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Visual Perception of Apparent Motion Follows Minimization Principles of Geometry, not Physics

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An abstract of
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#### Abstract

Visual Perception of Apparent Motion Follows Minimization Principles of Geometry, not Physics By: Yaxin Liu


Apparent motion is a robust perceptual phenomenon in which observers perceive a stimulus traversing the vacant visual space between two flashed stimuli. It is known that the "filling-in" of apparent motion perception tends to favor the simplest and most economical paths of motion (Wagemans et al., 2012). However, the principles that underlie such computations remain largely unknown. Here, we tested whether the path of apparent motion is favored by the principle of kinematic geometry or Newtonian mechanics. In Experiment 1, a Pacman-shaped object was presented in succession across two positions differing by $0^{\circ}$ to $120^{\circ}$ in increments of $30^{\circ}$. Participants adjusted the position of a gap to indicate their perception of apparent motion. We found that adjusted gap position increased linearly with angular disparity, consistent with a curved path of motion defined by the unique center of rotation, as predicted by kinematic geometry, rather than a straight path constrained by objects' centroids, as predicted by Newtonian mechanics. To test this conclusion more directly, we gave participants a target detection task in conjunction with concurrent apparent motion in Experiment 2. Similar to Experiment 1, a Pacman-shaped object was briefly presented in alternation ( $90^{\circ}$ differences). Participants were instructed to respond as soon as they detected the target. We found that participants' RTs were significantly longer when a target appeared on a curved path, compared to a straight path, suggesting that apparent motion, as predicted by geometry, disrupted target detection, even after controlling for attentional biases and Pacman position. Taken together, our findings suggest that the "filling-in" perception of apparent motion is guided by kinematic geometry, not physics.

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# Visual Perception of Apparent Motion Follows Minimization Principles of Geometry, not Physics 

## Introduction

The human visual system faces enormous computational challenges in representing dynamic events. In a world where motion is ubiquitous, human vision appears to have evolved to favor the simplest and most economical paths of motion (Restle, 1979; Wagemans et al., 2012). When an object travels behind an occluder, we predict a straight-line path of motion, not a circuitous path that deviates from the original path. Despite a long history on the 'organizational' principles of perception, such as the Law of Prägnanz, in which the simplest possible computation on an external stimulus is favored (Wagemans et al., 2012), there is much unknown about the computations employed by the visual system to achieve such regularity.

Perceptual illusions are crucial for informing our understanding of the computations underlying visual perception. In the present study, we focus on the phenomenon of apparent motion because it provides a strong test of how vision solves a constructive 'filling in' process. Apparent motion, which consists of a stimulus traversing the vacant visual space between two positions, involves interpolation between the endpoints. Evidence from neuroimaging suggests that this 'filling in' occurs via feedback from areas MT and MST to V1 (Goebel, Khorram-Sefat, Muckli, Hacker, \& Singer, 1998). However, the principles that constrain this process remain elusive. Here we test between two candidates: Newtonian mechanics and kinematic geometry.

The physical regularities in the real world follow the conservation law and the principle of least action (Hanc \& Taylor, 2004), such that there is a minimization of overall energy expenses,
according to Newtonian mechanics (Buckley, Kim, McGregor, \& Seth, 2017; Mousavi \& Sunder, 2019). Studies have found that human gait minimizes the metabolic costs (Rebula \& Kuo, 2015), and other natural movements such as motor drawing movements follow the least action (e.g., the two-thirds power law) according to the shortest path in energy (Lebedev, Tsui, \& Van Gelder, 2001; Ville et al., 2008), suggesting that biological systems are subject to the same laws governing everyday motion in physical systems. It seems viable that visual perception of motion, as a part of biological systems, may also be subject to the minimization principle of least action according to Newtonian mechanics. However, another possibility for implementing a minimization principle is kinematic geometry (Carlton \& Shepard, 1990). Kinematic geometry is a branch of mathematics that accounts for movement only using geometric properties. Mass, forces, and dynamics are not considered in kinematic geometry. Thus, geometric regularities are governed by fewer rules than physical regularities; it may be more costly to compute Newtonian mechanics as compared to kinematic geometry in visual perception. Indeed, optimal coding of geometric properties has been shown in fMRI studies. It has been found that cortical motion representations are tuned optimally to kinematic invariants (Dayan et al., 2007), providing strong evidence that brain representation favors geometric properties. Moreover, developmental studies with infants demonstrate an early sensitivity to geometric properties, such as shapes and angles (Lourenco \& Huttenlocher, 2008; Schwartz, Day, \& Cohen, 1979), which contrasts with a relatively late-developing sensitivity to gravity and forces (Kim \& Spelke, 1999; Spelke, Katz, Purcell, Ehrlich, \& Breinlinger, 1994). Additionally, it has been argued that the constraints of geometry are unavoidable in any physical systems in the world, and therefore they are universal constraints. Geometric properties are more invariant than physics, such that a triangle maintains its triangular shape no matter where it is positioned on Earth or the Moon. By contrast,

Newtonian mechanics may vary across physical systems. For example, gravity is strong on Earth, but much less so on the Moon, where people and objects float in space. Thus, although Newtonian mechanics characterizes our world, it would seem more adaptive for perceptual systems to favor universal geometric constraints rather than specific physical constraints (Shepard, 2001).

Newtonian mechanics and kinematic geometry, as viable minimization principles for perception, are not dissociable constraints in the real, physical environment. As forces always act on physical objects, it becomes difficult to dissociate movements governed by physics and the geometric constraints within motion. Apparent motion, however, is a closed system without real forces, relying on the mental construction of motion, such that, here, it is possible to dissociate perceptual constraints that are either consistent with Newtonian mechanics or kinematic geometry. Newtonian mechanics describes motion by minimizing energy expenses, whereas kinematic geometry describes motion by minimizing variants of geometric properties. Moreover, and crucially, although the minimization principle is assumed under both accounts, it is instantiated uniquely by each. When two endpoints are clearly present, Newtonian mechanics minimizes the cost of energy involved in constructing the path of motion, whereas kinematic geometry minimizes the path itself, producing the shortest possible path of motion that most accurately preserves the invariant features of the object. In cases of pure translation and pure rotation ${ }^{1}$, kinematic geometry and Newtonian mechanics do not dissociate the paths of motion;

[^0]the path defined by kinematic geometry coincides with the path defined by Newtonian mechanics. For both pure rotation and pure translation, there is only one possible path of motion. However, when there is both rotation and translation (e.g., a rolling wheel of a moving car), Newtonian mechanics describes a straight path with concurrent rotation (see Figure 1A), whereas kinematic geometry describes a curved path based on the unique center of rotation (see Figures 1 B and C ) given by the endpoints of position and orientation of the object (Carlton \& Shepard, 1990; Papaxanthis, Paizis, White, Pozzo, \& Stucchi, 2012). It is important to note that although the absolute curved arc defined by kinematic geometry is longer than the length of a straight path defined by Newtonian mechanics, the length of the arc provides the kinematically shortest path (i.e., the shortest path as defined by the unique center of rotation), given the position and orientation of the objects (compare red and blue paths in Figures 1B and C).

A


B


C


Figure 1. An illustration of the paths predicted by Newtonian mechanics (A) and kinematic geometry (B and C). A. The path defined by Newtonian mechanics is a straight line with simultaneous rotation and
translation. B and C. The paths defined by kinematic geometry are curved. Point $P$ is the unique center of rotation for the $90^{\circ}$ disparity between the Pacman-shaped objects. The red paths - curved upward (B) and downward (C) - are the kinematically shorter paths compared to the blue paths.

## The Present Study

Despite a wealth of neuroimaging studies in the perception of apparent motion (Chong, Familiar, \& Shim, 2016; Kaneoke, Bundou, Koyama, Suzuki, \& Kakigi, 1997; Tse, 2006), it remains elusive how the "filling-in" of visual construction is achieved and what the governing principle of minimization may be. Here we argue that kinematic geometry and Newtonian mechanics, as two competing accounts of minimization in apparent motion, have the potential to shed light on the computational mechanism of the "filling-in" process of apparent motion, as well as the principles that underlie perception more generally. We contrasted kinematic geometry with Newtonian mechanics by testing whether the kinematically shortest path, as predicted by kinematic geometry, or the least amount of energy expenses, as predicted by Newtonian mechanics, provided a better characterization of participants' perceptions of apparent motion. Although the comparison of kinematic geometry and Newtonian mechanics in the present study is necessarily approximate, we were nevertheless in a position to provide a direct contrast of the two principles in the case of apparent motion perception.

We conducted two experiments to test this contrast. In both experiments, we examined whether the percept of apparent motion followed the path defined by kinematic geometry (i.e., a curved path) or the path defined by Newtonian mechanics (i.e., a straight path). We used an explicit judgment task in Experiment 1 to examine whether participants' reported percepts dissociated
between these two possibilities. In Experiment 2, we adopted an implicit attention paradigm to further examine the dissociation between kinematic geometry and Newtonian mechanics in the case of apparent motion.

## Experiment 1

In this first experiment, we adopted a paradigm previously used by McBeath and Shepard (1989). In this paradigm, participants were asked to adjust the position of a gap in a vertical bar to indicate their perception of the path of apparent motion for a Pacman-shaped object. We hypothesized that if the path of apparent motion followed the minimization principle of Newtonian mechanics, the adjustment would reflect a consistent straight path, whereas if the path of apparent motion followed the minimization of kinematic geometry, the adjustment should follow a curved path. More specifically, we predicted that the curved path defined by kinematic geometry would vary as a function of the angular disparity of the Pacman-shaped objects.

Previous studies have shown that both apparent motion and motion imagery share similar neural mechanisms. More specifically, it has been found that apparent motion perception and imagined motion both activate areas MT and MST, which is not the case for flickering stimuli (Goebel et al., 1998). Recent evidence also suggests shared ventral visual stream functioning that underlies both perception and mental imagery (Dijkstra, Bosch, \& van Gerven, 2019). In the current experiment, we included a static condition in which participants imagined the motion between two static objects to compare with the apparent motion condition in which participants judged
their perceived path of motion. We predicted that if both apparent motion and imagined motion follow the same principles of minimization, then there should be no difference between apparent and imagined motion conditions.

## Methods

## Participants

A total of 56 undergraduate participants between 18 and 22 years of age ( $M=19.0$ years) participated for course credit. All participants reported normal or corrected-to-normal vision, and all participants were included in the data analyses. Participants provided written informed consent, and experimental procedures were approved by the Institutional Review Board (IRB) at Emory University.

## Stimuli and Procedure

Participants were tested in a dimly lit room at a viewing distance of $\sim 50 \mathrm{~cm}$ from a Dell OptiPlex 990 monitor ( $1270 \times 768$ pixels, 60 Hz ). Stimuli were Pacman-shaped objects (diameter: 1.4 cm , $1.5^{\circ}$ ) created using Adobe Illustrator 2017. Experimental procedures were programmed in PsychoPy2 (Peirce, 2009). All participants completed both apparent motion and static conditions (condition order counterbalanced across participants).

In the apparent motion condition, the Pacman-shaped objects were presented in alternation to create the perception of illusory motion from one side to the other. Each trial began with a fixation cross $\left(1^{\circ}\right)$ presented for 700 ms . The fixation period was followed by the presentation of the Pacman stimuli and a brown vertical bar ( $0.7^{\circ}$ wide, $1.5^{\circ}$ tall). The Pacman stimuli were presented in alternation for 550 ms (center-to-center horizontal distance of $6^{\circ}$ ). Angular disparity between left and right stimuli differed by $0^{\circ}$ to $120^{\circ}$ in increments of $30^{\circ}$. Based on participant ratings of the quality of motion (see Appendix A), the interstimulus interval (ISI) was set to 0 ms. Participants were instructed to adjust the position of the gap in the vertical bar to allow for continuous movement of the object. There was no time limit for each adjustment.

The static condition was identical to the apparent motion condition except that the Pacman stimuli appeared onscreen simultaneously and remained static until participants adjusted the bar. In this condition, participants were instructed to imagine motion between the two objects and to indicate the path of motion by adjusting the bar to the height that would allow for continuous movement.

In each condition, the vertical bar appeared at a fixed position. On half the trials, the center of the object was aligned with the lower margin of the gap; on the other half of trials, the center of the object was aligned with the upper margin of the gap. This manipulation was adopted to eliminate response biases associated with a particular direction. There were a total of 120 trials ( 60 trials per condition), with an equal number of trials for each disparity $\left(0^{\circ}, 30^{\circ}, 60^{\circ}, 90^{\circ}\right.$, and $\left.120^{\circ}\right)$. In each condition, the left and right sides of the object were counterbalanced across trials, such that
half of the trials curved upward as a shorter path defined by kinematic geometry and half of the trials curved downward as a shorter path defined by kinematic geometry.


Figure 2. Illustrations of Apparent Motion (A) and Static (B) conditions. A. In the Apparent Motion condition, the Pacman stimuli were presented in alternation and participants adjusted the gap to allow for a smooth path of perceived motion. B. In the Static condition, the Pacman stimuli, appeared simultaneously onscreen and participants adjusted the gap to allow for a smooth path of imagined motion for the specific objects in the trial.

Disparity between the left and right objects


Figure 3. An illustration of upward (top) and downward (bottom) directions of motion, as predicted by kinematic geometry. The colored dotted lines represent the curved paths for $30^{\circ}$ (green), $60^{\circ}$ (yellow), $90^{\circ}$ (blue), and $120^{\circ}$ (red) disparities, respectively. The black dashed lines represent the line that connects the center of each object (original position of the bar). These straight lines represent paths of motion defined by Newtonian mechanics. Black arrows represent the displacement away from the origin in the case of a $30^{\circ}$ disparity defined by kinematic geometry.

## Results

The dependent variable in all analyses was the displacement of the vertical bar from the original position (measured in visual angle; see Figure 3). Trials of $0^{\circ}$ disparity were included in the task, but they were excluded from the analyses because they do not dissociate paths defined by kinematic geometry and Newtonian mechanics. In a preliminary analyses, we examined whether condition order affected performance. There was no effect of condition order on displacement, $F(1,53.9)=.006, p=.935$, and thus, the remaining analyses were conducted separately for
apparent motion and static conditions. There was a total of $1.3 \%$ of missing data in the static condition.

## Apparent Motion Condition

We performed linear mixed effects analyses to test for the effect of angular disparity on participants' displacement responses. The fixed effects were angular disparity and direction of motion. Random intercepts of participants were entered as a random effect. Models were fit by restricted maximum likelihood (REML). For ease of comparison, absolute values of displacement were used for upward and downward trials. In a separate analysis, to test for a linear trend, angular disparity was entered with a polynomial contrast. P-values were obtained by likelihood ratio tests. Analyses were conducted in R (version 3.5) and lme4 package (Bates, Maechler \& Bolker, 2012). Satterthwaite methods were used for degrees of freedom for all analyses (Kuznetsova, Brockhoff, \& Christensen, 2017).

The overall model revealed significant main effects of angular disparity, $F(3,385)=12.19, p$ $<.001$, and direction, $F(1,385)=51.76, p<.001$, as well as a significant interaction between angular disparity and direction, $F(3,385)=4.60, p=.004$. Moreover, the linear trend analysis revealed an interaction between angular disparity and direction $(\beta=.045, S E=.013), t(385)=$ $3.37, p<.001,95 \%$ CI $[.019, .07]$ (quadratic: $p=.135$; cubic: $p=.661$ ). Visual inspection of Figure 4A suggested an interaction effect that was primarily driven by the $120^{\circ}$ disparity trials in the downward condition. To test this possibility directly, we conducted simple effects analyses, which confirmed that the linear trend was modulated by one of the angular disparities that
moderated direction in the downward trials $(\beta=-.046, S E=.019, t(55)=-2.45, p=.015)$.
More specifically, when trials of $120^{\circ}$ disparity were excluded from a separate analysis, as expected, the model revealed significant main effects of angular disparity, $F(2,275)=18.31, p$ $<.001$, and direction, $F(1,275)=26.24, p<.001$. Importantly, there was no interaction between the angular disparity and direction, $F(2,275)=.61, p=.545$. Moreover, the linear trend of displacement as predicted by angular disparity remains, $\beta=.069, t(55)=6.05, p<.001$. Taken together, these effects indicate that participants' adjustments varied linearly as a function of angular disparity, consistent with paths of motion that follow the minimization principles defined by kinematic geometry.



Figure 4. A. Mean displacement from original position of the gap for Apparent motion (A) and Static (B) conditions. A. For both upward and downward trials, mean displacement increased as a function of angular disparity. This effect was linear for both upward and downward trials when $120^{\circ}$ disparity trials were
excluded. B. For both upward and downward trials, mean displacement increased linearly as a function of angular disparity. Error bars represent $95 \%$ CI.

## Static Condition

As in the apparent motion condition, we conducted mixed-effect analysis for the static trials with the same model specification (see Figure 4B). The overall model revealed a significant main effect of angular disparity, $F(3,379)=16.21, p<.001$, and separate trend analyses revealed a linear increase of displacement as a function of angular disparity $(\beta=.09, S E=.015), t(54.6)=$ $6.27, p<.001,95 \% \mathrm{CI}[.06, .12]$. The linear relationship indicates that the displacement is consistent with kinematic geometry. There was no significant main effect of direction ( $p=.277$ ) nor a significant interaction between angular disparity and direction ( $p=.052$ ). However, since the interaction was marginal, we again excluded trials of $120^{\circ}$ disparity in a separate analysis to ensure that the interaction effect was primarily driven by the $120^{\circ}$ disparity trials. The updated model revealed a significant main effect of angular disparity, $F(2,273)=19.85, p<.001$. Furthermore, there was no main effect of didreciton, $F(1,273)=.11, p=.743$, nor an interaction between angular disparity and direction after excluding $120^{\circ}$ disparity trials, $F(2,273)=.916, p$ $=.401$. Thus, participants' imagined motion was consistent with kinematic geometry, as in the apparent motion condition.

## Discussion

We sought to differentiate between kinematic geometry and Newtonian mechanics in the case of apparent motion. To this end, we examined whether the displacement of the gap changed as a function of angular disparity, as defined by kinematic geometry. Previous research suggested that
kinematic geometry captured individuals' perceptions more accurately (Shepard, 2001). The current data demonstrate that participants' judgments when presented with apparent motion and when reflecting imagined motion were consistent with paths of motion predicted by kinematic geometry. That is, angular disparity linearly predicted the displacement away from the straight line for both upward and downward trajectories and whether apparent or imagined motion, providing support for a pervasive effect of path computation according to kinematic geometry instead of Newtonian mechanics. A potential concern with the current findings may be that the displacement at $120^{\circ}$ disparity deviated from the linear trend, at least in the downward trials of the apparent motion condition (marginally so in static condition). Since the adjustments for both upward and downward directions were equally effortful for participants, the obtained difference cannot be attributed to measurement error. We suggest two potential explanations for this effect. First, previous research has shown that the degree of symmetry may affect the perception of the paths of apparent motion (Proffitt, Cutting, \& Stier, 1979). It is possible that objects with $120^{\circ}$ disparity have a more salient vertical symmetrical axis than other disparities, and such an axis may bias perception toward paths defined by Newtonian mechanics. The fact that the static condition showed a marginal, though non-significant, interaction between disparity and direction suggests that imagined motion is also influenced by a symmetrical axis. However, if the $120^{\circ}$ disparity is indeed biased toward paths defined by Newtonian mechanics, it does not sufficiently explain why direction only interacts with downward, but not upward, trials. Another explanation for the deviation at the $120^{\circ}$ disparity is the representation that the gravitational field exerts on what was considered a possible path of motion. Although the initial motion status was approximately comparable in which the upward trials go through a projectile-like path, the downward trials go through a parabola-like path, which is less commonly observed in the natural
world due to the constraint of gravity, suggesting that jugdments of apparent motion percepts may be influenced by such external factors.

In summary, the present experiment provides preliminary support that both apparent and imagined motion are best characterized by kinematic geometry rather than Newtonian mechanics. However, the explicit nature of the task used here leaves open the possibly that participants employed higher-level, more strategic processes to estimate the paths of movement. Although there is evidence that apparent and imagined motion recruit comparable neural regions, including MT/MST (Goebel et al., 1998), it is also possible that this task involved more explicit mechanisms such as mental rotation, which could be characterized by principles of kinematic geometry rather than Newtonian mechanics. Thus, in the next experiment, we adopted a more implicit measure to test whether perception of apparent motion indeed follows kinematic geometry.

## Experiment 2

In Experiment 2, we removed the explicit gap adjustment used in the previous experiment. We modified a paradigm used previously by Yantis and Nakama (1998) to more directly test the perception of apparent motion. We reasoned that if the perception of apparent motion follows the principles of kinematic geometry, then the path of motion, as predicted by kinematic geometry, should interfere with an object along that path. Yantis and Nakama (1998) previously found that apparent motion interfered with target detection of an object along the path. That is, when tasked with detecting a target, participants showed longer RTs when the target was positioned along the perceived path of motion compared to when it was off the path of motion.

In the present experiments, we hypothesized that the representation of the path of motion defined by kinematic geometry should disrupt a physically presented target on its path of motion, leading to longer RTs during target detection. Hence, if a curved path defined by kinematic geometry is computationally preferred, then detection of the physically presented target on the kinematically curved path should be slower than when it is presented on a straight path defined by Newtonian mechanics. In contrast, if a straight path is computationally preferred, then detection of the physically presented target on the straight path should be slower than the kinematically curved path.

Below we present Experiment 2A in which we first tested for an attentional bias in the current task. We then conducted Experiment 2B as a preliminary test of whether perceived motion interfered with target detection, as previously reported by Yantis and Nakama (1998). Experiment 2C was a replication and extension of Experiment 2B, in which; we demonstrated that participants perceived apparent motion according to the path predicted by kinematic geometry and, crucially, that the perception of this motion interfered with the detection of a target along the path.

## Methods

## Participants

A total of 123 undergraduates participated for course credit ( $M=18.5$ years; Exp. 2A: $N=40$;
Exp. 2B: $N=43$; Exp. 2C: $N=40$ ). All participants had normal or corrected-to-normal vision.

Participants provided written informed consent, and experimental procedures were approved by the IRB at Emory University.

## Stimuli and Procedure

All stimuli were presented on a Dell OptiPlex 990 monitor (1270 x 768 pixels, 60Hz) at viewing distance of $\sim 50 \mathrm{~cm}$.

In Experiment 2A (see Figure 5A), each trial began with a fixation cross presented for 700 ms , which was then followed by the presentation of a blank screen for 150 ms . Three placeholders oriented vertically (each $0.25^{\circ}$ wide, $0.5^{\circ}$ tall) appeared. After 200 ms , a target (the digit 2 or 5) was revealed at one of the three placeholders by removing two line-segments from the placeholders. Participants were instructed to press a customized key on the computer keyboard as soon as they detected the target. The target remained onscreen until participants responded. There was a total of 24 trials, with an equal number of trials presented in random order at each target position.

In Experiment 2B (see Figure 5B), each trial began with a fixation cross presented for 700 ms , which was then followed by the presentation of a Pacman-shaped object $\left(1.5^{\circ}\right)$ that appeared in succession to the left and right sides ( $90^{\circ}$ disparity) of the placeholders. After 25 frames ( $\sim 450$ ms ), three placeholders oriented vertically appeared (each $0.25^{\circ}$ wide, $0.5^{\circ}$ tall). After 200 ms , a target (the digit 2 or 5) was revealed at one of the three placeholders by removing two linesegments from the placeholders. As in Experiment 2A, participants pressed a key as soon as they
detected the target. Participants completed a total of 24 trials, with an equal number of upward and downward trials. The location of the target was counterbalanced across trials (randomized order). To ensure that participants were familiar with the target, participants were given two practice trials with apparent motion stimuli. Each target, 2 or 5, was presented, and the location was randomized across trials.

Experiment 2C differed from Experiment 2B in that there were five, instead of three, placeholders (see Figure 5C). The procedure was identical to Experiment 2B except that the Pacman-shaped object appeared on the sides of one of the $2^{\text {nd }}, 3^{\text {rd }}$, and $4^{\text {th }}$ placeholders (equal number of trials at each placeholder). There were 30 trials in total, with an equal number of upward and downward trials. The location of the target at one of the five placeholders was counterbalanced across trials.


Figure 5. Experiment 2. In all three experiments, participants were instructed to press a customized key on the computer keyboard as soon as they detected the target.

## Results

## Experiment 2A

We aimed to ensure that there were no attentional biases across the three target positions by examining RTs across target position (top, bottom, middle). Error rate was 3.3\%. Only correct trials were included in the analyses. An Analysis of Variance (ANOVA) revealed no main effect of target position, $F(2,117)=1.95, p=.147$, such that participants were equally fast at detecting the target regardless of the position (see Figure 6). Thus, this experiment provided support for the lack of an attentional bias across vertical position in this paradigm.


Figure 6. RTs for target appeared at bottom ( $M=594 \mathrm{~ms}, S D=98.8 \mathrm{~ms}$ ), middle ( $M=564 \mathrm{~ms}, S D=102$ ms ), and top ( $M=607 \mathrm{~ms}, S D=102 \mathrm{~ms}$ ) positions were not significantly different from each other.

## Experiment 2B

Three participants were excluded from the analyses for mean RTs exceeding 3000 ms . RT data were trimmed per participant based on the criterion of $M+/-2 S D$. An ANOVA, with target position (straight-path, curved-path, or the opposite path), direction (upward or downward), and the interaction between the two as fixed factors, was conducted to determine whether RTs varied for target detection. Sphericity tests revealed no violation of assumption. Only correct trials were analyzed. Error rate was $4.68 \%$.

This analysis revealed a significant main effect of target position, $F(2,234)=7.91, p<.001$, partial $\eta 2=.063$, but no main effect of direction $(p=.315)$, nor an interaction between direction and target position ( $p=.740$ ). Pairwise comparisons (Bonferroni corrected) revealed that when a target appeared on the curved-paths defined by kinematic geometry ( $M=841 \mathrm{~ms}, S D=295 \mathrm{~ms}$ ), participants showed significantly longer RTs than when the target appeared on straight-path ( $M=$ $693 \mathrm{~ms}, S D=169 \mathrm{~ms}), M_{\text {difference }}=148 \mathrm{~ms}, t(39)=3.97, p<.001,95 \% \mathrm{CI}[60.27,237.1], d=.62$ (see Figure 7), suggesting that the percept of the paths defined by kinematic geometry, but not paths defined by Newtonian mechanics, interfered with target detection. There was no difference in RT between straight- $(M=693 \mathrm{~ms}, S D=169 \mathrm{~ms})$ and opposite- $(M=694 \mathrm{~ms}, S D=228 \mathrm{~ms})$ paths, $p=.253$.

Although the results of Experiment 2B are consistent with apparent motion, as constrained by kinematic geometry, an alternative possibility is that target detection for the central target was
enhanced by the Pacman-shaped stimuli appearing at fixed positions. To rule out this possibility, we varied the positions of the Pacman-shaped stimuli in Experiment 2C.


Figure 7. Results of Experiment 2B. Longer RTs were found when the target appeared on curved-path compared to the straight-path.

## Experiment 2C

Following from Experiment 2B, we predicted that target detection on the curved paths defined by kinematic geometry would show longer RTs than target detection appearing on the straightpath defined by Newtonian mechanics. Importantly, in this experiment, the Pacman stimuli varied in position across trials, such that any effects could not be due to the constant placement of these stimuli. Only correct trials were analyzed. Error rate was 4.5\%.

Two participants' data were excluded due to unusual patterns of key responses. Preliminary analyses revealed that RTs for the two outer placeholders were slower $(M=861 \mathrm{~ms})$ than the three placeholders in the middle $(M=702 \mathrm{~ms}), t(37)=8.95, p<.001$. Thus, subsequent analyses were conducted on the middle placeholders ( $2^{\text {nd }}, 3^{\text {rd }}$, and $4^{\text {th }}$ position $)$. There was no difference in RTs between upward and downward trials ( $p=.211$ ). Thus, subsequent analyses were aggregated across these the two trial types.

An ANOVA, with target position and Pacman position as fixed factors, revealed a significant main effect of target position, $F(2,327)=4.36, p=.014$, partial $\eta 2=.026$ (see Figure 8). There was no main effect of Pacman position, $F(2,327)=1.79, p=.168$, partial $\eta 2=.011$, nor a significant interaction between Pacman and target position, $F(4,327)=.054, p=.642$, partial $\eta 2$ $=.008$. Post hoc tests (Bonferroni corrected) revealed a significant difference between participant RTs on straight- and curved paths. RTs on the curved-path ( $M=772 \mathrm{~ms}$ ) were significantly longer than RTs on the straight-path $(M=701 \mathrm{~ms}), M_{\text {difference }}=71.6 \mathrm{~ms}, S E=25.5, t(37)=2.81$, $p=.016$. As in Experiment 2A, these results suggest that target detection was interfered with by a curved-path of apparent motion defined by kinematic geometry, not Newtonian mechanics. In addition, there was no difference between RTs on adjacent- ( $M=756 \mathrm{~ms}$ ) and straight- ( $M=701$ $\mathrm{ms})$ paths, $M_{\text {difference }}=55.6 \mathrm{~ms}, t(37)=2.19, p=.087$, suggesting that it was less likely that RT enhancement only occurred along the straight-path.



Figure 8. Mean RTs for the different Pacman positions. Target detection along the curved-path was significantly slower than along the straight-path. There was no difference between adjacent- and straightpaths. Error bars represent $95 \%$ CI.

## Discussion

Building on our findings from Experiment 1, the current data provide additional support for kinematic geometry as a constraint on apparent motion perception. In the current experiment, we found that perception of a curved path defined by kinematic geometry, which was absent in the retinal input but are interpolated during apparent motion, interrupted target detection along this path.

Our findings are interpreted in light of consistent theories of motion masking (Yantis \& Nakama, 1998; Hidaka, Nagai, Sekuler, Bennett, \& Gyoba, 2011), such that targets are less easily to be detected within of the represented path of apparent motion. There are counter theories such as motion enhancement, in which a target is more visible on the represented path of motion. In the case of motion enhancement, the effect is more likely to occur when the shapes of the target and stimuli were similar (Lenkic \& Enns, 2013). Given the dissimilar shapes between the Pacmanshaped stimuli and digit-shaped target, we suggest that the likelihood of motion enhancement is unlikely in our experiments.

## General Discussion

In two experiments, we provide evidence that the perception of apparent motion, as well as imagined motion, follows the minimization principle consistent with kinematic geometry, not Newtonian mechanics. In Experiment 1, we found that angular disparity linearly predicted the displacement consistent with the curved-path of kinematic geometry, not the straight-path of Newtonian mechanics. In Experiment 2, we found that target detection was interrupted by the representation of the curved-path defined by kinematic geometry, not the straight-path defined by Newtonian mechanics, and this effect could not be explained by attentional biases or the placement of the moving objects. Importantly, our study was a systematic investigation of the minimization principles of apparent motion using both explicit and implicit measures. The results of our study shed new light on the computational cost of perceptual organization, suggesting that the principles governing the perception of apparent motion, as well as imagined motion, follow abstract principles that resemble to Law of Prägnanz, not the energy cost consistent with physical regularities.

Shepard (2001) argued that geometry is internalized in mental representation more than physics. Although we do not attempt to argue for the innate nature of geometry, we want to point out that our results suggest that the perception of apparent motion is not concerned with the minimization of energy costs of stimulus objects. Why does the perceptual system compute a curved-path, when a straight path that conserves energy is readily available? It is likely that for fast, efficient visual processing, it is computationally more convenient to only consider geometry, unless action is needed. It may be particularly interesting to look into how dorsal and ventral pathways differ in terms of different computational mechanisms. While the dorsal stream primarily deals with action, the ventral stream primarily deals with object perception (Goodale, Milner, \& Melvyn, 1992). Evidence has suggested that perception and imagery share similar ventral stream processes, and visual features are coded in the ventral stream (Dijkstra et al., 2019). Therefore, as the dorsal stream may contribute to the computing of spatial attention and motion that likely involves Newtonian mechanics, it is likely that the ventral stream may contribute to the orientation encoding and the computation of kinematic geometry.

It is less clear how the orientation encoding in visual perception is influenced by the symmetrical axis of objects. Although Experiment 2 did not probe further into the computation of the $120^{\circ}$ disparity, future research may consider focusing on the relationship between symmetry groups and perceptual invariants (see, Pizlo, 2019) in studying the computational mechanism of kinematic geometry.

Taken together, our study offers strong evidence in support of the perceptual organization of kinematic geometry rather than Newtonian mechanics in both apparent and imagined motion.

We encourage future behavioral research tapping into specific geometric constraints on other perceptual phenomenon and neuroimaging research focus on tapping into the neural mechanisms of the computation of kinematic geometry.

## Appendix A. Rating Experiment

To ensure apparent motion stimuli have the best subjective quality of motion, we conducted a rating experiment to determine the best interstimulus interval (ISI) for the main experiments. We manipulated seven types of Stimulus-onset asynchrony (SOA). The best SOA was used in the main experiments.

## Methods

## Participants

A total of 34 undergraduate participants with normal or corrected-to-normal vision ( $M=18.5$ years).

## Stimuli and Procedure

Half participants were randomly assigned to the upward condition (105 trials), the other half were randomly assigned to the downward condition (105 trials).

Each participant completed the task in a dimly lit room. As in the main experiment, participants were presented with Pacman-object with a viewing distance of $\sim 50 \mathrm{~cm}$. Each object appeared in alternation for 550 ms (or 33 frames). The Stimulus-onset asynchrony (SOA) ranges from 0 to 300 ms (every 50 ms ). The angular disparity ranges from $0^{\circ}$ to $120^{\circ}$ (every $30^{\circ}$ ). The trials include all possible combinations of $0^{\circ}, 30^{\circ}, 60^{\circ}, 90^{\circ}, 120^{\circ}$ disparity given the stimulus set. The SOA followed stimulus off-set on the left side and precedes stimulus on-set on the right side. A rating scale appeared after participants became familiar with the object ( $\sim 4 \mathrm{~s}$ ). Then, participants were prompted to use mouse to rate their perception of the apparent motion stimuli on a 5-point rating scale ( $1=$ least natural motion; $5=$ most natural motion $)$.

## Results and Discussion

There were no missing data. Data were analyzed using a 2 (condition) X 7 (SOA) X 5 (angular disparity) ANOVA.

The analyses revealed a significant main effect of SOA, $F(6,1190)=3.89, p<.001$, partial $\eta 2=$ 0.019. A comparison of mean rating revealed that $0 \mathrm{~ms}(M=3.26)$ was rated most highly (See Figure 9). To examine whether 0 ms was significantly different from others, pairwise comparisons (Bonferroni corrected) revealed that 0 ms SOA was rated significantly higher than most other SOAs, including $150 \mathrm{~ms} \mathrm{SOA}(t(33)=3.50, p=.028), 200 \mathrm{~ms} \operatorname{SOA}(t(33)=4.28, p$
$=.003), 250 \mathrm{~ms} \mathrm{SOA}(t(33)=5.13, p<.001)$, and $300 \mathrm{~ms} \mathrm{SOA}(t(33)=5.09, p<.001)$, except 50 ms SOA and $100 \mathrm{~ms} \mathrm{SOA}(p>.05)$. There was also a significant main effect of angular disparity, $F(4,1190)=2.89, p=.02$, partial $\eta 2=.01$, and a significant interaction between angular disparity and condition, $F(4,1190)=7.95, p<.001$, partial $\eta 2=.026$ (see Figure 10). No difference in mean ratings between the two conditions was found, $p=.083$. Since our goal with these ratings was to determine the optimal SOA for the apparent motion task, we did not probe the interaction effects further.

Taken together, given that 0 ms SOA has the highest mean rating, we selected the 0 ms SOA as the optimal SOA for both Experiments 1 and 2.


Figure 9. The SOA with $0 \mathrm{~ms}(M=3.26)$ has the highest rating across disparities.


Figure 10. A significant interaction between angular disparity and condition.

## Appendix B. Supplementary video

https://osf.io/xkm9y/?view_only=72c3d4a039364e2e84028888cef77ace

## Appendix C. Data

## https://github.com/yliu668/AM

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[^0]:    ${ }^{1}$ Pure translation occurs when an object moves without rotational movement, whereas pure rotation refers to an object that itself rotates without translational movement.

