Quantity Constrained Duopoly With A Market For Quotas

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Abstract

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As the control of common pool resources becomes increasingly important, there have been many proposed ways to manage these resources. One such way is through an individual transferable quota (ITQ) system where each firm is allocated a certain amount of quotas that can then be traded in a market for quotas. The goal of this paper is to examine how quota markets work in principle, in the absence of externalities. Working with a duopoly situation, I create two different models. The first model assumes that each firm has different, but constant, marginal costs. In this model, I find that it is always in the firms' best interest to trade quota, although the exact price at which quota is traded is indeterminate. The second model of this paper examines the case in which each firm has increasing marginal costs. When marginal costs are increasing, there is an endogenously determined price and quantity at which quota is traded. Furthermore, it is shown that there is a welfare-maximizing quota level for different demand and cost parameters. Understanding the theoretical basis of a quota market and its impact on social welfare in a duopoly situation, even without externalities, helps to provide insight on how effective an ITQ system is at managing common pool resources.

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Table of Contents

Introduction

With a growing world population, the growing demand for certain world resources have left many of these integral resources uncontrolled for and exposed to overexploitation. Such is the case for various fisheries throughout the world, as the low cost of inputs and large potential profits from fishing have tremendously expanded the fish market. Now more than ever, fish stocks are overfished, stressing the importance of finding a sustainable solution through which fish stocks can be kept at a sustainable level, while simultaneously ensuring that firms can still profit off of fishing.

Fisheries, like many environmental goods, are a common pool resource. Therefore, while it is difficult to exclude people from benefiting from such a good, at the same time, consumption of the good by one individual detracts from the amount available to others (Ostrom, Gardner, & Walker, 1997). As a common pool resource, fisheries are thereby also subject to the "tragedy of the commons," as coined by Hardin (1968). In his article, Hardin claims that should everybody be granted the freedom to use as much of a common resource as possible, people will all act in their own self-interest, ultimately leading to the destruction of the commons (Hardin, 1968).

In an attempt to prevent fish stocks from becoming overfished, different countries have over time engaged in a wide variety of methods to keep fish stocks at a healthy, sustainable level. While some of these methods aim to make the participating parties responsible through forms of co-management and community-based management (Pomeroy, 1995), other countries use taxes and fines to prevent the overexploitation of such resources. Furthermore, there are also countries that use individual transferable quotas (ITQs) as a way of managing fisheries. In this system, a quota caps the total allowable catch, thereby permitting a sustainable stock of fish. In this strategy, the involved firms are each given quotas that can be traded with other firms, and through a market for quotas, a socially beneficial equilibrium quantity and price can then be endogenously determined (Holland & Brazee, 1996).

Using a market-based solution to enhance welfare has often proven to be relatively successful. Numerous empirical studies examine individual transferable quotas in their ability to increase economic gains as well as to prevent the common pool resource from being overused (Annala, 1996; Newell, Sanchirico, & Kerr, 2005; Batstone & Sharp, 1999; etc.). However, despite the apparent success of managing common-pool resources through a quota market, few researchers have considered the theoretical implications of this system. Adelaja, Menzo, and McCay, suggest that markets for quotas lead to less competition and increased market concentration (1998), and support their argument using data from the Mid-Atlantic Surf Clam and Ocean Quahog fishery.

In efforts to fill the void between the ideal goals of an ITQ system and the empirical evidence supporting such, this paper examines the theory behind the individual transferable quota system as a way of managing common pool resources. In this paper, I assume that there is an obvious need to limit the use of the common pool resource, e.g., the amount of fish being caught. I further assume that a “regulator” chooses to reduce the catch by allocating quotas to each participating firm, and then allowing the firms to trade their quotas amongst one another, thereby forming a market for quotas. In this paper, I examine a duopoly situation with two firms, Firm 1 and Firm 2. The simplicity of the duopoly model allows a market for quota to exist in its most basic form. While externalities can arguably impact common pool resources, I will abstract from including the effects of externalities and will assume that the amount of fish caught by Firm 1 does not impact Firm 2, as to reduce potentially confounding factors. Even so, the underlying concepts developed in this paper will serve as guidance to future findings when externalities are included.

In this paper, I examine two different models that are plausible when dealing with a market for quota. The first model takes into account two firms each with different, yet constant, marginal costs. When these two firms are allowed to trade their exogenously determined quota with one another, a market for quota is formed. Through this model, I determine that it is always in the best interests of the firms to trade. Furthermore, when the firm with the lower marginal cost chooses to purchase quota from the second firm, it chooses to purchase an exact quantity that can be endogenously determined. However, an exact price is indeterminate.

The second model of this paper also examines a duopoly situation; but here, both firms now have different increasing marginal costs. In the quota market of this model, I find that there are different amounts of quota that can be traded depending on the demand and cost parameters, as in Model 1. However, the price at which quota is purchased is now endogenously determined. I further establish that there is an optimal quota level that maximizes social welfare for different cost and demand parameters.

Model 1: A Basic Duopoly Model with Constant Marginal Costs

Duopoly Without and With Quotas

This model examines a classic asymmetric duopoly situation, where there are two firms, Firm 1 and Firm 2. Suppose the marginal cost for Firm 1 is c_1 while that for Firm 2 is c_2, where $c_1 < c_2$. The market demand equation is

$$p = a - q_1 - q_2,$$

where p is the price, a is a constant, and q_1 and q_2 are the quantities demanded by each of the firms.

With these simple assumptions, the total revenue for Firm i = 1, 2 is

$$TR_i = aq_i - q_i^2 - q_i q_j$$

when j = 1, 2 and $i \neq j$. Then the total profits for Firm i, is

$$\pi_i = aq_i - q_i^2 - q_i q_j - c_i q_i.$$

Finding the derivative of π_i and solving for 0 generates the best response for Firm i, which is

$$q_i = \frac{a - c_i - q_j}{2}.$$

The Nash Equilibrium quantities for Firm 1 and Firm 2 are

$$q_1{}^* = \frac{a - 2c_1 + c_2}{3} \text{ and } q_2{}^* = \frac{a + c_1 - 2c_2}{3}.$$

Then naturally,

$$p^* = \frac{a + c_1 + c_2}{3},$$

and firm profits are

$$\pi_1 = \left(\frac{a - 2c_1 + c_2}{3}\right)^2 \text{ and } \pi_2 = \left(\frac{a + c_1 - 2c_2}{3}\right)^2.$$

Figure 1 graphically depicts the best responses for both firms as labeled by BR_1 and BR_2. The Nash equilibrium (q_1*, q_2*) is labeled *NE*.

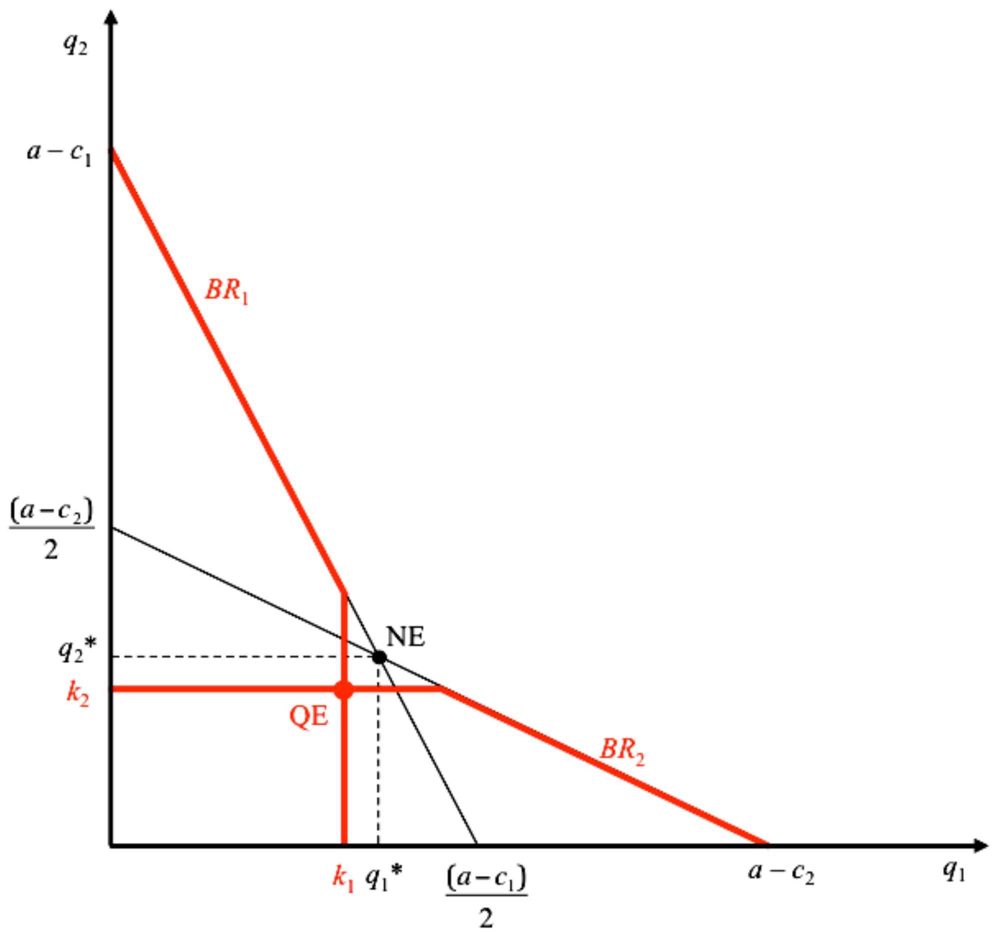

Figure 1: Best Responses Without and With Quotas

Suppose that the government imposes a total catch of $K = k_1 + k_2$, where $k_i < q_i^*$ are individual firm-level quotas. If the firms are not allowed to trade any of their quotas, the best responses for each firm are truncated as shown by the red lines in Figure 1, resulting in the quota equilibrium, *QE*. The quota equilibrium is obviously southeast of the Nash equilibrium by design. In this particular quantity-constrained model where the

QE quantities are slightly smaller than the *NE* quantities, both firms become more profitable. However, it must be noted that this is not necessarily always the case. In fact, there can be many quota combinations that result in decreased profits for either Firm 1, Firm 2, or possibly for both firms.

In Figure 2, the green and blue lines represent the isoprofit curves of the firms, where the arrows show the direction in which the profit level increases. With these assumptions, the *QE* can be in one of four regions. If the (k_1, k_2) quota allocation falls in region I, both firms will be better off; in region IV, both firms will be worse off. In region II, the quota favors Firm 1, causing it to be better off, while Firm 2 would be worse off. Similarly, if the quota is set in region III, Firm 2 would profit at Firm 1's expense.

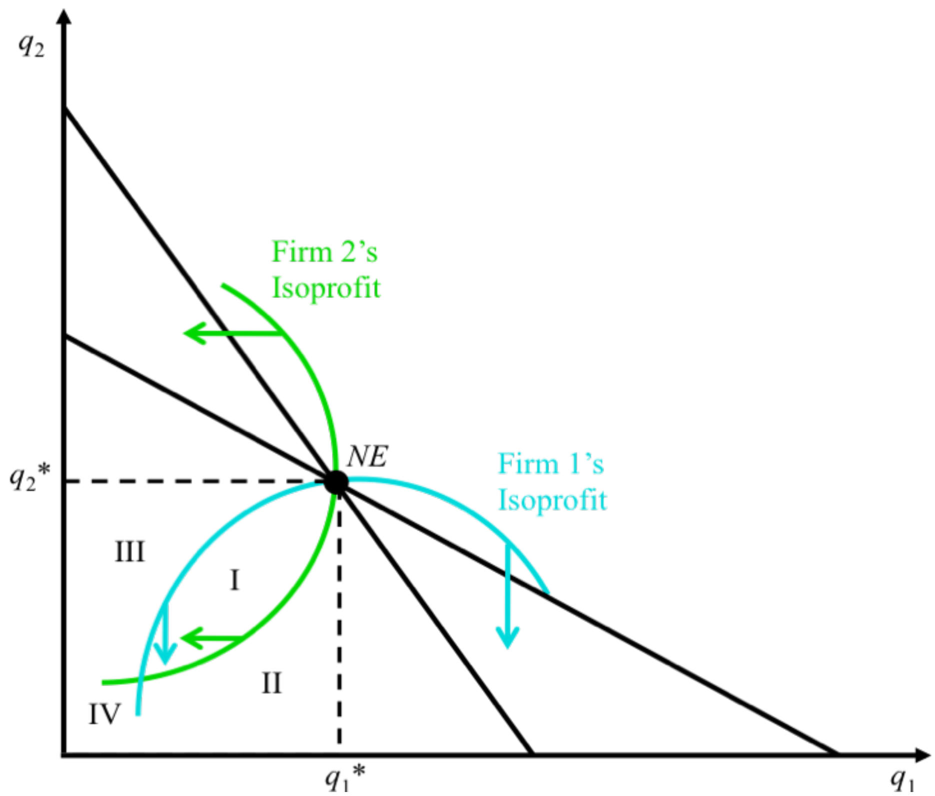

Figure 2: Firm Isoprofits

Duopoly with a Market for Quota

Suppose that the *QE* lies in regions I – III and trade in quotas is permitted. The following proposition can then be established.

Proposition 1: When trade in quotas is possible, it is always in the firms' best interest to trade.

Proof: At the quota equilibrium, *QE*, the value of a quota to Firm 1 is given by the profit margin $p^* - c_1$, where $p^* = a - (k_1 + k_2) = a - K$. Similarly, the value of a quota to Firm 2 is given by its profit margin, $p^* - c_2$. Since $c_1 < c_2$ by assumption, it follows that $p^* - c_1 > p^* - c_2$. Therefore, Firm 1 can offer a quota price, r, where $p^* - c_2 \leq r \leq p^* - c_1$ to ensure that quota will be traded. □

Now assume that Firm 2 is willing to part with its quota for a price of $r = p^* - c_2$, which is the dollar-value of a single unit of quota. I assume that firms behave competitively in the quota market taking the price r as given. Therefore Firm 2 does not exercise its monopoly market power as it agrees to sell its quota at the lowest per-unit price that it can possibly accept. Figure 3 and Proposition 2 demonstrate what happens when trade in quotas is allowed.

In Figure 3, beginning from the quota equilibrium point at (k_1, k_2), if Firm 1 purchases quotas from Firm 2, then the new equilibrium after the trade must lie on the blue 45° line that goes through the *QE* point to the *B* point. This blue line graphically depicts all output combinations for which the quota level is K. If Firm 1 purchases all of k_2, it will own all K quotas; however, it will not produce at K, but instead, after driving Firm 2 out of the market, Firm 1 will produce the monopoly quantity, $(a - c_1)/2$.

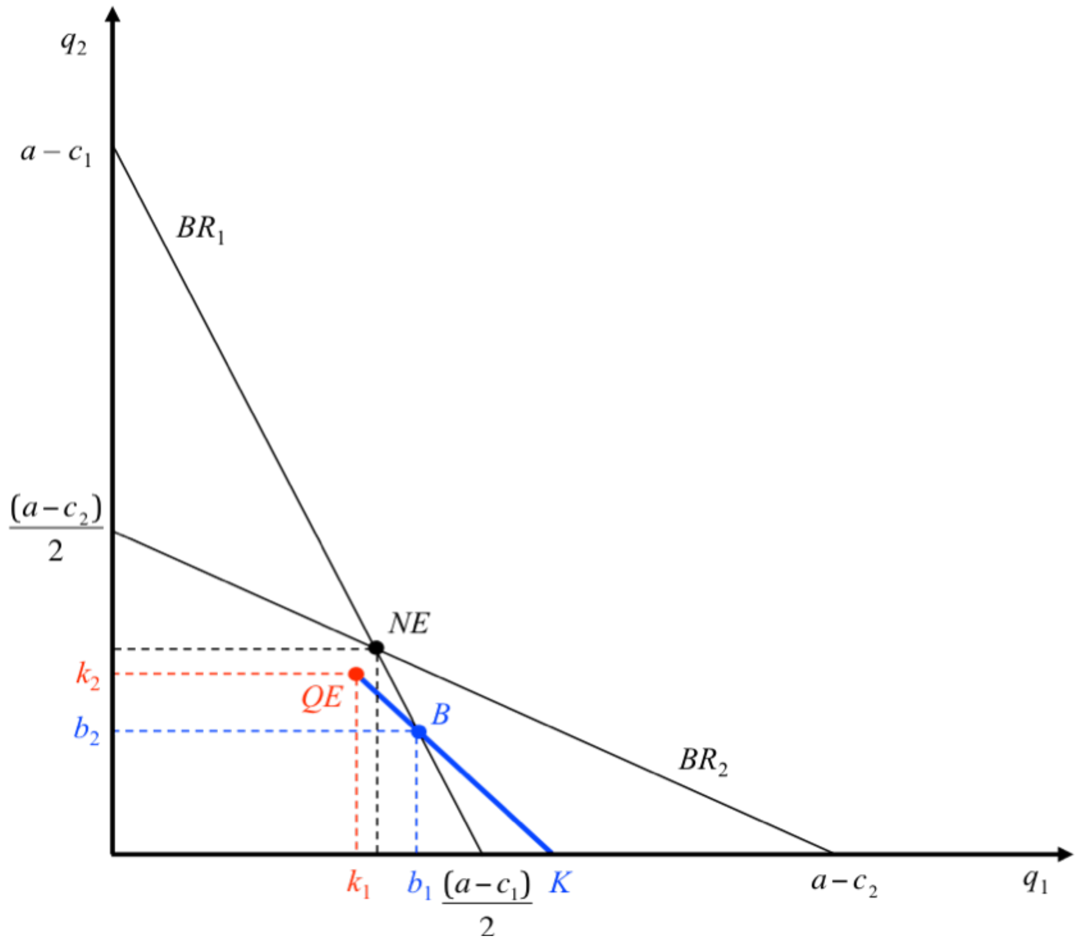

Figure 3: Quota Equilibrium With Trade in Quotas

Proposition 2: After quotas are set so that $k_i < q_i^*$ for $i = 1, 2$, either:

(a) Firm 1 purchases quotas up to its best response, represented by point B in Figure 3, or

(b) Firm 1 purchases k_2 entirely, drives Firm 2 out of the market, and produces the monopoly quantity, $(a - c_1)/2$.

Proof: To show (a), note that Firm 1 owns a quota level of k_1 and wants to be at b_1. Therefore, Firm 1 needs to purchase $b_1 - k_1$ units of quota from Firm 2. Then, Firm 1's profit at B is

$$\pi_1^B = (p^* - c_1)b_1 - (p^* - c_2)(b_1 - k_1),$$

where $p^* = a - K$ and $b_1 = a - c_1 - K$. The first term of the profit equation, $(p^* - c_1)b_1$, is the profit made from producing b_1 units of output. The second term, $(p^* - c_2)(b_1 - k_1)$, is the cost of buying $(b_1 - k_1)$ quotas from Firm 2. Simplifying, I get

$$\pi_1^B = (p^* - c_2)k_1 + (c_2 - c_1)b_1.$$

This can easily be shown to be greater than the profit at *QE*, $(p^* - c_1)k_1$. Therefore, $\pi_1 (b_1, b_2) > \pi_1 (k_1, k_2)$. This establishes that profits increase as Firm 1 moves from the original quota equilibrium to its new equilibrium at *B* when there is a market for quotas.

For part (b), note that Firm 1's profit when producing at the monopoly quantity of $(a - c_1)/2$ is

$$\pi_1^M = \frac{(a - c_1)^2}{4} - (p^* - c)(k_2) ,$$

where the first term is the gross profit from the monopoly output and the second term is the cost of purchasing all of Firm 2's quota. From this derivation, it is not obvious that Firm 1 prefers to produce b_1 instead of $(a - c_1)/2$. The following numerical example shows that this is indeed possible for suitable parameter values. When $a = 120$, $c_1 = 10$, $c_2 = 20$, the Nash equilibrium quantities are $q_1^* = 40$ and $q_2^* = 30$. Then if the quotas are set at $k_1 = 36$, and $k_2 = 20$, $\pi_1^B = 2{,}124$ and $\pi_1^M = 2{,}145$, showing that Firm 1 prefers to drive Firm 2 out of the market and behave as a monopoly. □

Figure 4 shows Firm 1's demand for quota in orange, which is from k_1 to b_1.[1] Because the decision to trade up until the best response output or the monopoly output is dependent upon the specific parameters of each situation, as shown in the second part of Proposition 2, it is uncertain as to exactly how much quota Firm 1 wants to purchase. The

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[1] Note that in Figure 4, the horizontal axis begins from k_1, and not zero.

supply of quota by Firm 2, k_2, is marked with the purple line in Figure 4. As established in Proposition 1, Firm 1 will purchase quota at a price r somewhere between $p^* - c_2$ and $p^* - c_1$, so the exact price at which quota will be purchased from Firm 2 is indeterminate, as shown in Figure 4. It is only assumed in this model that Firm 2 is willing to sell its quota for $p^* - c_2$ per unit.

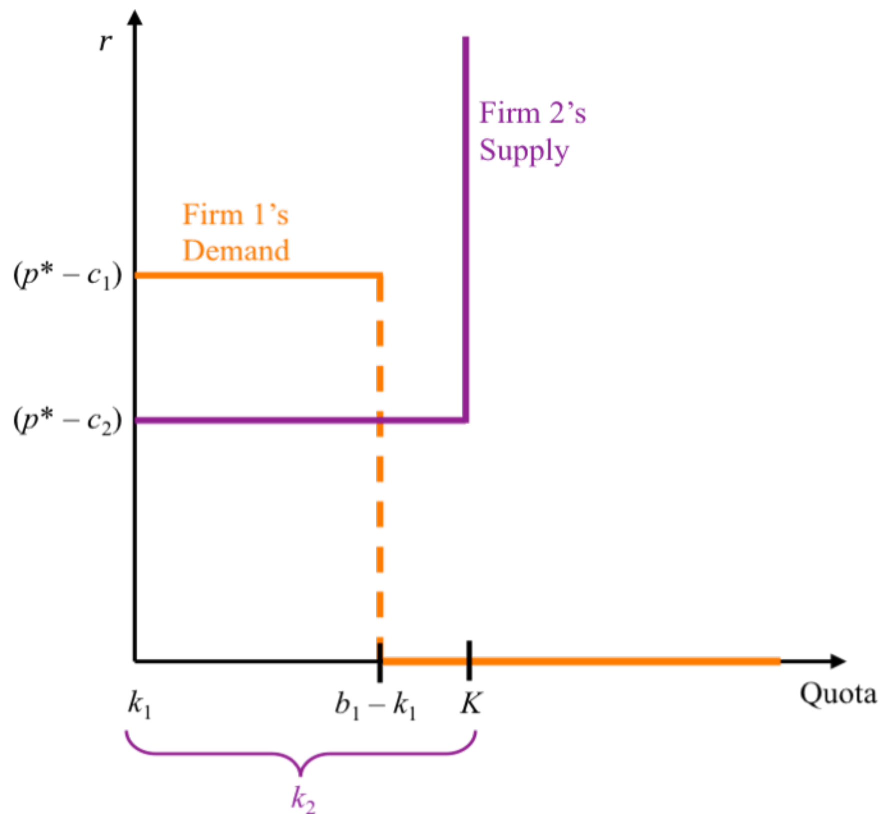

Figure 4: Supply and Demand of Quota with Constant Marginal Costs

Model 2: A Basic Duopoly Model with Increasing Marginal Costs

Duopoly Without and With Quotas

In Model 1, the supply and demand for quotas resulted in step-functions (see Figure 4) due to the constant marginal costs. For Model 2, I assume that there are increasing marginal costs, $MC_1 = c_1q_1$ and $MC_2 = c_2q_2$, where $c_1 < c_2$. So naturally, MC_1 lies below MC_2.

As before, the market demand function is $p = a - q_1 - q_2$, and the total revenue for Firm $i = 1, 2$ is then $TR_i = aq_i - q_i^2 - q_iq_j$, $j = 1, 2$ and $i \neq j$. The total profits for Firm i is now

$$\pi_i = aq_i - q_i^2 - q_iq_j - ½\, c_iq_i^2.$$

From the derivative of π_i, the best response for Firm i is then

$$q_i = \frac{a - q_j}{2 + c_i}.$$

When I solve for the Nash equilibrium quantity produced by each firm, I get

$$q_1{*} = \frac{a(1 + c_2)}{3 + 2c_1 + 2c_2 + c_1c_2} \text{ and } q_2{*} = \frac{a(1 + c_1)}{3 + 2c_1 + 2c_2 + c_1c_2}.$$

The market price of each quota is then

$$p = \frac{a(1 + c_1 + c_2 + c_1c_2)}{3 + 2c_1 + 2c_2 + c_1c_2},$$

and the firm profits are

$$\pi_1 = \frac{a^2(1 + c_2)(2 + c_1 + c_1c_2 + 2c_2)}{2(3 + 2c_1 + c_1c_2 + 2c_2)^2} \text{ and } \pi_2 = \frac{a^2(1 + c_2)(2 + 2c_1 + c_1c_2 + c_2)}{2(3 + 2c_1 + c_1c_2 + 2c_2)^2}.$$

Duopoly with a Market For Quotas

Assume that quotas were implemented as before, i.e., $k_i < q_i$*, and a market for trading quotas is developed. Figure 5 graphically depicts the best responses for both firms as labeled by BR_1 and BR_2. As in Model 1, any equilibrium after quotas have been traded will lie on the 45° blue line crossing through point B, since the post-trade quantities sum up to K. Because $K = b_1 + b_2$, and the best response of Firm 1 is $q_1 = (a - q_2)/(2 + c_1)$, the value of b_1 is

$$b_1 = \frac{a - K}{1 + c_1}.$$

Furthermore, because the amount of quota traded, $t°$, is the difference between where Firm 1 is currently producing after the trade of quota, b_1, and its original allocation of quota, k_1, the total amount of quota that Firm 1 purchases from Firm 2 is

$$t° = \frac{a - K}{1 + c_1} - k_1.$$

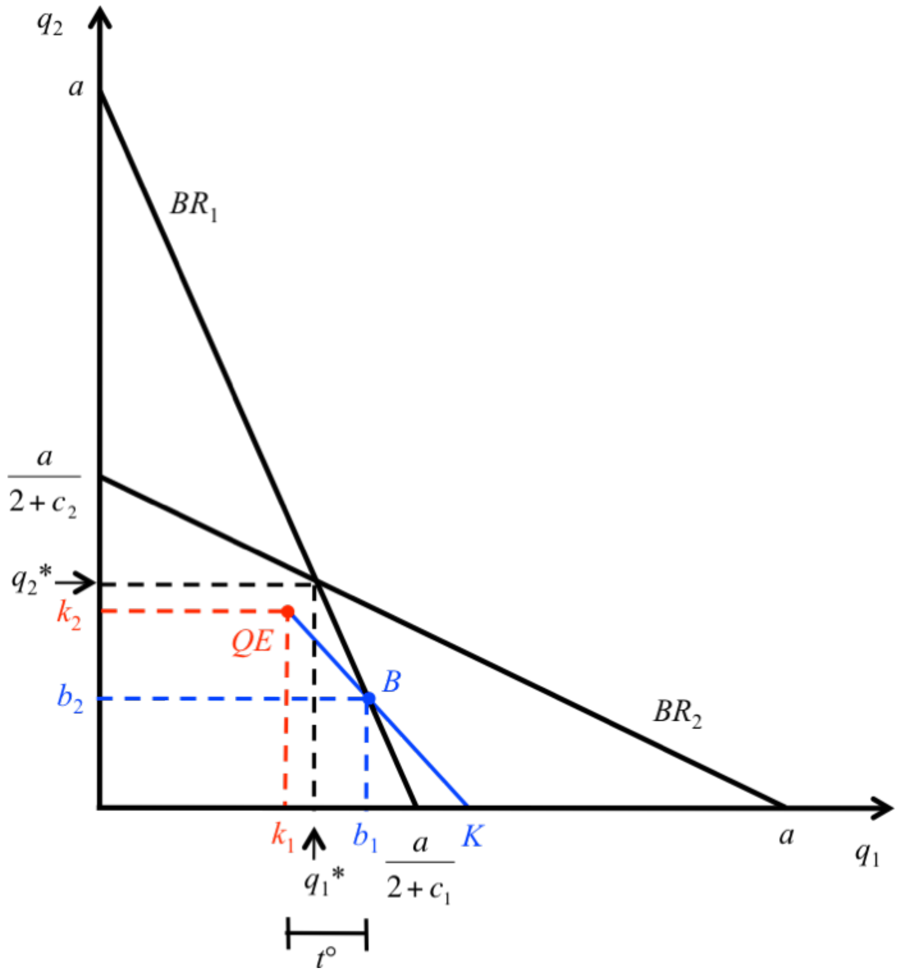

Figure 5: Quota Equilibrium with Trade in Quotas

When Firm 1 purchases t quotas from Firm 2, the new profit equations for the two firms would be as follows:

$$\pi_1 = p^*(k_1 + t) - \tfrac{1}{2}\, c_1(k_1 + t)^2 - rt$$
$$\pi_2 = p^*(k_2 - t) - \tfrac{1}{2}\, c_2(k_2 - t)^2 + rt,$$

where $p^* = a - (k_1 + k_2) = a - K$. The marginal profits of Firm 1 and Firm 2 are

$$\frac{\partial \pi_1}{\partial t} = p^* - c_1(k_1 + t) - r \text{ and } \frac{\partial \pi_2}{\partial t} = -p^* + c_2(k_2 - t) - r.$$

I set the marginal profits of Firm 1 equal to zero to derive the equation for the demand of quotas

$$r = p^* - c_1k_1 - c_1t,$$

where $p^* - c_1k_1$ is the marginal gain for purchasing quota and c_1t is the marginal cost of purchasing additional units of quota. From Firm 2's marginal profit equation, the supply of quota is

$$r = p^* - c_2k_2 + c_2t.$$

This results in a quota price of

$$r^* = \frac{c_2(p^* - c_1k_1) + c_1(p^* - c_2k_2)}{c_1 + c_2},$$

and the corresponding amount of quota traded, t^*, is

$$t^* = \frac{c_2k_2 - c_1k_1}{c_1 + c_2}.$$

Note that for t^* to hold true, I need to assume that $c_2k_2 - c_1k_1 > 0$. The unique value for r^* shows that when there are increasing marginal costs, there is an endogenously determined market-clearing price at which quota is traded. There are three market-clearing possibilities for the quota market as explained in Proposition 3.

Proposition 3: When there are increasing marginal costs and perfect competition in the market for quota, three scenarios are possible:

(a) Firm 1 will purchase t^* units of quota,

(b) Firm 1 will purchase t° units of quota, or

(c) Firm 1 will purchase all of k_2, drive Firm 2 out of the market, and then produce a monopoly quantity.

Proof: To show (a), note that the supply and demand curves of the quota market intersect at (t^*, r^*). Since $t^* < t^\circ$, it is in the best interest of the firms to trade up until the market equilibrium quantity t^*, but not until t°, as shown in Figure 6a. This corresponds to a movement along the blue 45° line from *QE* towards *B* in Figure 5, however, not enough quota is traded to actually reach *B*.

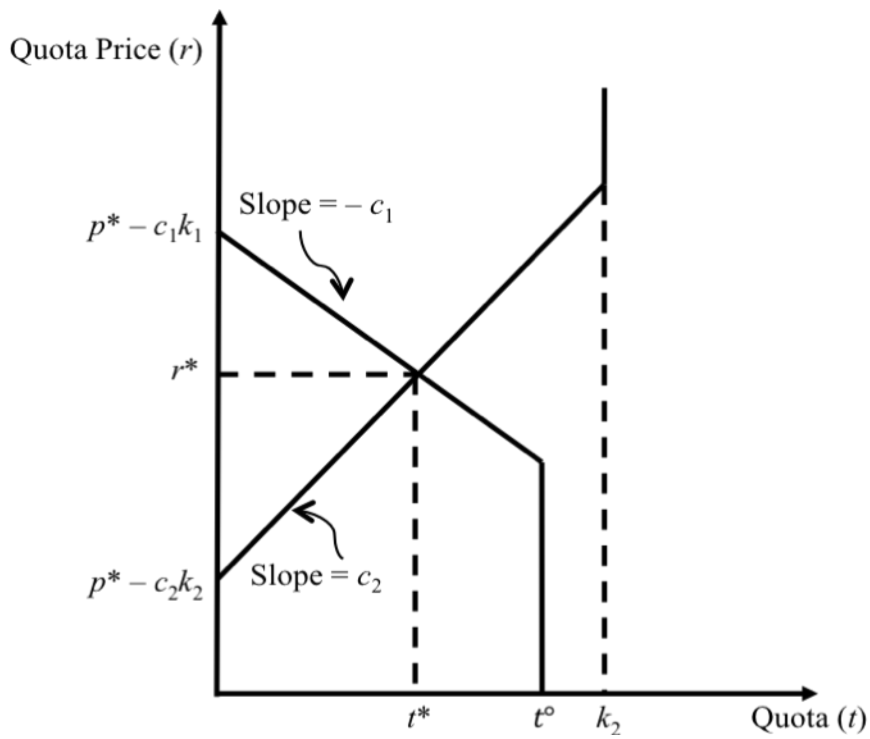

Figure 6a: Quota Market Equilibrium, Case (a)

To show (b), consider a situation where the amount of quota demanded by Firm 1 is significantly smaller than the amount of quota supplied by Firm 2. In this case, the supply and demand curves do not intersect until Firm 1 purchases *t*° units of quota. This scenario is shown in Figure 6b.

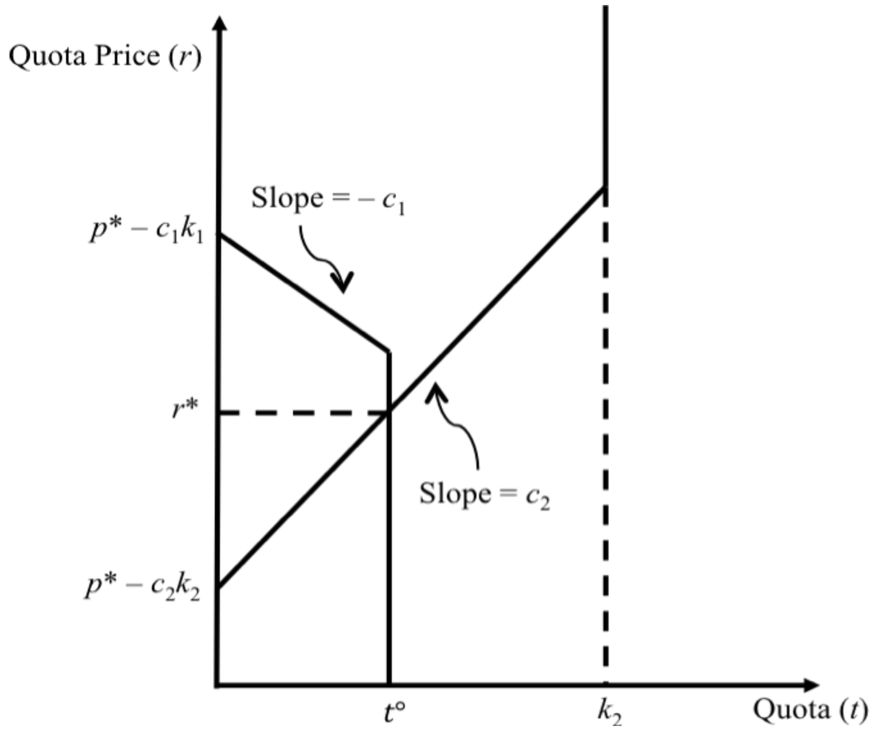

Figure 6b: Quota Market Equilibrium, Case (b)

To show (c), consider a situation where the amount of quota demanded by Firm 1 exceeds the total amount of quota given to Firm 2, k_2. When this situation occurs, the supply and demand indicates that Firm 1 will purchase all of k_2. Therefore, Firm 1 can drive Firm 2 out of the market, and produce a monopoly quantity. This possibility is shown in Figure 6c.

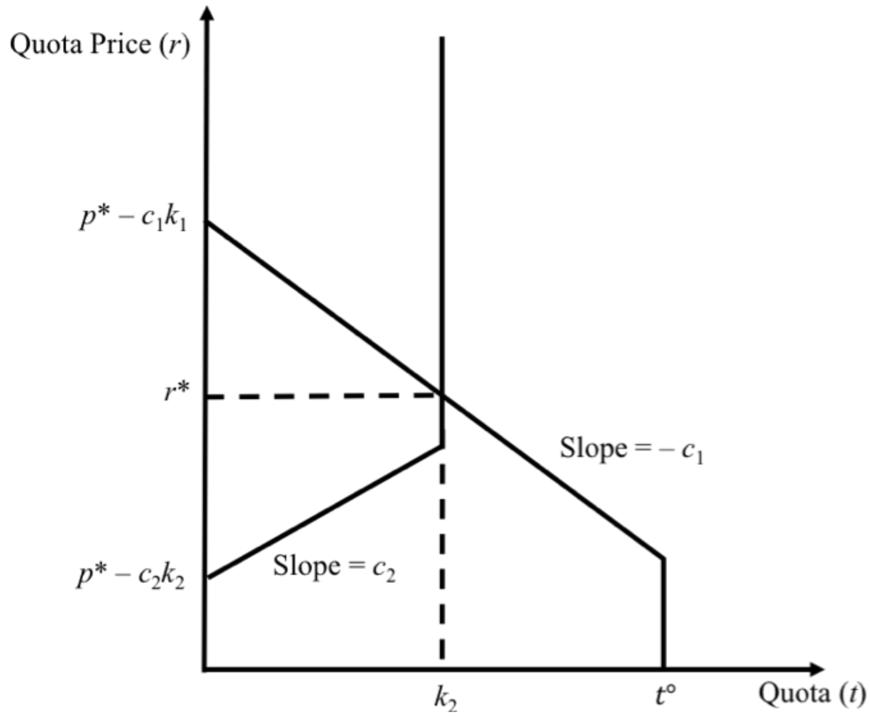

Figure 6c: Quota Market Equilibrium, Case (c)

□

In the remainder of this paper I will only examine scenarios (a) and (b) from Proposition 3. Scenario (c) is less compelling because while a monopoly situation may theoretically arise in a duopoly, this is not very likely in an oligopoly.

When determining where the quota level should be set, the regulator tries to maximize total welfare, W. By definition, welfare maximization is when the sum of consumer surplus, CS, and industry profits, $\pi_1 + \pi_2$, is maximized, i.e., $W = CS + \pi_1 + \pi_2$. Figure 7 shows that $CS = K^2/2$. Industry profits can also be written in terms of K. From above,

$$\pi_1 = p^*(k_1 + t) - \tfrac{1}{2}\, c_1\, (k_1 + t)^2 - rt \text{ and}$$
$$\pi_2 = p^*(k_2 - t) - \tfrac{1}{2}\, c_2\, (k_2 - t)^2 + rt,$$

where $(k_1 + t)$ and $(k_2 - t)$ simply represent the different proportions of K that the two firms produce. Suppose that $k_1 + t = \alpha_1 K$ while $k_2 - t = \alpha_2 K$, where $\alpha_1 + \alpha_2 = 1$. Then

$$\pi_1 + \pi_2 = p^*(\alpha_1 K) - \frac{1}{2}c_1(\alpha_1 K)^2 + p^*(\alpha_2 K) - \frac{1}{2}c_2(\alpha_2 K)^2,$$

and the social welfare equation is given by

$$W(K) = \frac{K^2}{2} + p^*(\alpha_1 K) - \frac{1}{2}c_1(\alpha_1 K)^2 + p^*(\alpha_2 K) - \frac{1}{2}c_2(\alpha_2 K)^2.$$

Because the entire social welfare equation can be written in terms of *K*, this suggests that there is a unique value of *K* that maximizes social welfare, shown by Proposition 4.

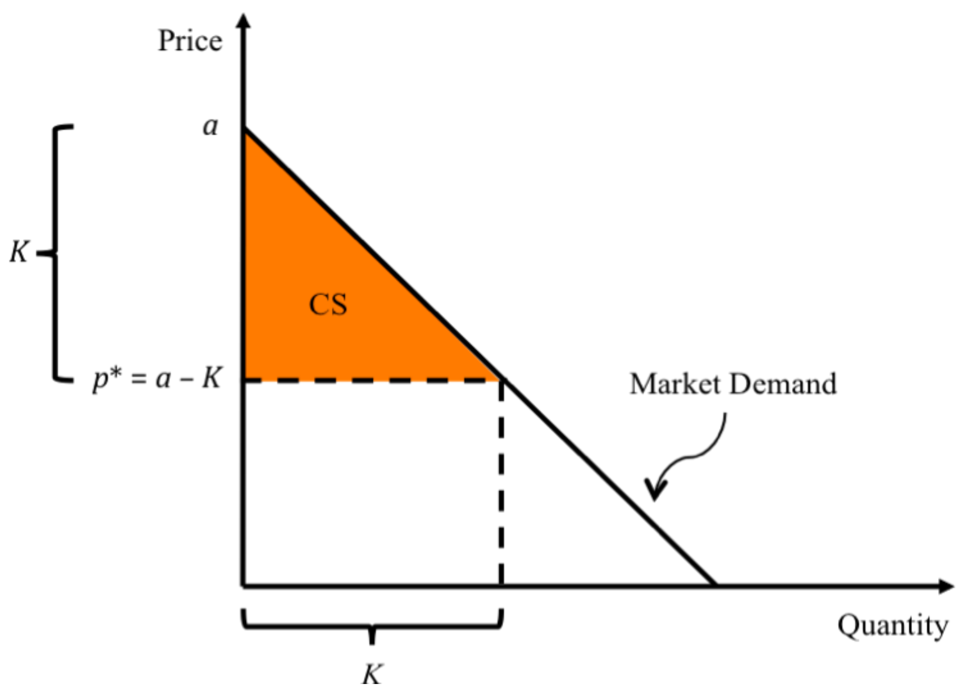

Figure 7: Welfare Maximization

Proposition 4: There is a level of *K* that maximizes social welfare.

Proof: Suppose a quantity cap, *K*, was implemented, and the regulator wants to maximize total social welfare

$$W(K) = CS + \pi_1 + \pi_2.$$

When t^* units of quota are traded,

$$CS = K^2/ 2,$$

$$\pi_1 = (a - k)(k_1+t^*) - ½c_1 (k_1+t^*)^2 - r^*t^*, \text{ and}$$
$$\pi_2 = (a - k)(k_2 - t^*) - ½c_2 (k_2-t^*)^2 + r^*t^*.$$

Therefore, by substituting t^* into the social welfare equation, and solving for the second derivative, I get

$$W''(K) = -\frac{2(3+3c_1+c_1^2)}{(1+c_1)^2}.$$

This equation shows that the regulator's objective function is strictly concave at any single value of K. Setting the first derivative of $W(K)$ equal to zero and solving for K, I get

$$K^* = \frac{a(4+3c_1+c_1^2)}{6+6c_1+2c_2}.$$

This distinct value of K^* is the point at which social welfare is maximized.

Similarly, when Firm 1 purchases $t°$ units of quota, the second derivative equation is given by

$$W''(K) = -\frac{2c_1^2+2c_1c_2+2c_2^2}{(c_1+c_2)^2}.$$

Since W is concave in K^*, by setting the first derivative of $W(K)$ equal to zero and solving for K, I get

$$K° = \frac{a(c_1+c_2)^2}{c_1^2+c_1c_2+c_2^2},$$

which is the quantity cap that maximizes social welfare when $t°$ units to quota are traded. □

Because there are two unique values of K^* and $K°$, this means that there are two different quantity caps that can arise depending on how much quota is traded.

Proposition 5 shows the relationship between these two different values of K are in fact unique quantities, and that $K^* > K°$.

Proposition 5: The quantity cap $K°$ is less than the quantity cap K^* if $c_1(c_1^2 + 2) > c_2$.

Proof: As shown above,

$$K^* = \frac{a(4 + 3c_1 + c_1^2)}{6 + 6c_1 + 2c_2} \text{ and } K° = \frac{a(c_1 + c_2)^2}{c_1^2 + c_1c_2 + c_2^2}.$$

For the proposition to hold true, it must be shown that $K° - K^* < 0$. Therefore, I need to know that

$$K° - K^* = \frac{a(c_1^2 - 2c_1c_2 + c_2^2 - 3c_1^2c_2 - c_1^3c_2)}{2(3 + 3c_1 + c_1^2)(c_1^2 + c_1c_2 + c_2^2)}$$

is a negative number. Because the denominator of this equation will obviously be positive, the numerator must be negative. By rearranging the terms of the numerator, it can be shown that

$$(c_1^2 - 3c_1^2c_2) - c_2(c_1^3 + 2c_1 - c_2) < 0.$$

The first term of this equation will clearly be negative since $c_1 < c_2$. The second term of this equation is also negative when $c_1(c_1^2 + 2) > c_2$. □

Conclusion and Extensions

This paper models the effects of a market for quota and formulates the solutions for optimal amounts of quota that should be allocated to, and traded between, different firms. Even in a simple duopoly situation where externalities are not accounted for, there are various ways by which firms can act depending on the specific scenario. I show in Model 1 that when Firms 1 and 2 have constant but different marginal costs, the two firms will always be better off when they choose to trade quotas. However, the amount of quota each firm should buy/sell, and whether a firm should produce at its best-response quantity or a monopoly quantity, is entirely dependent on values of the demand and cost parameters. Furthermore, in this model, the price at which quota is bought and sold, r^*, cannot be endogenously determined.

In Model 2 of this paper, the firms were assumed to have increasing marginal costs. With this model, I was able to endogenously generate the price and quantity at which quotas are bought and sold. Depending on the parameters of the specific situation, there are three different market-clearing opportunities for the quota market. I also determine that there is a single value of K that maximizes social welfare for different demand and cost parameters. With different optimal values of the quantity cap, K, I then show that the quantity caps are set at different levels when firms choose to trade t^* units of quota, versus when firms trade t° units of quota.

While this paper offers some insight into the workings of a market for quota, there are several ways to further expand upon the ideas of this thesis. To begin, this paper uses a Cournot duopoly structure to analyze the effects of a quota market, which examines firms as they compete in output produced. The Cournot competition model is appropriate because the objective of the regulator is to limit overall production, so it is reasonable to assume that each firm chooses quantities, while price is determined in the market. However, in a study performed by Adelaja, Menzo, and McCay (1998), a Bertrand competition model, where firms choose the price at which they want to sell, was used to examine market concentration after a market for quotas is introduced. While the objective of their paper is different from mine, it does introduce the idea that the impact of a quota market can also be examined when firms are assumed to be price competitors. Therefore, a future extension of this paper could instead use a Bertrand competition model to examine a market for quotas. Such results could then be juxtaposed to the findings of this paper to provide a more complete understanding of the impact of quota markets.

Further extensions of this paper may also examine the effect of an output tax, contrary to limiting output by using a quota market. Theoretically, a tax should be able to limit output by the same amount as the quota, with the added advantage of being able to generate tax revenue for the regulator. The effectiveness of this Pigovian tax levied to manage the negative externality of common pool resources can then be compared to introducing a market for quota in order to determine which might be a better way to manage common pool resources.

Additionally, this paper does not take into account the externalities that are generally involved with common pool resources. The very definition of a common pool resource suggests that the use of quotas by one firm could have unintentional consequences on the other firms. However, such externalities were not modeled for the sake of simplicity, and future extensions of this paper could incorporate these effects.

While my results may be modified when externalities are introduced, the basic insights of this model are still likely to be relevant.

Assuming that the negative externality is only a product of competition between firms, the introduction of negative externalities in the model suggests that increased market power may ameliorate the common pool resource problem. In the absence of negative externalities, increased market concentration is generally viewed as being bad for the market, as it can hinder competition between firms. However, when externalities are introduced, a greater market concentration reduces the external effects, and may make it less likely for firms to over-utilize the resource.

Should a negative externality be introduced, the social welfare equation, in addition to including consumer surplus and industry profits, would also need to subtract the dollar value of the externality costs, to be

$$\text{Welfare} = CS + \pi_1 + \pi_2 - \text{Negative Externality Costs.}$$

However, determining how to capture the precise cost of a negative externality and exactly how it impacts social welfare is beyond the scope of this paper.

In additional efforts to simplify the theoretical model, I only examine a market for quotas in a duopoly situation. However, in simplifying the model I also reduced the applicability of this model to real-world scenarios. Though the findings of this paper seem to suggest how firms could respond to a situation with more than two firms, the complexity of an oligopoly may alter the results. For example, in a duopoly, there is always a buyer, the firm with lower marginal costs, and a seller, the firm with higher marginal costs. In an oligopoly, it is unclear as to which firms are buyers and which firms are sellers without knowing how the firms differ in their costs. Therefore, depending on the cost parameters of each firm, the equilibrium price and quantity of quota traded could vary drastically. Also, because this paper only examines a market for quotas for a duopoly, I was able to show in Case (c) of Proposition 3 that it is theoretically possible to purchase all quotas from the second firm and produce at a monopoly quantity. While this result might technically be possible in an oligopoly, it would be less likely.

Another potential issue with this paper is that I assumed firms to be strategic in the product market but not in the quota market. In the second model of this paper, firms are understood to be price-takers in the quota market, and therefore do not exercise their ability to affect prices through their monopoly/monopsony power. In actuality, however, if there were only one buyer and one seller, each firm would most likely utilize its market power. While this is less likely in the case of an oligopoly, the precise workings of such a model would have policy implications for quota markets and should be explored in future studies.

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