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The Relation between Number Acuity and Mathematical Ability in Young Children

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The Relation between Number Acuity and Mathematical Ability in Young Children

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An abstract of
A thesis submitted to the Faculty of the
James T. Laney School of Graduate Studies of Emory University
in partial fulfillment of the requirements for the degree of
Master of Arts
in Psychology
2010

Abstract

The Relation between Number Acuity and Mathematical Ability in Young Children
By Justin W. Bonny

A system of number that detects differences in large quantities has been found to be active in young infants and adults. Over development, the precision of this number system increases (Halberda & Feigenson, 2008). Recent research focusing on whether mathematical reasoning is founded on this system of number representation has produced conflicting results. This research, however, has focused on how this number system relates to math understanding broadly rather than whether the relation is to specific components of math understanding. Additionally, the age ranges used for these studies focus on children who have already developed some mature number and math systems, leaving open the question of whether a relation exists in younger children who are still learning these skills. The current study explored these issues using a task where 3- to 5-year-olds decided which of two numerical quantities was larger and then completed a standardized math test. Extending previous research with a younger age range, children who showed more precision (i.e. acuity) in their discrimination scored higher on the math test, even when age and verbal comprehension were controlled. An item analysis revealed that correlations between number acuity and specific components of math understanding (i.e., arithmetic with physical objects and cardinal understanding) were driving the association between number acuity and math. These results suggest that for some math abilities, the approximate number system is activated as children develop symbolic number and math abilities.

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TABLE OF CONTENTS

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A system of number that detects differences in large quantities has been found to be active in young infants and adults. Over development, the precision of this number system increases (Halberda & Feigenson, 2008). Recent research focusing on whether mathematical reasoning is founded on this system of number representation has produced conflicting results. This research, however, has focused on how this number system relates to math understanding broadly rather than whether the relation is to specific components of math understanding. Additionally, the age ranges used for these studies focus on children who have already developed some mature number and math systems, leaving open the question of whether a relation exists in younger children who are still learning these skills. The current study explored these issues using a task where 3- to 5-year-olds decided which of two numerical quantities was larger and then completed a standardized math test. Extending previous research with a younger age range, children who showed more precision (i.e. acuity) in their discrimination scored higher on the math test, even when age and verbal comprehension were controlled. An item analysis revealed that correlations between number acuity and specific components of math understanding (i.e., arithmetic with physical objects and cardinal understanding) were driving the association between number acuity and math. These results suggest that for some math abilities, the approximate number system is activated as children develop symbolic number and math abilities.

Keywords: number, math, development

LIST OF TABLES

Number *Page*

LIST OF FIGURES

The Relation between Number Acuity and Mathematical Ability in Young Children

Introduction

A hallmark of human cognition is the development of a symbolic number system. This system serves as the foundation for formal mathematics, which, in turn, serves as the basis for fields of study ranging from economics to nuclear physics. Components of this number system, which take years of training to reach a mature state, can be found early in life. Evidence suggests that infants have rudimentary numerical abilities, such as the ability to detect a change in non-symbolic number forms (e.g., an array of dots; Starkey & Cooper, 1980; Strauss & Curtis, 1981; Xu & Spelke, 2000; Lipton & Spelke, 2002), to perform simple arithmetic calculations (e.g., addition and subtraction; Wynn, 1992b; Simon, Hespos, & Rochat, 1995), and to expect either a larger or smaller quantity based on the relations of an ordinal sequence (e.g., ascending versus descending sequences; Brannon, 2002; Suanda, Tompson, & Brannon, 2008).

Studies investigating preschooler performance on tasks that tap these abilities have shown a connection to performance on tests of mathematical understanding, though this connection varies with task properties, such as range of numbers and types of math questions used (Halberda, Mazzocco, & Feigenson, 2008; Booth & Siegler, 2006; Mundy & Gilmore, 2009; Gilmore, Mundy, & Spelke, 2010; Holloway & Ansari, 2009). These results suggest a relation between early emerging non-symbolic quantitative abilities and more formal mathematical ability. However, these studies have focused on general math skills with children who have substantial development in number and math ability (older than 5 years of age). There may be differences in the relation between early number representations and specific math skills, such as counting and arithmetic, rather than math ability in general. Additionally, there may be more pronounced individual differences in younger children that are just beginning to understand symbolic number (e.g., Arabic numerals) and math abilities (e.g., addition). The current study investigated the relation between the ability to detect a difference in two quantities (i.e., number acuity) of young children (3- to 5-year-olds) and specific components of number and math abilities to determine which components of mathematical understanding are related to the early-developing approximate number system.

Systems of Visual Number

For children and adults to make use of mathematical abilities across development, they must first be able to detect quantities. Evidence of two dissociable visual systems of quantification is present in the distance effect as number varies from small (e.g., ≤ 4) to large (e.g., > 4) quantities (Chi & Klahr, 1975; Mandler & Shebo, 1982; Trick & Pylyshyn, 1994). The distance effect refers to the decrease in comparison performance (i.e., slower reaction times and less accurate judgments) when the numerical difference between two quantities becomes smaller (Kaufman, Lord, Reese, & Volkmann 1949; Moyer & Lauder, 1967). When examining the rate at which performance decreases, there is a distinct change between small numbers (1 – 4) and large numbers (> 4), suggesting that there are different systems for these two numerical ranges (Chi & Klahr, 1975; Mandler & Shebo, 1982; Trick & Pylyshyn, 1994). The rapid, parallel, enumeration of small numerical arrays with little error is characteristic of "subitizing," which is theorized to be a pre-attentive visual process that is present across development (Chi & Klahr, 1975; Trick & Pylyshyn, 1994; Leslie, Xu, Tremoulet, & Scholl, 1998). An example situation of when this system would be engaged is if you were asked how many wheels are on a child's bike that had training wheels. You would not have to count the wheels, the subitizing system would rapidly identify a total of four objects in your visual field.

A separate system is engaged for estimating the quantity of arrays that contain more than 4 objects, dubbed the "approximate number system" (ANS) (Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004). The ANS involves noisy representations, which abide by Weber's law, where the difference needed between two numerosities to detect a difference increases proportionally as the numerosities increase (Dehaene, 1992). The variability of number estimations is present across species as demonstrated by the Gaussian distribution of responses made by rats and humans when trained to make lever presses until a target number is reached (Mechner, 1958; Whalen, Gallistel, & Gelman, 1999; Cordes, Gelman, Gallistel, & Whalen, 2001). An example of when this system would be engaged is if a group of people had to guess how many legs a caterpillar has without counting. For each person the ANS would provide an estimate based on the visual information, but each estimate would vary, with some being more accurate than others. Unlike subitizing, there are developmental, and individual, differences in the variability of the estimation, with increasing precision over development (Pica, Lemer, Izard, & Dehaene, 2004; Halberda & Feigenson, 2008). The early development of the two number systems has been investigated using young infants' preference for novelty and preschoolers' ability to detect similarities and differences in number.

Exact and Approximate Number in Infancy and Childhood

Early studies exploring number representations in infants began with quantities that were within subitizing range (i.e., 1 – 4). To determine the numerical competence of young infants, looking time methods have been employed to examine whether infants preferentially look to a novel number. After becoming familiarized, or habituated, to a specific numerosity via repeated presentation, infants' longer looking to novel, rather than familiar, numbers suggests they can discriminate between two numerosities. These studies suggest that young infants (5 to 12 months of age) were able to detect a change in the number of objects, at least when in subitizing range, for a variety of objects including visually homogeneous (Starkey & Cooper, 1980; Strauss & Curtis, 1981) and heterogeneous sets (Starkey, Spelke, & Gelman, 1990), as well as temporal sets of events (Wynn, 1996), which suggests that infants can extract numerosity from a range of sensory modalities (although see Clearfield & Mix, 1999). Further evidence suggesting that infants are able to create representations of numerosity has been gathered using cross-modal (visual vs. auditory, visual vs. tactile) matching paradigms. When infants viewed arrays of two and three heterogeneous objects while presented simultaneously with two or three sounds (Starkey et al., 1990; Jordan & Brannon, 2006) or touches (Féron, Gentaz, & Streri, 2006), they preferentially looked to the visual array with the matching number of objects.

For sets of larger numerosities, it is was unclear whether infants would be able to notice a difference between large numbers, due to the variability in ANS representations. Using habituation to large arrays (above subitizing range), by 6 months, infants could only discriminate 2:1 ratio changes (e.g., 16 vs. 8) or larger, and older infants, around 10 months, could detect a more fine grained 3:2 change (e.g., 12 vs. 8) using either visual arrays of dots or auditory sequences of tones (Xu & Spelke, 2000; Lipton & Spelke, 2002; Xu & Arriga, 2007). These studies suggest that with development, the acuity of the ANS increases by reducing the variability of the representations.

Until recently, research on ANS acuity has only focused on development in infancy. The precision of ANS representations of 3- to 6-year-olds and adults was compared in a recent study by Halberda and Feigenson (2008). Participants judged which of two numerical arrays had a greater number as the ratio between the two arrays was varied from coarse (2:1) to fine (10:9). The extent to which participants in each age group could discriminate arrays was captured by calculating a Weber fraction for each individual in each group. The Weber fraction expresses the amount of uncertainty an individual has about the approximate quantity of a numerical array (Pica, Lemer, Izard, & Dehaene, 2004). For 3-, 4-, 5-, and 6-year-olds the highest number ratio they could reliably discriminate was 3:2, 4:3, 5:4, and 7:6, respectively (Halberda & Feigenson, 2008). American adults were accurate up to a 10:9 ratio, which replicates previous results collected from French adults (Pica et al., 2004). This increase in ANS acuity across the preschool years allows children to distinguish between numerical arrays that are close in value and may enable the mapping of number words onto approximate representations (Gallistel & Gelman, 2000). The question of what role early number systems have in learning formal number and math skills has focused on whether the approximate number system can express the formal principles and properties of counting and math.

Development of Formal Counting and Arithmetic

In everyday life, individuals invoke formal principles of counting and arithmetic. For children to become proficient counters they must first be able to extract the relevant information from the environment to be enumerated and understand three key principles of counting: one-to-one correspondence, ordinality, and cardinality (Gelman & Gallistel, 1978, Gallistel & Gelman, 1992). The principle of one-to-one correspondence implies that each item within a set of objects is present once in a number representation. Research demonstrating that infants can match visual and auditory number (Starkey et al., 1990; Wynn, 1998) and that preschoolers can say whether two sets of objects have the same quantity (Gelman & Gallistel, 1978; Gelman & Meck, 1983) suggests that the principle of one-to-one correspondence is present prior to, and does not require, the learning of verbal counting. The principle of ordinality, or ordered sequences of quantity, allows for the inference of more versus less relations among amounts. Research demonstrating that 10- and 12-month-old infants choose the larger quantity of desirable crackers (Feigenson, Carey, & Hauser, 2002) along with evidence that 11-month-olds can detect a difference between a series of numerical arrays that increase or decrease in quantity (Brannon, 2002; Suanda, Thompson, & Brannon, 2008) suggest that children at a young age can make ordinal judgments. The principle of cardinality, or that the integer last counted refers to the numerosity of the set, is composed of two concepts, exact mapping and the successor function (Gelman & Gallistel, 1978; Wynn, 1990, 1992a; Gallistel & Gelman, 1992; Carey, 2001; Izard, Pica, Spelke, & Dehaene, 2009). Exact mapping involves understanding that the last number counted refers to the same number of objects in the set just enumerated. The successor function refers to the knowledge that the next numeral in a count list is exactly one more than the previous and is one less than the number next in the list. When asked to give a number of items to a puppet, the difference between children's correct and incorrect responses suggest that there may be a long acquisition period where it is only around 42 months that most children display an understanding of the cardinal principle (Wynn, 1990; Wynn, 1992a). However, poor performance in younger children may be due to high task demands, masking a true understanding of cardinality (Gelman & Gallistel, 1978; Gelman, Meck, & Merkin, 1986; Gallistel & Gelman, 1992).

To create a system of mathematics, children must also understand how numerical representations can be manipulated by arithmetic operations. In order to have an understanding of arithmetic, individuals must recognize that adding units to a target set increases the quantity of the target and that the inverse is true for subtraction. Using a violation of expectation paradigm, arithmetic operations of exact quantities were presented to 5-month-old infants. Infants looked longer when the revealed number of puppets did not match the arithmetic answer presented during a familiarization phase, suggesting that young infants are capable of simple addition and subtraction operations (Wynn, 1992b). These results were later replicated (Simon, Hespos, & Rochat, 1995) and extended using approximate number quantities (McCrink & Wynn, 2004), though the strong arithmetic interpretation has been challenged as instead being due to differences in interest to stimuli and the computation of non-numerical cues (Mix, Huttenlocher, & Levine, 2002).

Arithmetic abilities have also been investigated with preschoolers using approximate representations. When 5-year-olds were shown two arrays of objects placed into a box and a comparison array was presented, children chose correctly whether the sum or the comparison array had more objects (Barth, La Mont, Lipton, & Spelke, 2005). Additional studies demonstrated that 5-year-olds could choose above chance when tones were used, and when subtraction and multiplication were the operators, suggesting that the basic structure of formal math operations may be present at a young age, and may emerge without specific formal instruction (Barth et al., 2006; Barth, Beckman, & Spelke, 2008; Barth, Baron, Spelke, & Carey, 2009; Izard et al., 2009).

Two theories have been proposed to explain how children learn the principles of counting and arithmetic. The main difference between these two theories is the specified role of the approximate number system. The continuity hypothesis, put forward by Gallistel, Gelman (1992, 2000) and colleagues, holds that the preverbal number representations contain all the counting principles due to the way the ANS creates number representations. Based on an accumulator model, number representations are created through an accumulator in which a neural pulse is added to a bin as each item in a set is counted or added. This is analogous to dropping marbles into a bag, the system keeps track of how many marbles to place in, or add to, the bag, and once that number has been reached, the bag is closed and the quantity fixed. Noise from memory processes add the characteristic variability of approximate representations. Hence, since a non-verbal counting process creates number representations, the quantity in the bin, the counting principles and arithmetic operations are inherent to the system (Gallistel & Gelman, 1992; Gallistel & Gelman, 2000; Leslie, Gelman, & Gallistel, 2008). When young children learn verbal counting, what they learn is how to map their non-verbal counting process to the verbal count list of the particular language they are immersed in. This suggests there is a strong connection between early approximate number representations and all symbolic number and mathematical understanding.

The discontinuity hypothesis holds that the approximate number system does not contain a full set of counting principles since the representations it creates are too noisy for distinctions between them to be clearly identified. Rather, as young children practice counting sets of objects that are within subitizing range, they are able to extract counting principles. In order to learn these principles, children need to compare and contrast the differences between sets of objects to notice rules that are present in counting. As children establish differences in small quantities, they are able to attach the values to the verbal count list and then learn the principle that the next item in the count list is exactly one more than the last and "boot-strap" this knowledge to quantities outside of subitizing range (Carey, 2001; Carey, 2004; Le Corre et al., 2006; Le Corre & Carey, 2007). This process entails a qualitative difference in number concepts between children who have and have not learned cardinality, and holds that specific linguistic experiences (e.g., counting) are critical in establishing the cardinal principle (Carey, 2004; Le Corre et al., 2006; Le Corre & Carey, 2007). Since this hypothesis focuses on a discontinuity in cardinal understanding, it would suggest a connection between the ANS and specific math skills, such as abilities that require one-to-one correspondence, ordinal judgments, and arithmetic, which evidence suggests emerge early in development, but not abilities such as an understanding of cardinality.

Relation between Approximate Number and Formal Math Ability

One view on the development of formal number and math ability holds that cultural knowledge used to construct these skills is built upon the early emerging number systems, in particular the ANS (Gallistel & Gelman, 1992; Dehaene, 1997, Butterworth, 1999). Previous research has suggested a relation between children's spatial representation of number and math performance. Evidence exploring the mental representation of number has found evidence of a mental number line, where quantities are arranged spatially in an increasing fashion, similar to a ruler (Dehaene, 1992; Dehaene, Bossini, & Giraux, 1993). Research investigating the development of the mental number line has found that when children are asked to mark where a target number is on a horizontal line there is a shift in the spatial layout of the responses during primary schooling from a compressed logarithmic layout to a more linear layout with equal intervals between numbers (Booth & Siegler, 2006). When considering math achievement scores, children who score higher on the tests have a more linear spatial representation (Booth & Siegler, 2006). In a similar task, the ability of 8-year-olds boys to ignore misleading spatial information while deciding which of two numerical distances was larger was correlated with their calculation ability (Lonnemann et al., 2008). While these studies suggest that the spatial representation of numbers is related to math ability, other studies have linked math scores to the precision of their approximate number representations.

Halberda, Mazzocco, and Feigenson (2008) sought to determine whether an individual's number acuity, characterized by a Weber fraction, was related to their math ability. For the task, adolescents determined whether there were more blue or yellow dots in a dot array as the ratio difference varied from 2:1 to 8:7 using quantities from five to sixteen. Adolescents who had scored high on previously collected standardized math tests (Test of Early Math Ability – 2nd edition, Woodcock–Johnson revised calculation) as children had a higher Weber fraction (i.e., greater acuity), while controlling for verbal scores, suggesting a connection between non-symbolic number acuity and symbolic math skills (Halberda et al., 2008). This result has also been suggested with 6-year-olds on an approximate arithmetic task (Gilmore et al., 2010). These results suggest that ANS acuity is not simply a measure of general intelligence, but is specifically related to math achievement.

There has also been evidence suggesting no relation between approximate number and math ability. Using quantities from one to nine, 6- to 8-year-olds had to choose which of two simultaneously displayed dot arrays or Arabic numerals was larger (Holloway & Ansari, 2009). When controlling for verbal ability, math scores from the Woodcock-Johnson Tests of Achievement (Math Fluency and Calculation subtests) were only correlated with a numerical distance effect, for symbolic numerals rather than non-symbolic dot arrays (Holloway & Ansari, 2009). The lack of a relation between non-symbolic number acuity and math ability has also been suggested with children of the same age and slightly younger children (4-to-7-year-olds) (Mundy & Gilmore, 2009; Soltész, Szücs, & Szücs, 2010). However, when accuracy was used to calculate the distance effects, there was a relation between math calculation ability and both Arabic numeral and dot array versions of the task (Mundy & Gilmore, 2009; Soltész, Szücs, & Szücs, 2010). The different results from studies investigating the connection between number acuity and general math skills may hide a different interpretation – specifically, that rather than acuity being related to all types of math skills, it may be that the relation exists for only specific facets of formal math understanding.

When considering the tasks and age ranges used to investigate the connection between number acuity and math skills, there are certain aspects that can influence the amount of difference in performance between individuals. In order for the discrimination task to accurately capture acuity of the ANS, only quantities that are out of subitizing range should be used. One of the hallmarks of subitizing is that performance is near perfect and there are very few, if any, differences across individuals. Additionally, the age ranges investigated by these studies have focused on children that have mastered the principles of counting and have begun to receive formal instruction in arithmetic. Younger children between the ages of 3 and 5 years are just beginning to acquire, or have just acquired, counting principles, in particular cardinality (Sarnecka & Carey, 2008). It is during this period that individual differences in components of formal counting and arithmetic may be most pronounced.

For the standardized math tests, several different types of number and arithmetic abilities are combined into a general score (e.g., addition, cardinality, timed calculation). As such, correlating performance from a number comparison task and general math scores may mask specific components of formal math and number skills that may be driving the relation. This is suggested by a recent study where a relation was observed specifically between distance effects and calculation scores, but not symbolic number knowledge, in 6- and 7-year-olds (Mundy & Gilmore, 2009). In order to gain a clearer understanding of the nature of the relation between number acuity and math skills, potential differences in specific math skills needs to be explored.

To address these issues, the current study investigated how approximate number acuity is related to a general score of math ability and specific facets of mathematical ability with 3- to 5-year-old children. Similar to previous studies, children completed a computerized number task and standardized tests of math ability (Test of Early Mathematics Ability-3rd edition, or TEMA-3) and verbal ability (Peabody Picture

Vocabulary Test – 4th edition, or PPVT-4). On the number task, children made ordinal judgments, deciding which of two visual arrays has a greater number of objects. Number acuity, calculated from number task performance, was then compared to scores on the TEMA-3 and PPVT-4. In addition, and unlike existing studies, items on the TEMA-3 were categorized into clusters of specific skills, and the relation between number acuity and performance on each cluster was compared. Based on previous research, two predictions were made. First, we predicted that number acuity would relate to a general math score, when controlled for vocabulary scores. Second, if the ANS is connected differentially to specific math skills, then there should be differences in the strength of the correlation between acuity and performance on math clusters.

Method

Participants

Children were recruited from the Atlanta metropolitan area using a database of families who had expressed an interest in participating in research at the Emory University Child Study Center. A total of 50 children from 3 to 5 years of age participated (3-year-olds: 9 boys, 6 girls, M = 42.3 months, SD = 2.9 ; 4-year-olds: 7 boys, 16 girls, M = 52.7 months, SD = 3.5; 5-year-olds: 7 boys, 5 girls, M = 65.7 months, SD = 3.9). Over two sessions, children completed the number task, TEMA-3, and PPVT-4. During the first session children were administered the number task and TEMA-3, and of the sample, 29 children completed the PPVT-4 during a second session. Children received a small prize at the end of each testing session. An additional 2 children did not complete the discrimination task due to fussiness and were not included in the analysis.

Materials

Children completed the Number Task on a Dell Vostro Laptop, equipped with a touch screen (MagicTouch, Keytec, Inc.). The program was developed using Visual Basic and children made their responses using a touch screen stylus. The TEMA-3 (Ginsberg & Baroody, 2003) and the PPVT-4 (Dunn & Dunn, 2007) were administered following a standardized protocol and all tasks were given in a quiet room. The TEMA-3 is a standardized math test, that contains various counting and math problems, such as giving a specific number of items to an experimenter's request and solving word problems and takes on average 40 minutes to complete. The test has been normed with 1,219 children from 3 to 8 years of age with internal consistency of $r = .94$ to .96 and the standardized scores have a mean of 100 and standard deviation of 15 (Ginsberg & Baroody, 2003). The PPVT-4 is a standardized vocabulary comprehension test that contains four pictures on a page and children are asked to choose which picture matches a word and takes on average 10 to 15 minutes to complete. The test has been normed with 3,540 children and adults from 2 to 90 years of age with an internal consistency of $r = .94$ to .95 and the standardized scores have a mean of 100 and a standard deviation of 15 (Dunn & Dunn, 2007).

Number Task

Insert Figure 1

On this task, children judged which of two dot arrays had the larger number of dots (see Figure 1). The arrays were presented simultaneously and organized vertically on the screen to avoid effects of left and right spatial associations (Dehaene, 1992; Dehaene et al., 1993). One of the arrays was a reference array, which had a fixed number of 8 dots, and was present across all trials while size and color varied. The comparison array had a number of dots varying from 4-7 and 9-12, yielding 8 different ratios (larger divided by smaller): 2.00, 1.60, 1.50, 1.38, 1.33, 1.25, 1.14, 1.13. A reference array was used similar to previous work in which participants were required to make an ordinal judgment with respect to a fixed reference (Dehane, 1992). To minimize the influence of cumulative contour length and area, each element of the array varied in size and the total area of the numerically larger array was either larger or smaller than the comparison array to create spatially congruent (i.e., more in number, more in area) and incongruent (i.e., more in number, less in area) trials. In an attempt to ensure that the distracting effect of cumulative surface area was constant across the different ratios, the ratio difference of the surface area mirrored the ratio difference in number. For example, if the ratio difference in number was 1.33, the ratio difference in surface area was also 1.33. The position of the correct array, as well as spatial congruity was counterbalanced across trials.

Procedure

Children were administered the number task and then the TEMA-3 over a 75 minute session, and the PPVT-4 in a second session that lasted up to 45 minutes. Prior to completing the number task, children played a short game to become familiar with the touchscreen and general procedure. This game involved catching a picture of a frog by touching it with the touchscreen stylus and starting trials by touching a box. For the number task, children were shown a short video where they observed two boxes that contained a different number of "bubbles" and were asked to choose which box had more bubbles. The training video served to help children become more familiar with the task and the instructions.

Children then received 4 practice trials, with large ratio differences not seen during the test trials, where the experimenter demonstrated how to play the game and corrective feedback was provided. Children were instructed to pick the box that had more bubbles in it as fast as they could before they popped. For each trial, children touched a center box to begin the trial and were shown the two arrays of dots while being asked, "Which box has more bubbles in it?" and touched either array to end the trial. If a child tried to count the bubbles, the experimenter discouraged the counting to ensure that only visual estimation was used, and told the child that this was not a counting game.

For test trials, the same procedure as the practice trials was used. No corrective feedback was given during these trials, and after 8 test trials, children were presented with an animation, which served as a reward, and were reminded of the instructions. There were 5 trials for each ratio for a total of 40 test trials. After children completed the number task, children took a break and were then given the TEMA-3.

Results

Number Task

Based on previous research, it was predicted that older children would perform better than younger children, performance would be higher on spatially congruent than incongruent trials, and performance would be higher for farther than closer ratio differences (Halberda & Feigenson, 2008; Lonnemann et al., 2008). Proportion correct was used as the dependent variable in these analyses. Comparisons to chance revealed that the overall accuracy (collapsed across ratio differences) for spatially congruent trials was above chance for all age groups: 3-year-olds, *M* = .664, *SD* = .173), $t(14) = 3.672$, $p = .003$, 4-year-olds, *M* = .817, *SD* = .169), $t(22) = 5.989$, $p < .001$, and 5-year-olds (*M*= .892, *SD* = .089), $t(11) = 15.237$, $p < .001$. For spatially incongruent trials performance was at chance for 3-year-olds (*M*= .540, *SD* = .147), $t(14) = 1.058$, $p = .308$, but significantly above chance for both 4-year-olds (*M*= .674, *SD* = .139), $t(22) = 5.989$, $p < .001$, and 5-year-olds (*M*= .799, *SD* = .143), $t(11) = 7.231$, $p < .001$.

A repeated-measures ANOVA with a between-subjects factor of age (3-, 4-, 5-years) and gender (male, female) and within-subject factors of spatial congruity (congruent, incongruent) and ratio difference (2.00, 1.60, 1.50, 1.38, 1.33, 1.25, 1.14, 1.13) revealed that gender was not a significant factor. Subsequent analyses without the factor of gender revealed the following effects. There were significant main effects of age, $F(2, 47) = 11.157$, $p < .001$, $\eta_p^2 = .322$, spatial congruity $F(1, 329) = 45.611$, $p < .001$, $\eta_p^2 = .492$, and ratio, $F(7, 329) = 3.301$, $p =.002$, $\eta_p^2 = .066$. Planned contrasts revealed that as the ratio difference increased, performance increased in a linear fashion, $F(1, 47) = 8.894$, $p = .005$ and as age increased, performance increased in a linear fashion, $F(1, 47) = 21.477$, $p < .001$. There was also a significant interaction between spatial congruity and ratio, $F(7, 329) = 2.127$, $p = .040$, $\eta_p^2 = .043$. Post-hoc analyses for the spatial congruity and ratio interaction revealed the effect of ratio was significant for spatially incongruent, $F(7, 343) = 4.305$, $p < .001$, but not for spatially congruent trials, $F(7, 343) = .434$, $p = .881$. These results indicate that for the spatially incongruent trials, as the ratio difference increased, performance increased in a linear fashion. This was not the case for spatially congruent trials in which there was no difference in performance as ratio increased.

Insert Figure 2

Relation Between Number Acuity and Math Ability

To capture individual differences in number acuity, the accuracy of each child was modeled as a function of ratio. To this end, a psychophysical function, which has been used in previous research, was used to fit a Gaussian curve to the performance of each child (Piazza, Pinel, Izard, Le Bihan, & Dehane, 2004). This model has been proposed to best represent the underlying number representation and behavior based on the representation (Pica et al., 2004; Piazza et al., 2004). The fitted curve was then used to predict the child's accuracy on a ratio discrimination that they were not been presented with (1.17), yielding a proportion score between chance (50%) and perfect performance (100%). While recent experiments have determined a cut-off point assumed to represent above chance performance, by using an unseen stimulus to predict performance this assumption can be avoided. This process has also been used in previous research to predict performance using models of verbal number comparison (Dehaene, 1989). The ratio of 1.17 was chosen since its placement is on the steepest slope of the Gaussian curve which allows for a wide variability in predicted performance. The predicted accuracy served as an indicator of their number acuity on the task and was used in the correlational analyses described below.

For the standardized tests, children were given a standardized score reflecting their performance on the TEMA-3 and PPVT-4. To explore performance differences on different types of number and math questions, children's performance on the TEMA was broken down into 6 different clusters by determining which math principles each item best fit (Gelman & Gallistel, 1978; Gallistel & Gelman 1992; Dehaene, 1992; see Appendix for TEMA item clusters). Items that required children to make an ordinal judgment about which of two numbers was more or less comprised a number comparison score. Items that required children to recite a count-list comprised a count-list score. Items that required children to assign a numerical value to a set of items or determine the value of the next number in a count list using the cardinal principle comprised a cardinal score. Items requiring children to identify or write a numeral comprised a symbolic knowledge score. Items requiring children to perform arithmetic with tokens, fingers, or written problems comprised a physical arithmetic score. Items requiring children to perform arithmetic without counting and/or physical tokens comprised a mental arithmetic score. Cluster scores were created using the proportion correct of items from below each child's basal level up to the ceiling level on the TEMA-3 (see Table 1).

Insert Table 1

Insert Figure 3

The relation between number task acuity and mathematical ability was investigated using a hierarchical regression., When age, β = -.017, $t(25)$ = -.084, p = .934, and PPVT-4 scores, β = .295, $t(25)$ = 1.733, p = .095, were controlled for, children's number acuity predicted a significant amount of variance in TEMA-3 scores, β = .571,

$t(25) = 2.885$, $p = .008$, suggesting that there is a relation between approximate number discrimination and math ability (see Table 2). To explore potential differences in the relation between task performance and specific number and math clusters, partial correlations between acuity and cluster scores, while controlling for children's age, were calculated. Significant partial correlations were found between number acuity and cardinal score, $r\ (47) = .311$, $p = .030$, and number acuity and physical arithmetic, $r\ (47) = .371$, $p = .009$, but not for number comparison ($p = .482$), count-list ($p = .\ 426$), symbolic knowledge ($p = .359$), and mental arithmetic ($p = .886$), suggesting that there are specific aspects of math ability that are related to ANS performance (see Table 3).

Insert Table 2

Insert Table 3

Discussion

The goal of the study was to investigate the relation between approximate number representations and different math abilities. Results from the number discrimination task demonstrated an increase in number acuity over the preschool years; children's ability to differentiate two numerical arrays improved across this developmental period, replicating previous studies (Halberda & Feigenson, 2008; Mundy & Gilmore, 2009; Soltész et al., 2010). Additionally, when controlling for age and verbal scores, number acuity predicted children's math performance, extending previous work to a younger age group (Halberda et al., 2008; Gilmore et al., 2010). The partial correlation analyses suggest that there are facets of math ability, specifically cardinal and physical arithmetic, that may be primarily responsible for the relation between math skills and number acuity.

Could other variables account for this relation? While one possibility is general intelligence, we would suggest that it can be ruled out as number acuity predicted TEMA-3 scores when age and verbal ability were controlled for. General intelligence can also be ruled out since there were differences in the partial correlations between task performance and math cluster scores. Another possibility is that working memory can account for the observed relation. While these results cannot definitively rule out this factor, it seems unlikely, since, during the number task, arrays were presented simultaneously and were kept on the screen until a response was made, which would have minimized working memory demands.

There were difficulties in using cluster scores comprised of TEMA-3 items. Due to different start points in the test based on age, there were differences in the number of items each child received. This led to some children not receiving many, if any, items of certain clusters (e.g., Mental Arithmetic). Additionally, as the test progressed there was a shift in the focus of the items to more arithmetic based questions that were also more difficult, leading to floor effects, specifically in the mental arithmetic cluster. While there were some differences in the mean performance across cluster scores, even with clusters that had similar mean and spread of performance (e.g., Count-List and Cardinal), there were differences in the relation to number acuity. Despite difficulties, this suggests that cluster scores were a valid means of capturing and testing specific mathematical skills.

While the current results replicate the findings of Halberda et al. (2008), they appear to be in conflict with Holloway and Ansari (2009). The differences in the relation between non-symbolic performance and math ability may be due to differences in the range of quantities used. Whereas in the current study and in Halberda et al. (2008), all numbers were above subitizing range, the range of numbers used by Holloway and Ansari (2009) spanned across subitizing and approximate number ranges. The use of numbers within subitizing range may have reduced variability, as an exact rather than an approximate quantity may be extracted, which may in turn have skewed results.

Neuroimaging research has explored whether the neural correlates of math operations reflect results of behavioral experiments. Numerical distance effects, where processing the difference between two numbers close in quantity takes longer than two numbers far in quantity, have been revealed whether the number format is a dot array, Arabic numeral or verbal word with adult participants (Moyer & Landauer, 1967; Buckley & Gillman, 1974; Dehaene, 1992). These results were suggested to indicate that all forms of number activate an analog magnitude, similar to dot arrays. This suggests that the relation between number acuity and math ability should be present regardless of the number format.

Using neuroimaging techniques, a neural region, or neural activity, that is modulated by the small and large differences in quantity has been taken to indicate a neural correlate of the distance effect. A particular region, the intraparietal sulcus (IPS) has been found to be modulated by the distance effect with non-symbolic and symbolic number with adult participants (Pinel et al., 2009; Ansari & Dhital, 2006; Piazza, Mechelli, Price, & Butterworth, 2006). When exploring neural correlates of number development, the IPS has been found to be sensitive to the distance effect when viewing analog and symbolic number. When passively viewing changes in the quantity of dot arrays, the IPS was similarly activated for both 4-year-olds and adults, suggesting a continuity in ANS processing across development (Cantlon, Brannon, Carter, & Pelphrey, 2006). Additionally, when choosing which of two dot arrays or numerals was larger, IPS activation was modulated by the distance between the two quantities, for both notations, for both 6- to 7-year-olds and adults (Cantlon, Libertus, Pinel, Dehaene, Brannon, & Pelphrey, 2009). These results suggest that across development, similar neural regions are activated when viewing number in different formats. Additionally, this suggests that the correlation between acuity and math skills in the current study is not limited to children, but should also be present in adults.

Studies exploring the neural correlates of arithmetic have focused on whether, similar to numerical comparison, parietal regions are activated during operations, such as addition. Using event-related potential (ERP) and fMRI neuroimaging techniques, the operations of approximate and exact arithmetic have been contrasted. When completing approximate arithmetic, where adult participants had to choose which of two answers were closer to an arithmetic answer, there was greater activation and quicker processing in visuo-spatial regions compared to language and frontal regions activated for exact calculation (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999). These results suggest that, compared to exact arithmetic, approximate arithmetic may more strongly activate neural regions that are also activated by number comparison (Dehane et al., 1999).

Differences in neural activation during arithmetic have also been reported as a result of task strategies. When participants reported solving arithmetic problems using fact retrieval (e.g. recalling a multiplication table) versus a procedural process, greater activation was present in the left angular gyrus, a region that has also been shown to be connected to language processes (Grabner et al., 2009). These results suggest that neural regions are differentially activated by approximate and exact arithmetic, a dissociation that was present in correlations in the current study.

The myelination of neural connections between frontal and parietal regions also suggest different pathways for approximate and exact calculation. A recent study used diffusion tensor imaging with children to determine the level of myelination in the neural connections between the IPS and frontal regions and its relation to performance scores on math knowledge and approximate and exact arithmetic problems (Tsang, Dougherty, Deutsch, Wandell, & Ben-Schachar, 2009). Children with higher levels of measured white matter scored higher on the approximate arithmetic problems, but there was no relation between white matter and exact calculation indicating the connection between the IPS and frontal regions plays a role in approximate arithmetic performance. The implication of the IPS during approximate, but not exact, arithmetic supports the specific relations between number acuity and facets of math ability observed in the current study.

Differences in numerical operations are also predicted by models of number processing. One model in particular, the triple-code model put forward by Dehaene (1992), postulates the presence of three different forms of number representations, each associated with specific number and math operations. In the model, number can either be represented as an analog magnitude (similar to representations of the ANS), an Arabic number, or an auditory / written word. Additionally, these representations are interconnected meaning that numerical information in one representation can be transformed into any other number representation. However, each representation has exclusive number procedures which can only be used in the native number format. In the model, the processes of number comparison and approximate arithmetic are exclusive to the analog magnitude representation. The process of determining parity is exclusive to the Arabic numeral representation. The process of accessing multiplication and addition tables is exclusively tied to the word representation. The model further postulates that the analog magnitude representation is present from an early age and the word representation is highly connected to general language processes.

The specific relations between cluster scores and number acuity reflect some of the subtleties predicted by the triple-code model. In the model, number acuity represents the precision of the analog magnitude representations and, in turn, would then be related to analog specific processes. The relation between number acuity and physical arithmetic suggests that this cluster is similar to the approximate arithmetic processes predicted to be part of the analog magnitude representation. Additionally, the sharp contrast between the relation of number acuity to physical and mental arithmetic is also supported by the model since calculating arithmetic answers mentally requires some use of arithmetic tables, which are part of the auditory / word number representation. The clusters of count-list and symbolic knowledge would also be part of the auditory / word representation and Arabic numeral representation, but not analog magnitude, which is reflected by a lack of a correlation between number acuity and these clusters. The lack of a relation between number acuity and the number comparison cluster at first does not seem to follow the model. However, the majority of the items in the cluster used Arabic numerals and number words, so this cluster may be more related to the ability to transfer between number representations, which may take longer to develop. The relation between number acuity and performance on the cardinal cluster is supported by the triple-code model. Since both subitizing and ANS are proposed to be inputs to the analog magnitude representation, it would predict a relation between cardinal understanding and number acuity.

While the different relations between number acuity and cluster scores support the triple-code model, they do not discriminate as to how the cardinal principle is learned. Based on the continuity theory, learning the cardinal principle requires mapping culturally-determined count lists to nonverbal counting mechanisms. Since number acuity is a measure of the ANS, in addition to a relation between number acuity and performance on the cardinal cluster, there should be a relation between number acuity and performance on the count list cluster, which did not occur. Based on the discontinuity theory, the cardinal principle is learned through the process of subitizing and experience with language. Since the ANS is not proposed to play a role in the cardinal principle, then there should be a weak, if any, correlation between number acuity and cardinal performance, which did not occur. These results, suggest that a hybrid theory may explain the role of the ANS in learning the cardinal principle. For children, the ANS may serve as an implicit source for counting principles, but children also need a large amount of practice with language to gain the experience needed to make these counting rules explicit. A representational re-description model of learning would fit this view and the observed relation between number acuity and performance on the cardinal cluster (Karlimoff-Smith, 1992).

Future research should investigate the nature of the connection between number acuity and mathematical ability. While number acuity predicts math performance, it is unclear whether math skills are built from the ANS. Rather, as children learn math skills, the ANS may be activated simply since it is part of a broader number and math system. Additionally, while the relation between number acuity and components of math skills are in line with the triple-code model, future research should investigate further how children's ability to transfer between the three number representations of the model is related to math ability. Research with older children already suggests that there is a relation, but the development of this skill between preschool and primary school years is relatively unknown (Mundy & Gilmore, 2009).

Future research should also explore how number acuity can be used as a index of early math development. The extension of the relation between acuity and math ability to young children opens the possibility that acuity tasks can be used as a fast means to gauge the math ability of children who have not yet entered primary schooling. The low verbal demand of these tasks ensures that preschool children who have yet to learn the vocabulary needed to perform arithmetic problems can complete the tasks. By using number acuity tasks in tandem with standardized testing, children who may need more instruction when learning math skills can be identified and reduce the possibility of children falling behind in the classroom.

Overall, this study suggests a specific, rather than general, relation between number acuity and select math skills. The differences in the relation between number acuity and components of mathematical abilities reflect a model of number processing and suggests the ANS plays a role in learning the cardinal principle. While the causal nature of the relation remains unclear, the specific correlation between number acuity and components of mathematical ability suggests that the ANS is activated as these skills are developed.

Table 1. Descriptive Statistics of TEMA-3 Clusters

Cluster	Example Item	N	M	SD
Number Comparison	*"Which is larger, five or six?"*	50	0.647	0.301
Count-List	*"Count to 10."*	50	0.725	0.238
Cardinal	*"Give me five tokens."*	50	0.717	0.212
Symbolic Knowledge	*"Can you write the number '5'?"*	45	0.544	0.310
Physical Arithmetic	*"Use your tokens to solve 2 plus 3."*	50	0.447	0.395
Mental Arithmetic	*"What is 4 plus 7?" (timed and cannot use fingers)*	32	0.244	0.298

Note. Each cluster score was created by averaging proportion correct on items that were determined to best fit the skill of focus for each cluster. A representative example of an item from each cluster is given.

Table 2. Hierarchical Regression Model for TEMA-3 Score

Step	Factor	R^2 Change	*p*
1	Age	0.052	0.234
2	PPVT-4	0.049	0.246
3	Number Acuity	0.225	0.008

Note. Using TEMA-3 scores as the dependent variable, the regression tested whether there was a significant increase in predicted variability when each factor was added.

Table 3. Partial Correlations of Number Acuity and TEMA Cluster Scores Controlling for Age

	PPVT -4	Number Comparison	Count List	Cardinal	Symbolic Knowledge	Physical Arithmetic	Mental Arithmetic
r_p	-0.129	0.103	0.116	**0.311***	0.142	**0.371****	0.027
p	0.512	0.482	0.426	0.03	0.359	0.009	0.886
df	26	47	47	47	42	47	29

*p < .05, **p < .01

Note. Reported is the significance of two-tailed partial correlations between cluster scores (as well as PPVT-4 standardized scores) and number acuity, while children's age is controlled.

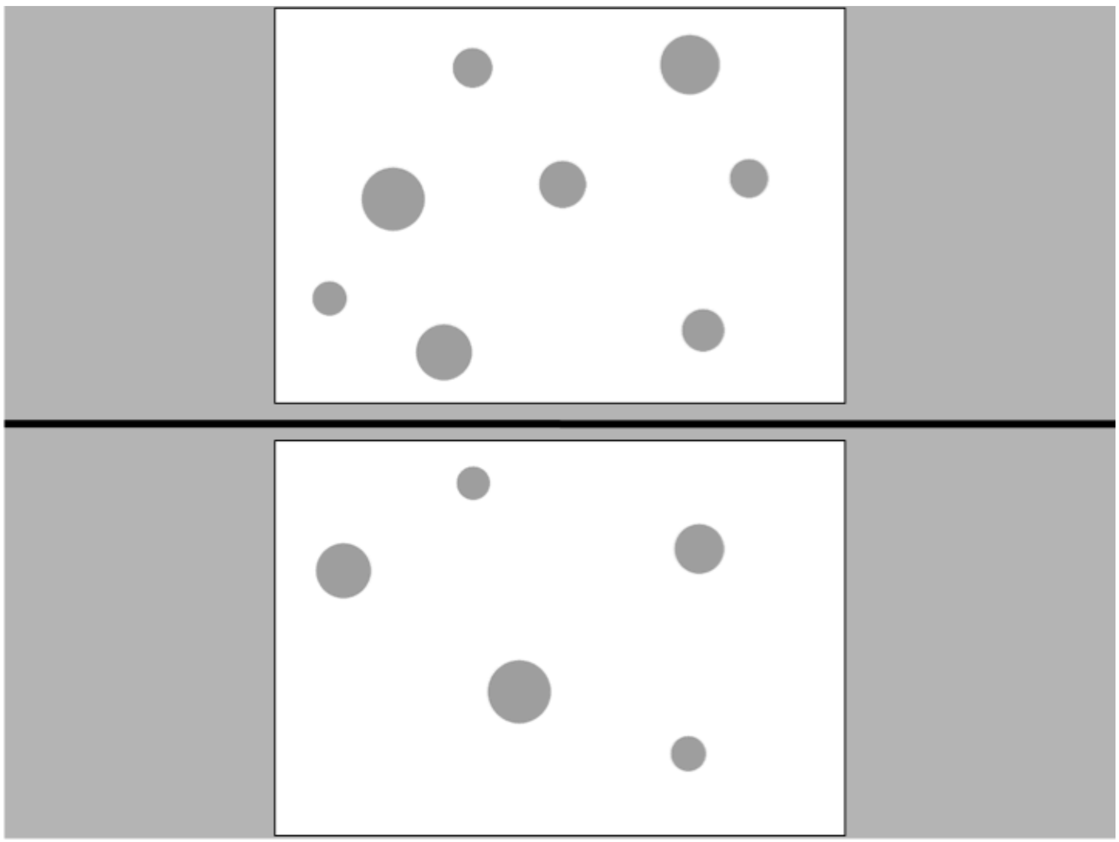

Figure 1. Sample trial presented to children during the number task. Children chose which box had more bubbles in it.

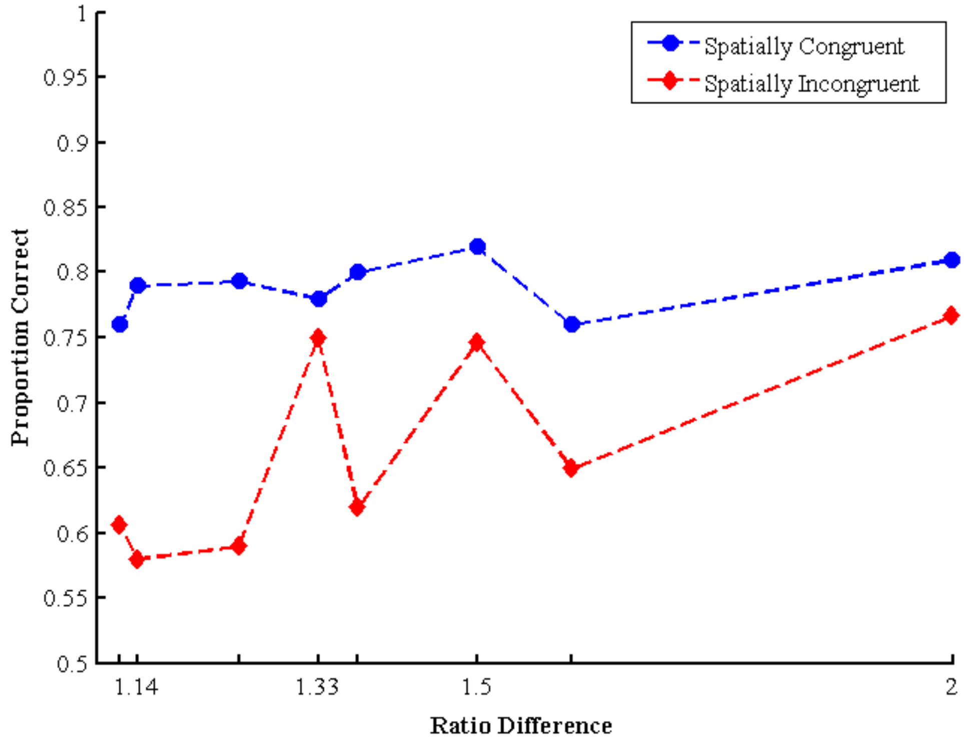

Figure 2. Performance on the number task by spatial congruity condition as measured by proportion correct.

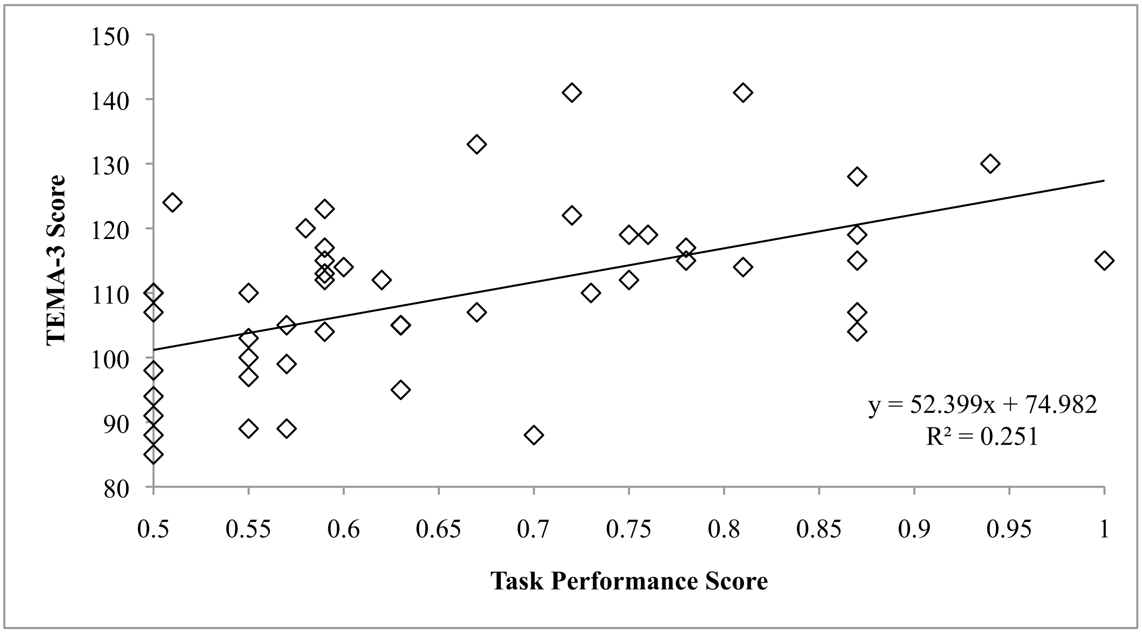

Figure 3. Scatter plot of predicted performance scores on the computerized number task and standardized TEMA-3 scores for all children.

Appendix

TEMA-3 Items used in Cluster Sub-scores:

Number Comparison: 4, 19, 20, 27, 37, 58

Count-List: 3, 6, 12, 21, 23, 24, 31, 33, 38, 40, 42, 68

Cardinal: 1, 2, 7, 9, 10, 11, 13, 18, 22, 28, 36, 39, 48

Symbolic Knowledge: 14, 15, 29, 30, 35, 44, 45, 60, 66

Physical Arithmetic: 8, 16, 17, 25, 49, 55, 59, 62, 63, 69, 71

Mental Arithmetic: 26, 32, 41, 43, 46, 47, 50, 51, 52, 53, 54, 56, 57, 61, 64, 65, 67, 70, 72

(Two items could not be categorized according to the criteria, 5 & 34)

Sub-score Creation Example: If a child's basal ended at item 24 and ceiling ended on item 36, items 24 and below would be counted as correct, correct items between 24 and 36 would be counted correct and items 36 and above would be counted as incorrect.

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