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Spatial Optimization of 4-Poster Feeders for Tick-Borne Disease Management

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Abstract

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Amblyomma americanum, the Lone Star tick, is the predominant tick species throughout the southeast United States. Its significance as a threat to human health was not realized until recently. Recognized as an important disease vector, Amblyomma carry a serious bacteria, Ehrlichia chaffeensis, that causes human monocytic ehrlichiosis. In 1995, eleven cases of ehrlichiosis due to E. chafeensis were identified in Fairfield Glade, a retirement golf community near Crossivlle, Tennessee. The placement of 4-poster acaricide feeders has be demonstrated to be a highly effective control method for eliminating Amblyomma populations. Here we formulate an economic criterion to evaluate various feeder placement scenarios within Fairfield Glade that minimize infected ticks and that tend toward future projects in optimization of this system.

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Chapter 1

Introduction

In recent years there has been growing concern over tick-borne diseases such as Lyme disease, rickettsiosis, and relapsing fever. These diseases are serious health problems that affect both humans as well as domestic animals. Thanks to rising awareness, our understanding of these diseases is constantly increasing. As a result, many different control efforts have been studied to eliminate the disease(s) from tick populations. This thesis optimizes one such method for a community in central Tennessee. In this chapter we present the biological background of the problem, and then discuss different approaches to modeling tick populations for effective disease management.

1.1 Biological Background

While Amblyomma americanum, the Lone Star tick, has long been the predominant tick species in the Southeast United States, its significance as a threat to human health was not realized until recently. Initially noted for its aggressive nature, this tick is now recognized as an important disease vector as well. Ten species of bacteria have been isolated from Amblyomma including Ehrlichia chaffeensis, Ehrlichia ewingii, Rickettsia rickettsii, Coxiella burnetii, Francisella tularensis, Borrelia lonestari, and four other unnamed agents [4]. Perhaps the most serious bacteria among this group

is E. chaffeensis, the causative agent of human monocytic ehrlichiosis (HME), due to both the nature of the disease and the role of the lone star tick as its primary means of transmission. The first human cases of ehrlichiosis were not described until 1987 [14], and the lone star tick was not identified as the vector of Ehrlichia chaffeensis until 1995 [12]. It is now known that HME results in a general febrile illness and leukopenia. Distinctive inclusion bodies of the rickettsial agent can be noted within white blood cells of infected patients [4].

More recently, white-tailed deer (Odocoileus virginianus) were identified as the natural reservoir host of E. chaffeensis, the only mammal known to date to transmit the rickettsia back to ticks once infected [13]. It is within this tick-deer life cycle that E. chaffeensis is preserved in the wild, resulting in occasional transmission to humans following the bite of an infected lone star tick. While these ticks will feed on a wide array of mammals, they demonstrate a predilection for white-tailed deer throughout all three life stages. Lone star ticks are aggressive, and actively seek out hosts. Eggs hatch in late spring, giving rise to larvae by mid-summer. Those larvae that successfully obtain a blood meal will molt and remain dormant until the following spring. During the spring, nymphs emerge, feed, and molt to adults. Male and female adults then mate on the host and die following egg deposition. Overwintering, or this period of dormancy, can be done by three of the life stages- unfed larvae, unfed adults, or fed adults gravid with eggs. This two-year life cycle results in the emergence of the more mature nymph and adult stages earlier in the year than the larval stage. All three stages require blood meals to continue the life cycle, creating the potential for disease transmission to the tick at any one of these stages [4].

Approximately 1000 HME cases have been identified over the past ten years since the disease became reportable [7]. Yet even in endemic areas, only 5% - 15% of lone star ticks carry the pathogen [4], and the annual incidence of human cases is

as low as 0.003% - 0.005% [21]. However, rare clusters of ehrlichiosis due to E. chafeensis have been reported. In 1995, eleven cases were identified in Fairfield Glade, a retirement golf community near Crossville in eastern Tennessee, at an attack rate nearly 100 times the national average for endemic areas. It was determined that this elevated rate of E. chafeensis infection was due to both the community's proximity to a large, protected population of white-tailed deer and its suitability for outdoor recreational activities [21]. Since then attempts have been made to control the tick population with the use of 4-poster feeder systems that passively apply acaricide to the deer hosts [18]. In research trials, this technique has been demonstrated as a highly effective control method against Amblyomma, with nearly 100% reduction rates of the nymphal population over five years in areas where deer repeatedly visit the feeders [3]. The efficacy of the technique for large-area tick control is unclear; presently the tick population at the site of interest near Crossville remains high (J. Harmon, unpubl. data). The distribution of E. chaffeensis is highly dependent on the presence of white-tailed deer, lone star ticks, and five critical variables: moisture, temperature, habitat type, day length, and host density [22]. Abundance and growth of the lone star tick are associated with warm months and wooded habitats such as those found in Fairfield Glade.

1.2   Mathematical Models

There are several different approaches to modeling tick populations in the literature, most of which involve disease dynamics. Haile and Mount (1987) have simulated population dynamics of the lone star tick with a discrete time model that are applicable to any environment depending on factors such as temperature, relative humidity, and host density [11]. Sandberg et al provide a matrix model that seasonally varies

population densities of questing ticks [19]. Awerbuch-Friedlander et al formulate a nonlinear system of difference equations that model the three stage life cycle of ticks and investigate consequences of environmental changes [1]. Ghosh and Pugliese present a semi-discrete model for tick population and disease dynamics [10]. More recent models such as that proposed by Barnard et al [17] have analyzed integrated management strategies for both short- and long-term control of these ticks. Proposed methods included area-wide acaricide application, vegetation reduction, acaricide self-treatment of white-tailed deer, deer population density reduction, or any combination of those techniques [17]. Here we consider self-treatment of deer via a topical rather than systemic route, since recent FDA regulations have made the deer population density reduction approach infeasible. The goal of this study is to optimize spatial arrangements of 4-poster feeders to control E. chafeensis within the Fairfield Glade retirement community. This thesis is a continuation of work done by a Research Experience for Undergraduates (REU) program at the National Institute for Mathematical and Biological Synthesis (NIMBioS) at the University of Tennessee.

This thesis is outlined as follows: The model developed by the REU group is described in Chapter 2. In Chapter 3 a new model is considered that gives more realistic results and is more manageable to optimize. Optimization methods, test cases and results are reported in Chapter 4. Finally, Chapter 5 discusses the results and offers suggestions for future work with this project.

Chapter 2

Original Method

In this section we present the model and findings developed by the NIMBioS REU group in the summer of 2009 at the University of Tennessee. The model has two components: a time model, and a space model.

2.1 Discrete Model

A discrete time and spatial model was developed by the REU group to model the tick life cycle. The time model was designed to represent the rather complex three-host life cycle of the Lone Star Tick (LST) and to investigate the optimal spatial arrangement of 4-poster feeders within the Fairfield Glade (FFG) retirement community. The REU group formulated an economic criterion to evaluate various feeder placement scenarios that allow recommendations to be made to FFG for 4-poster feeder arrangements that minimize both cost and ehrlichia cases, and that tend toward future projects in optimization of this system.

2.1.1 Time model

A discrete time model was designed first to represent the rather complex three-host life cycle of the LST. This model tracks the flow of individuals among each of the seven life stages, as shown in Figure 2.1. Data collected by J. Harmon (unpubl.)

are used to estimate some of the parameters, while others are based on previous LST modeling work by Haile and Mount [11]. Because Haile and Mount's model parameters vary on habitat type, temperature and humidity, the values we used were calculated based on the assumption that FFG is an upland wooded environment [2]. Average temperature and humidity for each month were obtained from FFG's website (www.fairfieldglade.net). There is no literature for LST on-host survival rates, so preliminary estimates were adjusted so that the model would better represent field data provided by M. Rosen (unpubl. data). A complete list of parameters and variables can be found in Table 2.1.

Table 2.1: Parameter values for disease model [2].

Parameter	Description
ε	eggs
λ	unfed larvae
$\hat{\lambda}$	fed larvae
ν	unfed nymphs
$\hat{\nu}$	fed nymphs
α	unfed adults
$\hat{\alpha}$	fed adults
b	mortality rates (mr)
h	host-finding rates (hfr)
σ	on-host survival rates (ohsr)
k	monthly time step

The model equations are listed below:

$$\varepsilon_{k+1} = \varepsilon_k(1 - b_\varepsilon)$$
$$\lambda_{k+1} = \lambda_k(1 - b_\lambda) - h_\lambda\lambda_k$$
$$\hat{\lambda}_{k+1} = \sigma_\lambda h_\lambda\lambda_k$$
$$\nu_{k+1} = \nu_k(1 - b_\nu) - h_\nu\nu_k + .85\hat{\lambda}_k \quad (2.1)$$
$$\hat{\nu}_{k+1} = \sigma_\nu h_\nu\nu_k$$
$$\alpha_{k+1} = \alpha_k(1 - b_\alpha) - h_\alpha\alpha_k + .85\hat{\nu}_k$$
$$\hat{\alpha}_{k+1} = \hat{\alpha}_k(1 - b_{\hat{\alpha}}) + \sigma_\alpha h_\alpha\alpha_k$$

Several assumptions were made with this model. A tick is not "fed" unless it
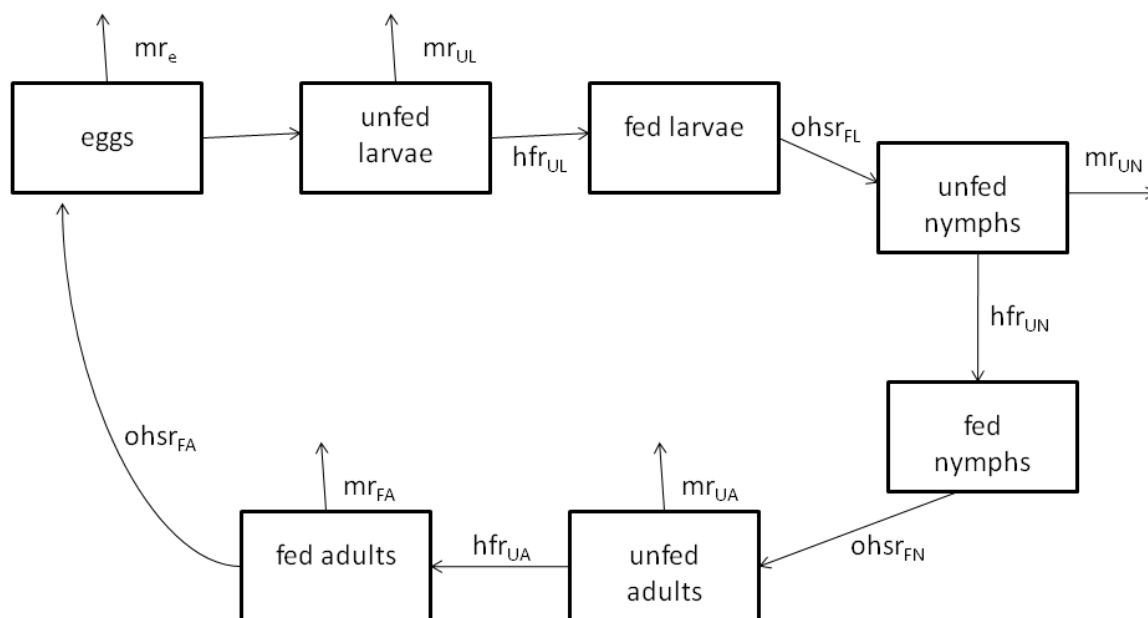
Figure 2.1: Diagram of the factors and rates determining movement between the LSTs seven life stages. The discrete time model equations were based upon this flow. The model's parameters are defined in Table 2.1 [2].

survives the complete feeding process. Over the course of a month, fed larvae and fed nmphs either moult and survive to the next stage, or they die. Fed adults can survive for more than a month. All engorged females lay eggs at the beginning of April, and 52% of those eggs hatch 2 months later. Survival rates remain constant throughout the year, whereas host-finding rates change based on month. The population at the beginning of each year (April) consists of: eggs, unfed adults, and unfed nymphs. In Figure 2.2 we show tick dynamics during a 50 month time period without the use of a feeder.

2.1.2 Space model

The model also takes into account the geography of the Fairfield Glade (FFG) retirement community (Figure 2.3). The property of FFG and the surrounding forest

Figure 2.2: Tick dynamics for an undeveloped patch without a feeder for 50 months. All life stages of the ticks are shown (e=eggs, uL=unfed larvae, fL=fed larvae, uN=unfed nymphs, fN=fed nymphs, uA=unfed adults and fA=fed adults). The '+' correspond to unfed populations, the '*' correspond to fed populations, and the 'o' correspond to the egg population.
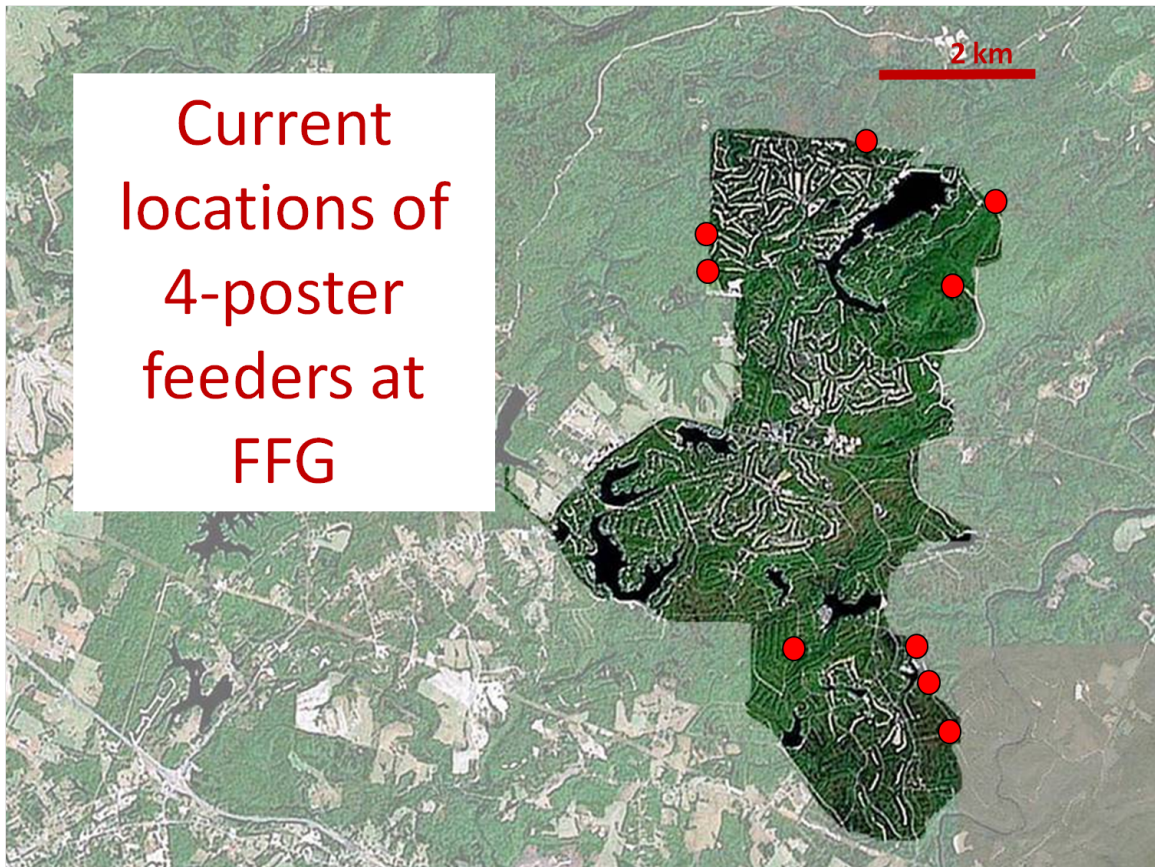
Figure 2.3: Fairfield Glade Retirement Community. Current feeder sites are labeled with red circles. This area was put on a grid where each square of the grid has its own tick population. Satellite image obtained from www.earth.google.com.

areas were put on a grid, with each square on the grid assumed to have its own tick population. Each square in the grid was given values to represent the type of area (developed or undeveloped) and the proximity of feeders (no feeder, feeder(s) in adjacent squares, or a feeder in the square). Note that for safety purposes, feeders are only allowed to be placed in undeveloped areas. Forest areas without any feeders result in the "baseline" tick population represented by the discrete time model described previously. The model is designed to predict how successful a given arrangement of 4-poster feeders would be at reducing tick populations and, consequently, cases of ehrlichiosis among Fairfield Glade residents. If the tick population was in a residential area, the initial conditions were decreased to 10,000 eggs, 80 unfed nymphs, and 35 unfed adults so that the tick population was smaller. If there is a feeder in the square, on-host survival rates for all life stages was 0.05; if there is a feeder in an adjacent square, on-host survival rates for all life stages was 0.50. We based these values on the assumption that 95% of deer will receive acaricide treatment from a feeder in their own grid square, and about 50% of deer will be treated by a feeder in a square adjacent to the one in which they primarily reside [2]. Figure 2.4 shows the same tick population from Figure 2.2, but now with acaricide feeders. The results show that the presence of feeders greatly reduces the numer of ticks over the 50 month time period.

2.1.3 Cost-Effectiveness Analysis

The output of the model is an estimate of the total cost of managing tick-borne disease at FFG. It comprises the cost associated with human cases of ehrlichiosis, plus the cost of acaricide feeder program. We assumed that without any feeders there would be, on average, 10 cases of ehrlichiosis among the FFG residents per year.
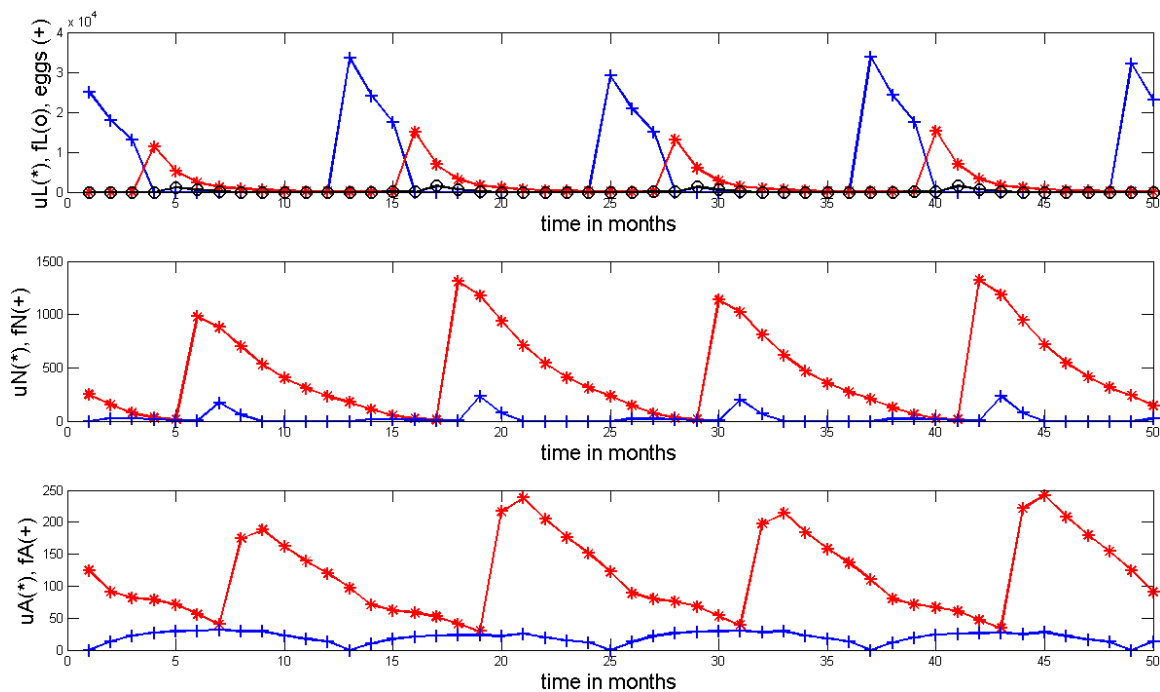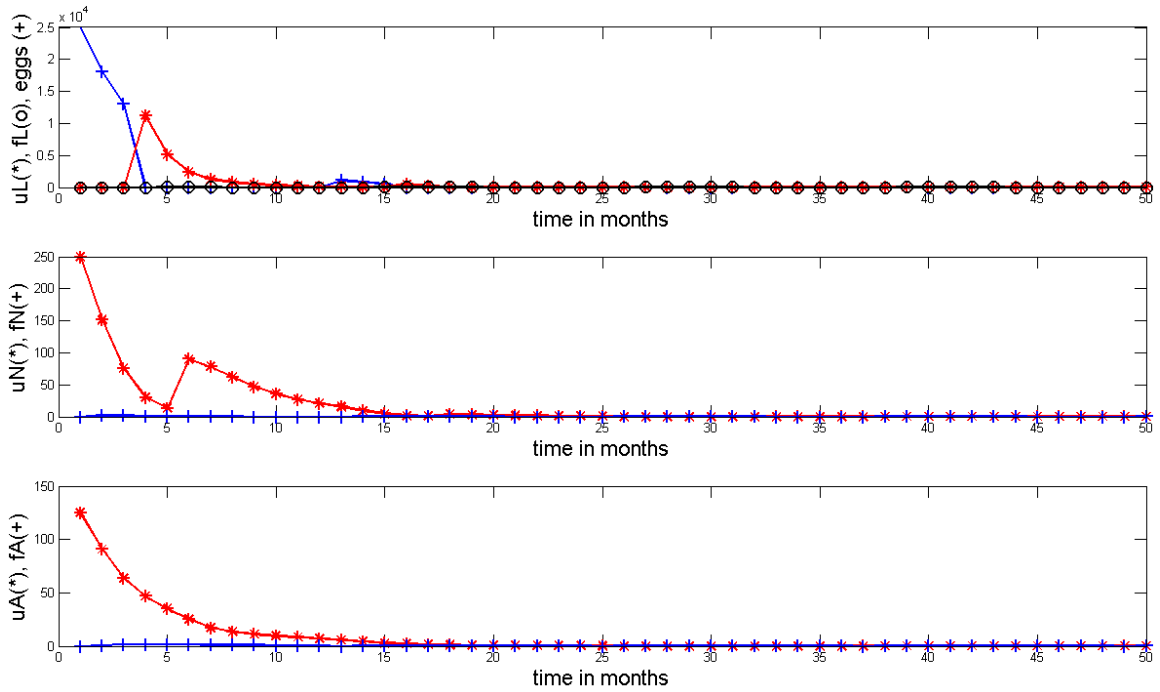
Figure 2.4: Tick dynamics for an undeveloped patch with a feeder for 50 months. All life stages of the ticks are shown (e=eggs, uL=unfed larvae, fL=fed larvae, uN=unfed nymphs, fN=fed nymphs, uA=unfed adults and fA=fed adults). The '+' correspond to unfed populations, the '*' correspond to fed populations, and the 'o' correspond to the egg population.

Each case of ehrlichiosis was assumed to cost $10,000 for medical treatment and lost productivity. Each feeder was assumed to cost $2,000 per year to construct, install and maintain [2]. The model compares the tick population in July of year 4 on the control grid (i.e. when feeders are present) versus the population on the grid when feeders are not used, such that $\frac{\text{controlled population}}{\text{uncontrolled population}}$ in that month represents the percent of ticks remaining despite the feeder program. Only ticks on developed grid squares are included in this calculation since ticks in undeveloped areas are assumed to have no contact with FFG residents. The REU group assumed that the number of
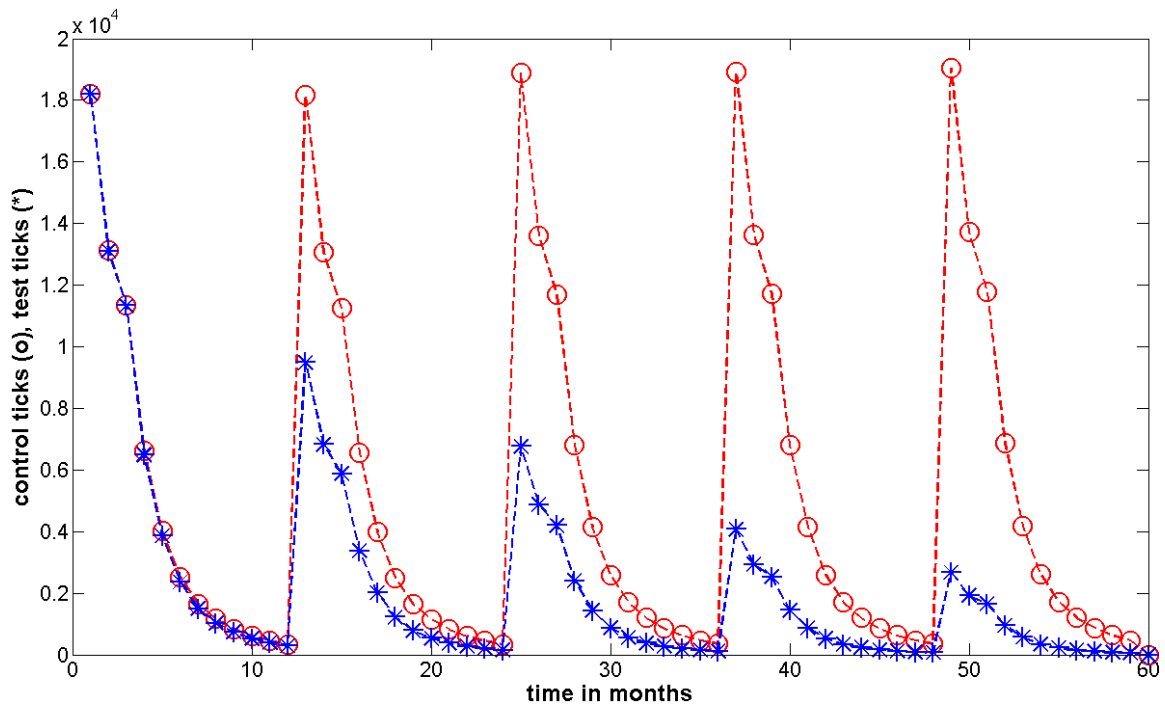
Figure 2.5: Total tick dynamics with current arrangement of feeders used by Fairfield Glade. The '*' represent the population with feeders in place, and the 'o' represent a control population without feeders.

cases is directly related to the number of ticks, so the expected number of cases per year will be (percent of ticks remaining)×10 [2]. Our resulting equation for cost per

year in the 4th year is:

$$\text{Cost per year} = \$2,000 \times \text{number of feeders} + \$10,000 \times \text{cases in year 4}$$

Since the model handles only one arrangement of 4-posters at a time, the REU group individually tested 6 different scenarios of feeder arrangements and then compared the cost and percent reduction of each scenario. For example, the current arrangement of feeders (see Figure 2.3) gives a 63% reduction and costs approximately $7700 per year. Figure 2.5 plots the sum of all tick populations in the grid. The initial goal of this thesis was to continue work on this model and spatially optimize the arrangement of feeders. However, as discussed in the next section, we found that better results could be obtained by considering an alternative model.

2.2 Seeking a New Model

The model described in the previous section has many advantages over other modeling methods. Since it is discrete in time and space, it does not require solving computationally expensive differential equations. This model also provides dynamics of each life stage of the tick, and successfully models the two year life cycle. However, several problems exist with the model. First, each square's tick population was independent from other populations. Ticks could not migrate between squares. This is not realistic because ticks could easily move amongst the squares while feeding on a host. Second, hosts and the disease itself were left out of the model. Minimizing the disease was assumed to correlate with minimizing the tick population. However, one may want to minimize the infected tick population and maximize the healthy tick population, so this assumption is not justifiable. Third, the dynamics of the tick population became

unrealistic as time progressed. For these reasons and to obtain more computationally efficient methods, we sought an alternate approach to the space-time model.

Chapter 3

Population Model

While researching other tick population models, we came across a paper discussing the same Fairfield Glade problem that was published prior to the REU group's work. Gaff and Gross developed a space-time model for ticks, hosts, and Ehrlichia chaffeensis in the retirement community. Instead of working to fix the old model, we decided to pick up where this model left off. The model is described in detail in [8], and summarized here. To model tick-borne disease, Gaff models the dynamics of host (N) and tick (V) population densities as well as the densities of infected hosts and ticks (Y and X, respectively) at six different sites in the Fairfield Glade community. The sites were chosen to mimic a study done by Marsland [15]. Marsland investigated the effect of feeding ivermectin treated corn to a free-ranging, unrestricted white-tailed deer population on free-living stages of the Lone Star Tick. The study found a 61% reduction in average number of adult females, 61% reduction in adult males, 80% reduction in nymphs and 44% reduction in larvae from 1994 to 1996 [15]. Each site in the model has a grassy patch and a wooded patch, with parameters varying depending on patch type. The sites are named as follows: CEM, BGSM, BPIT, CHAT, GRAV and LAKE. Note that Gaff does not break the ticks up into separate life stages, but models only the dynamics of a single tick population. This simplification is justifiable given that all life stages of the tick prefer the white-tailed deer as their host [8]. Therefore individuals of different locations, ages, and sizes are equivalent,

and it is unnecessary to model the tick life cycle. This is an important distinction between the aforementioned model.

3.1 Parameter Estimation

Because many model parameters for tick populations are seasonally variable [1], the growth and the external death rate of ticks vary over the course of the year (see Figure 3.1 for external death rates for ticks in both wooded and grassy patches). The growth and death rates are impacted by the following factors: the annual reproductive output of ticks, host finding rates, on-host and off-host survival rates. Host-finding rates and weather conditions are the two most influential factors of lone-star tick dynamics [5]. Host-finding rates depend on host density, area type, tick and host movement patterns. Haile and Mount [11] point out that ticks in grassy areas are much more susceptible to temperature and humidity changes than ticks in wooded areas. Depending on habitat type, deer population densities can vary from 7.5 to 40.0 per km^2. Lockhart et al [13] and Mount et al [16] estimate the average density of ticks per deer to be 50 and 400, respectively. As Gaff and Gross [8] point out, there is no consensus for any of the parameters used in this model, so many were estimated by averaging values from geographic regions similar to the Fairfield Glade retirement community. The tick growth rate was calculated as a product of many factors present in the model presented by Haile and Mount [11]. The deer growth rate is arbitrary, and is allowed to be non-zero assuming the area allows hunting [8]. Disease transmission rates have not been explicitly estimated, and are estimated here based on research by Ewing et al, 1995 [6]. More detailed explanation of parameter estimation can be found in [8].

The population densities in the ith patch are described as follows:

$$\frac{dN_i}{dt} = \beta_i \left(\frac{K_i - N_i}{K_i} \right) N_i - b_i N_i + \sum_j m_{ij}(N_j - N_i) \tag{3.1}$$

$$\frac{dV_i}{dt} = \hat{\beta}_i \left(\frac{M_i N_i - V_i}{M_i N_i} \right) V_i - b_i V_i + \sum_j m_{ij}(V_j - V_i) \tag{3.2}$$

$$\frac{dY_i}{dt} = A_i \left(\frac{N_i - Y_i}{N_i} \right) X_i - \beta_i \frac{N_i Y_i}{K_i} - (b_i + v_i)Y_i + \sum_j m_{ij}(Y_j - Y_i) \tag{3.3}$$

$$\frac{dX_i}{dt} = \hat{A}_i \left(\frac{Y_i}{N_i} \right) (V_i - X_i) - \hat{\beta}_i \frac{V_i X_i}{M_i N_i} - \hat{b}_i X_i + \sum_j m_{ij}(X_j - X_i) \tag{3.4}$$

where m_{ij} is the migration term as described in Table 3.2. Each patch can have its own unique set of parameters, but for simplification all grass patch parameters are equivalent and likewise for wooded patches. For a complete list of parameters see Table 3.1.

Table 3.1: Parameter values for disease model taken from [8].

Name	Description	Value
β	Growth rate for hosts	0.200
$\hat{\beta}$	Growth rate for ticks	varies (see 3.1)
K	Carrying capacity for hosts per m^2	0.002 (woods), 0.001 (grass)
M	Maximum number of ticks per host	225
b	External death rate of hosts	0.010
$\hat{b}$	External death rate of ticks	varies (see 3.1)
A	Transmission rate from hosts to ticks	0.020
$\hat{A}$	Transmission rate from ticks to hosts	0.070
v	Recovery rate of hosts	0.000
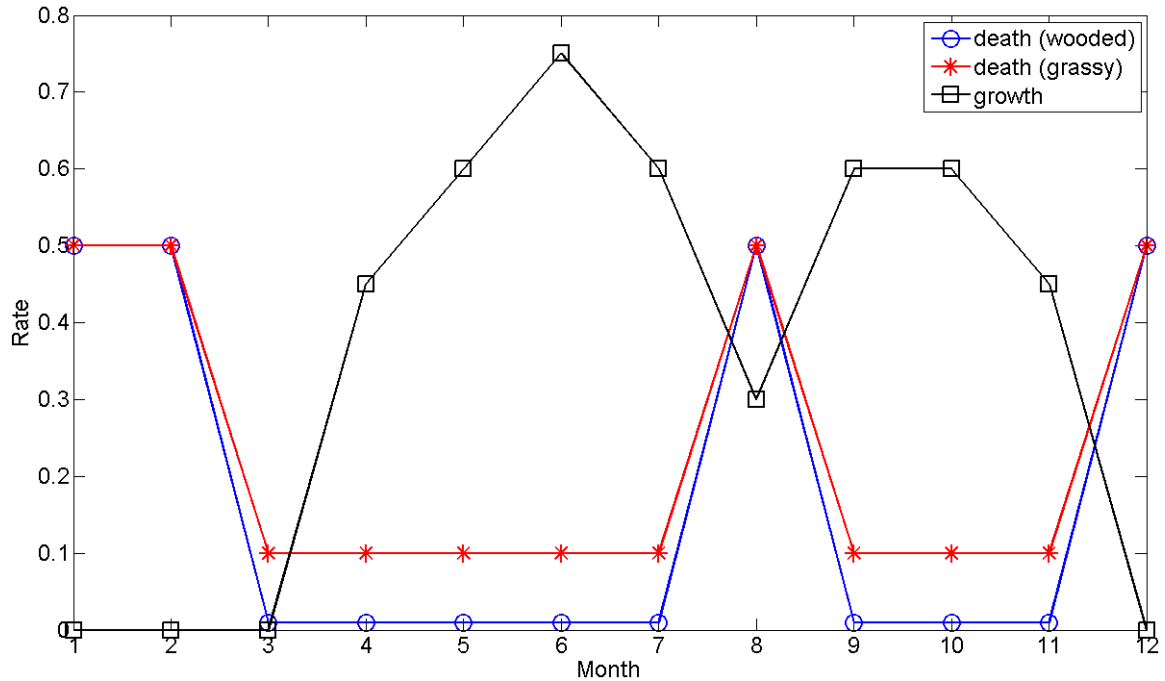
Figure 3.1: Graph of monthly growth and death rates for the Lone Star Tick reconstructed from [8]. Death rates differ depending on patch type; death rates in grassy patches are slightly higher than those in wooded patches.

Table 3.2: Migration rates for the six data collection sites. These rates are based upon the geographic distance between sites and the presence of Lake Dartmoor. The migration rates between the grass and wooded patch at each site are assumed to be 90%, and the migration rates are applied to both patches at each site.

Site	CEM	BGSM	BPIT	CHAT	GRAV	LAKE
CEM	1.00	0.75	0.00	0.01	0.01	0.01
BGSM	0.75	1.00	0.75	0.01	0.01	0.01
BPIT	0.00	0.75	1.00	0.01	0.01	0.01
CHAT	0.01	0.01	0.01	1.00	0.80	0.80
GRAV	0.01	0.01	0.01	0.80	1.00	0.80
LAKE	0.01	0.01	0.01	0.80	0.80	1.00

3.2   Without Control

This model describes the interaction of lone star ticks and white-tailed deer, and tracks infections of E. chaffeensis ricksettsia in both populations. The time unit is 1 month, and the spatial unit is per m². The disease is transmitted from tick to host or from host to tick during a tick's blood meal on a host. The model assumes that the disease does not spread from tick to tick or from deer to deer. Also, the ticks and deer do not transmit the disease from one generation to the next, and neither ticks nor hosts can recover from the disease. Because the system of ODEs given in equations (3.1–3.4) is fairly stiff, that is, the solution consists of a slowing varying part and a quickly varying part, the system is solved using MATLAB's ode15s solver. ode15s is based on numerical differentiation formulas and is a multistep solver, and is recommended by Shampine et al. when ode45 fails or is inefficient, as it is here [20]. The solution computed by ode45 produces undesirable oscillations as the population densities approach 0, and ode15s successfully eliminates this problem. Each patch had equivalent initial conditions of .00001 hosts, .0025 ticks, 0 infected hosts, and .000025 infected ticks. While each patch could have unique initial conditions, they are kept uniform here for simplicity.

As shown in Figure 3.2, the population takes around 300 months, or roughly 25 years to reach equilibrium. At equilibrium, the tick densities oscillate between .04 and .18 ticks per m² and infected ticks oscillate 0.004 and 0.007 ticks per m². This implies that between 4% and 12% of ticks are infected at any given time of year, with peaks in the summer. The more alarming result of this model is that the percent of infected deer densities oscillate between approximately 28% and 31%. Figure 3.3 shows tick densities during a year after the model has reached equilibrium.
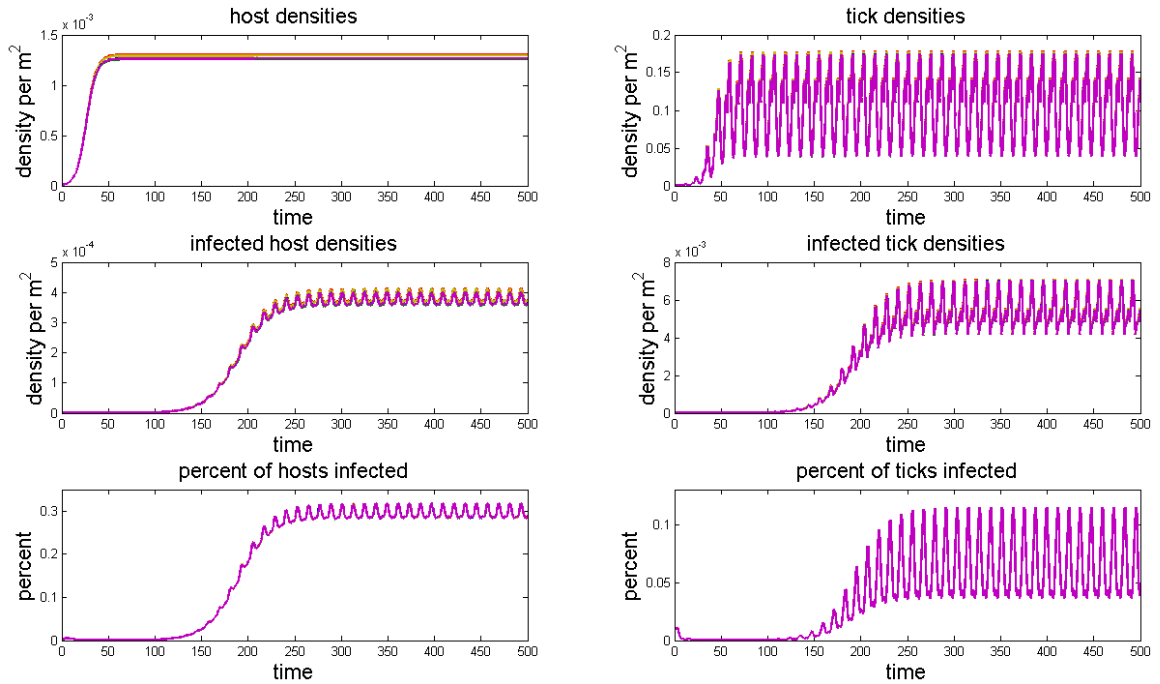
Figure 3.2: Population dynamics for ticks, host and disease without 4-poster feeders using the 12-patch model.
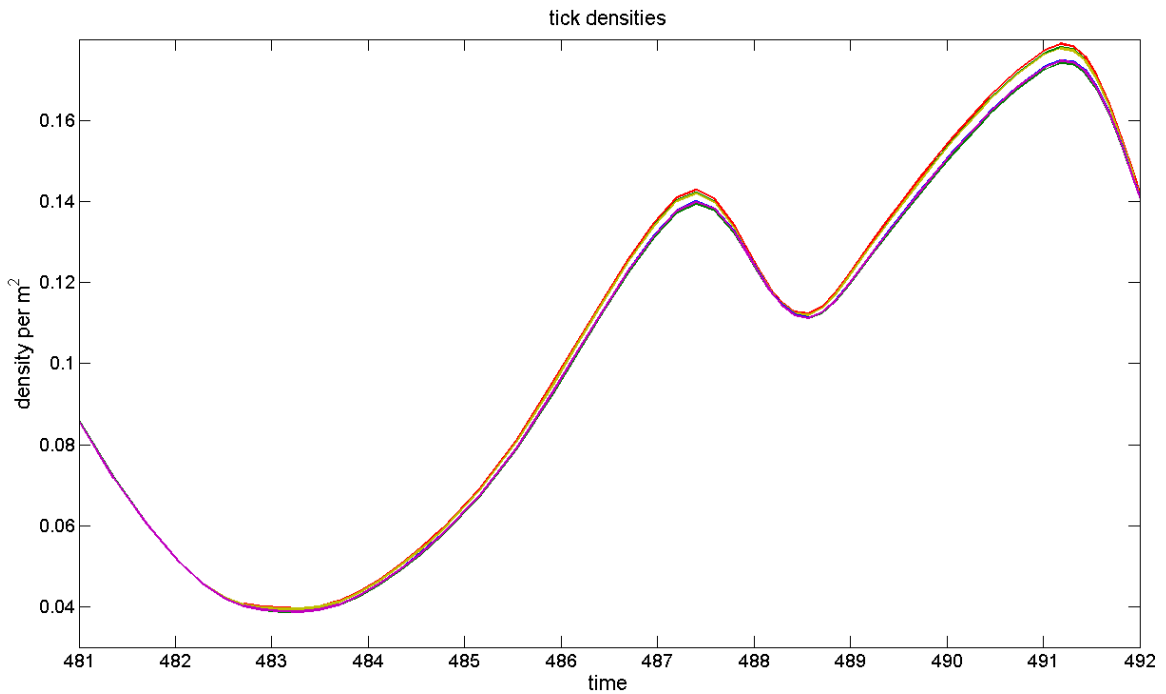
Figure 3.3: A year of tick population dynamics after the model reaches equilibrium.

3.3   With Control

By varying the external death rate term, $\hat{b}$, we can introduce time-dependent control into the system. If a site uses control, both the wooded and grassy patches at that site have 4-poster feeders, and thus control can be applied for different amounts of time in wooded and grassy patches. Since the natural tick death rate is higher in open fields, one may not need to apply acaricide in grassy patches for as long as wooded patches. The acaricide has a decay time as it leaves the hosts' systems that was not included in the model. Instead, the death rate, $\hat{b}$, returns to the original value 1 month after the feeding is stopped. We allow the system to reach equilibrium before the control is applied.

With acaricide feeders in place it is clear that infected tick and host densities are significantly reduced (see Figure 3.4). For this case the control was applied in
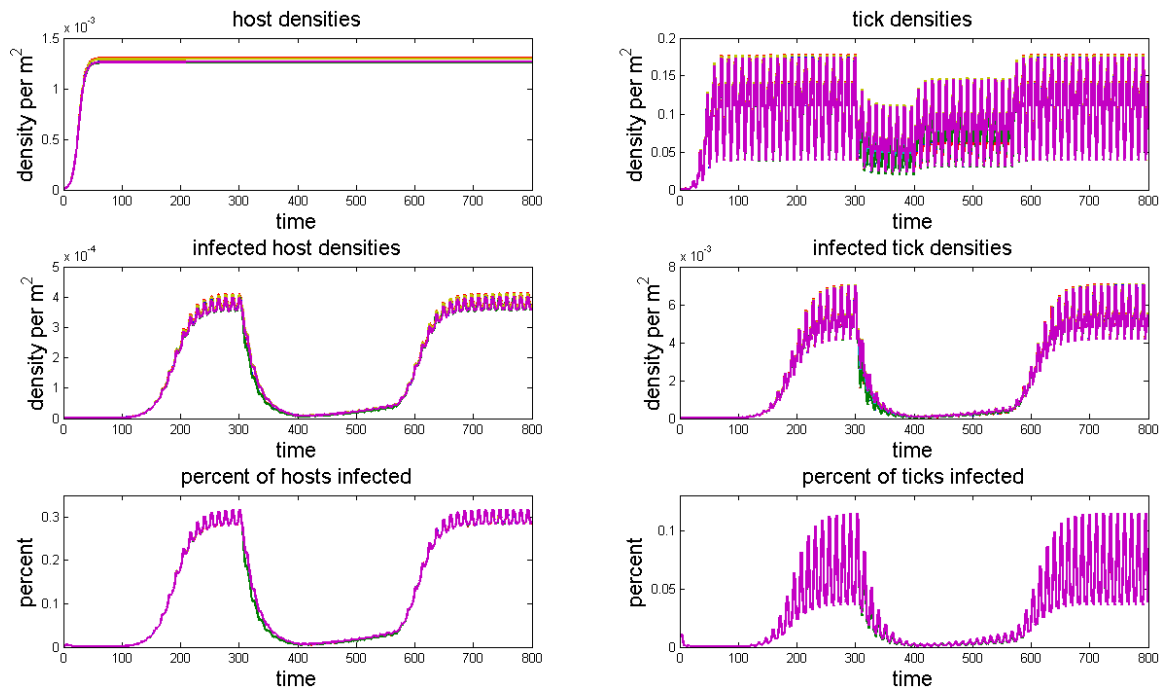
Figure 3.4: Population dynamics for ticks, host and disease with 4-poster feeders in 3 of 6 sites using the 12-patch model. Acaricide was applied for arbitrary values of 22 years in wooded patches and 8 years in grassy patches.

3 random sites for arbitrary values of 22 years in wooded patches, and for 8 years in grassy patches. After acaricide is removed from the system, tick densities return to equilibrium approximately 400 months (33.3 years) before infected tick densities return to equilibrium. Gaff and Gross observe that varying the number of years the control is applied affects the time it takes for the disease to return, and thus an optimal length of time may exist. In [9], Gaff et al optimize the length of time to apply control in one patch with three different objective functionals, and this work will be summarized in the next chapter. There is no published work for optimizing the 12-patch case. As one could imagine, adding patches to the model could affect the optimal length of time, but for the purposes of this study we assume that it would not.

Chapter 4

Optimization

There are two components of optimization for this model. First, the temporal aspect will tell us how long to apply acaricide in grassy and wooded patches for optimal results. Second, the spatial aspect tells us in which of the six sites we should place feeders. The two components are done independently for this study. As described in the following section, the temporal optimization was computed using only a single patch. The spatial optimization is computed for 12 patches using results of the temporal optimization for a single patch.

4.1   Temporal Optimization

Gaff et al. investigate a variety of frameworks for optimal control for a single patch model [9]. The results of the study show similar strategies are created across all frameworks and objective functionals within the closed environment, and that tick-borne disease risk can be minimized without eliminating the tick population. Here we chose two of the frameworks tested in Gaff's study as our control level and duration. Both scenarios chosen use a bang-bang control; the control is applied at maximum for a certain number of months, then oscillates seasonally between the maximum and minimum until stopping all treatment [9]. Other frameworks presented by Gaff et al. involve varying the strength of the control. The bang-bang scenario is perhaps

easier to manage since implementers of the 4-poster feeders only have to worry about "when?" as opposed to "when and how much?".

First we will summarize the work by Gaff et al. that was used to optimize acaricide application in a single patch. Gaff et al. investigate results of three different objective functionals, but the equation used in both of the scenarios mentioned above is as follows:

$$J(\delta(t)) = \int_0^T (C_0 X(t) - C_1 V(t) + C_2 \delta(t)) dt \tag{4.1}$$

where C_1, C_2 and C_3 are balance terms and $\delta(t)$ is the control. The equation was minimized using an iterative method developed by W. Hackbush for solving parabolic equations with opposite orientations [9] . In their results for all scenarios with seasonal rates, control needed to be applied for as little as 11 years with an average of 13.5 years at maximum treatment [9].

Scenario 1: This scenario's objective is to maximize disease-free ticks. For grassy patches, the optimal number of years to apply control at maximum is 3.5833, and all treatment is stopped after 13.0833 years. For wooded patches, the optimal number of years to apply control at maximum is 20.5000, and all treatment is stopped after 27.0000 years [9].

Scenario 2: This scenario's objective is to minimize diseased ticks. For grassy patches, the optimal number of years to apply control at maximum is 3.5833, and all treatment is stopped after 12.0000 years. For wooded patches, the optimal number of years to apply control at maximum is 22.3333, and all treatment is stopped after 22.2500 years [9].

The model for this thesis was designed to apply the control at maximum for 1 year at a time, so we simplified Gaff et al. 's results by applying the control at maximum for 9 years in wooded patches and 24 years in grassy patches and stopping all treatment

afterwards for Scenario 1, and for 8 and 22 years analogously for Scenario 2. These were found by averaging the two numbers for each patch type and rounding to the nearest integer. Since the optimal number of years were computed using only a single patch, they may not be optimal for the full 12 patch model. However, they will suffice for the purposes of this study.

4.2   Spatial Optimization

As mentioned in the model description, there are six sites that were chosen to mimic a study done by Marsland [15] (See Figure 4.1). With six sites there are a total of 64 possible combinations of feeder arrangements, ranging from 0 feeders to 6 feeders. It is important to note that Gaff labels 3 sites as "treated" and the other 3 as "untreated" for experiments in [8]. We make no such distinction here, as each site is a possible location for a feeder. Since there are a finite number of possibilities, it is feasible to test each scenario and compare them directly to find the optimal arrangement. For each arrangement the model calculates the percent of infected ticks, $p_j = \frac{X_j}{V_j}$, for each patch. The average value of p_j, $\bar{p}_j$, is then calculated on the entire time interval for each patch. The average percent of ticks infected for all 12 patches, η, of each arrangement was evaluated by averaging each patch's value of $\bar{p}$. Thus, the average percent of ticks infected for the ith arrangement of feeders is:

$$\eta_i = \frac{1}{12} \sum_{j=1}^{12} \bar{p}_j$$

We want to minimize η over all possible arrangements of feeders. This is done by a brute force approach; model each arrangement individually then directly compare the results. The cost for each scenario is also computed by multiplying the number
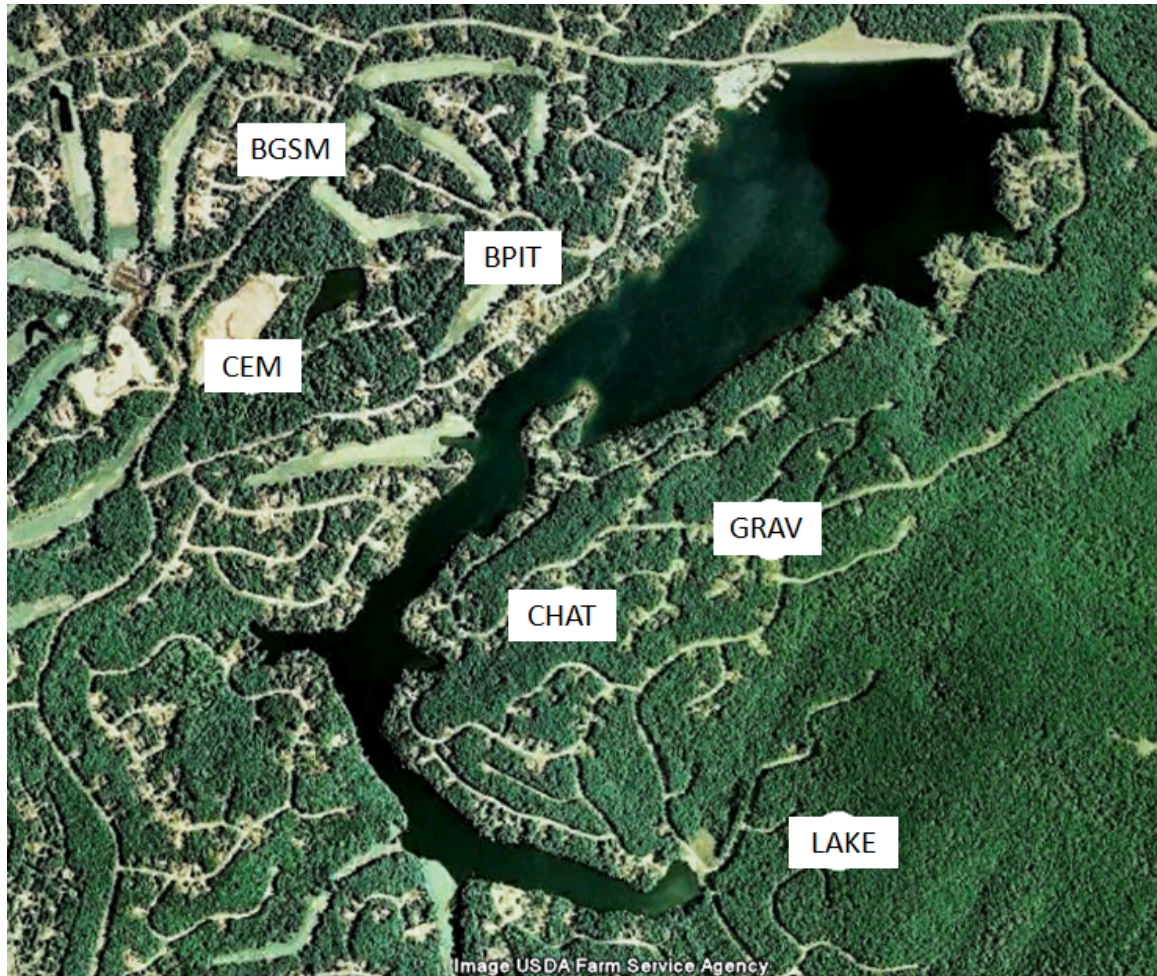
Figure 4.1: 6 feeder sites, labeled by white circles, in Fairfield Glade Retirement Community. These sites were chosen to mimic a study done by Marsland [15]. Satellite image obtained from www.earth.google.com.

of years acaricide is applied in each patch, the number of feeders and $2,000, the estimated cost to maintain a feeder for one year. Keep in mind that if a site has feeders there is a feeder in both its grassy patch and wooded patch.

Table 4.1: Number of years acaricide was applied in each patch for different scenarios

Scenario	Duration (grassy patches)	Duration (wooded patches)
1	9	24
2	8	22
3	17	17
4	15	15
5	10	10

We tested 5 scenarios: the 2 derived from [9] in the previous section and 3 more. For scenarios 3, 4 and 5 we apply acaricide for equal amounts of time in grassy and wooded patches. The durations for scenarios 3 and 4 are found by averaging the durations from scenarios 1 and 2, respectively. Scenario 5 is an arbitrary number of years. See Table 4.1 above for a complete list of scenarios. To compute each scenario's optimal η, the model is run for 84 years and acaricide is applied after 25 years.

4.3   Results

Table 4.2: Optimal arrangement of feeders and average value of percent of ticks infected (allowing 6 feeders).

Scenario	Optimal Arrangement	η	Cost (thousands)
1	All sites	.0146	$792
2	All sites	.0199	$720
3	All sites	.0135	$816
4	All sites	.0130	$720
5	All sites	.0290	$480

For all 5 scenarios, our results show that placing feeders in all 6 sites is optimal

(see Table 4.2). As we can see from Figures 4.3, 4.5, 4.7, 4.9 and 4.11 the disease is almost completely eliminated from the tick population with the optimal arrangements. However, the disease does eventually come back because the infected tick densities of each patch will never equal 0. Note also that η is the average value of the functions in the aforementioned Figures. One might expect that applying the acaricide too long would cause too many healthy ticks to die, and thus result in a suboptimal η. While each individual scenario does not show this, scenarios 3 and 4 exemplify this result. Scenario 3 applies acaricide in all 6 sites for 17 years for both patch types and results in $\eta = .0135$. Scenario 4 applies acaricide for two years less in both patch types and results in a slightly lower $\eta = .0130$. Thus our expectations are confirmed.

This model can also be used to find an optimal arrangement of feeders given cost constraints. For example, if we must limit the number of feeders because placing feeders in all six sites is too expensive, the results also give us the optimal arrangement for each number of feeders. Figures 4.2, 4.4, 4.6, 4.8 and 4.10 show the η values for each arrangement, and the solid line connects the optimal arrangement for each number of feeders. As an example, Table 4.3 shows results excluding feeders in all 6 sites as a possibility. The optimal η for each scenario is roughly twice the optimal η from Table 4.2, but the cost is significantly reduced in each scenario. Since the η values are still relatively small, using only 5 feeders may be more favorable.

Table 4.3: Optimal arrangement of feeders and average value of percent of ticks infected (not allowing 6 feeders).

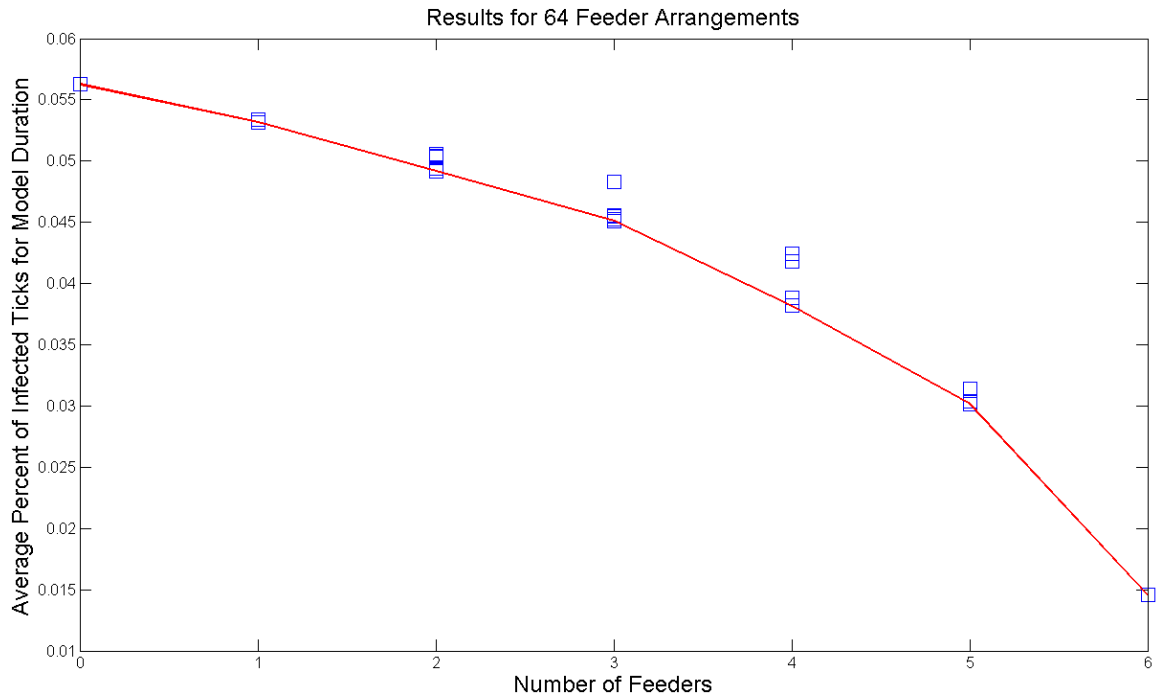| Scenario	Optimal Arrangement	η	Cost (thousands)
1	All but LAKE	.0302	$660
2	All but LAKE	.0222	$600
3	All but CHAT	.0220	$680
4	All but GRAV	.0272	$600
5	All but LAKE	.0388	$400

Figure 4.2: Average percent of infected ticks vs. number of feeders for Scenario 1. Each point on the graph represents a feeder arrangement, and the solid line connects the optimal arrangement for each number of feeders. From these results it is clear that an arrangement with 6 feeders is optimal. The optimal arrangement gives an average percent infected tick value of .0146
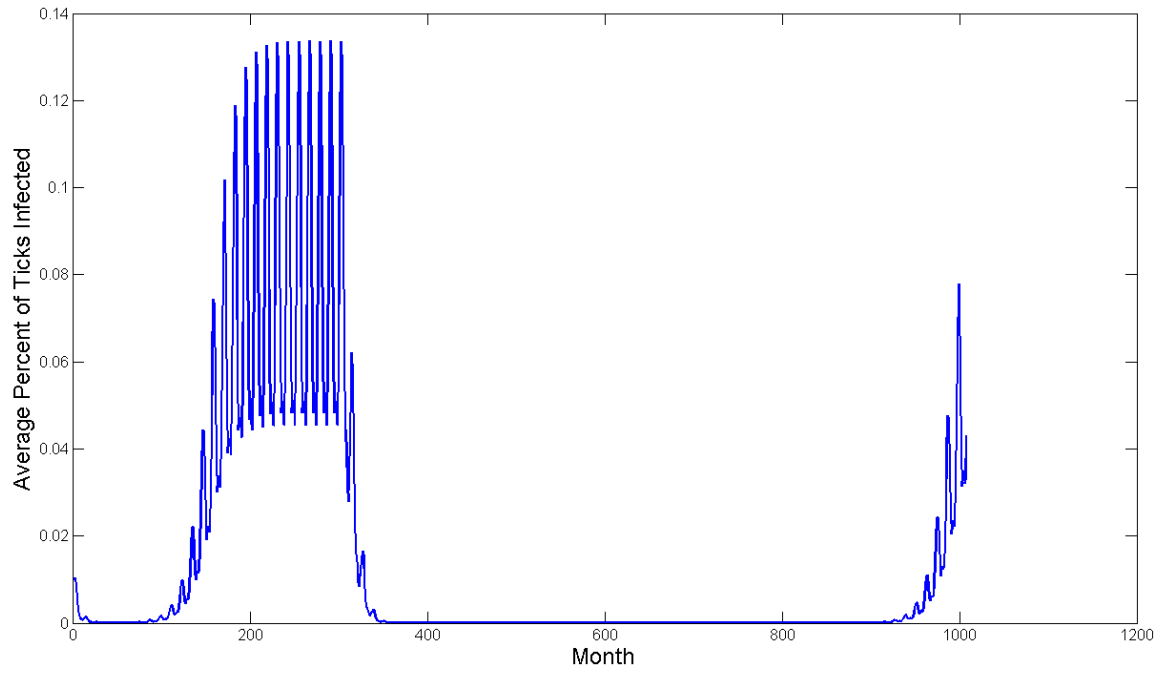
Figure 4.3: Percent of ticks infected using optimal feeder arrangement for Scenario 1.
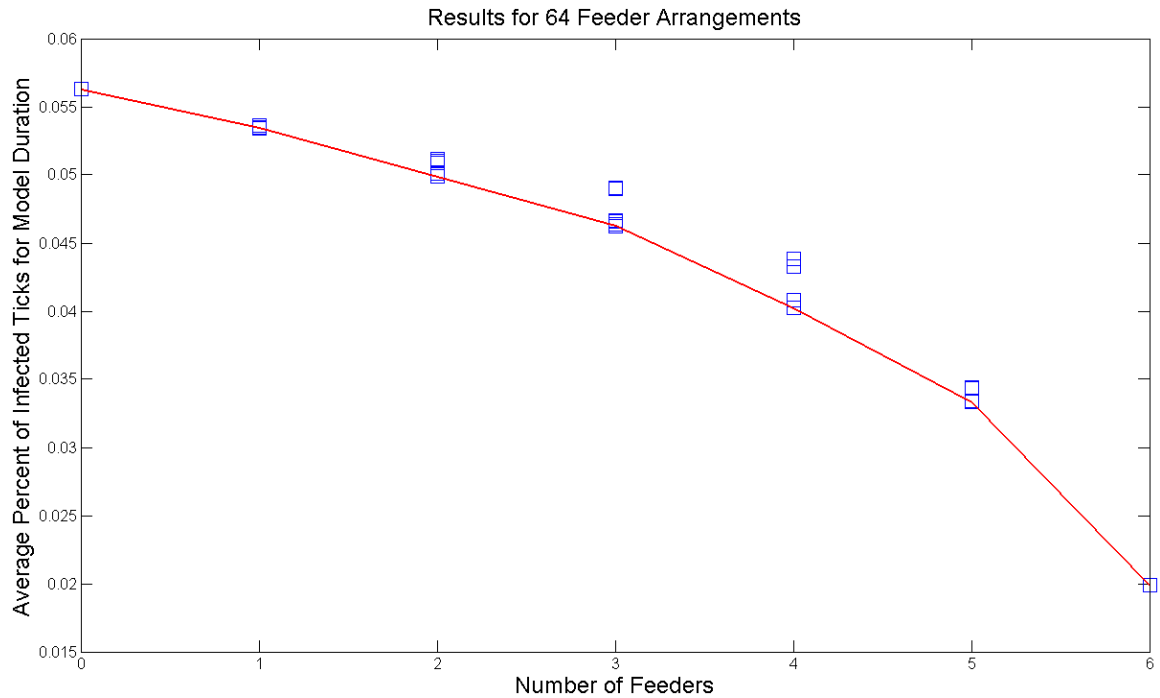
Figure 4.4: Average percent of infected ticks vs. number of feeders for Scenario 2. Each point on the graph represents a feeder arrangement, and the solid line connects the optimal arrangement for each number of feeders. From these results it is clear that an arrangement with 6 feeders is optimal. The optimal arrangement gives an average percent infected tick value of .0199.
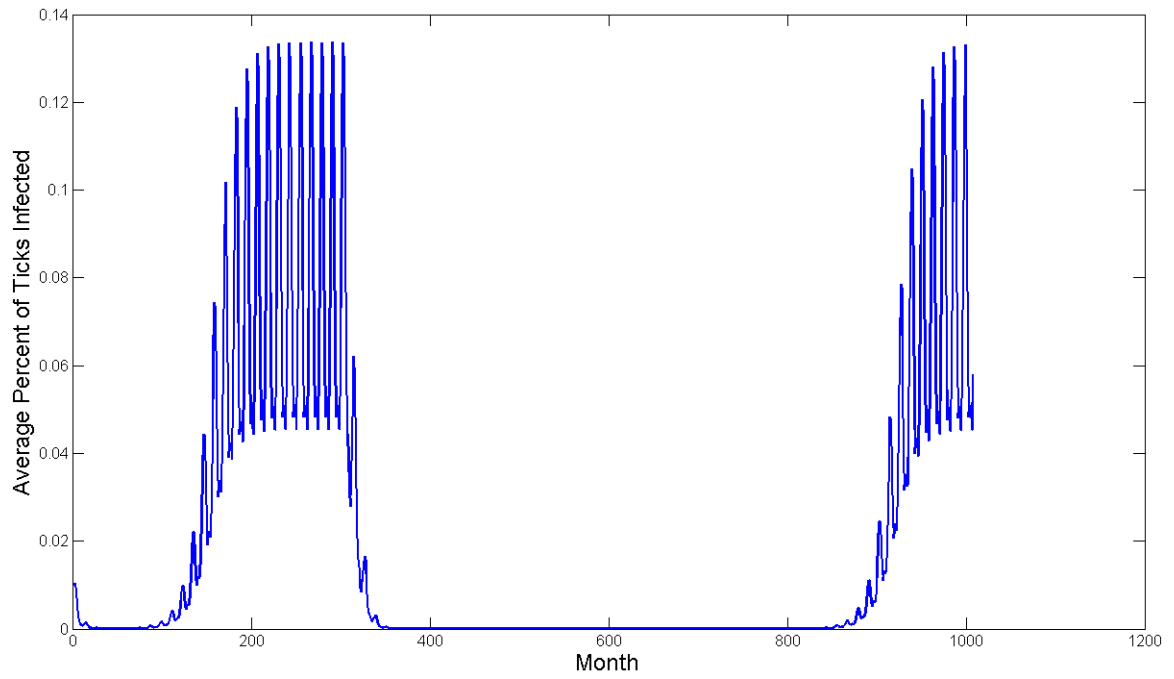
Figure 4.5: Percent of ticks infected using optimal feeder arrangement in Scenario 2.
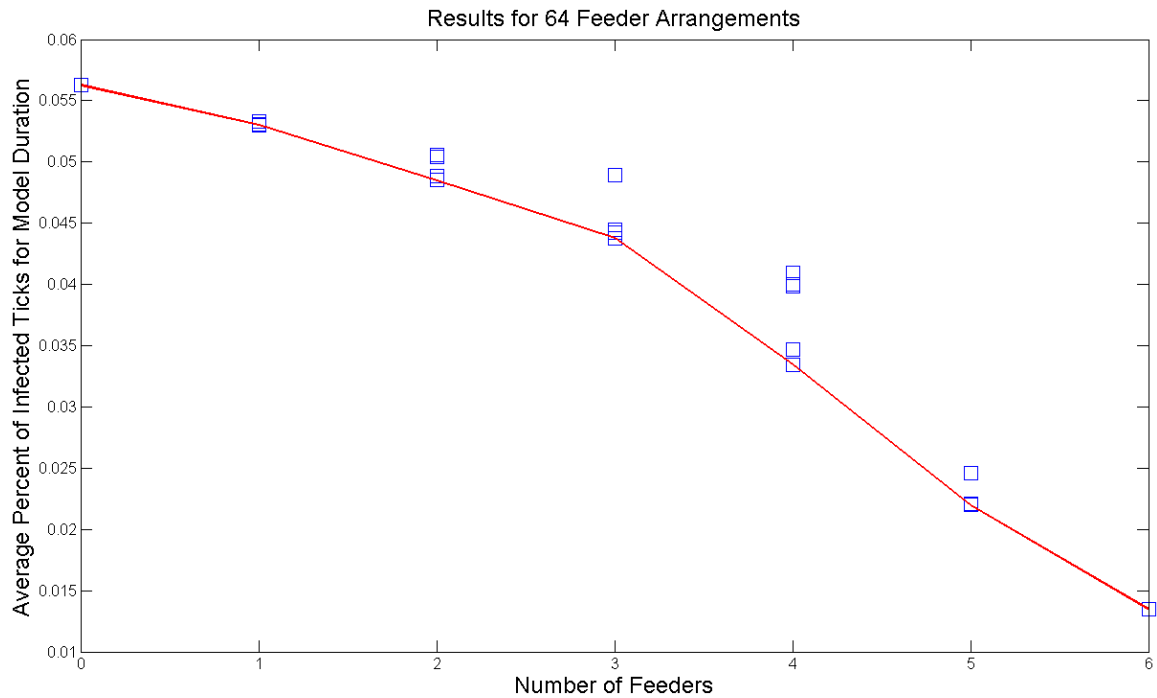
Figure 4.6: Average percent of infected ticks vs. number of feeders for Scenario 3. Each point on the graph represents a feeder arrangement, and the solid line connects the optimal arrangement for each number of feeders. From these results it is clear that an arrangement with 6 feeders is optimal. The optimal arrangement gives an average percent infected tick value of .0135, slightly better than Scenario 1's optimal value.
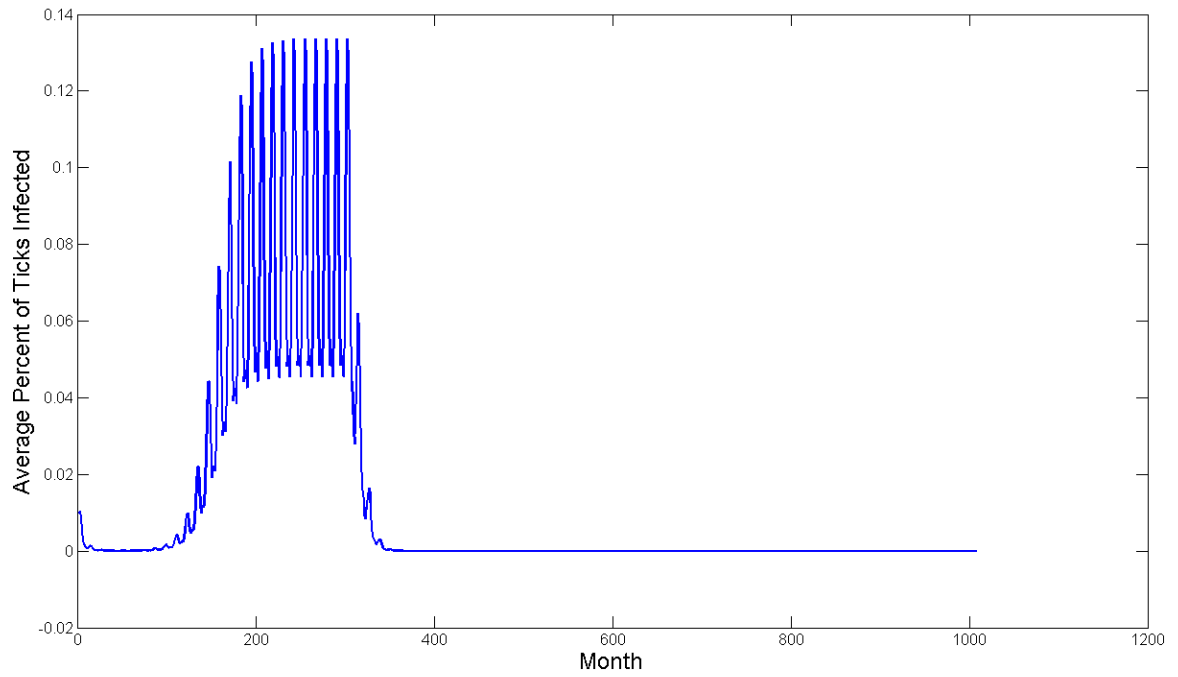
Figure 4.7: Percent of ticks infected using optimal feeder arrangement in Scenario 3.

Figure 4.8: Average percent of infected ticks vs. number of feeders for Scenario 4. Each point on the graph represents a feeder arrangement, and the solid line connects the optimal arrangement for each number of feeders. From these results it is clear that an arrangement with 6 feeders is optimal. The optimal arrangement gives an average percent infected tick value of .0130.

Figure 4.9: Percent of ticks infected using optimal feeder arrangement in Scenario 4.
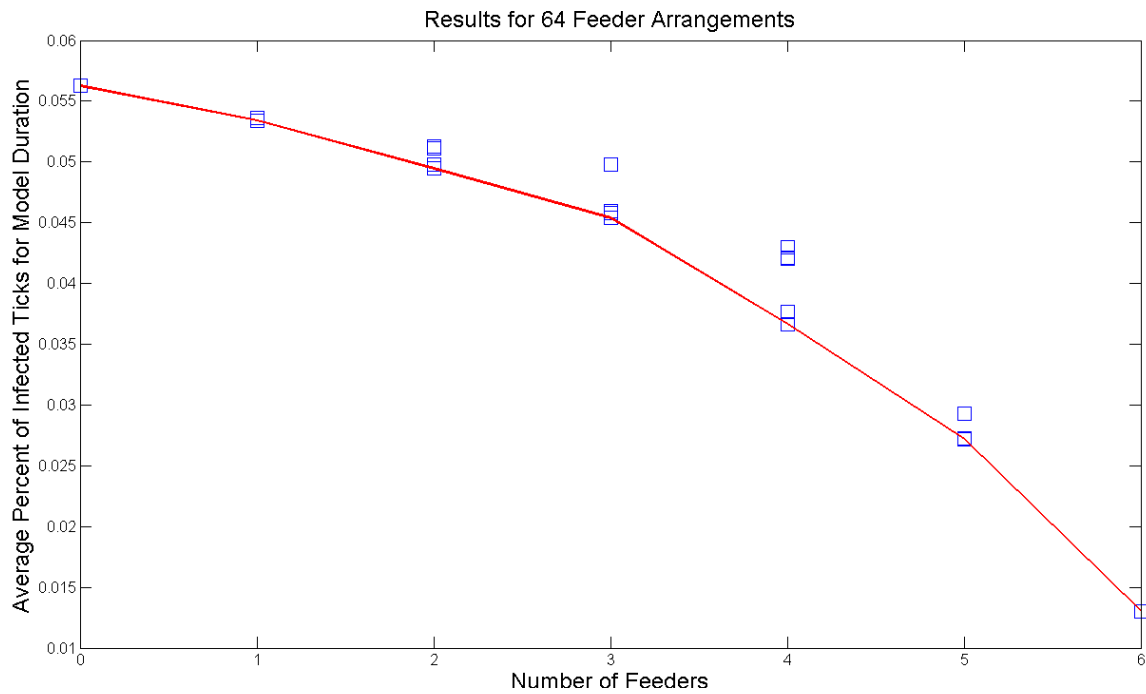
Figure 4.10: Average percent of infected ticks vs. number of feeders for Scenario 5. Each point on the graph represents a feeder arrangement, and the solid line connects the optimal arrangement for each number of feeders. From these results it is clear that an arrangement with 6 feeders is optimal. The optimal arrangement gives an average percent infected tick value of .0290.
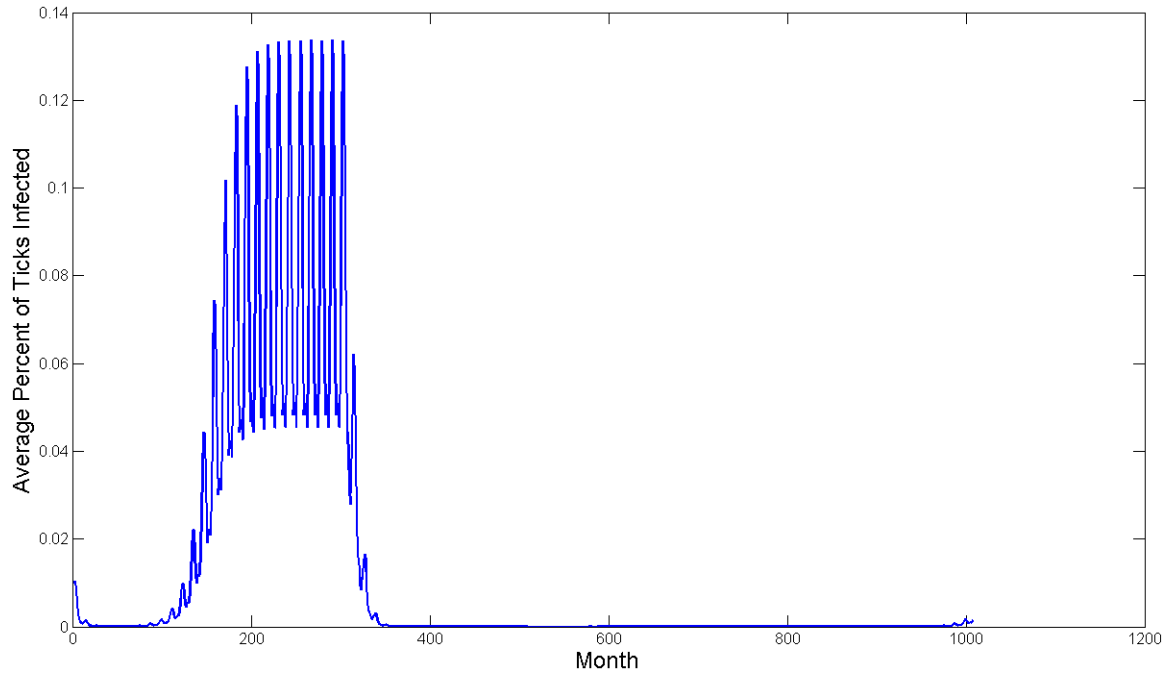
Figure 4.11: Percent of ticks infected using optimal feeder arrangement in Scenario 5.
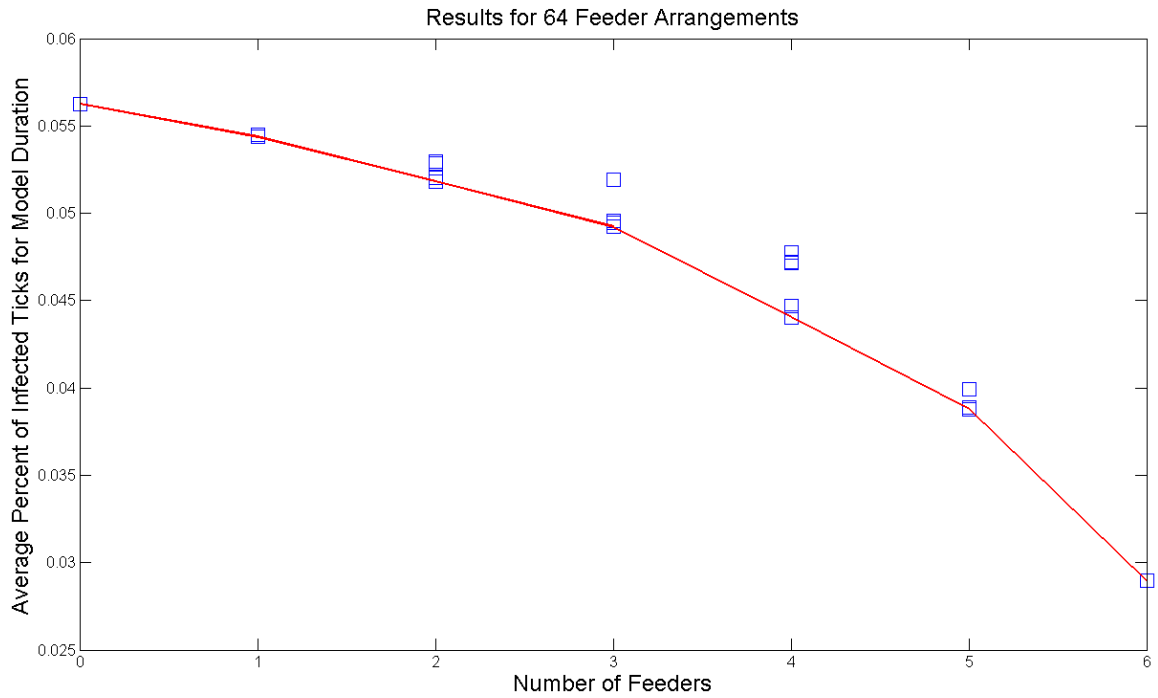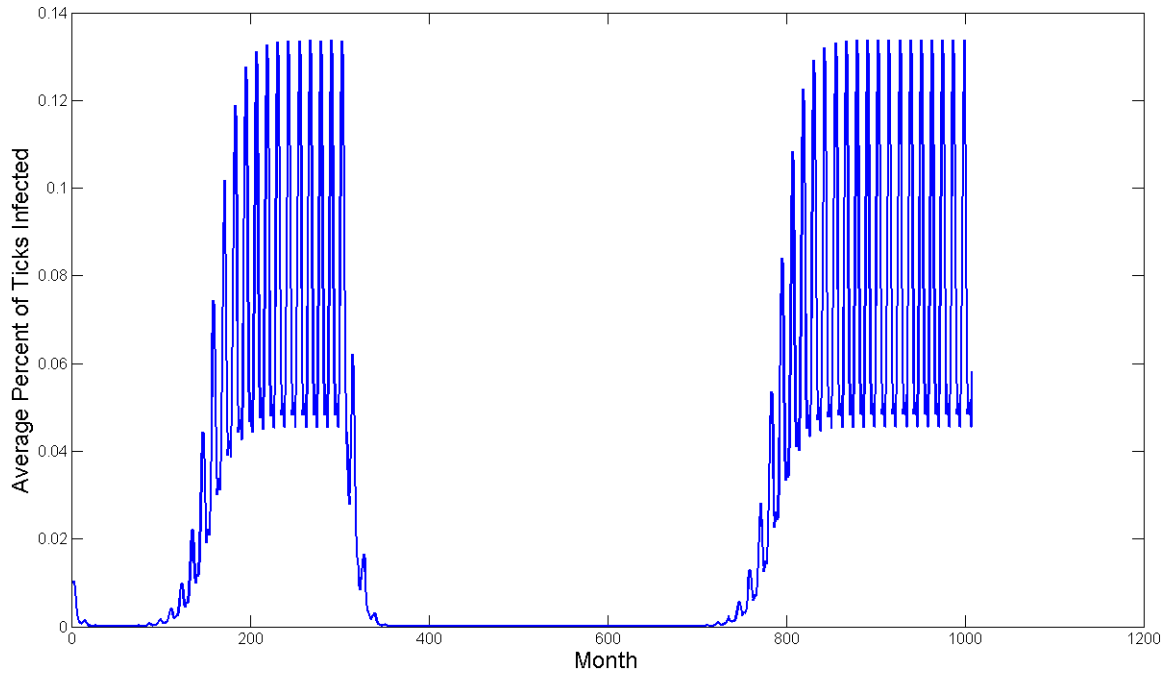
Chapter 5

Conclusion

We have used a previous difference model developed by Gaff and Gross to find the optimal arrangement of feeders given 6 different sites and varying durations of acaricide application estimated by Gaff et al.'s optimal control for a single patch. For each case we found the optimal arrangement by a brute force method, computing the efficiency of each arrangement and then comparing them directly. Other numerical experiments were conducted to compare accuracy of the optimal length of application time derived from the single patch model used in [9]. All test cases resulted in all 6 sites containing feeders as the optimal arrangement. However, the results allow us to calculate the optimal arrangement given any number of sites with feeders in the case that the optimal arrangement with feeders in all 6 sites is too expensive to implement.

Future work on this project could include adding more patches so that more of the Fairfield Glade retirement community is covered by acaricide application. The area of Fairfield Glade from Gaff et al.'s model is a subsection of the area from the REU group's model. This complicates computation, however, as each run of the model already takes around 5 minutes. The brute force method for each case takes between 2 and 3 hours. Methods can be developed to reduce the running time of the model, and to simplify the spatial optimization process. The use of parallel processing could dramatically improve computation time. Also, temporal optimization of each feeder arrangement should be considered. For this work we derived application times from

temporal optimization of a single-patch model [9]. From our test cases it is clear that these results do not carry over to the 12-patch model, and thus spatial and temporal optimization of feeder arrangements and application durations should be solved for simultaneously.

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