-\rho}{1-\sigma}} \mathbb{E}_t \frac{\lambda_{t+1}}{V_{t+1}^{\sigma}}, \tag{1.6}$$

$$q_t^l \lambda_t = \beta_h V_{h,t}^{\rho} \left(\mathbb{E}_t V_{h,t+1}^{1-\sigma} \right)^{\frac{\sigma-\rho}{1-\sigma}} \mathbb{E}_t \frac{\lambda_{t+1}}{\epsilon_{t+1} V_{t+1}^{\sigma}}, \tag{1.7}$$

where λ_t is the Lagrange multiplier of the household budget constraint.

Experts Expert are identical and atomic. They have log preferences over consuming tradable goods

$$\mathbb{E} \sum_{t=0}^{\infty} \beta_e^t \log \left(c_{e,t}^T \right) \tag{1.8}$$

Only experts can hold capital and convert tradable goods into capital. In each period t, experts enter with a predetermined level of capital, k_{t-1}, and combine labor offered by households to produce nontradable goods by the Cobb-Douglas production function for households to purchase and consume:

$$\begin{aligned} y_t^N &= F(k_{t-1}, h_t) \\ &= k_{t-1}^{\alpha} h_t^{1-\alpha}. \end{aligned} \tag{1.9}$$

Experts repay their existing debt denominated in foreign and local currencies, d_t^f and D_t^l. They also issue new debt in both currencies, d_{t+1}^f and D_{t+1}^l, to finance their tradable goods consumption and capital investment together with the net worth, n_t. The sequential budget constraint of experts is

$$P_t^T c_{e,t}^T + P_t^T(1+\tau_t^k)k_t = E_t n_t + (1-\tau_t^f)E_t q_t^f d_{t+1}^f + (1-\tau_t^l)q_t^l D_{t+1}^l + T_{e,t}, \tag{1.10}$$

where $T_{e,t}$ is the lump-sum fiscal transfer payment received from the government, and n_t is the value of experts' net worth in terms of the foreign currency

$$n_t = p_t y_t^N - w_t h_t + (1-\delta)k_{t-1} - d_t^f - \frac{D_t^l}{E_t}. \tag{1.11}$$

We introduce physical capital investment tax, τ_t^k, and capital control tax τ_t^f and τ_t^l on foreign and local currency denominated debt such that later we can decentralized the (constraint) planner's choice of denomination.

In the financial market, experts are subject to a borrowing constraint which requires experts' debt can not be more that θ fraction of the capital in each period

$$d_{t+1}^f + \frac{D_{t+1}^l}{E_{t+1}} \leq \theta(1-\delta)k_t. \tag{1.12}$$

An optimal allocation of the expert's problem satisfies the first order conditions

$$\frac{1}{(1-\beta^e)n_t} = \beta_e \mathbb{E}_t\left[\frac{1}{(1-\beta_e)n_{t+1}}\left[\alpha p_{t+1}\left(\frac{k_t}{l_{t+1}}\right)^{\alpha-1} + (1-\delta)\right]\right] + \beta_e\theta(1-\delta)\mathbb{E}_t\left[\mu_{t+1}\right], \tag{1.13}$$

$$\frac{q_t^f}{(1-\beta^e)n_t} = \beta_e \mathbb{E}_t\left(\frac{1}{(1-\beta_e)n_{t+1}} + \mu_{t+1}\right), \tag{1.14}$$

$$\frac{q_t^l}{(1-\beta^e)n_t} = \beta_e \mathbb{E}_t \frac{1}{\epsilon_{t+1}}\left(\frac{1}{(1-\beta_e)n_{t+1}} + \mu_{t+1}\right), \tag{1.15}$$

$$w_t = (1-\alpha)p_t\left(\frac{k_{t-1}}{h_t}\right)^{\alpha}, \tag{1.16}$$

where μ_{t+1} are Lagrangian multiplier associated with experts' borrowing constraint, and we plug in expert's optimal decision on tradable goods consumption, $c_{e,t}^T = (1-\beta_e)n_t$.

Downward Wage Rigidity In the labor market, there exists a lower bound, $\gamma > 0$, on the growth rate of nominal wage

$$W_t \geq \gamma W_{t-1}. \tag{1.17}$$

If the lower bound on the nominal wage is not binding, $W_t > \gamma W_{t-1}$, labor market is in full employment, $h_t = \bar{h}$. Otherwise, involuntary unemployment, $h_t < \bar{h}$, will happen in the economy. These two cases can be summarized as the labor market slackness condition

$$(W_t - \gamma W_{t-1}) \left(\bar{h} - h_t\right) = 0. \tag{1.18}$$

The Government We assume the government can decide and commit on the state-contingent nominal exchange rate, E_t. The government can also choose the capital control tax on both foreign currency and local currency denominated debt, τ_t^f and τ_t^l, and lump-sum transfers, $T_{h,t}$ and $T_{e,t}$.

Competitive Equilibrium of the Nominal Economy

Definition 1 (Competitive Equilibrium)**.** A competitive equilibrium of the nominal economy is a set of sequential allocations $\{c_{h,t}^T, c_{h,t}^N, c_{e,t}^T, k_t, h_t, b_t^f, B_t^l, d_t^f, D_t^l\}$ and prices $\{P_t^T, P_t^N, W_t, p_t, w_t, q_t^f, q_t^l\}$ such that (1) households and experts optimization problems are both solved; (2) goods and financial market are clear, given exogenous processes $\{y_t^T\}$.

1.2.2 The Real Economy with State-contingent Claims

In this section, I first describe a corresponding real economy with state-contingent claims but without the labor market friction. The planer can intervene for one period by choosing consumption, investment, and labor input allocations. The planner is the constraint because the intervention is one-period only, and she takes the equilibrium outcome of all following periods as given. Based on this real economy, I define a constraint planner's problem. Then, I illustrate how to implement the constraint efficient allocation in the nominal economy with non-state contingent claims.

In the real economy, the sequential budget constraint of the households is

$$c_h^T\left(s^t\right)+p(s^t)c_h^N(s^t)+\sum_{s_{t+1}}q\left(s_{t+1}|s^t\right)a\left(s_{t+1}|s^t\right)=y^T(s^t)+w\left(s^t\right)l\left(s^t\right)+a\left(s^t\right), \quad (1.19)$$

where $q\left(s_{t+1}|s_t\right)$ is the price of the state-contingent claim. The optimal condition of household's choice for $a(s_{t+1}|s_t)$ satisfies

$$q(s_{t+1}|s^t)=\beta_h\pi(s_{t+1}|s^t)\frac{\frac{\partial A(s^{t+1})}{\partial c_h^T}}{\frac{\partial A(s^t)}{\partial c_h^T}}\left(\frac{A_h(s^t)}{A_h(s^{t+1})}\right)^{\rho}\left(\frac{\tilde{V}(s^t)}{V_h(s^{t+1})}\right)^{\sigma-\rho}, \quad (1.20)$$

where $\tilde{V}(s^t)=\mathbb{E}_t\left[V_h(s^t,s_{t+1})^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$. Expert's budget constraint is

$$\begin{aligned}c_e^T\left(s^t\right)&+w\left(s^t\right)l(s^t)+k\left(s^t\right)+b\left(s^t\right)=\\&p(s^t)k(s^{t-1})^{\alpha}l(s^t)^{1-\alpha}+(1-\delta)\,k\left(s^{t-1}\right)+\sum_{s_{t+1}}q\left(s_{t+1}|s^t\right)b\left(s_{t+1}|s^t\right).\end{aligned} \quad (1.21)$$

The collateral constraint of borrowing against state s_{t+1} is

$$b\left(s_{t+1}|s^t\right)\leq\theta\,(1-\delta)\,k\left(s^t\right). \quad (1.22)$$

The optimal condition of expert's choice for $b(s_{t+1}|s_t)$ satisfies

$$q(s_{t+1}|s^t)\frac{1}{(1-\beta^e)N(s^t)}=\beta_e\pi(s_{t+1}|s^t)\left(\frac{1}{(1-\beta^e)N(s^{t+1})}+\mu(s^{t+1})\right), \quad (1.23)$$

where $\mu(s^{t+1})$ is the Lagrange multiplier associated with the collateral constraint in the state s^{t+1}.

To understand the constraint efficient allocation For the convenience to discuss the planner's problem, we define the recursive competitive equilibrium. The aggregate state in the economy is $\mathbf{s} = \left[K, B, y^T\right]$. I rewrite household's optimization problem as

$$\max_{c_h, a'} \left\{(1-\beta_h) c_h^{1-\rho} + \beta_h \left[\mathbb{E}_{\mathbf{s}} V_h^{1-\sigma}(\mathbf{s}')\right]^{\frac{1-\rho}{1-\sigma}}\right\}^{\frac{1}{1-\rho}},$$

$$c_h^T + p(\mathbf{s}) c_h^N + \sum_{\mathbf{s}'} q\left(\mathbf{s}'|\mathbf{s}\right) a\left(\mathbf{s}'|\mathbf{s}\right) = y^T + w(s)l + a,$$

and expert's problem as

$$\max_{c_e^T, b', k'} \log c_e^T + \beta_e \mathbb{E}_{\mathbf{s}} V_e(\mathbf{s}'),$$

$$c_e^T + w\left(\mathbf{s}\right) l + k' + b = p(\mathbf{s}) k^\alpha l^{1-\alpha} + (1-\delta)\, k + \sum_{\mathbf{s}'} q\left(\mathbf{s}'|\mathbf{s}\right) b\left(\mathbf{s}'|\mathbf{s}\right).$$

With the recursive notation of household's and expert's optimization problem, the constraint planner's allocation can be solved by the following primal problem

$$\max_{K', B'(s'), L(s')} \left((1-\beta) C_h^{1-\rho} + \beta^h [\mathbb{E}_{\mathbf{s}} V_h(B'(s'), L'(s'), K', s'))^{1-\sigma}]^{\frac{1-\rho}{1-\sigma}}\right)^{\frac{1}{1-\rho}},$$

$$C_h^T + C_e^T + K' \leq Y^T + (1-\delta)K,$$

$$C_h^N = K^\alpha L^{1-\alpha},$$

$$\log(C_e^T) + \beta^e \mathbb{E} V^e(B'(s'), L'(s'), K', s') \geq V^e(B, K, s).$$

Lemma 1. *If an allocation in the initial period, $\{C_{h,0}, C_{e,0}, K_1, B_1(s)\}$, is constraint efficient, then it must satisfy the following conditions*

$$\beta_e C_e^T \left(\frac{1}{C_e^T(s)} + \mu(s)\right) - \beta_h \frac{\frac{\partial A(s')}{\partial C_h^T}}{\frac{\partial A}{\partial C_h^T}} \left(\frac{A}{A(s')}\right)^\rho \left(\frac{\tilde{V}_{h0}}{V^h(s)}\right)^{\sigma-\rho}$$
$$= \beta_e C_e^T \frac{\partial V_1^e}{\partial B_1(s)} + \beta_h \frac{A^\rho \tilde{V}_{h0}^{\sigma-\rho} V_h(s)^{-\sigma}}{(1-\beta_h)\frac{\partial A}{\partial C_h^T}} \frac{\partial V_h}{\partial B_1(s)},$$

$$-\beta_h \frac{\frac{\partial A(s')}{\partial C_h^T}}{\frac{\partial A}{\partial C_h^T}} \left(\frac{A}{A(s')}\right)^\rho \left(\frac{\tilde{V}_{h0}}{V_h(s)}\right)^{\sigma-\rho} w(s) = \beta_e C_e^T \frac{\partial V_1^e}{\partial L_1(s)} + \beta^h \frac{A^\rho \tilde{V}_{h0}^{\sigma-\rho} V_h(s)^{-\sigma}}{(1-\beta_h)\frac{\partial A}{\partial C_h^T}} \frac{\partial V_h}{\partial L_1(s)},$$

where all variables after the initial period are determined in the recursive competitive equilibrium.

The above lemma states the necessary conditions for the constraint planner to choose state-contingent claims and labor supply to achieve efficient risk sharing between households and experts. The left-hand side of the first condition is the risk sharing condition in the competitive equilibrium. The externality terms on the right-hand side imply that competitive equilibrium outcome is not constraint efficient in general. The second equation states the constraint planner's trade-off on the choice of the labor input. The left-hand side of the condition is the utility loss of households from reduced labor income, while the right-hand side is the externalities induced by the labor input. In the next section, a simplified version of the model is listed to illustrate the externalities of the individual choices.

1.2.3 Implementation of the Constraint Efficient Allocation

In the last section, I characterize the constraint planner's choice of state-contingent claim and labor input across states. Now, I consider how the government can implement the constraint efficient allocation in the nominal economy described in section 2.1.

The constraint planner defined above has two tools to reallocate wealth between households and experts. However, the replication in the nominal economy may encounter a conflict between these two margins. To illustrate the possible conflict, I first consider replicating the constraint efficient allocation in which the planner chooses full employment across all states. The following proposition illustrates how replication can be done.

Proposition 1. *The constraint efficient allocation can be implemented with* $\left\{D^l\left(s^t\right), \tau^l\left(s^t\right), E\left(s^{t+1}|s^t\right)\right\}$ *if* $L(s^{t+1}|s^t) = 1$ *for all* s_{t+1}, *and*

$\left\{D^l\left(s^t\right), \tau^l\left(s^t\right), q^l\left(s^t\right), E\left(s^{t+1}|s^t\right)\right\}$ *should satisfy the following conditions*

$$\frac{D^l\left(s^t\right)}{E\left(s^{t+1}|s^t\right)} = B\left(s_{t+1} = s_i|s^t\right) \tag{1.24}$$

$$\tau^l\left(s^t\right) = \mathbb{E}_{s^t}\tau\left(s^{t+1}|s^t\right)\frac{E\left(s^t\right)}{E\left(s^{t+1}|s^t\right)} \tag{1.25}$$

$$q^l\left(s^t\right) = \mathbb{E}_{s^t}q\left(s^{t+1}|s^t\right)\frac{E\left(s^t\right)}{E\left(s^{t+1}|s^t\right)} \tag{1.26}$$

given $E\left(s^t\right), B\left(s^t\right), y^T\left(s^t\right), K\left(s^t\right)$.

Proposition 1 states the sufficiency of the single currency denomination when it is optimal to have full employment in all states. In this case, the government can achieve full employment across states by setting up exchange rates that ensure nominal wages across states are not binding. The downward wage rigidity only restricts the lower bound in each state to achieve full employment. Thus, it leaves the government enough freedom to adjust the real value of the non-state contingent claim across states via choosing state-contingent exchange rate or depreciation rate. The nominal exchange rates are set so that the implied asset return of households and debt repayment of experts mimic the state-contingent claim in the real economy. In this case of full employment, there is no conflict for government to achieve the constraint efficient allocation with the single currency denomination.

The other implication of the Proposition 1 is that the single denomination may be insufficient for replication if involuntary unemployment exists in some states of the constraint efficient allocation. With involuntary unemployment in some states, the nominal wage would be binding. The unemployment level implies the wage rate through labor market slackness condition, and only one specific exchange rate would be consistent with this unemployment level. However, this specific exchange rate is not necessarily consistent with the planner's choice of the state contingent claim in the same state. With a single currency denomination, the government may not find a solution to the state-contingent exchange rates. It satisfies the labor market slackness condition and replicates debt value simultaneously. In the case of involuntary unemployment, the following proposition characterizes the policy choice of the government.

Proposition 2. *In an economy with binary state,* $s_{t+1} \in \{s_1, s_2\}$, *the constraint efficient allocation can be implemented with dollarization, if* $\left\{D^l(s^t), D^f(s^t), \tau^l(s^t), \tau^f(s^t), q^l(s^t), q^f(s^t), E(s^{t+1}|s^t)\right\}$ *satisfy the following conditions*

$$\frac{E(s_{t+1}=s_1|s^t)}{E(s_{t+1}=s_1|s^t)} = \frac{B(s_{t+1}=s_1|s^t) - D^f(s^t)}{B(s_{t+1}=s_2|s^t) - D^f(s^t)} \tag{1.27}$$

$$\left(\frac{L(s_1)}{L(s_2)}\right)^{-\alpha} = \frac{B(s_1|s^t) - D^f(s^t)}{B(s_2|s^t) - D^f(s^t)} \tag{1.28}$$

$$q^l\left(s^t\right) = \mathbb{E}_{s^t} q\left(s^{t+1}|s^t\right) \frac{E(s^t)}{E(s^{t+1}|s^t)} \tag{1.29}$$

$$q^f\left(s^t\right) = \mathbb{E}_{s^t} q\left(s^{t+1}|s^t\right) \tag{1.30}$$

$$\tau^l\left(s^t\right) = \mathbb{E}_{s^t} \tau\left(s^{t+1}|s^t\right) \frac{E(s^t)}{E(s^{t+1}|s^t)} \tag{1.31}$$

$$\tau^f\left(s^t\right) = \mathbb{E}_{s^t} \tau\left(s^{t+1}|s^t\right) \tag{1.32}$$

given $E(s^t), B(s^t), y^T(s^t), K(s^t)$.

The above Proposition 2 utilizes the additional currency denomination to ameliorate the conflict between debt value adjustment and labor market friction lifting. The first two equations in the Proposition 2 illustrate how the dollar-denominated debt acts as if a wedge to reconcile two policy targets.

1.3 A Simple Case

In this section, I give a simple example economy to illustrate the mechanism described in the last section. The example economy is infinity horizon, but there is only risk in period 1. From period 2 and on, the tradable endowment is deterministic and constant. Besides, I also make further assumptions that households have the same discount factor as experts, and the elasticity of intertemporal substitution is infinite.

This simple economy can be solved backward. The only state variable from period 1 and on is the net worth of experts, n_t. There is a threshold value, n_1, of the expert's net worth in period 1. If an expert's net worth is above the threshold, the economy stays at the first-best level. Otherwise, a financial amplification happens. The risk sharing condition in a competitive equilibrium is

$$\frac{n_0}{n_1(s)} = \frac{\frac{\partial A(s')}{\partial C_h^T}}{\frac{\partial A}{\partial C_h^T}} \left(\frac{\tilde{V}_{h0}}{V^h(s)} \right)^{\sigma}.$$

Given that the only risk is on the tradable endowment, the condition connects the expert's net worth with the tradable endowment received by households. The more risk averse households are, the more larger spread of expert's net worth distribution across states. A planner may choose borrowing and lending differently from the individual optimal choice. The planner would increase experts' net worth in states with the low realization of tradable goods endowment by restricting experts from borrowing against those states. The planner also has the other option to increase the real exchange rate (nontradable goods price in this economy) and the expert's net worth.

Lemma 2. *If $\xi < 1$, expert's net worth, n_1, are decreasing in labor supply, l_1, for any given initial state $\left(y^T(s), k_0, b_0(s)\right)$ entering $t = 1$.*

When there is unemployment, the output of nontradable goods decreases, resulting in an increase in nontradable goods price. The impact of unemployment on an expert's net worth depends on which one of the quantity and price effects dominate. In Lemma 2, the price effect dominates when intratemporal substitution's elasticity is below 1. The reduction of the nontradable goods consumption final leads to an increase in the net worth.

Given that the planner can adjust borrowing and lending in each state and also labor input, the following lemma characterizes the necessary condition for an allocation to be constraint efficient or how the planner should choose its policy margins

Lemma 3. *A necessary condition for an allocation to be constraint efficient in the simple* *case we consider here is*

$$\frac{\left(\frac{\tilde{V}_{h0}}{V_1(s)}\right)^{\sigma} w(s)}{\left(\frac{\tilde{V}_{h0}}{V_1(s)}\right)^{\sigma} - \frac{n_0}{n_1(s)} \frac{\frac{\partial A_0}{\partial c_h^T}}{\frac{\partial A_1(s)}{\partial c_h^T}}} = \frac{\frac{\partial n_1}{\partial L_1}}{\frac{\partial n_1}{\partial B_1}}$$

for all states s.

An interpretation of the condition is through the marginal cost and benefit of the planner adjusting borrowing/lending and labor input. The left-hand side is the marginal cost ratio of choosing less labor input and less borrowing in some states, while the left-hand side is the benefit ratio. In optimum, the planner would adjust its margin until two policy tools result in equalized marginal effects. The other implication of the lemma is that full employment across the state does not hold in general, which echos the point in the last section.

1.4 Conclusion

This chapter has discussed why many emerging economies allow for domestic dollarization or dual currency denomination in the domestic financial system. I propose a microfounded model with currency denomination determined by domestic households and experts to rationalize the government's incentive for accommodating dollarization. I first set up a heterogeneous agent nominal economy with downward wage rigidity and collateral borrowing constraint. Then I characterize the constraint efficient allocation through a corresponding real economy and the implementation of the allocation in the nominal economy. Through the implementation exercise, the necessity of dollarization arises from the conflict between the government lifting labor market friction and adjusting the real value of debt between households and experts. Dollarization effectively ameliorates the conflict and allows the government to gain more freedom in exchange rate policy choice. A quantitative model calibrated to some emerging economies' data would be required to apply the theory developed in this chapter.

Chapter 2

Housing Market Policy and Intergenerational Wealth Transfer[1]

[1]This chapter is based on a joint research project with one committee member, Kaiji Chen, Ph.D.. With this dissertation committee's approval, I included my contribution as this chapter in the dissertation.

2.1 Introduction

In many emerging economies, the rapid economic growth in recent decades is also accompanied by the booming financial market. For example, China's real gross domestic product has grown over 22 times from the year 1994 to the year 2020, while the average housing price growth rate reached the level of approximately 8.0%. The distribution of economic prosperity is not equal across generations. During this period, the younger households benefited from fast wage growth, and the older generation gained significantly less. However, the older generation with housing assets still retained capital gain due to the fast growing housing price. The government carried out housing market reforms in the 1990s, which were a series of market interventions favoring older generations in purchasing housing assets and renting housing services. In this paper, we study the role of government housing market interventions in intergenerational transfer from the young generations to the initial old generations during a transition stage featuring fast wage growth.

With China's housing market reform (privatization), housing privatization allows those initial generations to purchase housing at a government-subsidized price. Since the state owned enterprise (SOE) reforms happen in the same time, older generations may not enjoy the full benefit of economic transition through rising wage income, as many of them are forced to retire early or still get a lower wage rate. However, housing subsidy would allow them to consume more housing services as a substitute for nondurable consumption, and obtain housing assets for potential capital gain.

When the initial generations retire, they can sell the houses to reap the capital gain to replace their portfolio with larger housing or finance retirement consumption. Hence, housing privatization in China serves as an intergenerational transfer from the young generation to the old generations, who are poor in wage income when the reform started and who faced the elevated risk of unemployment due to the SOE reform.

The transfer plays a similar role as the social security system in the U.S. as intergenerational transfers were initially established in the 1930s. The government establishes the social security system to tax the younger generations and transfer the wealth to the older generations. Since the young are entitled to social security benefits, social security is essentially international transfer across generations.

The difference is that housing provides service flows and its role as durable goods. So the initial generations, old or young, all benefit from housing reforms. Specifically, by selling housing at a discounted price to the initial generations, it transfers resources to the initial generations in two ways (1) allow them to consume housing service before retirement (2) allow them to reap the capital gain when they retire. The initial young's entitlement to social security benefits is not accessible since the government taxes them to finance the initial old.

The younger generation needs to purchase housing at a higher price as the land becomes more scarce (controlled by the government) and the population grows. However, these generations also enjoy the fast growth of the economy. Hence, it is optimal for the social planner to tax them and transfer resources to the initial generations. This mechanism in [26] is the public pension system. However, higher housing price now is another way to transfer wealth from the younger generations to the elder.

A rising housing price affects the welfare of different cohorts through intergenerational transfer and impacts welfare across income groups within cohorts (intra-generational transfer). Anticipating of risking housing prices, potential home buyers may be forced to purchase housing early in life with a high down payment, thus reducing nondurable consumption, see [15]. Furthermore, this would increase their monthly mortgage payment as a fraction of income if they purchase housing early in life for mortgage borrowers. Lastly, for those relatively poor young households, rising housing prices may force them to be out of the housing market and be a renter. Another related research, [29] finds that China's housing market reform (privatization) allows the household to switch their consumption to housing consumption and therefore increases equilibrium housing prices. The price response to the market reform depends on the degree of misallocation before the privatization.

The rest of the paper is organized as follows. Section 2 describes a simple model to illustrate the intergenerational transfer induced from the housing market. We set up the constraint planner's problem to solve the optimal intergeneration transfer and discuss the implementation of the constraint efficient allocation, assuming the government can intervene in the housing market. Section 3 develops and calibrates a quantitative overlapping generation model with intragenerational heterogeneity to quantify the welfare impact of government intervention in the housing market with China's housing market reform as the empirical target. Section 5 is the conclusion.

2.2 A Simple Model

In this section, we construct a simple dynamic model that illustrates the the main point of the paper is that in emerging economies with fast but declining wage growth, even a planner with a low discount rate would find it optimal to redistribute resources from future to current generations. Moreover, the optimal redistribution can be implemented by subsidizing the housing purchase of the current generations (and tax housing expenditure by future generations).

Consider an economy populated by two-period lived overlapping-generations of households who work when young and live off savings when old. For simplicity, we assume that the household value both nondurable consumption and housing service when young and only nondurable consumption when old.

In period 0, the planner is endowed with a fixed stock of public housing, $\overline{h}$, to be sold to the initial cohort of the household.

We capture the notion of temporarily high wage growth by assuming that wages grow at steady state rate g from period $t = 1$ and onward, while the growth rate is high in the first period: $w_1 > (1 + g)\, w_0$. At this stage, we assume a constant population growth rate n (which later matters for housing price growth. To endogenize housing prices, we assume that each period the government provides (inelastically) a fraction n of new housing such that the total housing supply, which is the sum of existing housing supplied by the old-generation and the new housing supplied by the government, grows at the population growth rate. As a result, per capita housing supply is constant, denoted as $\overline{h}$.

The preference of a household born at time $t \geq 0$, indexed as cohort t, is given by

$$U_t = \log c_t^y + \alpha \log h_t + \beta \log c_{t+1}^o \tag{2.1}$$

where c_t^y and c_{t+1}^o denote the nondurable consumption when young and old, respectively. h_t denotes housing services produced by housing stock on a one-to-one basis; Note that households retire when old.

We assume that the planner could not affect the housing supply at this stage. Accordingly, both the planner and the households in the decentralized economy take housing prices as exogenously given. After implementing the planner's solution with housing policies in the decentralized economy, we solve for the equilibrium housing price.

2.2.1 The Planner's Solution

We first solve the planner's problem. In this simple model, the planner takes housing prices and wage rates as given. The planner is endowed with an initial stock of wealth, A_0, and can borrow and lend in an international bond market at the gross interest rate R. In addition, the planned is endowed with a fixed stock of land/housing each period $\overline{h}$. We assume that all land sales revenues are used for government consumption, which corresponds to the idea that, in reality, the land sales revenues are used for infrastructure investment. In other words, the government could not finance the transfer from the revenue from selling the public housing. The planner's resource constraint is given by

$$\sum_{t=0}^{\infty} \left(\frac{1+n}{R} \right)^t \left[c_t^y + \frac{c_{t+1}^o}{R} + \left(p_t - \frac{p_{t+1}}{R} \right) h_t - w_t \right] \leq A_0 \tag{2.2}$$

where $p_t - \frac{p_{t+1}}{R}$ is the user cost of one unit of housing consumption.

Moreover, given the inelastic supply of public housing in the initial period and the government's use of lower prices than market price to subsidize housing, it is likely that the demand for housing at the subsidized price is higher than the supply. Hence we impose the constraint $h_0 \leq \overline{h}$ to clear the market.

The planner cares about all present and future generations, and discount future generations' utilities with a discount factor $\phi \in (0,1)$. For the resource constraint to be well-defined, we assume $R > (1+g)(1+n)$. Moreover, we assume $\phi(1+n) < 1$, such that the planner's problem is well defined.

The planner's problem is

$$\max_{\left\{c_t^y, c_{t+1}^o, h_t, l_t\right\}_{t=0\infty}} \sum_{t=0}^{\infty} (\phi(1+n))^t \left[\log c_t^y + \alpha \log h_t + \beta \log c_{t+1}^o\right]$$

subject to (2.2) and

$$h_0 \leq \overline{h}. \tag{2.3}$$

The first-order conditions for consumption at all $t \geq 0$,

$$c_t^y = \lambda^{-1} (\phi R)^t \tag{2.4}$$

$$c_{t+1}^o = \beta R \lambda^{-1} (\phi R)^t \tag{2.5}$$

and for housing

$$\frac{\alpha}{h_0} - \kappa = \lambda \left(p_0 - \frac{p_1}{R}\right) \tag{2.6}$$

$$\left(p_t - \frac{p_{t+1}}{R}\right) h_t = \alpha \lambda^{-1} (\phi R)^t \text{ for } t \geq 1 \tag{2.7}$$

where λ is the Lagrangian multiplier for the resource constraint, capturing the shadow value of wealth and κ is the Lagrangian multiplier for (2.3).

Both (2.4), (2.7) reflects the planner's incentive for inter-generational redistribution. Note that the optimal sequence of both nondurable consumption and housing expenditure is independent of the wage sequence and only depends on planner's discount factor and interest rate, ϕR. Over the life cycle, the planner chooses the same consumption growth as individual households, where across cohorts, consumption growth by the factor ϕR. Similarly, housing expenditure across cohorts grows by the factor ϕR.

2.2.2 Decentralization

The planner's solution can be decentralized in the competitive equilibrium. For simplicity, assume that there is no rental market for housing. A household purchases housing when young, and sells it when old. In addition to housings, household can save in bank with a fixed interest rate R.

At the initial period, the government holds public housing. Hence, we assume that the policy instrument for the government at period 0 is housing prices it sells to the households (public housing). This may capture, in reality, that in the initial stage housing market privatization (1994-2000), the Chinese government allowed the initial generations to purchase housing at a discount rate (the so-called "standard price").

For generality, we assume that the government can impose a lump-sum transfer (or tax) for each of the cohorts born or after $t \geq 1$.To avoid distorting the consumption allocation between nondurable goods and housing services, we further assume housing subsidy takes the form of lump-sum transfer (or tax) that is proportional to the housing expenditure:

$$T_t = \varsigma_t \left(p_t - \frac{p_{t+1}}{R} \right) h_t \tag{2.8}$$

where ς_t denotes the housing subsidy rate. Here, for tractability, the magnitude of transfers depend on the user cost of consuming housing h_t, rather than the purchasing cost $p_t h_t$.The lifetime budget constraint of a household born at time t is

$$c_t^y + \frac{c_{t+1}^o}{R} + \left(p_t - \frac{p_{t+1}}{R}\right) h_t = w_t + T_t \tag{2.9}$$

Maximizing the household problem (2.1) subject to (2.9) give the following first-order condition on housing

$$\left(p_t - \frac{p_{t+1}}{R}\right) h_t = \frac{\alpha w_t}{1 + \alpha + \beta - \alpha \varsigma_t} \tag{2.10}$$

Note that the higher is ς_t, the housing subsidy rate, the higher is the housing expenditure. In addition, we have consumption for each cohort as

$$\begin{aligned} c_t^y &= \frac{1}{\alpha}\left(p_t - \frac{p_{t+1}}{R}\right) h_t \\ c_{t+1}^o &= \frac{\beta R}{\alpha}\left(p_t - \frac{p_{t+1}}{R}\right) h_t \end{aligned}$$

The following proposition establishes that the first-best solution can be implemented by setting a suitable sequence of housing subsidy rates.

Proposition 3. *The first-best solution is implemented by setting the sequence of housing subsidy rate as*

$$\varsigma_t = \frac{1 + \alpha + \beta}{\alpha} - \frac{w_1 \lambda}{(1+g)\,\alpha}\left(\frac{\phi R}{1+g}\right)^{-t} \tag{2.11}$$

where λ is the Lagrangian multiplier associated with the planner's resource constraint.

Proof: See Appendix.

Note that for $t \geq 1$, the housing subsidy rate may increase or decrease over time depending on whether $\phi(1+g) - R$ is positive or negative. In particular, when $\phi = (1+g)/R$, $\varsigma_t = \frac{1+\alpha+\beta}{\alpha} - \frac{w_1\lambda}{(1+g)\alpha}$ is constant, implying that the planner has no incentive for inter-generational redistribution in steady state. Under this particular value of planner's discount factor, we have the following optimal housing subsidy policy. We may solve for p_t by plugging (2.11) into (2.10).

$$\left(p_t - \frac{p_{t+1}}{R}\right) h_t = \frac{\alpha w_t (1+g)}{\lambda w_1}$$

And forward iterate the above equation, we get for $t \geq 1$

$$p_t = \frac{\alpha}{\lambda \overline{h}} \frac{1}{1 - (1+g)/R} \tag{2.12}$$

By assumption, cohort 0 has no cash subsidy. Instead, the government chooses p_0 as implicit transfer, which might lead to binding housing demand constraint. Hence the constraints for cohort 0 are

$$c_0^y + \frac{c_1^o}{R} + \left(p_0 - \frac{p_1}{R}\right) h_0 = w_0 \tag{2.13}$$

and (2.3). In the decentralized economy, (2.3) captures the fact that in reality, the Chinese government only allows a SOE employee to purchase the house she currently lives at a subsidized price, which essentially impose a demand constraint under the subsidized housing prices. The first order conditions are

$$\begin{aligned} \frac{1}{c_0^y} &= \beta R \frac{1}{c_1^o} \\ \frac{\alpha}{h_0} - \mu &= \frac{1}{c_0^y}\left(p_0 - \frac{p_1}{R}\right) \end{aligned} \tag{2.14}$$

where μ is Lagrangian multiplier associated with the housing demand constraint. Since p_0 is a policy variable and in equilibrium $h_0 = \overline{h}$, we have

$$c_0^y = \frac{w_0 - \left(p_0 - \frac{p_1}{R}\right) \overline{h}}{1+\beta} \tag{2.15}$$

To solve for the level of housing prices, we detrend all per capital variables $\widehat{x}_t = \frac{x_t}{(1+g)^{t-1}}$. After detrending, we have

$$\widehat{c}_0^y = \frac{\widehat{w}_0 - (\widehat{p}_0 - \widehat{p}_1 (1+g)/R) \overline{h}}{1+\beta}$$

Now, we establish the optimal housing price $\widehat{p}_0$.

Corollary 1. *The optimal housing price chosen by the government at period 0 is*

$$\widehat{p}_0 = \frac{\left[\widehat{w}_0 - \lambda^{-1}(1+\beta)\right]}{\overline{h}} + \frac{\alpha}{\lambda \overline{h}} \frac{(1+g)/R}{1-(1+g)/R} \tag{2.16}$$

In (2.16), $\widehat{w}_0 - \lambda^{-1}(1+\beta)$ is the user cost of housing for cohort 0. According to (2.16), the lower is $\widehat{w}_0$, the lower is the user cost of housing for initial cohort and the lower should be $\widehat{p}_0$ chosen by the government. Plugging (2.16) into (2.6), we can see for the demand constraint to be binding ($k > 0$), it is necessary that

$$\lambda \leq \frac{1+\alpha+\beta}{w_0}. \tag{2.17}$$

Finally, we can solve for λ using the planner's intertemporal budget constraint. Note for period 0, since there is no explicit transfer, (2.13) holds. Hence, plugging (2.4), (2.5) and (2.7) into the the intertemporal budget constraint becomes

$$\sum_{t=1}^{\infty} \left(\frac{1+n}{R}\right)^t \left[(1+\alpha+\beta)\lambda^{-1}(\phi R)^t - w_t\right] = A_0$$

which gives

$$\lambda = \frac{(1+n)\phi(1+\alpha+\beta)}{1-(1+n)\phi}\left[A_0 + \sum_{t=1}^{\infty}\left(\frac{1+n}{R}\right)^t\right] \tag{2.18}$$

Transfer Comparison

We can construct the implicit transfer for cohort 0 via subsidized housing price as the difference of user cost for housing between the case where $\widehat{p}_0$ is chosen by the government and the case $\widehat{p}_0$ is decided by the market equilibrium. This difference is used to finance the consumption when young and old when young. Hence, the implicit transfer, denoted as $\widehat{T}_{r0}$, is

$$\begin{aligned}
\widehat{T}_{r0} &= \left(\widehat{p}_0^F - \frac{\widehat{p}_1^F\left(1+g\right)}{R}\right)\overline{h} - \left(\widehat{p}_0 - \frac{\widehat{p}_1\left(1+g\right)}{R}\right)\overline{h} - \\
&= \frac{\alpha\widehat{w}_0}{1+\alpha+\beta} - \left[\widehat{w}_0 - \lambda^{-1}\left(1+\beta\right)\right] \\
&= \lambda^{-1}\left(1+\beta\right) - \frac{1+\beta}{1+\alpha+\beta}\widehat{w}_0,
\end{aligned}$$

where $\widehat{p}_t^F$ denote the housing prices in an economy without government intervention. The first argument on the third row of right hand side is the user cost of housing for initial cohort under the equilibrium price for $\widehat{p}_0$. To understand the source of implicit transfer via subsidized housing price, we compute the return for housing investment under decentralized economy.

$$\begin{aligned}
& \frac{\widehat{p}_1\left(1+g\right)}{\widehat{p}_0 - \frac{\alpha\widehat{c}_0^y}{h_0}} \\
=\ & \frac{\frac{\alpha\lambda^{-1}}{\overline{h}}\frac{R(1+g)}{R-(1+g)}}{\left[\widehat{w}_0 - \lambda^{-1}\left(1+\beta\right)\right]/\overline{h} + \frac{\alpha}{\lambda\overline{h}}\frac{(1+g)/R}{1-(1+g)/R} - \frac{\alpha}{\lambda\overline{h}}} \\
>\ & \frac{\frac{\alpha\lambda^{-1}}{\overline{h}}\frac{R(1+g)}{R-(1+g)}}{\frac{\alpha}{\lambda\overline{h}} + \frac{\alpha}{\lambda\overline{h}}\frac{(1+g)/R}{1-(1+g)/R} - \frac{\alpha}{\lambda\overline{h}}} \\
=\ & R
\end{aligned}$$

where the first equality is obtained by plugging in (2.16) and $(B.3)$, the second inequality comes from the assumption (2.17).

Given the only policy tool available to the government for transferring to the initial cohort is the subsidized housing prices, the government would choose a lower housing price than the market equilibrium price as implicit transfer to the initial cohort Accordingly, the initial cohort benefits from a higher return to housing investment than R by selling the public housing purchased at subsidized price later to future cohorts at the market equilibrium prices. Such a capital gain would allow the initial cohort to enjoy higher consumption despite their low wage income.

Now we would like to compare the implicit transfer for cohort 0 with the transfer for cohorts born at $t \geq 1$. Note that for cohorts born at $t \geq 1$, part of the transfer is used to purchase housing services, while for the initial cohort, the implicit transfers are used to finance consumption only. Hence, make the transfers between cohort 0 and the rest of the cohorts comparable, we need to construct the transfer measure by excluding the part of transfers for future cohorts that are used to finance the user cost of housing from the total transfer they receive is denoted as

$$\begin{aligned}
\widehat{T}_{rt} &= \widehat{T}_t - \left\{ \left(\widehat{p}_t - \frac{\widehat{p}_{t+1}(1+g)}{R} \right) \overline{h} - \left(\widehat{p}_t^F - \frac{\widehat{p}_{t+1}^F(1+g)}{R} \right) \right\} \\
&= \widehat{T}_t - \left[\alpha\lambda^{-1} \left(\frac{\phi R}{1+g} \right)^{-t} - \frac{\alpha \widehat{w}_1}{1+\alpha+\beta+\gamma} \right]
\end{aligned}$$

where the argument in the solid bracket on is the difference of user cost for housing with and without government transfer. With (2.8) and (2.11), we have

$$\widehat{T}_{rt} = (1+\alpha+\beta)\lambda^{-1} \left(\frac{\phi R}{1+g} \right)^{-t} - \widehat{w}_1 \left(\frac{\phi R}{1+g} \right)^{-t} - \left[\alpha\lambda^{-1} \left(\frac{\phi R}{1+g} \right)^{-t} - \frac{\alpha \widehat{w}_1}{1+\alpha+\beta} \right] \quad (2.19)$$

Proposition 4. *If the planner's discount factor is $\phi = (1+g)/R$, then the housing subsidy for $t \geq 1$ less than the implicit housing subsidy for the initial generation via subsidized housing price.*

Proof. Plugging $\phi = (1+g)/R$ into (2.19), we have

$$\widehat{T}_{rt} = \lambda^{-1}(1+\beta) - \frac{1+\beta}{1+\alpha+\beta}\widehat{w}_1$$

for all $t \geq 1$. Since by assumption $\widehat{w}_1 > \widehat{w}_0$, $\widehat{T}_{r0} > \widehat{T}_{rt}$ for all $t \geq 1$. □

Intuitively, as the initial generation receives low wage income, even a utilitarian planner that has no incentive to redistribute across generations in steady state will find it optimal to transfer resources from the future to the current generations. The transfer is achieved by charging a housing price lower than the equilibrium price as implicit transfer to the initial cohort.

2.2.3 Second-best Ramsey Solution

In reality, the government may not be able to provide negative transfers to future cohorts (or tax future cohorts) based on their housing expenditure. That is, the government may face the constraint that $\varsigma_t \geq 0$, for $t \geq 1$. We now discuss whether this constraint changes our main results. According to (2.10), this is equivalent to put a constraint on the lower bound of h_t for the planner's problem.

$$h_t \geq \widetilde{h}_t \text{ for } t \geq 1 \tag{2.20}$$

Therefore, the planner solves the following problem

$$\max_{\left\{c_t^y, c_{t+1}^o, h_t\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left(\phi\left(1+n\right)\right)^t \left[\log c_t^y + \alpha \log h_t + \beta \log c_{t+1}^o\right]$$

subject to (2.2) , (2.3) and (2.20). The first-order conditions for consumption at all $t \geq 0$,

$$\begin{aligned} c_t^y &= \lambda^{-1} \left(\phi R\right)^t && (2.21) \\ c_{t+1}^o &= \beta R \lambda^{-1} \left(\phi R\right)^t && (2.22) \end{aligned}$$

The first order conditions for h_t is

$$\begin{aligned} \frac{\alpha}{h_0} - \kappa &= \lambda \left(p_0 - \frac{p_1}{R}\right) && (2.23) \\ \frac{\alpha}{h_t} &= \left(p_t - \frac{p_{t+1}}{R}\right) \lambda \left(\phi R\right)^{-t} - \mu_t \text{ for } t \geq 1 && (2.24) \end{aligned}$$

where κ and μ_t are the Lagrangian multiplier associated with the constraints (2.3) and (2.20), respectively.

Now, we assume that in the decentralized economy, the non-negative constraint for housing transfer is binding, $\varsigma_t = 0$, for $t \geq 1$. Hence, (2.10) becomes

$$\left(p_t - \frac{p_{t+1}}{R}\right) h_t = \frac{\alpha w_t}{1+\alpha+\beta} \geq \alpha \lambda^{-1} (\phi R)^t \tag{2.25}$$

This will give a lower bound for λ

$$\begin{aligned} \lambda &\geq \left(\frac{\phi R}{1+g}\right)^t \frac{(1+g)(1+\alpha+\beta)}{w_1} \\ &= \frac{(1+g)(1+\alpha+\beta)}{w_1} \end{aligned}$$

Similar to the logic above, for the initial cohort to receive a discounted housing price ($\kappa > 0$), it is necessary that

$$\lambda \leq \frac{1+\alpha+\beta}{w_0}. \tag{2.26}$$

Therefore, we have the following proposition

Proposition 5. *Suppose that $\phi = (1+g)/R$ and the planner cannot set negative housing subsidy. Assume further that $\lambda \in \left[\frac{(1+g)(1+\alpha+\beta)}{w_1}, \frac{1+\alpha+\beta}{w_0}\right]$, where λ is the Lagrangian multiplier for the planner's resource constraint:*

$$\lambda = \frac{(1+n)\phi(1+\alpha+\beta)}{1-(1+n)\phi}\left[A_0 + \sum_{t=1}^{\infty}\left(\frac{1+n}{R}\right)^t\right]. \tag{2.27}$$

Then, the Ramsey constrained (second-best) allocation is implemented by setting the following sequence of policy.

$$\begin{aligned} \widehat{p}_0 &= \frac{\left[\widehat{w}_0 - \lambda^{-1}(1+\beta)\right]}{\overline{h}} + \frac{\alpha}{\lambda \overline{h}} \frac{(1+g)/R}{1-(1+g)/R} && (2.28) \\ \varsigma_t &= 0, \text{ for all } t \geq 1. && (2.29) \end{aligned}$$

In other words, in the second best solution, the planner applies the discounted housing price for the initial cohort and does not tax future generations. The derived housing subsidy coincides what the housing policy reform in reality.

2.3 The Full-Blown Model

2.3.1 Households

Demographic Structure

Each period a continuum of household is born. The new-born population grows at a exogenous rate n_t. Households live a maximum J periods, and each period represents one year. They enter the economy as adults, and are active as workers until age J_w. All households face a probability S_j of surviving up to age j, and they die at age J with certainty. $S_j = \prod_{k=1}^{j} \psi_k$, and ψ_k is the conditional survival probability from age $k-1$ to age k. Given this information, at the steady state, the fraction of households of age j, denoted as μ_j is

$$\mu_j = \frac{\psi_j}{1+n}\mu_{j-1}$$

where $\sum_{j=1}^{J} \mu_j = 1$. In the transitional dynamics, the equation becomes

$$\mu_{t,j} = \frac{\psi_j}{1+n_t}\mu_{t-1,j-1}$$

t is the calendar time, and j is the age.

Preference

All agents have the identical form of preference over the aggregation of goods consumption and housing service,

$$U_t^i = \sum_{j=1}^{J} S_j \beta^{j-1} u\left(c_{t,j}^i, s_{t,j}^i\right),$$

where the utility function is separable in terms of goods and housing,

$$u\left(c_{t,j}^i, h_{t,j}^i\right) = \log c_{t,j}^i + \phi \log\left(s_{t,j}^i - \underline{h}\right),$$

where $\underline{h}$ is the subsistence level of housing, which we introduce to drive a difference of housing value of income ratio across households of different permanent income levels.

$e_{t,j}^i = \epsilon^i \varepsilon_j \eta_{j,t}^i$, ϵ^i is the permanent efficiency shock to household i when he was born, ε_j is life-cycle income profile, and $\eta_{t,j}^i$ is the idiosyncratic efficiency shock to household i in age j; w_t is the wage rate per efficiency unit at t; τ_t is the labor income tax;

Finally, for individual household i, the social security benefit each period is determined as

$$b_{t,j}^i = \theta(0.6\widetilde{y}_{t-j+J_w,J_w}^i + 0.4\bar{y}_t),$$

where θ is the replacement rate at the time of retirement, and $\widetilde{y}_{t-j+J_w,J_w}^i = w_{t-j+J_w} e_{t-j+J_w,J_w}$ is the before-tax wage earnings of the household i before retirement. $\bar{y}_t$ is the current average yearly earning of workers.

$$\bar{y}_t = \frac{\sum_{j=1}^{J_w} \sum_i \mu_{t,j} w_t e_{t,j}^i}{\sum_{j=1}^{J_w} \mu_{t,j}}$$

Note that the above formula only allows the partial indexation of pension benefits to the current period earnings. We also assume that the social security program is self-financing where the social security payroll tax rate τ_t^{ss} each period is endogenously determined to balance the social security budget each period. In other words, the social security tax rate is determined as

$$\tau_t^{ss} = \frac{\sum_{j=1}^{J_w} \sum_i \mu_{t,j} b_{t,j}^i}{\sum_{j=1}^{J_w} \sum_i \mu_{t,j} w_t e_{t,j}^i}$$

Finally, denote the after-tax income of a household as

$$y_{t,j}^i = \begin{cases} (1 - \tau_t - \tau_t^{ss}) w_t e_{t,j}^i \text{ for } j \leq J_w \\ b_{t,j}^i \text{ for } j > J_w \end{cases}$$

2.3.2 The Housing Reform

Assume at the beginning of period 1 (1994), there is a housing reform, under which the government sells rental housing $h_{i,t}$ to existing households age j at a discounted price. All households born before the year 1994 are eligible to purchase at the discounted price at any point of their life unless they choose to move out of rental housing and move into commercial housing. In particular, the discounted price computed as

$$p_{t,j}^g = p_t \left[1 - \frac{\min[\text{Age in 1994}, J_w]}{J_w} \times 65 \times 0.9\% \right]$$

This formula is a simplified rule of the discounted price in practice, where p_t is the market price of the current period. J_w is the maximum working age. According to this rule, the discounted price is age-specific: if a household was retired before 1994, then he can enjoy 58.5% (65*0.9%) of the price discount. The longer a household worked before 1994, the more discount he or she can enjoy. The subsidy setup reflects the government's idea to transfer resources for future cohorts to these cohorts who have fewer years to enjoy the higher wage growth. At the same time, all households revised their expectation about TFP growth and land supply in 2003. The first shock increased the resources of future generations, and the second shocks help to increase the land prices, which further increase the housing prices. Note that the newborn household has to purchase the housing from the market, which also creates the housing markets since 1994. So we need to have market supply. The household who are eligible (alive in the year 1994) for reformed housing can choose to purchase the housing at the discounted price, or keep renting the housing at the pre-reform prices or purchase housing from the market.

Compared to purchasing reformed housing versus purchasing commercial housing, the trade-off is that reformed housing is cheaper than the market price but fixed amount. On the other hand, commercial housing does not have size restrictions. Therefore, the high income household expecting high income growth in the future and fast housing price growth may choose to purchase commercial housing for more capital gain. Low income households, whose optimal housing size is small, may purchase reformed housing. However, the low income household may be subject to the credit constraint and is not capable of purchasing discounted housing in the year 1994.

Now, since higher TFP growth and lower land supply shocks imply higher future prices, households may prefer the reformed housing to the government rental housing for the potential capital gain in the future. However, the down payment constraint and the transaction cost would make the reformed housing only affordable to the middle income households. Also, since they expect that the discounted housing purchase option would last throughout their life-cycle, they may postpone purchasing the discounted housing in a later stage of life.

The 2nd stage is an expected reform in period 5 (the year 1998), with the Stage 1 reform as the period 1, the government declared to eliminate the government rental housing for households born after period 5. All households born between periods 1 and 5 are still eligible for government rental discount until they move out.

In summary, all households born no later than the year 1998 can keep renting the government assigned rental housing until they moved out to a commercial housing. Households born before the year 1994 have an additional option of purchasing rental housing with a discounted price, and the option is valid as long as they live in government-assigned rental housing.

2.3.3 Households' Decision

Housing service can be obtained from the market by either renting a house at a rental rate ρ_h or buying houses at a price p. Housing sizes for both rental and purchased housing are discrete. For purchased houses, housing size belongs to the set H, while for rental housing, housing size belongs to the set $\tilde{\mathcal{H}}$.

To capture rental market friction on the demand side, we assume that renting generates service less than one-for-one with the size of the house, i.e. $s = \omega h$. $0 < \omega < 1$. When a household sells its home, it incurs a proportional transaction cost $\kappa_h ph$, that is proportional to its housing value, and a fixed transaction cost, κ_0^j. The renter can adjust their housing size without cost. For renters who purchase commercial housing or government housing (if qualified), they incur a fixed cost of initiating the mortgage, m_j, which is time and age-dependent. Households are subject to down payment constraint; $h^i_{t+1,j+1}$ is the the stock of housing enjoyed by the household in t; households need to pay for the housing maintenance cost each period, $\delta_h p_t h^i_{t,j}$;

At the beginning of period 1, all transitional households hold no housing. At each period since period 2, the transitional household born before the year 1994 started the period with three possible states: 1. renting government housing (Ng), 2. renting commercial housing (N), 3. owning housing (H). If the transitional household holds no housing, it can continue to rent public housing (r) at the government-subsidized rental rate or purchase discounted (reformed) housing or commercial housing or rent from the market. If the household holds the discounted housing, it can keep the discounted housing or sell it and purchase commercial housing (or rent from the market). If the household holds the commercial housing, it can only choose to keep its commercial housing or sell it and purchase another commercial housing (or rent from the market). Households can not hold discounted housing and market housing at the same time. Denote the index for the beginning of period housing status as $I \in \{N_g, N, H\}$.

We now write the households' problem in recursive form. Each period, a household's

Beginning of Period State		
w/o house	w/o house	w house
Ng	N	H

Intermediate/Choice					
rent gov't H	buy gov't H	buy com. H	keep H	sell H	rent com. H
rg	bg	bm	p	s	r

$$N_g \longrightarrow \begin{cases} \text{rent gov't H}\rightarrow N_g \\ \text{buy gov't H}\rightarrow\text{H} \\ \text{buy com. H}\rightarrow\text{H} \\ \text{rent com. H}\rightarrow\text{N} \end{cases}$$

$$N \longrightarrow \begin{cases} \text{buy com. H}\rightarrow\text{H} \\ \text{rent com. H}\rightarrow\text{N} \end{cases}$$

$$H \longrightarrow \left\{ \begin{array}{l} \text{keep H}\rightarrow\text{H} \\ \text{sell H}\longrightarrow \left\{ \begin{array}{l} \text{buy com. H}\rightarrow\text{H} \\ \text{rent com. H}\rightarrow\text{N} \end{array} \right\} \end{array} \right.$$

Table 2.1: Diagram I: Beginning-of-period housing status and housing choices for transition cohorts born before 1994

idiosyncratic state vector is $\chi = (a, y, h)$. Denote the measure of households across individual states as $\Phi_t(\chi)$. The above household problem can be rewritten recursively as

- The choice of a transitional household without house and renting government housing (Ng) at the beginning of the period is

$$V_{t,j}^{Ng}(a, y) = \max \left\{ V_{t,j}^{r}(a, e), V_{t,j}^{bg}(a, e), V_{t,j}^{bm}(a, e), V_{t,j}^{rg}(a, y) \right\}$$

- The choice of a transitional household without house and renting commercial housing (N) at the beginning of the period is

$$V_{t,j}^{N}(a, y) = \max \left\{ V_{t,j}^{r}(a, e), V_{t,j}^{bm}(a, e) \right\}$$

- The intermediate optimization problems for transitional household renting government or commercial housing are as follows

  - For a renter of government house, its problem is therefore

$$V_{t,j}^{rg}(a, y) = \max u\left(c_{t,j}^{i}, s_{t,j}^{i}\right) + \beta \psi_{j+1} E\left[V_{t+1,j+1}^{N}(a, y)\right]$$

    subject to

$$\begin{aligned} c_{t,j}^{i} + q a_{t+1,j+1}^{i} + R_{c,t} s_{t,j}^{i} &\leq y_{j,t}^{i} + a_{t,j}^{i}, \\ s_{t,j}^{i} &= \omega \widetilde{h}_{1,1+j-t}^{i} \end{aligned} \tag{2.30}$$

    where $a_{t+1,j+1}^{i}$ is the amount of asset purchased by a household of age j at t, which has an exogenous price $q = \frac{1}{1+r}.h_{1,j-t,}^{i}$ $\widetilde{h}_{1,1+j-t}^{i}$ is the rental housing occupied by an agent i in the initial steady state

  - For a renter of commercial housing, its problem is

$$V_{t,j}^{r}(a, y) = \max u\left(c_{t,j}^{i}, s_{t,j}^{i}\right) + \beta \psi_{j+1} E\left[V_{t+1,j+1}^{N}(a', y')\right]$$

subject to

$$
\begin{aligned}
c^i_{t,j} + qa^i_{t+1,j+1} + \rho_{h,t}\widetilde{h}^{i\prime}_{t+1,j+1} &\leq y^i_{j,t} + a^i_{t,j}, \\
s^i_{t,j} &= \omega\widetilde{h}^{i\prime}_{t+1,t+1},\ \widetilde{h}^{i\prime}_{t+1,j+1} \in \widetilde{H}
\end{aligned}
\tag{2.31}
$$

– For a household that decides to buy the discounted government housing, the problem is

$$
V^{bg}_{t,j}(a,y) = \max u\left(c^i_{t,j}, s^i_{t,j}\right) + \beta\psi_{j+1}E\left[V^H_{t+1,j+1}(a', y', h')\right]
$$

subject to

$$
\begin{gathered}
c^i_{t,j} + qa^i_{t+1,j+1} + p^g_{t,j}h^i_{t+1,j+1} + m_j\mathbb{1}(a^i_{t+1,j+1} < 0) \leq y^i_{j,t} + a^i_{t,j}, \\
s^i_{t,j} = h^i_{t+1,j+1}, h^i_{t+1,j+1} = \widetilde{h}^i_{1,1+j-t}. \\
a^i_{t+1,j+1} \geq -\Upsilon p^g_{t,j}h^i_{t+1,j+1},
\end{gathered}
\tag{2.32}
$$

– For a household that chooses to buy commercial housing, the problem is

$$
V^{bm}_{t,j}(a,y) = \max u\left(c^i_{t,j}, s^i_{t,j}\right) + \beta\psi_{j+1}E\left[V^H_{t+1,j+1}(a', y', h')\right]
$$

subject to

$$
\begin{gathered}
c^i_{t,j} + qa^i_{t+1,j+1} + p_t h^i_{t+1,j+1} m_j\mathbb{1}(a^i_{t+1,j+1} < 0) \leq y^i_{j,t} + a^i_{t,j}, \\
s^i_{t,j} = h^i_{t+1,j+1}, h^i_{t+1,j+1} \in H \\
a^i_{t+1,j+1} \geq -\Upsilon p_t h_{t+1,j+1},
\end{gathered}
\tag{2.33}
$$

- A household that holds housing at the beginning of the period (no matter government housing or commercial housing) can make the following choice, keep the housing (government housing or commercial housing) or sell it and purchase new commercial housing.

$$V_{t,j}^{H}(a,y,h) = \max\left\{V_{t,j}^{p}(a,e,h), V_{t,j}^{s}(a,y,h)\right\}$$

– For a household that keeps the government housing, in the next period, it still own the government housing. As a result, the problem is

$$V_{t,j}^{p}(a,y,h) = u\left(c_{t,j}^{i}, s_{t,j}^{i}\right) + \beta\psi_{j+1}E\left[V_{t+1,j+1}^{H}(a',y',h')\right]$$

$$\begin{aligned} c_{t,j}^{i} + qa_{t+1,j+1}^{i} + \delta p_t h_{t,j}^{i} &\leq y_{j,t}^{i} + a_{t,j}^{i}, \\ s_{t,j}^{i} &= h_{t+1,j+1}^{i}, h_{t+1,j+1}^{i} = h_{1994}^{i}, \\ a_{t+1,j+1}^{i} &\geq -\Upsilon p_t h_{t+1,j+1}^{i}, \end{aligned} \tag{2.34}$$

Note that keeping the government housing means that selling the government housing at the market price and repurchase the same house at the market price.

– For a household that sells the housing, then it can only purchase the commercial housing or rent from the housing market.

$$V_{t,j}^{s}(a,y,h) = \max\left\{V_{t,j}^{sr}(a_n,e), V_{t,j}^{sb}(a_n,y)\right\} \tag{2.35}$$

subject to

$$a_{t,j}^{i,n} = a_{t,j}^{i} + (1-\delta_h-\kappa)p_t h_{t,j}^{i} - \kappa_0$$

where the fixed moving cost, κ_0, can depend on household specific types.

– For a household that chooses to sell the housing and purchase commercial housing, the problem is

$$V_{t,j}^{sb}(a^n,y) = u\left(c_{t,j}^{i}, s_{t,j}^{i}\right) + \beta\psi_{j+1}E\left[V_{t+1,j+1}^{H}(a',y',h')\right]$$

subject to

$$c^i_{t,j} + qa^i_{t+1,j+1} + p_t h^i_{t+1,j+1} \leq y^i_{j,t} + a^{i,n}_{t,j}, \tag{2.36}$$
$$s^i_{t,j} = h^i_{t+1,j+1},\ h^i_{t+1,j+1} \in H$$
$$a^i_{t+1,j+1} \geq -\Upsilon p_t h^i_{t+1,j+1}.$$

– For a household that chooses to sell the housing and rent commercial housing, the problem is

$$V^{sr}_{t,j}(a,y) = u\left(c^i_{t,j}, s^i_{t,j}\right) + \beta\psi_{j+1} E\left[V^N_{t+1,j+1}(a',y')\right]$$

subject to

$$c^i_{t,j} + qa^i_{t+1,j+1} + \rho_{ht}\widetilde{h}^{i\prime}_{t+1,j+1} \leq y^i_{j,t} + a^{i,n}_{t,j}, \tag{2.37}$$
$$s^i_{t,j} = \omega\widetilde{h}^{i\prime}_{t+1,j+1},\ \widetilde{h}^{i\prime}_{t+1,j+1} \in \widetilde{H}$$

For a household born between period 1 (1994) and period 5 (1998), their problem is similar to households born before 1994. The difference is that they do not have the option of purchasing government rental housing, although they can choose to rent until they move out. The intermediate optimization problems are the same as above.

For a household born after period 5 (1998), their problem is described in Table 2.2.

2.3.4 Prereform Steady State

Before the year 1994, we assume the economy is in the prereform steady state, and the government budget is in balance. We assume household's idiosyncratic shocks perfectly insured in the prereform stage, i.e. households with the same age and permanent efficiency shock are identical, including the level of housing. This setup is equivalent to a deterministic

Beginning of Period State		
w/o house	w/o house	w house
Ng	N	H

Intermediate/Choice				
rent gov't H	buy com. H	keep H	sell H	rent com. H
rg	bm	p	s	r

$$N_g \longrightarrow \begin{cases} \text{rent gov't H} \to N_g \\ \text{buy com. H} \to \text{H} \\ \text{rent com. H} \to \text{N} \end{cases}$$

$$N \longrightarrow \begin{cases} \text{buy com. H} \to \text{H} \\ \text{rent com. H} \to \text{N} \end{cases}$$

$$H \longrightarrow \begin{cases} \text{keep H} \to \text{H} \\ \text{sell H} \longrightarrow \left\{ \begin{array}{l} \text{buy com. H} \to H \\ \text{rent com. H} \to N \end{array} \right\} \end{cases}$$

Table 2.2: Diagram II: Beginning-of-period housing status and housing choices for cohorts born between 1994 and 1998.

Beginning of Period State	
w/o house	w H
N	H

Intermediate/Choice			
rent com. H	buy com. H	keep H	sell H
r	bm	p	s

$$N \longrightarrow \begin{cases} \text{buy com. H} \rightarrow \text{H} \\ \text{rent com. H} \rightarrow \text{N} \end{cases}$$

$$H \longrightarrow \begin{cases} \text{keep H} \rightarrow \text{H} \\ \text{sell H} \longrightarrow \begin{cases} \text{buy com. H} & \rightarrow H \\ \text{rent com. H} & \rightarrow N \end{cases} \end{cases}$$

Table 2.3: Diagram III: Beginning-of-period housing status and housing choices for cohorts born at or after 1998.

case. For notation concision, we drop the time subscript.

For a household born before period 1 (1994), their problem is

$$V_{t,j}^{rg}(a,y) = \max u\left(c_{t,j}^{i}, s_{t,j}^{i}\right) + \beta\psi_{j+1}E\left[V_{t+1,j+1}^{rg}(a,y)\right]$$

subject to

$$c_{t,j}^{i} + qa_{t+1,j+1}^{i} + R_{c,t}s_{t,j}^{i} \leq y_{j,t}^{i} + a_{t,j}^{i},$$
$$s_{t,j}^{i} = \widetilde{h}_{1,1+j-t}^{i}$$

We use the households' choice of government rental housing in the prereform steady state as the government plan of assigning rental housing to households born between periods 1 and 5.

For housing production, we assume that the government use land and labor to produce housing and then rent it out to household at a given rental rate.

2.3.5 Production Sectors

There are two production sectors in the economy: consumption goods sector and a construction sector.

The competitive final goods sector operates a the constant return to scale technology

$$Y_t = z_t N_{ct}$$

where N_{ct} is the total units of labor services used to produce consumption goods, and $w_t = z_t$. z_t is an exogenous process.

The construction sector operates the production technology

$$I_t = (z_t N_{ht})^{\alpha} (L_t)^{1-\alpha}$$

A developer solves the static problem

$$\begin{aligned} &\max_{N_h} p_t I_t - w_t N_{ht} \\ &s.t. \\ &I_t = (z_t N_{ht})^{\alpha} (L_t)^{1-\alpha} \end{aligned} \tag{2.38}$$

We have the housing investment function

$$I_t = (\alpha p_t)^{\frac{\alpha}{1-\alpha}} L_t$$

The labor demand from the construction sector is

$$N_{ht} = (\alpha p_t)^{\frac{1}{1-\alpha}} \frac{L_t}{z_t} \tag{2.39}$$

The profit of the government leasing the land is

$$\pi_t = \frac{1-\alpha}{\alpha}\left(\alpha p_t\right)^{\frac{1}{1-\alpha}} L_t$$

Combine the labor demand of two sectors, $N_{dt} = N_{ct} + N_{ht}$, and the total labor supply is given by $N_{st} = \int e\Phi_t(s)ds$.

2.3.6 Rental Sector

To simplify the rental market, we assume that there is a representative real estate developer. Each period, a representative rental company purchases housing in the housing market and rents them to renters. The rental company can frictionlessly buy and sell housing units subject to an operating cost, ψ for each housing unit rented out. The problem of the representative rental company is

$$J(\tilde{H};\Phi) = \max_{\tilde{H}'}\, [\rho_h(\Phi) - \psi]\tilde{H}' - p(\Phi)[\tilde{H}' - (1-\delta_h)\tilde{H}] + \frac{1}{1+r}E_{\Phi'|\Phi}J(\tilde{H}';\Phi')$$

The zero profit condition gives the equilibrium rental rate as

$$\rho_h(\Phi) = \psi + p(\Phi) - \frac{1-\delta_h}{1+r}E_{\Phi'}[p(\Phi')|\Phi].$$

In other words, the rent is simply the user cost of housing plus the operating cost.

2.3.7 The Government

We assume all the revenues by the governments by either renting or selling the public houses or land consumed by the government. In the transition path (in and after 1994), the government's budget is more complicated. The expenditure includes discounted housing subsidies and rental housing subsidies. The revenue consists of income tax.

$$\sum_{t=1}^{\infty} R^{-t} \left[\begin{array}{c} \sum_{j=t+1}^{J} \mu_{j,t} \int_{I\in bg} \left(p_t - p_{t,j}^g\right) h_{1,1+j-t}^i \Phi_t(s) ds \\ + \sum_{j=t+1}^{J} \mu_{j,t} \int_{I\in rg} \left(\rho_t - R_{c,t}\right) h_{1,1+j-t}^i \Phi_t(s) ds \\ + \sum_{j=1}^{t} \mu_{j,t} \int_{I\in rg} \left(\rho_t - R_{c,t}\right) h_{1,j}^i \Phi_t(s) ds \\ - \sum_{j=1}^{J_w} \mu_{j,t} \tau_t w_t \int_s e \Phi_t(s) ds \end{array} \right] N_t = A_0.$$

where $h_{1,1+j-t}^i$ denotes the reformed housing purchase by a household age j at period t, which is the size of house resided by this household at period 1 when he was aged $1+j-t$. This intertemporal government budget constraint implies that all transfer to the transitional cohorts need to be financed by future generations.

2.3.8 Equilibrium

Denote $\chi^H = (a, y, h)$ and $\chi^N = (a, y)$ as the idiosyncratic state vectors for homeowners and non-homeowners.

A recursive competitive equilibrium is a sequence of age-dependent value functions of $\left(\chi^N; \Phi\right)$, including households' value functions $\left\{V_j^N, V_j^H, V_j^r, V_j^{rg}, V_j^{bm}, V_j^{bg}, V_j^p, V_j^s\right\}_{j=1}^{J}$, households' decision rules, aggregate function for construction labor $N_h(\Phi)$, rental unit stocks $\widetilde{H}'$, home buyers' housing stock H', housing investment $I_h(\Phi)$, rental prices, $\rho_h(\Phi)$, housing price $p(\Phi)$, and a law of motion for the aggregate state Φ such that:

1. Given the price functions and aggregate law of motion, the value function solve the recursive problem of the households, and are the associated policy functions.

2. Construction sector firms maximize profit with associated labor demand and housing investment function (N_h, I_h)

3. The labor market clears at wage rate $w = z$.

4. The rental market clears at price ρ_h.

5. Housing market clears at price p

$$\widetilde{H}' + H' = (1 - \delta_h)\left(\widetilde{H} + H\right) + I$$

6. The aggregate law of motion is induced by the exogenous stochastic processes and all the decision rules, and it is consistent with individual behavior.

7. Government intertemporal budget constraint is satisfied.

2.4 Main Results

2.4.1 Calibration

We calibrate the benchmark economy to match China's economy's aggregate and cross-sectional statistics during 2003-2010. In the model, each period represents one year. Our model matches well the empirical moments. In Figure 2.1, we compare the simulated home-ownership rate with the rate based on the UHS dataset from the year 2003 to the year 2010. The homeownership rate arises from around 82% to almost 90%. In terms of cross-sectional distribution, Figure 2.2 and Figure 2.3 focus on the proportion of households who purchased government housing in the total population. The proportion is decomposition by age and by education group. The model matches closely with date in both decomposed proportions.

2.4.2 Welfare measures

We are interested in the welfare effects of housing reform (stage 1 and 2 together) for the initial cohorts alive in 1994. The welfare cost can be evaluated by computing the consumption equivalent variation (CEV) between two transitional paths. One is the transition path in the above benchmark model. The other is a counterfactual transition path, in which one

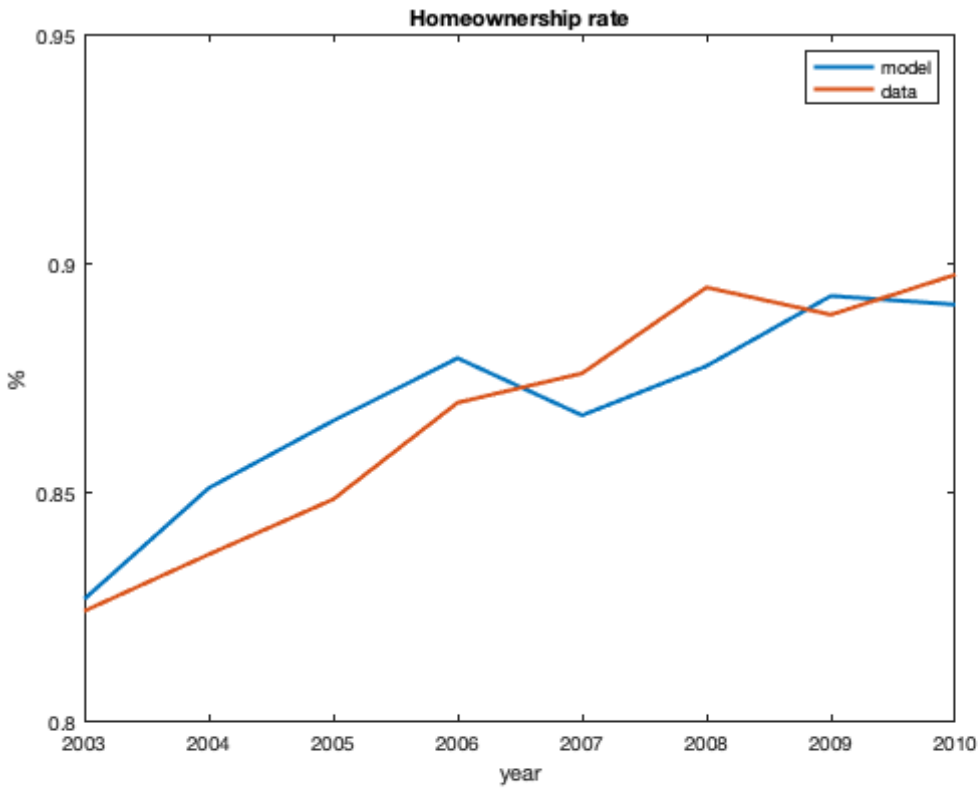

Figure 2.1: Homeownership rate, 2003 - 2010

ingredient of the policy reform is absent.

$$\Delta c_t = \left[\left(\frac{V_t^l}{V_t^{\text{base}}} \right)^{\frac{1}{1-\sigma}} - 1 \right] \times 100\%,$$

where V_t^l is the life time utility of households born in year t along the transitional path l. The path l is a counterfactual experiment. V_t^{base} is the life time utility along the benchmark path.

Specifically, we can conduct the following three counterfactual experiments to compute the welfare implications of housing reforms. These three experiments are to turn off one or both stages of the policy reform such that we can compare the welfare of each alternative versus our benchmark policy reform. The economy still experienced a transition with the expected increase in TFP growth and land supply decrease, assuming that these funda-

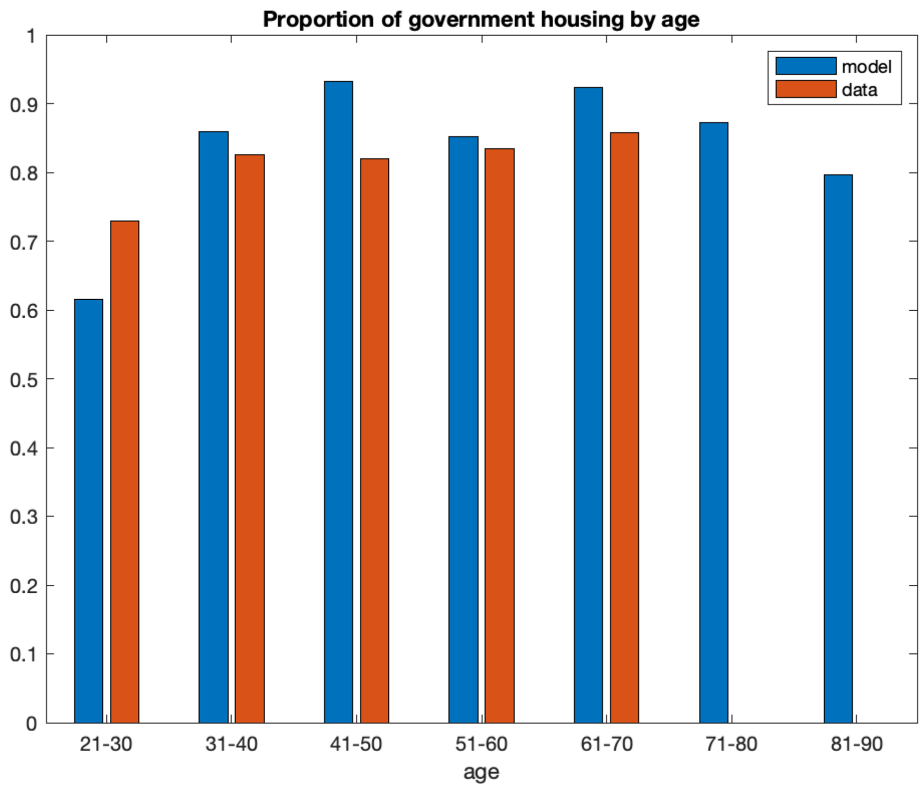

Figure 2.2: Proportion of households with government housing by age, 2003

mental changes are independent of housing reforms. Also, throughout these counterfactual experiments, we assume that households of all cohorts can purchase houses or rent houses at market prices. The difference is the pace of housing market privatization. Our benchmark policy reform is gradual because cohorts alive in an initial steady state can enjoy discounted housing. We can compute the CEV between these alternative paths versus our benchmark policy reform.

- Remove stages 1 and 2 (Transition path A): In a transitional path where there is no discounted housing purchase and no elimination of government subsidized housing for cohorts born after 1998. All households can live in government subsidized rental housing or purchase or rent houses from the market. This counterfactual economy differs from the benchmark economy for the initial cohorts in the following sense:

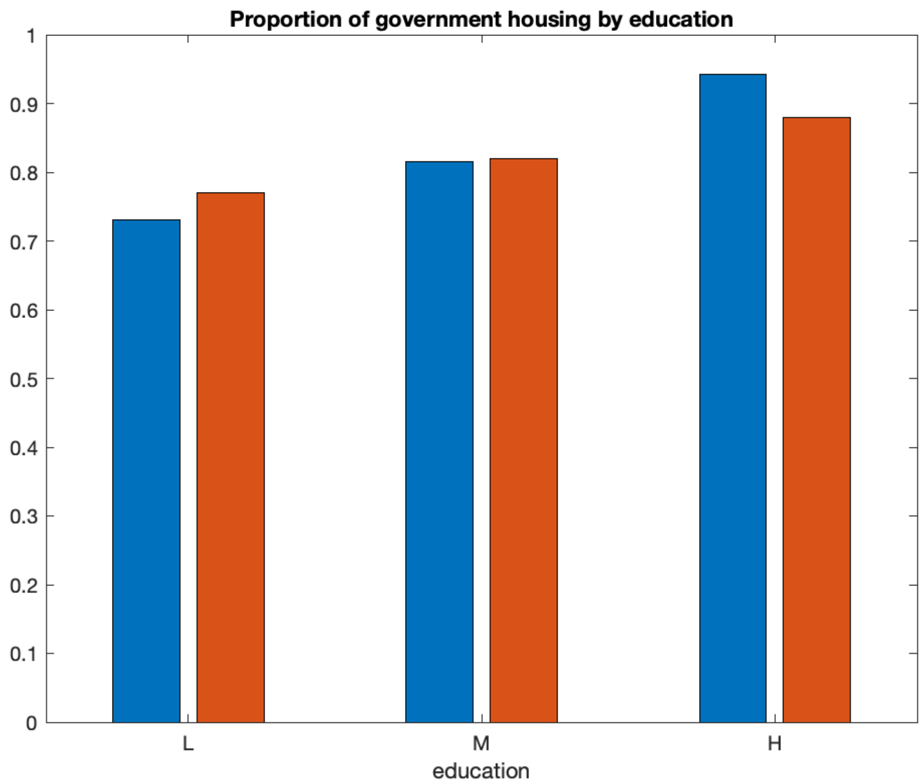

Figure 2.3: Proportion of households with government housing by education, 2003

(1) they are deprived of the chances for purchasing housing at discounted prices; (2) cohorts born after 1998 can still choose to rent the government's subsidized housing, which may affect the overall housing demand and housing prices.

- Remove stage 1 (Transition path B): In a transitional path where there is no government subsidized housing purchase at the year 1994. However, the 1998 reform is still implemented in that the government subsidized rental housing is only available for all households born before the year 1998.

- Remove stage 2 (Transition path C): In a transitional path where there is no expected change in housing policy in the year 1998. The government still allows households born after 1998 to rent housing at predetermined prices.

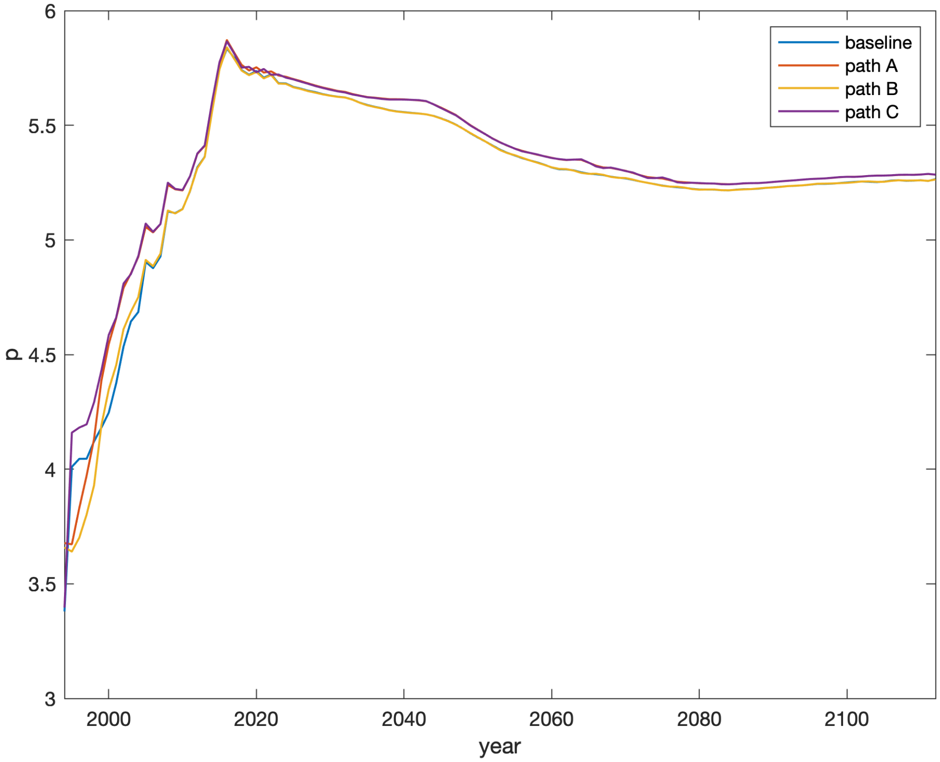

Figure 2.4: Housing price over year of all simulated paths, 1994 - 2112

Figure 2.4 shows the housing prices on all paths. By comparing either baseline with path B or path A with path C, the first stage of housing market reform effectively reduces the housing price for the initial old generations to purchase the government housing they have been living in. The price reduction induced by the reform in 1994 is almost 10% of the baseline path. The second stage of the reform in the year 1998, on the contrary, boosts housing prices by comparing path A with path B. It is even more significant to consider the combined effect of both stages of housing on the housing market capital return. Despite the price effect, we demonstrate the quantity effect of the housing reform. Figure 2.5 shows the sequences of simulated homeownership rates on all paths. The first stage reform leads to an increase in the homeownership rate of more than 60% regardless of the existence of the second stage reform. While the second stage reform directly changes the homeownership rate in the

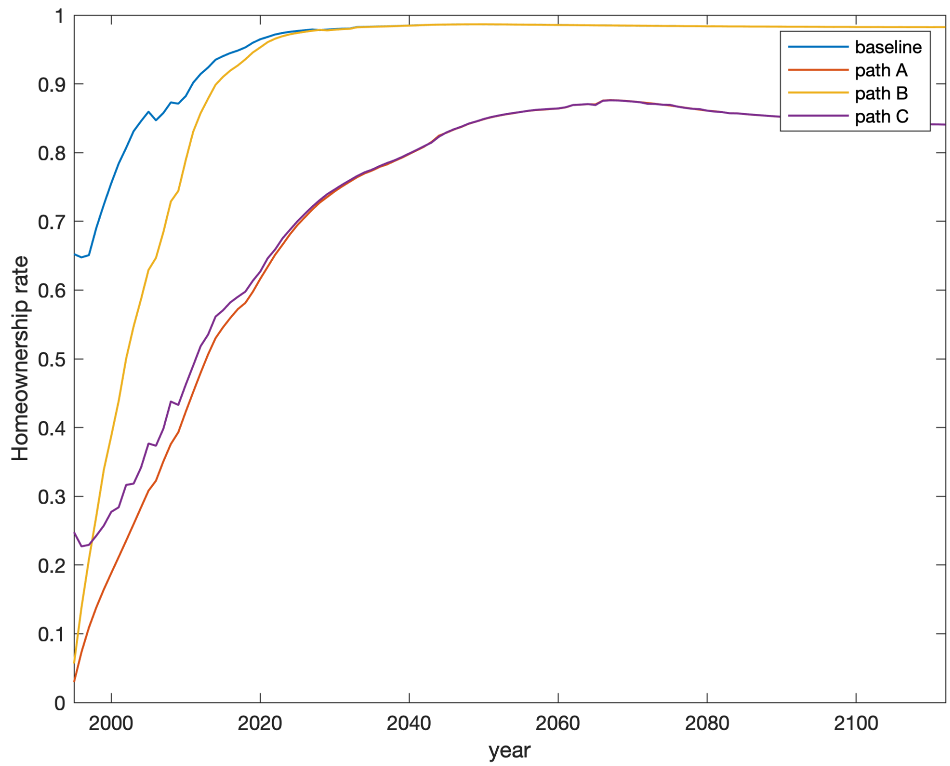

Figure 2.5: Homeownership rate over year of all simulated paths, 1995 - 2112

long run. On the paths with government subsidies to renting government housing, around 14% fewer households are willing to purchase houses than the paths without rental subsidies. The initial old households react to government subsidies by purchasing more government housing in the first stage of the reform. Due to the stimulated housing demand, the general equilibrium effects lead to faster housing price growth and higher capital gain in the housing market for the initial old households. The initial old households who purchase government housing benefit from government subsidies and capital gain from the housing market. Table 2.4 lists the CEV of all alternative paths relative to the baseline path. All alternative paths reduce the initial old generations' welfare because of missing one or both stages of the reform. Without stage 1 of the reform, the initial old generations lose the government's direct subsidy while still gaining from the housing market booming if purchasing housing in the early stage.

Without stage 2 of the reform, the housing price growth rate is lower than the baseline, which diminishes the capital gain from the housing market. The welfare impact of the two stages of the reform is complementary, given the welfare cost of path A exceeds the costs of two other paths together. On path A, the initial old generations are worse off than the baseline due to the direct cost of no subsidy and the reduced capital gain.

Path	Welfare cost
A	9.83%
B	2.54%
C	4.71%

Table 2.4: The average welfare cost of initial old generations relative to the baseline path

2.5 Conclusion

This paper studies the impact of the government housing market intervention on intergenerational transfer. We set up the constraint planner's problem through a simple life-cycle equilibrium model to characterize the optimal intergenerational allocation. To implement this constraint efficient allocation, the government would need to subsidize the initial old generations' purchase of housing and ration the purchased quantity.

Based on the theoretical finding, we build up a quantitative overlapping generation model to evaluate the welfare impact of the government's housing market intervention. China's housing market reform in the 1990s is chosen to quantify the impact. We calibrate the quantitative model to match micro-level data's aggregate and cross-sectional empirical moments. By comparing the simulation of the baseline model and several counterfactual policy experiments, we find government's subsidy effectively stimulus initial old generations' housing purchase. They benefit from the direct subsidy and the capital gain from the housing market in the general equilibrium. Given the housing market intervention, government transfers a significant amount of wealth from younger generations enjoying higher wage income to the initial old generations.

Chapter 3

Terms-of-Trade and Sovereign Debt Crisis

3.1 Introduction

This paper extends a sovereign default model with a single tradable good by introducing a distinction between tradable goods from the domestic and foreign economies and the relative price between two tradable goods (terms-of-trade). The extension contributes to the fundamental question in the sovereign default literature, or "Why do countries pay their international debts?". Two main reasons why the countries repay the international or external debts are economic sanction and reputation, according to [28]. The reputation motive states that the debtor countries need to use the external debt to smooth consumption, but defaulting on the external debt would exclude them from the international financial market. Thus, repaying the existing external debt is necessary if countries would like to maintain access to the international financial market. This paper provides further evidence to support this argument by showing that debtor countries can gain extra benefit from active terms-of-trade management via current account adjustment, which is only possible if maintaining a good reputation.

The domestic economy faces a downward sloping demand for goods from foreign economies that are domestically tradable, see [25] and [19]. Even with a monopolistic supplier for the domestic tradable goods, the domestic economy's market power can be regulated if it is involved in the international trade agreements, which have restricted the tariff levied by their member countries, see [27], and [3]. Nevertheless, there also exists a way to curve around the tariff restriction. Instead of direct intervention on terms-of-trade using the tariff, the government can actively manage the supply of domestic tradable goods to foreign economies. The government can manipulate the domestic absorption of domestic tradable goods by influencing the external debt position, hence having some degree of control over the supply to foreign economies to some degree. Given the condition that there exist restrictions on tariff, the motive of managing the terms-of-trade requires the government to adjust the domestic economy's external borrowing or capital inflow, which is possible only if the domestic economy remains credible on the international financial market.

The government's intervention on the capital flow has been more recognized as a standard policy tool ever since the financial crisis in 2008. Some international institutions, for example, IMF, reversed their views on the government's intervention on the capital account by acknowledging the role of capital or prudential measures in safeguarding the macroeconomic and financial stability, see [5]. In reality, researchers have shown that the government's deployment of capital control measures is relatively stable through newly constructed data set across different countries and types of transactions, [16]. In theory, there have been studies on how capital control measures can reconcile individual optimal decisions with the efficient outcome of the social planner. Specifically, in the literature about the sovereign default, [23] analyze how the optimal external borrowing and default outcome can be implemented in a decentralized economy if the capital control measures (tax on external borrowing) are available. This paper aims to combine the government's motive of managing the terms-of-trade into a sovereign default model. This paper also studies how the additional motive interacts with the financial stabling role of the capital control measures and the optimal defaulting decisions.

Following [23], this paper develops a sovereign default model. Individual households choose how much external debt to borrow and how much domestic and foreign tradable goods to consume. The government decides whether to declare default on the external debt and conduct fiscal policies. The government's objective is to maximize the aggregate welfare of the representative household, but it does not commit to its future decisions, including repayment of external debt and fiscal policies. The first question this paper studies is whether an individual's decision deviates from the efficient outcomes if there is no government intervention. This paper finds evidence that individuals fail to endogenize the externalities of their borrowing decisions on the debt price and the terms-of-trade. The findings endorse the necessity of government intervention to achieve a more efficient outcome. The social planner's decision is only on external borrowing in the sovereign default literature. The household consumption is settled with external borrowing. When setting up the social planner's problem in this paper, the modeler needs to assume whether or not the social planner can decide on allocation the domestic tradable goods. Casting this modeling choice into the optimal policy problem of the government, it echos with whether the government can use tariffs to intervene in the terms-of-trade. This paper then shows how to connect the social planner's allocation with the optimal policy problem under different assumptions on the planning capacity. Through quantitative analysis of the calibrated model, this paper finds that the allocation of a social planner without market power on domestic tradable is superior to the planner with market power in terms of household welfare. The result is counter-intuitive at first glance. How can a social planner with more planning capacity or a government with more policy tools achieve a worse outcome than the less powerful one with fewer tools? The reason lies in the lack of commitment of the planner or the government in the sovereign default model. Imagine the default decision by the planner of a high leverage economy - if it has no market power on the domestic tradable goods, defaulting exchanges the benefit of alleviating the debt burden right now with the benefit of consumption smoothing terms-of-trade management in the future. If it has market power, losing the terms-of-trade management is outside its consideration. As a result, the more regulated planner or government is more credible and has cheaper external financing. This result implies the rationale of committing to an outside trade agreement while actively using capital control to achieve efficient allocations.

This paper relates to the literature studying the optimal sovereign default. After the seminal work by [13], many papers have improved the quantitative performance of the sovereign default model and established the essential features of economic dynamics around the sovereign default crisis, see [1] and [2]. [18] and [8] study the long maturity debt in the soverieng default. [22] combine business cycle study with the default crisis. [21] study the externality of individual borrowing on the debt price. [23] explains why default and devaluation usually happen hand in hand. In terms of modeling, this paper is close to [21] and [23] in setting up the decentralized version of the sovereign default model. This paper contributes to this literature by showing an additional source of externality associated with external borrowing and connecting the terms-of-trade management with the optimal borrowing and defaulting decision. It also contributes to the literature studying terms-of-trade management with capital control policies. [10] considers the optimal use of capital control to manage intertemporal terms-of-trade in a two-country model with free trade. [4] study the interaction between tariff and capital control tax in a two-country model without aggregate uncertainty. In the literature of terms-of-trade management, there is no default risk on the external debt; thus, no interaction between tariff and optimal default decision.

The remainder of the paper is organized as follows. Section 2 presents the model's environment and derives the competitive equilibrium and social planner's problems. Section 3 introduces the government policy tools and shows how to replicate the social planner's allocation given the availability of policy tools. Section 4 conducts a quantitative analysis to investigate the difference between competitive equilibrium and social planner's problem and evaluates household welfare and dynamics of key macroeconomic variables. The last section concludes.

3.2 The Model

This section describes the environment of the model economy. It considers a small open economy with three types of agents - households, the government, and the foreign lender. The economy is small because domestic residents take the terms-of-trade and the interest rate on the external debt as given. This section then defines the competitive equilibrium and the social planner's problem for the convenience of the discussion in later sections on the source of externalities and how the government can exert its intervention given different set up of policy tools.

3.2.1 Households

The economy is populated by a continuum of identical households $j \in [0, 1]$ that maximize utility from consuming domestic and foreign tradable goods. Households live for infinite periods. Their preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{3.1}$$

where β is the discount factor of the households and c_t is the aggregate consumption.

Consumption is a composition of domestic and foreign tradable goods, with an Armington aggregator,

$$c_t = A(c_t^H, c_t^F),$$

where c_t^H and c_t^F denote consumption of domestic and foreign tradable goods, and A is an increasing, concave, and linearly homogeneous function.

Households receive an endowment of domestic tradable goods each period and access a state non-contingent one-period debt denominated in foreign tradable goods. The sequential budget constraint is given by

$$P_t^H c_t^H + P_t^F c_t^F + P_t^F d_t = P_t^H \tilde{y}_t^H + P_t^F q_t d_{t+1},$$

where P_t^H denotes the price of domestic tradable goods, P_t^F denotes the price of foreign tradable goods. The endowment variable $\tilde{y}_t^H$ is a stochastic process and taken as given by the household. Due to the assumption of a small economy, the price of foreign tradable goods is exogenous to the domestic households. After dividing both sides of the household's sequential budget constraint by the price of foreign tradable goods, the budget constraint in real terms can be written as

$$p_t c_t^H + c_t^F + d_t = p_t \tilde{y}_t^H + q_t d_{t+1}, \tag{3.2}$$

where $p_t \equiv P_t^H / P_t^F$ is the relative price of domestic tradable goods in terms of foreign tradable goods, and it is often referred as the terms-of-trade.

The budget constraint reflects the currency mismatch of the small open economy's balance sheet. The borrowing is denominated in terms of foreign tradable goods, while the equity or income is denominated by domestic tradable goods. The debt burden is decided by the external debt level, endowment level, and the relative price or terms-of-trade in this paper.

The household maximizes utility subject to the budget constraint expressed in units of foreign tradable goods and a no-Ponzi-game constraint by choosing sequences $\{c_t, c_t^H, c_t^F, d_{t+1}\}_{t=0}^{\infty}$. Household's optimal decisions can be characterized by the following first order conditions:

$$p_t = \frac{A_1(c_t^H, c_t^F)}{A_2(c_t^H, c_t^F)}, \tag{3.3}$$

$$\lambda_t = U'(c_t) A_2(c_t^H, c_t^F), \tag{3.4}$$

$$q_t \lambda_t = \beta \mathbb{E}_t \lambda_{t+1}. \tag{3.5}$$

Households export part of their endowments to meet the foreign demand for domestic tradable goods. Following [17], this paper assumes that the functional form of the foreign demand is

$$c_t^{H*} = p_t^{-\gamma} y^*, \tag{3.6}$$

where γ is the trade elasticity, and y^* is the factor reflecting the overall demand of foreign countries.

3.2.2 The Government

The benevolent government decides whether to honor the existing external debt at the beginning of each period in order to maximize the welfare of the representative household, $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$. The domestic economy can be in two states in each period, in good or bad financial standing. The dummy variable, I_t, denotes the economy's financial standing at time t, which takes the value of 1 if the economy is in a good state. If the economy starts the time t with good financial standing, $I_{t-1} = 1$, the government can choose to repay the debt which leads to good financial standing, $I_t = 1$, or repudiate all the external debt which leads to bad financial standing, $I_t = 0$. It is assumed that the government has necessary measures to enforce the private agents into defaulting on their external debt, see [21] for more discussion.

The default on external debt comes with a direct loss of endowment. In the bad financial standing, the domestic economy is also isolated from the international financial market without access to international borrowing. In reality, many countries that defaulted on their external debt regained access to the external debt after some time. To match this observation, it is assumed the country in the bad credit standing returns to the state of good standing with a probability, θ.

3.2.3 The Foreign Lender

The international financial market is perfectly competitive, and foreign lenders are risk neutral, who discount their future income by the risk free rate, r^*. The price of debt borrowed by domestic households must satisfy the foreign lenders' zero profit condition,

$$q_t = \frac{\mathbb{E}_t I_{t+1}}{1 + r^*} \tag{3.7}$$

3.2.4 Competitive Equilibrium

To close the model, two more conditions on domestic tradale goods need to be satisfied. Firstly, the market for domestic tradable goods is always clear,

$$\tilde{y}_t^H = c_t^H + c_t^{H*}. \tag{3.8}$$

Secondly, a stochastic endowment process, y_t^H, of domestic tradable goods is exogenously given. The endowment received by domestic households, $\tilde{y}_t^H$, is equal with the stochastic process y_t^H if the economy is in good financial standing. Otherwise, households suffer from an output loss, $L(y_t^H)$,

$$\tilde{y}_t^H = \begin{cases} y_t^H, & \text{if } I_t = 1 \\ y_t^H - L(y_t^H), & \text{if } I_t = 0. \end{cases} \tag{3.9}$$

With all agents' objective and constraints described above, the competitive equilibrium is ready to be defined.

Definition 2. A competitive equilibrium is a set of stochastic processes $\{c_t^H, c_t^{H*}, c_t^F, p_t, d_{t+1}, \lambda_t, q_t\}_{t=0}^{\infty}$ that satisfy (2) - (9), given processes $\{y_t^H, I_t\}$ and initial condition d_0.

To maximize the welfare of domestic households, the government chooses to default optimally subject to optimization conditions of households and other equilibrium conditions. The government is assumed to lack commitment to external debt repayment. Thus it takes as given the decisions of the future government. This paper considers the model environment with decentralized borrowing and centralized default, as in Kim and Zhang (2012) and Na (2018). Later, this paper sets up the social planner's problem to derive efficient allocations as in the standard Eaton-Gersovitz model. By comparing allocations, the externalities related to external borrowing can be identified. To solve for the competitive equilibrium quantitatively, this paper defines a recursive competitive equilibrium in which individual households with debt position, d, take as given the aggregate debt position, D.

In the periods of being in the good financial standing with aggregate debt, D_t, households with endowment level, y_t^H, debt position, d_t, solve the following optimization problem,

$$\begin{aligned} v^g(y_t^H, d_t, D_t) = \max_{c_t^F, c_t^H, d_{t+1}} & U(A(c_t^F, c_t^H)) + \\ & \beta E_t \left[I(y_{t+1}^H, D_{t+1}) v^g(y_{t+1}^H, d_{t+1}, D_{t+1}) + (1 - I(y_{t+1}^H, D_{t+1}) v^b(y_{t+1}^H)) \right], \end{aligned} \tag{3.10}$$

subject to the household budget constraint (2), and aggregate debt law of motion, $D_{t+1} = \Gamma(y_t^H, D_t)$, where v^b is the welfare of individual households being i the bad financial standing, and $I(y_{t+1}^H, D_{t+1})$ is the indicator that takes the value of one if the government defaults given states y_{t+1}^H and D_{t+1} in period $t+1$.

In the periods of being in the bad financial standing, households are financially autarky, and their endowment level drops from y_t^H to $y_t^H - L(y_t^H)$. The economy has a probability θ of regaining access to the financial market. The household value function is given by

$$\begin{aligned} & v^b(y_t^H) = \max_{c_t^F, c_t^H} U(A(c_t^F, c_t^H)) + \beta E_t \left[\theta v^g(y_{t+1}^H, 0, 0) + (1-\theta) v^b(y_{t+1}^H)) \right], \\ & s.t. \ p_t c_t^H + c_t^F = p_t(y_t^H - L(y_t^H)). \end{aligned} \tag{3.11}$$

The benevolent government maximizes the lifetime utility of households by choosing to default or not. We impose the equilibrium condition that $d_t = D_t$, because households are representative and the aggregate measure of the household sector is one.

$$I(y_t^H, D_t) = \begin{cases} 1 \ if \ v^g(y_t^H, D_t, D_t) > v^b(y_t^H) \\ 0 \ otherwise. \end{cases}$$

As described above, the price of debt needs to satisfy the condition with risk neutral lenders

$$q(y_t^H, D_{t+1}) = \frac{1 - E_t I(y_{t+1}^H, D_{t+1})}{1 + r^*}.$$

Definition 3. A recursive competitive equilibrium of this economy is a list of (1) households' value functions, $v^g(y^H, d, D)$ and $v^b(y^H)$; (2) households' policy functions $d(y^H, d, D)$, $c^H(y^H, d, D)$, and $c^F(y^H, d, D)$; (3) a government default decision function, $I(y^H, D)$; (4) price functions, $q(y^H, D')$ and $p(y^H, D, D')$; (5) a law of motion of aggregate external debt, $D' = \Gamma(y^H, D)$, such that

1. value functions and policy functions solve the household's optimization problem (9) and (10), given prices and laws of motion of aggregate variables;
2. government's default decision satisfies its objective of maximizing households' lifetime utility;
3. domestic tradable goods market are clear;
4. foreign leaders earn the zero profit;
5. individual policy function is consistent with the aggregate law of motion of external debt.

From a model perspective, there are two sources of pecuniary externalities in this small open economy. In the households ' optimization problem, they are both related to the aggregate debt level, D. Households treat aggregate debt level and the law of motion of external debt as given, although they are endogenous in the equilibrium. Thus, the price of debt and the terms-of-trade, which are functions of aggregate variables, are taken as given. The debt price, or the discounted probability of defaulting, depends on current endowment level and aggregate debt level next period, $q(y_t^H, D_{t+1})$. The terms-of-trade depends on current endowment level, and current and next period's aggregate debt level, $p(y_t^H, D_t, D_{t+1})$. Assuming the domestic tradable goods and the foreign tradable goods are complimentary, it can be found that $\frac{\partial p}{\partial D_t} < 0$ and $\frac{\partial p}{\partial D_{t+1}} > 0$. Intuitively, the high current debt level depresses the domestic consumption of foreign tradable goods, which raises the marginal utility of foreign tradable goods and dampens the relative price of domestic tradable goods.

On the contrary, the high insurance of new external debt results in boomed domestic demand and elevating the relative price or the terms-of-trade. Because households internalize none of the price effects of external borrowing, one would expect the level of external debt in a competitive equilibrium to be different from the efficient allocation. However, the difference can be positive or negative in equilibrium, as households neglect the negative marginal effect of extra borrowing on debt price, $\frac{\partial q}{\partial D_{t+1}}$, but also positive marginal effect on terms-of-trade, $\frac{\partial p}{\partial D_{t+1}}$. Besides, the supply of external debt may be shifted in the equilibrium as in [21].

3.2.5 Social Planner's Problem

To justify the use of policy intervention, this paper defines a benevolent social planner's problem. By comparing the social planner's allocations and competitive equilibrium, the externalities of the atomic household's decision in the competitive equilibrium are more clear qualitatively and quantitatively. The social planner maximizes the aggregate social welfare with limited planning abilities. First of all, following the literature, this paper assumes that the social planner can not commit to repaying the external debt. Then this paper starts by considering the case in which social planners can choose the level of borrowing while accepting the domestic tradable goods market clearing competitively. This paper also considers the case in which the social planner chooses external borrowing and household consumption. In the rest of the paper, the former case is called the constraint social planner's problem, and the latter one is the unconstraint social planner's problem. The constraint social planner is the price taker on the domestic tradable goods market. In reality, governments may be involved in trade agreements with other countries to not interfere with international trade. The setup of an unconstraint social planner's problem is more comparable with the social planner's problem in Eaton and Gersovitz model. After that, this paper distinguishes the two social planner's problems. In the next section, this paper shows how the social planner's problems correspond to optimal policy problems, and the planning capacity reflects policymakers' freedom.

In the competitive equilibrium, the borrowing and consumption decisions are made by individual households, who do not internalize their impact on market prices. On the contrary, the social planner collectively decides how much external debt to borrow and how much domestic tradable goods to consume. By choosing the domestic tradable goods' domestic absorption, the social planner determines the exportation to foreign households. With the knowledge of the foreign demand, equation cite here,

The constraint social planner's objective is to maximize the lifetime utility. Instead of households, the social planner decides on the level of external borrowing in the periods of good financial standing. Due to the constraint planning capacity, the social planner takes the household consumption decisions as given. Specifically, the constraint social planner solves the following optimization problem

$$\begin{aligned} V_c^g(y_t^H, D_t) = \max_{D_{t+1}} U(A(c_t^F, c_t^H)) + \\ \beta E_t \left[I(y_{t+1}^H, D_{t+1}) V_c^g(y_{t+1}^H, D_{t+1}) + (1 - I(y_{t+1}^H, D_{t+1}) V_c^b(y_{t+1}^H)) \right], \end{aligned} \quad (3.12)$$

subject to the budget constraint

$$p_t c_t^H + c_t^F + D_t = p_t y_t^H + q_t D_{t+1},$$

market clearing condition

$$y_t^H = c_t^H + (p_t)^{-\gamma} y^*,$$

and the household first order condition

$$p_t = \frac{A_1(c_t^H, c_t^F)}{A_2(c_t^H, c_t^F)},$$

where the constraint social planner's value function, V_c^b, in the bad financial standing is defined as

$$
\begin{aligned}
V_c^b(y_t^H) &= U(A(c_t^F, c_t^H)) + \beta E_t \left[\theta V_c^g(y_{t+1}^H, 0, 0) + (1-\theta) V_c^b(y_{t+1}^H)\right], \\
s.t.\ p_t c_t^H + c_t^F &= p_t(y_t^H - L(y_t^H)), \\
y_t^H &= c_t^H + (p_t)^{-\gamma} y^*, \\
p_t &= \frac{A_1(c_t^H, c_t^F)}{A_2(c_t^H, c_t^F)}.
\end{aligned}
\tag{3.13}
$$

At the beginning of the period in the good financial standing, the social planner chooses to default or not by comparing the value of continuing the good standing, V_c^g, and the value of defaulting on the external debt, V_c^b. In the equilibrium, the price of the domestic tradable goods should clear the goods market, and the price of the external debt ensures foreign lenders earn zero profit.

Definition 4. A recursive equilibrium of a constraint social planner's problem is a list of (1) value functions, $V_c^g(y^H, D)$ and $V_c^b(y^H)$; (2) policy function $d(y^H, D)$; (3) a default decision function, $I(y^H, D)$; (4) price functions, $q(y^H, D')$ and $p(y^H, D, D')$, such that

1. value functions and policy functions solve the constraint planner's optimization problem (11) and (12), given prices of debt and terms-of-trade;
2. domestic tradable goods market are clear;
3. foreign lenders earn zero profit.

To illustrate the difference between the optimal borrowing decision of households in competitive equilibrium and the one of the constraint social planner, this paper treats the value functions as if they are differentiable to present the Euler equation of household's problem in this section. The purpose is to show is how the pecuniary externalities deter the households' optimal borrowing decision from being efficient. This paper's computation and numerical results do not rely on the differentiability of the value functions. The first-order condition of external borrowing in a constraint social planner's problem is

$$U_{c^F}(c^H,c^F)(\underbrace{p^{-\gamma}y^*\frac{\partial p(y^H,D,D')}{\partial D'}}_{\text{terms-of-trade effect}}+q(y^H,D')+\underbrace{\frac{\partial q(y^H,D')}{\partial D'}D'}_{\text{debt price effect}})$$
$$=\beta E_{y^{H'}|y^H}I(y^{H'},D')U_{c^F}(c^{H'},c^{F'})(1-\underbrace{(p')^{-\gamma}y^*\frac{\partial p'(y^{H'},D',D'')}{\partial D'}}_{\text{terms-of-trade effect}}). \quad (3.14)$$

The first order condition of individual household's external borrowing in a competitive equilibrium is

$$U_{c^F}(c^H,c^F)q(y^H,D')=\beta E_{y^{H'}|y^H}I(y^{H'},D')U_{c^F}(c^{H'},c^{F'}). \quad (3.15)$$

The equations show that the constraint social planner has a more deliberate intertemporal trade-off than households. The bracket terms are the pecuniary externalities in a competitive equilibrium that the planner internalizes. The left-hand side of equation (13) is the marginal benefit of borrowing external debt. The constraint social planner measures the marginal benefit by incorporating two price effects: the debt price effect and the terms-of-trade effect. The two price effects have opposite directions, as mentioned previously. The additional borrowing decreases the price of debt, but it also boosts the domestic demand and improves the terms-of-trade. On the right-hand side of the equation (13) is the marginal cost of borrowing, which reduces the consumption measure in terms of the expected marginal utility. The social planner is aware of the effect of additional debt repayment on the domestic demand next period, which results in a worsening terms-of-trade and further consumption reduction next period. None of these price effects appear in the individual households' consideration as they are price takers in the competitive economy. Although this paper shows the difference in optimal borrowing between the social planner and households, the two first-order conditions are not comparable quantitatively. In both first-order conditions, the debt price and the default decision are endogenous and jointly decided in the equilibrium.

At the end of this subsection, this paper defines the unconstraint social planner's problem. Unlike constraint one, the unconstraint social planner can decide on the households' consumption of foreign tradable and domestic tradable goods. The social planner is subject to the resource constraints of tradable goods when it maximizes the household's lifetime utility. In the good financial standing periods, social planner's value of repaying the external debt given the state y_t^H and D_t is

$$\begin{aligned} V_u^g(y_t^H, D_t) = \max_{c_t^F, c_t^H, D_{t+1}} & U(A(c_t^F, c_t^H)) + \\ & \beta E_t \left[I(y_{t+1}^H, D_{t+1}) V_u^g(y_{t+1}^H, D_{t+1}) + (1 - I(y_{t+1}^H, D_{t+1}) V_u^b(y_{t+1}^H)) \right], \end{aligned} \tag{3.16}$$

subject to the budget constraint

$$p_t c_t^H + c_t^F + D_t = p_t(y_t^H) + q_t D_{t+1},$$

and the market clearing condition

$$y_t^H = c_t^H + p_t^{-\gamma} y^*,$$

where the social planner's value function, V_u^b, in the bad financial standing is defined as

$$\begin{aligned} V_u^b(y_t^H) &= \max_{c_t^F, c_t^H} U(A(c_t^F, c_t^H)) + \beta E_t \left[\theta V_u^g(y_{t+1}^H, 0, 0) + (1 - \theta) V_u^b(y_{t+1}^H)) \right], \\ s.t.\ & p_t c_t^H + c_t^F = p_t(y_t^H - L(y_t^H)), \\ & y_t^H = c_t^H + p_t^{-\gamma} y^*. \end{aligned} \tag{3.17}$$

The unconstraint social planner is not subject to the intratemporal optimization condition of households but is subject to the market clearing condition. It is no longer a price taker on the goods market but a monopoly supplier of domestic tradable goods. The price of the tradable goods is pinned down by the export of domestic tradable goods chosen by the planner or leftover after the domestic absorption, given the exogenous foreign demand. One can consolidate the two constraints into a single resource constraint that the unconstraint social planner is subject to. Even in the bad financial standing, the unconstraint social planner maintains its market power in the goods market.

3.3 The Optimal Policies

By comparing a competitive equilibrium and social planner's problem, this paper demonstrates that the competitive economy and scope for the policy intervention are inefficient. Given a variate of policy instruments, this paper narrows down policy makers' choice to the tax on external borrowing and tax on the consumption of foreign tradable goods.

The first policy tool, τ_t^d, is the tax ($\tau_t^d > 0$) or subsidy ($\tau_t^d < 0$) on the one period of external debt issued at time t. In the literature, the tax on external debt is also interpreted as the capital control measures adopted by the government. The second policy tool, τ_t^F, is the tax ($\tau_t^F > 0$) or subsidy ($\tau_t^F < 0$) on the household consumption of the foreign tradable goods at time t. The consumption tax on foreign goods can be considered the tariff in reality. Similarly, one can consider the tax on exports of domestic tradable goods. It can be easily shown that tariffs imposed on imports and exports are equivalent in the model environment of this paper.

The government is assumed to have access to, T_t, the lump sum transfer to households ($T_t > 0$) or tax ($T_t < 0$) and maintain a balanced budget in each period, thus

$$\tau_t^d q_t d_{t+1} + \tau_t^F c_t^F = T_t.$$

The assumption that a lump-sum transfer exists ensures that government interventions can be implemented with the minimum cost manner, i.e., without introducing additional distortion. The balanced government budget assumption implies that the household's endowment is intact. The household budget constraint is

$$p_t c_t^H + (1+\tau_t^F)c_t^F + d_t = p_t \tilde{y}_t^H + (1-\tau_t^d)q_t d_{t+1} + T_t. \tag{3.18}$$

Given the processes of government policy $\{\tau_t^d, \tau_t^F, T_t, I_t\}_{t=0}^{\infty}$, the household maximizes utility subject to the budget constraint (17) by choosing sequences $\{c_t, c_t^H, c_t^F, d_{t+1}\}_{t=0}^{\infty}$. Household's optimal decisions under government's intervention can be characterized by the following first order conditions:

$$p_t = (1+\tau_t^F)\frac{A_1(c_t^H, c_t^F)}{A_2(c_t^H, c_t^F)}, \tag{3.19}$$

$$(1+\tau_t^F)\lambda_t = U'(c_t)A_2(c_t^H, c_t^F), \tag{3.20}$$

$$(1-\tau_t^d)q_t\lambda_t = \beta\mathbb{E}_t\lambda_{t+1}. \tag{3.21}$$

Lemma 4. *Given the endowment process $\{y_t^H, I_t\}_{t=0}^{\infty}$, and initial condition, d_0, stochastic processes $\{c_t^H, c_t^F, d_{t+1}, q_t, p_t\}_{t=0}^{\infty}$ can be supported as a competitive equilibrium if and only if they satisfy the equilibrium condition (7), and*

$$c_t^H = y_t^H - (1-I_t)L(y_t^H) - (p_t)^{-\gamma}y^*, \tag{3.22}$$

$$c_t^F = (p_t)^{1-\gamma}y^* + q_t d_{t+1} - d_t, \tag{3.23}$$

$$0 = (1-I_t)d_{t+1}. \tag{3.24}$$

Proof. It can be proved by construction.

The key steps are showing the conditions (7), and (22) - (24) can be expanded to competitive equilibrium conditions (6) - (9), and (18) - (21). From stochastic processes $\{c_t^H, c_t^F, d_{t+1}, q_t, p_t\}_{t=0}^{\infty}$, $\{\tau_t^d, \tau_t^F\}$ can be uniquely chosen to satisfy the condition (19) - (21). The household budget constraints hold by combining (22), (23), and government budget constraint. □

The above lemma helps to simplify the equilibrium conditions into a more compact form. In the following subsections, this paper firstly restricts the government's policy tool on capital control only by setting $\tau_t^F = 0$ all the time. The reason for considering this specific scenario is similar to setting up a constraint social planner's problem. In reality, the government may involve itself in an international trade agreement that regulates the member countries' tariffs. Then, this restriction is relaxed, and this paper studies the optimal policy mix given the availability of capital control and tariff. It is worth noting that the optimal policy studied in this paper is not Ramsey's optimal policy due to the government's lack of commitment. Instead, this paper considers how to replicate the social planner's allocation in a competitive economy through available tax rates. In each period, the government chooses the default decision and tax rates while taking as given the government's policy decisions in the next period.

3.3.1 The Optimal Capital Control

This subsection describes the optimal tax rate on external borrowing when the government can freely choose the tax rate on debt but not the tariff.

Lemma 5. *In the competitive economy defined above, if the government can freely set the tax on the external borrowing, τ_t^d, the constraint social planner's allocation can be replicated as a competitive equilibrium by choosing τ_t^d such that (21) holds. The optimal policy is time consistent.*

Proof. It can be proved by construction.

When the tax rate on foreign tradable goods, $\tau_t^F = 0$ for all t, condition (19) and (20) can be written as $p_t = \frac{A_1(c_t^H, c_t^F)}{A_2(c_t^H, c_t^F)}$ and $\lambda_t = U'(c_t)A_2(c_t^H, c_t^F)$. The allocation of a constraint social planner's problem satisfies the condition (19). After choosing τ_t^d such that (21) always holds, the rest part of the proof is the same with the lemma 3.1. $\square$

With the capital control tax as the only policy instrument, the government can intervene in the market price of external debt, thus disciplining the households' intertemporal decision to coincide with the constraint social planner's allocation. The tax on household borrowing only exists in periods with good financial standing. The government is silent about household decisions in a period of bad financial standing.

3.3.2 The Optimal Policy Mix

To enlarge the scope of government intervention, this subsection endorses the government with an additional policy instrument, tax on the foreign tradable goods, so that the government can intervene in household's intratemporal decisions in good and bad financial standing.

Lemma 6. *In the competitive economy defined above, if the government can set the tax on the external borrowing, τ_t^d freely, and the tax on the foreign tradable goods, τ_t^F, the unconstraint social planner's allocation can be replicated as a competitive equilibrium by choosing τ_t^d and τ_t^F such that (19) - (21) hold. The optimal policy is time consistent.*

Proof. It can be proved by construction.
The unconstraint social planner is not subject to any condition among (19) - (21). Thus, tax rates τ_t^d and τ_t^F can be chosen specifically to satisfy the conditions. The process of construction is the same with the proof of the lemma 3.1. □

A remark follows the lemma of optimal policy mix. Compare the equation (19) with the unconstraint social planner's first order condition

$$p_t = \frac{\gamma}{\gamma - 1} \frac{A_1(c_t^H, c_t^F)}{A_2(c_t^H, c_t^F)}.$$

It is clear that the tariff of the optimal policy mix is a constant, $\tau_t^F = \frac{1}{\gamma}$. With the capital control tax available, the optimal tariff is as if a solution to the static problem. Consolidating equation (19) - (21), there exists following equilibrium restriction on the optimal capital control tax

$$1-\tau_t^d = \beta \frac{\mathbb{E}_t U'(c_{t+1}) A_2(c_{t+1}^H, c_{t+1}^F)}{q_t U'(c_t) A_2(c_t^H, c_t^F)},$$

government chooses τ_t^F to reconcile the household intratemporal optimal decision with the unconstraint social planner's decision. It decides on τ_t^d in the period of the good financial performance standing to adjust the household's intertemporal optimal condition.

With the results that each social planner's allocation can be replicated given a specific set of policy tools, this paper treats the optimal policy problems and social planner's problems as equivalent in the rest of the paper.

3.4 Quantitative Analysis

This section evaluates the quantitative implication of the sovereign default model with the endogenous terms-of-trade. This paper first compares the equilibrium outcome of the decentralized economy with two social planner's problems, especially the equilibrium distribution of external debt, to investigate whether individual households tend to underborrow or overborrow. If either deviation from the level of external debt chosen by the social planner happens, the existence of the government intervention is justified. Then this paper demonstrates the dynamics of the optimal policies and the dynamics of other key variables around the default episode. Finally, this paper also ranks the outcomes of competitive equilibrium and two social planner's problems in terms of social welfare.

3.4.1 Calibration

This paper follows the conventional calibration strategy used by the majority of the literature. The underlying endowment process, y_t^H, of the model is assumed to be an AR(1) process in logarithm,

$$\log y_t^H = \rho \log y_{t-1}^H + \epsilon.$$

It is calibrated to match the Argentine economy at a quarterly frequency, which generates the first order correlation, $\rho = 0.9317$, and standard deviation of the innovation, $\sigma = 0.037$. The period utility function takes the form of the constant relative risk aversion type,

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma},$$

where the risk averse parameter is $\sigma = 2$. The aggregate consumption c is a Cobb–Douglas function of the foreign and domestic tradable consumption,

$$c = (c^H)^\alpha (c^F)^{1-\alpha}.$$

The output cost of defaulting is asymmetric as in Chatterjee and Eyigungor (2012),

$$L(y_t^H) = \max\{0, \delta_1 y_t^H + \delta_2 (y_t^H)^2\},$$

The assumption of the default cost has a large impact on the equilibrium default decision and debt price. The reason to choose an asymmetric default cost is to capture the empirical finding that governments rarely default on external debt when economic conditions are booming. The trade elasticity of substitution, $\gamma = 5$, in line with recent estimates in the literature, e.g. Simonovska and Waugh (2014); Imbs and Mejean (2015). The probability of reentering the international financial market is set to $\theta = 0.0385$, to match the average length of the periods in financial autarky. The international lender is assumed to be the risk free asset with a return, $r^* = 0.01$, in each quarter. In other words, the foreign lender is more patient than the households in the small open economy. The assumption ensures that households or social planners borrow to consume in equilibrium instead of saving.

Models are solved recursive problems numerically by the value function iteration on the discrete state space. The endowment process is approximated by 150 points. The number of the grid on the external debt for the social planner's problems is 200, and 1,500 for the recursive competitive equilibrium problem. The computation burden of solving the recursive competitive equilibrium is much heavier because (1) there is an additional dimension of households problem; (2) the aggregate law of motion in equilibrium requires a fine state space grid to be accurately approximated. The details of the computation procedure are documented in the Appendix. For the summary of parameter values used in the calibration, please refer to Table 3.1.

Parameter	Value
σ	2
γ	5
ρ	0.9317
σ	0.037
α	0.62
β	0.85
δ_1	-0.35
δ_2	0.4403
θ	0.0385
r^*	0.01
Computation parameter	
n_y	150
n_d	1500
n_D	200
$[\underline{d}, \overline{d}]$	[0,1.5]
$[\underline{D}, \overline{D}]$	[0,1.5]

Table 3.1: Parameters in calibration

3.4.2 External Debt Distribution

To understand the necessity of the policy intervention, Figure 3.1 illustrates the unconditional distribution of the aggregate external debt, D_t, under competitive equilibrium, constraint social planner, and unconstraint social planner's problem. First, the equilibrium distributions show that the economy is underborrowing in the competitive equilibrium than both the social planner's allocations. Several factors are contributing to this difference. Under competitive equilibrium, households fail to internalize the externalities of their borrowing decision on debt price and terms-of-trade of the current period and the next period. Households underestimate the marginal cost of the extra borrowing due to the marginal decreases in debt price and expected terms-of-trade of the next period. Meanwhile, households tend to underestimate the marginal benefit of the extra borrowing because they neglect the boosting effect of their borrowing on the current terms-of-trade. Besides, the foreign lender adjusts the debt price based on their expectation on the defaulting probability, which shifts the supply of the external debt. Nonetheless, it is a quantitative question whether households overborrow or underborrow.

The other result shown in Figure 3.1 is that the unconstraint social planner is not as leveraged as the constraint social planner in the equilibrium. The constraint social planner is more credible than the unconstraint social planner given the same state, (y^H, D). From equation (14), the constraint social planner's decision on the household consumption path is only through borrowing and defaulting. Once the external debt defaults, the constraint social planner has no access to affect the household's consumption choice, for the unconstraint social planner can still decide on the intratemporal consumption of households. Due to the loss of planning capacity in the bad financial standing, the difference of welfare is more significant for the constraint social planner between being in the goods financial standing and bad financial standing. The rational foreign lender anticipated the government's credibility in the equilibrium. Thus the price of the external debt is higher in the constraint social planner's problem. Accordingly, the equilibrium borrowing of the constraint social planner is higher than the unconstraint social planner, as seen in Figure 3.2 and 3.3.

3.4.3 Social Welfare

Analyzing the external debt distribution in equilibrium has demonstrated the difference among the three economies in terms of borrowing and default decisions. The households borrow more in the constraint social planner's problem than in the other two economies. From this perspective, the constraint social planner's economy seems superior to the others because it is efficient for impatient households to tilt the consumption by borrowing from the

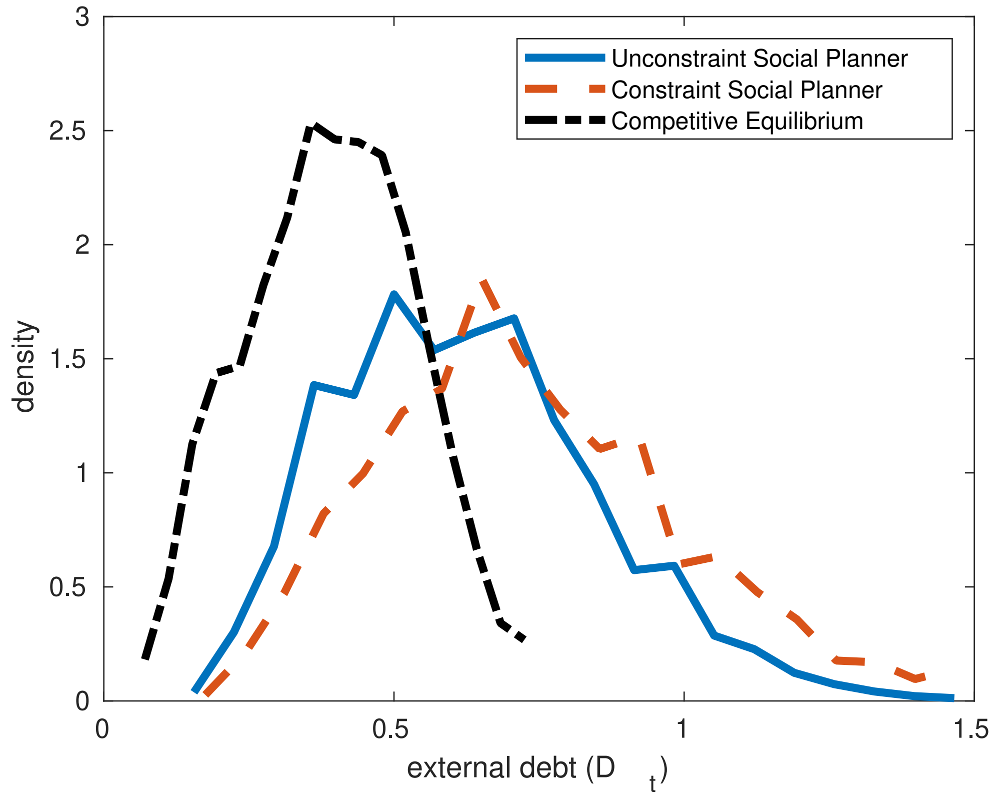

Figure 3.1: Equilibrium distribution of external debt.

patient foreign lender against the future income. However, the households in the unconstraint social planner's problem are better off in the bad financial standing due to the optimal management of the terms-of-trade. In this section, the comparison focuses on the welfare of households along the infinite horizon. The welfare of households living in each model economy is calculated to rank the corresponding economy. The welfare is measured by the lifetime utility from consuming domestic and foreign tradable goods. The unconstraint social planner's problem is chosen to evaluate the relative performance of other economies.

Given the state of the economy, (y^H, D), the welfare cost of living in the competitive equilibrium economy or constraint social planner's economy is defined as the proportional increase of aggregate consumption of households living in each economy such that they are indifferent with living in the unconstraint social planner's economy. In more accurate tone, the welfare cost of living in economy i, ($i = 1$ if competitive equilibrium, and $i = 2$ if the constraint social planner problem) is define as

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t^{\text{basis}})^{1-\sigma}}{1-\sigma} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{((1+\lambda(y^H, D))c_t^i)^{1-\sigma}}{1-\sigma},$$

where c_t^{basis} is the aggregate consumption in the unconstraint social planner's problem. By the definition of the value function, the solution of the welfare cost is

$$\lambda(y^H, D) = \left[\frac{v^{\text{basis}}(y^H, D)(1-\sigma)(1-\beta)+1}{v^i(y^H, D)(1-\sigma)(1-\beta)+1}\right]^{\frac{1}{1-\sigma}} - 1,$$

where $v^{\text{basis}}(y^H, D) = \max(V_u^g(y^H, D), V_u^b(y^H))$ is the welfare of households in the unconstraint social planner's problem, and the welfare in the competitive equilibrium is defined as $v^1(y^H, D) = \max(v^g(y^H, D), v^b(y^H))$ and in the constraint social planner's problem $v^2(y^H, D) = \max(V_c^g(y^H, D), V_c^b(y^H))$. The welfare cost $\lambda(y^H, D)$ is conditional on the state (y^H, D). To evaluate the overall welfare loss or gain, the unconditional moments of the $\lambda(y^H, D)$ is calculated based on the stationary distribution of state (y^H, D) in the unconstraint social planner's problem.

Model	Welfare cost
Competitive Equilibrium	6.25%
Constraint Social Planner	-5.50%

Table 3.2: Unconditional welfare cost relative to the Unconstraint Social Planner

The unconditional expectation of $\lambda(y^H, D)$ measures the welfare loss of switching from the unconstraint social planer's economy to other economies in aggregate consumption. Table 3.2 shows that the households enjoy higher welfare if the economy switches from the one defined in the unconstraint social planner's problem to the constraint social planner's problem. This result is in line with the equilibrium distribution of the external debt. The constraint social planner understands it would lose all the planning capacity if it chose to default, and it is more credible than the unconstraint social planner. The constraint social planner can sustain a higher leverage ratio in the equilibrium.

3.4.4 Default Episode

With knowledge of the overall performance, this paper looks into the behavior of each economy around the default crisis. Figure 3.3 and Figure 3.4 show the dynamics of major macroeconomic variables of the social planner's problems around the sovereign default crisis. The sovereign default crises are identified from 10^6 periods simulation, and a time window tracks the dynamics of variables with 12 quarters before the crisis (at period 0) and 12 quarters after. In each figure, it is shown the median value of these variables in each period. In both figures or for both social planner's problems, the sovereign default crisis is triggered by a series of adverse endowment shocks. The domestic tradable endowment is in severe contraction by more than 10 percent for the social planner's problems. During the economic contraction, the consumption of domestic and foreign tradable goods drops with a similar magnitude with the endowment. However, the foreign tradable consumption cut is more than the domestic one. Thus, the relative price of domestic tradable goods decreases before the crisis. The annualized interest rate premium of borrowing external debt is soaring from around 3 percent to over 5 percent. The planner is also deleveraging in the process. The level of the external decreases until the planner chooses to default at time 0.

After the default crisis, the economy starts to recover due to the mean-reverting property of the endowment process. Nevertheless, a discontinuous output loss exists during the recovery, and the loss is persistent. Correspondingly, the relative price increases discontinuously because the output loss of the sovereign default crisis reduces the supply of domestic tradable goods. The price drops gradually with the increase of the endowment.

Comparing the dynamics of the unconstraint and constraint social planner's problem, the basic pattern is almost identical. The pattern is consistent with evidence and facts in the sovereign default literature. However, it is also found that constraint social planner is more leveraged before the default crisis than the other. Accordingly, consumption of foreign tradable goods is higher, and so does the relative price of domestic tradable goods. The default episode demonstrates that the constraint social planner defaults with a relatively higher leverage rate than the unconstraint social planner when they both experience large economic contraction.

3.4.5 Optimal Tax Rate

The final subsection of the quantitative analysis investigates the dynamics of the tax on external borrowing, which replicate the allocations of social planner's problems. Table 3.3 summarises the key moments of the tax on borrowing. It is shown that the tax rate is countercyclical with the endowment process in both cases, or the tightness of the capital control is countercyclical. The government should encourage borrowing in the economic boom and tighten up in the economic recession. However, the average tax rate on external borrowing is positive, and the average tax rate is lower. From Figure 3.4, given the same level of endowment, the optimal tax rate increases as external debt increases, but the tax rate soars up even faster if the tariff is available. The cyclicality is coherent with the finding before that the unconstraint social planner deleverage faster than the constraint social planner. Such a pattern holds for other income levels. Figure 3.5 shows the dynamics of the tax rate around the default episode.

	Constraint Social Planner	Unconstraint Social Planner
Mean	7.53%	8.96%
Standard deviation	11.66%	12.41%
Corr(y^H, τ^d)	-0.33	-0.42

Table 3.3: Summary statistics of the capital control tax

3.5 Conclusion

This paper studies the terms-of-trade management motive of an indebted small open economy that lacks commitment to debt repayment and future policies and how the objective of manipulating terms-of-trade interacts with the optimal external borrowing and sovereign defaulting decisions. This paper extends the space of consumption goods in the workhorse model of the sovereign default literature. It generates an endogenous terms-of-trade by assuming there exists the demand for domestic tradable goods by foreign households. In this environment, the individual external borrowing results in the externality on the debt price and terms-of-trade, which justifies the government's intervention in household decisions.

This paper first shows that households in the competitive equilibrium are under borrowing relative to the social planner because the individual households are incapable of managing neither debt price nor terms-of-trade. For the social planner, this paper finds it can achieve a more efficient allocation by accepting the competitive outcome of the domestic tradable goods market instead of exerting its market power. The regulation or limit of planning capacity ensures that maintaining access to the international financial market is valuable to the social planner because it loses the planning capacity in bad financial standing. This finding maps to the policy implication. It is better for social welfare that the government commit to zero tariffs and only use capital control tax.

Next, this paper can be extended to include the long-maturity debt, which adds a more realistic feature to countries' external borrowing. Under the assumption of the one-period debt, the debt price schedule drops from the risk free price to zero too fast as the leverage ratio increases. By assuming the long-maturity debt, one may explore the interaction between terms-of-trade management and debt price. The other direction is to consider the optimal tariff only in a dynamic view. The computation of the exercise is challenging because it can not be mapped to a social planner's problem.

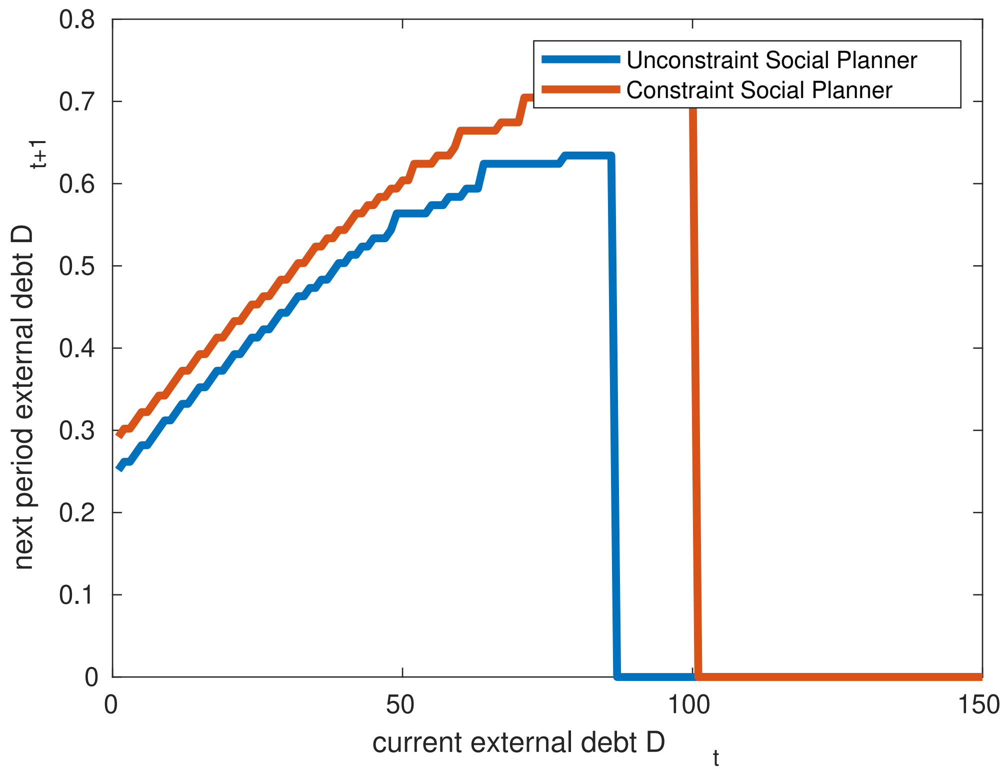

Figure 3.2: External Debt Borrowing in Next Period

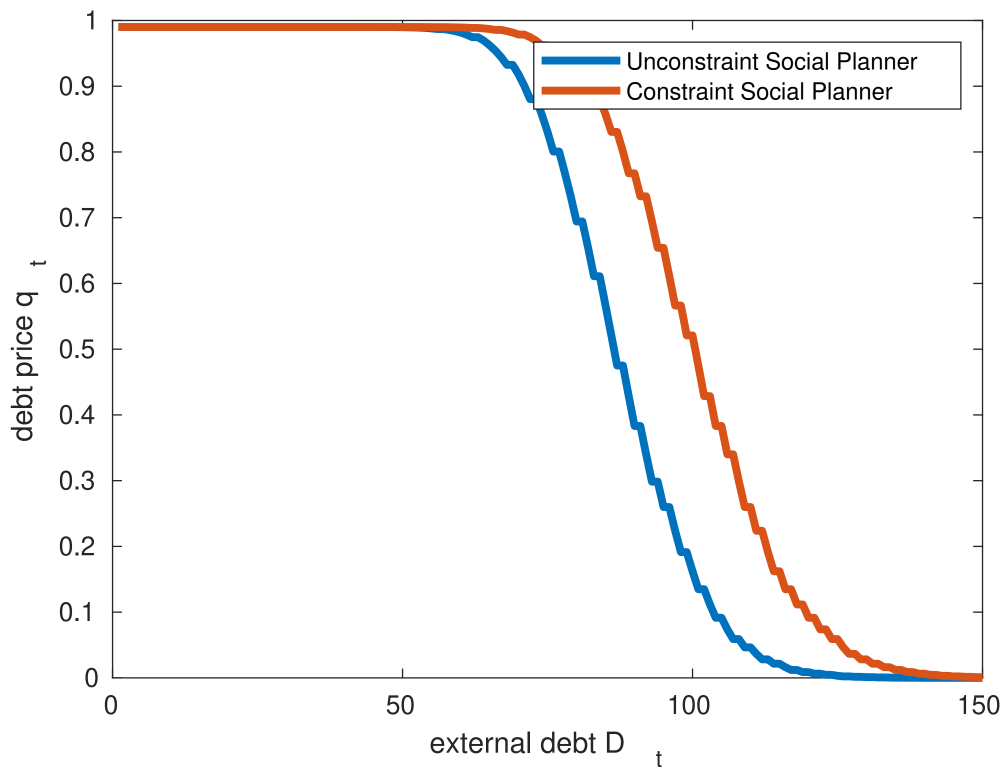

Figure 3.3: Debt Price Schedule

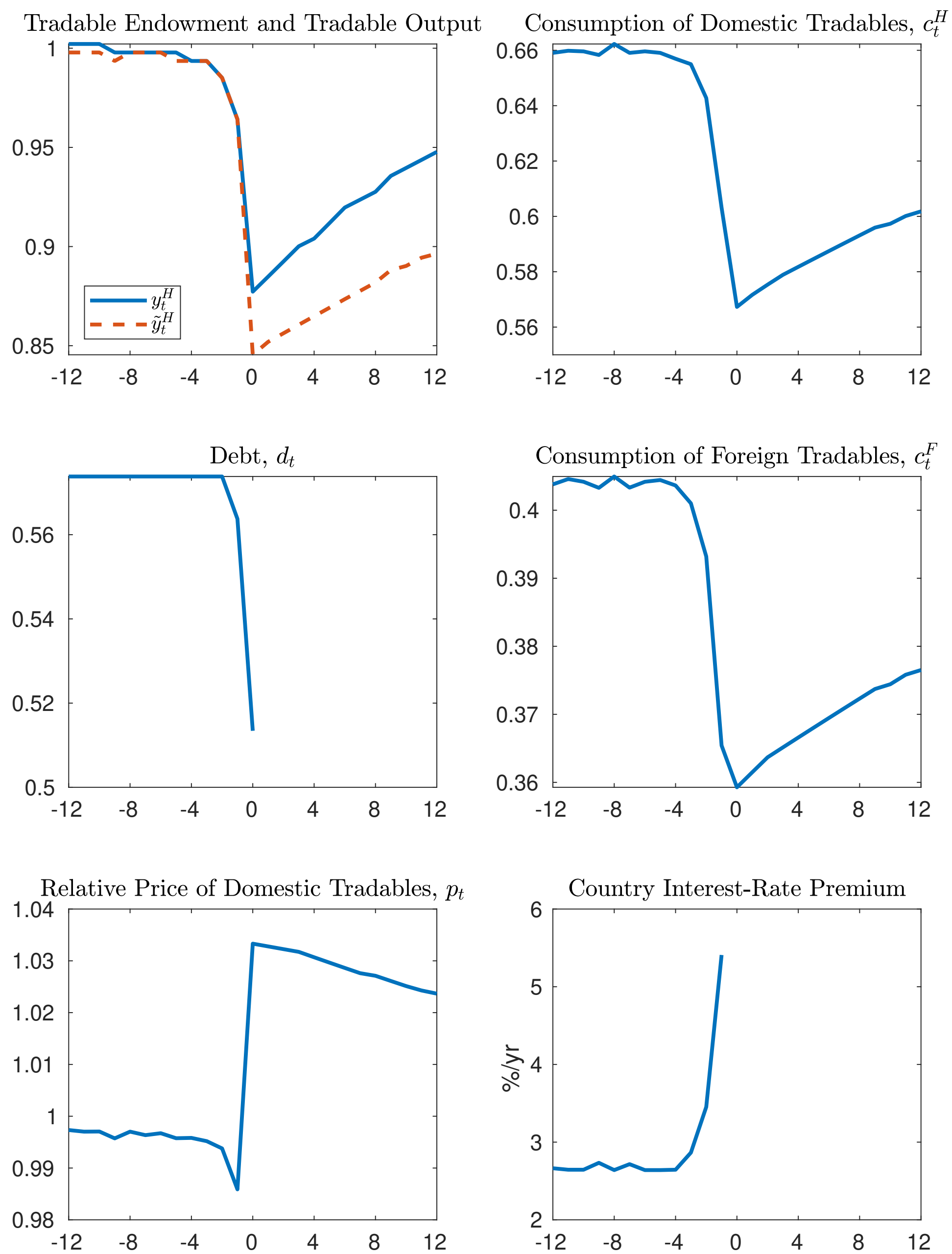

Figure 3.4: Default Episode of Unconstraint Social Planner.

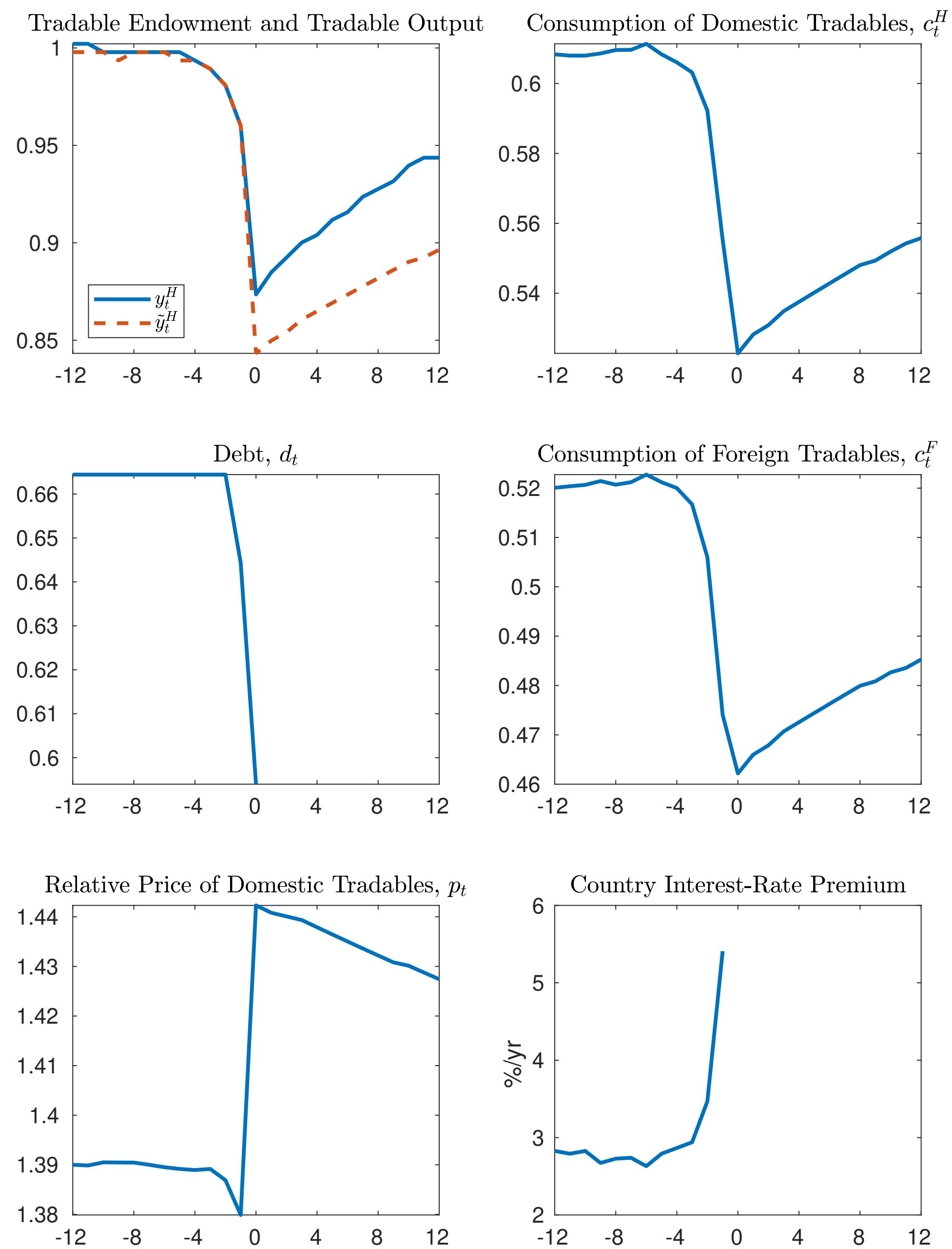

Figure 3.5: Default Episode of Constraint Social Planner.

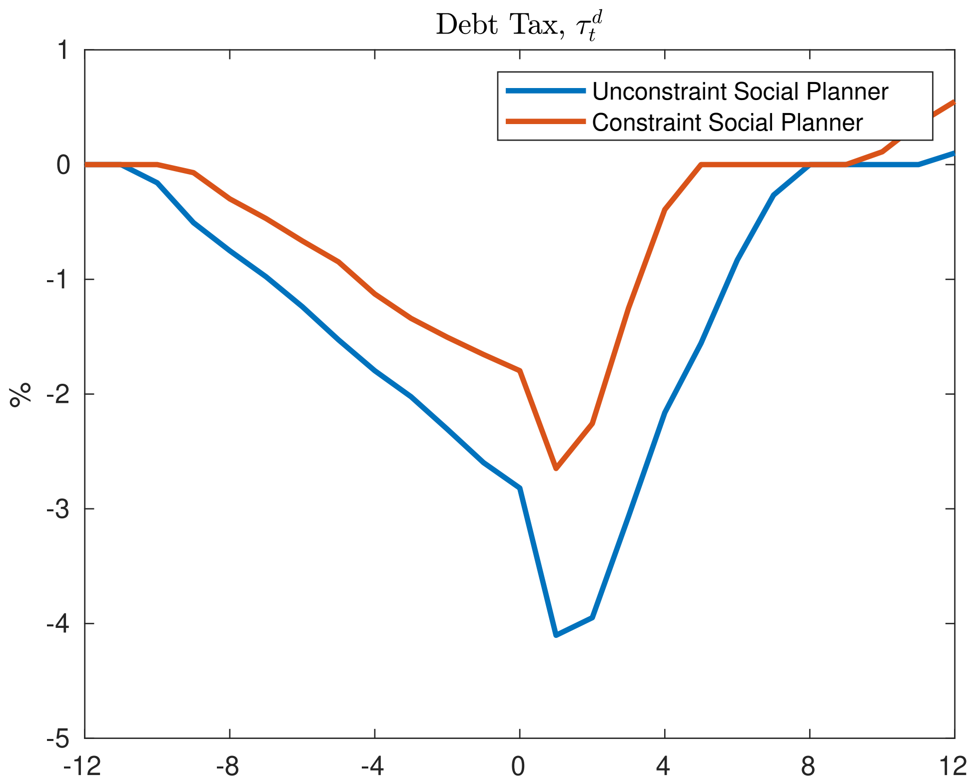

Figure 3.6: Debt Tax Rate in the Economic Boom.

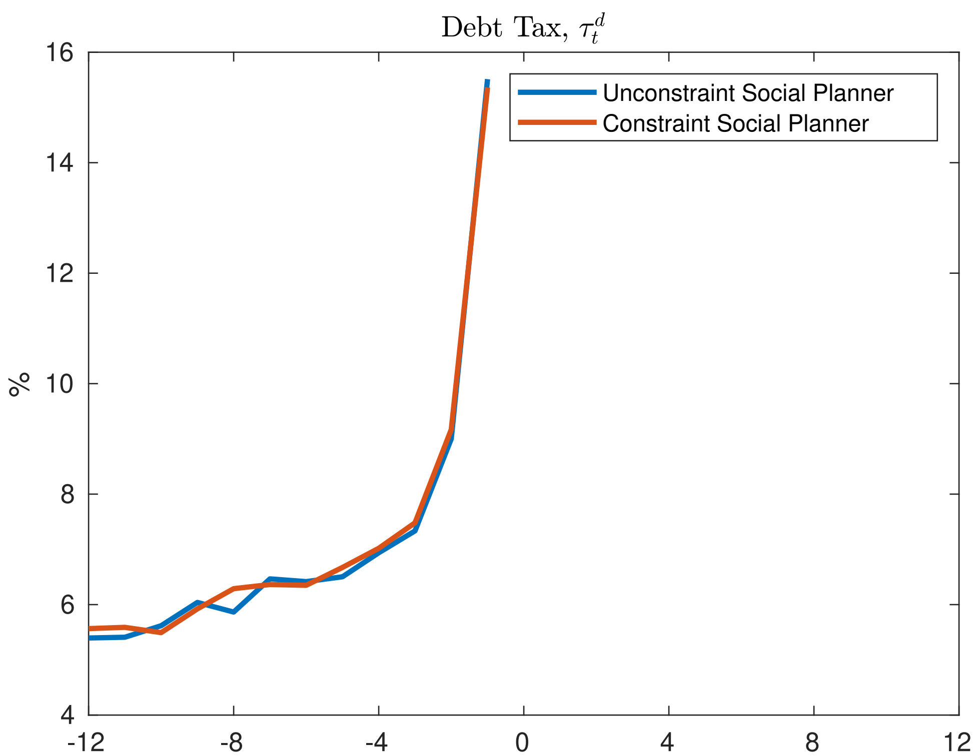

Figure 3.7: Debt Tax Rate around the Default Episode.

Appendix A

Chapter 1

A.0.1 Proofs of Propositions and Lemmas

Proof of Lemma 1: Take the derivative of the constraint planner's problem on $B_1(s)$ and $L_1(s)$, and the conditions follow.

Proof of Proposition 1: Take households as example: 1. the constraint efficient allocation satisfies households' budget constraints in the nominal economy.

$$
\begin{aligned}
&\mathbb{E}_{s^t}\left[\left(q\left(s^{t+1}|s^t\right)+\tau\left(s^{t+1}|s^t\right)\right)\frac{1}{\epsilon\left(s^{t+1}|s^t\right)}\right]\frac{D^l\left(s^t\right)}{E\left(s^t\right)}\\
=&\mathbb{E}_{s^t}\left[\left(q\left(s^{t+1}|s^t\right)+\tau\left(s^{t+1}|s^t\right)\right)\frac{E\left(s^t\right)}{E\left(s^{t+1}\right)}\frac{D^l\left(s^t\right)}{E\left(s^t\right)}\right]\\
=&\mathbb{E}_{s^t}\left[\left(q\left(s^{t+1}|s^t\right)+\tau\left(s^{t+1}|s^t\right)\right)A\left(s_{t+1}=s_i|s^t\right)\right]
\end{aligned}
$$

2. Households' optimal conditions in the real economy are

$$
\begin{aligned}
&\left(q\left(s^{t+1}|s^t\right)+\tau\left(s^{t+1}|s^t\right)\right)\lambda^h\left(s^t\right)=\beta\lambda^h\left(s^{t+1}|s^t\right)\\
\Longrightarrow&\left(q\left(s^{t+1}|s^t\right)+\tau\left(s^{t+1}|s^t\right)\right)\lambda^h\left(s^t\right)\frac{E\left(s^t\right)}{E\left(s^{t+1}|s^t\right)}=\beta\lambda^h\left(s^{t+1}|s^t\right)\frac{E\left(s^t\right)}{E\left(s^{t+1}|s^t\right)}\\
\Longrightarrow&\mathbb{E}_{s^t}\left(q\left(s^{t+1}|s^t\right)+\tau\left(s^{t+1}|s^t\right)\right)\lambda^h\left(s^t\right)\frac{E\left(s^t\right)}{E\left(s^{t+1}|s^t\right)}=\mathbb{E}_{s^t}\beta\lambda^h\left(s^{t+1}|s^t\right)\frac{E\left(s^t\right)}{E\left(s^{t+1}|s^t\right)}\\
\Longrightarrow&\left(q^l\left(s^t\right)+\tau^l\left(s^t\right)\right)\lambda^h\left(s^t\right)=\mathbb{E}_{s^t}\beta\lambda^h\left(s^{t+1}|s^t\right)\frac{E\left(s^t\right)}{E\left(s^{t+1}|s^t\right)}
\end{aligned}
$$

From the first equation in the proposition, we have for $\forall s_i, s_j$

$$
\begin{aligned}
B\left(s_{t+1}=s_i|s^t\right)E\left(s_{t+1}=s_i|s^t\right)=B\left(s_{t+1}=s_j|s^t\right)E\left(s_{t+1}=s_j|s^t\right),\\
\frac{E\left(s_{t+1}=s_i|s^t\right)}{E\left(s_{t+1}=s_j|s^t\right)}=\frac{B\left(s_{t+1}=s_j|s^t\right)}{B\left(s_{t+1}=s_i|s^t\right)},\\
\frac{\epsilon\left(s_{t+1}=s_i|s^t\right)}{\epsilon\left(s_{t+1}=s_j|s^t\right)}=\frac{B\left(s_{t+1}=s_j|s^t\right)}{B\left(s_{t+1}=s_i|s^t\right)}.
\end{aligned}
$$

Define depreciation rate $\epsilon\left(s^{t+1}|s^t\right)$ as

$$
\epsilon\left(s^{t+1}|s^t\right)=\frac{E\left(s^{t+1}|s^t\right)}{E\left(s^t\right)}
$$

The state-contingent depreciation rates $\{\epsilon\left(s_{t+1}|s^t\right)\}$ would replicate the constraint efficient allocation if

$$
\epsilon\left(s_{t+1}|s^t\right)=\frac{B\left(s_{t+1}=\underline{s}|s^t\right)}{B\left(s_{t+1}|s^t\right)}\epsilon\left(s_{t+1}=\underline{s}|s^t\right)
$$

From downward wage rigidity, we have the condition of achieving full employment

$$
\begin{aligned}
&\epsilon\left(s^{t+1}|s^t\right)\geq\gamma\frac{w\left(s^t\right)}{w\left(s^{t+1}\right)}\\
\Longrightarrow&\epsilon\left(s^{t+1}|s^t\right)\geq\gamma\left(\frac{z\left(s^t\right)k\left(s^{t-1}\right)}{z\left(s^{t+1}\right)k\left(s^t\right)}\right)^{\alpha}
\end{aligned}
$$

State space S is discrete. Thus, $\exists\tilde{\epsilon}$, such that $\tilde{\epsilon}\geq\gamma\frac{w\left(s^t\right)}{w\left(s^{t+1}\right)}$ in any state.

Let $\hat{\epsilon} = \max_{s_{t+1}} \left\{ \tilde{\epsilon} \frac{B\left(s_{t+1}|s^t\right)}{B\left(s_{t+1}=\underline{s}|s^t\right)} \right\}$. We can set $\epsilon\left(s_{t+1}=\underline{s}|s^t\right) = \hat{\epsilon}$, then the following inequality holds uniformly across states

$$\epsilon\left(s_{t+1}|s^t\right) = \frac{B\left(s_{t+1}=\underline{s}|s^t\right)}{B\left(s_{t+1}|s^t\right)} \epsilon\left(s_{t+1}=\underline{s}|s^t\right) \geq \frac{B\left(s_{t+1}=\underline{s}|s^t\right)}{B\left(s_{t+1}|s^t\right)} \frac{B\left(s_{t+1}|s^t\right)}{B\left(s_{t+1}=\underline{s}|s^t\right)} \tilde{\epsilon} = \tilde{\epsilon}.$$

Proof of Proposition 2: The proof is similar to the proof of Proposition 1. Repeat the same construction process with D^f.

Proof of Lemma 2: The impact of labor supply on expert's net worth

$$\frac{dn_1}{dl_1}\left(\frac{a\alpha}{1-a}\left(\frac{c_h^N}{c_h^T}\right)^{\frac{1}{\xi}} + \left((1-\beta)+\frac{\beta}{1-\beta\theta(1-\delta)}\right)\frac{1}{\xi}\frac{c_h^N}{c_h^T}\right) = \frac{\xi-1}{\xi}(1-\alpha)k_0^{\alpha}l_1^{-\alpha}$$

Given the parameters $(\alpha, \theta, \beta, \delta)$ all less than 1, if $\xi < 1$, we have $\frac{dn_1}{dl_1} < 0$.

Proof of Lemma 3: Plug the parameters of the simple case into conditions in Proposition 1. Combine the two conditions, and the conclusion follows.

Appendix B

Chapter 2

B.0.1 Proofs of Propositions and Lemmas

In this section, we prove the propositions in the main text.

Proof of Proposition 1: The strategy of the proof is to show that the housing subsidy policies is consistent with the planner's allocation. Plug subsidy policies into first order conditions,

$$\begin{aligned}
c_t^y &= \frac{w_t}{1+\alpha+\beta-\alpha\varsigma_t} \\
&= \lambda^{-1}(\phi R)^t \\
c_{t+1}^o &= \frac{\beta R w_t}{1+\alpha+\beta-\alpha\varsigma_t} \\
&= \beta R \lambda^{-1}(\phi R)^t \\
\left(p_t - \frac{p_{t+1}}{R}\right) h_t &= \alpha\lambda^{-1}(\phi R)^t
\end{aligned}$$

which is the same as the first-best allocation.

Proof of Corollary 2: Plugging $h_t = \overline{h}$ into the first order condition and reordering, we obtain the law of motion for equilibrium housing prices

$$p_t = \frac{w_t}{\overline{h}\left(1+\alpha+\beta-\alpha\varsigma_t\right)} + \frac{p_{t+1}}{R} \tag{B.1}$$

Plugging future subsidy $(B.1)$, we have

$$p_t = \frac{\alpha(1+g)w_t}{\overline{h}\lambda w_1} + \frac{p_{t+1}}{R} \text{ for } t \geq 1 \tag{B.2}$$

Forward iterating $(B.2)$, we get

$$\widehat{p}_t = \frac{\alpha}{\overline{h}\lambda} \frac{R}{R-(1+g)}, \text{ for } t \geq 1. \tag{B.3}$$

Hence, p_t grows at a constant rate $(1+g)$ for $t \geq 1$.

Proof of Corollary 3: Equating young households' consumption for the planner's solution and the one for the decentralized economy, we have

$$\lambda^{-1} = \frac{\widehat{w}_0 - \left(\widehat{p}_0 - \widehat{p}_1 (1+g)/R\right)\overline{h}}{1+\beta}$$

Hence,

$$\widehat{p}_0 = \left[\widehat{w}_0 - \lambda^{-1}(1+\beta)\right]/\overline{h} + \widehat{p}_1 (1+g)/R$$

Plugging $(B.3)$ into the above equation, we obtain the optimal price.

B.0.2 Algorithm

We have two algorithms here. One algorithm is to solve the prereform steady state. The other is to solve jointly the steady state after 2013 and the transition path from some initial state of the economy to the steady state after 2013. For example, in the benchmark, we need to use transition path algorithm twice. Firstly, we solve the transition path from the prereform to the steady state after 2013. Secondly, we use the economy in year 1998 as the initial state and solve for the transition path again.

Prereform steady state:

To solve for the prereform steady state, we assume households taking the assigned housing service as given, and government plans for the life cycle consumption profile across permanent efficiency shock groups.

Given government policy, τ, b, and interest rate r and wage rate w in prereform steady state.

1. Parameterize the model, and calculate the density of retired workers in the population, μ_t, $t = J_{w+1}, ..., J$.

2. Given the government expenditure, $\bar{g}$, guess the prereform rental rate R_c;

3. Given the policy function (analytical) of households,

   (a) guess the initial bequest.

   (b) simulate the optimal path for consumption and saving for the new born generation by forward induction given the initial bequest.

   (c) aggregate household's decision, and calculate the bequest leftover on the path.

   (d) update the guess of the initial bequest until it converges. (Guass-Seidel method)

4. Aggregate government's tax revenue, renting revenue, and pension expenditure. Check whether government's intertemporal budget is balanced, and update the guess of the rental rate R_c.

5. Check whether R_c match the calibration target, if not, update $\bar{g}$. Derive the allocation of $\overline{H}$.

Transition path:

We need to find the equilibrium path of housing price and the tax rate in the final steady state. Assume we know the state of the economy at $t = 1$, and the economy reaches the final steady state after some periods T. (T is larger than the three times of the maximum lifespan.)

To solve the transition path, we have the following steps:

Given government policy, $\{\tau_t, b_t\}_{t=1}^{\infty}$, discount housing policy, land supply, $\{H_t\}_{t=1}^{\infty}$, and interest rate $\{r_t\}_{t=1}^{\infty}$ and wage rate $\{w_t\}_{t=1}^{\infty}$ on the path, and the initial distribution of household on the state space (initial state).

1. Choose the number of transition periods T.

2. De-trend the economy by the time T variables.

3. Provide an initial guess for tax rate in the steady state, τ.

4. Given all policy variables, solve for the final steady state housing price that clear the housing market by bisection method.

5. Provide an initial guess for housing price on the path, $\{p_t\}_{t=0}^{T}$, and solve household's problem backwards:

   At period t, compute the value functions and policy functions for the new born at t, which has a perfect foresight.

6. Compute the transition path: Compute the optimal path for consumption, housing, and saving by forward induction given the initial state in period $t = 1$. In initial state, households receive assignment of public housing from the government, $\bar{H}$. The bequests for period t newborn are collected from the household passing away at period t.

7. Aggregate household's net housing demand each period. Check if housing market in each period is clear. If not, update the guess of $\{p_t\}_{t=1}^{T}$, and go to step 5.

8. Aggregate government tax revenue, housing sale revenue, pension expenditure on the path. Combined with government's deficit/surplus in the steady state, check whether government's intertemporal budget is balanced. If not, update the guess of τ, and go to step 4.

9. Check whether p_T is close enough with the final steady state housing market price. If not, increase T, and go to step 2.

Long run equilibrium:

The price adjustment step. Because it is the long run equilibrium

$$I_t = \delta H$$

$$p_0 \Longrightarrow H\left(p_0\right) + \widetilde{H}\left(p_0\right) \Longrightarrow I_t\left(p_0\right) \Longrightarrow p_1 = 0.2 * \frac{1}{\alpha}\left(\frac{I_t\left(p_0\right)}{L}\right)^{\frac{1-\alpha}{\alpha}} + 0.8 * p_0$$

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