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The Great Reversal of Inflation Inequality

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Abstract

The Great Reversal of Inflation Inequality

By LOLA CLEAVELAND

This study finds that there was a reversal of inflation inequality during the Covid-19 pandemic. From 2012 to 2021, inflation varied inversely with income US households. In mid-2021, amidst the recent inflationary period, this trend reversed and the lowest income group experienced the lowest inflation rate of any income quintile. Key drivers of this disruption were spending on leisure and travel goods such as rental vehicles and lodging away from home which drove up inflation for high income households. Using an intertemporal optimization model, this study shows that the reversal of inflation inequality stemmed from the interaction between lockdown constraints on luxury services during the pandemic and non-homothetic preferences for normal consumption as well as luxury services. Lockdown constraints created deferred demand for luxury services in the post-pandemic period, contributing to a reversal of inflation inequality.

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The Great Reversal of Inflation Inequality

LOLA CLEAVELAND

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1 Introduction

1.1 Motivation and Research Question

Inflation has a drastic impact on the health and functioning of economies. At the micro level, inflation impacts households' ability to consume goods and maintain their quality of life. At the macro level, inflation impacts interest rates and the supply of money through monetary policy, trade through exchange rate markets, and stock prices. Given that the movement of inflation rates triggers powerful policy levers that impact financial stability, it is crucial that the measurement of inflation is accurate and representative of the relevant population.

In the US, the most widely utilized measures of inflation are the Consumer Price Index (CPI), maintained by the Bureau of Labor Statistics (BLS), and the Personal Consumer Expenditures Price Index (PCE), maintained by the Federal Reserve. While the two vary in their calculation, both attempt to distill the vast experiences of inflation into an aggregate index. This process inherently dilutes the specificity of the indices, calling into question the

validity of these measures in policy discussions. In the wake of an economic crisis that necessitates fiscal and monetary policy action, inaccuracy in economic statistics can have drastic impacts (Blanchett et al., 2022). While the BLS has begun reporting specific CPIs for various geographic regions and urban-versus-rural designations, a gap in our understanding lies in the CPI's lack of attention to measuring inflation across income levels. Understanding inflation's impact across the income distribution is especially important because the CPI is commonly used to "adjust income eligibility levels for government assistance, federal tax brackets, federally mandated cost-of-living increases, private sector wage and salary increases, poverty measures, and consumer and commercial rent escalations" (BLS, 2023). Furthermore, interest rate targeting to combat inflation impacts outcomes such as employment rates and savings that often have the highest relevancy for low-income households.

It has become increasingly apparent that understanding how inflation changes with income is of first order importance. In April, 2022, Vice Chair Lael Brainard of the Federal Reserve highlighted that "...We are only beginning to understand the ways in which inflation experiences vary from household to household, how this variation correlates with income and demographic information, and how these divergent inflation experiences change over time" (Brainard, 2022). If inflation is known to correlate systematically with income, this would either exacerbate or mitigate the distributional impacts of interest rate hikes, necessitating that policymakers augment the metrics used to evaluate inflation to safeguard the most vulnerable households from abnormal price level changes.

This paper aims to create income-specific measures of inflation at the income quintile level from 2012-2022, implying non-homothetic preferences for different categories of consumption. Inherent to this discussion of inflation from 2012-2022 are the economic and behavioral shocks that ensued as a result of the Covid-19 pandemic. This paper pays special attention to changes in heterogeneous inflation rates across the income distribution during this period, finding that—in the second half of 2021 and the first half of 2022—there was a significant reversal of the common trend that inflation varies inversely with income. This paper relates these findings to a model of consumption and the inter-temporal elasticity of substitution to determine how non-homothetic expenditure choices adapted to economic factors, impacting the different experiences of inflation across the income distribution. This paper aims to be explanatory in its analysis rather than predictive. Given the significant disruptions to economic trends in the past 3 years, it is important to expand understanding through descriptive models that relate behavior to economic outcomes.

The rest of this paper is structured as follows. Section 2 contains a literature review, surveying academic research related to inflation inequality and varying consumption baskets across the income distribution. Section 3 outlines this paper's empirical strategy, describing hypotheses, empirical methodology, and data and variables. Section 4 contains empirical results. Section 5 uses a theoretical model to explain the reversal of inflation inequality that was empirically observed. Following the model in section 5, this paper discusses the implications of findings on policy discussions, exploring how a more detailed understanding of inflation could impact monetary policy decisions. Section 7 presents a conclusion on this research, discusses limitations, and poses recommendations for further inquiry into this area of research.

2 Literature Review

Recent literature has linked inflation to income inequality. A study of survey data from 2001 revealed that lower-income Americans were more likely to mention inflation as a top national concern (Easterly and Fisher, 2001), and periods of worsened inflation from 1887 to 2014 have been correlated with worsened income inequality (Law and Soon, 2020). A study of inflation rates for different income deciles in the UK between 1979 and 1992 established that the lowest and highest income deciles consistently deviated from average inflation in opposite directions, but their position above or below average inflation switched periodically (Crawford, 1994). Decreases in the prices of normal goods such as food that is eaten at home

and gas have been correlated with lower than average inflation for lower income groups while decreases in the prices of luxury goods such as eating out or traveling have been associated with lower than average inflation for higher income groups (Crawford, 1994). Preferences for goods have been established as non-homothetic, systematically impacting how different income levels experience inflation. Specifically, tt is known that higher-income individuals consume more luxury goods while lower-income individuals consume more normal goods (Henry, 2014). Studies have shown that taking this non-homotheticity into account when modeling inflation can reduce the amount of upward bias present in the CPI, making the measurement more accurate (Hamilton, 2001) (Almas et al., 2018).

Furthermore, systematic variations in consumption interact with changing inflationary conditions to exacerbate or mitigate already prevalent levels of income inequality. In recent years, literature has skewed towards the conclusion that lower income groups experience higher inflation rates. This conclusion has important implications for the understanding of real, price-level adjusted, income and on monetary policy decisions. Studies have shown that monetary policy decisions produce heterogenous and redistributive outcomes with respect to income, creating a further need to understand income differentiation in inflation rates a priori to making monetary policy decisions (Doepke et al., 2019). In 2021, a study conducted by Xavier Jaravel showed that between 2004 and 2015, inflation rates consistently increased as income deciles decreased (Jaravel, 2021). Jaravel created inflation indices using individually recorded price and consumption data as well as national price data and consumption data aggregated by income decile. Through this Jaravel found evidence of inflation inequality as well as aggregation bias when using national data that diluted the variance of inflation across the income distribution.

A similar 2008 study modeled inflation for the lowest and highest 10% of the income distribution in Ireland, using group-specific inflation rates to examine how price changes impact welfare (Murphy et al., 2008). This study found that from 1996 to 2001, the lowest-income decile experienced higher than average inflation rates. More recently, researchers at

the BLS have attempted to create an experimental CPI for low and high-income households, much like this paper does. In a 2021 report released by the BLS, this experimental CPI showed faster inflation growth for the lowest income quartile compared to the highest income quartile when looking at inflation from 2003 to 2018 (Click and Stockburger, 2021).

Systematic price level variation that exploits heterogeneity in consumption baskets (as this paper studies) is one possible reason for inflation inequality. However, macroeconomic conditions have also been pointed to as factors that may systematically exacerbate inflation rates for the poor. A 2017 study of inflation inequality in Mexico established that poor households in Mexico spent more on lower-priced and heavily traded goods when compared to higher-income groups. This left lower-income populations' consumption baskets vulnerable to rising prices in imports during periods of currency devaluation (Cravino and Levchenko, 2017). Researchers replicated this study in a Brazilian context, finding that following the 2002 currency devaluation, low-income households experienced inflation that was 11 percent higher than high-income households (Gouvea, 2022). These studies point out that low-income households may be more susceptible to adverse inflation in countries whose currencies are at risk of devaluation.

In the absence of devaluation-spurred inflation inequality, it is unclear what economic and behavioral dynamics have caused the persistent levels of inflation inequality that have scaled down real income levels for low-income households in the past 20 years. Changes in prices interact with changes in consumption baskets due to income and substitution effects to create different distributions of inflation rates each year, making it difficult to opine as to the specific cause of inflation inequality. Murphy et al. isolated the weighted inflationary impact of different goods in a consumption basket to examine which market trends and goods exacerbated inflation inequality (Murphy et al., 2008). This paper reasoned that if above average contributions to inflation rates are persistent for certain goods in certain baskets, that may point to a systematic driver behind inflation inequality. This would indicate that inflation inequality is a phenomenon that necessitates adjustments to the inflation rate targeted by central banks. Murphy et al. shows that when studying the persistence of inflation inequality, it is important to analyze if there is a difference in the ability of different income groups to adjust their consumption preferences to mitigate inflation inequality (Murphy et al., 2008). Failure to account for changes in consumption preferences has long been the cause for critiques of inflation measurements and has been found to introduce bias into estimates of inflation (Redding and Weinstein, 2020)(Martin, 2021).

When exploring consumption preference changes across income, researchers have found that high-income groups are more rigid in their preferences due to heightened time costs associated with substitution, less behavioral concerns with optimizing income, and more rigid preferences (Murphy et al., 2008). Substitution due to changes in inflation is closely related to the intertemporal elasticity of substitution (IES)—the theory that households adjust their consumption and savings preferences in response to changes in the real interest rate. Not only is inflation a component of the real interest rate, it classically leads to hikes in the real interest rate if it rises above a central bank's desired target. Literature has established that intertemporal elasticity of substitution is not zero (Beaudry and van Wincoop, 1996). However, there is conflicting literature regarding wealth-varying IES models—how intertemporal elasticity of substitution varies across the income distribution. Ogaki and Atkeson, when studying consumption patterns in India, found mixed evidence that intertemporal elasticity of substitution rose or fell as income increased. Their study theorized that higher-income households sometimes possessed more volatile consumption patterns than low-income households in response to changing interest rates and sometimes the opposite occurred (Ogaki and Atkeson, 1997). A previous study by Hamori on consumption preferences across Japan found the opposite—that intertemporal elasticity of substitution decreased as income levels increased, and higher-income households were less likely to alter consumption in response to economic stimuli (Hamori, 1996).

More recently, researchers have explored ways to model the IES for non-homothetic preferences, showing how IES estimates may change when referring to durable, non-durable, luxury, or normal goods (Okubo, 2008). A similar framework is used in this paper to explore changing preferences for different classes of goods throughout the pandemic period.

It is natural that in an effort to maximize utility subject to a budget constraint, households will continually substitute away from more expensive goods and towards less expensive ones, decreasing the inflation rate they experience. Their ability to do this relies on whether a good is durable or non-durable, whether a good is a luxury or a necessity, and the availability of substitutes. Their propensity to do so is linked to the subjective utility that they obtain from consuming a certain class of goods. Atop of this, consumers' income levels impact how much households prioritize the optimization of their income as well as their ability to optimize their expenditures. Low-income households may desire to substitute away from more expensive goods more than other income levels, but they may find this difficult if they are already optimizing their income by consuming the cheapest goods possible (Brainard, 2022).

Literature also points to the role that consumers' expectations about future inflation rates and earning potential play in present consumption preferences (Chan, 1994). Risk aversion, which could vary systematically with income, might cause lower-income individuals to be more prudent when it comes to spending in periods of anticipated inflation.

Each of these complex interactions between non-homothetic preferences and macroeconomic factors have interesting impacts on consumption. While a number of studies have been done on inflation inequality, analysis of how non-homothetic preferences lead to varied experiences of inflation over time is relatively unexplored. This study conducts a descriptive analysis of how non-homothetic preferences lead to varied inflation rates across the income distribution and theoretically explores the impact of heterogenous consumption preferences in the face of Covid-19 spending constraints.

3 Empirical Strategy

3.1 Hypotheses

This study aims to discern whether inflation varies systematically with income given the understanding that consumption behaviors systematically vary with income. This paper anticipates that prior to the pandemic, inflation rates are higher for lower-income groups— as previous studies have found. Hypotheses 1 and 2 test the significance of differences in inflation rates between different income quintiles during the non-Covid period and the Covid period. This study anticipates that there may be disruptions to the predicted trend that inflation varies inversely with income due to the Covid-19 pandemic and the concurrent high period of inflation.

This study hypothesizes that during the Covid-19 period, exogenous spending constraints had an impact on experienced levels of inflation. To explore this, this paper utilizes an intertemporal Euler equation that models consumption choices across two representative baskets of goods. This analysis attempts to discern the impact that changes in consumption preferences have on a household's experienced price level, getting at their propensity to substitute their consumption during a period of heightened inflation. This study explores how abnormal supply and demand shocks during the Covid-19 period impacted consumption behaviors and therefore the incidence of inflation across the income distribution.

3.1.1 Hypothesis 1

H0 1: Between 2012 and April 2021, inflation rates are the same or significantly lower for households with incomes in the bottom 20% of American households when compared to households with incomes in the top 20% of the income distribution.

HA 1: Between 2012 and April 2021, inflation rates are significantly higher for households with incomes in the bottom 20% of American households when compared to households with incomes in the top 20% of the income distribution.

3.1.2 Hypothesis 2

H0 2: Between April 2021 and April 2022, inflation rates are the same or significantly higher for households with incomes in the bottom 20% of American households when compared to households with incomes in the top 20% of the income distribution.

HA 2: Between April 2021 and April 2022, inflation rates are significantly lower for households with incomes in the bottom 20% of American households when compared to households with incomes in the top 20% of the income distribution.

3.2 Empirical Methodology

To test hypotheses, this paper uses disaggregated price level and consumption data to model consumption baskets for each quintile of the income distribution from January 2012 to December 2022. Using Consumer Expenditure Survey (CEX) data from the Bureau of Labor Statistics (BLS), this paper created re-weighted consumption baskets that model the preferences of American households for 36 representative goods over five income quintiles. Then, monthly year-over-year inflation rates from the BLS for the 36 categories were applied to the consumption baskets to construct a monthly inflation rate for each income quintile between 2012 and 2022 using a Laspeyres formula (1). Next, hypothesis testing was used to determine the presence of statistically significant differences in inflation rates between income quintiles. All relevant measurements of inflation were created for this paper using 30 R scripts amounting to over 10,000 lines of code. Functionally, this code was used for data cleaning, data matching, inflation index aggregation, hypothesis testing, and figure making.

$$I = \sum_{i} \frac{P_{i,t} W_{i,t}}{P_{i,t-1} W_{i,t-1}}$$
(1)

After obtaining initial results, this paper chose to focus on the anomalistic period of 2020-2022. Nine expenditure categories were isolated using R scripts to help conduct a descriptive analysis of the differences in inflation between income quintiles. From this data, this paper developed an intertemporal model that explores changes in consumption preferences over two periods—before the pandemic and during the pandemic.

3.3 Data and Variables

The Bureau of Labor Statistics which is responsible for collecting the data for and calculating the Consumer Price Index for all urban consumers provides open-source price level data for categories of goods purchased in relatively urban areas. The CPI-U is collected based on a multi-staged sample of prices from 9 geographic regions in the US, weighted based on population. Price level data is primarily collected physically from establishments on either a monthly or bi-monthly basis and then sorted into more than 200 categories which are then aggregated into larger expenditure categories.

The BLS also maintains consumption data, called the Consumer Expenditure Survey (CEX). CEX data is aggregated based on various demographic characteristics of a consumer unit (CU) which can be loosely defined as a household. The CEX is collected 4 times a year using household sampling either in person or over the phone. This Consumer Expenditure Survey data (CEX) is provided going back to 2003 and disaggregates consumption data for CUs based on income quintiles and deciles, region (North, South, Midwest, and West), highest education level obtained in CU, the number of earners in CU, housing status of CU, age, and race. CEX data is provided in midyear measures as well as full-year mea-

sures in the form of expenditure shares and average dollar expenditures. This study uses dollar expenditure measures from 2012 to 2021 that are reported yearly. The CEX reports consumption data for 14 different categories of goods: food, alcoholic beverages, housing, apparel and services, transportation, healthcare, entertainment, personal care products and services, reading, education, tobacco products and smoking supplies, miscellaneous, cash contributions, and personal insurance and pensions.

This paper matched 36 expenditure categories in the CEX to 36 categories in the CPI data. This study then weighted monthly year-over-year inflation rates from the CPI using yearly CEX weights for each income quintile. From 2012-2021, the corresponding year's CEX data was used to construct an inflation measurement. Because the 2022 CEX data has not been released, 2022 inflation rates were weighted using 2021 expenditure shares. Because of this, analysis conducted using 2022 data should be considered preliminary and should be made subject to future scrutiny once 2022 CEX data is released.

While the CEX and the CPI are both maintained by the BLS, there are a few key discrepancies between the ways they categorize goods that should be noted. The largest discrepancy comes from the way the two datasets track owned dwellings. The CPI treats expenditure on owned housing as a capital investment, and therefore, does not include it in the normal CPI. Instead' BLS tracks Owner's Equivalent Rent (OER)—how much owners could rent their homes for—as a proxy for expenditure on owned housing. While BLS weights OER as roughly 20-25% of household expenditure in the CPI, in the CEX they weigh owned dwellings as roughly 15% of household expenditure. To test the accuracy of this paper's model, it was compared to the BLS less-shelter inflation index as well as the CPI-U. This paper's model closely tracks the BLS less-shelter CPI in 2021 and 2020 when high inflation occurs (see appendix). However, because this paper models inflation with the purpose of discussing differences between income groups rather than targeting inflation in the aggregate, the main model used in this paper includes shelter.

Another discrepancy between this paper's model and the CPI comes from the CEX's lack

of separation of spending on goods versus services relative to the CPI which separates goods and services as they aggregate spending. This made it difficult to match CPI and CEX categories seamlessly, which is why this paper uses 36 categories for calculation. Utilizing this level of detail allowed for heightened accuracy when matching categories across the CPI and CEX.

To best match the categories that are recorded in the CPI, this study excluded cash contributions as well as personal insurance and pensions from calculations. Each year, those two categories comprise around 10-15% of consumers' expenditure baskets. Because of this, this study re-weighted yearly consumption (around 85% of total expenditure) to make up 100% of consumption. This re-weighting slightly exaggerated upward and downward movements in inflation rates.

Still, given these discrepancies, this paper's estimates of aggregate inflation closely track the year-over-year monthly inflation rate published by the CPI. Given the re-weighting as well as the discrepancies in data organization between the CPI and the CEX, estimates in this paper should be used as a guide to model variation between in income-specific inflation rates rather than as estimates of inflation.

4 Empirical Results

4.1 Differences in Inflation Rates Experienced by Various Income Quintiles

Between 2012 and 2021, there was substantial differentiation in the inflation rates experienced by different income quintiles. A one-sided t-test established that the inflation rate modeled for the lowest 20% of US households was significantly higher than the inflation rate for the top 20% of US households between 2012 and April 2021. While this difference is estimated



2012–2022 Inflation: Model vs. CPI–U Year–year inflation rates collected monthly

Figure 1: This graph compares the CPI-U monthly year-over-year index to the aggregate inflation index created as a part of this study using BLS data. This study's model tracks inflation closely up until around 2019 when more significant deviation occurs. Much of this deviation stems from discrepancies in the measurement of housing. See the data and variables section for a discussion of the discrepancies as well as the appendix for a comparison of the CPI-U less shelter against this paper's model.

to be only 0.27% on average each year, that difference compounds upon itself to create a larger gap in the overall price level that impacts each income group. A 0.27% difference in inflation across 9 years (the time frame of this study) would become about a 2% difference in the overall price level. If this directional difference is shown to be consistent over time, that difference could become problematic, deflating real income more for those who have the least to spare. This finding fails to reject null - hypothesis 1 and affirms theories that argue

that lower-income Americans consistently experience higher inflation, further exacerbating income inequality.

T-test statistic	df	P-value	Sample Estimate
17.639	110	2.2e-16	0.002748633

2012–2022 Inflation by Income Quintile

Figure 2: Inflation rates by income quintile from 2012-2022 as modeled by this paper. These inflation rates were created using programs written in R for this thesis that match BLS CPI and CEX data to create measurements of inflation specific to different income quintiles.

While overall, the lowest income quintile experienced higher inflation than the highest income quintile, this trend waivers in 2021 when the highest income quintile's inflation rate surpassed the lowest income quintile. A one-sided t-test established that the inflation rate modeled for the lowest 20% of US households was significantly lower than the inflation

rate for the top 20% of US households between April 2021 and April 2022, failing to reject null-hypothesis 2. This complete reversal of a trend that held consistently for over 8 years coincides with record-high inflation rates that began to rise at the end of 2020. On average, the lowest income quintile experienced an inflation rate that was .6% lower than what the highest income quintile experienced. At its highest, in July 2021, the highest income quintile's inflation rate was modeled to be 1.155% greater than the lowest income quintile.

T-test statistic	df	P-value	Sample Estimate
-7.2095	12	5.363e-06	-0.006085955

Interactions between each income quintile's consumption preferences and the dramatic variation in inflation rates in 2021 and 2022 elucidate why this flip occurred. In the past two years, inflation was highly volatile and idiosyncratic with year-over-year inflation for goods like motor fuel hitting 60.2% in Jun of 2022 and hotels/motels peaking at over 25% in February of 2022. Analyzing the differences in expenditure shares devoted to these highly volatile goods between income levels shows that higher-income households' consumption baskets have been highly susceptible to the most recent period of inflation.

4.2 What Expenditure Patterns Lead to Differences in Inflation Across Income Quintiles?

Between 2012 and 2022, the difference in monthly inflation rates for the top and bottom income quintiles fluctuates between 0 and about 1.5%. Because this study does not explore price-heterogenetiy across income groups, these differences stem from differences in consumption preferences. Analysis of how different goods contribute to this gap can be used to build theories about how time dimensions contribute to the utility that consumers get from certain goods and their subsequent propensity to consume those goods.

The lowest and highest income quintiles consistently differ from each other. Prior to April

Difference in Inflation between the Lowest and Highest Income Quintile Lowest Income Quintile Inflation Rate – Highest Income Quintile Inflation Rate

Figure 3: This graph shows the difference between inflation rates for the lowest and highest income quintiles. This graph shows that before April 2021, inflation for the lower income quintile is consistently higher. From April 2021 until April 2022, the lowest income quintile falls below the highest income quintile.

2021, this study shows that the lowest income quintile experienced higher inflation rates. As inflation spiked in 2021, this trend reversed, leading to a more than 1% difference between the groups in July 2021. Analysis of consumption baskets shows that goods with relatively high inflation represented an outsized proportion of high-income consumption baskets, and goods with relatively low inflation represented a larger portion of low-income consumption baskets.

The primary culprits that drove up the inflation rate for the top 20% of the income quintile in July 2021, contributing to the 1.1% difference between the lowest and highest

Difference in Inflation between the Lowest and Highest Income Quintile, 2021–2022 Lowest Income Quintile Inflation Rate – Highest Income Quintile Inflation Rate

Figure 4: This graph zooms in on data depicted in figure 3 to give a more detailed picture of the reversal of inflation inequality that ocurred between April 2021 and April 2022.

groups were: transportation expenditure (0.01), leisure expenditure (0.042), owned housing (0.002), and health expenditure (0.002). Spending on travel-related goods and services such as rental vehicles and lodging away from home (hotels, motels, and vacation rentals) made up a significant portion of this upward shock to top-quintile inflation. Concurrently, spending on motor fuel (0.003), rent (0.002), and food at home (0.001) countered these effects and contributed more to inflation of lower income baskets. In April 2022, the lowest income quintile's inflation rate once again surpassed that of the highest income quintile. By September 2022, the lowest income quintile's inflation rate were expenditure 2022, the top contributors to the lowest income quintile's inflation rate were expenditure on rent (0.009), spending on gas and fuel (0.006),

health expenditures (0.005), and food (0.004).

Difference in Inflation between the Lowest and Highest Income Quintile, 2021–2022 Contribution to Difference by Expenditure Categories

Figure 5: Depending on their relative inflation rates and consumption shares, expenditure categories contribute differently to the inflation rates of various groups. Categories represented by bars below Y=0 represent categories that contributed more heavily to the inflation rate experienced by the highest income quintile. Categories represented by bars above Y=0 represent categories that contributed more heavily to the lowest income quintile. Where the lower bars have a larger absolute value (between April 2021 and April 2022), inflation is overall higher for the highest income quintile.

Which goods drive up inflation for the top and bottom income quintiles elucidate a key difference in each groups' consumption preferences. Typically, luxury goods like hotels, rental cars, and eating out drive up the top quintile's inflation rate while necessities like rent, groceries, and electricity drive up the lowest's. This observation points to a possible theory behind the persistent difference between high income inflation and low income inflation. In the wake of inflation, it is much easier for households to reduce luxury spending to mitigate their cost of living than it is to reduce necessity spending. Historically, high income groups have been able to augment their consumption baskets depending on inflation, driving their cost of living down (Murphy et al., 2008) (Hamori, 1996). Consumption patterns during the period of 2021, however, contradict this trend. Certain luxury goods were subject to high levels of inflation, but expenditure on them by the highest income quintile did not waiver. Expenditure on luxury services such as hotels, vacation rentals, and transportation contributed the most towards the unusually high-income rates experienced by the highest 20% of the income distribution between April 2021 and April 2022. These luxury services constituted high portions of high-income consumption baskets throughout 2021, despite their unusually exorbitant price levels. An analysis of the intertemporal dimensions of utility along with contextual socio-political factors point to why this might have been the case.

5 Theoretical Explanation

5.1 Model Environment

Augmenting the Euler equation to reflect changing constraints and preferences throughout the pandemic provides a framework through which to examine the empirically observed phenomenon. This model focuses on demand-side impacts on consumption because it aims to model consumption preferences in the wake of exogenous consumption constraints and price level changes. In this model, a few key assumptions are made:

- 1. Consumption baskets are split into two categories. Normal consumption goods (C) and luxury services (sometimes referred to as market leisure) (S) make up the entire basket of goods desired by households.
- 2. Preferences are non-homothetic.

- Preferences are modeled for households in the lowest quintile of the income distribution (L), and households in the highest quintile of the income distribution (H).
- 4. <u>S</u> represents home leisure which can be conceptualized as low-cost leisure like going to the park or making dinner at home. S above <u>S</u> is market leisure or services which is represented by S_1 and S_2 in the model. S above <u>S</u> is a luxury good and can be conceptualized as travel, eating out, or entertainment spending. The lowest income households (L) can be conceptualized as those who only engage in low-cost or free leisure activities like going to the park. The highest income households (H) consume closer to \overline{S} on leisure expenditures such as vacations and concerts.
- 5. When using the model to establish its theoretical basis, relevant time periods can be considered wholly pre-pandemic.

5.2 Normal Period

Equation 2 reflects the log-linearized utility function for C and S in pre-pandemic times subject to budget constraints for period 1 and period 2 (Equations 3 and 4) or an intertemporal budget constraint (Equation 5). Equations 2-5 allow for the optimization of the consumption of two types of goods—luxury services and other consumer goods—to examine how the lowest and highest income levels respond to constraints intertemporally.

$$\max_{C_1, C_2, S_1, S_2} \frac{\left[C_1^{\alpha} (S_1 + \underline{S})^{1-\alpha}\right]^{1-\sigma} - 1}{1 - \sigma} + \beta \frac{\left[C_2^{\alpha} (S_2 + \underline{S})^{1-\alpha}\right]^{1-\sigma} - 1}{1 - \sigma}$$
(2)

$$C_1 + P_1 S_1 + A_2 = A_1 (1+r) + Y_1$$
(3)

$$C_2 + P_2 S_2 = A_2 (1+r) \tag{4}$$

Equations 3 and 4 can be combined to create an intertemporal budget constraint (equation 5). Equation 2 can be optimized subject to equation 5.

$$C_1 + P_1 S_1 + \frac{C_2 + P_2 S_2}{(1+r)} = A_1 (1+r) + Y_1$$
(5)

Equation 6 is obtained after optimizing the Euler equation subject to the inter-temporal budget constraint to obtain an expression for optimal luxury service consumption in periods 1 and 2. The optimal consumption of services is a ratio equal to a price level ratio multiplied by the real interest rate and a constant multiplier (β).

$$\frac{S_2' + \underline{S}}{S_1' + \underline{S}} = \frac{P_1}{P_2} \times \beta(1+r) \tag{6}$$

Equation 7 sets the left side of equation 6 for the higher income group equal to the left side of equation 6 for the lower income group following the assumption that price level, realinterest rates, and β for the two income quintiles are equal. Terms are then rearranged to obtain equation 8 which relates the amount of luxury services a household consumes above the base level of home leisure to the change in luxury service consumption between periods. In equation 8 this, the rate of change between the amount of expenditure devoted to luxury services times the current ratio of expenditure devoted to services is equal for the highest income quintile (H) and the lowest income quintile (L). In this scenario, H consistently demands luxury services above \underline{S} (close to or at \overline{S}), increasing their ratio of services demanded in period 1 to be greater than L. To make up for this gap, the equation predicts that in period 2, the lowest income quintile will experience a proportionally greater increase in their luxury service expenditure. Therefore, this model predicts that for the highest income quintile, luxury service expenditures are consistently high while for the lowest income quintiles, the growth rate for luxury service expenditures between period 1 and period 2 outpaces that of the highest income quintile (Equation 9).

$$\frac{S_2^H + \underline{S}}{S_1^H + \underline{S}} - 1 = \frac{S_2^L + \underline{S}}{S_1^L + \underline{S}} - 1 \tag{7}$$

$$\frac{S_2^H + \underline{S}}{S_1^H + \underline{S}} - 1 = \frac{S_1^H}{S_1^H + \underline{S}} \times \widehat{S^H} = \frac{S_2^L + \underline{S}}{S_1^L + \underline{S}} - 1 = \frac{S_1^L}{S_1^L + \underline{S}} \times \widehat{S^L}$$
(8)

$$\frac{S_1^H}{S_1^H + \underline{S}} \ge \frac{S_1^L}{S_1^L + \underline{S}} \longrightarrow \widehat{S^H} < \widehat{S^L}$$
(9)

where
$$\widehat{S^{H}} = \frac{S_{2}^{H} - S_{1}^{H}}{S_{1}^{H}}$$
 and $\widehat{S^{L}} = \frac{S_{2}^{L} - S_{1}^{L}}{S_{1}^{L}}$ (10)

This means that while the highest income quintile consumes more luxury services above the minimum from year to year, growth in demand for luxury services between two periods does not typically outpace the lower income quintile. This finding is consistent with other papers such as Hamori et al. which found that the value of the inter-temporal elasticity of substitution is related inversely to income (Hamori, 1996). While low-income households save during period one to support consumption in period two, high-income households are less likely to respond to inter-temporal constraints on consumption, tending towards more consistent consumption patterns over time.

5.3 Pandemic Period

The Covid-19 pandemic provides an opportunity to explore whether this finding differs under the pressure of external constraints on consumption. During the pandemic, market services were constrained so that $S_1 \leq \overline{S}$. For the lowest income households who do not consume very many market services, this constraint is not binding. For the highest income households, this constraint is binding and therefore significantly altered consumption baskets during the pandemic.

Equations 11 - 15 explore the inter-temporal model established in sections 5.1 and 5.2 in the context of Covid-19 when lockdown policies placed binding constraints on luxury service expenditures. In this context, Time period 1 is the most heavily restricted period of the pandemic (2020 - beginning of 2021) while time period 2 begins in Spring 2021 as restrictions lessened significantly.

In this scenario, the intertemporal utility function (equation 11) is optimized subject to the intertemporal budget constraint (equation 14) and the downward constraint on luxury consumption (equation 15). Because Equations 11 12, depict the inter-temporal optimization equation and a budget constraint, both constrained by $S_1 \leq \overline{S}$.

$$\max_{C_1, C_2, S_1, S_2} \frac{\left[C_1^{\alpha} (S_1 + \underline{S})^{1-\alpha}\right]^{1-\sigma} - 1}{1 - \sigma} + \beta \frac{\left[C_2^{\alpha} (S_2 + \underline{S})^{1-\alpha}\right]^{1-\sigma} - 1}{1 - \sigma} \tag{11}$$

$$C_1 + P_1 S_1 + A_2 = A_1 (1+r) + Y_1$$
(12)

$$C_2 + P_2 S_2 = A_2 (1+r) \tag{13}$$

Equations 12 and 13 can be combined to create an intertemporal budget constraint (equation

14). Equation 11 can be optimized subject to equation 14.

$$C_1 + P_1 S_1 + \frac{C_2 + P_2 S_2}{(1+r)} = A_1 (1+r) + Y_1$$
(14)

$$S_1 \le \overline{S} \tag{15}$$

The result of optimization is equation 16.

$$L = \alpha log C_1 + (1 - \alpha) log (S_1 + \underline{S}) + \beta [\alpha log C_2 + (1 - \alpha) log (S_2 + \underline{S}) - \left[\lambda ((C_1 + P_1 S_1 + \frac{C_2 + P_2 S_2}{(1 + r)} - A_1 (1 + r) + Y_1) + \lambda_2 (S_1 - \overline{S}))\right]$$
(16)

The lagrangian above (equation 16) includes $\lambda_2(S_1 - \overline{S})$. This term represents the inequality between demand and realized consumption for households in the highest income quintile. $\lambda_2(S_1 - \overline{S})$ is binding only for the highest income quintile because they consistently consume close to \overline{S} while the lowest income quintile consumes close to \underline{S} . Additionally, in a period where consumption is not externally constrained, $\lambda_2(S_1 - \overline{S})$ would be equal to zero and therefore would not have any effect on consumption for either group.

Simplification of equation 16 shows that the growth rate of luxury service consumption between period 1 (during Covid-19 lockdown) and 2 (after the Covid-19 lockdown) is greater for the higher income group (equation 17). Because $\lambda_2(S_1 - \overline{S})$ is binding for the highest income quintile but not for the lowest income quintile, an inequality arises in equation 17 between the two groups in the optimized intertemporal equation. The left side of the equation (for the higher income group) is impacted by λ_2^H being ≥ 0 but the right side of the equation is not impacted because $\lambda_2^L = 0$ (See appendix 8.3 for a full derivation).

Equations 18 and 19 show how the inequality present in equation 17 offsets the diminishing impact that higher price levels in period 2 would have on consumption for services in period 2.

Because service consumption was constrained in period 1, the higher income group must play catchup in period two to optimize their utility from luxury service consumption. Therefore, the consumption growth with regards to S for the higher income group is greater than for the lower income group (Equation 19).

$$\frac{S_2^H + \underline{S}}{S_1^H + \underline{S}} \ge \frac{P_1}{P_2} \beta (1+r) = \frac{S_2^L + \underline{S}}{S_1^L + \underline{S}}$$
(17)

$$\frac{S_1^H}{S_1^H + \underline{S}} \times \widehat{S_2^H} \ge \frac{S_1^L}{S_1^L + \underline{S}} \times \widehat{S_2^L}$$
(18)

$$\widehat{S}^{\widehat{H}} > \widehat{S}^{\widehat{L}} \tag{19}$$

To show the validity of equation 16, it is helpful to test it within a pre-lockdown framework (See appendix section 8.3 for proof). This yields an identical equation to equation 7. In this exercise, λ_2 is not binding on either H or L, as there is no constraint on luxury service consumption ($S_1 \leq \overline{S}$). Therefore, λ_2 does not impact the growth rate of luxury services for the lowest quintile versus the highest quintile. In this scenario, luxury service consumption grows more for the lowest income quintile over the two periods.

However, in the instance where constraints on luxury services are binding for the higher income group, constrained consumption of S in period 1 defers demand for S to period 2, increasing the willingness of high-income households to consume more services despite price increases. Distinguishing between the demand for luxury services that were restricted until the second half of 2021 and other goods allows for an inter-temporal model that explains the anomalous inflation patterns across the income distribution between April 2021 and April 2022. The inequality present in equation 19 shows the heightened propensity of high income households to consume luxury services or market leisure goods in the post pandemic period, despite high prices. This model shows the heightened flexibility that high income households have to adjust consumption based on their preferences. This finding explains the empirical results of this paper in the conjunction with previous research which finds that higher-income households tend to be more rigid in their preferences and therefore would be willing to pay a premium to maintain them (Murphy et al., 2008).

6 Discussion: Policy Implications

Policymakers have a heightened interest in examining heterogeneous inflation rates in order to select interest rates that will be optimal to combat the distributional effects of suboptimal inflation. Knowing that inflation rates have historically been higher for low-income Americans suggests that policymakers would typically want to take a more aggressive stance to ward off higher inflation to protect those most vulnerable against rising price levels.

Alternatively, given the reversal of this trend in 2021, it would be logical to assume that a more aggressive stance against inflation—than what is warranted by the aggregate marker would be necessary to guard higher-income groups against high inflation. The model proposed in this paper shows that heightened inflation for high-income Americans was driven by deferred demand which increased voluntary expenditure on luxury services. Therefore, the aforementioned policy concerns do not apply to high-income Americans in the same way that they typically apply to low-income Americans. Additionally, as expenditure caught up to deferred demand in late 2021 and early 2022, inflationary patterns between income quintiles returned to normal, corroborating the theory that this period was anomalistic.

Therefore, the distributional impact of inflation from April 2021 until April 2022 should

not be regarded as a counter to the conception that low-income families suffer more greatly from inflation. Studies have shown that low-income households experience higher inflation due to systemic consumption tendencies as well as systemic price level differences (Brainard, 2022) (Jaravel, 2021). This study shows that high-income households suffered higher inflation rates due to self-inflicted consumption preferences. This does not reduce the onus of policymakers to reduce the price level for low-income American families who are more constrained in their consumption abilities by inflation and income.

7 Conclusion

This study shows how inflation varies across different income groups. Empirically, this study shows that lower income households typically face higher inflation rates. However, the results of this study show that there is a significant reversal of inflation inequality between April 2021 and April 2022, coinciding with the most strict period of lockdown in the Covid-19 pandemic. To explain this reversal, this paper used a model of consumption for a high and low income household across two periods with constrained and un-constrained consumption. This model shows that high income households deferred their demand for luxury services from the lockdown period onto the next period, causing them to consume more luxury services. This theory explains the high inflation rate they experienced, as inflation highly impacted luxury services.

This study contributes to the growing body of research around inflation inequality, and it exposes the current challenges when trying to accurately model inflationary experiences for different income levels. As discussed in the data and methods portion of this paper, the available CES and CPI data from the BLS has a few fundamental differences in the ways it conducts sampling and organizes data, especially with regard to expenditure on shelter. CES data is also updated yearly while CPI data is updated monthly. This limits the specificity of this study and hinders the study's ability to give a highly detailed account of consumption changes—especially during the Covid-19 period. Additionally, price level data is aggregated up to the categories listed in the CPI meaning that this study is unable to capture inflation inequality due to disparate price levels experienced by different income groups.

Future studies should be conducted on this time period using scanner-level price and expenditure data. Additionally, the BLS should continue its efforts to model inflation across the income distribution (Click and Stockburger, 2021). Not only will this create the impetus to increase the consistency between the CPI and CES, but it will also hopefully lead to regular publishing of an income-specific CPI which will inform policymakers of the potential threats of inflation inequality.

While this study strives to explore a descriptive analysis to build a theoretical understanding of how consumption preferences interact with constraints and inflation across the income quintile, it is recommended that the model presented in this paper be empirically tested. This study recommends that as future research into inflation inequality is done, other models like this one be tested and refined for empirical and predictive validity.

8 Appendix

8.1 Figures

Figure 6: BLS CPI measures for all urban consumers with shelter and less shelter compared to the model created for this paper

8.2 Derivation of Equation 6 Using Partial Derivatives

$$\max_{C_1, C_2, S_1, S_2} = \log C_1 + (1 - \alpha) \log(S_1 + \underline{S}) + \beta [\log C_2 + (1 - \alpha) \log(S_2 + \underline{S})]$$
(20)

$$C_1 + P_1 S_1 - A_2 = A_1 (1+r) + Y_1$$
(21)

$$C_2 + P_2 S_2 = A_2 (1+r) \tag{22}$$

Substitute C_1 into the utility function.

$$\max_{C_1, C_2, S_1, S_2} = \alpha log(A_1(1+r) + Y_1 - P_1S_1 - \frac{C_2 + P_2S_2}{(1+r)}) + (1-\alpha)log(S_1 + \underline{S}) + \beta [logC_2 + (1-\alpha)log(S_2 + \underline{S})]$$
(23)

Take partial derivatives of the above equation with respect to $S_1, C_2, \text{and} S_2$ and set equal to zero to optimize the utility function.

Optimize C_1

$$\frac{\partial U}{\partial C_1} = \frac{-\alpha}{C_1} = 0 \tag{24}$$

Optimize S_1

$$\frac{\partial U}{\partial S_1} = \frac{-\alpha}{C_1} P_1 + \frac{1-\alpha}{S_1 + \underline{S}} = 0$$

$$\frac{\alpha}{C_1} P_1 = \frac{1-\alpha}{S_1 + \underline{S}}$$

$$\frac{P_1(S_1 + \underline{S})}{(1-\alpha)} = \frac{C_1}{\alpha} = \overline{Y}$$
(25)

Optimize C_2

$$\frac{\partial U}{\partial C_2} = \frac{-\alpha}{C_1} \times \frac{1}{1+r} + \frac{\alpha\beta}{C_2} = 0$$
$$\frac{\alpha\beta}{C_2} = \frac{\alpha}{C_1} \times \frac{1}{1+r}$$
$$\frac{C_1}{\alpha} = \frac{1}{\beta(1+r)} \times \frac{C_2}{\alpha}$$
(26)

Optimize S_2

$$\frac{\partial U}{\partial S_2} = \frac{-\alpha P_2}{1+r} \times \frac{1}{C_1} + \frac{\beta(1-\alpha)}{S_2 + \underline{S}} = 0$$

$$\frac{\alpha P_2}{1+r} \times \frac{1}{C_1} = \frac{\beta(1-\alpha)}{S_2 + \underline{S}}$$

$$\frac{P_2(S_2 + \underline{S})}{1-\alpha} = \frac{\beta C_1(1+r)}{\alpha}$$

$$\frac{P_2(S_2 + \underline{S})}{1-\alpha} = \frac{C_2}{\alpha}$$
(27)

Combine the three equations

$$\frac{P_1(S_1 + \underline{S})}{(1 - \alpha)} = \frac{C_1}{\alpha} = \frac{1}{\beta(1 + r)} \times \frac{C_2}{\alpha} = \frac{1}{\beta(1 + r)} \times \frac{P_2(S_2 + \underline{S})}{1 - \alpha}$$
(28)

Combine like terms and simplify to get the optimized form of the utility function subject to the intertemporal budget constraint.

$$\frac{S_2' + \underline{S}}{S_1' + \underline{S}} = \frac{P_1 \beta (1+r)}{P_2} \tag{29}$$

8.3 Derivation of Equation 7 and 17 using Lagrangian Optimization

Derivation of optimized consumption given the downward constraint on S consumption using Lagrangian optimization.

$$L = \alpha log C_1 + (1 - \alpha) log (S_1 + \underline{S}) + \beta [\alpha log C_2 + (1 - \alpha) log (S_2 + \underline{S}) - \left[\lambda ((C_1 + P_1 S_1 + \frac{C_2 + P_2 S_2}{(1 + r)} - A_1 (1 + r) + Y_1) + \lambda_2 (S_1 - \overline{S}))\right]$$
(30)

Take partial derivatives of the above equation with respect to S_1, C_2 , and S_2 and set equal

to zero to optimize the utility function.

Optimize C_1

$$\frac{\partial L}{\partial C_1} = \frac{\alpha}{C_1} - \lambda_1 = 0 \tag{31}$$

$$\lambda_1 = \frac{\alpha}{C_1} \tag{32}$$

Optimize S_1

$$\frac{\partial L}{\partial S_1} = (1 - \alpha) \frac{1}{S_1 + \underline{S}} - \lambda_1(P_1) - \lambda_2 = 0 \tag{33}$$

 λ_2 would equal 0 in a non-constrained period, and it would be greater than 0 in a period with constrained consumption that is binding.

Optimize C_2

$$\frac{\partial L}{\partial C_2} = \beta \frac{\alpha}{C_2} - \frac{\lambda}{1+r} = 0 \tag{34}$$

Optimize S_2

$$\frac{\partial L}{\partial S_2} = \frac{\beta(1-\alpha)}{S_2 - \overline{S}} - \lambda_1 \left(\frac{P_2}{1+r}\right) = 0 \tag{35}$$

Combine the four equations and simplify like terms with the assumption that $\lambda_2 = 0$. This scenario occurs if there is no binding constraint on consumption in period 1. First, combine equations 32 and 34.

$$\frac{1}{C_1} = \beta (1+r) \frac{1}{C_2} \tag{36}$$

Next, combine equations 33 and 35.

$$\lambda_1 = \frac{1 - \alpha}{(S_1 + \underline{S})P_1} \tag{37}$$

Then, combine the above two equations to reach the optimized intertemporal function.

$$\frac{S_2' + \underline{S}}{S_1' + \underline{S}} = \frac{P_1 \beta (1+r)}{P_2}$$
(38)

Equation 39 is the same as equation 7. In the case where there is not a binding constraint on consumption, this equation can be applied to the lowest and highest income group discussed in this paper.

$$\frac{S_2^H + \underline{S}}{S_1^H + \underline{S}} = \frac{P_1 \beta (1+r)}{P_2} = \frac{S_2^L + \underline{S}}{S_1^L + \underline{S}}$$
(39)

In the case of a binding constraint on S $S < \overline{S}$, $\lambda_2 > 0$. In this scenario, equations 32, 33, 34, 35 can be combined to create two equations.

$$\frac{1}{C_1} = \beta (1+r) \frac{1}{C_2} \tag{40}$$

and

$$\lambda_1 \le \frac{1 - \alpha}{(S_1 + \underline{S})P_1} \tag{41}$$

These two equations can then be simplified into an intertemporal optimization function that accounts for a constraint on S.

$$\frac{S_2' + \underline{S}}{S_1' + \underline{S}} \ge \frac{P_1 \beta (1+r)}{P_2} \tag{42}$$

In this paper, the constraint is only binding on the higher income group. Therefore, this equation can be used to show the inequality between the two groups.

$$\frac{S_2^H + \underline{S}}{S_1^H + \underline{S}} \ge \frac{P_1 \beta (1+r)}{P_2} = \frac{S_2^L + \underline{S}}{S_1^L + \underline{S}}$$
(43)

8.4 Appendix of all equations used in main body text

$$I = \sum_{i} \frac{P_{i,t} W_{i,t}}{P_{i,t-1} W_{i,t-1}}$$
(44)

$$\max_{C_1, C_2, S_1, S_2} \frac{[C_1^{\alpha} (S_1 + \underline{S})^{1-\alpha}]^{1-\sigma} - 1}{1 - \sigma} + \beta \frac{[C_2^{\alpha} (S_2 + \underline{S})^{1-\alpha}]^{1-\sigma} - 1}{1 - \sigma}$$
(45)

$$C_1 + P_1 S_1 + A_2 = A_1 (1+r) + Y_1$$
(46)

$$C_2 + P_2 S_2 = A_2 (1+r) \tag{47}$$

$$C_1 + P_1 S_1 + \frac{C_2 + P_2 S_2}{(1+r)} = A_1 (1+r) + Y_1$$
(48)

$$\frac{S_2 + \underline{S}}{S_1 + \underline{S}} = \frac{P_1}{P_2} \times \beta(1+r) \tag{49}$$

$$\frac{S_2^H + \underline{S}}{S_1^H + \underline{S}} - 1 = \frac{S_2^L + \underline{S}}{S_1^L + \underline{S}} - 1$$
(50)

$$\frac{S_2^H + \underline{S}}{S_1^H + \underline{S}} - 1 = \frac{S_1^H}{S_1^H + \underline{S}} \times \widehat{S^H} = \frac{S_2^L + \underline{S}}{S_1^L + \underline{S}} - 1 = \frac{S_1^L}{S_1^L + \underline{S}} \times \widehat{S^L}$$
(51)

$$\frac{S_1^H}{S_1^H + \underline{S}} \ge \frac{S_1^L}{S_1^L + \underline{S}} \longrightarrow \widehat{S^H} < \widehat{S^L}$$
(52)

where
$$\widehat{S^{H}} = \frac{S_{2}^{H} - S_{1}^{H}}{S_{1}^{H}}$$
 and $\widehat{S^{L}} = \frac{S_{2}^{L} - S_{1}^{L}}{S_{1}^{L}}$ (53)

$$\max_{C_1, C_2, S_1, S_2} \frac{[C_1^{\alpha} (S_1 + \underline{S})^{1-\alpha}]^{1-\sigma} - 1}{1 - \sigma} + \beta \frac{[C_2^{\alpha} (S_2 + \underline{S})^{1-\alpha}]^{1-\sigma} - 1}{1 - \sigma}$$
(54)

$$\lambda(C_1 + P_1S_1 + \frac{C_2 + P_2S_2}{(1+r)} = A_1(1+r) + Y_1) - \lambda_2(S_1 - \overline{S})$$
(55)

$$\frac{S_2^H + \underline{S}}{S_1^H + \underline{S}} \ge \frac{P_1}{P_2} \beta (1+r) = \frac{S_2^L + \underline{S}}{S_1^L + \underline{S}}$$
(56)

$$\frac{S_1^H}{S_1^H + S} \times \widehat{S_2^H} > \frac{S_1^L}{S_1^L + S} \times \widehat{S_2^L}$$

$$(57)$$

$$\widehat{S_2^H} > \widehat{S_2^L} \tag{58}$$

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