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Abstract

Understanding the Mental Number Line: spatial-numerical associations across reference frames

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For two decades, psychologists have investigated the idea of a Mental Number Line (MNL) wherein numbers are systematically mapped onto—and represented in—a particular side of space (e.g., Dehaene, Bossini, & Giraux). Nevertheless, the mechanisms of this putative MNL number line remain unclear: what does it mean to say that small numbers are mapped onto the left side of space—to the left of what? The present study examines spatial-numerical associations (SNAs) and the notion of a MNL in two egocentric (eyes, body) and two allocentric (keyboard, screen) frames of reference. In a first experiment, we test all four reference frames in unison and find evidence of a SNA only in the allocentric, keyboard-centered frame of reference. In two follow-up experiments, we isolated the screen-centered and eye-centered reference frames, finding evidence of a SNA only in the former. We discuss how these results relate to the idea of a possible "hierarchy of reference frames" (e.g., Viarouge, Hubbard, & Dehaene, 2014) and consider the implications of these findings in regards to the existence of a MNL. Furthermore, we discuss the implications of this work for spatial representation (i.e., frames of reference) more broadly.

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Introduction

Decades of research have highlighted the deep relationship between spatial and numerical cognition (for review, see Mix & Cheng, 2011). Indeed, spatial abilities such as mental rotation have been shown to predict mathematical performance in young children (Gathercole & Pickering, 2000; Guay & McDaniel, 1977), spatial and mathematical competency are correlated in adulthood (e.g., Delgado & Prieto, 2004; Lubinski & Benbow, 1992; Robinson et al., 1996), and both spatial and numerical magnitudes are mapped topographically in overlapping areas of parietal cortex (Harvey, Fracasso, Petridou, Dumoulin, 2015; Harvey, Klein, Petridou, & Dumoulin, 2013). Additionally, it has been shown that human subjects demonstrate a spatial-numerical mapping wherein numbers are associated with a particular side of space (see Dehaene, Bossini, & Giraux, 1993). This ostensible Mental Number Line (MNL) has been extensively studied over the past two decades (for review, see Fias & Fischer, 2005; Fischer & Shaki, 2014; Hubbard, Piazza, Pinel, & Dehaene, 2005; Wood, Willmes, Nuerk, & Fischer, 2008), though surprisingly little is known about the cognitive mechanisms underlying such spatial-numerical associations (SNAs).

In a classic study, Dehaene and colleagues (1993) showed that, when responding to the parity of Arabic numerals, Western participants were faster to respond to smaller numerals with their left hand and to larger numerals with their right hand. This effect, now known as the Spatial-Numerical Association of Response Codes (SNARC), propelled the notion of a MNL and has proven to be quite robust: since the original study, this classic effect has been replicated many times (see Wood et al., 2008), demonstrated in multiple modalities (e.g., audition; see Nuerk, Wood, & Willmes, 2005), and extended to other magnitudes (e.g., the extent of emotional expression; Holmes & Lourenco, 2011). Notably, this mapping is thought to be modulated in part by reading direction, such that English-speaking individuals will exhibit a left-to-right (i.e., smaller numbers on the left, larger numbers on the right) MNL (see Fias & Fischer, 2005; Fischer & Shaki, 2014; Hubbard, Piazza, Pinel, & Dehaene, 2005). However, some evidence exists that a consistent association between number and space exists in preliterate children and non-human animals (Adachi, 2014; De Hevia & Spelke, 2010; Rugani, Vallortigara, Priftis, Regolin, 2015).

Nevertheless, the underlying mechanisms that result in such spatial-numerical associations (SNAs) remain unclear. Traditionally, SNAs have been interpreted as evidence of a stable MNL (Dehaene et al., 1993; Holmes & Lourenco, 2011). Others have argued that SNAs are a consequence of polarity correspondence (i.e., the idea that poles of two orthogonal dimensions still share representational status; Proctor & Cho, 2006), or that they are transient, task-specific mappings that occur in working memory (van Dijck & Fias, 2011). This debate has critical implications for numerical cognition. On the one hand, the existence of a MNL may be the foundation of the space-math relationship (e.g., Mix & Cheng, 2011); perhaps math learning is facilitated by a spatial-numerical mapping. On the other hand, a working memory account of SNAs has no theoretical implications for math learning. According to this account, SNAs are little more than an artifact of the task demands.

Similarly, it is unclear *how* numbers are mapped onto space: what does it mean to say that small numbers are represented on the left—to the left of what? To better understand SNAs and the potential stability of the MNL, it is necessary to understand how SNAs manifest across various reference frames. Lending credence to the possibility of a stable MNL, SNAs have been demonstrated using a variety of paradigms (e.g., Calabria and Rossetti, 2005; Cheung et al., 2015; Dehaene et al., 1993; Fischer et al., 2005; Lavidor et al., 2004), and recent work has demonstrated intraindividual consistency across various SNA tasks (Cheung et al., 2015). The authors found relationships among only some SNA tasks, however, leading to the proposal that correlations across tasks may be driven by the different spatial reference frames activated in each task. The importance of reference frames in relation to SNAs has been discussed previously (e.g., Viarouge, Hubbard, & Dehaene, 2014) though surprisingly little is known about the relationship between the two.

In their original study, Dehaene and colleagues (1993) addressed the impact of spatial reference frames by having participants cross their hands as they completed the task. In this instance, participants continued to exhibit a typical SNA in relation to the buttons on the keyboard (i.e., participants were faster to respond to lower numbers when responding with the left button and to higher numbers when responding with the right button) but not their hands. Accordingly, it may be argued that the SNA is driven by a single allocentric, keyboard-centered reference frame (buttons) rather than an egocentric, hand-centered reference frame. Notably, however, others have failed to replicate the effect in the crossed-hands condition (Wood, Nuerk, & Willmes, 2006), though it has been suggested that this failure to replicate is a matter of task instructions (see Viarouge et al., 2014).

In this vein, several studies have since addressed spatial frames of reference in relation to SNAs. Marghetis, Kanwal, and Bergen (2013) had participants complete parity and magnitude judgment tasks with a single, centered response button in a go/no-go paradigm (e.g., participants might be instructed to press spacebar in response to odd numbers but do nothing in response to even numbers). They found that participants continued to exhibit left-to-right MNL even when the spatial information provided by button responses was no longer a factor. Riello and Rusconi (2011) manipulated finger-based reference frames, having participants complete parity judgment tasks with their index and middle fingers. Additionally, they had participants complete the task in palm-up and palm-down postures. They found that SNAs varied in accordance with both the hand used (i.e., right hand or left hand) and the hand posture (i.e. palm-up or palm-down), leading them to conclude that reference frames must play a significant part in the manifestation of the SNA. In the most systematic manipulation of spatial reference frames to date, Viarouge and colleagues (2014) identified the many possible reference frames that may be implicated in SNAs, laying out a "dynamic hierarchy of reference frames". In their view, there exists a dominant reference frame (perhaps the allocentric, keyboard-centered reference frame described above) that will be preferentially activated depending on the context. In addition, they postulate that other aspects of the experimental design (e.g., instructions given to the participant, block order) may activate secondary reference frames. They claim that these secondary reference frames may include hand-based (see Conson, Mazzarella, & Trojano, 2009), finger-based (see Brozzoli et al., 2008), or eye-centered (see Schwarz & Keus, 2004) reference frames. Testing a combination of egocentric and object-centered reference frames in a novel SNA task, they failed to provide evidence that secondary reference frames impacted SNAs, casting doubt on the feasibility of their framework.

Understanding spatial reference frames

Valuable insight on the nature of spatial reference frames can be gleaned from patients with hemispatial neglect. Originally described by Holmes (1918), hemispatial neglect is often associated with damage to the parietal cortex and manifests as an inability to attend to the side of space contralateral to the damage. Naturally, this raises the question: how are the left and right sides of space construed? Farrah (1990) systematically manipulated spatial reference frames and demonstrated subjects' use of "viewer-centered" and "environment-centered" reference frames to represent space. For example, subjects laying down perpendicular to a computer screen would more frequently attend to the rightward quadrants—whether in reference to the subject's body or the screen itself, even when the two were incongruent. Subsequent work has emphasized the role of "stimulus-centered" reference frames (e.g., Arguin & Bub, 1993, Ota et al., 2001), egocentric reference frames (Beschin, Cubelli, Della Sala, & Spinazzola, 1997) and other ambiguously-defined reference frames that exist somewhere in between (Behrmann & Tipper, 1999). Accordingly, it appears that no single reference frame governs spatial attention and that the activation of certain reference frames may be highly context-dependent. It is worth considering, then, the extent to which the same is true of SNAs: is it possible that SNAs will manifest in multiple reference frames when the reference frames are properly isolated, or is it the case that SNAs, as some have suggested, will disappear in the absence of certain task demands?

Notably, the previous work refers to a plethora of reference frames (e.g., viewer-centered, stimulus-centered, object-centered, environment-centered, location-centered, egocentric, etc.), which are not always independent of one another. Henceforth, for the sake of clarity, we refer broadly to either egocentric or allocentric reference frames. Specifically, in the current study, we tested two allocentric frames (keyboard and screen), and two egocentric frames (eyes and body).

The current study

Drawing on the work with hemispatial neglect patients, the current study aimed to assess the role of reference frames in SNAs by systematically manipulating and subsequently isolating various frames. In a first experiment, we tested multiple reference frames. Participants completed a parity judgment task in which four reference frames were invoked simultaneously. Numerals appeared in different locations on the screen (activating an allocentric, screen-centered reference frame), in different locations relative to prior eye movements (activating an egocentric, eye-centered reference frame), and in different positions relative to the body (activating an egocentric, body-centered reference frame; see Figure 1). Our goal with the first experiment was two-fold. First, we wanted to determine whether the traditional SNA effect would persist while other frames of reference (i.e., screen, eye, and body) varied. Second, we wanted to test whether these additional reference frames (the "secondary reference frames" postulated by Viarouge and colleagues) would exhibit left-right spatial-numerical mappings. To anticipate the results of the first experiment, we replicated the traditional parity-judgment finding in the keyboard-based frame but failed to show evidence of a SNA in any other frame.

However, if the framework laid out by Viarouge and colleagues is accurate, then it is possible we failed to detect effects in other reference frames due to the presence of a dominant reference frame (e.g., keyboard). For this reason, it was necessary to test the alternate reference frames in isolation. In two follow-up experiments, we attempted to isolate an allocentric, screen-centered reference frame, and an egocentric, eye-centered reference frame, using a no/go-no tasks similar to that used by Marghetis and colleagues (2013; see Figure 1). In the screen condition, participants responded to the parity of a digit with the spacebar while the location of digits on the screen varied between each trial. In the eye condition, the location of the digits was stable in the center of the screen, but a stimulus appeared prior to the presentation of the digit to guide participants' eye movements either to the left or right.

The relationship between spatial and numerical cognition is of key interest to cognitive, developmental, and educational psychologists. Beyond the spatial-numerical relationship *per se*, SNAs reveal the ability of the human mind to utilize knowledge in one domain (i.e., space) to support representations in a separate domain (i.e., number). As such, a deeper understanding of the cognitive mechanisms underlying SNAs may provide valuable insight into the spatial-numerical relationships that exist across development and, moreover, inform our understanding of the attentional mechanisms in the human mind. More specifically, a clearer understanding of spatial reference frames and their relation to SNAs may help to answer two fundamental questions. First, how reliable are SNAs? Critics of SNAs have claimed that they are nothing more than a consequence of task demands (e.g., Proctor & Cho, 2006; van Dijck & Fias, 2011). However, demonstrations of SNAs across modalities (e.g., Nuerk, Wood, & Willmes,), across domains (e.g., Holmes & Lourenco, 2011), and across reference frames help to enhance the notion of a stable MNL. Indeed, the discovery of SNAs in paradigms requiring only a single button response (e.g., Marghetis et al., 2013), in relation to both fingers and hands (Riello & Rusconi, 2011), and in numerous other instances (see Wood et al., 2006) casts doubt on the notion that task demands alone are responsible for the observed associations. Second, where do SNAs arise? Schwarz and Keus (2004) have suggested that saccadic eye movements exhibit a SNA similar to manual responses. Accordingly, an understanding of SNAs in an eye-centered reference frame, for example, may shed light on this idea. Finally, what does it mean to have a "dynamic hierarchy" of reference frames? Investigating spatial reference frames both in combination and in isolation will allow us to assess the feasibility of the supposed hierarchy. In the following three experiments, we hope to shed light on each of these questions.

Experiment 1

Method

Participants

36 undergraduates (27 female) participated for course credit. Five additional subjects were tested but failed to complete the task and were consequently excluded from all analyses. As assessed by the Edinburgh Handedness Inventory (EHI), the sample was predominantly right-

handed (Oldfield, 1971). Participants reported normal or corrected-to-normal vision. Study procedures were approved by the local Institutional Review Board (IRB), and all participants provided written consent.

Stimuli and procedure

The experiment was programmed using E-prime software on a Dell desktop computer. Stimuli were Arabic numerals between 1 and 9, excluding five. The numerals were Times New Roman, size 18 (visual angle: $.7^{o}$). Participants completed a parity judgment task (see Dehaene et al., 1993), consisting of 6 blocks of 64 trials each. For three of the blocks, participants were instructed to respond by pressing the left key ("Q") to indicate an even response and the right key ("P") to indicate an odd response. For the other three blocks, these responses were reversed. These blocks were presented in an alternating order and were counterbalanced such that an equal number of participants received each block first. At the beginning of the experiment, participants completed a set of eight practice trials in which they received feedback about the accuracy of their parity judgments. There was no feedback on the remaining trials. Throughout the experiment, participants sat in a chinrest positioned 43 cm from the screen.

On each trial, a 500-ms fixation cross was presented centrally, immediately followed by the presentation of a vertical line (visual angle: 11.3^{o}). This line appeared either to the left or right of center (visual angle: 12.0^{o} in either direction) and remained onscreen for 1000 ms. An Arabic numeral followed that appeared either directly to the left or directly to the right of the previously presented line (4.7^{o} in either direction). The numeral remained on screen until participants indicated a response. Prior to the task, participants were instructed to respond "as quickly but as accurately as possible." An example of the experimental procedure can be seen in Figure 1.

Results

Reaction times (RTs) were analyzed only for correct responses (92.8% of trials). Additionally, outliers were removed on select trials where RTs were greater than 2.5 standard deviations from an individual's mean RT. Mean RT on remaining trials was 643 ms (*SD* = 146 ms). For each participant, mean RTs were calculated for right- and left-side responses (dependent on the frame of reference being tested) for each digit. A difference score (dRT = right RT – left RT) was calculated for each digit in each reference frame, and these dRTs were then regressed on the corresponding digit (as in previous research; see Fischer & Shaki, 2014). Separate analyses for each reference frame are presented below.

Keyboard-centered reference frame

Mean RTs were calculated for right- and left-side responses (corresponding to the "P" and "Q" keys, respectively), and a difference score (dRT) was calculated for each digit and regressed on the corresponding digits.

Averaged across all participants, dRTs regressed on corresponding digit revealed a significant linear effect indicating that participants responded faster to smaller numbers with the left button and faster to larger numbers with the right button, $R^2 = .541$, $F(1, 6) = 7.08$, $p < .05$ (see Figure 2A). A significant number of participants (29 of 36) demonstrated an effect in this direction, (binomial test, $p < .001$; see Figure 2B). Order of blocks (whether participants responded odd-P/even-Q or even-P/odd-Q first) did not significantly affect participant slopes (dRT regressed on corresponding digit), $t(34) = 1.975$, $p > .05$. There was no effect of parity (i.e., participants responded no faster to odd or even numbers with either button), $t(35) = 1.90$, $p = .061$.

Screen-centered reference frame

Mean RTs were calculated for right- and left-side responses (corresponding to the right and left sides of the screen), and a difference score (dRT) was calculated for each digit and regressed on the corresponding digits.

Averaged across all participants, dRTs based on screen position regressed on corresponding digit revealed no significant linear effect, $R^2 = .263$, $F(1, 6) = 2.14$, $p > .15$. Only 21 of the 36 participants tested showed an effect in this direction (binomial test, p = .08). Task instructions (whether participants responded odd-P/even-Q or even-P/odd-Q first) did not significantly affect participant slopes (dRT regressed on corresponding digit), $t(34) = .22$, $p > .80$. There was no effect of parity in relation to the screen, $t(35) = .10$, $p > .90$.

Eye-centered reference frame

Mean RTs were calculated for right- and left-side responses (corresponding to a right or left movement of the eyes prior to responding), and a difference score (dRT) was calculated for each digit and regressed on the corresponding digits.

Averaged across all participants, dRTs based on eye movements regressed on corresponding digit revealed no significant linear effect, $R^2 = .44$, $F(1, 6) = 4.80$, $p = .08$. 22 of the 36 participants tested showed an effect in this direction (binomial test, p = .06). Task instructions did not significantly affect participant slopes (dRT regressed on corresponding digit), $t(34) = .49$, $p > .60$. There was no effect of parity in relation to the screen, $t(35) = .58$, $p > .50$.

Body-centered reference frame

Mean RTs were calculated for right- and left-side responses (corresponding to the right and left side of the participants body), and a difference score (dRT) was calculated for each digit and regressed on the corresponding digits.

Averaged across all participants, dRTs based on body position regressed on corresponding digit revealed no significant linear effect, $R^2 = .01$, $F(1, 6) = .40$, $p > .80$. Only 15 of the 36 participants tested showed an effect in this direction (binomial test, $p > .50$). Task instructions (whether they respond odd-P/even-Q or even-P/odd-Q first) did not significantly affect participant slopes (dRT regressed on corresponding digit), $t(34) = .34$, $p > .70$. There was no effect of parity in relation to the screen, $t(35) = .89$, $p > .30$.

Cross-condition comparisons

Comparisons were made across reference frame using a Bonferroni-corrected alpha ($\alpha = .016$). Participant slopes differed across the conditions such that slopes for the keyboard reference frame were significantly greater than slopes in the screen-centered reference frame ($t[35] = 4.73$, $p < .001$), the eye-centered reference frame ($t[35] = 3.48$, $p < .01$), and the body-centered reference frame ($t[35] = 2.60$, $p = .014$). No other slopes differed significantly from one another (*p*s > .40)

Discussion

Consistent with previous results (e.g., Dehaene et al., 1993), we found evidence of a SNA in a keyboard-centered reference frame but failed to demonstrate a SNA in any of the other frames or reference that were tested. On the one hand, this may provide evidence that SNAs uniquely manifests only in that frame of reference, lending credence to the claim that reference frames may not be created equal (e.g., Viarouge et al., 2014). Another interpretation of our results may be that the keyboard-centered reference frame is simply the dominant reference frame postulated by Viarouge and colleagues. Recall that all frames of reference varied independently on a trial-by-trial basis *except* the keyboard frame of reference, which varied on a block-by-block basis. Thus, it is possible that the "dominant" reference frame is simply more salient than the other frames. Nevertheless, it remains unclear whether the possible dominance of the keyboard frame is merely a consequence of task demands (in that this frame varies less frequently than the others) or an indication of a hierarchy as Viarouge and colleagues suggested. Similarly, it is worth noting that the other three reference frames had the potential to counteract each other such that on a given trial a response may be congruent with respect to one reference frame but incongruent with respect to another.

Taking these two matters into account, then, it is possible that SNAs would manifest in the other frames tested in the absence of such confounds. In the following experiments, we attempted to isolate other reference frames (Experiment 2: screen; Experiment 3: eyes). We used the same paradigm used in Experiment 1 except that participants responded using a single key and with only one hand at a time. If it is true that the keyboard frame of reference is dominant over (or else more salient than) the other reference frames, then we would expect to find a SNA when that frame of reference is eliminated. If we find no significant SNAs, then this suggests that SNAs may in fact be driven by task demands.

Experiment 2

Method

Participants

56 undergraduates (40 female) participated for course credit. Six additional subjects were excluded from analyses because they failed to complete the tasks. The sample was

predominantly right-handed as assessed by the EHI. Participants reported normal or corrected-to-normal vision. Study procedures were approved by the local Institutional Review Board (IRB), and all participants provided written informed consent.

Stimuli and procedure

The experiment was completed using the same apparatus and stimuli as the previous experiment. Rather than responding with the “P” and “Q” keys, however, participants responded using the spacebar in a go/no-go paradigm (see Marghetis et al., 2013). For two of the blocks, participants were told to press the spacebar only when they were presented with an even number. For the other two blocks, participants were told to press the spacebar only when they were presented with an odd number. Similarly, participants were asked to use their right hand for two blocks and their left hand for the other two blocks. Accordingly, there were four possible blocks (odd/left-hand, odd/right-hand, even/left-hand, even/right-hand), which were counterbalanced across participants. At the beginning of the experiment, participants completed a set of eight practice trials during which they received feedback. Throughout the experiment, participants were constrained by a chinrest positioned 43 cm from the screen.

All aspects of the experimental design were identical to Experiment 1 except that participants saw no line on the screen between the fixation and the presentation of the digit. Instead, there was a 1000 ms delay (mirroring the 1000 ms of the line). Similar to Experiment 1, the digit could appear in one of four locations: far left (visual angle: 16.7^{o} left of fixation), near left (visual angle: 7.3^{o} left of fixation), near right (visual angle: 7.3^{o} right of fixation), and far right (visual angle: 16.7^{o} right of fixation). Additionally, there was a 1000 ms response window, beginning at the presentation of the digit, during which participants were able to indicate their response. Prior to the task, participants were instructed to respond “as quickly but as accurately as possible.” An example of the experimental procedure can be seen in Figure 1.

Results

Reaction times (RTs) were analyzed only for correct responses (97.0% of trials). Additionally, outliers were removed on select trials where RTs were greater than 2.5 standard deviations from an individual’s mean RT. Mean RT on remaining trials was 535 ms (*SD* = 40 ms). For each participant, mean RTs were calculated for right- and left-side responses (corresponding to the right and left sides of the screen) for each digit. A difference score (dRT= right RT – left RT) was calculated for each digit, and these dRTs were then regressed on the corresponding digit, as in the previous experiment.

Collapsed across all participants and all blocks, dRTs regressed on corresponding digit revealed no significant linear effect, $R^2 = .30$, $F(1, 6) = 2.56$, $p > .10$ (see Figure 3A). To understand the impact of block order on the magnitude of the participants’ slopes, we conducted a 2 × 2 analysis of variance (ANOVA) with order of parity instructions (whether participants responded odd or even in the first block) and order of hand instructions (whether participants responded with the left or right hand in the first block) as between-subject variables. Doing so revealed no main effect of the block order of parity ($p > .20$) or hand ($p > .30$), nor an interaction ($p > .20$). A separate test revealed an effect of parity on RTs (though in the opposite direction of what is typically observed; see Dehaene et al., 1993; Fitousi, Shaki, & Algom, 2009) such that participants were faster to respond to odd numbers when they appeared on the right side of the screen and even numbers when they appeared on the left side of the screen, $t(55) = 2.4$, $p < .05$. Moreover, this effect was consistent across participants: 35 of the 56 participants tested displayed this effect (binomial test, $p < .05$) and this effect was consistent across blocks (left hand blocks: 33 of 56 participants, $p < .05$; right hand blocks: 35 of 56 participants, $p < .05$). Accordingly, it is possible that the linear effect of dRT in relation to the screen is being underestimated as a consequence of this effect. To address this, we conducted a separate linear regression controlling for parity which revealed a significant linear relationship, $R^2 = .54$, $F(1, 6) = 7.00$, $p < .05$ such that participants were faster to respond to low numbers on the left side of the screen and high numbers on the right side of the screen[1]. Notably, a significant number of participants (35 of 56) exhibited slopes in this direction, (binomial test, $p < .05$;see Figure 3B).[2]

Discussion

The results here lead to a few notable conclusions. First, the linear effect reveals that it is possible to detect SNAs a reference frame other than the keyboard-centered frame, providing support for the hierarchical framework postulated by Viarouge and colleagues. That we demonstrated an SNA for screen in Experiment 2, but not Experiment 1, suggests that this effect may have indeed been masked by a separate, dominant reference frame (in this case, the keyboard-centered reference frame).

An alternative account of our results may be that the screen-centered reference frame was not completely isolated. The results of this experiment could be accounted for in terms of either eye-movements (insofar as subjects fixated at center between each trial) or body position (insofar as participants were situated such that the center of the body aligned with the center of the screen). However, to control for these reference frames would be to make them salient: had we

[1] We regressed dRT on parity and used the resulting residuals as parity-corrected dRTs. These parity-corrected dRTs were then averaged across participants and were regressed on the corresponding digit, as in previous analyses.
[2] The same analysis in the keyboard-centered reference frame of Experiment 1 revealed a larger—though not significantly larger—linear effect when accounting for parity, $R^2 = .791$, $F(2, 5) = 9.47$, $p < .05$. The same analyses performed on the other three reference frames revealed no significant effects (ps > .05).

moved participants' bodies relative to the screen to control for the body-centered reference frame, we may have been inadvertently activating it.

While the results of this experiment may be interpreted in multiple ways, the critical takeaway here is that we detected a SNA in a reference frame other than the keyboard frame. Accordingly, we may expect to find effects in other reference frames when they are separated from the keyboard-centered frame. If it is true, for example, that eye movements are sufficient to account for the effects in this experiment, then we would expect to see a SNA when an eye-centered reference frame is isolated. If not, it would suggest that the results of this experiment may be accounted for solely in terms of a screen-centered reference frame. To that end, Experiment 3 builds on Experiment 2 and again utilizes a go/no-go paradigm in an attempt to isolate an eye-centered frame of reference.

Experiment 3

Method

Participants

56 undergraduates (36 female) participated for course credit. Twelve additional subjects were excluded from analyses because they failed to complete the task. The sample was predominantly right-handed as assessed by the EHI. Participants reported normal or corrected-to-normal vision. Study procedures were approved by the local Institutional Review Board (IRB), and all participants provided written consent.

Stimuli and procedure

The experiment was completed using the same apparatus, stimuli, and procedure as Experiment 2, except that participants saw a line presented on the screen instead of experiencing a delay (similar to Experiment 1). This line could appear in one of four locations: far left (16.7^{o}

left of fixation), near left (visual angle: 7.3° left of fixation), near right (visual angle: 7.3° right of fixation), and far right (visual angle: 16.7° right of fixation). The line disappeared after 1000 ms, immediately followed by the presentation of an Arabic digit. The Arabic digit always appeared in the center of the screen. Accordingly, the only spatial information that varied across trials was the direction of eye-movements as determined by the position of the line. There was a 1000 ms response window, beginning at the presentation of the digit, during which participants were able to indicate their response. Prior to the task, participants were instructed to respond "as quickly but as accurately as possible." An example of the experimental procedure can be seen in Figure 1.

Results

Reaction times (RTs) were analyzed only for correct responses (97.0% of trials). Additionally, outliers were removed on select trials where RTs were greater than 2.5 standard deviations from an individual's mean RT. Mean RT on remaining trials was 491 ms (*SD* = 62 ms). For each participant, mean RTs were calculated for right- and left-side responses (corresponding to the rightward or leftward eye movements prior to responding) for each digit. A difference score (right RT – left RT) was calculated for each digit, and these dRTs were then regressed on the corresponding digit (as in the previous two experiments).

Collapsed across all participants and all blocks, dRTs regressed on corresponding digit revealed no significant linear effect, $R^2 = .02$, $F(1, 6) = .14$, $p > .70$ (see Figure 4). To understand the impact of block order on the magnitude of the participants' slopes, we conducted a 2 × 2 ANOVA with block order of parity (responding odd or even in the first block) and block order of hand (responding with right hand or left hand in the first block) as between-subject variables. Doing so revealed no main effect of parity ($p > .50$) or hand ($p > .50$) and no interaction ($p >$

.40). There was no significant effect of parity, $p > .10$. To be sure that parity was not a factor, a regression controlling for parity (as in Experiment 2) was conducted but nevertheless revealed an insignificant linear effect ($p > .50$).

Discussion

We detected no evidence of an SNA in the eye-centered frame of reference. Thus, we have demonstrated effects in two reference frames (keyboard and screen), both of which may be considered allocentric or "object-centered" reference frames. It is possible that SNAs manifest only in non-egocentric reference frames. This is consistent with the original work of Dehaene and colleagues (1993), which showed that, when in conflict, participants mapped numbers onto space using the keyboard as a reference frame rather than their hands. While there are conflicting results in this task, there is *no* evidence that this effect is tied exclusively to the hands—or, for that matter, evidence of SNAs in *any* tasks that exclusively involve egocentric frames.[3] Thus, on balance, it seems more likely that this effect is supported by an allocentric, keyboard-centered reference frame. Here, similarly, we have shown that SNAs can be found in two allocentric reference frames (keyboard and screen; Experiments 1 and 2) but not in an egocentric frame (eyes; Experiment 3).

However, our results here appear to be at odds with work that suggests that the origins of SNAs are related to saccadic eye movements (Schwarz & Keus, 2004), and other work that has related eye-movements and SNAs (Holmes, Ayzenberg, & Lourenco, under review; Loetscher, Schwarz, Shubiger, & Brugger, 2008). What may account for this difference? The most likely explanation is the *timing* of the eye movements. In the three experiments cited above, it is eye movement *after* the presentation of a number that exhibit a SNA. In our experiment, we are

[3] In fact, Viarouge and colleagues detected a typical SNARC effect in the keyboard-centered reference frame even when participants were given instructions that specifically emphasized the movement of the hands.

manipulating eye movements prior to the presentation of the number. Perhaps eye movements are related to SNAs (insofar as they guide spatial attention) but only after the presentation of a number.

An alternate possibility is that our manipulation failed to ensure that participants attended to the vertical lines on each trial. While the position of the vertical lines in Experiment 1 were predictive of the location of the digit, that was not the case here. The digits would appear in the center of the screen on every trial, leaving subjects with no reason to shift their attention in response to the appearance of the line.

General Discussion

Here, we have provided evidence for SNAs across multiple reference frames. In Experiment 1, consistent with past findings, we showed that an SNA exists in relation to button responses on a keyboard. In Experiment 2, we demonstrated an SNA in relation to the screen. Notably, both of these effects occurred in allocentric reference frames and may fall into the category of object-centered or stimulus-centered reference frames described in other research (e.g., Arguin & Bub, 1993; Behrmann & Tipper, 1999; Ota et al., 2001). We fully acknowledge, however, that these reference frames were not perfectly isolated (i.e., some egocentric information was available in each task). That said, when trying to understand the effect of a given reference frame, it is important to consider evidence across multiple tasks and paradigms.

Previous research has provided evidence of SNAs in multiple allocentric reference frames (keyboard-centered frame: Dehaene et al., 1993; screen-centered: Lavidor, Brinksman, & Göbel, 2004). Additionally, there is evidence of egocentric, finger-centered reference frames (Marghetis et al., 2013; Riello & Rusconi, 2011), and egocentric, eye-centered reference frames (Holmes, et al., under review; Loetscher, et al., 2008; Schwarz & Keus, 2004). However, there are also many instances in which certain egocentric and allocentric reference frames appear irrelevant (e.g., Wood et al., 2006; Viarouge et al., 2014), raising questions about the salience and stability of different spatial reference frames.

On the one hand, Viarouge and colleagues (2014) propose a framework for thinking about spatial reference frames that emphasizes the details of the experimental procedure. Indeed, decades of work with neglect patients has revealed the striking flexibility in the activation of spatial reference frames (see Halligan et al., 2003). In this way, we think the work of Viarouge and colleagues (and all related work on SNAs and reference frames) reaffirms what has been gleaned from neglect patients, namely that *spatial reference frames are malleable and context-dependent*. Certainly, they may be correct that inconsistent results across reference frames are due to subtle differences in experimental methods. However, a dynamic hierarchical framework is largely unfalsifiable: how are we to disprove the use of a particular frame of reference if any null result can be dismissed as a failure of the task itself to properly instantiate a given frame of reference? Here, we propose an alternate framework that attempts to make sense of the apparent inconsistency of SNAs across reference frames.

First and foremost, we think it is critical to consider the instances in which a stable effect has been observed. In the original work of Dehaene and colleagues (1993) and in numerous other experiments, stable SNAs have been observed when numerous reference frames are congruent with one another (see Cheung et al., 2015; Fischer & Shaki, 2014; Riello & Rusconi, 2011; Wood et al., 2008). This is true of our data as well. In Experiment 1, the putative keyboard-centered reference frame is confounded with the spatial information of the subject’s hands. In Experiment 2, the screen-centered effect could, instead, be explained by an egocentric frame either in relation to the eyes or the body. In Experiment 3, the only spatial information that varied across trials was the movement of the eyes, and in this instance we failed to detect an effect.

Perhaps it is the case, then, that there is no hierarchy of spatial reference frames. Instead, perhaps all spatial information is weighted to form a stable representation of “left” or “right” in a context-dependent manner. When spatial information is inconsistent (say, when hands and buttons provide incongruent spatial information in a parity judgment task) it becomes difficult to form a stable MNL. Yet, when information is congruent (in Experiments 1 and 2, for example), a stable MNL is formed and a SNA is observed.

The critical differences between this perspective and that of Viarouge and colleagues are that in this view: 1) no reference frame is inherently preferred to any other and 2) no single reference frame is responsible for the observed SNAs. Instead, our framework proposes that the establishment of a MNL is at all times reliant on multiple sources of spatial information. This is consistent with the work on neglect patients, wherein there is never a complete masking of one reference frame over another. Rather, when multiple sources of spatial information are incongruent, they all appear to have an influence (e.g., Farrah, 1990). Thus, the notion of spatial reference frames may be construed in an entirely different way: namely, that there are not multiple reference frames implicated in SNAs at all, and that the individual “reference frames” which we have referred to here—and have been referred to in other work—are not reference frames *per se* but merely sources of spatial information that contribute to a single context-dependent representation of left and right.

Consider the cross-hand parity-judgment experiments (wherein participants crossed their hands such that they pressed the left button with their right hand and vice versa; e.g., Dehaene et al., 1993; Viarouge et al., 2014; Wood et al., 2006). The mixed results in this paradigm speak for themselves: according to the dynamic hierarchy framework, if hands and keyboard are two dissociable reference frames, then both should *independently* yield SNAs when made salient. Yet this is not the case. Indeed, Viarouge and colleagues replicated the classic parity-judgment effect (with respect to buttons) *in spite of* directions that emphasized the hands. Moreover, it must be noted that while the hierarchical framework predicts that factors such as block order or task instructions will activate secondary reference frames, we found no such effects in any of the experiments presented here.

Per our view, SNAs may only be observed in instances where there is sufficient spatial information to form a stable representation of what is “left” or “right”—regardless of superficial elements of the task such as block order. In Experiment 1, for example, the position of the buttons, the hands, and the body (relative to the buttons, at least) were all congruent, providing enough consistent spatial information to form such a representation. That said, while we think Viarouge and colleagues are correct to suggest that context influences the salience of spatial cues, there appears to be no clear evidence that independent reference frames can account for SNAs. What this suggests is not any sort of flexibility of a MNL with respect to spatial reference frames. Rather, our view proposes that the MNL is supported by a single representation that necessarily integrates all available spatial information. Recall that the SNAs observed here—and in related work—occur only when multiple sources of spatial information are congruent and in spite of task-specific elements.

Critically, this framework makes many clear predictions with respect to SNAs and the MNL. First and foremost, one would expect that, so long as the spatial information is equally-salient, multiple congruent spatial cues should result in stronger SNAs than incongruent spatial information. Consistent with this prediction, it is worth noting that the size of the effect we observed in Experiment 1 (*d* = .6) is smaller than what is typically observed in a parity judgment task (95% Confidence Interval for *d*: [.9, 1]; Wood et al., 2008). Furthermore, this provides a falsifiable framework for assessing spatial reference frames. Rather than speculating about the relative dominance of spatial cues, we can ask what spatial information is sufficient to produce a SNA and use this knowledge to understand the relative weight of different sources of spatial information. Nevertheless, more work would be necessary to disentangle the framework proposed here from that of Viarouge and colleagues (2014).

In sum, our work adds to the existing literature that suggests that space and number are deeply related. Insofar as we have shown that SNAs persists across a variety of experimental procedures (e.g., in the go/no-go paradigm of Experiment 2), we have reason to believe that SNAs are not a product of arbitrary, task-specific mappings but, rather, are a consequence of systematic, stable mappings of number onto space. While more work is needed to better understand the nature of spatial reference frames—or else, the weighting of spatial information to support the formation of a single left-right representation—our work has shown that multiple sources of spatial information are relevant in the materialization of a mental number line.

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Appendix

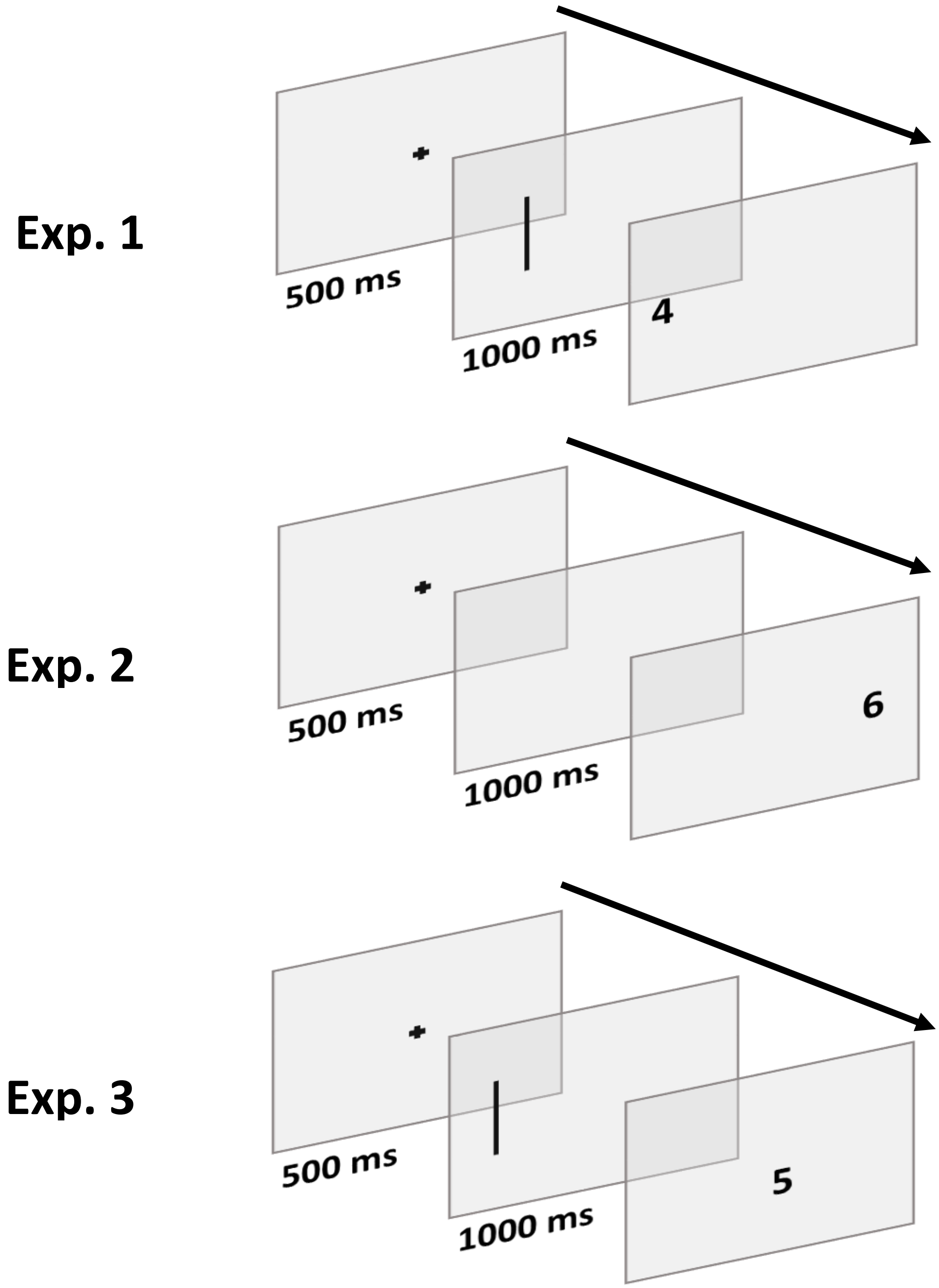

Fig. 1: Design for Experiments 1-3. In Experiment 1, the vertical line could appear in one of two locations (centered on either the left or right side of screen) and the numbers could appear in one of four locations (to the left or right of the line on either side of the screen). Participants indicated their responses with the "P" and "Q" keys. In Experiment 2, there was a 1000ms delay between fixation and present of the digit. The number could appear in one of four locations (identical to Exp. 1). Participants indicated their responses using the spacebar. In Experiment 3, the vertical line could appear in one of four locations (matched with the locations of the digits in Exps. 1 and 2), but the digits would always appear in the center of the screen. Participants indicated their responses using the spacebar.

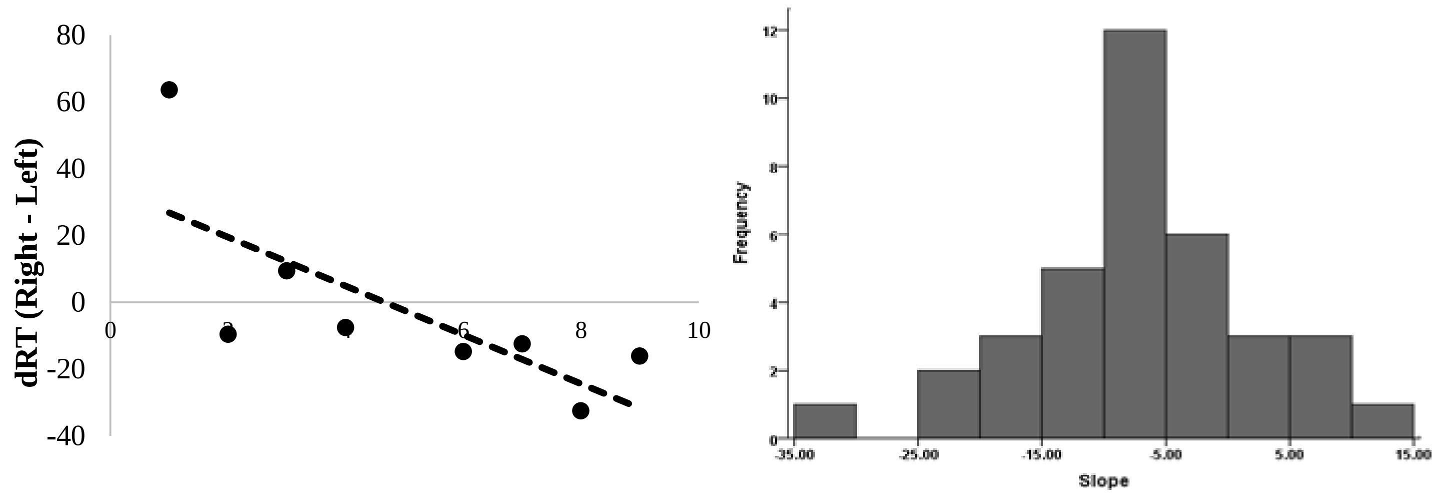

Fig. 2: SNA for Experiment 1 (left), where x-axis corresponds to digit and y-axis corresponds to dRT for right button minus left button. R^2 = .54. Distribution of participant slopes (right). 29 of 36 participants exhibited a negative slope.

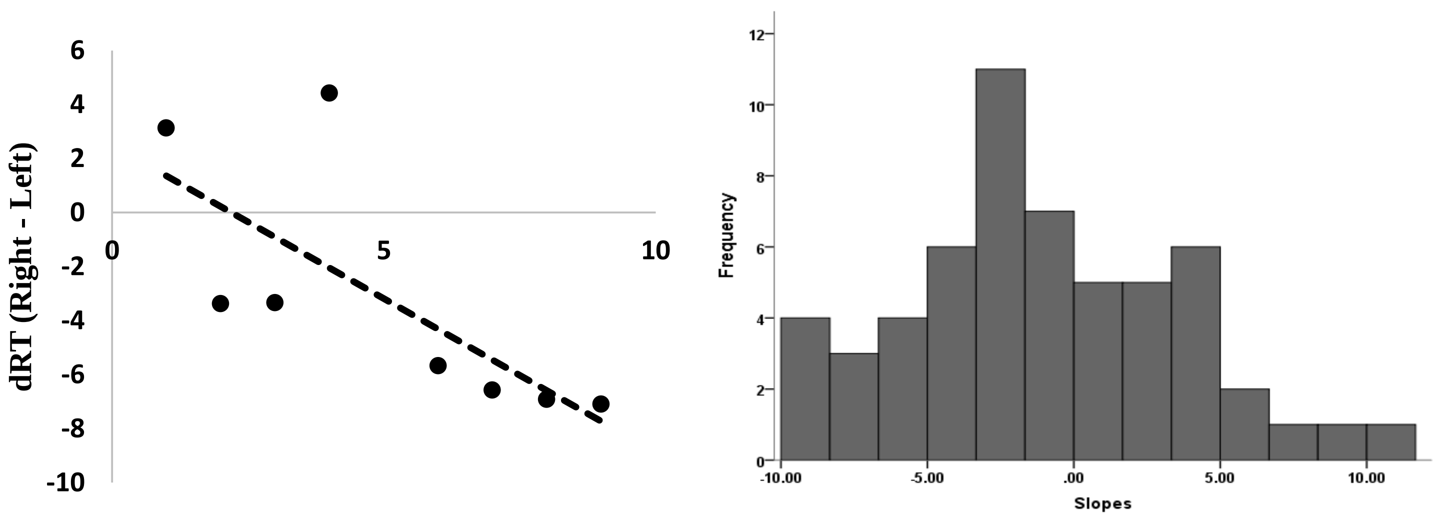

Fig. 3: Parity-corrected SNA for Experiment 2 (left), where x-axis corresponds to digit and y-axis corresponds to dRT for right side of screen minus left side of screen. R^2 = .54. Distribution of participant slopes (right). 35 of 56 participants exhibited a negative slope.

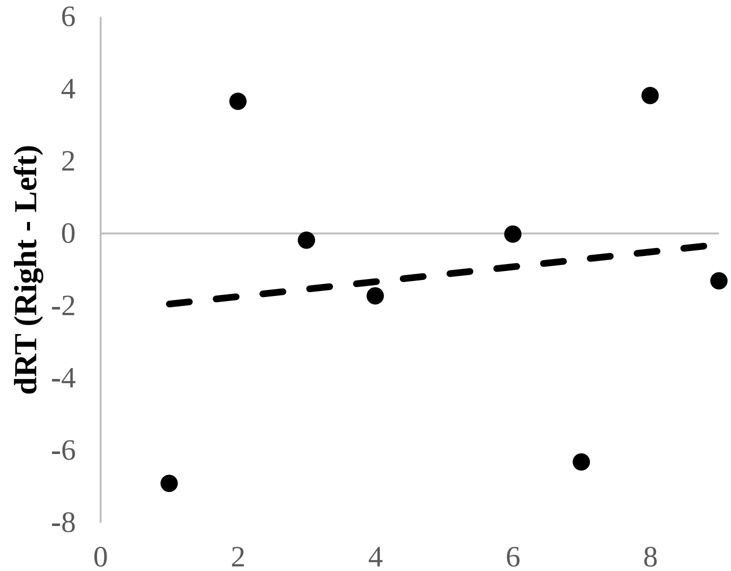

Fig. 4: SNA for Experiment 3 where x-axis corresponds to digit and y-axis corresponds to dRT for rightward eye movements minus leftward eye movements. R^2 = .02.