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Hasse Principle for Hermitian Spaces

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Abstract

Hasse Principle for Hermitian Spaces
By Zhengyao Wu

This dissertation proves new results on Hasse principle for Hermitian spaces. Let p be an odd prime. Let F be the function field of a curve over a p-adic field.

In a recent paper, Colliot-Thélène, Parimala and Suresh conjectured that a local-global principle holds for projective homogeneous spaces of connected linear algebraic groups over function fields of p-adic curves for $p \neq 2$. The first main result of this dissertation proves the following: Let A be a finite-dimensional simple F-algebra with an involution σ such that $F = Z(A)^\sigma$. Let $\varepsilon \in \{1, -1\}$ and $h\colon V \times V \to A$ an ε-hermitian space over (A, σ). Let X be a projective homogeneous space under

$$G = \begin{cases} \mathrm{SU}(A, \sigma, h) & \text{if } \sigma \text{ is of the first kind;} \\ \mathrm{U}(A, \sigma, h) & \text{if } \sigma \text{ is of the second kind.} \end{cases}$$

Let Ω be the set of all rank one discrete valuations on F. For each $v \in \Omega$, let F_v be the completion of F at v. Then

$$\prod_{v \in \Omega} X(F_v) \neq \emptyset \implies X(F) \neq \emptyset.$$

The proof implements patching techniques of Harbater, Hartmann and Krashen. As an application, we obtain a Springer-type theorem for isotropy of hermitian forms over odd degree extensions of function fields of p-adic curves.

Parihar and Suresh provided upper bounds for the u-invariant of hermitian spaces over division algebras over function fields of p-adic curves for $p \neq 2$. It was an open problem what their exact values are. The second main result of this dissertation proves the following: Let D be a central division algebra over F.

(1) If D is quaternion, then $u^+(D) = 6$ and $u^-(D) = 2$.

(2) Let L/F be a quadratic extension. If D is quaternion and $D \otimes_F L$ is division, then $u^0(D \otimes_F L) = 4$.

(3) If D is biquaternion, then $u^+(D) = 5$ and $u^-(D) = 3$.

The proof implements Larmour's theorem on Hermitian spaces over division algebras over complete discrete valued fields.

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