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The Role of Dynamic Incentives in Customer Engagement: Applications in Gaming and Gamification

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An abstract of
A dissertation submitted to the Faculty of the
James T. Laney School of Graduate Studies of Emory University
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy
in Business
2019

Abstract

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Dynamic incentive schemes are rewards structures widely used to motivate and engage customers in marketing practice. Compared to the traditional loyalty programs, dynamic incentives in the digital era are more flexible and accessible with more features at firms' disposal (outcome uncertainty, social status etc.). This evolvement of dynamic incentives raises academically and managerially important questions.

My first essay studies how customer decisions are influenced by their past investments in a product and by expectations of future rewards. Investments in learning product related knowledge, purchases of add-ons or the accumulation of loyalty points can all create switching costs that result in behavioral loyalty. Similarly, dynamic incentive schemes like loyalty programs can increase loyalty measures because consumers base current decisions on expectations of future benefits. With a data set from the video game industry, I find that the effectiveness of rewards varies based on reward types and the level of customer investments. Rewards that help players explore the game become less attractive after commitment while rewards that help players progress are effective with or without commitment. Furthermore, I find that players' decisions to invest in the game depend on the breadth of game content experienced. These findings have implications for designing rewards systems in product categories involving customer learning.

My second essay focuses on the impact of outcome uncertainty in gamification: the usage of gaming principles and elements in non-gaming contexts. While sharing some common elements with loyalty programs including dynamic incentives and status, games and gamification are unique in their outcome uncertainty, rendering consumers less confident of whether they can achieve goals. Using data from a mobile app, I find that points pressure effects exist in gamification settings: as players approach the next prize level, their motivation to participate increases. I also find that for some customers multiple losses motivate continued play, consistent with Gambler's fallacy. Moreover, I find status comparisons can only motivate behavior to a point: positive status comparisons can fuel continued effort until players achieve the highest status, after which status decreases future effort. My results have important implications for firms who apply game concepts to nongaming applications.

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Acknowledgments

PhD has been an incredible journey. I cannot say I never regretted starting the journey pursuing the highest and the most difficult degree, but the view near the end is stunning and the valuable memories along the road would be something I cherish for the rest of my life. I could not have reached this point without the wonderful people around me. Thanks to them, I was able to keep my head up and fight my way through.

I am forever indebted to my advisor, Dr. Mike Lewis. Becoming his student is the best thing that happened to me in my entire PhD study. I deeply appreciate all of his guidance and support when I was unconfident, confused and frustrated. Thanks for having faith in me when the doctoral degree seems a mission impossible. With his help and encouragement, I am able to explore my research interests in the video game industry I deeply love. I could not thank him enough for being an amazing mentor not only in research but also in life.

I would also like to extend my deepest gratitude to my committee members, Dr. Yi Zhao, Dr. Michelle Andrews and Dr. Panagiotis Adamopoulos. In my second year I decided to take a modeling seminar offered by Dr. Zhao and this was one of the best decisions I have made in the program. His knowledge and insights on empirical analysis, especially in Bayesian methods, helped me immensely in my research. Thanks for being so patient and thorough in answering my numerous questions about empirical analysis and giving me great advice and suggestions. Dr. Michelle Andrews has been extremely supportive and responsive for which I am immensely grateful. Her feedbacks and suggestions helped me perfect my work which made the intense job market process much easier than it could have been. Thanks for always being there and willing to help. I would like to thank Dr. Panagiotis Adamopoulos for being so encouraging and responsive. I deeply appreciate all the profound insights and helpful suggestions for my work.

I am also grateful to all the marketing faculties at Emory for their guidance and help. I benefited immensely from the seminars hosted by Dr. Doug Bowman, Dr. Sandy Jap, Dr. Ryan Hamilton and Dr. David Schweidel. Their seminars build a solid foundation for my research and always will be an invaluable treasure for my road ahead. I would also like to thank the other marketing faculties for their helpful feedbacks and constant encouragement: Dr. Jag Sheth, Dr. Inyoung Chae, Dr. TI Kim, Dr. Morgan Ward, Dr. Daniel McCarthy, Dr. Edgar Leonard, Dr. Omar Rodríguez-Vilá, Dr. Manish Tripathi. I would also like to thank the Associate Deans of the PhD program at Goizueta, Dr. Anand Swaminathan and Dr. Kathryn Kadous for creating a caring and supportive environment in the PhD program which all the PhD students benefit deeply.

Thanks also go to the wonderful staff members at Goizueta who have made my stressful PhD life a lot easier. Special thanks to Ms. Allison Gilmore, our beloved program administrator who has the magic to make all administrative tasks so smooth and takes great care of all the PhD students. I would also like to thank Mr. Ron Harris for his great technical support and patience in helping students. Many thanks to Ms. Sonya Owens, our beloved program assistant, for all the coordination efforts.

Many thanks to all my friends in the program. Thanks for making my journey a great one full of invaluable memories: Yanwen Wang, Nuo Xu, Beth Fossen, Anthony Koschmann, Ning Zhong, Suhyeon Kim, Dayoung Kim, Emily Ko, Hulya Karaman, Buffy Mosley, Karen Wallach, Wooyong Jo, Jihyeon Ha, Weishi Jia, Yuepin Zhou, Xin Zheng, Pranay Jinna, Zhenping Wang, Ai He, Cong Wang, Chandra Sekhar Mangipudi, Yanbin Wu, Rohit Allena and many more.

At last I would like to extend my deepest gratitude to my parents Fengrui Han and Yuemei Li. Without their encouragement and unconditional support, I never would have had the courage to embark on this amazing journey. They are my heroes.

Table of Contents

List of Tables

List of Figures

List of Appendices

Chapter 1

Introduction

Dynamic incentives are rewards structures that have been used widely in loyalty programs (punch cards, airline mileage programs and chain store membership programs) to engage and motivate customers in marketing practice. Loyalty program dynamic incentives are comprised of goals and tiers associated with rewards ranging from coupons, gifts, privileges and exclusive access to special promotions. The goals and tiers in loyalty programs are usually based on spending and customers need to meet prespecified spending thresholds to obtain rewards and tier status. Through loyalty programs with dynamic incentives firms strive to build long and mutually rewarding relationships with their customers.

Technology breakthroughs have spawned the flourishing online and mobile market. The virtual channels provide increased reach in breadth and depth for firms to get involved in customers' purchase and consumption of their products. Firms specializing in digital products interact with customers almost exclusively online and they are able to observe and participate in customers' complete consumption process. Firms which mainly focus on tangible products also have started to utilize the online and mobile channels as supplementing or substituting platforms for sales, customer feedback and loyalty program management. Compared to the dynamic incentive schemes in the traditional loyalty programs, firms that are utilizing the online and mobile channels now have more elements at their disposal when it comes to the design of dynamic incentives schemes. This development presents both great opportunities and challenges to modern firms. Loyalty programs research is silent on how customers may react to new elements that are not seen in traditional loyalty programs (for example, the role of luck or chance, social comparison facilitated by competition, etc.) and how various features may interact with each other and jointly influence customers' decisions.

In my first essay, which is titled "The Interplay between Consumer Investments and Response to Dynamic Incentives: An Empirical Study of Video Game Player Behaviors", I study how future rewards from dynamic incentives and past customer efforts can jointly influence customer behavior. Digital products like software and video games have started to employ a freemium business model where the base product is free with add-on features that come with charges. This freemium or free-to-play strategy aims at reducing customer risk in purchases by letting consumers try products for free such that they can make more informed decisions afterwards. The dynamic incentives in digital products have the flexibility of using consumption or usage for points accumulation thus making the rewards system accessible to users even before purchase. This new development poses interesting questions on how customer investment may interact with dynamic incentives. Also, almost all customer actions are exclusively online where firms have access to the whole process, a privilege that is not shared by most of the tangible products. These unique features make digital products like video games a great context to study how potential future rewards provided by dynamic incentives and past customers' investment in the product in various prospects (time, money and relationship) may jointly influence customers' decisions. With a data set from the video game industry I develop a joint state-space model to study players' play and purchases under the interactions of dynamic incentives and customer investment with a latent "preference" structure that captures the correlation across behaviors.

In my second essay, which is titled "Gamification: The Interplay between Dynamic Incentives and Outcome Uncertainty", I take a look at how customers react to outcome uncertainty using a data set from a mobile application that utilizes gamification. Gamification refers to the usage of gaming principles and elements in non-game contexts (Blohm and Leimeister 2013). Dynamic incentive scheme is a common feature that is shared by games and loyalty programs. However, uncertainty of outcome can be often found in dynamic incentives in games while rewards in loyalty programs rarely involves chance in their designs. The introduction of randomness in the design of dynamic incentive schemes raises interesting questions. In traditional loyalty programs, the customers have more control over their progress in the program. The rewards are deterministic such that customer will know for sure they will obtain the prizes once certain spending threshold is met. However, in gamification procedures with uncertain outcomes, customers are not sure about the payout after certain amount of effort is exerted. This lack of control may demotivate customers from pursuing the rewards but the feeling of “winning” or being “lucky” will provide psychological benefits that do not exist in traditional loyalty programs. I develop a nonhomogeneous Hidden Markov Model to capture the dynamics of the relationship between customer and firm in a gamification procedure featuring outcome uncertainty.

Chapter 2

The Interplay between Consumer Investments and Response to Dynamic Incentives: An Empirical Study of Video Game Player Behaviors

2.1 Introduction

Consumer behavior is often influenced by both forward-looking factors and consumer past actions. For example, loyalty programs provide dynamic incentives that motivate consumers to purchase during the current period based on the promise of future rewards (Taylor and Neslin 2005; Lewis 2004). Consumer behavior is also often influenced by past consumer actions or investments (Burnham, Frels, and Mahajan 2003) such as investments in platforms or base products. For example, investment in platforms, such as operating systems on smart phones (IOS vs. Android), video game consoles (Nintendo vs. Play Station vs. Xbox), dictates much of future consumption in category. Once a consumer commits to or invests in a product, switching to a competitive option involves forgoing the value of the previous investment. Loyalty programs may also create a form of switching costs whereby accumulated points represent an investment in the program (Klemperer 1995). The key insight for these types of products and even promotions such as loyalty programs is that after a consumer commits to an option, the time and money that is then invested in that product creates hurdles for switching.

In this research, I focus on the interplay between dynamic incentives and consumer commitments using data on video game playing and in-game purchasing provided by a leading video game producer. The gaming sector is a significant and rapidly growing sector that generates in excess of $140 billion in annual revenues (Kellie Ell 2018; Ingraham 2018; Wilburn 2018). The video game category is an especially interesting sector for studying consumer behavior. For instance, video games include significant elements of gamification (Hofacker et al. 2016) such as rewards for progress, leveling-up systems and opportunities for status. These games also involve significant opportunities for customer investments and commitments such as the accumulation of game specific knowledge and, in some business models, purchases of premium content.

Gamification structures (Hofacker et al. 2016) are especially prevalent in the video game category. These structures involve dynamic incentives designed to increase playing rates and customer retention. For example, these products often include opportunities to increase status, earn rewards, and unlock game features by acquiring game-play-based experience points. In some respects, these gaming elements are analogous to the frequency or cumulative buying-based rewards common in loyalty programs. The game designer uses the promise of rewards or access to new levels to motivate consumers to play incremental games.

In addition, to motivate behavior through future rewards video games often create a form of switching costs because the rewards generated through play represent an investment in the game. Players may devote considerable time to learning how to play and to gaining access to content. Significantly, the video game sector also features several innovative business models that may influence the degree to which consumers feel invested in a game. For example, many game producers have adopted variations of Free-To-Play (FTP) business models. FTP business models create revenue through voluntary purchases of in-game currency, aesthetic items or access to premium features. Purchases within an FTP game represent an explicit and voluntary investment in the game by the consumer.

In this research, I empirically study consumer behavior using data provided by a video game producer who uses an FTP model. In my application, the game producer attempts to generate revenues through access to different playable characters[1]. The game is a Multiplayer Online Battle Arena game that allows consumers to play as characters with different abilities.[2] Players may access additional characters beyond a core group of free characters through earning in-game rewards, buying access to individual characters or by purchasing an expansion package that unlocks all current and future characters. The game also includes an extensive system of rewards that include experience-based bonuses and opportunities to progress through levels. My specific research interest is in how consumer investments in the game interact with the dynamic incentives and rewards. Specifically, I am interested in how consumers' response to dynamic incentives changes based on whether the player has invested in the character expansion pack.

The scope of observable consumer information and the mechanisms that can be used to influence consumer behavior in the digital world provide opportunities for marketing researchers, but also present nontrivial challenges to empirical analyses. Multiple dimensions of consumer behavior are observable but they are rarely independent from each other. Empirical models that study one single decision without considering others can suffer from serious endogeneity issues and lead to erroneous conclusions. I use a joint state space model of players' game play behaviors and investment decisions to study the interplay between dynamic incentives and consumer investment simultaneously. Playing and purchasing decisions may be correlated since these decisions may be influenced by underlying preference levels. To account for this endogeneity issue,

[1] The game also generates revenue through sales of aesthetic items (skins).
[2] While character abilities differ, the game itself emphasizes competitive balance. Different characters provide alternative abilities and game play but do not provide a competitive advantage.

I model players' unobserved level of "preference" and allow this preference to influence play and purchase decisions.

In the next section, I will provide further background on the video game industry and review select literature on dynamic incentives and consumer investment. Then I will discuss the data and provide model free evidence that highlights several key relationships. I then describe my modeling approach and present results. The results are then more fully explored via several simulation studies that illustrate the potential impact of alternative managerial policies. The essay concludes with a discussion of managerial implications, research opportunities and limitations.

2.2 Related Literature

In this section I will first take a quick glance at the video game industry and then go through scholarly works that have been done on the subjects of dynamic incentives and consumer investment. By reviewing the literature, I hope to spot the gap in the current literature and describe the contribution of my work.

2.1.1 Background

The video game category is an increasingly prominent component of the entertainment sector. Over 2 billion people worldwide (more than 30% of the global population) and 215 million people in US (more than 60% of the US population) play video games (Ukie 2018; Nielsen 2018). While much of the growth of the video game industry is attributable to technological advancements, the industry has also been able to extend its appeal beyond its traditional core of younger men. Video games also enjoy appeal across demographic segments. Survey data suggests that games are played by players of all ages and 45% of gamers are female (Entertainment Software Association 2018).

The video game utilizes multiple business models. While early and many current video game makers still generate revenue through game purchases, mobile and online games are increasingly adopting FTP models that use different mechanisms for generating revenue. Some mobile games have adopted systems where the game is FTP but players can accelerate their progress via the purchase of in-game currency. These games are frequently labeled as PTW or Pay-To-Win (McKinney 2017; Goethe 2017). In multi-player online games, revenue models are more passive. In the MOBA category (Multiplayer Online Battle Arena) the gaming community contains a culture that emphasizes competitive balance in games (Palm and Noren 2015; Katkoff 2014). In these games, video game makers derive revenue from the sale of additional content such as incremental characters and aesthetic elements such as character "skins" (Kilkku 2015).

Multiplayer Online Battle Arena games are a type of action real-time strategy game. In a MOBA game, a player will control one character in a team competing with an opposing team comprised of other players. The usual objective in these games is to "kill" the opposing team members and destroy opponent team's main structure (Katkoff 2014). In contrast to many mobile FTP games, in-game purchases in MOBA games like characters and skins typically do not provide a competitive advantage. The paid characters provide alternative ways of playing the games but they are explicitly designed to not provide an advantage relative to the free characters (Palm and Noren 2015; Barnes 2017). Aesthetic items like skins only change in-game appearance of playable characters. In general, the gaming industry is moving towards the FTP model and revenue from FTP games dominates traditional pay-to-plays games in all markets. In 2016, the "Pay to Play" market in Asia was about 12.5% of the size of the FTP market in terms of revenues. In North America, PTP revenues were about 30% of FTP revenues (Clairfield International 2018).

My research employs data from a game in the MOBA category. The game generates revenue through in-game purchases that includes an option to purchase an add-on package that unlocks all the playable characters. This add-on package is referred to as the "God Pack" because the playable characters are Gods and Goddesses from different cultures. The character pack may have interesting implications in terms of consumer behavior. On the positive side, the purchase of the character pack represents a direct investment in the game. Once consumers have invested in this bonus content, the game allows for more variety in play and a consumer may perceive a switching costs since the value of the package is lost if the game is no longer played. However, the purchase of the character pack can also negatively affect consumer response to the game's reward system. In particular, rewards that provide access to bonus characters are unlikely to be motivational to consumers who have purchased access to all characters. My research provides an empirical investigation into how these potentially conflicting forces affect the game studio's customer relationships.

I next consider selected literature that inform my empirical analyses. First, I consider literature focused on consumer response to dynamic incentive schemes such as loyalty programs. The marketing literature contains an extensive body of findings related to how future rewards can motivate consumers. This material is especially relevant to studying consumer consumption decisions in video games since these games include extensive reward systems. I then cover selected literature related to consumers' investments in products. The accumulation of points or the purchase of add-on product features represent within product assets that can affect future customer loyalty.

My goal is to understand how these dynamic reward systems interact with customers' commitments or investments in a product. The gaming sector is ideally suited for this investigation.

The reward or gamification systems are extensive and consumer investments are voluntary. The complex interactions among these factors provide a great opportunity to study how consumers invest in games and their responses to the in-game dynamic incentive schemes. My investigation is designed to both produce findings related to consumer behavior and findings that will help managers create improved reward systems.

2.2.2 Dynamic Incentive Schemes

Many games include incentive schemes that provide players with a series of goals related to progressing through game levels and content. These incentive schemes are dynamic in that players need to exert effort over multiple periods to achieve goals and earn rewards. Rewards range from items that provide symbolic or psychological value such as levels, badges, and trophies to items that provide tangible value such as new characters, aesthetic items (skins) and in-game currency. While these reward systems are similar in spirit to the loyalty-based rewards popular in traditional marketing categories, the rewards systems are so prevalent in gaming contexts that they have spawned the term "gamification" (Blohm and Leimeister 2013; Groh 2012).

The marketing literature has devoted significant attention to the study of loyalty programs. One relevant aspect of this literature is work that investigates how long-term incentives can influence current and future consumption decisions (Lewis 2004; Kopalle et al. 2012). Lewis (2004) models consumer response to a reward program using an individual level dynamic optimization model. This model explicitly considers how expectations of future rewards motivate current consumption. The rewards systems featured in games often use similar dynamic structures as in traditional marketing contexts. Similar to how consumers earn rewards by accumulating points based on past spending, game players earn rewards by accumulating in-game points.

The loyalty program literature has noted that the accumulation of points can act as a motivator for consumer activity. This "points based" effect has been termed "points pressure" (Lewis 2004; Taylor and Neslin 2005; Kopalle et al. 2012). Points pressure, also known by the goal gradient effect in the behavioral literature, suggests that as consumers get closer to a pre-specified consumption goal, their motivation to achieve the goal becomes stronger (Kivetz, Urminsky, and Zheng 2006; Nunes and Drèze 2006). Many researchers have been able to identify the existence of "points pressure" in loyalty programs with dynamic incentives (Taylor and Neslin 2005; Kivetz, Urminsky, and Zheng 2006; Lewis 2004). Using grocery chain store data Taylor and Neslin (2005) found that as consumers get closer to rewards they increase their purchasing rate. Kivetz, Urminsky, and Zheng (2006) documented and analyzed the points pressure effect using multiple methods (field experiments, secondary customer data, etc.). In the context of gaming, players are given series of goals related to achieving new levels or earning rewards, and the games continually update players on their progress. From a modeling perspective the robust nature of points pressure findings suggests that any empirical specification of player behavior should incorporate structures that capture points pressure effects.

In general terms, the rewards in games provide players with goals. The use of goals is important for managing customer relationships. At a basic level, goal achievement and failure may affect customer preferences. Goal success within loyalty programs has been found to motivate subsequent effort exerted by consumers (Drèze and Nunes 2011; Wang et al. 2016), while goal failures usually led to poor subsequent performances (Soman and Cheema 2004; Wang et al. 2016). Beyond the basic notion of goal success or failure, it may be important to consider how various types of rewards may differentially affect consumer behavior. Some goals may promise very utilitarian benefits such as in-game currency while others may provide status-based benefits

(Kivets and Simonson 2002; Siddiqui et al. 2018; Suh and Yi 2012). In the gaming context, different types of rewards might represent different benefits to players. Some rewards may be more symbolic or mainly markers of progress while other rewards might unlock new content. It is also possible that at different stages of game playing players find different kinds of reward motivating. Developers can really leverage on this information and design the rewards system such that players will be provided with the best type of rewards that engage them the most at all stages of play.

In addition to establishing forward-looking goals, points-based rewards systems can potentially create customer-switching costs. The idea is that points earned in a game or loyalty program represent an investment in the program or game. If consumers or players are reluctant to forgo the value of these investments then participation in a points-based program can create a form of switching costs. Theoretical work suggests that loyalty programs should be able to increase consumer switching cost (Klemperer 1987, 1995; Kim, Shi, and Srinivasan 2001). However, empirical studies have yielded mixed results. Hartmann and Viard (2008) found that the rewards offered by dynamic incentive schemes in loyalty programs are negligible for heavy users, but effective for less frequent buyers who may face substantial switching costs when they are close to a reward, a stage which they rarely attain. Rossi (2017) showed that the reward program of a group of gas stations was able to generate switching cost only for a small group of consumers who were price-insensitive and reward seeking.

Although dynamic incentives in video games resemble rewards structure offered by loyalty programs in many ways, the gaming context includes several features that may influence whether findings will replicate in the gaming context. For example, while loyalty programs often require significant expenditures to earn rewards, in FTP games many reward requirements are based on usage rather than purchase. This means that purchase is no longer a condition for rewards from dynamic incentive schemes. Therefore, in contrast to traditional consumer-loyalty programs the rewards systems in FTP gaming environments often require effort rather than expenditures. This may be an important difference because in traditional loyalty programs consumers may lack the financial means to participate in a program. In gaming, participation is based on effort, time and interest rather than financial constraints. How customers or players react to rewards structures that are based on usage rather than spending remains an empirical question wanting answers. The removal of financial constraints may increase the effectiveness of dynamic reward systems since players may engage with the reward program by playing games rather than spending.

Another feature that separates incentive schemes in games and loyalty programs is the nature of the rewards offered. Rewards offered in games are almost always related to the future consumption or game play experience. In standard loyalty programs, rewards are often an incremental product such as a free flight. In games, rewards are usually related to the game experience. Symbolic rewards like levels and badges, and the rewards with a cash value like aesthetic items and new characters change the gaming experience. These rewards may operate more similarly to status or tier rewards in traditional loyalty programs that change the quality of the consumption experience. These differences across customer loyalty programs and game reward systems reveal opportunities for new research. While the empirical literature on switching costs within loyalty programs has yielded mixed results, the unique features of the gaming industry give me reason to question whether it may be easier to create perceived switching costs within games.

2.2.3 Customer Investment and Switching Costs

While the accumulation of loyalty or experience points may be one source of switching cost for consumers, customer switching costs come from various sources during the consumption process, with examples like search cost for alternatives, financial investment made, potential discount as a loyal customer, time and effort spent learning how to use the product, emotional cost, psychological risks etc. (Fornell 1992). Disciplines like marketing, economics and strategy (Burnham, Frels, and Mahajan 2003; Klemperer 1987; Porter 1980) have long recognized and conducted research on antecedents and consequences of customer switching cost. Switching costs are defined as onetime costs that consumers face when they switch from one product or service supplier to another (Porter 1980; Burnham, Frels, and Mahajan 2003; Jones, Mothersbaugh, and Beatty 2000). Several frameworks had been suggested in the purpose of covering and categorizing the various sources of switching cost for consumers. Klemperer (1987) focused on costs that can be imposed by the seller and the nature of the product and he identified three kinds of switching costs: transactions costs, learning costs and contractual costs. Guiltinan (1989) suggested four kinds of switching cost: contractual, set-up (a combination of learning costs and transaction costs in Klemperer’s framework), psychological commitment (sunk cost) and continuity cost (opportunity cost and risk of switching). Burnham, Frels, and Mahajan (2003) brought together a framework applicable for both tangible products and services, where three types of consumer switching cost were identified: financial switching cost (financial resources invested), procedural switching cost (time and effort invested), and relational switching cost (identity and relationships built).

For my empirical context of the gaming industry, the framework proposed by Burnham et al. (2003) to a large extent describes how switching costs can be created through consumer or player investment in different aspects of the game. For FTP games players can play without any down payment. The in-game purchases of add-on features can help enhance players’ game experience. Once purchased, these features seldomly can be sold back to the firm. For some games there are marketplaces where players can trade in-game items even accounts with other players.

However, the discount rates are usually very high and only a small portion of the initial investment can be recovered. Once committed financially, in-game expenditures become sunk costs that can potentially prevent players from turning to alternative options. As experiential products video games require players to put into substantial time and effort, and progression measurement is a popular built-in feature in games at multiple aspects. Players will forgo all the progress they have made in the game once they decide to switch, thus in-game progress and effort and time committed serve as a barrier for player switching behavior. Social features like online friends and in-game community and out-of-game communities on popular social platforms are also prevalent. Many players make in-game friends and some even manage to establish identities in game-specific communities. If players want to switch to other games, the in-game friends and any identity or status they have earned in communities will be lost. For my empirical setting, unfortunately I do not observe players' social activities in the game. However, it is highly likely that players who are more dedicated and better in skill are also the ones who engage in social activities more. By including past effort and performance I hope to at least partially capture the impact of relational switching cost on game play decisions. All these costs will increase the obstacle for switching and developers have started to realize the important roles these factors can play in retaining players.

Consequences of switching costs from various sources have been studied by multiple empirical works in marketing. The direct impact of switching cost on consumer behavior is customer retention. Most of the literature on this topic suggests a positive relationship (Pick and Eisend 2014). Through a survey of customers in the credit card industry and long-distance industry, Burnham, Frels, and Mahajan (2003) found that all three types of switching costs (financial, procedural and relational) significantly improve customer retention. Under the same framework, Blut, Frennea, Mittal, and Mothersbaugh (2015) conducted a meta analysis on over 133,000 customers and showed that relational switching cost have the strongest association with repurchase intention and behavior while financial switching cost enhance the association between satisfaction and repurchase. Using survey data on organizational buyers in high-tech market, Heide and Weiss (1995) showed that switching costs limit buyers' intention to switch. In a B2B context Wathne, Biong and Heide (2001) found evidence of switching cost being a barrier for switching behavior. Lam, Ahearne, Hu, and Schillewaert (2010) suggested that switching costs induced by financial investment together with customer-brand identification and relative perceived value of the incumbent make it less likely for consumers to switch when a radically new brand is introduced using data from the smart phone industry. Jones, Mothersbaugh, and Beatty (2000) studied the interaction between satisfaction and switching cost on repurchase intentions. They found that although perceived switching cost had no influence on repurchase intentions when satisfaction is high, switching cost boosted purchase intention when satisfaction is low. Kim and Son (2009) suggests that when online service provider encourage their customers to customize their services, consumers become more dedicated to the provider due to the increased switching cost from nontransferable investment in personalization. Using data from the financial service industry Dong and Chintagunta (2015) found that customer with higher financial investment have higher switching cost and they are more likely to stay with their current financial service provider.

2.2.4 Summary

Literature from loyalty program studies has documented a points pressure effect for consumers during goal pursuing processes. Also, literature has touched on whether and how incentive schemes and customer investment are able to impose switching cost on consumers. However, the question of how incentive schemes and customer investment interact and jointly influence consumption decisions is under researched. This is largely due to the fact that in loyalty programs purchase is a condition in the rewards structure. The gaming industry and the rise of gamification has indicated new possibilities for the use of dynamic incentive schemes in engaging consumers. Games and gamification processes employs usage as conditions rather than purchase, thus rewards structures become more accessible for all customer segments. Also, the FTP or freemium business model endows consumers with the flexibility of how much and when they would like to invest in the products financially. Understanding the interplay between incentive schemes and customer investment possesses both academic and managerial value.

2.3 Data and Preliminary Analysis

In this section, I will provide additional details related to my empirical setting. Specifically, I will describe the game that provides the data. I will also present basic descriptive statistics and preliminary analyses that illustrate salient patterns in the data.

2.3.1 The Game

The data comes from one of the top games in the MOBA category. In this category of games, the emphasis is on team versus team competition. Players are put into teams of five that compete on a map where the bases of the two opposing forces are located on the opposite sides. The ultimate task is to take down the enemy's base by eliminating enemy players. Each game session generally lasts from 20 to 40 minutes. Although players can enter into queues for game sessions with friends, the majority of the players play solo. Teammates and opponents are determined by a matching algorithm. In each game session, players choose a character to play from a pool of available characters.

The pool of available characters for players is determined by their levels and previous purchases of characters. When players start to play this game for the first time, they have a character pool of ten characters as free users. Five of those characters are permanently free (players can play them any time) while the other five are on rotation on a weekly basis. Every week five characters that are not in the free character pool will be made available for free users to try out. This character pool of players can be expanded by either purchasing the character pack which unlocks all characters or purchasing single characters with the in-game currency called gold.

The character pack is a one-time investment that gives players access to all current and future characters. It should be noted that access to the full range of characters is not intended to provide a competitive advantage to players. MOBA games, unlike mobile games that frequently employ pay-to-win models, emphasize skill-based competition. For the competitive games between teams to be enjoyable, MOBA developers try to maintain a balance among all the characters available[3]. In addition to the character pack, players can also purchase aesthetic items in the game, which are called “skins”. Skins only change the look of characters during game sessions but do not impact game play. All skins can be purchased with real cash.

Rewards and other incentives are based on the accumulation of experience points that are acquired through finishing game sessions. As experience points are accumulated players’ accounts level up. Figure 1 depicts the dynamic incentive scheme deployed in the game. The three types of rewards associated with certain levels are rental rewards, in-game currency rewards and progress accelerator rewards. Rental rewards allow players access to certain characters or aesthetic items for one to two weeks. The rental reward of characters can temporarily expand character pool by granting player temporary access to a certain character. Like rental rewards for characters, rental rewards for skins allow players access to a certain skin for a short amount of time. Upon claiming

[3] By an analysis of 22,191 games played in current patch, the win rate of paid character is 49.91% while the win rate of free characters is 48.88%. The difference between win rates of paid and free characters are not significant (p=0.99)

rental rewards players should feel propelled to try these new contents out by playing sessions with them. Also, these sessions should help players gather further information about the game which will aid players in their further purchase decisions.

As an in-game currency, gold can be earned by claiming currency rewards associated with certain levels and by finishing game sessions. The amount of in-game currency players can obtain by finishing sessions (with the winning team getting a little more than the losing team) is much smaller compared to the currency rewards. The currency can be accumulated and used to purchase characters. Since there are a relatively large number of characters available for purchase, collecting all characters through in-game currency accumulated requires an amount of game play far in excess of normal players. The currency rewards associated with levels also serve as reminders reaffirming players the possibility of acquiring characters through more play. Characters cost much more than a single currency reward provides. Thus, players should be motivated to play more sessions in order to accumulate the currency needed for character purchases. For the purchase decision of the character pack, the impact of currency rewards is less obvious. The sessions players are motivated to play to accumulate the in-game currency can help players gather more information about the entire game which should aid their character pack purchase decisions. However, offering players a way of collecting their favorite characters through just game play may decrease their intention to purchase the whole character package.

Progress accelerator rewards provide access to an in-game item (booster) that increases the amount of experience points and in-game currency earned by players from each game session. A progress accelerator typically last from 1 to 3 days. The count down for an accelerator starts the moment player attain the corresponding levels and the clock cannot be stopped. Progress accelerators can boost players' progress in the game and help them get to higher levels and rewards faster. With its limited effective period, players should feel motivated to engage in more game sessions to take advantage of the increased payout. The increased play should help players improve and progress, which might facilitate players' purchase decisions concerning the character pack.

2.3.2 Sample

For my analysis, I collected data for the cohort of players who registered between January 11, 2016 and February 10, 2016. I tracked the activity of this cohort from acquisition until January 06, 2017. Player activities includes game session histories and purchases of the character expansion pack. I summarized players' purchase and play history at the daily level and generated a random sample of 2,518 players[4]. A list of variables and corresponding definitions are provided in Table 1. Table 2 presents the descriptive statistics and Table 3 is a correlation matrix of all the variables.

A unique aspect of online video gaming is that detailed information on game consumption is available. For example, player performance is a key factor that may influence players' interest in the game. Performance measurements reflect players' experience from game sessions which will directly impact their feeling and preference towards the game. It is likely that the players do well in sessions feel that the game is made for them and they might want to invest more time and possibly money into the game.

There are many indicators of one player's in-game performance and the most recognized in the MOBA sector is KDA (kills, deaths and assists), which is calculated as the sum of player kills and assists divided by player deaths. Kills refers to the number of times a player has killed enemy players while assists refers to the number of times a player has participated in the taking

[4] Initially I generated a random sample of 3,000 players from the total batch. Then the players who only played 1 day are dropped and the sample reduced to 2,518 players.

down of an enemy player without dealing the killing blow. Deaths are the number of times a player has been killed by enemies. KDA has been widely used in the MOBA category as an indicator for player performance and almost all players, from novice amateurs (newbies) to professionals (pros), are aware of the term and judge their own and other players' performance by their KDAs. At the end of each game, a scoreboard is presented to all participants and KDA is one of the key statistics displayed, so the players are aware of their KDAs after each game session.

On the reward side I have four dummy variables for three types of the rewards (rental reward, currency reward and progress accelerator reward) and level-up which equals one if a player claims a specific type of reward or levels up on that day. To study points pressure I also include progression variables for each type of reward and level-up. The distance between two rewards of the same type and two levels can be measured by experience points needed to get to the next from the current one. The progression variables are calculated by dividing the number of experience points players earned since they get to the current reward or level by the total number of experience points needed from this reward or level to the next one. The progression variables measure the progress players have made towards rewards and levels. If points pressure does play a role, I would observe players playing more games when their progression variables are at higher values.

CharPack is a dummy variable that is used to signal character package ownership. The character pack owners might be more engaged with the game since they have a much bigger pool of characters to choose from and that they have made a financial investment which would be a frustration if they don't put the purchased characters in use.

As control variables I have included character pack price (*CharPackPrice*), number of characters played (*CharPlayed*), cumulative spending on skins (*SkinExp*) and days since registration (*RegistDays*) in the panel. The price of character pack is set at $29.99 and from time to time the developer would put the character pack on sale with discount rates from 10% to 50%, which provides enough variation to measure players' price sensitivity. Number of characters played serves as a measurement of breath of content players have experienced, which I think would influence players' purchase decisions. Cumulative spending on skins measures how much money players have spent on aesthetic items in the game cumulatively, which can also influence play and character pack purchase decisions as a different kind of investment in the game. Days since registration counts the number of days passed since player registered with the game and it is used to take any time trend into account.

2.3.3 Preliminary Analysis

Prior to presenting a detailed statistical model, I first would like to show some model free evidence that illustrates key patterns in the data. In particular, I am interested in the relationships between playing, earning rewards and purchasing. The goal is to understand how purchasing decisions and reward structures influence player's decisions to continue to consume the game. The statistics presented in this section come from the random sample of 2,518 players.

Play and Purchase

On the play side, on average players in the sample play 0.91 game sessions daily. Figure 2 presents the evolution of average number of game sessions played daily over time. On the first several days after registration players are likely to engage in multiple game sessions and this number quickly drops to less than 1 game session and remain relatively steady through time. On the purchase side, the character pack was purchased by 6.87% of players. This pay ratio is relatively high compared to the industry average of 4.02% for North America and 2.43% for Europe (DeltaDNA 2018). The relatively low conversion from free players to pay players in the FTP industry highlights the importance of understanding consumer consumption behavior. Figure 3 shows the timing of character pack purchases. Interestingly, the majority of character pack purchases occur early in the consumer lifecycle. Most of the purchases happen within the first month with the highest purchase intention during the first week, and purchase rates decrease over time. An important observation is that players' decisions to invest in the game tend to occur relatively quickly.

In order to understand the relationship between play and purchase by players, I calculated the average number of game sessions played for the group of player day combinations without the character pack and the group of player day combinations with the character pack purchased (see Table 4). For free users, on average .67 game sessions are played while the number for pack owners is 1.82. At a minimum there is a correlation between purchase and participation.

One can argue that this result can be driven by the subset of players who just like the game better and play more games before and after their purchases. To see if it is the case, I constructed a sub sample comprised of all the players who ended up buying the character pack and calculated the average number of game sessions played before and after their commitment (see Table 4). Before purchases are made selected players play 1.3 game sessions on average, and this number goes up to 1.82 after commitment ($p < .01$). It looks like players who choose to invest are the ones who like the game better (they play more before their purchases comparing to other players), however, I still see a boost in play for those players after they commit.

Dynamic Incentives and Play

Incentive schemes are used to motivate and engage players, so I summarized players' game play decisions with regard to reward claims and level-ups. Take rental rewards as an example, I separate the player and day combinations into two groups: one includes the days players do not claim a rental reward while the other includes the days players claim a rental reward. For each group the average number of game sessions played daily is calculated. Table 5 shows that on days players claim rental rewards they play 8.35 game sessions on average while on days they do not only 0.74 game sessions are played. Players are more engaged when rental rewards are within arms' reach. Similar comparisons can be found for currency rewards, progress accelerator rewards and level-ups (see Table 5).

Dynamic Incentives and Purchase

Another focus of this research is the interactions between incentive schemes and consumers' decisions to invest in the game. To get a glance of their interplay, similar to the case with game play behavior, I compare players' purchase decisions with regards to reward claims and level-ups. Again, using rental rewards as an example, player and day combinations are categorized into two groups: one includes the days players claim a rental reward while the other includes days they do not. Then I accumulated number of purchases happened in each group. In total 140 purchases happened on days player claim rental rewards while only 36 happened on days they do not (see Table 6). Table 6 also presents comparisons made for currency rewards and progress accelerator rewards where similar results are found. For level-ups the case is different. There are 71 purchases on the days when players level up while 105 purchases happen on the days they do not. Although the trend seems to be reversed for level-ups, it is worth noting that there are far fewer level-up days. After calculating the percentage of purchases with regard to total number of days in the level-up and no level-up group respectively, the data suggests that .63% for the level-up group and .04% for the no level-up group, the difference of which is statistically significant ($p < .01$) (see Table 6 and note). Thus, players are more likely to purchase the character pack on level-up days.

Summary

The model free evidence summarizes players' play and purchase behaviors and the impact of the incentive scheme deployed. Players play about one session daily on average and this number is higher initially and quickly drops to the average level and stays there steadily. The conversion rate for the game is a little less than 7%, and most of the purchases happen at early stages of game play. From the tables pertaining the interactions of incentive schemes with play and purchase, it is quite straightforward that claiming rewards from the in-game incentive scheme and leveling-up have relatively strong positive correlations with both game play and character pack purchases. It seems like the incentive scheme is doing the job it is designed to do: motivate and engage players. However, this is not considering the observed and unobserved factors that might have influenced both play and pay decisions made by players. To account for the myriad of factors in play I include observed elements like player progression towards different type of positive feedbacks, character pack ownership and reward claims. It is also possible that some unobserved factors drive both play and pay decisions, the neglect of which may lead to serious endogeneity concerns. In the next session of model setup, I will talk about the joint state-space model by which the intricate relationships among the key factors in the gaming setting will be captured.

2.4 Model

In this section I will develop a model of consumer behavior in the FTP category. In this setting, consumers make two distinct but related decisions daily. First, players make consumption decisions. Specifically, I am interested in how many game sessions players choose to play. Second, players also decide whether and when to purchase or invest in the game. In this specific setting, I am primarily interested in the decision of character pack purchases. These decisions are likely related as both may be driven by some measure of preference for the game. For unobserved factors like players' feelings for the game, it would not be surprising to find state dependence in those variables. Players' feeling for the game possibly depends on how they felt about the game last period and their experience with the game this period. With these features in mind, I decide to use a joint state-space model to analyze players' play and purchase decisions.

State-space models are a type of Markov model that have been used to handle sequential data both in the statistics literature (Carlin, Polson, and Stoffer 1992; Kim, Menzefricke, and Feinburg 2005) and the machine learning literature (Bishop 2006; Murphy 2012). State-space models include two types of equations: observation equation and state equation. In my setting, since the joint decisions of play and purchase are of interest, both of which are observed, I use two observation equations for play and purchase respectively. For the state equation, I assume a latent variable which is named "preference" (p), and this preference should capture the unobserved factors that drive both play and purchase decisions, i.e., play and purchase are conditionally independent when preference is given. I believe that the latent preference should depend on preference from last period and it should also be adjusted by players' experience from the current period. Thus, preference should follow an AR(1) process and the measurement of players' experience with the game in the current period should also enter the state equation. Figure 4 provides a visual illustration of the model structure. In the following subsections, I will talk about the specifications of each of the three equations in the state-space model.

2.4.1 Observation Equations

Play Equation

My model is based on the idea that consumers make a daily decision about how many games to play that day. Thus, I specify a play equation that includes the number of games played each day as the dependent variable. Given that the number of games is left censored at zero I decide to use a Tobit model where the dependent variable follows a truncated normal distribution. In terms of notation, I use i to denote player and t to indicate time. The specification for games played, y, by player i on day t is given in equation 1.

$$\begin{aligned} y^*_{it} &= p_{it} + \beta_1 * CharPack_{it} + \beta_2 * RentProg_{it} \\ &+ \beta_3 * CurProg_{it} + \beta_4 * AcclProg_{it} + \beta_5 * LvlupProg_{it} \\ &+ \beta_6 * CharPack_{it} * RentProg_{it} + \beta_7 * CharPack_{it} * CurProg_{it} \\ &+ \beta_8 * CharPack_{it} * AcclProg_{it} + \beta_9 * CharPack_{it} * LvlupProg_{it} \\ &+ \beta_{10} * SkinExp_{it} + \beta_{11} * RegistDays_{it} + \varepsilon_{it} \end{aligned} \tag{1}$$

$$y_{it} = \max\{y^*_{it}, 0\} \tag{2}$$

$$\varepsilon_{it} \sim N(0, \delta^2) \tag{3}$$

Equation 1 includes a variety of variables related to incentives and player investment. *CharPack* is a dummy variable that signals ownership of the character pack. I would expect players who own the character pack to be more engaged with the game since they have invested financially and they have more options for game play. Similarly, players with more money invested in aesthetic items like skins should also be more engaged to make their skin purchases more worthy. Progression variables for different types of rewards describe how much progress players have made towards the next reward of each type. Similarly, level progression measures how much progress players have made towards the next level. Those variables are included to see if players exhibit points pressure to rewards and levels in the dynamic incentive scheme.

I am also curious about how customer investment might change the level of appeal offered by different types of rewards and level-ups. To do this, I include interaction terms between character pack and progression variables for each type of reward and level-up. The main effect and the interactions together can tell whether the rewards and levels are effective in motivating and engaging players at different stages of game play. The latent preference variable p_{it} is the intercept which captures the player's intrinsic preference for the game. Instead of assuming a time-invariant preference, I allow player preference towards the game to evolve over time such that dynamic changes in player intrinsic preference are captured by the model. To account for time trend, I include number of days passed since registration as a control variable.

Purchase Equation

In terms of customer investments in the game, I focus on character pack purchase decisions. This character pack is the most significant expenditure made by most customers. Purchase of the character pack is a one-time investment decision such that character pack purchase data follows an event-timing data structure. Hence for the analysis of purchases I choose to use a proportional hazard model which has been used widely to characterize purchase-timing behavior of households (Seetharaman and Chintagunta 2003).

In proportional hazard model the hazard function specifies the instantaneous probability of making a purchase conditional on elapsed time. The hazard function for the purchase decision by player *i* on day *t* is given in equation 4:5

$$\begin{aligned} h_i(t) = h_0(t) \exp\big(&\gamma_1 * p_{it} + \gamma_2 * RentRwd_{it} + \gamma_3 * CurRwd_{it} + \gamma_4 * AcclRwd_{it} \\ &+ \gamma_5 * Lvlup_{it} + \gamma_6 * CharPlayed_{it} + \gamma_7 * CharPlayed^2{}_{it} \\ &+ \gamma_8 * CharPackPrice_{it} + \gamma_9 * SkinExp_{it}\big) \end{aligned} \tag{4}$$

with a Weibull baseline hazard:

$$h_0(t) = \lambda_1 * \lambda_2 * (\lambda_1 * t)^{\lambda_2 - 1} \tag{5}$$

where λ_1 is the rate parameter and λ_2 is the shape parameter.

The hazard function can be multiplicatively broken down into two components: the baseline hazard, which captures player population's intrinsic purchase pattern, and the covariate function, where covariates like pricing information, incentive scheme related variables can be introduced into the model. For baseline hazard I choose to use the Weibull baseline hazard (Equation 5). Weibull baseline hazard is a fairly flexible baseline hazard with a rate parameter (λ_1) and a shape parameter (λ_2). When shape parameter (λ_2) equals one, the Weibull baseline hazard reduces to exponential which is a flat line. The Weibull baseline hazard is monotonically decreasing when the shape parameter (λ_2) is smaller than 1, and it is monotonically increasing when the shape parameter (λ_2) is greater than 1.

I included various variables in the covariate function to study the important factors that influence purchase decisions. Like the play decisions, it is highly likely that the purchase decisions are also influenced by players' preference (p_{it}) towards the game. The latent preference variable is included in the purchase equation as a covariate and through intrinsic preference players' play and purchase decisions are connected. To study the interplay between the incentive scheme and customer investment decisions I include dummy variables for claims of three types of rewards and level-ups. Achieving goals provided by the incentive scheme and getting associated prizes are positive feedbacks from the game. Studies from the loyalty program literature suggest that goal success generally leads to more subsequent effort exerted by consumers (Drèze and Nunes 2011), so I predict that claiming rewards and leveling up might increase players' interest in the game which can lead to higher purchase intentions. As control variables I include number of characters played and its square term. Number of characters played is a measurement of breadth of game content experienced by players so far and I believe that information gathered about the game plays an important role in purchase decision making. Another important control variable is the price of the character pack which is set at $29.99 in game. Occasional promotional campaigns set character pack price to 50% to 90% of its original, which offers the variation needed to detect players' price sensitivities. Players' cumulative spending on aesthetic items like skins are also included to see whether players who have purchased more skins are more likely to invest in character packs. For skins to be worthy players need to own corresponding characters first. Thus, it is more likely that skin purchases are contingent on character ownership rather than the other way around.

2.4.2 State Equation

The state equation describes how unobserved preference (p_{it}) which captures players' interest levels evolve over time. The following state equation is used to model preference evolution:

$$p_{it} = \rho_i + \theta * p_{i,t-1} + \alpha_1 * Perf_{i,t-1} + \alpha_2 * Perf^2{}_{i,t-1} + \eta_{it} \quad (6)$$

$$\rho_i \sim N\left(\mu_\rho, \delta_\rho{}^2\right) \quad (7)$$

$$\eta_{it} \sim N\left(0, \delta_p{}^2\right) \quad (8)$$

where ρ_i's are random effects included to take care of player unobserved heterogeneity.

The preference variable is the latent variable that links purchase and play decisions. I assume that preference captures the correlation between play and purchase decisions so that these two decisions are conditionally independent. Players' interest level will likely be highly correlated with their interest level from last period. Thus, I assume that preference *p* follows an AR(1) process. Another factor that potentially can influence preference is players' experience with the game in the current period. To capture this effect, I introduce players' daily performance measures into the state equation. If a player plays the game and does well in the sessions today, he/she might like the game more at the end of the day. Players' performance is measured by KDA (kills, deaths and assists), a performance measure used widely in this category. A quadratic term is included in the equation to test for the possibility of a non-linear relationship.

2.4.3 Estimation

The proposed joint state-space model is estimated using a hierarchical Bayesian approach. Within each iteration the parameters are drawn from their corresponding posteriors using a Monte Carlo

Markov Chain method designed with a hybrid of Metropolis-Hastings algorithm and Gibbs Sampling method. For details of every step in the MCMC procedure, please refer to Appendix 1. Here I will elaborate on a key step in the estimation process.

In every period, players decide on how many sessions they want to play and whether they would like to purchase the character pack. These two decisions are connected by the latent preference variable that evolves over time. I argued that preference is influenced by players' past experience and assumed that it follows an AR(1) process. Thus, posterior for p_{it} is conditional on both $p_{i,t-1}$ and $p_{i,t+1}$, character pack purchases, game sessions played and other relevant parameters. For the initial condition p_0 I assume that it has a non-informative normal prior *N* (mu_0, sig_0) with mu_0=0 and sig_0=100. Assume that covariates in the play equation are *x*, covariates in the purchase equation are *z* and covariates in the state equation are *w*, I establish the conditional posterior density for p_{it}:

$$p(p_{it} \mid \theta, \alpha, {\delta_p}^2, \rho_i, \beta, \delta^2, {y^*}_{it}, p_{i,t-1}, p_{i,t+1}, \lambda_1, \lambda_2, \gamma)$$

$$\propto N\,(F_{it} * f_{it}, F_{it}) * p_i(t|\lambda_1, \lambda_2, \gamma, p_{it}) \tag{9}$$

where

$${F_{it}}^{-1} = \begin{cases} \dfrac{1}{sig_0} + \dfrac{\theta^2}{{\delta_p}^2}, & t = 0 \\ \dfrac{1}{{\delta_p}^2} * (1 + \theta^2) + \dfrac{1}{\delta^2}\,, & t = 1, \dots, T_i - 1 \\ \dfrac{1}{{\delta_p}^2} + \dfrac{1}{\delta^2}\,, & t = T_i \end{cases} \tag{10}$$

and

$$f_{it}=\begin{cases}\dfrac{mu_0}{sig_0}+\dfrac{\theta*(p_{i1}-w_{i1}{}'\alpha-\rho_i)}{{\delta_p}^2}, & t=0\\ \dfrac{\theta*p_{i,t-1}+w_{it}{}'\alpha+\rho}{{\delta_p}^2}+\dfrac{\theta*\left(p_{i,t+1}-w_{i,t+1}{}'\alpha-\rho_i\right)}{{\delta_p}^2}+\dfrac{{y^*}_{it}-x_{it}'\beta}{\delta^2}, & t=1,\dots,T_i-1\\ \dfrac{\theta*p_{i,T_i-1}+w_{iT_i}{}'\alpha+\rho}{{\delta_p}^2}+\dfrac{{y^*}_{iT_i}-x_{iT_i}'\beta}{\delta^2}, & t=T_i\end{cases}\qquad(11)$$

The character pack purchase likelihood for player *i* on day *t* is $p_i(t|\lambda_1,\lambda_2,\gamma,p_{it})$ which can be formulated as:

$$p_i(t|\lambda_1,\lambda_2,\gamma,p_{it})=\left(q_{it}(t,z_t)^{b_{it}}*(1-q_{it}(t,z_t))^{(1-b_{it})}\right)^{(1-CharPack_{i,t-1})}\qquad(12)$$

with $q_{it}(t,z_t)$ as the hazard rate:

$$q_{it}(t,z_t)=1-\frac{S(t,z_{it})}{S\left(t-1,z_{i,t-1}\right)}\qquad(13)$$

and $S(t,z_{it})$ can be calculated from the survival function in equation 14:

$$S(t,z_{it})=exp\left(-\sum_{v=1}^{t}exp(z_{iv}\gamma)\int_{v-1}^{v}h_0(u)du\right)\qquad(14)$$

The component of $N\ (F_{it}*f_{it},F_{it})$ incorporates information from play and other relevant preference variables ($p_{i,t-1}$ and $p_{i,t+1}$) while $p_i(t|\lambda_1,\lambda_2,\gamma,p_{it})$ incorporates information from character pack purchases. The multiplicative form of the conditional posterior density is the result of the assumption that play and purchase are independent when preference is known. To draw from this posterior density, a Metropolis-Hastings algorithm step will be conducted. First, using the most recently updated value I can generate a new value for p_{it} using this equation:

$${p_{it}}^{new}={p_{it}}^{old}+\Delta_{p_i}\qquad(15)$$

Here $\Delta_{p_i}\sim N(0,\phi_p)$ and ϕ_p is a fixed tuning constant. Then acceptance probability can be calculated as:

$$Pr\ (acceptance)_{it}\ =\ min\left\{\frac{\left(N({p_{it}}^{new}|F_{it}*f_{it},f_{it})\right)*p_i(t|\lambda_1,\lambda_2,\gamma,{p_{it}}^{new})}{\left(N({p_{it}}^{old}|F_{it}*f_{it},f_{it})\right)*p_i(t|\lambda_1,\lambda_2,\gamma,{p_{it}}^{old})},1\right\} \tag{16}$$

With this acceptance probability I can decide whether the new value will be accepted or not and this decision will be repeated at each iteration.

The processes for drawing other parameters from their corresponding posterior densities follow relatively standard Bayesian approach. In Appendix 1 I describe the non-informative priors chosen for parameters and formulated posterior densities for each parameter with Metropolis-Hastings steps generated for parameters I cannot sample with Gibbs Sampling method. A total of 45,000 draws were generated and convergence was checked by monitoring the time series of the parameters. In the next section I will review the model results and discuss their implications.

2.5 Results and Analysis

In this section, I will present the estimation results of the proposed joint state-space model and some simulation studies are conducted to show how changes in rewards structure influence players' play and purchase decisions.

2.5.1 Model Results

Estimation results of the proposed state-space model is shown in Table 7. In the following subsections I will discuss players' play and purchase decisions with the presence of in-game dynamic incentive structure.

Play Equation

Using game sessions played daily as the dependent variable, I hope to figure out the important factors influencing player's play decisions with the play equation. The character pack ownership dummy has a positive significant relationship with number of sessions played ($b = 3.38$, $p < .01$). This implies that owners of the character pack are more engaged with the game and they play more game sessions compared to free players. Similarly, the coefficient for player spending on skins is also positive and significant ($b = .14$, $p < .05$). Players who have purchased more aesthetic items are also more engaged.

Progression variables for each type of the reward (rental, currency and progress accelerator) and level-up were included to test for points pressure effects. The main effects of all four progression variables are positive and significant. The rental rewards have the strongest points pressure effect ($b = .38$, $p < .01$) while the progress accelerator rewards have the weakest points pressure effect ($b = .13$, $p < .05$). For currency rewards the coefficient is .22 ($p < .01$) and for level-ups the coefficient is .19 ($p < .01$). These results imply that when players are closer to the three types of rewards or the next level, they play more games, i.e. evidence for the existence of points pressure for players is identified.

The main effects of the progression variables described the case of free players. The interaction terms between character package ownership and the progression variables describe how players reactions to rewards and levels change after their character pack purchases. The interaction terms for rental and currency rewards are negatively significant ($b = -.53$, $p < .01$ for rental rewards and $b = -.23$, $p < .05$ for currency rewards). Therefore, rental and currency rewards are much less appealing to players who have purchased the character packs. For progress accelerator rewards and level-ups, although the coefficients are negative ($b = -.22$ for booster and $b = -.16$ for level-ups), neither of them is statistically significant. It looks like progress accelerator rewards and level-ups are still valuable goals players want to pursue after their financial commitment.

Some rewards remain to be appealing to players while others lost value after players purchase the character pack, and I think this is possibly due to the nature of different rewards. The rental rewards give players temporary access to characters and skins, so it is a reward that helps players explore the game and figure out whether they want to commit with real money. Similarly, players can purchase characters with currency accumulated from sessions and currency rewards. Each character takes a big amount of in-game currency. For players who likes multiple characters it makes sense to use the in-game currency to sample some characters before making the commitment decision, rather than trying to collect all characters of interest by pure in-game effort. These rewards lose value after a player purchases the character pack since they will have access to all characters. The rental rewards also provide previews for selected skins. However, skins only change the appearance of characters and the effect can be previewed on other platforms like YouTube. Thus, the value provided by rental rewards for skins are compromised. Progress accelerator rewards and level-ups on the other hand, will provide players with benefits with or without the purchase. Levels are milestones in game progression and accelerator rewards facilitate this leveling up process by increasing the experience points gained from every game session. The conclusion based on these results is that after purchase, rewards that help players explore the game become less attractive while rewards that help players progress remain relevant.

The game has a very active player community and on websites like Reddit I have seen multiple posts from players complaining about more than half of the rewards provided by the incentive scheme become almost useless after players purchase the character pack. These players feel that the developer only cares about converting and the paid users get ignored after purchase. It is pretty clear that the developer missed an opportunity to engage the more dedicated users after their commitment with real money and this definitely is harmful to the relationship between pay users and the developer in the long run.

Purchase Equation

In the purchase equation, the dependent variable of character pack purchases is used to study the factors influencing player financial commitment. The latent preference variable has a coefficient of .10 ($p < .01$) which implies that the higher the player preference the more likely they will purchase the character pack. For the three types of rewards and level-ups, it looks like their influence on purchase decisions vary. The coefficient for the rental reward is slightly negative ($b = -.01$) but not statistically significant. The coefficient for progress accelerator rewards and level-ups are both positive and significant ($b = .93$, $p < .01$ for progress accelerator rewards and $b = 1.69$, $p < .01$ for level-ups). When players get progress accelerators and when they level up, their purchase intention increases, which is consistent with my prediction based on loyalty program literature findings.

Surprisingly, the coefficient for currency rewards are negative and significant ($b = -.5$, $p < .05$). Claiming a reward should be a positive event, however, the claim of currency rewards actually dampens players' intention to invest. This effect is probably the result of how the currency can be used. As an alternative to the character pack for players to expand their character pool permanently, the group of players who likes only a very limited number of characters find the in-game currency a substitute for the character pack. For these players they more currency rewards they claim, the less likely they would like to invest in the character pack since they already own what they want. For other players who find the in-game currency insufficient for their demand for content, the currency rewards also can serve as a distraction which will drive players attention away from making the purchase decision. When players claim currency rewards, they are reminded of the possibility of sampling some characters with the currency and the information provided by these try-outs can aid the purchase decision in the end. A single character costs much more than a single currency reward and players need to exert effort to achieve this goal. The pursuit of single character try-outs thus may lead to players postponing the purchase decisions of the character pack ("let me buy this character first and see if I like it. Then I may decide whether I want to purchase the whole pack"). The above situations might be the reasons why players' purchase intention towards the character pack decreases when they claim currency rewards. The shape parameter for the Weibull baseline hazard is smaller than 1($\lambda_2 = .79$), which implies a monotonically decreasing hazard function. Thus, the hazard rate (purchase probability) falls as time goes by after registration, i.e. players' purchase intention is higher at the beginning and drops rapidly over time (see Figure 5 and 6). Under this circumstance triggering a goal that potentially will postpone character package purchase decisions can be harmful for the developer since players will lose interest in the game over time. So, currency rewards can be seen as a type of reward that is conflictive with player investment decisions.

Character pack price has a negative and significant coefficient ($b = -.61$, $p < .01$), as expected. Number of characters played, as a measure of breadth of game content players have experienced, has an inverted U relationship with purchases (for the quadratic term $b = -.48$ $p < .01$ and for the linear term $b = .67$, $p < .01$). The turning point is between 18 and 19 characters. This result implies that in general players purchase intention are the highest when they have tried out about 18 or 19 characters of the game. It looks like the players will first gather information from playing multiple characters. When the number goes up to 18 or 19, they feel like they have the information needed to make the purchase decision. This result has strong implications on how the incentive scheme should be designed to best provide players enough information at the right time to help players make purchase decisions since rewards can be used as ways to feed players with characters to try. The coefficient for players' cumulative spending on skins is not significant (b = -.02), so players' investment in the character pack is not influenced by their skin purchases.

State Equation

With the latent preference variable, I hope to capture the unobserved factors that drives both play and purchase decisions. The coefficient for preference from last period is positive and significant (b = .94, p < .01). This is evidence for a pretty strong carry-over effect. Players who liked the game last period are highly likely to stay as engaged players this period while players that are not very interested in the last period probably will not become avid fans all of a sudden. The quadratic term for performance is negative and significant (b = -.01, p < .01) while the linear term for performance is positive and significant (b = .12, p < .01). These coefficients imply an inverted U relationship between performance and preference. The turning point is beyond majority of the data so the relationship is a positive one with diminishing return. This implies that preference benefit from performance with a decreasing marginal return. This finding speaks to a phenomenon well identified in the behavioral literature – human beings prefer moderately challenging tasks (Atkinson 1958; Mathwick and Rigdon 2004; Csikszentmihalyi 1990). Players don't enjoy games where they got stomped by players of much higher skill level or the games where they crush inexperienced players. Games players enjoy the most are the ones where players are matched with opponents of similar skill levels. This finding emphasizes the importance of having an effective matching algorithm which can find players worthy opponents.

Table 8 and 9 provide the correlations between the recovered latent preference variable and covariates in play and purchase equations respectively. The correlations are relatively low. I also regressed the preference variable on play and purchase variables respectively and both regressions yield low R-squares (0.09 and 0.29). These results highlight the importance of bringing in the latent preference variable in the process of understanding play and purchase behaviors. Figure 7 depicts the evolution of preference over time. The dashed line is the averaged player preference which shows a clear downward trend. The solid lines come from preferences of 30 random players in the sample. The preferences in general have downward trends but fluctuate over time as a result of combined influence from players' past preference and interactions with the game in the current period.

2.5.2 Simulation Studies

Results from the state space model suggest that rewards that help players explore the game become less attractive after purchase and currency rewards will decrease players' purchase intention. Also, I have found empirical evidence of points pressure for all three types of rewards and level-ups. In this section I seek to gauge the impact of some adjustments of the rewards structure according to the findings on players' play and purchase decisions. Adjustments in the rewards structures I propose are easy to implement for the developer and I will show how those changes influence player conversion and engagement.

The results from the play equation showed that rental rewards and currency rewards become not very appealing to pay players. So, in the first simulation study I want to use customer information (whether players have purchased the character pack) to customize the reward structure. Specifically, for players who own the character pack several rental rewards are replaced by progress accelerators since results from play equation results suggest that rewards that help players progress stays relevant while rewards that help players become less influential. The impact of replacing 1/2/3 rental rewards by progress accelerator rewards on players' play activity are compared to the case with the original rewards structure and the results are displayed in Table 10.

By changing one rental reward to an accelerator reward after players' purchase, the number of game sessions played daily by character pack owners increased by about 4.95%. This number goes up to 11.54% when I replace 3 rental rewards instead. The model predicts that by simply adjusting the reward type based on player investment status the developer can have a decent boost in customer engagement for the group of committed players.

Results of the purchase equation imply that currency rewards work against players' character pack purchase intentions. However, the results of the play equation show that progress towards currency rewards can motivate free users more than accelerators. In the second simulation study I try to gauge the impact of replacing several currency rewards with accelerator rewards. The impact of this change on player purchase and play are shown in Table 11. When I replace one currency reward by an accelerator the decrease in sessions played daily is only 1.45% while the increase in number of character pack purchases is about 4%. If two currency rewards are replaced, the percentage decrease in play is 2.48% while the percentage increase in purchase is over 7%. The model predicts that the decrease in play activities caused by the proposed change is much smaller comparing to the boost in purchase intention. By replacing several currency rewards by progress accelerators, the developer can help convert more free users to pay users without critically hurting players' play intention.

The results of the progression variables in the play equation provided empirical evidence of points pressure, especially for free players. If the developer can somehow make players feel they have made more progress towards their goals, the points pressure effect should lead to higher engagement level. Both Kivetz, Urminsky, and Zheng (2006) and Nunes and Drèze (2006) studied the impact of endowed or illusionary progress on goal pursuit and they both identified increased effort after endowed progress was implemented. Follow their studies I try to measure the impact of endowed or illusionary progress using the model results. A critical level in the rewards structure was chosen and I changed the experience points needed to get to the next level to 120% of the original. When players get to this level, they will receive an experience points bonus at the amount of 20% of the original distance between this level and the next. The impact of percentage progress implied by this manipulation is illustrated in Figure 8. In this way the absolute number of experience points needed for next level are kept untouched while the percentage progress towards the next level is increased. The predicted results of this change on players at this critical level are shown in Table 12. This change brought a decent increase in number of game sessions played daily (from .48 to .82) while players' purchase intention remains largely untouched. The joint state space model predicts a significant increase in player engagement if the developer can increase players' perception of progress by employing endowed progress.

2.6 Implications and Limitations

Firms possess a variety of options for managing customer relationships. Two important factors that influence consumers' decisions over the course of the customer lifecycle are customer commitments and forward-looking goals. The gaming industry is a useful context for studying the interplay between dynamic incentive schemes and customer investments. Video games have, in particular, extensive gamification elements designed to engage and motivate players. These systems often include goals related to rewards and require consumers to invest time and effort. In addition, recent video game business models such as "Free to Play" systems often include elements that encourage consumers to invest in the games. Beyond these design elements, online games are also useful research contexts because the researcher is able to observe detailed consumption data in addition to transaction data.

I use data from an online FTP video game to explore how incentive schemes designed to encourage forward-looking behaviors interact with past consumer investments in a game. Many of my results are consistent with findings from the loyalty program literature. For example, I found evidence for points pressure. Specifically, players are more active when they are closer to earning rewards or to reaching the next level.

The most interesting results are related to the interplay between past consumer investments in the game and response to future rewards. I find that consumer investment in the game moderates player response across different types of incentives. Specifically, I find that rewards that help players explore the game become less appealing after they purchase the character pack while rewards that help players progress in the game stay relevant. This finding has important customer management implications. The key insight is that the incentives that facilitate initial customer expenditures may be ineffective for managing established customers. If the goal is to maximize customer lifetime value then the incentives available to customers may need to change over the customer lifecycle.

I also find that certain incentives may actually deter purchasing. In particular, rewards that provide in-game currency appear to discourage purchasing the character pack. This type of result highlights the complexity of customer management efforts. In game currency rewards increase preference by encouraging play but in-game currency can also act as a substitute for the purchase of the character pack.

The preceding point illustrates the complexity of designing consumer incentive schemes. While some rewards might increase engagement with the game, these rewards might deter purchasing. I illustrated this complexity with a simulation that evaluates the effects of a change in policy that replaces an in-game currency reward with a progress accelerator reward. This switch is predicted to increase purchasing while potentially decreasing game play. This occurs because the in-game currency reward reduces the need to purchase the character pack because in-game currency can be used to acquire characters. In terms of the impact on game play, I find a very minor decrease in play from this change. This is an important type of analysis as there is a need to balance incentives that build engagement with policies that increase purchasing.

I also explored the value of changing the incentive structure for players who have invested in the game through purchase of the character pack. For this simulation, I examine the effects of switching rental rewards with progress accelerator rewards. I find that a simple change to a reward that has value for this class of customers increases the number of sessions played by about 5%.

My findings may have applicability beyond the gaming industry. Loyalty programs have been using incentive schemes widely and the rise of the mobile platform brings about a wave of gamification. Dynamic incentive schemes are picked up by more and more software and application developers as a customer engagement tool. My findings can shed some light on how dynamic incentive schemes in gamification processes should be designed. The proposed model also provides a template for the analyses of these systems. There is often a need to consider product usage and purchasing decisions separately since some incentives may differentially impact these decisions. The model also includes an explicit component devoted to preference development. This is critical in categories where product usage may create positive feedback effects.

While the video game space offers several significant advantages in terms of consumer level data such as detailed product usage statistics, there are other aspects of the data that created limitations that should be acknowledged. For example, the single game nature of the data limits my ability to understand the role of competition. Switching between games is a particular issue in mobile and console gaming because consumers may switch through just a few clicks. It would be interesting to understand how consumer investment and incentives influence switching rates.

An important limitation to my analysis is that I only focus on the purchase of the character pack. As noted, free to play games often also generate revenues through the sale of “skins” or aesthetic items that alter what a game looks like but do not impact game play. The proposed model could be extended to also include the purchase of aesthetic items as investment decisions. This would require extending the model to include an equation for these types of incremental purchases. This is a challenging extension because detailed data on available skins at any moment in time is not currently available. However, this type of extension would make the model more relevant as a tool for examining how incentives change customer lifetime value.

Another interesting question is the relationship between in-game and real currencies. Many games frequently feature in-game currencies (gold or gems) that may be earned through play or acquired via purchases. The determination of the optimal exchange rate between in-game effort and real dollars would be useful information when designing a gamification system.

Chapter 3

Gamification: The Interplay between Dynamic Incentives and Outcome Uncertainty

3.1 Introduction

Gamification is the application of game concepts such as challenges, scoring points and competition between individuals to engage consumers in non-gaming contexts (Blohm and Leimeister 2013). Fueled by the desire to boost customer retention and create reasons for repeat play, firms are increasingly incorporating gamification into their offerings, leading global gamification market projections to exceed $11 billion by 2020 (BusinessWire 2016). While gamification systems are becoming an important marketing tool, incentive programs have long leveraged similar game design elements to motivate behavior. For instance, loyalty programs use dynamic incentives and status recognition to reward members for repeat consumption (Drèze and Nunes 2008; Lewis 2004). Although the loyalty program literature can enlighten managers interested in gamification systems, there are significant outstanding research questions related to how common game structures such as uncertain outcomes affect responses to dynamic incentive structures. The purpose of this research is therefore to investigate how gamification features drive customer engagement and retention.

A key distinguishing characteristic of gamification is the amount of uncertainty involved in task accomplishment. For example, predicting the outcome of an athletic event is a probabilistic task subject to chance variation, which decreases the ability of gamified app players to control their performance outcomes. In traditional loyalty programs, though, tasks are deterministic. Since loyalty program members can earn rewards by meeting predefined purchasing levels, they have a better understanding of whether they can reach task completion milestones. Rewards for participating in gamification systems are hence more uncertain than rewards for participating in loyalty programs.

A related feature of gamification is the degree of uncertainty surrounding status attainment. In gamification settings, player status often depends on some form of relative performance. In traditional loyalty programs, acquiring membership levels is based on individual purchasing levels relative to pre-defined cumulative buying thresholds, so anyone who accomplishes a specified task achieves status. In contrast, status in games is typically based on relative performance, as status is accorded based on ranking relative to other players. The use of status recognition based on relative standings may increase the importance of interpersonal competition in gamification programs compared with loyalty programs.

An open research question related to the differences between gamified processes and loyalty programs is the effect of goal-related outcomes on subsequent behavior. Since failing to achieve goals generally has negative consequences, loyalty programs tend to minimize the frustration of goal failure by designing programs in which everyone can win. Losing is an inherent feature of games, though, especially when players compete against each other, and thus is a generally accepted part of participating. In contrast to the more guaranteed success of winning in loyalty programs, losing or fear of losing in gamification programs may demotivate players, discouraging them from entering games or accelerating their exit from games. But, the possibility of losing renders winning exceptionally rewarding and attractive. In repeated games, it is even possible that losing in a prior round will motivate players to try again after witnessing the rewards bestowed on winners.

I will empirically explore how gamification elements engage consumers using data from a mobile gaming application. Players of the mobile application can predict the results of athletic competitions on a daily basis. Players who make correct predictions consecutively begin winning streaks and can redeem streaks for prizes corresponding to the streak length. Players are ranked according to their own performance (their historical performance) and their relative performance (their performance relative to all other active players).

My findings reveal several effects of gamification design elements. Using a Hidden Markov Model, I identified several segments where reactions to gamification elements are different and consumers are found to be moving amongst those states over time. For some segments I find evidence of points pressure: motivation to participate increases as players approach the next prize level. This finding is consistent with that of the loyalty program literature, which suggests that evidence of progress in goal pursuit, such as proximity to a goal, can boost motivation and involvement (Kivetz, Urminsky, and Zheng 2006; Lewis 2004). I also find evidence that the inclusion of uncertainty can generate unexpected results. For example, for one segment I find that a series of losses motivates players to continue playing, suggesting that players do not view the gaming application to be skill-based, but rather probability-based. While the loyalty program literature would not predict this finding, I reconcile it with ideas consistent in the gambling literature. In terms of status effects, in some segments I find evidence that positive status comparisons can motivate continued effort, but only to a point. For many players, once they achieve the highest status, status comparison can decrease future effort, implying that status comparisons have wear-out effects once players accomplish the social comparison goal of "being the best."

My findings provide actionable insights for the gamification design decisions of marketers. Practitioners and scholars have lamented the failure of most gamified mobile apps to meet the goals they aim to achieve (Hofacker et al. 2016). As marketer interest in applying game concepts to non-gaming applications increases, my work sheds light on the design of such systems. In particular, my findings relate to how outcome uncertainty and goal achievement change the anticipated impacts of dynamic incentive structures and status levels. For example, while loyalty program designers might leverage promotion to an elite status tier as a means to grow customer loyalty, in a gamified system with competition across players, such promotion can catalyze the end of the game and reduce retention.

3.2 Related Literature

Multiple streams of research inform my investigation into gamification. Prior work on loyalty programs, gambling and goals each discuss how gamification may influence consumer behavior. In what follows, I will identify key findings from these literatures that help explain how consumers should respond to gamification programs that combine dynamic incentive schemes with outcome uncertainty.

3.2.1 Uncertainty

Much research has examined how uncertainty drives behavior. Prospect theory holds that consumers are risk-averse to outcomes obtained with uncertainty (Kahneman and Tversky 1979). However, it is readily apparent that uncertainty may not always negatively affect consumer choice. Observations from the gambling industry demonstrate how even moderately uncertain outcomes can motivate risk-seeking behavior. The tendency of consumers to overweigh very small probabilities can partially explain this contradiction (Kahneman and Tversky 1979). Another explanatory factor is the value of recreational activities such as gamification that moderate uncertainty and stakes can affect (Senters 1971). Consumer elaboration over possible outcomes of an event can prolong positive moods and intensify how pleasant events feel (Bar-Anan, Wilson, and Gilbert 2009; Lee and Qiu 2009). As such, people who enjoy surprises appreciate uncertainty when decisions involve affect (Laran and Tsiros 2013). Uncertainty about positive outcomes can thus stimulate positive feelings and arousal, which in turn increase motivation (Shen, Fishbach, and Hsee 2014). This motivation strengthens when consumers focus on the pursuit, rather than the result, of a reward, since the former tends to be a consummatory, affect-rich experience, while the latter tends to be an affect-poor experience. In gamified applications designed to provide game-like experiences rich in affect, the process matters as much as, if not more than, the ultimate outcome, in contrast to loyalty programs where pure economic benefits are more salient.

Moreover, consumer preferences for a certain reward over an uncertain one with greater expected value decrease when consumers focus on the details of choice options (Duke, Goldsmith, and Amir 2018). This decrease occurs especially when rewards are of low value, which tend to make consumers pay attention to the details of choice options due to the accessibility of low values. Gamification programs often provide small or purely symbolic rewards and demand focus on details, since strategic optimization can only occur with a thorough understanding of the rules. Hence, gamification processes may explicitly require players to focus on the details of choice options.

3.2.2 Uncertainty and Reward Proximity

The inclusion of elements of uncertainty comprises a major point of distinction between gamification systems and traditional loyalty programs. Compared to loyalty programs where consumer outcomes are usually explicitly defined based on spending levels, how players of gamified applications behave, in which outcomes are more uncertain, is unknown. Standard loyalty programs reward consumers with points for their purchases of a firm's product, which consumers can accumulate to earn status or other benchmarks of achievement. Many researchers have examined how "points pressure" influences response within dynamic incentive schemes (Taylor and Neslin 2005; Kivetz, Urminsky, and Zheng 2006; Lewis 2004). Since the benefits of loyalty programs become more salient when members reach pre-specified thresholds, members are motivated to increase their expenditures the closer they are to such thresholds (Nunes and Drèze 2006; Taylor and Neslin 2005). This points pressure effect is due to economic and psychological reasons. Reward proximity boosts consumer motivation by increasing the expected value of participating in the program, even if consumer heuristics, rather than absolute expected value calculations, drive such expectations. Reward proximity can also increase effort and commitment towards achieving a goal since it becomes easier to visualize, and, consumers tend to infer higher goal values from higher goal progress (Zhang and Huang 2010).

While both loyalty programs and gamification systems involve the collection of points, participation rewards in gamification systems are contingent on game performance. The uncertainty of such reward outcomes begs the question of how goal proximity affects player motivations in gamified applications. An argument could be made that accumulated points may be less impactful in contexts where it is uncertain whether consumers will be able to acquire points in the future. My first research question for gamification is therefore as follows:

RQ1: In gamification settings, what is the effect of proximity to the next reward level on player motivation to participate?

3.2.3 Uncertainty and Status

The loyalty program literature has also focused on the dynamic incentive of status recognition. Loyalty programs often assign members into status tiers based on accumulated points. Achieving status unlocks benefits such as preferential treatment, upgrades and/or access to amenities. Status can motivate loyalty members to continue or increase efforts to obtain or maintain status. The motivating effect of status may have diminishing returns, though. In multi-tiered loyalty programs, for instance, the number of members with elite status can diminish perceptions of status (Drèze and Nunes 2008). These differential effects of promotion in loyalty programs across status tiers are echoed by Wang et al. (2016). In gamification, status measures can occur on absolute scales as markers of personal progress and milestones, and on relative scales as markers of social standing. In contrast to loyalty programs where status is based on individual performance, relative rankings such as leaderboards in gamified systems can facilitate and even encourage social comparison, which in turn can boost player competition.

Moreover, player progress relative to others in gamification programs may influence their behavior. This is because performance goals are more salient in gamification processes compared to traditional loyalty programs due to comparisons facilitated by features such as rankings and leaderboards. More specifically, as gamification players progress towards goals, they may adopt performance approach goals since they have nothing to lose. Winning or defeating others motivates them to increase their status. However, top status may lead players to adopt performance avoidance goals, since they cannot go higher but must still expend effort to maintain their status. While performance avoidance goals have been shown to decrease motivation and performance (Elliot and Church 1997), how status in gamification systems influence player participation nevertheless remains an empirical question. Formally stated:

RQ2: In gamification settings, what is the effect of status comparisons on player motivation to participate?

3.2.4 Uncertainty in Performance Outcomes

The effect of performance outcomes on subsequent behavior has also received scholarly attention. Achieving a goal can reinforce behavior, such as losing weight and working out, since the behavior emphasizes the importance of the goal (Shah, Kruglanski, and Friedman 2002). Goal achievement can even increase subsequent effort as consumers learn more about themselves and the system in which they participate (Drèze and Nunes 2011). In the case of loyalty programs, consumers may learn the value of participating by experiencing the reward program benefits that their purchasing efforts translate into. Failing to achieve goals, though, can decrease subsequent performance (Soman and Cheema 2004). These differences in subsequent behaviors may reflect the attributions consumers make about outcomes. In deterministic systems such as standard loyalty programs, consumers tend to attribute goal failure to their lack of effort or unrealistic task requirements. Since loyalty program members accumulate points precisely according to their spending decisions and earn status by meeting pre-specified purchasing requirements, they explicitly control their progress. Thus, if loyalty members believe their efforts failed to achieve status, they would simply need to increase their future effort levels to the amount explicitly required. Believing the task is difficult to achieve, though, will lead members to decrease their commitment and motivation towards the goal (Zhang and Huang 2010).

In probabilistic systems such as gamified applications, the attributions consumers engage in that govern their subsequent behavior may depend on how they interpret results (Ayton and Fischer 2004; Burns and Corpus 2004; Clotfelter and Cook 1993). When consumers believe the underlying process is random, they tend to follow the “Gambler’s fallacy,” allowing recently observed outcomes to drive their expectations of future outcomes. For example, if one player outcome occurred for an unusual amount of time in the past several rounds of a series of independent events, consumers will believe the probability of that particular outcome occurring in the near future will be lower since it already experienced its “share.” When consumers believe a task is skill-based, they tend to follow the “Hot-hand fallacy”, believing the abnormal trend described above will persist into the near future because the player is on a hot streak. In gamification settings, outcomes are uncertain by design, rendering how players react to outcomes an empirical question. I formalize this inquiry via the following:

RQ3: In gamification settings, what is the effect of realized outcomes on player motivation to participate?

In summary, this work contributes to the above streams of research in several ways. First, I examine the effect of uncertainty on consumer participation outside of gambling contexts. Second, I study how proximity to reaching a goal affects consumers in game contexts where economic rewards are lower than in traditional loyalty programs. Third, I investigate how status recognition affects behavior in systems where status is relative, not absolute. Fourth, I explore how realized outcomes drive subsequent behavior in applications where outcomes are uncertain by design.

3.3 Data and Model Free Analysis

In this section I will first describe the gamification elements of the mobile application where my data comes from. Then I will present the details of the sample collected for empirical application along with some model free analyses concerning my research questions.

3.3.1 The Mobile Application

The data comes from a mobile application in which players predict the outcome of sporting events. The game application is free to play and provides benefits based on a player's ability to predict the outcomes of five pre-selected sporting events each day. The events involve binary predictions (which team will win, will a player reach a certain point level, etc.), of sporting events including professional and collegiate football, professional and collegiate basketball, hockey, baseball, soccer and Olympic events. If a player correctly predicts the outcomes of at least three of the day's five events, the player's "streak" is incremented by one. If a player predicts two or fewer events or does not play, the players' streak resets to zero. The majority of prediction questions relate to picking the winner of athletic events. The prediction tasks may also involve predictions of individual player performance such as predicting which of two players will score more points in an event.

Whenever players reach milestone streaks, they can redeem their streaks for corresponding prizes. For example, a winning streak of four can be redeemed for $1, a streak of eight can be redeemed for $3 and a streak of twelve can be redeemed for $10. Achieving a winning streak of sixty consecutive days (during which a player successfully picks at least three events correctly each day) yields a $1 million reward. Prizes can be redeemed in one of two currencies. One currency is virtual and can be used to obtain discount coupons from third party merchants or physical prizes such as sports themed apparel. The other currency is hard and can be transferred from the app to a designated bank or PayPal account. However, withdrawals are only permitted when a player reaches a predefined threshold. The virtual currency is only available for streaks of one and three. All other milestone streak levels are redeemable for hard cash. Streaks are cleared once redemptions are made and players can begin another streak after redemption. The app tracks how much virtual and real money a player earns and the redemptions a player makes . Table 13 describes the value of each level of reward and corresponding streaks required.

The app also calculates the percentage of correct picks made by a player during the past 30 days and provides a corresponding percentile among all active players who played at least once in the last 30 days. Guru status (novice, intermediate, expert, etc.) is a value that reflects the percentile range a player falls into, which compares a player's performance to all other active players' performance in the last 30 days. A player's percentile rankings and Guru status are updated daily and players are informed of any promotions and demotions of their Guru status through app notifications. Table 14 presents details of the Guru status feature of the mobile app.

My data provides an ideal context for investigating gamification due to the unique features the mobile app possesses. First, various game elements are used to engage customers, such as points/streaks and status variables including Guru status and percentile that reflect a player's accumulated achievements and within-game social position. Second, uncertainty plays a key role in engaging players. Departing from typical loyalty programs where reward structures are deterministic, in my gamification context players face more uncertainty from both the results of sports games and other players competing in the game when they attempt to climb the streak and Guru ladder. The feeling of winning prizes and defeating others creates enjoyment, which in turn motivates players to play more. The sports app also enables me to isolate gamification effects, whereas in contexts of gamified apps tied to firms whose primary service is not gamification, user behavior may be driven in part by loyalty to the firm or its goods.

3.3.2 Sample

My sample consists of 2,000 users randomly selected from the cohort of players who registered between February 9, 2016 and February 29, 2016. Daily picks and results for those players from the day they registered until the last day of the observation window (April 27, 2016) were recorded. Prize variables involving virtual currency are converted into cash with a conversion rate calculated from an in-app item labeled both in virtual currency and hard cash. (See Table 15 for a description of the variables and Table 16 and 17 for the descriptive statistics and correlations of those variables).

The key factor that influences players' level of interest is the streaks since the rewards (either hard cash or virtual currency) are exclusively based on streaks accumulated by players. As such, streak related variables comprise the key decision-making factors for players. It would be straightforward to expect players to be more motivated when their streak increases, since a higher prize is at stake. In addition to players' current streak, I also included variables of losing streaks to test my research question regarding how a series of outcomes from events with uncertainty influences subsequent effort. Compared to winning streaks, loss streaks provide a cleaner context to test probability perception biases like the Gambler's fallacy and Hot-hand, since no prize is involved for loss streaks. For my sample the success rate in getting one streak by players is 54% and the distribution of success rate resembles a normal distribution. Thus, I think the process is more likely to be a random process. If players subscribe to the Gambler's fallacy, I would expect them to play more when they incur a series of losses, since they would believe that losses had their share and now it is time for wins. On the other hand, if players subscribe to the Hot-hand, they would believe they have a "cold hand" with a series of losses, and motivation to play wanes accordingly. Wins needed to reach the next prize level can help test for points pressure effects or the goal gradient hypothesis. If points pressure is salient, I would observe that players are more motivated the fewer wins they need to reach the next prize level. I do not include variables such as longest winning streaks or similar measures because they are highly correlated with other prize variables.

As a dynamic incentive scheme, another important factor that influences players' engagement level overall are prizes redeemed and lost. Prizes accumulated or missed denote goal achievement and failure players experience in the game and as such should affect further participation decisions. This is because feedback from the system should help players understand the system and themselves better through learning. Prize redeemed and lost in total help testing for any long-term impact of success and failure, while prize claimed and lost yesterday should reflect the immediate impact of obtainment and miss of rewards. Findings from the loyalty program literature lead to the expectation that in both the short and long-term, goal success will lead to strengthened motivation, while goal failure will lead to player frustration. The conclusions from loyalty program literature about goal success and failure are drawn from situations with much less uncertainty compared to gamification programs. My study can therefore shed light on how uncertainty can influence the impact of goal success or failure on player motivation.

Another feature that sets this gamification process apart from the traditional loyalty programs is the social status system which is called "Guru stats" in the application. Guru status represents players' accumulated achievements and within-game social position, which serves as another major motivation for players to continue to participate. The status literature suggests that customers engage in both upward and downward social comparisons spontaneously, leading me to expect players with higher status to show more interest in playing the game compared with players with lower status. Research on goal achievement type suggests that different types of goals may have different effects on player motivation. When players climb to the top of the status ladder, their achievement goal type may have changed from approach type to avoidance type.

Performance-avoidance goals have been shown to be inimical to both intrinsic motivation and performance (Elliot and Church 1997; Burns and Corpus 2004). I therefore expect player motivation to play the game decreases after a player reaches the top. I also take time trends into consideration by controlling for the number of days elapsed since registration.

3.3.3 Model Free Analysis

Evidence on how consumers behave in gamification systems comes from my model-free analyses. Specifically, Figure 9 depicts the effect of points pressure. I plot players' playing rates versus the distance to the next reward threshold and include streaks greater than 4, because at those streak levels, the impact of nearness to goals can be seen more clearly. Players appear to show more interest in the game as they approach a goal. A downward trend line is spotted when I regress play rate on wins to next prize level ($p < .01$). This observation suggests that points pressure has a positive influence on participation, a finding consistent with the findings from the loyalty program literature (Kivetz, Urminsky, and Zheng 2006; Lewis 2004).

To see how feedback from the gamification system influences player motivation and whether there is evidence for either Gambler's fallacy or Hot-hand, I plot play decisions against loss streaks. As shown in Figure 10, a mild upward trend ($p < .01$) is spotted, implying that as players' loss streaks increase, they become more interested in playing. Note that I only include loss streaks of up to 5 consecutive losses, because observations for loss streaks of greater lengths are limited. This evidence provides some support for Gambler's fallacy rather than the Hot-hand. This implies that players are more likely to perceive this game as an inanimate random process.

Next, I plot play decisions against player status level. Figure 11 shows playing rates across the four status levels. Status appears to positively affect motivation to play for players with ongoing streaks, especially the higher the status. By regressing play rate on status an upward trend is detected ($p < .01$). This model free evidence supports my speculation that in general players with higher status are more active. This model free analysis does not render support for my argument of potential detrimental impact of the top status, and I will use a statistical model for a deeper understanding of the data.

The model free evidence provides a rough picture of relationships between factors of interest and player's play intentions. However, this is not considering many observed and unobserved factors that potentially play crucial roles in the process. In the next section I will present the statistical model to tackle this issue.

3.4 Model

One key feature of dynamic incentive schemes is that consumers or players will be interacting with the rewards system and the firm over a relatively long time span. The possibility of relationship development and dynamics over the life cycle of customers should not be ignored. It is likely that there exist several customer states where their reactions to certain features of the incentive schemes are different and consumers may move between these states across time based on their past and present interactions with the incentive structure. Unfortunately, these customers states are not directly observable and they can only be inferred from observed customer behaviors. To explicitly model the dynamics in relationships between customers and the gamification process, I choose to employ the Hidden Markov Model which has been widely used in relationship marketing literature (Netzer et al. 2008; Montoya et al. 2010; Ascarza et al. 2018). Hidden Markov Models explicitly specify hidden states where separate set of parameters that gauge customers' reactions to key factors are allowed for each state and customers can move between states according to transition probabilities which describe how likely customer may move. Hidden Markov Models have three main components: (1) the transition matrix; (2) the initial state distribution; and (3) the state dependent choice. In the following subsections I will in detail discuss the specification of each component and describe my estimation strategy.

3.4.1 The Nonhomogeneous Hidden Markov Model

Transition Matrix

Hidden Markov Models (HMM) assume that customers move between unobservable states in each period and their movement follow a Markov process where future probabilities are independent of the past given the present. Transition matrix Q_{it} describes the probability that a player *i* moves from state *k* in period *t* - 1 to state *k'* in period *t*

$$Q_{it} = \begin{pmatrix} q_{it11} & \cdots & q_{it1K} \\ \vdots & \ddots & \vdots \\ q_{itK1} & \cdots & q_{itKK} \end{pmatrix} \tag{17}$$

$$q_{itkk'} = P(S_{it} = \ k' \mid S_{i,t-1} = k, x_{i,t-1}) \tag{18}$$

for $k, k' \in \{1, \dots, K\}$, where S_{it} denotes the state of player *i* in period *t* and $x_{i,t-1}$ is a vector of time-varying covariates influencing customers' state transition from period *t* – 1 to *t*.

In my application different states represent different levels of intrinsic interest customers have towards the gamification process. Thus, those states can be ranked from low to high. A customer that is in a lower state generally has limited interest in the gamification process thus unlikely to engage with the product compared to a customer at a higher state. The state status of customers is influenced by relationship factors. With this in mind I decide to use a threshold model to describe state transition probabilities. In a threshold model the transition probabilities are decided by the comparisons of satisfaction level to thresholds. Specifically, I use an ordered logit model to calculate transitions probabilities. It is also highly likely that each customer may have their unique thresholds and to take the unobserved heterogeneity coming from this factor into account I decide to allow thresholds to be individual specific by adding random effects. The transition probabilities are also influenced by time-varying covariates which measure relationship factors, hence the proposed HMM is a nonhomogeneous HMM. The transition probabilities in equation 17 can be written as:

$$q_{itk1} = \frac{\exp(\mu(1)_{ik} - x_{i,t-1}{}'\rho_k)}{1 + \exp(\mu(1)_{ik} - x_{i,t-1}{}'\rho_k)}$$

$$q_{itk k'} = \frac{\exp(\mu(k')_{ik} - x_{i,t-1}{}'\rho_k)}{1 + \exp(\mu(k')_{ik} - x_{i,t-1}{}'\rho_k)} - \frac{\exp(\mu(k'-1)_{ik} - x_{i,t-1}{}'\rho_k)}{1 + \exp(\mu(k'-1)_{ik} - x_{i,t-1}{}'\rho_k)}$$

$$q_{itkK} = 1 - \frac{\exp(\mu(K-1)_{ik} - x_{i,t-1}{}'\rho_k)}{1 + \exp(\mu(K-1)_{ik} - x_{i,t-1}{}'\rho_k)} \tag{19}$$

for $k \in \{1, \ldots, K\}$ and $k' \in \{2, \ldots, K-1\}$, where $\mu(k')_{ik}$ is the k'th ordered logit threshold for individual *i* at state *k*, ρ_k is a vector of parameters that captures the impact of time-varying factors that can potentially influence consumer's transition propensity from state k and x_{it} are the corresponding time-varying covariates. By having state specific parameters (ρ_k), the impact of those time-varying factors can differ based on specific states customers are at in a certain period.

Netzer et al. (2008) suggests that the time-varying covariates included in the transition probabilities are the factors having enduring impact on customers' interest level. Following this logic, I decide to include prize redemptions and misses together with relative status variables in the transition probabilities. *Value of Streak Redemption* and *Value of Streak Reset* measure the value of prize player claims or fails to claim (either as a result of failing to make 3 correct picks out of all 5 choices or failing to participate). These two variables represent performance outcomes players obtain after putting in consistent effort and either getting a reward or missing a reward should have an impact on player's intrinsic interest towards the gamification process. In addition to redemption or miss on the current day, I also include *Total Prize Redeemed* and *Total Prize Lost with Reset* to see how accumulated gains or losses influence the relationship between customers and the mobile app. On the status side, I include four status dummies (low, middle, high and top) to study the impact of relative status on engagement intentions. The social status which is represented by Guru status in the app does not directly generate tangible benefits like rewards from redeeming streaks. However, as the basis for social comparison and recognition status may have long enduring impact on players' interest in the gamification process. As a control variable I include *Days since Registration* to model any time trend that possibly exists and it is expected that players will gradually lose interest in the gamification app as time goes by.

Initial State Distribution

The transition matrices describe how customers may swing between states and how time-varying factors can play an important role. However, I still need to specify where customers start, i.e. their initial state distribution. For nonhomogeneous HMM I can either use the stationary distribution of the transition matrix where all the time-varying covariates are set to their respective mean, or the stationary distribution of the transition matrix where all the covariates are set to zero. Since my data is not left-truncated (records of players cover their registration), I choose to use the stationary distribution of the transition matrix with zero for all covariates (Netzer et al. 2008). I use π_i to denote the initial state distribution for player *i*.

The State Dependent Choice

The initial state distribution determines players' starting point and the transition matrix in each period will decide which state players will land in each period. After players' states are decided they will make their decisions. The decisions are conditionally independent when state is given. In my case engagement is measured by a binary variable which equals one when player decided to participate and zero otherwise. To fit this data structure, I choose to use a logit model to describe players' state dependent choices (equation 20). The probability of consumer *i* participating in period *t* at state *k* is:

$$m_{it|k} = \frac{\exp(\tilde{\beta}_{0k} + z_{it}'\beta_k)}{1 + \exp(\tilde{\beta}_{0k} + z_{it}'\beta_k)} \tag{20}$$

for $k \in \{1, \dots, K\}$, where $\tilde{\beta}_{0k}$ are state specific intercepts, z_{it} are time-varying covariates that influence participation decisions and β_k are the corresponding set of parameters measuring the impact of those factors.

In the transition matrices I used a threshold model to accommodate the assumption that the states can be ranked from low to high. Similarly, in the state dependent choice part I choose to restrict the state specific intercepts to be non-decreasing such that those states can be identified. Specifically, state specific intercepts are structured in a way such that $\tilde{\beta}_{01} \leq \tilde{\beta}_{02} \leq \dots \leq \tilde{\beta}_{0K}$ is ensured (equation 21):

$$\tilde{\beta}_{01} = \beta_{01}$$

$$\tilde{\beta}_{0k} = \beta_{01} + \sum_{k'=2}^{k} \exp(\beta_{0k'}) \tag{21}$$

for $k \in \{2, \dots, K\}$.

Different from the variables in the transition matrices, the variables included in the state dependent choice only influence players' short-term decisions rather than long-term changes like state shifts. I choose to include streak related variables in the state dependent choice since rewards (either hard cash or virtual currency) are exclusively based on streaks accumulated by players. As such, streak related variables comprise the key decision-making factors for players. I would expect players to be more motivated when their streaks increase, since higher prizes are at stake. In addition to players' current streaks, I also included *Loss Streak* to test my research question regarding how performance outcomes from uncertain events influence future effort. With no prize involved, loss streaks appear to be a better context to test probability perception biases, which in my case are Gambler's fallacy and Hot-hand. Specifically, if players believe the process is random and fall for the Gambler's fallacy, they are expected to be motivated by a series of losses, since they would believe that losses had their share and the tide will turn. On the other hand, if players subscribe to the Hot-hand, a series of losses would result in doubts about their capability of making right predictions and their motivation to participate will decrease accordingly. Wins needed to reach the next prize level can help test for points pressure effects or the goal gradient hypothesis. If points pressure is salient, I would observe that players are more motivated the fewer wins they need to reach the next prize level. Other streak related variables are not included because they are highly correlated with prize variables. I use $m_{it} = [m_{it|1}, m_{it|2}, \ldots, m_{it|K}]'$ to denote the vector of choice probabilities for player *i* in period *t*.

3.4.2 Likelihood and Estimation

Likelihood

For each player *i* I observe his/her participation decisions in each period from the registration day till the end of the observation window. Since it is assumed that players' transition between states follow a Markov process, the individual likelihood for any player is the sum of all possible routes (equation 22)

$$P_i(Y_{i1}, Y_{i2}, \ldots, Y_{iT}) = \sum_{k_1=1}^{K} \sum_{k_2=1}^{K} \ldots \sum_{k_T=1}^{K} \Big[P(S_{i1} = k_1) * \prod_{\tau=2}^{T} P\big(S_{i\tau} = k_\tau \mid S_{i,\tau-1} = s_{\tau-1}\big) * \prod_{v=1}^{T} P(Y_{iv} = y_{iv} \mid S_{iv} = s_v) \Big] \tag{22}$$

Although this equation is intuitive and straightforward, it is computationally intractable. Following MacDonald and Zucchini (1997), the individual likelihood can be rewritten into equation 23:

$$L_{iT} = P(Y_{i1} = y_{i1}, \dots, Y_{iT} = y_{iT}) = \pi_i{}' \widetilde{m}_{i1} \prod_{\tau=2}^{T} Q_{i\tau} \widetilde{m}_{i\tau} \mathbf{1} \tag{23}$$

where $\widetilde{m}_{it|k} = m_{it|k}{}^{y_{it}} * (1 - m_{it|k})^{(1-y_{it})}$ for $k \in \{1, \dots, K\}$, $\widetilde{m}_{it}$ is the diagonal matrix where $\widetilde{m}_{it|k}$ are diagonal elements and $\mathbf{1}$ is a K by 1 vector of ones. The likelihood for all players is the product of individual likelihood:

$$L = \prod_{i=1}^{N} P(Y_{i1} = y_{i1}, \dots, Y_{iT} = y_{iT}) \tag{24}$$

To calculate the likelihood presented above, one important issue needs attention is underflow since the likelihood is a product of multiple probabilities. Following MacDonald and Zucchini (1997), in the calculation of individual likelihood at each time period, I divide the joint state likelihood by L_{it}/K while the logarithms of those scaling factors are accumulated and added to the log likelihood. By following this procedure, I make sure the likelihood or the log likelihood can be computed properly.

Estimation

I use a standard Hierarchical Bayesian approach to estimate the proposed nonhomogeneous Hidden Markov Model. The parameters that need to be estimated can be put into the following groups: (1) state thresholds which are random effects parameters $\mu(k')_{ik}$; (2) means of state thresholds δ; (3) variance-covariance matrix of the state thresholds Σ; (4) transition probability parameters ρ_k; (5) state dependent choice intercepts $\tilde{\beta}_{0k}$ and (6) state dependent choice parameters β_k. In each iteration each set of parameters are drawn sequentially from their corresponding posterior distributions. Specifically, the first set of parameters will be drawn and updated, then the second set of parameters will be drawn using the updated values for the first group and original values from the last iteration for other sets of parameters. Each set of parameters will be updated sequentially following this procedure. For means and variance covariance matrix of state thresholds I am able to update the values use Gibbs Sampling method. For the other four set of parameters their posterior distributions do not have close forms hence I use the Metropolis Hastings algorithm to draw from their posterior distributions. I completed the estimation program in the statistical software R. In total 50,000 iterations were run with the first 30,000 as burn-in. The results of the proposed model will be discussed in the next section.

3.5 Empirical Analysis

In this section I will report the results from the proposed nonhomogeneous Hidden Markov Model together with a static model which did not consider the relationship dynamics between players and the mobile application. I will especially focus on the part of results concerning my three research questions on points pressure, Gambler's fallacy and Hot-hand, and the impact of social status.

3.5.1 Static Model Results

Before running the proposed HMM I first run a static model as a base model and a comparison. Specifically, I use a mixed logit model with random effects for each individual player to study their participation in the gamification process. I combine the state transition covariates and state dependent choice covariates into one matrix $\mathrm{w}_{it} = [x_{it}\ z_{it}]$ and the probability of player *i* playing the game in period t is:

$$r_{it} = \frac{\exp(\gamma_{0i} + w_{it}'\gamma)}{1 + \exp(\gamma_{0i} + w_{it}'\gamma)} \qquad (25)$$

This static model is also estimated using a Hierarchical Bayesian approach where parameters are sequentially drawn from their corresponding posterior distributions. The results of this model are presented in table 18.

Number of streaks has a positive and significant relationship with playing, so players are more engaged with higher prizes at stake. Interestingly the loss streak also has a positive and significant relationship. This is evidence for Gambler's fallacy. This result suggests that players are more likely to play when they have a loss streak at hand. To my surprise number of wins needed to get to the next prize level also has a positive and significant relationship with playing and this result does not support the points pressure effect.

The immediate impact of streak redemptions is as expected. Streak redemptions lead to higher interest while streak misses are frustrating events for the players. Surprisingly total prize missed has a negative and significant relationship with playing intentions. It seems that in the long run players are not discouraged by missed prizes and the losses in a way make winning more attractive such that players are more engaged. A negative time trend is also spotted, which is consistent with my speculation that players will gradually lose interests.

The four Guru status variables all have positive and significant coefficients, which means that comparing to no status, players with any status are more engaged. The coefficients for the low, middle and high status are increasing, signaling higher engagement level at higher status. However, the coefficient for the top status is smaller than high status. This down-going trend from the high to the top status suggests that when players reach the top status, they become less engaged with the gamification procedure. Note that the results above are from a static model which does not consider dynamics in relationships over time. Failing to incorporate dynamics in the relationship can lead to erroneous conclusions and in the next subsection I will present the results for the proposed nonhomogeneous HMM.

3.5.2 Nonhomogeneous HMM Results

State Selection

The first stage in estimating an HMM is to select the number of states. Unlike other parameters in the model, the number of states has to be selected manually. This is done by comparing statistics of models with different number of states using different model selection criteria. The results of the comparisons of log-marginal density, log Bayes factor and DIC all suggest a three state HMM (Table 19).

Dynamics in State Membership

The results for the selected three state nonhomogeneous HMM are presented in Table 20 and Table 21. Since three states are ranked in the ordered logit threshold model, I name the three states low (state 1), middle (state 2) and high (state 3) where customers' interest levels are higher than the previous state. Figure 12 depicts the evolution of shares of each segment. In the beginning majority of the customers are in the high state where interest level towards the gamification application is the highest. As time goes by some customers fall from the high state to the middle and low state and at the end of the observation window the percentage of customers in the low state has surpassed that of the high state. The middle state possesses a relatively small but steady share of 10 – 15%. The dynamics of state membership over time meets my expectation that players have more interest in the gamification process in the beginning and their interest gradually decrease over time. Each state possesses its own set of parameters and I will discuss how customers in different states react to gamification features differently.

Points Pressure

For state dependent choices I included the variables most crucial for the short-term decision of whether to participate, which are the streak related variables. *Wins to Next Prize Level* is particularly included to test for points pressure effects. For the low and high states, the coefficients for *Wins to Next Prize Level* are negative and significant (b = -3.90 $p < .01$ for the low state and b = -.41 $p < .05$ for the high state). This is evidence for existence of points pressure. Specifically, customers in the low and high states (which together actually cover the majority of players) are more motivated to play when they are closer to the next reward level. And the points pressure effect is much stronger in the low state giving the magnitude of the effect. It seems points pressure plays a more important role in engaging players when their intrinsic interest is low. The coefficient of *Wins to Next Prize Level* in the middle state is positive and significant to my surprise. I think the reason behind this is the unique feature of the laddering reward system. The laddering structure of the reward system means that when players reach a higher reward level, they are quite some distance away from the next while the prize at stake has increased significantly. These two factors are conflictive towards player motivation and I think in the middle state players are more inspired by the bigger prize money compared to the effort needed to obtain it.

Gambler's Fallacy vs. Hot-Hand

To see whether players view the gamification process at hand as more of a "cold" inanimate random process or a task that depends on their ability, *Loss Streak* is included in the state dependent choice equation. The coefficient of cumulative losses in the low and high states are negative and significant (b = -2.63 $p < .01$ for the low state and b = -.52 $p < .01$ for the high state). It seems that players in these two states view the process as a task that requires skill such that when they lose consecutively, they start to doubt their ability and become frustrated. This is the evidence for Hot-hand. Interestingly the coefficient for the middle state is positive and significant (b = .31 $p < .01$).

For players in the middle state they actually view the procedure as an inanimate random process and they fall for the Gambler's fallacy. When they have a series of losses, they believe that the tide will turn and they will start to win. It is also worth noting that players move between states which suggests that their perceptions of the gamification procedure may actually change over time.

Social Status

The status variables represented by *Guru Low, Middle, High* and *Top* are included in the transition probabilities of states since they are more likely to have an enduring impact on players' intention to play. For players in the low state all the status coefficients are positive and significant except for *Guru Top*. Magnitude-wise *Guru Middle* has the highest coefficient ($b = .94$ $p < .01$). It looks like the impact of social status will increase to a point and start to drop off afterwards. A similar trend can be found in the middle state. *Guru Low and Guru Middle* have positive relationships with potential upward movements in states ($b = .38$ $p < .01$ for *Guru Low* and $b = 1.69$ $p < .01$ for *Guru Middle*) while the coefficient for *Guru High* is only marginally significant ($b = .19$ $p < .1$). Moreover, the coefficient for *Guru Top* is negative and significant ($b = -.24$ $p < .01$). Interestingly the situation for the high state is very different. Only *Guru Top* in the high state has a positive and significant coefficient ($b = .68$, $p < .01$). *Guru High* is not significant while *Guru Low* and *Guru Middle* both have negative and significant coefficients ($b = -.55$ $p < .01$ for *Guru Low* and $b = -.83$ $p < .01$ for *Guru Middle*). It seems that when players' interest levels are high, they aim for the best.

In both the low and middle state, I find that players start to lose interest when they reach a very high status in the game, which I call the "finishing the game effect". Unlike loyalty programs where consumers will strive to keep their status level for the associated benefits, a big proportion of the benefits of the gamification processes are psychological about winning and achieving rather than the tangible rewards associated (which are usually low in value). The top status in the game

(> 99%) is hard to reach and the majority of the players may feel they have beat the game when they reach the high status (> 90%). Once they are in the high status, they are likely to feel they have achieved everything within their capacity and start to say goodbye and move on. In comparison players in the high state are the group of most active players aiming for the best and they enjoy the challenge of obtaining and preserving the highest title in the gamification procedure.

Other Findings

The results for the prize redemptions and misses again emphasizes the difference between loyalty programs and gamification procedures. The immediate impact of streak redemption is found to be negative and significant for the middle and high state ($b = -.64$ $p < .01$ for the middle state and $b = -.14$ $p < .01$ for the high state). Although redeemed prizes in the reward system serve as positive feedbacks on performances, it is also worth noting that by redeeming a streak players also finish one "round" in the game. The psychological benefits for winning comprise a significant part of enjoyment in playing a game. Repeating the game by starting new rounds will generate decreased level of thrill since players have already experienced the game mechanism in full and this could be the reason why players are less motivated after streaks are redeemed. On the other hand, losing in games or gamification procedures may not necessarily be a negative event. The coefficients of *Value of Streak Reset* are positive and significant for the low and middle state ($b = .53$, $p < .01$ for the low state and $b = .29$ $p <$,01 for the middle state). Losing may render players the feeling of incompletion and make prizes from winning more salient, thus pushes players to start a new round. These results further set gamification procedures apart from traditional loyalty programs. The time trend is in general negative suggesting players lose interest gradually over time.

Transition Propensity

To see how likely players are moving between the three states I calculated the mean posterior transition matrix (Table 22). It seems that the low and high states are quite sticky and players tend to remain there. The middle state is a more transient state with players less likely to stay and more likely to move to the low state rather than the high state, which is consistent with the decreasing trend of player interest over time. This is only the mean of the transition probability and I have plotted the posterior distributions of the transition probabilities in Table 22 to get a better understanding of transition propensities in Figure 13 – 16. Figure 13 depicts players' tendency to state in current state for the three states. The low state appears to be the stickiest state where players will be trapped, followed by the high state. It looks like there exists a very loyal fanbase for this mobile app that will remain in the high state for quite a long time. The middle state serves as a transition between the low and high states with relatively low intention to stay. Figure 14 describes players' intention to move up from the low state. It is clear that players are unlikely to move when they reach the low state (trapped). Figure 15 presents players' intention to move up or down from the middle state. When players reach the middle state, they are more likely to move down to the low state rather than move up to the high state. Lastly, Figure 16 presents players' propensity to move down from the high state. It is more likely for players to move from the high state to the middle state rather than directly to the low state which is like a sudden death. However, the difference is marginal.

3.6 Discussion

By definition, gamification involves the application of game mechanics to customer focused marketing or employee management. In practice, gamification usually involves creating dynamic incentive schemes that provide rewards and recognitions for the achievement of goals. Given that marketing academics have devoted considerable attention to issues related to loyalty programs, the academic literature includes several insights relevant to gamification. However, my conjecture is that additional research is warranted because gamification systems tend to include significant levels of uncertainty that are not included in standard loyalty programs.

My results suggest that some findings from the loyalty program literature are robust to game environments to a large extent. In particular, points pressure seems to exist within gamification procedures for two of the three segments. I also found a positive relationship between participation and status levels. These are important results because they speak to the ability of gamification systems to incentivize consumers with both monetary rewards and achievements that provide psychological benefits. The points-based achievement systems are the core of these programs and nearness to rewards seems to increase participation levels.

Interestingly, there are certain elements of the game under study that yield empirical findings that are not consistent with the loyalty program literature. For example, I find that for two of the three segments identified, the high status does not motivate players as much as the middle status and being in the top status does not motivate players. This finding has critical implications for gamification systems and highlights the differences between loyalty programs and games. In contrast to standard loyalty programs in which consumers may strive to maintain a position in a top status tier, in a game-like environment consumers may lose interest after “winning” the game and achieving high status level. Also, I find that losing might not be the worst thing. In gamification programs losing is recognized by players as an essential part of the game and losing may even inspire players to play more by making winning more attractive.

In terms of limitations and future research opportunities, it is important to emphasize that my research is exploratory and that my findings are correlational in nature. The data was collected in the course of normal operations of the firm. This raises issues related to the endogeneity of program design elements. However, it should be noted that the firm has altered its game design over time and that the game continued to evolve in the time period following data collection.

A second limitation is that the data is sourced from a single firm. As with almost all customer relationship datasets there is a tradeoff between having detailed longitudinal customer histories for consumers but having limited or no ability to observe behavior outside of the program. In these types of environments, it is important to note that generalizability may be limited by the idiosyncrasies involved in the data generation process. As noted, gamification techniques are employed in a wide range of categories such as marketing promotions, employee management systems and even fitness trackers.

The data is sourced from a mobile application that includes explicit aspects of gaming and gambling. Consumers seek to achieve “winning streaks” and can “win cash prizes.” However, the application is very much a marketing promotion. The firm’s business model is to generate advertising revenue through customer acquisition and retention rather than to generate profit through the surcharge or “vig” attached to a wager. When game mechanics are used in different settings that look and feel less like gambling to consumers, the impact of the structures I study may be different. Future research that examines game mechanics in different categories would be useful. Specifically, research that focuses on how gamification works across categories would be useful.

Another limitation and opportunity for additional research is related to the lack of variation in terms of reward levels and thresholds in my data. This is a consistent issue in loyalty program research as well. In most loyalty program research, the reward levels and thresholds are fixed

(Lewis 2004). This means that the analysis is only able to say whether a reward alters behavior rather than how reward levels continuously alter response.

Chapter 4

Discussion

The Internet and mobile devices have enabled firms to interact with their customers at any time over various aspects concerning purchase and consumption. Along with many other marketing tools, dynamic incentive schemes have been updated with new dimensions and elements. The existing marketing literature on dynamic incentives which mainly resides in the loyalty program literature has become insufficient for providing guidance for marketing practice. In the purpose of filling this gap, I focus on research questions about dynamic incentives raised in the digital era.

In my first essay I studied how customer investment and dynamic incentive features jointly influence consumers' purchase and play decision in the context of the video game industry. I find that rewards provided by dynamic incentive schemes have different impact on players based on their commitment status which is represented by the purchase of the character pack. The rewards that help players explore become less attractive after player commitment while rewards that help players progress stay relevant. The rewards system is designed towards converting free users to pay users. For players who have committed, the dynamic incentive scheme does not fulfill their needs. This finding actually is quite common in digital products. Firms specializing on digital products oftentimes put great emphasis on acquiring new users with existing users being ignored to some extent. This could hurt established relationships and can be detrimental to firm growth in the long run. Through this research I hope to shed some light on the proper design of dynamic incentive schemes for digital products like video games, an area that has not attracted enough attention from marketing scholars.

Gaming principles and elements like dynamic incentives have long been used in many non-gaming occasions like education, health and wellness, employee training etc. However, virtual platforms have brought gamification to a whole new level. Some new gaming elements that are rarely found in traditional loyalty programs are enabled in gamification procedures. These new developments provide many exciting research opportunities for marketing researchers. In my second essay I look at how consumers react to uncertainties in gamification dynamic incentive schemes. In my case uncertainty resides both in the accumulation of points/streaks and customers' social status which is relative to all active players. A key finding is that unlike top status or tiers in loyalty programs which customers try hard to maintain, top status in gamification procedure serve more like a closure to many players. When players reach the top, they may feel they have achieved everything they can within their capacity and they are ready to move on to new challenges. This distinction sets gamification apart from loyalty programs and firms should be aware of the different meaning of status in game-like procedures. Through this work I hope to reveal some interesting and unique properties dynamic incentives in gamification procedures possess, which can be crucial in guiding the design of such processes and applications.

Beside the two topics that I have touched upon in the two essays, there are many other interesting opportunities concerning dynamic incentive schemes in digital marketing. One of those topics is the issue of switching in digital platforms. Switching in online and mobile channels may take only a few clicks while in tangible channels switching usually takes much more effort. Also, in the digital world it is hard to differentiate one's product since the cost of making me-too products are relatively low. These factors suggest that competition among digital products can be very intense and understanding crucial factors that help firms build switching barriers can be crucial for a firm's survival and flourishing.

Another topic that needs more attention from marketing researchers is the design of tasks or goals in dynamic incentive schemes. In modern life attention from customers has become a scarce resource that many firms compete for on a daily basis. This means that if goals in dynamic incentives cannot provide the benefits customers look for, they will lose interest rapidly. Easy tasks hardly can provide enough challenge and thrill while hard tasks can potentially frustrate customers which might lead to churn as well. In addition to difficulty levels, other elements like reward types and timing, effort types (money vs. time), social features (cooperative tasks vs. solo tasks, competition) can all be important factors that decide whether customers would enjoy the goal pursuing process. Research on this topic can potentially offer great value in guiding marketing practice in the design of dynamic incentive schemes.

Bibliography

Ascarza, E., Netzer, O., & Hardie, B. G. (2018). Some customers would rather leave without saying goodbye. *Marketing Science*, 37(1), 54-77.

Atkinson, John W (1958). Towards experimental analysis of human motivation in terms of motives, expectancies, and incentives. *Motives in fantasy, action and society* : 288-305.

Ayton, Peter, and Ilan Fischer (2004). The hot hand fallacy and the gambler's fallacy: Two faces of subjective randomness? *Memory & cognition*, 32: 1369-78.

Bar-Anan, Yoav, Timothy D Wilson, and Daniel T Gilbert (2009). The feeling of uncertainty intensifies affective reactions. *Emotion*, 9: 123.

Barnes, Adam (2017). The character conundrum: can there be too many? Retrived from: https://ginx.tv/overwatch/character-conundrum-too-many/

Bishop, C. M. (2006). Pattern recognition and machine learning. *springer New York*

Blohm, Ivo, and Jan Marco Leimeister (2013). Gamification. *Business & Information Systems Engineering* 5, no. 4: 275-278.

Blut, Markus, Carly M. Frennea, Vikas Mittal, and David L. Mothersbaugh (2015). How procedural, financial and relational switching costs affect customer satisfaction, repurchase intentions, and repurchase behavior: A meta-analysis. *International Journal of Research in Marketing* 32, no. 2: 226-229.

Burnham, Thomas A., Judy K. Frels, and Vijay Mahajan (2003). Consumer switching costs: A typology, antecedents, and consequences. *Journal of the Academy of Marketing Science*, 31: 109-26.

Burns, Bruce D, and Bryan Corpus (2004). Randomness and inductions from streaks:“Gambler’s fallacy” versus” hot hand “. *Psychonomic Bulletin & Review*, 11: 179-84.

BusinessWire (2016). Global Gamification Market Worth USD 11.10 Billion by 2020 - Analysis, Technologies & Forecast Report 2016-2020 - Key Vendors: Faya Corp, Microsoft & Bunchball - Research and Markets. Retrived from http://www.businesswire.com/news/home/20160218006350/en/Global-Gamification-Market-Worth-USD-11.10-Billion.

Carlin, Bradley P., Nicholas G. Polson, and David S. Stoffer (1992). A Monte Carlo approach to nonnormal and nonlinear state-space modeling. *Journal of the American Statistical Association* 87.418: 493-500.

Caruana, Albert (2003). The impact of switching costs on customer loyalty: A study among corporate customers of mobile telephony. *Journal of Targeting, Measurement and Analysis for marketing* 12, no. 3: 256-268.

Clairfield International (2018). Gaming industry - facts, figures and trends. Revtrieved from http://www.clairfield.com/wp-content/uploads/2017/02/Gaming-Industry-and-Market-Report-2018.01-2.pdf

Clotfelter, Charles T, and Philip J Cook (1993). The “gambler's fallacy” in lottery play. *Management Science*, 39: 1521-25.

Csikszentmihalyi, Mihaly (1990). Flow: The Psychology of Optimal Experience. *New York: Harper & Row*

DeltaDNA (2018). Game Profitability Index – Changing Trends in Monetization. Revrieved from https://deltadna.com/blog/changing-trends-monetization/

Dong, Xiaojing, and Pradeep K. Chintagunta (2015). Satisfaction spillovers across categories. *Marketing Science* 35, no. 2: 275-283.

Drèze, Xavier, and Joseph C Nunes (2008). Feeling superior: The impact of loyalty program structure on consumers' perceptions of status. *Journal of Consumer Research*, 35: 890-905.

——— (2011). Recurring goals and learning: The impact of successful reward attainment on purchase behavior. *Journal of Marketing Research*, 48: 268-81.

Duke, Kristen E, Kelly Goldsmith, and On Amir (2018). Is the Preference for Certainty Always So Certain? *Journal of the Association for Consumer Research*, 3: 000-00.

Elliot, Andrew J, and Marcy A Church (1997). A hierarchical model of approach and avoidance achievement motivation. *Journal of personality and social psychology*, 72: 218.

Ell, Kellie (2018). Video game industry is booming with continued revenue. Retrived from: https://www.cnbc.com/2018/07/18/video-game-industry-is-booming-with-continued-revenue.html

Entertainment Software Association (2018). Essential facts about the computer and video game industry. Retrived from: http://www.theesa.com/wp-content/uploads/2018/05/EF2018_FINAL.pdf\

Fornell, Claes (1992). A national customer satisfaction barometer: The Swedish experience. *Journal of Marketing* : 6-21.

Goethe, Taylor S. (2017). Pay to win: Monetization in gaming. Retrieved from: https://reporter.rit.edu/tech/pay-win-monetization-gaming

Groh, Fabian (2012). Gamification: State of the art definition and utilization. *Institute of Media Informatics Ulm University,* 39: 31.

Guiltinan, Joseph P (1989). A classification of switching costs with implications for relationship marketing. In *AMA winter educators' conference: Marketing theory and practice*, pp. 216-20. Chicago: American Medical Association.

Hartmann, Wesley R., and V. Brian Viard (2008). Do frequency reward programs create switching costs? A dynamic structural analysis of demand in a reward program. *Quantitative Marketing and Economics*, 6: 109-37.

Heide, Jan B., and Allen M. Weiss (1995). Vendor consideration and switching behavior for buyers in high-technology markets *The Journal of Marketing* : 30-43.

Heilizer, Fred (1977). A review of theory and research on the assumptions of Miller's response competition (conflict) models: Response gradients. *The Journal of General Psychology* 97.1 : 17-71.

Hull, Clark L. (1932). The goal-gradient hypothesis and maze learning. *Psychological Review* 39.1: 25.

Hofacker, Charles F., Ko De Ruyter, Nicholas H. Lurie, Puneet Manchanda, and Jeff Donaldson (2016). "Gamification and mobile marketing effectiveness." *Journal of Interactive Marketing* 34: 25-36.

Ingraham, Christopher (2018). The massive popularity of esports, in charts. Retrieved from: https://www.washingtonpost.com/business/2018/08/27/massive-popularity-esports-charts/?utm_term=.9614dea7c83f

Jones, Michael A., David L. Mothersbaugh, and Sharon E. Beatty (2000). Switching barriers and repurchase intentions in services. *Journal of retailing* 76, no. 2 : 259-274.

Kahneman, Daniel, and Amos Tversky (1979). Prospect theory: An analysis of decision under risk. *Econometrica: Journal of the econometric society*: 263-91.

Katkoff, Michail (2014). The MOBA market. Retrived from: https://www.deconstructoroffun.com/blog//2014/06/moba-success-series-market.html

Kilkku, Ville (2015). MOBA monetization and the power of the crowd. Retrived from: http://www.kilkku.com/blog/2015/09/moba-monetization-and-the-power-of-the-crowd/

Kim, Byung-Do, Mengze Shi, and Kannan Srinivasan (2001). Reward programs and tacit collusion. *Marketing Science*, 20: 99-120.

Kim, Jin Gyo, Ulrich Menzefricke, and Fred M. Feinberg (2005). Modeling parametric evolution in a random utility framework. *Journal of Business & Economic Statistics* 23.3: 282-294.

Kim, Sung S., and Jai-Yeol Son (2009). Out of dedication or constraint? A dual model of post-adoption phenomena and its empirical test in the context of online services. *MIS quarterly*: 49-70.

Kivetz, Ran, Oleg Urminsky, and Yuhuang Zheng (2006) The goal-gradient hypothesis resurrected: Purchase acceleration, illusionary goal progress, and customer retention. *Journal of Marketing Research*, 43: 39-58.

Kivetz, R., & Simonson, I. (2002). Earning the right to indulge: Effort as a determinant of customer preferences toward frequency program rewards. *Journal of Marketing Research*, 39(2), 155-170.

Klemperer, Paul (1987). The competitiveness of markets with switching costs. *The RAND Journal of Economics*: 138-50.

——— (1995). Competition when consumers have switching costs: An overview with applications to industrial organization, macroeconomics, and international trade. *The Review of Economic Studies*, 62: 515-39.

Kopalle, Praveen K., Yacheng Sun, Scott A. Neslin, Baohong Sun, and Vanitha Swaminathan (2012). The joint sales impact of frequency reward and customer tier components of loyalty programs. *Marketing Science* 31, no. 2: 216-235.

Lam, Son K., Michael Ahearne, Ye Hu, and Niels Schillewaert (2010). Resistance to brand switching when a radically new brand is introduced: A social identity theory perspective. *Journal of Marketing* 74, no. 6: 128-146.

Laran, Juliano, and Michael Tsiros (2013). An investigation of the effectiveness of uncertainty in marketing promotions involving free gifts. *Journal of Marketing*, 77: 112-23.

Lee, Yih Hwai, and Cheng Qiu (2009). When uncertainty brings pleasure: The role of prospect imageability and mental imagery. *Journal of Consumer Research*, 36: 624-33.

Lewis, Michael (2004). The influence of loyalty programs and short-term promotions on customer retention. *Journal of Marketing Research*, 41: 281-92.

MacDonald, I. L., & Zucchini, W. (1997). Hidden Markov and other models for discrete-valued time series (Vol. 110). *CRC Press*.

Mathwick, Charla, and Edward Rigdon (2004). Play, flow, and the online search experience. *Journal of Consumer Research* 31, no. 2: 324-332.

McKinney, Justin (2017). The rise of pay to win in video games. Retrived from: https://www.theodysseyonline.com/the-rise-of-pay-to-win-in-video-games

Montoya, R., Netzer, O., & Jedidi, K. (2010). Dynamic allocation of pharmaceutical detailing and sampling for long-term profitability. *Marketing Science*, 29(5), 909-924.

Murphy, Kevin P. (2012). Machine learning: A probabilistic perspective. *MIT Press*

Netzer, O., Lattin, J. M., & Srinivasan, V. (2008). A hidden Markov model of customer relationship dynamics. *Marketing science*, 27(2),

Nielsen (2018). U.S. games 360 report: 2018. Retrieved from: https://www.nielsen.com/us/en/insights/reports/2018/us-games-360-report-2018.html#

Nunes, Joseph C., and Xavier Dreze (2006). The endowed progress effect: How artificial advancement increases effort. *Journal of Consumer Research* 32.4: 504-512.

Palm, Emanuel, and Teodor Norén (2015). Character balance in MOBA games. Retrived from: https://www.diva-portal.org/smash/get/diva2:818292/FULLTEXT01.pdf

Pick, Doreén, and Martin Eisend (2014). Buyers' perceived switching costs and switching: a meta-analytic assessment of their antecedents. *Journal of the Academy of Marketing Science* 42, no. 2 : 186-204.

Porter, Michael E (1980). Competitive strategy: Techniques for analyzing industries. *Competitors, The Free Pres,. New York* .

Rossi, Federico (2017). Lower price or higher reward? Measuring the effect of consumers' preferences on reward programs. *Management Science*.

Seetharaman, P. B., and Pradeep K. Chintagunta (2003). The proportional hazard model for purchase timing: A comparison of alternative specifications. *Journal of Business & Economic Statistics* 21, no. 3: 368-382.

Senters, Jo M (1971). A function of uncertainty and stakes in recreation. *Pacific Sociological Review*, 14: 259-69.

Shah, James, Arie Kruglanski, and Ron Friedman (2002). Goal systems theory: Integrating the cognitive and motivational aspects of self-regulation. In *Motivated social perception: The Ontario symposium*, 247.

Shen, Luxi, Ayelet Fishbach, and Christopher K Hsee (2014). The motivating-uncertainty effect: Uncertainty increases resource investment in the process of reward pursuit. *Journal of Consumer Research*, 41: 1301-15.

Siddiqui, R. A., Monga, A., & Buechel, E. C. (2018). When Intertemporal Rewards Are Hedonic, Larger Units of Wait Time Boost Patience. *Journal of Consumer Psychology*, 28(4), 612-628.

Soman, Dilip, and Amar Cheema (2004). When goals are counterproductive: The effects of violation of a behavioral goal on subsequent performance. *Journal of Consumer Research*, 31: 52-62.

Suh, J. C., & Yi, Y. (2012). Do consumption goals matter? The effects of online loyalty programs in the satisfaction-loyalty relation. *Psychology & Marketing*, 29(8), 549-557.

Taylor, Gail Ayala, and Scott A. Neslin (2005). The current and future sales impact of a retail frequency reward program. *Journal of Retailing*, 81: 293-305.

Wang, Yanwen, Michael Lewis, Cynthia Cryder, and Jim Sprigg (2016). Enduring effects of goal achievement and failure within customer loyalty programs: A large-scale field experiment. *Marketing Science* 35, no. 4: 565-575.

Wathne, Kenneth H., Harald Biong, and Jan B. Heide (2001). Choice of supplier in embedded markets: relationship and marketing program effects. *Journal of Marketing* 65, no. 2: 54-66.

Wilburn, Yates (2018). The multi-billion dollar, free-market regulated video gaming industry. Retrieved from https://www.realclearmarkets.com/articles/2018/11/19/the_multi-billion_dollar_free-market_regulated_video_gaming_industry_103501.html

Zhang, Ying, and Szu-Chi Huang (2010). How endowed versus earned progress affects consumer goal commitment and motivation. *Journal of Consumer Research*, 37: 641-54.

Table 1. Variable Definition

Variable	Definition
Perf	Player performance measured by KDA (kills, deaths and assists) averaged across all games played that day
CharPack	Dummy variable for character pack ownership
RentRwd	Dummy variable for claims of rental rewards by player
RentProg	Percentage progression towards next rental reward
CurRwd	Dummy variable for claims of currency rewards by players
CurProg	Percentage progression towards next currency reward
AcclRwd	Dummy variable for claims of progress accelerator reward by player
AcclProg	Percentage progression towards next progress accelerator reward
Lvlup	Dummy variable for level-ups
LvlupProg	Percentage progression towards next level
CharPackPrice	Price of the character pack
CharPlayed	Number of characters played by player so far
SkinExp	Player’s cumulative spending (in $) on in-game aesthetic items (skins)
RegistDays	Days passed since player registered

Table 2. Descriptive Statistics

Variable	Mean	S.D.
Perf	4.41	11.66
CharPack	0.14	0.35
RentRwd	0.02	0.15
RentProg	0.31	0.30
CurRwd	0.01	0.11
CurProg	0.27	0.30
AcclRwd	0.01	0.10
AcclProg	0.20	0.28
Lvlup	0.05	0.22
LvlupProg	0.36	0.34
CharPackPrice	28.39	4.03
CharPlayed	11.09	13.33
SkinExp	12.52	60.51
RegistDays	133.22	98.01

Table 3. Correlation Table

#	Correlation	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Perf	1.00													
2	CharPack	0.12	1.00												
3	RentRwd	0.11	0.01	1.00											
4	RentProg	0.00	0.02	0.14	1.00										
5	CurRwd	0.05	-0.02	0.22	-0.01	1.00									
6	CurProg	-0.06	-0.08	-0.01	0.18	0.13	1.00								
7	AcclRwd	0.01	-0.03	0.16	-0.04	0.78	0.07	1.00							
8	AcclProg	-0.07	-0.11	-0.02	0.25	0.09	0.57	0.10	1.00						
9	Lvlup	0.20	0.00	0.56	0.11	0.44	0.06	0.34	0.04	1.00					
10	LvlupProg	-0.04	-0.04	0.07	0.52	0.06	0.48	0.03	0.41	0.16	1.00				
11	CharPackPrice	0.00	-0.02	0.00	0.00	0.00	0.01	0.00	0.02	0.01	0.01	1.00			
12	CharPlayed	0.28	0.51	0.02	-0.01	-0.03	-0.18	-0.05	-0.21	-0.01	-0.11	-0.05	1.00		
13	SkinExp	0.11	0.35	0.00	-0.08	-0.02	-0.14	-0.01	-0.12	-0.02	-0.10	-0.04	0.43	1.00	
14	RegistDays	-0.05	0.16	-0.09	-0.03	-0.09	-0.08	-0.08	-0.09	-0.17	-0.08	-0.14	0.34	0.17	1.00

Table 4. Model Free Evidence: Character Pack and Play

Group	Game Sessions Played Daily before Package	Game Sessions Played Daily after Package
All Players	0.67	1.82
Character Pack Owners	1.30	1.82

Table 5. Model Free Evidence: Incentives and Play

Incentive Type	Game Sessions Played on Claim Days	Game Sessions Played on Non-Claim Days
Rental Rewards	8.35	.74
Currency Rewards	7.05	.79
Accelerator Rewards	5.44	.82
Level-ups	7.10	.57

Table 6. Model Free Evidence: Incentives and Purchase

Incentive Type	Char. Packs Purchased on Claim Days	Character Packs Purchased on Non-Claim Days
Rental Rewards	140	36
Currency Rewards	152	24
Accelerator Rewards	161	15
Level-ups	71	105

Note: for level-up group: *Ratio: .63% vs. .04%, p-value < .01*

(Ratios are calculated as the number of purchases divided by level-up days and other days respectively.)

Table 7. Play and Purchase Joint State Space Model Results

Play Equation			Purchase Equation			Preference (Latent State)		
Variable	**Est.**	**S.E.**	**Variable**	**Est.**	**S.E.**	**Variable**	**Est.**	**S.E.**
CharPack	3.38	(.13)***	Preference	0.10	(.01)***	Preference_{t-1}	0.94	(.001)***
Reward Progression			*Reward Claim*			*Performance*		
Rental	0.38	(.04)***	Rental	-0.01	(.19)	Performance_{t-1}	0.12	(.02)***
Currency	0.22	(.06)***	Currency	-0.50	(.25)**	$\text{Performance}_{t-1}^2$	-0.01	(.002)***
Accelerator	0.13	(.05)**	Accelerator	0.93	(.15)***	*Random Effects*		
Level-up	0.19	(.04)***	Level-up	1.69	(.12)***	$E(\rho_i)$	-0.83	(.02)***
Interactions			*Controls*			$Var(\rho_i)$	0.13	(.01)***
CharPack x Rental	-0.53	(.10)***	CharPlayed	0.67	(.2)***	*Error Var.*		
CharPack x Currency	-0.23	(.11)**	CharPlayed^2	-0.48	(.10)***	δ_p^2	8.20	(.13)***
CharPack x Accelerator	-0.22	(.15)	CharPackPrice	-0.61	(.05)***			
CharPack x Level	-0.16	(.11)	SkinExp	-0.02	(.24)			
Controls			*Weibull Par.*					
RegistDays	0.37	(.06)***	λ_1 (Rate)	0.0006	(.0001)***			
SkinExp	0.14	(.05)**	λ_2 (Shape)	0.79	(.04)***			
Error Var.								
δ^2	22.63	(.25)***						

*Note: a)***<.01<**<.05<*<.1 b) # of obs. :270,717 c) Log Likelihood: -315,349.24*

Table 8. Correlation between Unobserved Preference and Play Covariates

#	Corr.	1	2	3	4	5	6	7
1	Preference	1.00						
2	CharPack	0.09	1.00					
3	RentProg	0.03	0.02	1.00				
4	CurProg	-0.05	-0.08	0.18	1.00			
5	AcclProg	-0.07	-0.11	0.25	0.57	1.00		
6	LvlupProg	-0.01	-0.04	0.52	0.48	0.41	1.00	
7	RegistDays	-0.25	0.16	-0.03	-0.08	-0.09	-0.08	1.00

Preference as DV R Square: 0.09

Table 9. Correlation between Unobserved Preference and Purchase Covariates

#	Corr.	1	2	3	4	5	6	7	8
1	Preference	1.00							
2	RentRwd	0.01	1.00						
3	CurRwd	0.21	0.00	1.00					
4	AcclRwd	0.15	0.00	0.22	1.00				
5	Lvlup	0.11	0.00	0.16	0.78	1.00			
6	CharPackPrice	0.35	0.01	0.56	0.44	0.34	1.00		
7	CharPlayed	0.40	-0.05	0.02	-0.03	-0.05	-0.01	1.00	
8	SkinExp	0.13	-0.04	0.00	-0.02	-0.01	-0.02	0.43	1.00

Purchase as DV R Square: 0.29

Table 10. Simulation Study: Replace Rental by Accelerator

Rental Rewards Replaced by Accelerator	Game Sessions Played Daily	% Increase to Original
0	1.82	-
1	1.91	4.95
2	1.97	8.24
3	2.03	11.54

Table 11. Simulation Study: Replace Currency by Accelerator

Currency Rewards Replaced by Accelerator	Game Sessions Played Daily	% Decrease in Play to Original
0	0.909	-
1	0.896	1.45
2	0.887	2.48
Currency Rewards Replaced by Accelerator	**Purchase**	**% Increase in Purchase to Original**
0	176	-
1	183	3.98
2	189	7.39

Table 12. Simulation Study: Endowed/Illusionary Progress

Progression	Game Sessions Played Daily	Purchase
Original	0.48	15
New	0.82	15.41

Table 13. Streaks and Prizes

Streak	Cash Prize ($)	Virtual Money Prize
1	0	500
2	0.25	0
3	0	2,500
4	1	0
8	3	0
12	10	0
16	40	0
20	125	0
25	300	0
30	1,000	0
40	10,000	0
50	100,000	0
60	1,000,000	0

Table 14. Status Tiers

Guru Status	Percentile
Novice	0% ~ <50%
Intermediate	50% ~ <75%
Skillful	75% ~ <90%
Expert	90% ~ <99%
Master	99% ~ 100%

Table 15. Variable Definition

Variable	Definition
Current Streak	Number of Consecutive wins (>=3 correct picks)
Loss Streak	Number of Consecutive losses (<3 correct picks)
Wins to Next Prize Level	Wins needed to reach next prize level
Value of Streak Redemption	Value of prize collected from streak redemption yesterday
Value of Streak Reset	Value of prize lost from either no participation or losing
Total Prize Redeemed	Total prize collected in the mobile app
Total Prize Lost with Reset	Total prize lost in the mobile app
Guru Low	Dummy, = 1 if player is in Guru level Intermediate (50%-75%)
Guru Middle	Dummy, = 1 if player is in Guru level Skillful (75%-90%)
Guru High	Dummy, = 1 if player is in Guru level Expert (90%-99%)
Guru Top	Dummy, = 1 if player is in Guru level Master (99% and above)
Days since Registration	Days lapsed since registration date

Table 16. Descriptive Statistics

Variable	Mean	S.D.
Current Streak	0.67	1.43
Loss Streak	0.31	0.73
Wins to Next Prize Level	0.90	0.72
Value of Streak Redemption	0.00	0.15
Value of Streak Reset	0.06	0.34
Total Prize Redeemed	0.08	0.94
Total Prize Lost with Reset	1.77	2.16
Days since Registration	25.43	14.80
Guru Low	0.19	0.39
Guru Middle	0.06	0.24
Guru High	0.03	0.18
Guru Top	0.003	0.06

Table 17. Correlation Table

#	Corr.	1	2	3	4	5	6	7	8	9	10	11	12
1	Current Streak	1.00											
2	Loss Streak	-0.20	1.00										
3	Wins to Next Prize Level	0.62	-0.54	1.00									
4	Value of Streak Redemption	-0.01	-0.01	0.06	1.00								
5	Value of Streak Reset	0.50	-0.08	0.32	0.00	1.00							
6	Total Prize Redeemed	0.00	0.01	0.00	0.16	0.00	1.00						
7	Total Prize Lost with Reset	0.17	0.10	0.01	0.00	0.19	0.01	1.00					
8	Days since Registration	-0.13	-0.14	0.06	0.00	-0.04	0.06	0.43	1.00				
9	Guru Low	0.12	0.09	-0.02	0.01	0.06	0.04	0.37	0.16	1.00			
10	Guru Middle	0.11	0.10	-0.01	0.00	0.05	0.03	0.36	0.21	-0.13	1.00		
11	Guru High	0.15	0.05	0.04	0.01	0.06	0.05	0.36	0.17	-0.09	-0.05	1.00	
12	Guru Top	0.06	0.01	0.02	0.00	0.02	0.05	0.14	0.05	-0.03	-0.01	-0.01	1.00

Table 18. Static Model

Variable	**Estimate**	**S.E.**
Current Streak	0.85	(0.02)***
Loss Streak	0.60	(0.03)***
Wins to Next Prize Level	0.34	(0.03)***
Value of Streak Redemption	0.79	(0.02)***
Value of Streak Reset	-0.83	(0.03)***
Total Prize Redeemed	0.02	(0.01)
Total Prize Lost with Reset	0.32	(0.01)***
Days since Registration	-1.69	(0.03)***
Guru Low	0.38	(0.03)***
Guru Middle	0.78	(0.03)***
Guru High	0.98	(0.03)***
Guru Top	0.77	(0.03)***
Random Effects		
$E(\rho_i)$	13.15	(0.22)***
$Var(\rho_i)$	4.84	(0.24)***

Note: a) ***<0.01<**<0.05<*<0.1 b) Number of Obs. : 98,261 c) Log Likelihood: -22,665.6

Table 19. Choosing the Number of States

Number of States	**-2 Marginal Log Density**	**Log Bayes Factor**	**DIC**
1	53484.20	-	54924.62
2	45471.00	4006.60	47360.37
3	**44645.08**	**412.96**	**45745.23**
4	44705.44	-30.18	45952.30

Table 20. Three-State Hidden Markov Model Results: State Transition and Choice

		State					
		Low		Medium		High	
Equation	**Variable**	**Estimate**	**S.E.**	**Estimate**	**S.E.**	**Estimate**	**S.E.**
State Dependent Choice	Current Streak	0.38	(1.00)	0.16	(0.01)***	0.09	(0.08)
	Loss Streak	-2.63	(0.94)***	0.31	(0.06)***	-0.52	(0.16)***
	Wins to Next Prize Level	-3.90	(1.41)***	0.27	(0.06)***	-0.41	(0.20)**
State Transition	Value of Streak Redemption	0.002	(0.09)	-0.64	(0.06)***	-0.14	(0.03)***
	Value of Streak Reset	0.53	(0.06)***	0.29	(0.04)***	-0.04	(0.02)**
	Total Prize Redeemed	0.10	(0.06)*	-0.08	(0.04)**	-0.01	(0.03)
	Total Prize Lost with Reset	-0.01	(0.05)	0.20	(0.05)***	-0.07	(0.03)**
	Days since Registration	-0.48	(0.07)***	-1.11	(0.06)***	0.03	(0.05)
	Guru Low	0.61	(0.16)***	0.38	(0.07)***	-0.55	(0.08)***
	Guru Middle	0.94	(0.11)***	1.69	(0.15)***	-0.83	(0.10)***
	Guru High	0.48	(0.19)**	0.19	(0.10)*	-0.13	(0.13)
	Guru Top	0.02	(0.17)	-0.24	(0.08)***	0.68	(0.12)***

*Note: a) ***<0.01<**<0.05<*<0.1 b) Number of Obs. : 98,261*

Table 21. Three-State Hidden Markov Model Results: State Intercept and Thresholds

		State					
		Low		**Medium**		**High**	
Variable		**Est.**	**S.E.**	**Est.**	**S.E.**	**Est.**	**S.E.**
State Intercept		-4.27	(0.34)***	1.36	(0.07)***	1.73	(0.07)***
μ(State Thre.)	Low	-1.42	(0.18)***	-1.88	(0.24)***	-3.16	(0.13)***
	High	0.46	(0.22)**	4.07	(0.51)***	-1.23	(0.14)***
v(State Thre.)	Low	6.61	(1.06)***	2.55	(0.19)***	1.58	(0.12)***
	High	2.05	(0.23)***	0.47	(0.10)***	0.22	(0.05)***

*Note: a) ***<0.01<**<0.05<*<0.1 b) Number of Obs. : 98,261*

Table 22. Mean Posterior Transition Matrix

		t (to State)		
	State	Low	Medium	High
	Low	0.89	0.09	0.02
t-1 (from State)	Medium	0.26	0.60	0.14
	High	0.02	0.09	0.89

Figure 1. In-game Dynamic Incentive Scheme

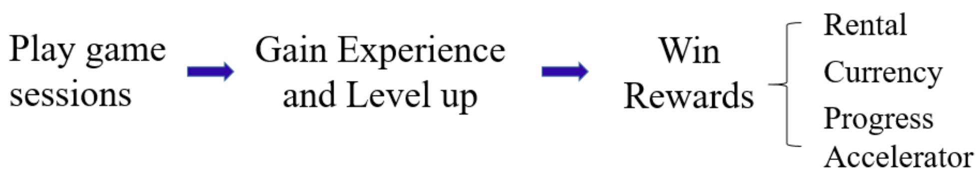


Figure 2. Average Game Sessions Played Over Time

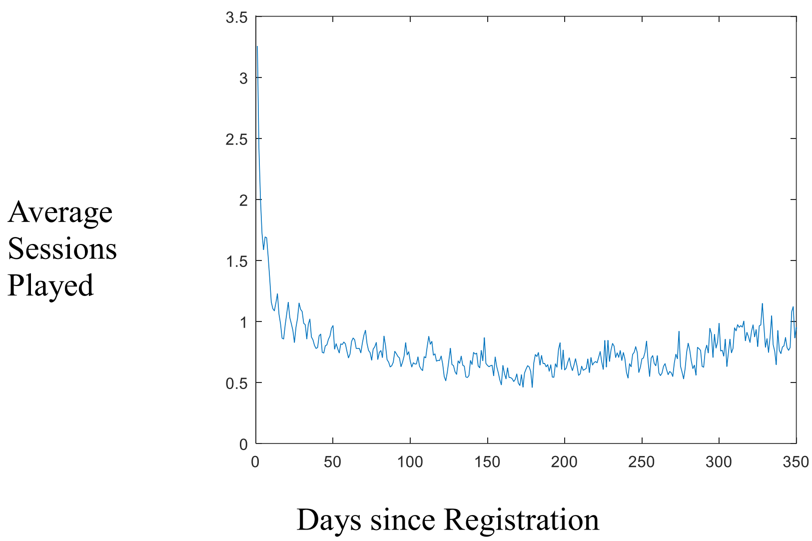


Figure 3. Timing of Character Pack Purchases

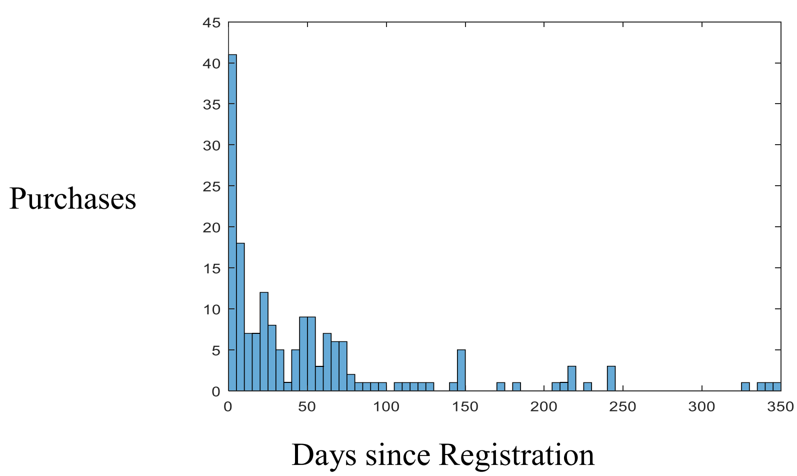


Figure 4. Play and Purchase Joint State Space Model Structure

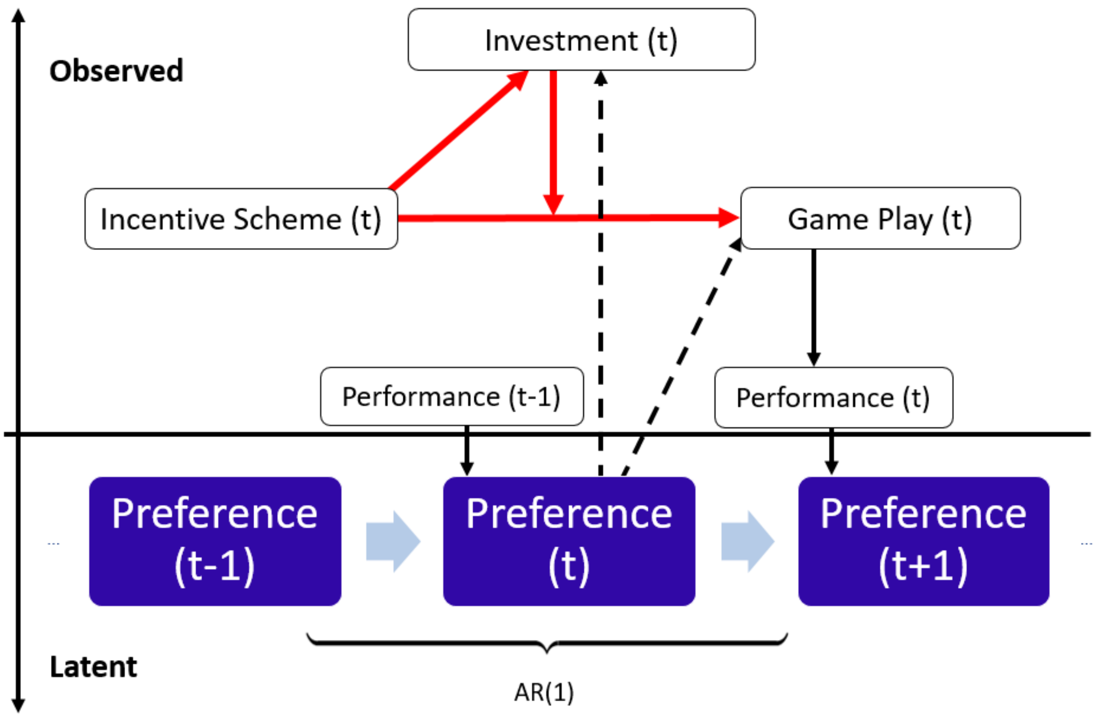


Figure 5. Weibull Baseline Hazard: Hazard Rate

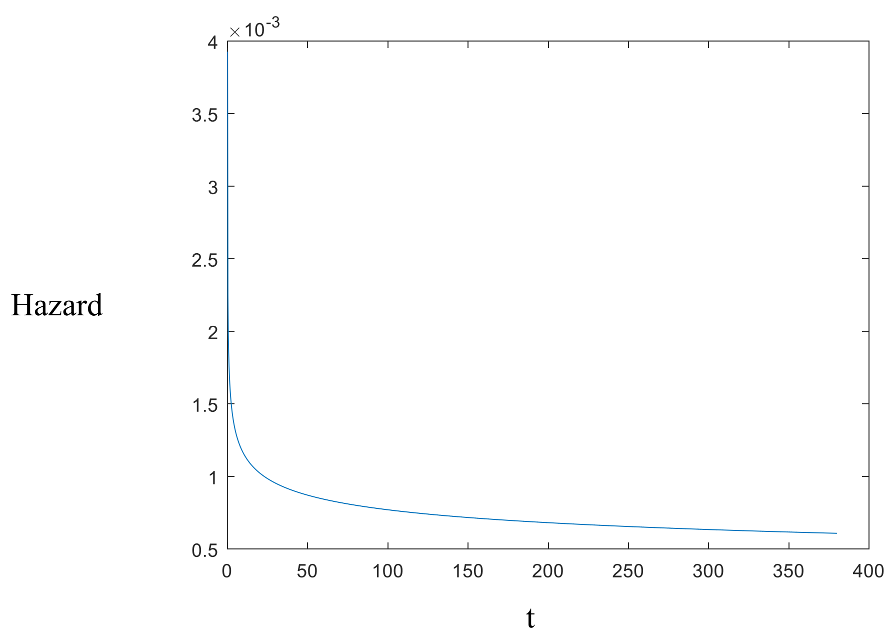


Figure 6. Weibull Baseline Hazard: Cumulative Hazard

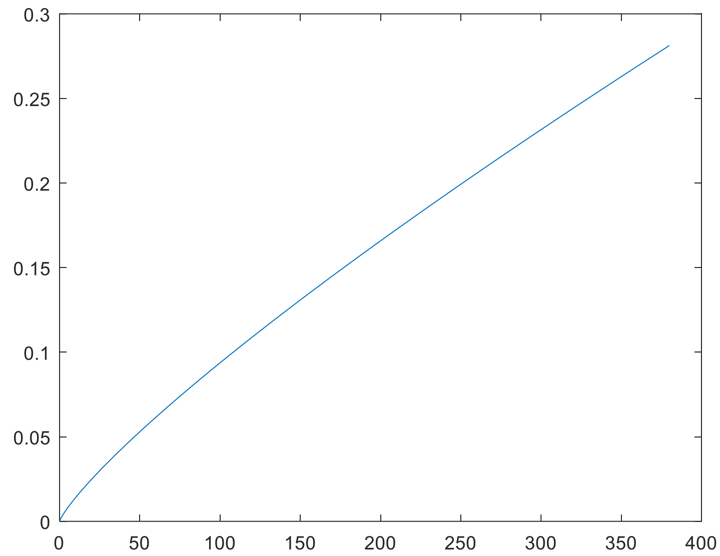


Figure 7. Preference Evolution: 30 Random Players (Solid Lines) and Mean across All Players (Dashed Line)

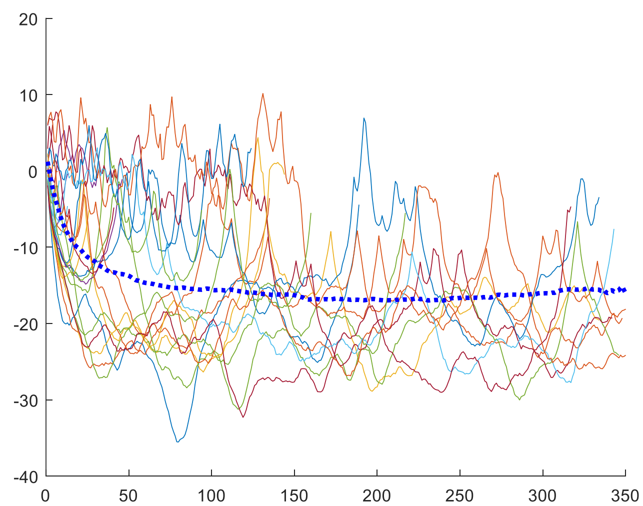
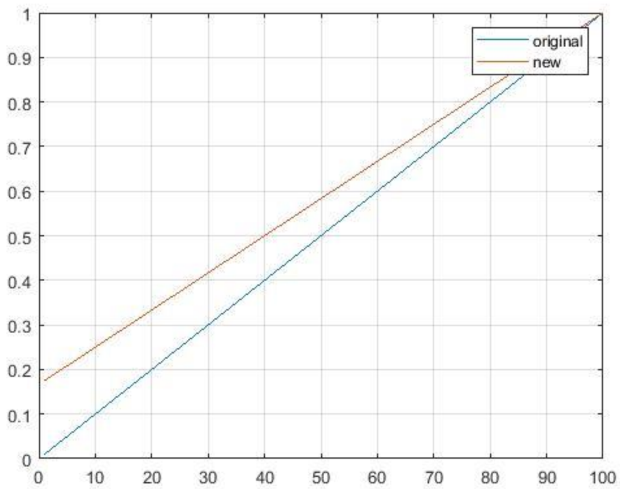


Figure 8. Endowed/Illusionary Progress Example



Progress

Experience Points

Figure 9. Model Free Analysis: Points Pressure

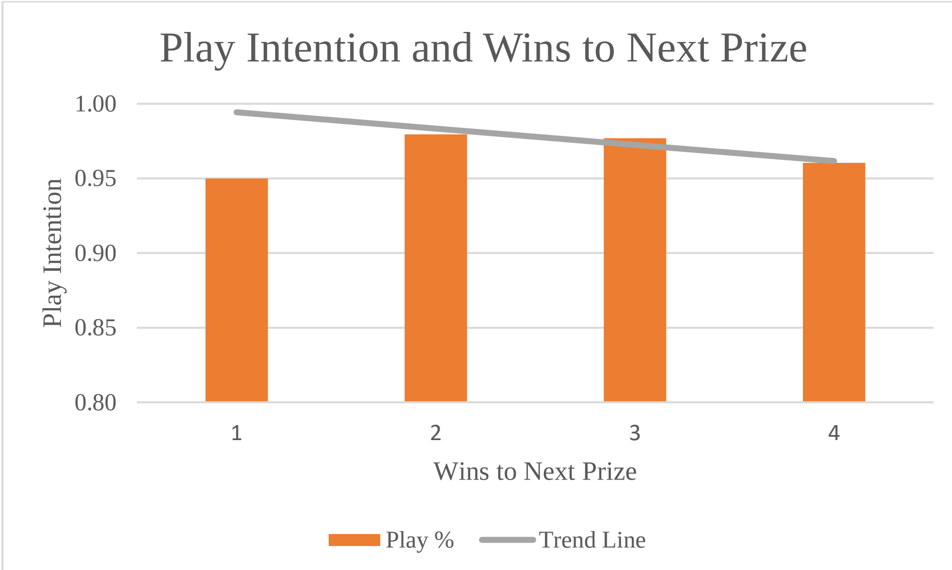


Figure 10. Model Free Analysis: Gambler's Fallacy

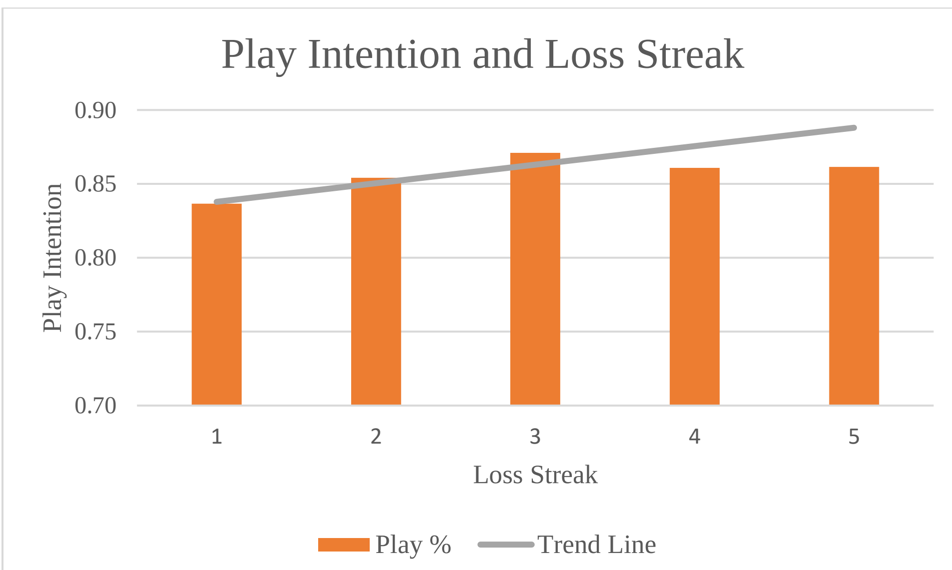


Figure 11. Model Free Analysis: Status

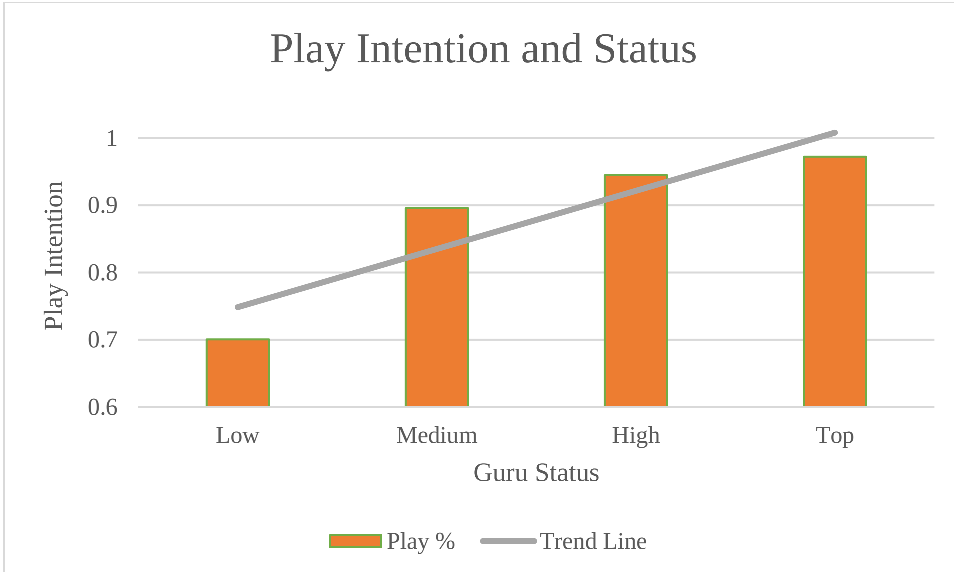


Figure 12. State Proportion Evolvement

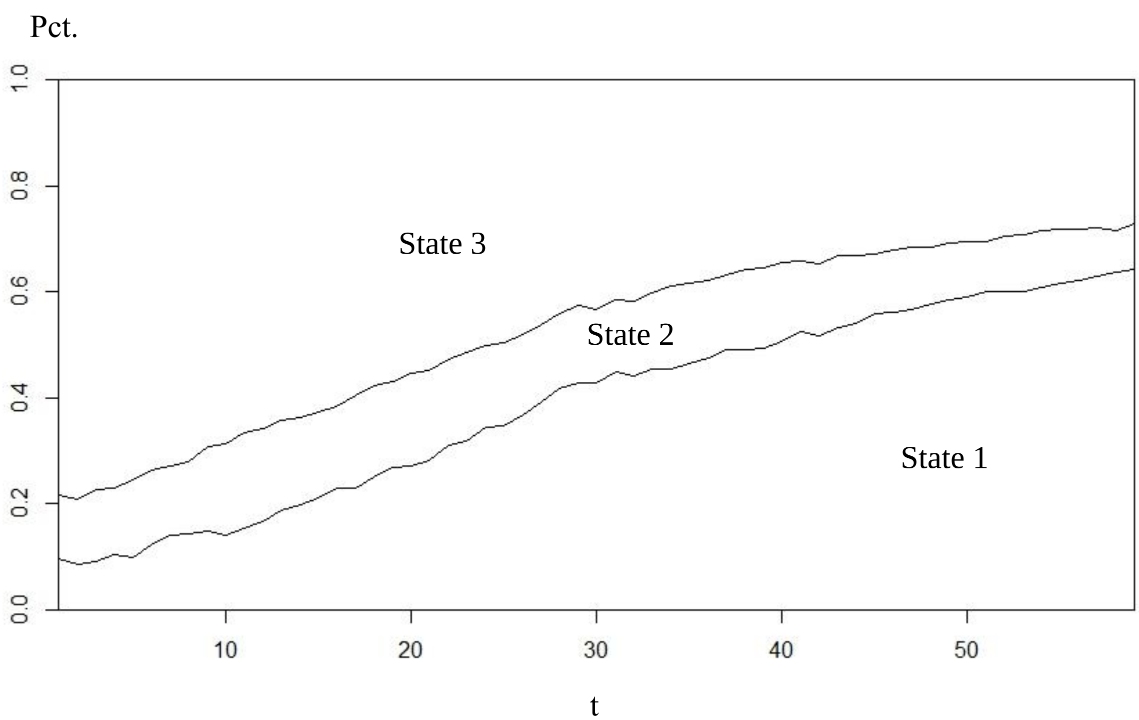
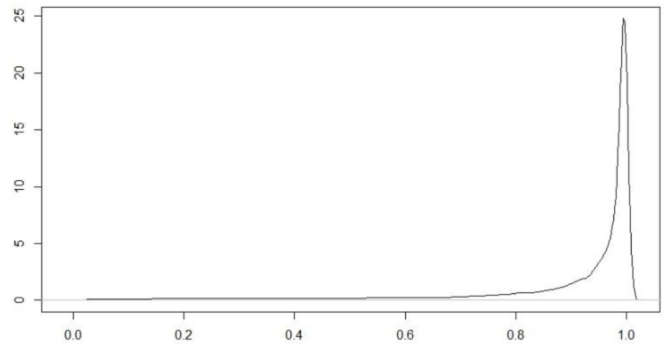


Figure 13. Posterior Distribution of the Propensity to Stay in Current States

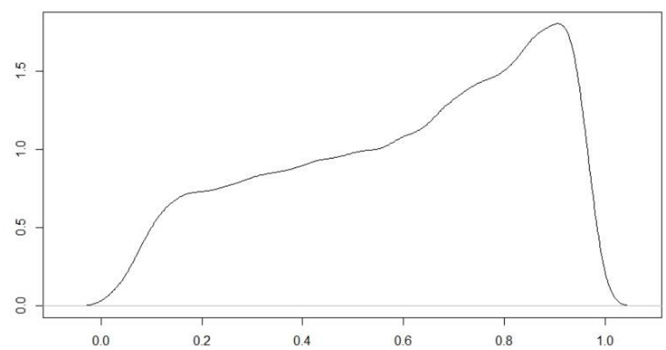
Density



Pct.

State Low

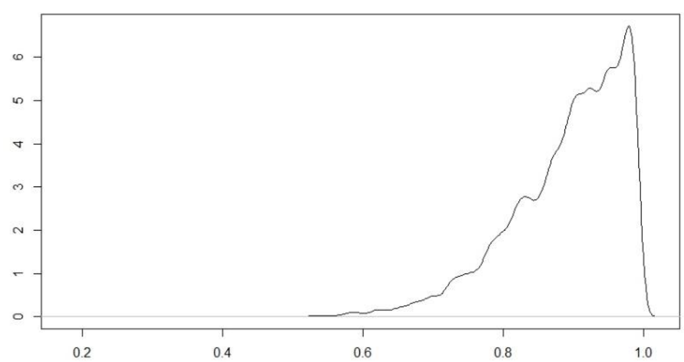
Density



Pct.

State Medium

Density

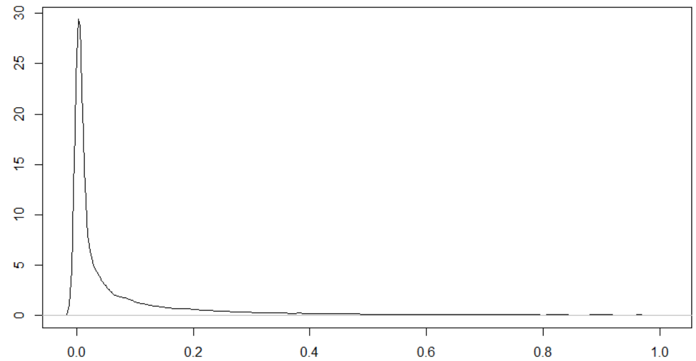


Pct.

State High

Figure 14. Posterior Distribution of the Propensity to Move in State Low

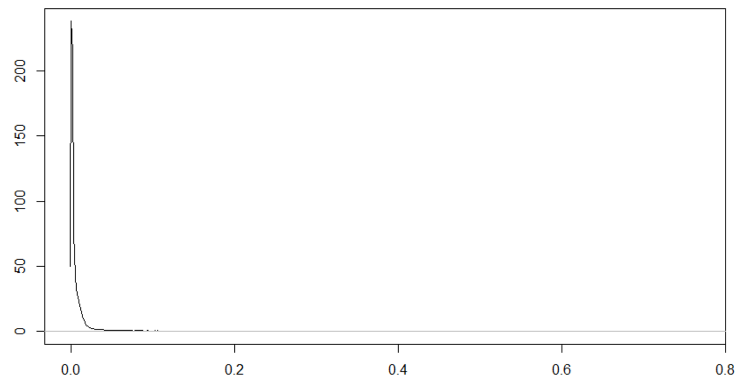
Density



Pct.

State Low to Medium

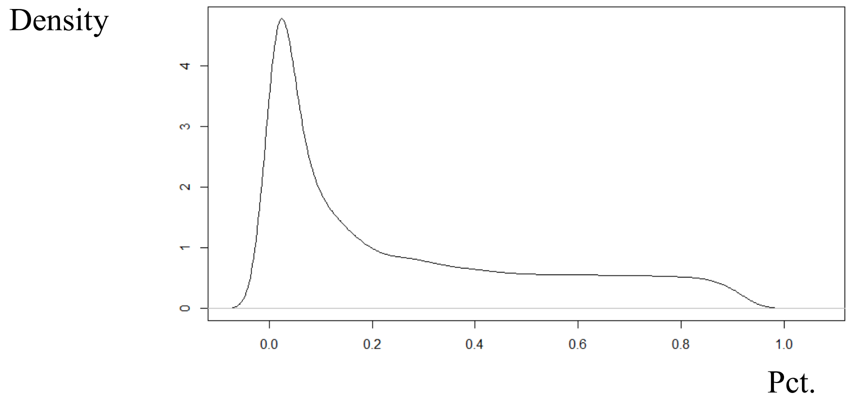
Density



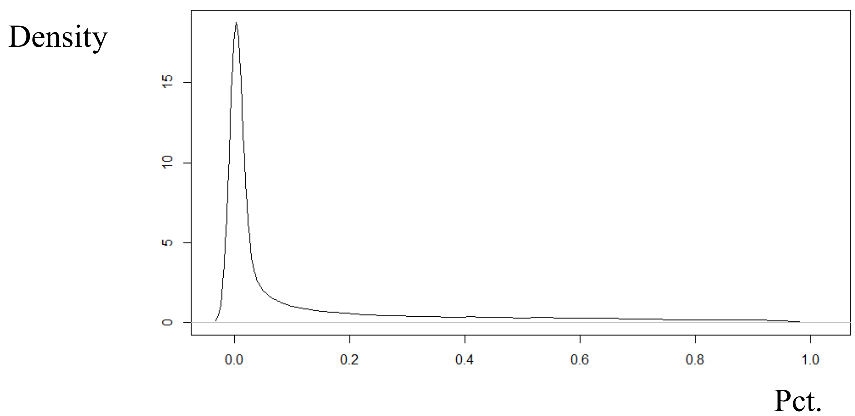
State Low to High

Pct.

Figure 15. Posterior Distribution of the Propensity to Move in State Middle



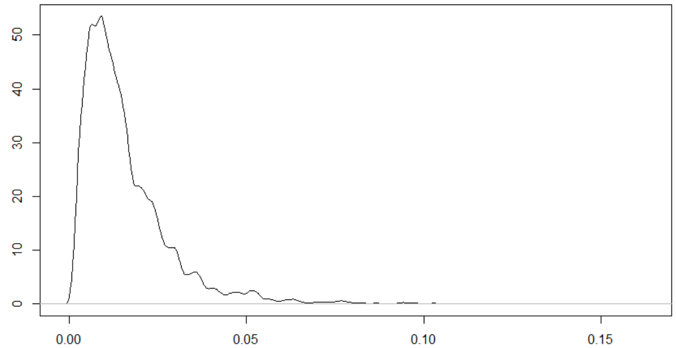
State Medium to Low



State Medium to High

Figure 16. Posterior Distribution of the Propensity to Move in State High

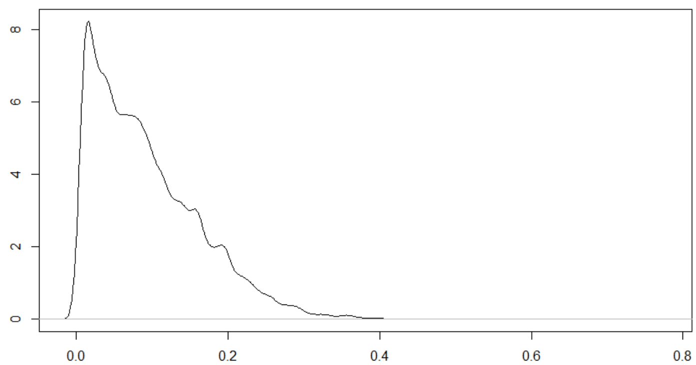
Density



Pct.

State High to Low

Density



Pct.

State High to Medium

Appendix 1. Hierarchical Bayes Estimation Algorithm for Joint State Space Model

Hierarchical Bayes Estimation Algorithm

I use *i* (*i = 1…n*) to denote player and *t* (*t = 1…T*) to denote time and T_i measures the length of player i's time span in the sample. Assume that covariates in the play equation are *x*, covariates in the purchase equation are *z* and covariates in the state equation are *w*.

1. ${\delta_\rho}^2 \mid \mu_\rho, \rho_i$

   (Variance of random effects in the state equation)

   Prior: ${\delta_\rho}^2$~ Inverse Gamma (a_ρ, b_ρ), where a_ρ is the shape parameter and b_ρ is the rate parameter and a_ρ = 0.001 and b_ρ = 0.001.

   Posterior: ${\delta_\rho}^2 \mid \mu_\rho, \rho_i \sim IG\left(a_\rho + \frac{n}{2}, b_\rho + \frac{1}{2}\sum_i(\rho_i - \mu_\rho)^2\right)$

2. $\mu_\rho \mid {\delta_\rho}^2, \rho_i$

   (Mean of random effects in state equation)

   Prior: μ_ρ ~ *N* ($mu_{\rho 0}$, $sig_{\rho 0}$) with $mu_{\rho 0}$ = 0 and $sig_{\rho 0}$ = 100.

   Posterior: $\mu_\rho \mid {\delta_\rho}^2, \rho_i \sim N\left(\left(\frac{1}{sig_{\rho 0}} + \frac{n}{{\delta_\rho}^2}\right)^{-1} * \left(\frac{mu_{\rho 0}}{sig_{\rho 0}} + \frac{\sum_i \rho_i}{{\delta_\rho}^2}\right), \left(\frac{1}{sig_{\rho 0}} + \frac{n}{{\delta_\rho}^2}\right)^{-1}\right)$

3. $\rho_i \mid \mu_\rho, {\delta_\rho}^2, p_{it}, \theta, {\delta_p}^2, \alpha$

   (Random effects in the state equation)

   Posterior: $\rho_i \mid \mu_\rho, {\delta_\rho}^2, p_{it}, \theta, {\delta_p}^2, \alpha \sim \mathrm{N}\,({\mu_\rho}^*_i, {\delta_\rho}^{2*}_i)$

   with $({\delta_\rho}^{2*}_i)^{-1} = \frac{1}{{\delta_p}^2} * T_i + \frac{1}{{\delta_\rho}^2}$ and ${\mu_\rho}^*_i = {\delta_\rho}^{2*}_i * \left(\frac{\mu_\rho}{{\delta_\rho}^2} + \frac{\sum_t(p_{it} - \theta * p_{i,t-1} - {w_{it}}'\alpha)}{{\delta_p}^2}\right)$.

4. ${\delta_p}^2 \mid \theta, \alpha, p_{it}, \rho_i$

   (Variance of error term in the state equation)

   Prior: ${\delta_p}^2$~ Inverse Gamma (a_0, b_0), with shape parameter a_0 = 0.001 and rate parameter b_0 = 0.001.

Posterior:

$${\delta_p}^2 \mid \theta, \alpha, p_{it}, \rho_i \sim IG\left(a_0 + \frac{n*T}{2}, b_0 + \frac{1}{2} * \sum_{it}(p_{it} - \theta * p_{i,t-1} - w_{it}\alpha - \rho_i)^2\right)$$

5. $\alpha \mid \theta, {\delta_p}^2, p_{it}, \rho_i$

(Parameter for covariates in the state equation other than lagged preference)

Prior: α ~ N (mu_a, sig_a) where mu_a = 0 and sig_a = 100*I

Posterior: $\alpha \mid \theta, {\delta_p}^2, p_{it}, \rho_i$ ~ N (${mu_a}^*, {sig_a}^*$), where $({sig_a}^*)^{-1} = \frac{1}{{\delta_p}^2} * w'w + \frac{1}{sig_a}$, and

$${mu_a}^* = {sig_a}^* * \left(\frac{mu_a}{sig_a} + \frac{1}{{\delta_p}^2} * \sum_{it}\left(w_{it}' * \left(p_{it} - \theta * p_{i,t-1} - \rho_i\right)\right)\right)$$

6. $\theta \mid \alpha, {\delta_p}^2, p_{it}, \rho_i$

(Parameter for lagged preference in the state equation)

Prior: θ ~ N (mu_θ, sig_θ), where mu_θ = 0 and sig_θ = 100.

Posterior: $\theta \mid \alpha, {\delta_p}^2, p_{it}, \rho_i$ ~ N (${mu_\theta}^*, {sig_\theta}^*$), where $({sig_\theta}^*)^{-1} = \frac{1}{{\delta_p}^2} * \sum_{it}\left({p_{i,t-1}}^2\right) + \frac{1}{sig_\theta}$,

and ${mu_\theta}^* = {sig_\theta}^* * \left(\frac{mu_\theta}{sig_\theta} + \frac{1}{{\delta_p}^2} * \sum_{it} p_{i,t-1} * (p_{it} - w_{it}'\alpha - \rho_i)\right)$

7. $p_{it} \mid \theta, \alpha, {\delta_p}^2, \rho_i, \beta, \delta^2, {y^*}_{it}, p_{i,t-1}, p_{i,t+1}, \lambda_1, \lambda_2, \gamma$

(Latent preference variable that links purchase and play)

Prior for $p_{i,0}$

N (mu_0, sig_0) where mu_0 = 0 and sig_0 = 100.

Posterior:

$p(p_{it} \mid \theta, \alpha, {\delta_p}^2, \rho_i, \beta, \delta^2, {y^*}_{it}, p_{i,t-1}, p_{i,t+1}, \lambda_1, \lambda_2, \gamma) \propto N\,(F_{it} * f_{it}, F_{it}) *$
$p_i(t|\lambda_1, \lambda_2, \gamma, p_{it})$

Where

$$F_{it}{}^{-1} = \begin{cases} \dfrac{1}{sig_0} + \dfrac{\theta^2}{\delta_p{}^2}, & t = 0 \\ \dfrac{1}{\delta_p{}^2} * (1+\theta^2) + \dfrac{1}{\delta^2}, & t = 1, \dots, T_i - 1 \\ \dfrac{1}{\delta_p{}^2} + \dfrac{1}{\delta^2}, & t = T_i \end{cases}$$

and

$$f_{it} = \begin{cases} \dfrac{mu_0}{sig_0} + \dfrac{\theta * (p_{i1} - w_{i1}{}'\alpha - \rho_i)}{\delta_p{}^2}, & t = 0 \\ \dfrac{\theta * p_{i,t-1} + w_{it}{}'\alpha + \rho}{\delta_p{}^2} + \dfrac{\theta * \left(p_{i,t+1} - w_{i,t+1}{}'\alpha - \rho_i\right)}{\delta_p{}^2} + \dfrac{y^*{}_{it} - x'_{it}\beta}{\delta^2}, & t = 1, \dots, T_i - 1 \\ \dfrac{\theta * p_{i,T_i-1} + w_{iT_i}{}'\alpha + \rho}{\delta_p{}^2} + \dfrac{y^*{}_{iT_i} - x'_{iT_i}\beta}{\delta^2}, & t = T_i \end{cases}$$

Suppose $b_{i,t}$ is a dummy variable for character pack purchase which equals 1 if a character pack is purchased by player *i* on day *t*. Here $p_i(t|\lambda_1, \lambda_2, \gamma, p_{it})$ is the likelihood for character pack purchase by player *i* on day *t*:

$$p_i(t|\lambda_1, \lambda_2, \gamma, p_{it}) = \left(q_{it}(t, z_t)^{b_{it}} * (1 - q_{it}(t, z_t))^{(1-b_{it})}\right)^{(1-CharPack_{i,t-1})}$$

where

$$q_{it}(t, z_t) = 1 - \frac{S(t, z_{it})}{S\left(t-1, z_{i,t-1}\right)}$$

$$S(t, z_{it}) = exp\left(-\sum_{v=1}^{t} exp(z_{iv}\gamma) \int_{v-1}^{v} h(u)du\right)$$

Metropolis Hastings steps:

$p_{it}{}^{new} = p_{it}{}^{old} + \Delta_{p_i}$ where $\Delta_{p_i} \sim N(0, \phi_p)$ and ϕ_p is chosen to make the acceptance rate about 20%

$$Pr\ (acceptance)_{it} = min\left\{\frac{\left(N(p_{it}{}^{new}|F_{it} * f_{it}, f_{it})\right) * p_i(t|\lambda_1, \lambda_2, \gamma, p_{it}{}^{new})}{\left(N(p_{it}{}^{old}|F_{it} * f_{it}, f_{it})\right) * p_i(t|\lambda_1, \lambda_2, \gamma, p_{it}{}^{old})}, 1\right\}$$

8. $\delta^2 \mid p, \beta,$ y*

   (Variance of error term in the play equation)

   Prior: $\delta^2 \sim \frac{v_0 {s_0}^2}{{X_{v_0}}^2}$ with v_0 = 5, ${s_0}^2$ = 0.1,

   Posterior: $\delta^2 \mid p, \beta,$ y* ~ $v_1 * {s_1}^2/{\chi^2}_{v_1}$

   $v_1 = v_0 + \sum_i T_i, {s_1}^2 = (v_0 * {s_0}^2 + ns^2)/(v_0 + \sum_i T_i)$

   $ns^2 = (y^* - p - x\tilde{\beta})'(y^* - p - x\tilde{\beta}) + (\tilde{\beta} - \bar{\beta})'A\,(\tilde{\beta} - \bar{\beta})$

   $\tilde{\beta} = (x'x + A)^{-1}(x'x\hat{\beta} + A\bar{\beta})$

   $\hat{\beta} = (x'x)^{-1}x'(y^* - p)$

   $\bar{\beta}$ = 0, A = 0.01*I

9. $\beta \mid p, \delta^2,$ y*

   (Parameters for covariates in the play equation)

   Prior: $\beta \mid \delta^2$ ~ $N\,(\bar{\beta}, \delta^2 A^{-1})$, $\bar{\beta}$ = 0, A = 0.01*I

   Posterior: $\beta \mid p, \delta^2,$ y* ~ $N\,(\tilde{\beta}, \delta^2\,(x'x + A)^{-1})$

   $\tilde{\beta} = (x'x + A)^{-1}(x'x\hat{\beta} + A\bar{\beta})$

   $\hat{\beta} = (x'x)^{-1}x'(y^* - p)$

10. y*$\mid p, \delta^2, \beta,$ y

    (Underlying normal variable for play which follows a truncated normal distribution)

    First I need to draw r (negative value that is truncated at 0 for y) from truncated normal distribution r_{it}~ $N_{-\infty,0}\,(p_{it}+x_{it}\beta, \delta^2)$ for all $i \in C$, where C is the index set of all 0 y's.

    Then replace 0's in y with r, I have y*.

11. $\gamma \mid \lambda_1, \lambda_2, p_{it}$

    (Parameters for covariates in the purchase equation) prior: γ ~$N(mu_\gamma, sig_\gamma)$, where mu_γ is 0 and sig_γ = 100*I

posterior: p ($\gamma|\lambda_1$, λ_2, p_{it}) ∝ ($\prod_{it} p_i(t \mid \gamma, \lambda_1, \lambda_2, p_{it})$)* $p(\gamma \mid mu_\gamma, sig_\gamma)$

MH steps:

$\gamma^{new} = \gamma^{old} + \Delta_\gamma$, where $\Delta_\gamma \sim N(0, \phi_\gamma I)$ and ϕ_γ is chosen to make accepance rate approximately 20%

$$p_i(t|\lambda_1, \lambda_2, \gamma, p_{it}) = \left(q_{it}(t, z_t)^{b_{it}} * (1 - q_{it}(t, z_t))^{(1-b_{it})}\right)^{(1-CharPack_{i,t-1})}$$

$$Pr(acceptance) = \quad min\left\{\frac{exp\left(-\frac{1}{2}*(\gamma^{new}-mu_\gamma)' sig_\gamma{}^{-1}(\gamma^{new}-mu_\gamma)\right)*(\prod_{it} p_i(t \mid \gamma^{new}, \lambda_1, \lambda_2, p_{it}))}{exp\left(-\frac{1}{2}*(\gamma^{old}-mu_\gamma)' sig_\gamma{}^{-1}(\gamma^{old}-mu_\gamma)\right)*(\prod_{it} p_i(t \mid \gamma^{old}, \lambda_1, \lambda_2, p_{it}))}, 1\right\}$$

12. $\lambda_1 \mid \gamma$, λ_2, p_{it}
    (Weibull rate parameter)

    prior: λ_1~ U (0, 1000)

    posterior: p ($\lambda_1 \mid \gamma$, λ_2, p_{it}) ∝ ($\prod_{it} p_i(t| \gamma, \lambda_1, \lambda_2, p_{it})$) * $p(\lambda_1)$

    MH steps:

    ${\lambda_1}^{new} = {\lambda_1}^{old} + \Delta_{\lambda_1}$, where $\Delta_{\lambda_1} \sim N(0, \phi_{\lambda_1})$ and ϕ_{λ_1} is chosen to make acceptance rate approximately 20%

$$Pr(acceptance) = min\left\{\frac{\prod_{it} p_i(t \mid \gamma, {\lambda_1}^{new}, \lambda_2, p_{it})}{\prod_{it} p_i(t \mid \gamma, {\lambda_1}^{old}, \lambda_2, p_{it})}, 1\right\}$$

13. $\lambda_2 \mid \gamma$, λ_1, p_{it}
    (Weibull shape parameter)

    prior: λ_2~ U (0, 1000)

    p ($\lambda_2 \mid \gamma$, λ_1, p_{it}) ∝ ($\prod_{it} p_i(t \mid \gamma, \lambda_1, \lambda_2, p_{it})$) * $p(\lambda_2)$

    MH steps:

    ${\lambda_2}^{new} = {\lambda_2}^{old} + \Delta_{\lambda_2}$, where $\Delta_{\lambda_2} \sim N(0, \phi_{\lambda_2})$ and ϕ_{λ_2} is chosen to make accepance rate approximately 20%

$$Pr(acceptance) = min\left\{\frac{\prod_{it} p_i(t\ |\gamma, \lambda_1, {\lambda_2}^{new}, p_{it})}{\prod_{it} p_i(t\ |\gamma\ , \lambda_1, {\lambda_2}^{old}, p_{it})}, 1\right\}$$