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Young Children's Concept of Middle in the Domain of Number
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An abstract of
A thesis submitted to the Faculty of the
James T. Laney School of Graduate Studies of Emory University
in partial fulfillment of the requirements for the degree of
Master of Arts
in Psychology
2012

Abstract

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The concept of middle is critical for a variety of everyday judgments, applicable across a variety of magnitude domains (e.g., physical size and number sequences). It may also serve as a precursor to advanced mathematical concepts such as interpolation. Young children's understanding of the middle concept in the domain of numerosity, however, is poorly understood. The current study examined 3- to 5-year-olds' ability to choose the midpoint value in a set of non-symbolic number arrays. We found that midpoint estimation emerges at the age of 4 years, and that this ability may be supported by the approximate number system (ANS), as accuracy is subject to ratio effects. Results also point to interactions between spatial and numerical concepts of middle. Specifically, all age groups were biased to choose the number arrays located in the middle spatial position, even though location was irrelevant to the task of judging the numerical midpoint. The interaction between spatial and numerical representations provides support for the mental number line model (Dehaene, Dupoux, & Mehler, 1990; Moyer & Landauer, 1967; Restle, 1970) and a general magnitude system (Walsh, 2003).

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Young Children's Concept of Middle in the Domain of Number

Middle is an important concept that is widely utilized across different domains. We can use this concept to talk about spatial locations (e.g., "middle of the street"), physical sizes (e.g., "medium sized sweater"), and numerical intervals (e.g., "midpoint between 6 and 22"). The everyday concept of middle may even be a precursor to more complex mathematical procedures such as interpolation. Given that this concept is so pervasive and perhaps even foundational for abstract math reasoning, we focus here on studying the origins and underlying developmental processes. The aim of the current study was to examine the nature of the middle concept in the domain of numerosity, and to begin to lay out developmental questions the relations between space and number as *middle* can be applied to both domains.

Representing the midpoint of numerical sequences

In the literature on mature mathematical cognition, the psychological midpoint of numerical intervals is often used to examine the relation between representations of number and space, given that, as mentioned above, the concept applies to numerical sequences and physical space (e.g. Beran, Johnson-Pynn, & Ready, 2008; Lourenco & Longo, 2009; Zorzi, Priftis, & Umilta, 2002). It has been suggested that if numbers are represented in the form of a mental number line in the brain, then finding the midpoint of a numerical interval involves the same mechanisms for finding the midpoint of a physical line. Empirical findings support this position. Zorzi et al. (2002), for example, asked patients with left hemispatial neglect to bisect physical lines and numerical intervals (i.e., indicate the perceived midpoint of both types of stimuli). When bisecting physical lines, patients bisected to the right of true center, ignoring the left side of the lines. In the numerical bisection task, participants estimated the midpoint between two numbers without explicit calculation. Patients' performance mirrored that with the physical lines;

they consistently overestimated the midpoint, as if they had ignored the left side of the mental number line. Other research with healthy adult participants has shown slight leftward bias (known as *pseudoneglect*) in both physical line and numerical bisection tasks, and, critically, has even reported that bias is correlated across the two tasks (Longo & Lourenco, 2007).

Moreover, the manipulation of spatial attention can affect the estimation of the numerical midpoint. After adapting to right-shifting prisms, patients with left neglect showed improvement in both physical line and numerical bisection (Rossetti et al., 2004). Similarly, pseudoneglect in healthy participants can be corrected by adaptation to prisms (Loftus, Nicholls, Mattingley, & Bradshaw, 2008) or presenting visual cues that draw attention to the right of the target stimulus (Nicholls & McIlroy, 2010). Studies have also shown that applying TMS to the right posterior parietal cortex can induce neglect-like behavior in healthy participants with bisection of physical lines (Fierro et al., 2000) and numerical sequences (Göbel, Calabria, Farnè, & Rossetti, 2006). Taken together, the findings suggest that midpoint estimation for physical lines and numerical intervals are mediated by common neural mechanisms.

Little is known about the development of the middle concept, or "midpoint", for numerical intervals in typically-developing children. One study by Droit-Volet, Clément, and Fayol (2003) used auditory stimuli to ask how number and duration information were processed. In this study, children (5- and 8-year-olds) and adults matched the presented stimuli to either a "short/few" anchor or a "long/many" anchor. When responding on the basis of number, it was found that the point of subjective equality (PSE) was at or about the arithmetic mean. This finding, however, was challenged by Jordan and Brannon (2006) who estimated the midpoint of 6-year-olds at the geometric mean. In another study, Beran, Johnson-Pynn, and Ready (2008) tested 4- and 5-year-olds with four sets of anchors (1 and 9, 1 and 16, 2 and 18, 3 and 12) and found that the PSE for the last 3 sets, but not the first set, was closer to the geometric mean. They suggested that large and small numbers may be processed by different format of spatial and numerical mapping (i.e. linear and logarithmic representation), allowing for different PSE values for different number pairs.

Although clearly useful for addressing questions concerned with precise spatial organization, studies that use psychophysical techniques to estimate the PSE are not designed for testing an explicit understanding of the *middle* concept of numerical sequences. Participants merely need to compare the target stimuli with the two anchors, and then choose the anchor that looks more similar to the target. This only requires the ability to order magnitude information. It does not rest on access to midpoint representation, nor does it rest on an explicit understanding of the middle concept, as it does for adults asked to bisect numerical intervals. Given that previous studies have already shown that infants are capable of representing ordinal sequences (Suanda, Tompson, & Brannon, 2008), it may not be surprising that the PSE is found in children as young as 4 years of age. Perhaps more importantly, it also does not provide direct evidence that 4-year-olds have an explicit understanding of the numerical *middle* concept.

Researchers have repeatedly pointed to a distinction between explicit and implicit knowledge (e.g. Alibali & Goldin-Meadow, 1993; Clements & Perner, 1994; Reber, 1967) . The psychological processes underlying these two kinds of knowledge are not the same, and the change in the level of explicitness is an important aspect of conceptual development (Crowley, Shrager, & Siegler, 1997; Karmiloff-Smith, 1992). This implicit-explicit distinction can also be found in the numerical domain. Priftis, Zorzi, Meneghello, Marenzi, and Umiltà (2006) argued that the numerical bisection task used with adults requires explicit knowledge, such that midpoint estimation demands deliberate manipulation of magnitude information. They found neglect patients showed deficits in the numerical bisection task (an explicit task), but patients performed comparably to healthy participants if the task tapped implicit processing of the mental number line. Following this approach, in the current study, we use a novel paradigm designed for use with children to ask about their explicit understanding of the numerical midpoint, bringing developmental research more in line with that of adults on this important mathematical concept.

Grounding the numerical *middle* concept onto the spatial *middle* concept

Researchers have argued that many abstract concepts in mathematics are made possible by their grounding in concrete spatial concepts (Barsalou, 2008; Fischer & Brugger, 2011; Lakoff & Núñez, 2000). For example, Lakoff and Núñez (2000) suggested that knowledge of arithmetic builds on the metaphor of motion along a physical line. Consider the case of addition. Adding number B to number A corresponds to moving from location A, in the direction away from the origin (which corresponds to zero), with the displacement equaling the distance between the origin and point B. The understanding of the *middle* concept in the spatial domain may similarly bootstrap the development of the *middle* concept in the numerical domain.

In the spatial domain, *middle* can be used for describing location. Although infants as young as 6 months have been shown to distinguish shapes that are in between or outside two reference bars (Quinn, Norris, Pasko, Schmader, & Mash, 1999), this sensitivity is highly imprecise, because it does not take metric distance into account. In contrast, the concept of an *exact midpoint* is, by definition, precise. An exact midpoint involves equal distances from all the endpoints. Research with children suggests that the concept of an *exact midpoint* emerges much later than the concept *between*, for which any value between two extremes is appropriate. Studies showed the ability to locate an object at the exact midpoint of two landmarks emerges at around 4 years of age (Simms & Gentner, 2008; Uttal, Sandstrom, & Newcombe, 2006). If search involves more than two landmarks (such that the search space is a plane instead of a single linear segment), however, even 9-year-olds show difficultly locating the exact center of a plane (MacDonald, Spetch, Kelly, & Cheng, 2004). The concept *middle* also applies to the physical size of an object or the volume of a container. Understanding middle (or in this case "medium") size seems to emerge later than the middle position. Sera and Smith (1987) examined the use of "big" and "little", asking 2- to 4-year-olds to label three objects that differed in size. Accuracy for the label "medium" was worse than that for the other two, with even 4-year-olds having difficulty labeling "medium" objects.

Given there are different kinds of spatial information (e.g. *middle* as middle location and as *medium size*), Lourenco and Longo (2011, p. 239) suggested there are two possible ways of conceptualizing the relation between spatial and numerical processing. The first one is to see both of them as magnitude (e.g. physical size and numerical magnitude), with both sharing "more versus less" structure. The second way is to see space as location. Numerical values are systematically arranged from left-to-right on a mental number line. Space can thus be a tool for organizing magnitude information.

Mental number line model (MNL): space is a tool for organizing number.

The view of conceptualizing space as a tool for organizing numerical information is one of the assumptions of the mental number line model (MNL) (Dehaene, Dupoux, & Mehler, 1990; Moyer & Landauer, 1967; Restle, 1970). MNL suggests that numbers are ordered according to their magnitude and the direction of number arrangement is aligned with the spatial orientation of the line. For Westerners, small numbers are represented on the left and large numbers are represented on the right side (Hubbard, Piazza, Pinel, & Dehaene, 2005). This mapping has been demonstrated by the well-known SNARC effect (Dehaene, Bossini, & Giraux, 1993). If spatial location is really used for organizing numbers, numerical midpoint estimation may involve applying the locational sense of middle onto the spatial frame of the mental number line (Priftis, et al., 2006). The current study allows for examining this kind of spatial-numerical interaction.

The MNL also suggests physical distance between two points on the number line reflects the psychological distance between two numerical values. This suggestion is supported by the distance effect (Dehaene, et al., 1990; Moyer & Landauer, 1967). When participants are required to compare the numerical values of two numbers, their responses are faster and more accurate if the numerical difference is greater. Related to the distance effect is the ratio effect. When asked to identify the larger number in a pair, participants' reaction times are better predicted by the ratio than the absolute difference of the number pairs **(**Moyer & Landauer, 1967). The ratio effect is a signature of the approximate number system (ANS; Feigenson, Dehaene, & Spelke, 2004). The ANS can automatically extract number information from different modalities (Izard, Sann, Spelke, & Streri, 2009; Lipton & Spelke, 2004)**,** so it may be involved in the process for numerical midpoint estimation. On the one hand, given the literature on ratio effects, it is reasonable to expect that ratio might affect the ability to identify the midpoint in a numerical interval. On the other hand, the explicit understanding of a concept seems to be dichotomous (i.e., there is either understanding or there isn't). If this is the case, ratio should have no effect on performance. The current study examines the nature of children's explicit understanding of numerical midpoint by varying the ratio among the stimuli.

A Theory of General magnitude system (ATOM): *middle* of different domains are related.

Another way to conceptualize the spatial-numerical relation is to see both space and number as magnitude. This view is implied by both the MNL and the theory of a general magnitude system (Walsh, 2003). According to this view, the spatial sense of *medium size* is related to the numerical midpoint concept because they both share the same representational structure. In his seminal paper,Walsh (2003) proposed that the dimensions of space, time and number are all 'prothetic', meaning that they all share common *more* versus *less* relations. Thus, seemingly distinct dimensions could be processed by a partially overlapping system using a common metric. One example is the *size congruity effect*. To find the larger number in a number pair, the reaction time is shorter if the number with the larger font size also has the larger numerical value. For example, the reaction time for number pair 1 9 would be shorter because of the congruity between font size and numerical value. Such effects are also elicited by non-symbolic dot arrays (Hurewitz, Gelman, & Schnitzer, 2006; Nys & Content, 2011). In the view of the close relation between number and physical size, the current study looks for possible relations between the understanding of *medium size* and midpoint of a numerical interval.

Overview of the current study

The primary objective of this study was to examine how an explicit understanding of the *middle* concept for abstract numerical sequences develops in 3- to 5-year-olds. Young children were presented with three numerical values and were asked to pick the middle value. The ratio of the triplets was varied systemically so as to test for potential distance effects. While the study was to get at an explicit understanding of the numerical midpoint, we also examined whether the spatial sense of *middle* plays a role in the development of the numerical *middle* concept.

Experiment 1: Age related development of *middle* in the domain of numerosity

This first experiment was designed to examine young children's understanding of the numerical midpoint. To assess how this concept comes to be represented, it is essential to construct a task that is sensitive enough to detect the earliest competency in young children. To this end, we first gave participants brief training on a Physical Size Task, in which they learned the structure of this task and next task. We then followed up with a Number Task, in which element size of the items constant within the triplets of the Number Task. The latter manipulation helps to avoid interference of conflicting information (e.g., from cumulative area).

Method

Participants. Sixty preschoolers participated in this study. There were 20 participants in each age group of 3-year-olds (12 were male, M_{age} = 42.6 months, age range: 36-47 months), 4-year-olds (11 were male, M_{age} = 54.15 months, age range: 48-59 months), and 5-year-olds (10 were male, M_{age} = 64.55 months, age range: 60-70 months). Participants came from diverse ethnic and socioeconomic backgrounds. At the end of the study, each participant received a toy as a gift. Data from one participant were excluded because of refusal to follow instructions.

Tasks and Materials.

Physical Size Task. Six sets of pictures depicting child-friendly and familiar objects were used as stimuli in this task. There were three pictures in each set, all of which depicted the same object but of different sizes. The first three trials used pictures that were printed on paper; the last three trials used pictures that were shown on a computer screen. Printed pictures were used because they looked more similar to real life objects. Computer presentation was used in the latter trials to help children transition to the Number Task, which was also computer administered.

Number Task. In this task, the stimulus set contained 21 sets of number triplets. The first three triplets were used as practice trials and the remaining triplets were used in the test phase. Test triplets (18 trials) were divided into three ratio groups: 2:1, 3:1 or 4:1 (6 triplets in each group). In the 2:1 group (e.g. 2, 4 and 8), the ratio between the middle number and the smallest number (i.e. 4 and 2) was two; the ratio between the largest and the middle number (i.e. 8 and 4) was also two. The same rule applied to the other ratio groups.

All numbers were represented in a non-symbolic format. In the practice trials, numbers were represented as arrays of oranges (1^{st} trial), lemons (2^{nd} trial), or squares (3^{rd} trial). In the test phase, all numbers were represented as arrays of squares, which allowed for controlling object familiarity. Within each trial, three separate arrays of squares (with the same color and element size) were used to represent the number triplet. The actual numbers and the attributes of the pictures are presented in Table 1. The color and size of individual squares varied from trial to trial. Numbers were arranged horizontally on the screen, with position (left, middle, right) counterbalanced across trials (see Figure 1). All pictures were created with Adobe Photoshop. E-prime was used for computer presentation.

Give-a-number Task. This task assesses children's ability to execute the counting routine, as well as their understanding of the cardinality of a set by asking them to put a certain number of objects in? a designated place (Wynn, 1990). This task was included to determine whether the understanding of the numerical midpoint is dependent on the development of cardinality, specifically knowledge of the count list and children's ability to represent exact number values. In this task, a set of plastic coins, a plastic box and a toy monkey were used as stimuli.

Assessment of expressive vocabulary. The size of participants' expressive vocabulary was assessed by the Developmental Vocabulary Assessment for Parents (DVAP; Libertus, Stevenson, Odic, Feigenson, & Halberda, 2011) . DVAP is a questionnaire that contains 212 words originated from the Peabody Picture Vocabulary Test (PPVT-4; Dunn & Dunn, 2007). Parents were asked to select all the words that they have heard their children use. Recent research showed that the DVAP significantly correlates with the PPVT (Libertus et al., 2011). It was thus used here as a proxy for non-numerical intelligence.

Procedure. All participants were tested individually. The order of the tasks was fixed: participants first completed the Physical Size Task, which was followed by the Number Task and then the Give-a-number task. Parents filled out the DVAP while children completed the other tasks. On the Physical Size Task, participants were shown three pictures of different size. They were required to look for a star-shaped sticker, which they were told was always behind the medium-sized object. The phrase "middle size picture" was used in the instructions because previous research suggests that children in this age range have difficulty with the word "medium"(Sera & Smith, 1987). As discussed in the results, children clearly understood these instructions, performing significantly above chance. Participants were rewarded with a small prize (e.g., silly band or sticker) whenever they found the star. In the first three trials, the stimulus pictures were printed on paper and the star could be revealed by turning over the picture. After the participants made their choice, the experimenter showed the small, medium, and large size pictures in the set. The last three trials were administered on a laptop computer. Before switching to the computer version of the task, the experimenter explained the computer game was the same as the game that they had just played. Children responded by pointing at the picture shown on the screen. The experimenter would use a mouse to click on the corresponding picture. If the choice was correct, a yellow star appeared on screen; otherwise, a white rectangle appeared, indicating that middle had not been selected. After providing children with feedback, the computer showed the set of stimulus pictures that they had just seen and the experimenter pointed to the smallest, largest and the middle size picture in the set.

At the beginning of the task, participants were told that the pictures in this task were different in their "number of things", and, as in Physical Size Task, that the star was always behind the picture containing the "middle number of things". The first three trials were practice trials. In the first two, the experimenter indicated which picture contained the smallest, middle and largest number of things before the participants made their choice. To ensure the participants paid attention to the relevant dimension, the positions of the pictures were swapped on these trials before the experimenter asked the participant to find the star. The feedback in the practice trials was similar to the Physical Size Task except the explanation was about number rather than physical size. The test phase followed the practice trials, which was identical to practice except that no explanation of the answer was provided in the test phase. Each test trial began with a reminder about the location of the star: "It is always behind the picture with the middle number of things". After the participants made their choice, they saw either a star or a white rectangle (depending on whether the choice was correct or not), and received a small prize if the choice was correct.

In the Give-a-number task, participants were asked to help a toy monkey count his coins. The experimenter would say "please put X coins into the box". Once the participant finished counting, the experimenter checked the number of coins in the box and decided whether the next number should be increased or decreased base on the response. The lowest target number was two and the highest was sixteen.

Results and Discussion

Physical Size Task. All age groups performed significantly above chance (i.e. 2 out of 6 trials) in this task: 3-years-olds (M = 3.3, SD = 1.84), $t(19) = 3.163$, $p = .005$, $d = 0.7$; 4-year-olds (M = 4.45, SD =1.432), $t(19)=7.653$, $p<.001$, d =1.71; 5-year-olds (M = 5.3, SD = 1.031), $t(19)$ =

14.313, $p < .001$, $d = 3.2$. These results show that even 3-year-olds responded reliably, suggesting an explicit understanding of the concept middle with respect to physical size. It also suggests that they can follow instructions, so that poor performance in the Number Task cannot be due to a problem understanding these instructions.

Overall performance on Number Task. Accuracy scores (see Figure 2) were analyzed using a 2 × 3 × 3 mixed-design ANOVA, with ratio (2:1, 3:1, 4:1) as the within-subjects factor, age group (3-, 4- and 5-year-olds) and gender (male and female) as between-subjects factors. This analysis revealed significant main effects of ratio (sphericity violated, Greenhouse-Geisser correction), $F(1.792, 96.756) = 20.381$, $p < .01$, $\eta_p^2 = .274$, and age, $F(2, 54) = 20.08$, $p < .01$, $\eta_p^2 = .427$. There was also a significant interaction between ratio and age, $F(3.584, 96.756) = 4.398$, $p = .004$, $\eta_p^2 = .140$. There was no main effect of gender or significant interactions with this factor.

Follow-up analyses (Bonferroni corrected, $\alpha = .05/3 = .016$) revealed that the strength of the ratio effect differed among age groups. In 3-year-olds, performance was comparable across the three ratios. Ratio effects were most pronounced in the 4-year-olds: specifically, performance differed significantly across each of the ratios, with greatest accuracy on trials that differed by 4:1, followed by 3:1, and then 2:1 (all *p*s < .016). The performance of 5-year-olds with the easiest ratio was significantly better than the hardest ratio ($p < .016$), but neither the performance difference between ratio 2:1 and 3:1 or ratio 3:1 and 4:1 reached significance. Because performance was high in this age group, the lack of significant differences across all three ratios could reflect a ceiling effect.

To examine the developmental trajectory of the explicit understanding of the numerical middle concept, additional analyses were conducted to compare performance to chance. Overall performance for 3-year-olds (*M* = 37.5%, *SD* = 24.5%) did not differ significantly from chance (33.3%), $t(19) = 0.760$, $p = .456$, $d = 0.17$. In contrast, both 4- ($t[19] = 6.34$, $p < .001$, $d = 1.42$, *M* = 67.2%, *SD* = 23.9%) and 5- year-olds ($t[19] = 11.282$, $p < .001$, $d = 2.52$, *M* = 85%, *SD* = 20.5%) performed significantly above chance. The analysis of performance was further broken down by ratio (Bonferroni corrected, $\alpha = .016$, see Table 2). For 3-year-olds, the accuracy was at chance at all ratios (all *p*s > .016). For 4- and 5-year-olds, performance was significantly above chance at all ratios (all *p*s < .001) except for 2:1 with 4-year-olds, $t(19) = 2.266$, $p = .035$. Taken together, the results suggest that children begin to understand the numerical *middle* concept at the age of four.

Effect of magnitude and interval. The effect of ratio could be driven by the effect of magnitude (i.e., absolute value) or the interval of the triplets. To test whether the relation between absolute numerical value and performance was significant in this task, the arithmetic means of all number triplets were calculated and their effects on accuracy were analyzed. The correlation between the two variables was not significant, $r(16) = -.215$, $p = .391$. As for the analysis of interval effect, interval of each triplet was obtained by subtracting the smallest number from the largest number. This analysis showed that the correlation between interval and accuracy was not significant, $r(16) = .057$, $p = .822$. Neither the effect of interval nor magnitude significantly affected the accuracy of the Number Task. In other words, the ratio effect was not driven by these two factors.

Error analysis on the Number Task. To better understand how participants performed on the Number Task, we analyzed their errors. Incorrect answers involved selecting either the smallest or largest value in the triplet. We calculated the proportion of either choice. The error of 20 participants in the youngest age group, 16 in the middle and 11 in the oldest age group was analyzed; some participants made no errors so they were not included in this analysis. A mixed design 2 × 3 ANOVA was conducted with error type (smallest number or largest number) as the within-subjects factor, and age group (3-, 4-, 5-year-olds) as between-subjects factors. There was a significant main effect of error type, $F(1, 44) = 36.34$, $p < .001$, $\eta_p^2 = 0.452$. The effect of age was not significant, $F(2, 44) < .001$, $p = 1$, $\eta_p^2 = 0$, nor was the interaction between error type and age, $F(2, 44) = 1.537$, $p = .226$, $\eta_p^2 = 0.065$. In general, children were more likely to choose the smallest value (*M* = 70.6%, *SD* = 24.8%) than the largest value (*M* = 29.4%, *SD* = 24.8%), suggesting that, at all ages, mistakes were more likely to involve confusing the smallest number with the middle number.

Effect of spatial location: Middle position bias? On each trial of the Number Task, there were three pictures of square arrays depicting three different numerical values. The pictures were arranged horizontally, with the position of the smallest, medium and largest values counterbalanced across trials. To determine whether there was a bias for choosing pictures located in the middle position, the number of trials in which the middle picture was picked was compared with the chance level of 33.3%. All age groups chose the middle picture more often than chance (see Figure 3): 3-year-olds (*M* = 43.6%, *SD* = 14.7%), $t(19) = 3.132$, $p = .005$, d = 0.7; 4-year-olds, *M* = 40.6%, *SD* = 9%, $t(19) = 3.577$, $p = .002$, $d = 0.8$. For 5-year-olds, *M* = 36.9%, *SD* = 4.9%, $t(19) = 3.322$, $p = .004$, $d = .74$. Result shows that children were biased to choose pictures located in the middle spatial position, even though position was irrelevant to the task.

Relation between different kinds of mental capacities. The relation between different kinds of mental capacities and the Number Task may provide important information about developmental mechanism. Therefore correlations across performance in all the tasks were analyzed (see Table 3). Age was controlled in these analyses. Neither the correlation between Number Task and Give-a-number task, $r(57) = .18$, $p = .172$, nor DVAP raw score, $r(57) = .111$, $p = .402$, was significant. Interestingly, the relation between DVAP and Give-a-number task was significant, $r(57) = .286$, $p = .028$. The DVAP and Give-a-number task assess abilities that belong to different domains (expressive vocabulary and cardinality). The correlation between these two distinct kinds of abilities is probably driven by a common cause – general intelligence. On the other hand, the Number Task was not correlated with these two abilities, but it was correlated with the Physical Size Task, $r(57) = .482$, $p < .001$, even when controlling for Give-a-number task and DVAP, $r(55) = 463$, $p < .001$, suggesting shared understanding of middle for different dimensions of magnitude (i.e., spatial extent and number), not explained by general intelligence.

Experiment 2: Ruling out the influence of continuous properties

Experiment 1 showed that 4-year-olds were capable of identifying the middle value in number triplets. One possible limitation of the previous task, however, is that participants' decisions may have been based on continuous properties (i.e., cumulative area) rather than number alone. In Experiment 1, the element size of the three pictures was held constant within the triplets, which means that cumulative area covaried with numerosity. Although children have been shown to display less sensitivity to differences in cumulative are (Brannon, Abbott, & Lutz, 2004; Cordes & Brannon, 2008), we more definitively address this issue in Experiment 2 by varying cumulative area so that it could no longer be a useful cue for midpoint estimation.

Method

Participants. Twenty (9 boys) 4-year-olds (M_{age} = 54.5 months, SD = 2.89, age range = 48-59 months) participated in this study. One of them was recruited from a preschool in Atlanta.

The remaining participants were recruited via a database in Atlanta, GA. Participants came from diverse ethnic and socioeconomic backgrounds. All children received a toy as a gift for participating.

Tasks and procedure. Tasks and procedure were the same as Experiment 1. The only difference was that the element sizes of pictures in the Number task was not held constant within a triplet, so that the cumulative area varied across numbers (see Figure 4). In some of the trials, two out of three stimuli pictures had practically the same cumulative area (it is impossible to control all three pictures because the ratio between the smallest and largest number was too large), so that it is impossible to find the correct answer just by processing cumulative area. In the rest of the trials, each pictures in the triplet had different element size, rendering information about element size useless for numerical midpoint estimation. The cumulative area and elements size were varied across the trials.

Results and discussion. The primary goal of Experiment 2 was to determine whether the 4-year-olds were truly capable of finding the middle numerical value, so we first focus on their performance on the Number Task. Their overall accuracy on the Number Task was compared to chance (33.3%). Analysis showed that the 4-year-olds (M = 56.4%, SD = 25.4%) performed significantly above chance $t(19) = 4.062$, $p = .001$, $d = 0.91$. Since the continuous properties were controlled in this study, the results suggest that 4-year-olds’ understanding about *middle* is about numerical value, not continuous properties like cumulative area or element size.

To evaluate whether controlling the continuous properties of the pictures had an aversive impact on performance, we compared overall accuracy of the 4-year-olds across the two experiments. The difference was not significant between the two studies, $t(38) = 1.39$, $p = .173$, d

= 0.44, providing further evidence that understanding of numerical midpoint is not driven exclusively by cumulative area.

To determine whether performance was affected by ratio, we conducted a one-way repeated measure ANOVA with ratio (2:1, 3:1, 4:1) as the within-subject factor. The main effect of ratio was significant, $F(2, 38) = 14.393$, $p < .001$, $\eta_p^2 = .431$. Bonferroni adjusted pairwise comparisons showed that the performance of ratio 2:1 and 3:1 was not significantly different from each other. Ratio 4:1 was significantly more accurate than the two other ratios. As compared to Experiment 1, both experiments found a significant ratio effect. Nevertheless, the pattern revealed by the pairwise comparison was slightly different. In Experiment 1, the 4-year-olds showed a pronounced ratio effect, all easier ratios were significantly more accurate than the harder ratios. But the ratio effect was less pronounced in Experiment 2, the performance of the two harder ratios was not significantly different from each other.

The errors made by 19 participants (the remaining one had perfect score) were analyzed by ANOVA with error type (smallest value, largest value) as the within-subject factor. Like Experiment 1, participants made significantly more errors in choosing the smallest value than the largest value, $F(1,18) = 13.713$, $p = .002$, $\eta_p^2 = 0.432$. Moreover, participants also showed a middle position bias in the current experiment. Their frequency of choosing the picture in the middle location (M = 38.3%, SD = 7.6%) was significantly higher than chance, $t(19) = 2.932$, $p = .009$, $d = 0.66$.

We also checked whether there were changes in other tasks. For the Physical Size Task (M = 90%, SD = 15%), participants again performed significantly above chance, $t(19) = 16.17$, $p < .001$, $d = 3.62$. But unlike Experiment 1, the correlation between Number Tasks and the Physical Task failed to reach significance, $r(18) = .377$, $p = .101$. This could be a result of ceiling effect. Among the 20 participants of the current experiment, 18 of them got at least 5 correct out of the 6 questions. The lack of variability made it difficult to achieve a significant correlation. For the Give-a-number task, like Experiment 1, its correlation with Number task was not significant, $r(18) = .284$, $p = .225$. As for the DVAP, its correlation with the Number Task was significant, $r(18) = .451$, $p = .046$. This is different from the result of Experiment 1. This difference between the two experiments may imply the ability to inhibit irrelevant continuous information could be part of the general intelligence.

General Discussion

The findings from the current study suggest an important milestone in the development of children's understanding of the numerical midpoint. Experiment 1 showed that beginning at age 4, children display explicit awareness of this concept across a range of numerical intervals. That is, they are able to identify the middle value in different number triplets. Moreover, their accuracy of midpoint estimation is affected by the ratio between numbers in the triplets, a signature of the ANS and magnitude representations more generally (Cohen Kadosh, Lammertyn, & Izard, 2008; Fulbright, Manson, Skudlarski, Lacadie, & Gore, 2003). Experiment 2 further showed that 4-year-olds can base their choices on numerosity per se, not just cumulative area (which covaried with number in Experiment 1) or individual element size (not relevant in Experiment 1). Another important finding was that children exhibited a middle position bias. That is, there was a clear and strong tendency to choose the numerical value that occupied the middle location, even though location was irrelevant to the task. Also, error analyses revealed that participants tended to confuse *small* with *middle* when they made mistakes, implying an asymmetry between the positive (*large*) and negative poles (*small*) of numerical continua.

Age-related Development

Previous studies concerned with the numerical midpoint in children have focused on the PSE (Beran, et al., 2008; Droit-Volet, et al., 2003; Jordan & Brannon, 2006), not any explicit sense of midpoint values across a range of intervals. Moreover, previous studies concerned with the developmental origins of midpoint representations have not tested children younger than 4 years. The current study showed that whereas 3-year-olds performed at chance when choosing the midpoints of number sequences, 4- and 5-year-olds' were reliably above chance, suggesting that an explicit understanding, at least when asked verbally, may only emerge at 4 years of age.

Three-year-olds performed above chance when asked to identify medium-sized objects (Physical Size task). So what might be responsible for their difficulty in estimating numerical midpoints (Number task)? One possibility is that this difference was simply a methodological artifact. Because the Physical Size task served to train children with the concept of middle, they were given additional feedback (i.e., explanations about which is the smallest, medium and largest object in set following each trial), which was not the case in the test trials of the Number task. Another possibility concerns their level of experience with the concept of middle in different domains. Parents may be more likely to teach children the concept for middle as it applies to single objects than groups of objects (e.g., toys that require fitting different sized objects in holes). Though previous studies have shown 4-year-olds experience difficulty in using the word *medium*, this may be caused by the low word frequency of the word *medium* (Sera & Smith, 1987). The problem was remedied by using a word with higher frequency (*middle*) in the instructions of the current experiment.

Yet another possible reason for 3-year-olds' greater difficulty with using the *middle* concept in the numerical, compared to the spatial domain, may have to do with differences in precision between different magnitude representations. In the Physical Size Task, each picture depicted a single object, such that size information could be extracted by processing one object at a time. In the Number Task, each picture consisted of an array of squares, and each trial involved comparisons for multiple complex pictures. The more complex stimuli in the Number task may have made it more difficult and less precise as compared to evaluating the size of one object, consistent with recent evidence showing that preschoolers may have more precise representations of individual element size than number (Bonny & Lourenco, 2011). As a result, the development of the *middle* concept may follow different trajectories in these two domains.

Young children's difficulty with understanding the numerical midpoint was not likely due to their inability to understand the task instructions. The Physical Size Task that preceded the Number Task had the same exact structure and yet 3-year-olds performed above chance there (though, as mentioned above, they were given more feedback). The extra support provided in the Physical Size Task was verbal explanation. If the problem with the 3-year-olds was about failure in understanding verbal instructions, they would not have benefited from the additional explanation provided in the Physical Size Task. And their success in the Physical Size Task showed that they at least understood the spatial sense of the word *middle*. We would thus suggest that 3-year-olds' difficulty might be due to greater imprecision of ANS compared to that of older preschoolers. Unlike 4- and 5-year-olds, the performance of 3-year-olds was not better with larger ratios, missing the signature of the ANS (Feigenson, Dehaene, & Spelke, 2004) and magnitude representations more generally (Cohen Kadosh, et al., 2008; Fulbright, et al., 2003). This does not imply that the ANS is non-functional at 3 years of age. Indeed, a plethora of existing studies have shown that the ANS is already operational in infancy (Izard, et al., 2009; Lipton & Spelke, 2003; Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005) and may even be innate (Izard, et al., 2009). In infancy, previous studies have shown that the ANS is capable of comparing two numbers at a time, as well as carrying out addition and subtraction on approximate numerical values (McCrink & Wynn, 2004; Xu & Spelke, 2000). Estimating the midpoint, however, requires additional processes. Unlike approximate arithmetic operations, finding the midpoint is not about simply increasing or decreasing a numerical value. And unlike the comparison between two numbers, the concept of middle requires the ANS to process three numbers simultaneously, which might increase the internal noise and affect the precision of the numerical representations. These added complexities may delay the emergence of midpoint estimation in the number domain.

Older children, in contrast, were able to identify the numerical midpoints of triplets regardless of absolute values or interval, and with accuracy increasing for larger ratios. Experiment 2 showed that 4-year-olds were capable of finding the middle value even when the cumulative area and elements size did not co-vary with numerosity, demonstrating that though the conceptual development of *middle* in these two domains is correlated, the success in the Number Task was not merely an artifact of conflated dimensions. Moreover, the performance in Experiment 1 and Experiment 2 did not differ significantly from each other, suggesting that children have the ability to represent midpoints for intervals of spatial extent (Physical Size Task) and number (Number Task). As for the effect of general intelligence, Lemer, Dehaene, Spelke, and Cohen (2003) showed that the processing of approximate number is largely independent from language and exact number knowledge. Our findings are generally consistent with this. The significant correlation between DVAP and Number Task in Experiment 2 may imply the ability to inhibit irrelevant continuous information is related to general intelligence.

Error analysis revealed that when participants made mistakes, they tended to confuse the smallest number with the middle number. An older literature showed that for gradable adjectives like *large-small*, *big-little, long-short*, the words for the negative pole (e.g. *small*, *little*, *short*) are more difficult than the words for the positive pole (e.g. *large*, *big*, *long*) (Clark, 1972; Klatzky, Clark, & Macken, 1973; Siegel, 1977). In describing number, Siegel (1977) used the words *big* and *little* to describe large and small numerical values. Confronted with two pictures which depict different number of dots, children were asked to point at the *big* or *small* one. Three and 4-year-olds found the word *big* easier to comprehend than the word *little* (Siegel, 1977). Klatzky et al. (1973) suggested that this asymmetry in representing positive and negative poles is due to conceptual structure, not merely word frequency. Moreover, McGonigle and Jones (1978) found that squirrel monkeys also confused *middle* with *small* when the task was to choose the medium size object in a set of three objects. Thus, this confusion could not be a result of language influence. We suggest our findings reflect the cognitive asymmetry between the positive and negative pole of dimension such that children may understand the *large* concept better than the *small* concept.

Relation between *middle* in the domain of space and numerosity

Current study provided evidence for the connection between numerical middle concept and two kinds of spatial *middle* concept (as location and as physical size). The middle position bias in estimating numerical midpoint demonstrates the link between the locational sense of *middle* and the numerical sense of *middle*. In the current study, the position of the pictures was completely irrelevant to the correct answer. Yet, at all ages, children were biased to choose the picture located in the middle. If location, as suggested by the mental number line model, is used for organizing numerical magnitude, then the process of finding the numerical midpoint inevitably involves the spatial middle concept, which makes it difficult for children to dissociate the two distinct meanings of middle. Indeed, healthy adults show pseudoneglect (leftward bias) in physical line and number bisection tasks, with the degree of leftward bias strongly correlated across the two tasks (Longo & Lourenco, 2010).

The *middle* concept of physical size was also found to be related to the midpoint of numerical intervals. Performance on the Physical Size Task, in which participants selected the middle sized object, significantly correlated with performance the Number Task in Experiment 1.Though this correlation was not significant in Experiment 2, this is likely to be a result of ceiling effect in the Physical Size Task. If the finding in the Experiment 1 is true, that would suggest the interaction between spatial extent and number is not restricted to "leakage" of activation, which could be an explanation for congruity effect as leakage can only happen when both kinds of representation are activated by external stimuli. Rather, the correlation would suggest there should be a long term, systematic link between the two kinds of representations. Theories of general magnitude representation would predict such overlap (Walsh, 2003). Future research interested in ATOM would thus do well to examine these connections systematically.

Limitation and future directions

As mentioned in the Introduction, the spatial sense of *middle* has two slightly different meanings. The first one is equivalent to *between*. Anything falling within the boundary can be considered as *in the middle of* or *in between*. The second meaning is about an exact midpoint. This concept requires metric information because an exact midpoint should be equal distance from all endpoints. These two meanings of *middle* apply similarly to numerical intervals. The current study used the geometric mean as the middle number in the triplets because previous studies have suggested that this is psychologically more probable than the arithmetic mean (Beran, et al., 2008; Jordan & Brannon, 2006). It is impossible to tell whether 4- and 5-year-olds master both kinds of *middle* concept or just one of the possible meanings (i.e., exact midpoint or between). Future studies should examine the performance difference if other intermediate numbers were used in the number triplets, and to map out the developmental sequence of the two kinds of numerical *middle* concepts.

To conclude, our findings establish the developmental milestone for the explicit understanding of *middle* in the numerical domain. We showed that the performance in identifying the midpoint value is subject to ratio effects, suggesting numerical magnitude is processed by approximate representations. The current study also illustrated how the development of the numerical sense of *middle* is related to the development of the *middle* concept in the spatial domain, suggesting broad connections of magnitude representations exist in the human mind, and their interaction could be a potential source of conceptual change.

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Table 1

Stimuli numbers used in Number Task

Question number	Smallest number	Middle number	Largest number	Ratio
1	1 (625)	4 (2500)	16 (10000)	4
2	2 (1352)	4 (2704)	8 (5408)	2
3	2 (800)	8 (3200)	32 (12800)	4
4	3 (1452)	9 (4356)	27 (13068)	3
5	3 (432)	12 (1728)	48 (6912)	4
6	4 (576)	16 (2304)	64 (9216)	4
7	5 (500)	20 (2000)	80 (8000)	4
8	6 (1944)	12 (3888)	24 (7776)	2
9	6 (384)	24 (1536)	96 (6144)	4
10	7 (700)	21 (2100)	63 (6300)	3
11	8 (1152)	24 (3456)	72 (10368)	3
12	9 (1764)	18 (3528)	36 (7056)	2
13	9 (324)	27 (972)	81 (2916)	3
14	10 (640)	30 (1920)	90 (5760)	3
15	11 (704)	33 (2112)	99 (6336)	3
16	12 (2700)	24 (5400)	48 (10800)	2
17	13 (1872)	26 (3744)	52 (7488)	2
18	18 (1152)	36 (2304)	72 (4608)	2
Mean	7.17	19.33	56	

Note. Cumulative area (in pixels) of the stimuli picture are indicated in parenthesis

Table 2

Comparisons between the performance on the Number Task to chance. Performance is broken down by age group and ratio.

Number of correct	*M*	*SD*	*t*(19)	*p*	95% CI LL	95% CI UL	Cohen's *d*
3 years old							
Ratio 2	2.15	1.31	0.513	.624	-0.46	-0.76	0.12
Ratio 3	2.15	1.81	0.37	.716	-0.7	-1	0.08
Ratio 4	2.45	1.85	1.088	.29	-0.42	1.32	0.24
4 years old							
Ratio 2	3	1.97	2.266	.035	0.08	1.92	0.51
Ratio 3	4.1	1.74	5.385	<.001*	1.28	2.92	1.2
Ratio 4	5	1.12	11.937	<.001*	2.47	3.53	2.67
5 years old							
Ratio 2	4.5	1.7	6.571	<.001*	1.7	3.3	1.47
Ratio 3	5.1	1.49	9.347	<.001*	2.41	3.79	2.09
Ratio 4	5.7	0.92	17.92	<.001*	3.27	4.13	4.01

Note. CI = confidence interval; *LL* = lower limit; *UL* = upper limit. Bonferroni adjusted p=.016.

*p<.016

Table 3

Summary of correlations, means, and standard deviation for scores of Number Task, Physical Size Task, Give-a-number task and DVAP

Tasks	1	2	3	4	*M*	*SD*
1. Number Task	-	.482*	.180	.111	11.38	5.41
2. Physical Size Task	-	-	.206	.067	4.35	1.67
3. Give-a-number Task	-	-	-	.286*	9.7	5.33
4. DVAP	-	-	-	-	106.82	30.4

Note. Age is controlled for all these analyses. *p<.05

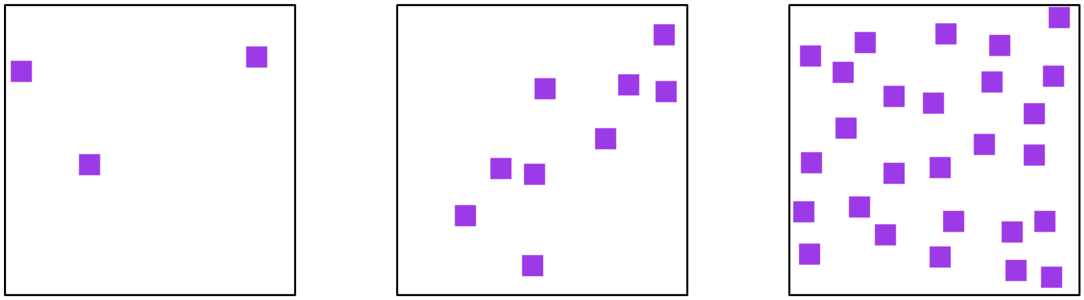

Figure 1. Example trial of the Number Task in Experiment 1

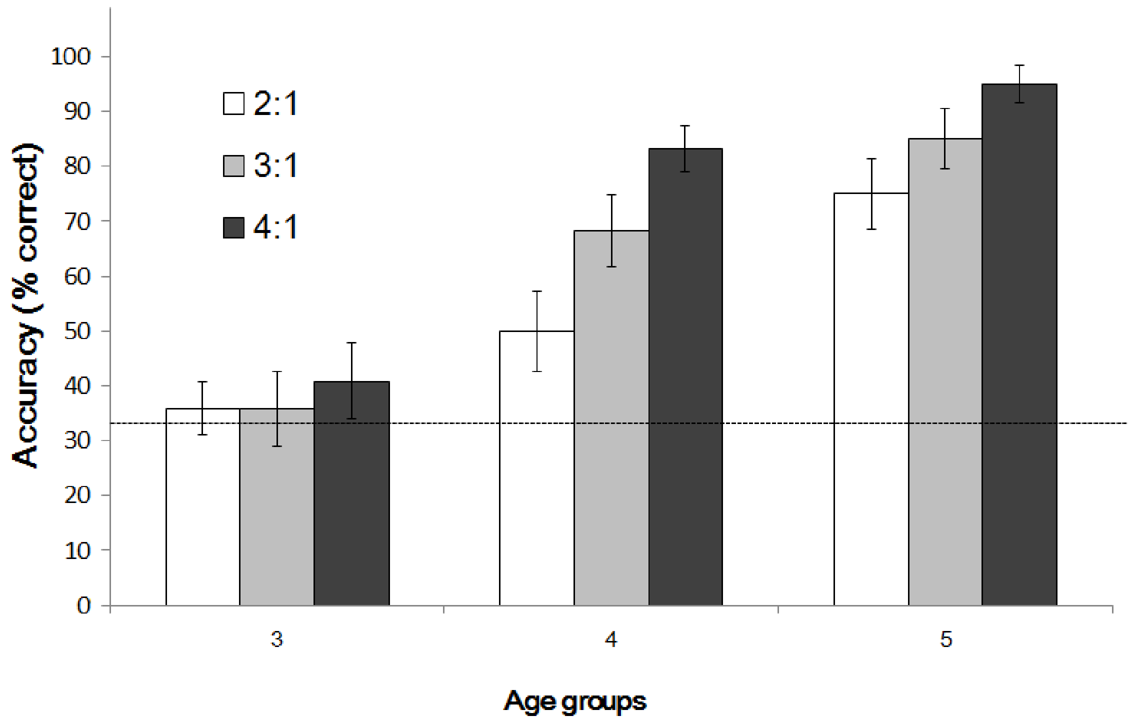

Figure 2. Mean accuracy in the Number Task as a function at age group as a function of ratio. Horizontal line represents chance level. Error bars represent standard error of the mean.

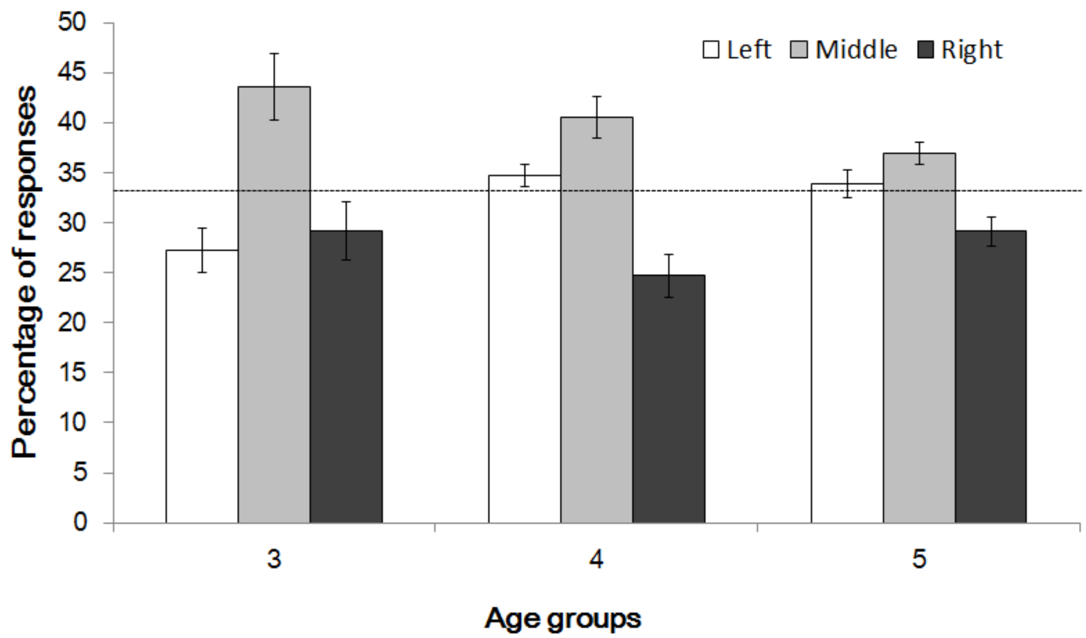

Figure 3. Choices as a function of array position/location for each age group. Horizontal line represents chance level. Error bars represent standard error of the mean.

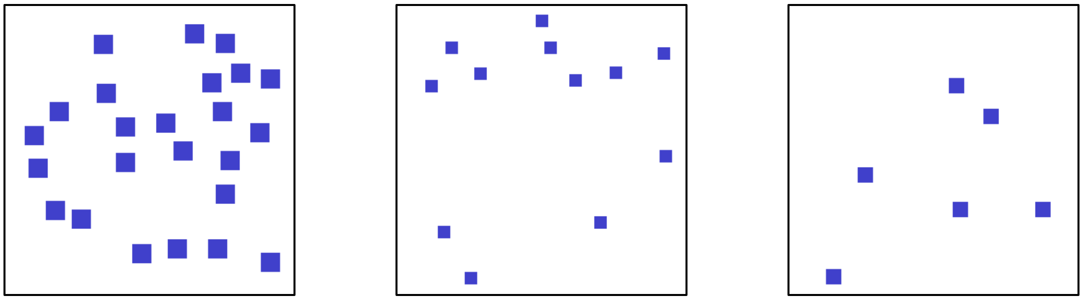

*Figure 4.*Example trial of the Number Task in Experiment 2