A Walrasian Voluntary Contribution to Public Goods

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Abstract

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This paper extends the standard model of voluntary provision of a public good. In a simple two-person economy with one public good which is provided through voluntary contributions and two private goods that are competitively traded in a general equilibrium setting, I find that the total level of the public good provided *and* the final allocation of the private goods are independent of income redistributions between consumers.

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Chapter 1

Introduction

The model of voluntary contributions to public goods plays an important role in contemporary society. This model has been applied by Bergstrom et al. (1986) to study the contribution of campaign funds for political parties and by Becker (1981) to study the economic behavior of households. Furthermore, issues of private donations to charity, collective bargaining of labor unions, allied defense, and multilateral environmental policies, such as restrictions on carbon dioxide emissions, are within the scope of this model.

This paper aims to provide an exposition to the private provision of a public good model with multiple private goods that are traded in competitive markets in a *general equilibrium* setting and a public good that is provided voluntarily by consumers. The extension from partial equilibrium to general equilibrium is primarily motivated by the fact that supply and demand in a single market is contingent on prices of other commodities.[1] Since prices and consumers' consumption bundles are endogenously determined in general equilibrium, the value of consumers' endowments is also endogenously determined. This implies that the consumers' demand functions exhibit wealth effects that link different markets.[2] Hence, a general equilibrium approach is more representative of consumers' economic behavior.[3]

[1] In reality, consumers consume multiple private goods that are traded in competitive markets.

[2] In contrast, demand functions in partial equilibrium do not exhibit wealth effects.

[3] The existence of competitive markets for private goods imply that consumers' maximum contributions are no longer bounded by their initial endowments of the input to the public good.

This paper presents an extension to the classic treatment of the theory of private provision of public goods by Warr (1983) and Bergstrom et al. (1986). Warr (1983) proved that for a single public good that is provided voluntarily by consumers, the total level of the public good provided is neutral of income redistributions in a *partial equilibrium* setting. Bergstrom et al. (1986) extended this neutrality result for a general model with a single private good and proved that government provision "crowds out" private contributions.[4] Ley (1996) showed the same results geometrically using the Kolm triangle.

The following are two main results of this model. Firstly, sufficiently small lump-sum wealth redistributions do not change the final allocation of both the private goods and the total level of the public good supplied in general equilibrium. This has two implications: (1) increased wealth equality in the presence of competitive markets does not lead to higher total contributions. That is, the neutrality result by Warr (1983) and Bergstrom et al. (1986) extends to my model. (2) The second fundamental welfare theorem[5] fails to hold in general equilibrium with public goods that are voluntarily supplied.[6] Secondly, government taxation in the interest of a specific public good cannot increase the total level of the public good supplied since there is a complete crowding out effect, just as in the model of Bergstrom et al. (1986).

The remainder of this paper is structured as follows: Chapter 2 introduces my basic model; Chapter 3 shows the effect of government provision on private contributions; Chapter 4 presents extensions from my model to multiple public goods, a single public good with Leontief production function, and an impure public good; and Chapter 5 concludes and describes future work.

[4] The neutrality theorem of Bergstrom et al. (1986) generalizes to several public goods.

[5] The second fundamental theorem of welfare economics, which applies to private goods, states that, under certain assumptions, every Pareto efficient allocation can be attained through a market-based (i.e., Walrasian) equilibrium using an appropriate lump-sum wealth redistribution policy.

[6] I am grateful to Professor Tilman Klumpp for pointing this out.

Chapter 2

The Basic Model

Consider a simple pure exchange economy[1] where there is one public good, two private goods and two consumers. Both private goods are traded in competitive markets and the public good is financed through voluntary contributions. Each consumer i consumes an amount x_i and y_i of the private goods and contributes an amount $g_i \geq 0$ to the supply of the public good. Suppose g_i is produced linearly from x_i. Let G denote the total amount of the public good contributed to by both consumers and G_{-i} denote the total contribution excluding consumer i's contribution. Each consumer i has a utility function of $u_i(x_i, y_i, G)$ and is endowed with x_i and y_i, indicated by ω_i^x and ω_i^y respectively.

Definition 1: A **Walrasian/voluntary contribution equilibrium** is an allocation (x_i^*, y_i^*, g_i^*) for $i = 1, 2$ and prices (p_x^*, p_y^*) satisfying the following conditions:

1. Feasibility condition

$$\sum_i x_i^* + \sum_i g_i^* = \sum_i \omega_i^x \text{ and } \sum_i y_i^* = \sum_i \omega_i^y.$$

[1] No production of private goods

2. Individual utility maximization condition

$$\forall i,\ \text{Given } g_i^* \text{ and } G_{-i}^*, x_i \text{ and } y_i \text{ maximizes } u_i(x_i, y_i, G^*)$$
$$\text{s.t. } p_x^* x_i + p_y^* y_i + p_x^* g_i \leq p_x^* \omega_i^x + p_y^* \omega_i^y.$$

3. Nash equilibrium condition

$$\forall i,\ \text{Given } G_{-i}^*, x_i^*, \text{ and } y_i^*, g_i \text{ maximizes } u_i(x_i^*, y_i^*, g_i + G_{-i}^*)$$
$$\text{s.t. } p_x^* x_i + p_y^* y_i + p_x^* g_i \leq p_x^* \omega_i^x + p_y^* \omega_i^y.$$

Note that there are two possible methods to solve for a Walrasian/voluntary contribution equilibrium using backward induction.[2] In the first, a sequential game, consumers choose how much to contribute toward the public good and then trade their private goods. The second case, called a simultaneous game, is the reverse. In this case, consumers initially trade their private goods and then choose how much to contribute toward the public good. Note that it is described as a simultaneous game because consumers indirectly determine their contributions by choosing their level of private goods consumption.

Now consider the utility maximization problem of the consumer under a simultaneous game. Each consumer i can indirectly choose the level of G by deciding how much to contribute g_i. Then each consumer i solves

$$\begin{aligned} &\max_{x_i, y_i, g_i} \quad u_i(x_i, y_i, g_i + G_{-i}^*) \\ &\text{s.t. } p_x x_i + p_y y_i + p_x g_i \leq p_x \omega_i^x + p_y \omega_i^y \\ &\text{where } x_i, y_i, g_i \geq 0. \end{aligned} \tag{2.1}$$

[2]The total level of public good supplied in a sequential game is the same in a simultaneous game under Cobb-Douglas utility function. See Appendix E.

2.1 An Extended Neutrality Result

Assumption 1: Each consumer's utility function satisfies $u_x > 0, u_y > 0, u_{xy} > 0, u_{xx} < 0$ and $u_{yy} < 0$.

Assumption 2: For each consumer, the marginal rate of substitution between x and g is independent of y.

Theorem 1. *If consumers have convex, strictly increasing and continuous preferences satisfying Assumptions 1 and 2, then in the Walras/voluntary contribution equilibrium, the final allocation of x^*, y^*, and G^* is independent of the initial allocation, provided that the exchange of x and y is appropriately small and does not change the set of contributors.*

Proof. This proof is divided into three steps. First, I will show that the final allocation of x_i and G is independent of the endowment ω_i^x and ω_i^y. In step two, I will show that given the total amount of contribution in the Nash equilibrium, denoted by G^*, the final allocation of x and y in this economy is constrained Pareto efficient. In the final step, I will show that the final allocation of y is independent of the endowments.

Step 1

I will show that the final allocation of x_i and G is independent of the endowment ω_i^x and ω_i^y following Ley's (1996) expositional framework with a Kolm triangle.[3] A Kolm triangle is an equilateral triangle representing the feasible allocations x_i and g_i for i = 1, 2. That is, for any arbitrary point e in Figure 2.1, each consumer i's consumption x_i is measured by the distance from e to OiO while the total level of the public good supplied G is measured by the distance from e to $O1O2$. It must follow that consumer one's utility is increasing in the northeast direction while consumer two's utility is increasing in the northwest direction. Suppose we select an arbitrary initial endowment allocation $W = (\omega_1^x, \omega_2^x)$, consumer one's

[3]Although Ley (1996) has proved this statement, I will briefly explain for the ease of reading through the proof.

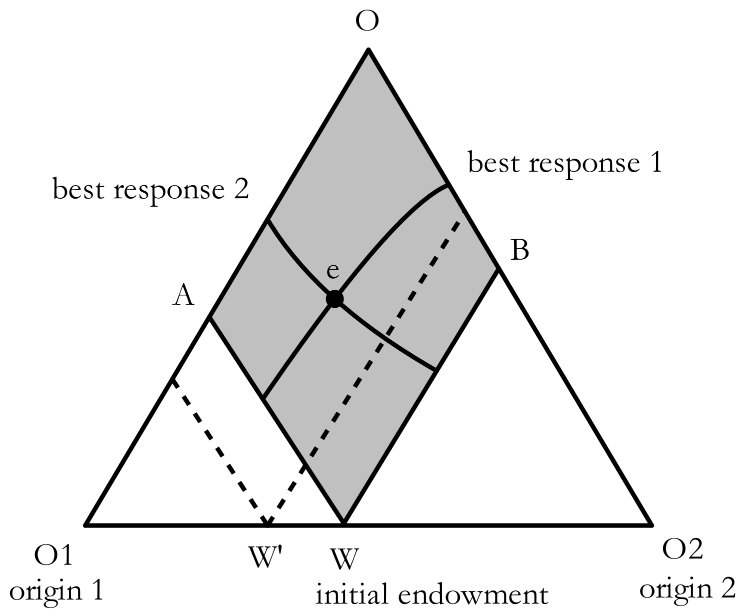

Figure 2.1: A Kolm triangle

budget line is represented by the distance from W to A while consumer two's budget line is represented by the distance from W to B. Each consumer chooses her consumption bundle where her indifference curve is tangent to her budget line so that the best response curves are increasing in the direction shown in Figure 2.1. Hence, a Nash equilibrium e must exist where the two best responses intersect each other in the shaded region. An income redistribution does not change the final allocation because changing the initial endowment from W to W' only shifts each consumer's budget line but not their best responses. Therefore the final allocations x_i and G are independent of endowments ω_i^x and ω_i^y, assuming that income redistributions are appropriately small.

Step 2

Given the total amount of contribution in the Nash equilibrium, denoted by G^*, the final allocation of x and y in this economy is constrained Pareto efficient. An allocation is constrained Pareto efficient if it is not possible to make at least one consumer better off without making any other consumer worse off. Suppose the market equilibrium is not constrained Pareto efficient, where there is a feasible allocation $\tilde{x_1}$, $\tilde{y_1}$, $\tilde{x_2}$, $\tilde{y_2}$ such that

$$\tilde{x_1} + \tilde{x_1} + G^* = \omega_1^x + \omega_2^x \tag{2.2}$$

$$\tilde{y_1} + \tilde{y_2} = \omega_1^y + \omega_2^y, \tag{2.3}$$

and is preferred by either or both consumers. That is,

$$u_1(\tilde{x_1}, \tilde{y_1}, G^*) \geq u_1(x_1, y_1, G^*) \tag{2.4}$$

$$u_2(\tilde{x_2}, \tilde{y_2}, G^*) \geq u_2(x_2, y_2, G^*), \tag{2.5}$$

with at least one strict inequality. However, by assumption, I have a market equilibrium where each consumer is purchasing the best bundle he or she can afford. If bundle $(\tilde{x_i}, \tilde{y_i}, G^*)$ is better than the bundle consumer i is choosing, then it must cost more than what consumer i can afford:

$$p_x\tilde{x_1} + p_y\tilde{y_1} + p_x g_1^* \geq p_x\omega_1^x + p_y\omega_1^y, \tag{2.6}$$

$$p_x\tilde{x_2} + p_y\tilde{y_2} + p_x g_2^* \geq p_x\omega_2^x + p_y\omega_2^y, \tag{2.7}$$

with at least one strict inequality. Adding equations (2.6) and (2.7) together I get

$$p_x(\tilde{x_1} + \tilde{x_2} + g_1^* + g_2^*) + p_y(\tilde{y_1} + \tilde{y_2}) > p_x(\omega_1^x + \omega_2^x) + p_y(\omega_1^y + \omega_2^y). \tag{2.8}$$

Substituting equations (2.2) and (2.3) into equation (2.8) yields

$$p_x(\omega_1^x + \omega_2^x) + p_y(\omega_1^y + \omega_2^y) > p_x(\omega_1^x + \omega_2^x) + p_y(\omega_1^y + \omega_2^y), \tag{2.9}$$

which is a contradiction. Therefore given G^*, the final allocation of x and y in this economy is constrained Pareto efficient.

Step 3

It has been shown above that the final allocation of x is independent of the endowments of x's. Let us denote the final allocation of x as x^*. Now I will show that the final allocation of y is independent of the endowments of y's. Any point on the contract curve satisfies

$$\frac{u_{1_x}}{u_{1_y}} = \frac{u_{2_x}}{u_{2_y}}. \tag{2.10}$$

Differentiate both sides implicitly with respect to y_1 to get the left-hand side

$$u_{1_{xy}} u_{2_y} + u_{1_{xx}} u_{2_y} \frac{dx_1}{dy_1} + u_{2_{xy}} u_{1_x} \frac{dx_2}{dy_1} + u_{2_{yy}} u_{1_x} \frac{dy_2}{dy_1} \tag{2.11}$$

and the right hand side

$$u_{1_{yy}} u_{2_x} + u_{1_{xy}} u_{2_x} \frac{dx_1}{dy_1} + u_{2_{xx}} u_{1_y} \frac{dx_2}{dy_1} + u_{2_{xy}} u_{1_y} \frac{dy_2}{dy_1}. \tag{2.12}$$

Then solve for dx_1/dy_1 to get

$$\frac{dx_1}{dy_1} = \frac{u_{1_x} u_{2_{yy}} - u_{2_y} u_{1_{xy}} + u_{2_x} u_{1_{yy}} - u_{1_y} u_{2_{xy}}}{u_{2_y} u_{1_{xx}} - u_{1_x} u_{2_{x}y} - u_{2_x} u_{1_{xy}} + u_{1_y} u_{2_{xx}}} \tag{2.13}$$

which will always be positive by Assumption 1. Given that the contract curve is increasing, for each x^*, there must exist a unique y such that the point (x^*, y) lies on the curve. Since I have proven that (x^*, y) is constrained Pareto efficient given g^*, it must follow that the point (x^*, y) lies on the contract curve. Therefore, y is unique given x^* and g^*. Since I have proven that the final allocation of x^* and g^* are independent of the initial endowments, then the final allocation of y must also be independent of the initial endowments. ■

To show that Theorem 1 is a sufficient but not necessary condition, suppose each consumer i solves the following constrained optimization problem in the following simple economy

$$\begin{aligned} &\max_{x_i,y_i,g_i} \quad u_i = (\sqrt{x_i} + \sqrt{y_i})(g_i + G^*_{-i}) \\ &\text{s.t. } p_x x_i + p_y y_i + p_x g_i \le p_x \omega_i^x + p_y \omega_i^y \\ &\text{where } x_i, y_i, g_i \ge 0. \end{aligned} \tag{2.14}$$

Then the first order necessary conditions with respect to x_i, y_i, g_i and λ respectively are

$$[x_i] \quad \frac{g_i + g_j}{2\sqrt{x_i}} = \lambda p_x \tag{2.15}$$

$$[y_i] \quad \frac{g_i + g_j}{2\sqrt{y_i}} = \lambda p_y \tag{2.16}$$

$$[g_i] \quad \sqrt{x_i} + \sqrt{y_i} = \lambda p_x \tag{2.17}$$

$$[\lambda_i] \quad p_x x_i + p_y y_i + p_x g_i = p_x \omega_i^x + p_y \omega_i^y. \tag{2.18}$$

Note that the marginal rate of substitution between x_i and g_i is dependent of y_i. Using the budget constraint to solve for g_i and g_j simultaneously in terms of prices (p_x, p_y) and endowments $(\omega_x^i, \omega_x^j, \omega_y^i, \omega_y^j)$, I obtain $g_i = (3p_x\omega_i^x + 3p_y\omega_i^y - p_x\omega_j^x - p_y\omega_j^y)/4p_x$, where g_j is symmetric to g_i. Subsequently, I normalize p_y and use the first feasibility requirement to solve for p_x^* in terms of the endowments $(\omega_x^i, \omega_x^j, \omega_y^i, \omega_y^j)$ to get

$$p_x^* = \frac{\omega_i^y + \omega_j^y + \left[(\omega_i^y + \omega_j^y)^2 + 8(\omega_i^x\omega_j^y + \omega_i^x\omega_i^y + \omega_j^x\omega_j^y + \omega_j^x\omega_i^y)\right]^{\frac{1}{2}}}{2(\omega_i^x + \omega_j^x)}. \tag{2.19}$$

Given the competitive prices (p_x^*, p_y^*), I can solve for the final allocation (x_i^*, y_i^*, G^*) in terms of the endowments $(\omega_x^i, \omega_x^j, \omega_y^i, \omega_y^j)$:

$$G^* = \frac{(3(\omega_i^y + \omega_j^y) + \alpha)(\omega_i^x + \omega_j^x)}{2(\omega_i^y + \omega_j^y + \alpha)} \tag{2.20}$$

$$x_i^* = \frac{(\omega_i^x + \omega_j^x)^2(3(\omega_i^y + \omega_j^y) + \alpha)}{4(\omega_i^x + \omega_j^x) + 2(\omega_i^y + \omega_j^y) + 2\alpha(\omega_i^y + \omega_j^y + 1)} \tag{2.21}$$

$$y_i^* = \frac{(\omega_i^y + \omega_j^y)(\alpha^2 + 3)}{16(\omega_i^x + \omega_j^x) + 8(\omega_i^y + \omega_j^y) + 8\alpha}, \tag{2.22}$$

where $\alpha = \left((\omega_i^y + \omega_j^y)(\omega_i^y + \omega_j^y + 8[\omega_i^x + \omega_j^x])\right)^{\frac{1}{2}}$. Since consumer j's final allocation of x_j and y_j is symmetric to consumer i's, I can ensure that the extended neutrality holds even when the marginal rate of substitution between x_i and g_i is dependent of y_i. Hence, I have established that Theorem 1 is a sufficient but not necessary condition to the extended neutrality result. Note that neutrality can be characterized conveniently as an allocation in the form of a function $f(\omega_i^x + \omega_j^x, \omega_i^y + \omega_j^y)$. Therefore, only changes in the *total* endowment can alter the final allocation.

Chapter 3

The Effect of Government Provision on Private Contributions

There are numerous instances where governments provide some amount of the public good financed through taxation in an attempt to increase the total level of public good in the economy. For example, a government may provide relief funds which are financed through tax revenues. However, can this government policy actually increase the total amount of public good supplied in the economy? Consider a Walrasian voluntary contribution model under which the government imposes a lump-sum tax in order to provide some quantity of the public good. Suppose that each consumer i has a Cobb-Douglas utility function and let t be the lump-sum tax imposed on each consumer's endowment of x's. Without loss of generality, each consumer i solves[1]

$$\begin{aligned} &\max_{x_i, y_i, g_i} \quad u_i = x_i^{\alpha} y_i^{\beta} (g_i + G_{-i}^{*} + 2t) \\ &\text{s.t. } p_x x_i + p_y y_i + p_x g_i \leq p_x(\omega_i^x - t) + p_y \omega_i^y \\ &\text{where } x_i, y_i, g_i \geq 0 \text{ and } 0 \leq \alpha, \beta \leq 1. \end{aligned} \tag{3.1}$$

[1]Consumer j's utility function is symmetric to consumer i's with coefficients γ and δ respectively.

Let G^*_{-t} denote the total level of public good supplied by the private sector. Solving the constraint optimization problem yields

$$x_i^* = \frac{\alpha(\omega_i^x + \omega_j^x)}{1+\alpha+\gamma} \tag{3.2}$$

$$y_i^* = \frac{\beta(\omega_i^y + \omega_j^y)}{\beta+\delta} \tag{3.3}$$

$$G^* = G^*_{-t} + 2t = \frac{-2t(\alpha+\gamma) + (\omega_i^x + \omega_j^x) - 2t}{1+\alpha+\gamma} + 2t = \frac{\omega_i^x + \omega_j^x}{1+\alpha+\gamma} \tag{3.4}$$

$$g_i^* = \frac{p_x[\omega_i^x(1+\gamma+\delta) - t\phi - \omega_j^x(\alpha+\beta)] - \omega_j^y(\alpha+\beta) + \omega_i^y(1+\gamma+\delta)}{p_x\phi} \tag{3.5}$$

where, $\phi = 1+\gamma+\beta+\alpha+\delta$. Since consumer j's final allocation is symmetric to consumer i's, I have established that the extended neutrality holds when the government provides some quantity of the public good.[2] The model further shows that there is a "dollar-for-dollar crowding-out" in private contributions when lump-sum tax is imposed. The one-to-one "crowding-out" effect is, in fact, consistent with the results shown by Bergstrom et al. (1986). Therefore, from a purely theoretical standpoint, government provision, in my model, cannot increase the total level of public good supplied in the economy.[3]

[2] See Appendix B for complete derivations.

[3] If the government instead taxes y's from consumers and can "exclusively" use y's to provide some quantity of the public good, then government provision via taxes can increase the total level of public good supplied. However, I cannot find a realistic scenario of this model.

Chapter 4

Extensions

4.1 Multiple Public Goods

In reality, consumers have the option of contributing to multiple public goods. Consider the Walrasian voluntary contribution model with two public goods: G and H. Each consumer i can voluntarily contribute some quantity of private good x_i toward G and some quantity of private good y_i toward H. For simplicity, suppose each consumer i has a simple utility function given by $u_i = x_i y_i + GH$. Then the constraint optimization problem for each consumer is given by

$$\begin{aligned} &\max_{x_i, y_i} \; u_i = x_i y_i + (g_i + G^*_{-i})(h_i + H^*_{-i}) \\ &\text{s.t. } p_x(x_i + g_i) + p_y(y_i + h_i) \leq p_x \omega_i^x + p_y \omega_i^y \\ &\text{where } x_i, y_i, g_i, h_i \geq 0. \end{aligned} \tag{4.1}$$

It appears that solving the maximization problem above yields multiple equilibria. Although there are four best response equations and four unknown variables g_i, g_j, h_i, h_j, the best responses are linearly dependent on each other. Consequently, one of the contributions needs to be determined exogenously to solve for a unique Walras/voluntary contribution equilibrium. Hence, solving the equations simultaneously I obtain

$$g_i = g_i \tag{4.2}$$

$$g_j = \frac{p_x(\omega_i^x + \omega_j^x) + p_y(\omega_i^y + \omega_j^y) - 6p_x g_i}{6p_x} \tag{4.3}$$

$$h_i = \frac{2(p_x\omega_i^x + p_y\omega_i^y) - (p_x\omega_j^x + p_y\omega_j^y) - 3p_x g_i}{3p_y} \tag{4.4}$$

$$h_j = \frac{p_x\omega_j^x + p_y\omega_j^y - (p_x\omega_i^x + p_y\omega_i^y) + 2p_x g_i}{2p_y}. \tag{4.5}$$

Despite the indeterminacy, the total level of public good G and H is given by

$$G^* = \frac{p_x(\omega_i^x + \omega_j^x) + p_y(\omega_i^y + \omega_j^y)}{6p_x} \tag{4.6}$$

$$H^* = \frac{p_x(\omega_i^x + \omega_j^x) + p_y(\omega_i^y + \omega_j^y)}{6p_y}, \tag{4.7}$$

in which I can conclude that the total level of public good G and H are both neutral to income redistributions. Since G and p_x^* are neutral, I have also established that the extended neutrality holds in this model.[1] Differentiating h_i^* with respect to g_i yields

$$\frac{\partial h_i^*}{\partial g_i} = -\frac{p_x^*}{p_y^*} = -\frac{\omega_i^y + \omega_j^y}{\omega_i^x + \omega_j^x} < 0, \tag{4.8}$$

which is the rate of exchange between consumer i's contribution g_i and h_i. Note that it may not always equal to one, depending on the total endowment of x's and y's. Suppose the total endowment of y's in the economy is much greater than that of x's, implying that the price of x is higher than the price of y. Then $\frac{\partial h_i^*}{\partial g_i}$ equals a negative number less than negative one (-1), meaning that a unit increase in g_i will reduce h_i by more than one unit. In other words, for a consumer to maintain her wealth, she needs to sacrifice more y's for a unit of x as the latter is more expensive than the former.

[1]See Appendix C for complete derivations.

4.2 Leontief Production Function for the Public Good

In this section, I extend my model by modifying the production function of the public good G, and allowing each consumer i to donate some monetary contribution $c_i \geq 0$ to the public good. Suppose the public good is produced by an independent entity under a Leontief production function. Hence, the entity solves

$$\begin{aligned} &\max_{x,y} \quad G = [\min\{\kappa x, y\}]^{\mu} \\ &\text{s.t. } p_x x + p_y y = c_i + c_j \\ &\text{where } 0 < \mu < \infty. \end{aligned} \tag{4.9}$$

Suppose that each consumer i consumes an amount x_i and y_i of the private goods, has a Cobb-Douglas utility function, and is endowed with x_i and y_i, indicated by ω_i^x and ω_i^y respectively. Without loss of generality, each consumer i solves[2]

$$\begin{aligned} &\max_{x_i,y_i} \quad u_i = x_i^{\alpha} y_i^{\beta} G^* \\ &\text{s.t. } p_x x_i + p_y y_i + c_i \leq p_x \omega_i^x + p_y \omega_i^y \\ &\text{where } x_i, y_i, c_i \geq 0. \end{aligned} \tag{4.10}$$

Given the production function for the public good, the entity's demand functions for x and y are given by $x = \frac{e^{\frac{\ln G}{\mu}}}{\kappa}$ and $y = e^{\frac{\ln G}{\mu}}$ respectively. Substituting the demand functions into the entity's budget constraint and isolating G I get $G = \left[\frac{\kappa(c_i+c_j)}{p_x+\kappa p_y}\right]^{\mu}$. Therefore, consumer i's new maximization problem can be written as

$$\begin{aligned} &\max_{x_i,y_i,c_i} \quad u_i = x_i^{\alpha} y_i^{\beta} \left[\frac{\kappa(c_i + c_j)}{p_x + \kappa p_y}\right]^{\mu} \\ &\text{s.t. } p_x x_i + p_y y_i + c_i \leq p_x \omega_i^x + p_y \omega_i^y \\ &\text{where } x_i, y_i, c_i \geq 0. \end{aligned} \tag{4.11}$$

[2]Consumer j's utility function is symmetric to consumer i's with coefficients γ and δ respectively.

Let C be the total monetary contributions by both consumers. From the first order necessary conditions, I get $x_i = \frac{\alpha C}{p_x \mu}$ and $y_i = \frac{\beta C}{p_y \mu}$. Normalizing p_y and solving for C from the constraint optimization problem above, I obtain

$$C^* = \frac{\mu[p_x(\omega_i^x + \omega_j^x) + (\omega_i^y + \omega_j^y)]}{\mu + \alpha + \beta + \gamma + \delta}, \tag{4.12}$$

which is neutral to income redistributions. Since p_x^* is also neutral to income redistributions, I have established that the extended neutrality holds under any general Leontief production function for the public good and at different types of returns to scale.[3]

4.3 Impure Public Goods

In reality, most public goods are instead impure public goods because consumptions of the public good, at some point, become rivalrous as many consume the public good simultaneously. Although a public park is generally considered a public good, a typical consumer's consumption becomes "congested" when many people are using the park at a particular time. Consider a model with one impure public good, two private goods and two consumers. Let G denote the total level of the impure public good contributed to by both consumers, and let c_i represent consumer i's *actual* consumption of the public good after congestion.

$$\begin{aligned} &\max_{x_i, y_i, g_i} \quad u_i(x_i, y_i, c_i) \\ &\text{s.t. } p_x x_i + p_y y_i + p_x g_i \leq p_x \omega_i^x + p_y \omega_i^y \\ &\text{where } x_i, y_i, g_i, c_i \geq 0. \end{aligned} \tag{4.13}$$

Assumption 3: The utility function for each consumer is multiplicatively separable.

Theorem 2. *If consumers have convex, strictly increasing and continuous preferences satisfying Assumptions 1, 2 and 3, then in the Walrasian/voluntary contribution equilibrium,*

[3] See Appendix D for complete derivations.

the final allocation of private goods x_i^**,* y_i^* *and the impure public good* G^* *is independent of the initial allocation, provided that the exchange of* x *and* y *is appropriately small and does not change the set of contributors.*

Proof. Let consumer i's post-congestion consumption of the impure public good be given by

$$c_i = G - \sum_{i \neq j}^{2} k_{ij} c_j, \tag{4.14}$$

where k_{ij} is a constant representing the congestion effect of consumer j imposed on consumer i. Since equation (4.14) is symmetric for each consumer, I can rearrange the linear equations into a matrix equation $\left(\begin{smallmatrix} 1 & k_{12} \\ k_{21} & 1 \end{smallmatrix}\right)\left(\begin{smallmatrix} c_1 \\ c_2 \end{smallmatrix}\right) = \left(\begin{smallmatrix} G \\ G \end{smallmatrix}\right)$. Using Cramer's rule to solve for c_i in terms of k's and G's I get $c_1 = det\left(\begin{smallmatrix} G & k_{12} \\ G & 1 \end{smallmatrix}\right)/det\left(\begin{smallmatrix} 1 & k_{12} \\ k_{21} & 1 \end{smallmatrix}\right)$ and $c_2 = det\left(\begin{smallmatrix} 1 & G \\ k_{21} & G \end{smallmatrix}\right)/det\left(\begin{smallmatrix} 1 & k_{12} \\ k_{21} & 1 \end{smallmatrix}\right)$. However, since there will always be a column of G's on the ith column for consumer i, basic properties of determinants allow us to, for any c_i, factor G such that

$$c_i^* = \gamma G, \tag{4.15}$$

where γ is a constant resulting from the k_{ij}'s. Substituting equation (4.15) into consumer i's maximization problem I get

$$\begin{aligned} &\max_{x_i, y_i, g_i} \quad u_i(x_i, y_i, \gamma G) \\ &\text{s.t. } p_x x_i + p_y y_i + p_x g_i \leq p_x \omega_i^x + p_y \omega_i^y \\ &\text{where } x_i, y_i, g_i, c_i \geq 0. \end{aligned} \tag{4.16}$$

Since the utility function is multiplicatively separable by Assumption 3, multiplying the utility function by a constant γ does not change the final allocation, and I have previously shown that the extended neutrality result holds for utility functions satisfying Assumptions 1 and 2. Hence, I have now established that the extended neutrality holds in a Walrasian voluntary contribution game under a single impure public good. ■

Furthermore, it is clear that this proof can be extended for n consumers and under a single private good voluntary contribution model by similar steps. In my model, the congestion effect is comparable to the effect of an increase in the price of the private good x; the existence of congestion makes it more expensive for consumers to contribute towards the public good.

Chapter 5

Conclusion and Discussion

This paper has provided an exposition to the theory of private provision to a public good using a general equilibrium framework for private goods. A general equilibrium analysis implies that consumers choose their private goods consumption levels by trading in an interdependent system of competitive markets. When consumers can competitively trade private goods, their contributions are no longer limited to their initial levels of endowment. A consumer who places a sufficiently high valuation for the public good can exchange their non-input private goods for more input private goods. A general equilibrium analysis also provides greater flexibility in determining the public good technology. Aside from having multiple private goods as inputs to the production of a public good, it is possible to characterize some private goods as time-intensive and some as commodity-intensive. All the above reasons suggest that a general equilibrium approach increases the realism of the model.

This paper reveals that, within a general equilibrium framework, in a simple pure exchange economy with a public good which is provided voluntarily and two private goods which are traded in competitive markets, the total level of the public good supplied and the final allocation of the private goods are independent of income redistributions between consumers. This extended neutrality result implies that wealth equality does not increase the total level of the public good provided by consumers, which is consistent with the neutrality result by Bergstrom et al. (1986). It also implies that the second welfare theorem of economics fails to hold when a public good is introduced to a general equilibrium model. That is to say, a social planner cannot achieve a desired Pareto-efficient allocation through a lump-sum wealth redistribution policy because consumers maximize their utility by changing their contributions by the exact amount of their change in wealth.

The extended neutrality is a persistent result. Chapter 4 shows that the extended neutrality result holds in the case of multiple public goods, a single public good with Leontief production function, and impure public goods.

This paper further shows that, consistent with the result by Bergstrom et al. (1986), government provision of public goods financed through lump-sum taxes cannot increase the total level of the public good supplied in a simple pure exchange economy. This complete crowding out effect arises since my model is limited to analyzing interior contributions. Consumers always contribute a positive amount so that the set of contributors never changes. It would be interesting to capture the effect of marginal contributors that changes the set of contributors.

Extending my pure exchange model into a Walrasian voluntary contribution with profit-maximizing producers may yield interesting results. When a consumer has a stake in a profit-maximizing firm that produces public goods and receives income according to her share of ownership, I hypothesize that extended neutrality will no longer hold due to the asymmetric wealth changes.

Chapter 6

Appendices

6.1 Appendix A

Basic Model

Let consumer i's maximization problem be expressed as a Lagrange function

$$\mathcal{L}_i = x_i^\alpha y_i^\beta (g_i + g_j) + \lambda \left[p_x \omega_i^x + p_y \omega_i^y - p_x(x_i + g_i) - p_y y_i \right]. \tag{6.1}$$

It follows that the first order necessary conditions are

$$[x_i] \quad \frac{\alpha x_i^\alpha y_i^\beta (g_i + g_j)}{x_i} - \lambda p_x = 0 \tag{6.2}$$

$$[y_i] \quad \frac{\beta x_i^\alpha y_i^\beta (g_i + g_j)}{y_i} - \lambda p_y = 0 \tag{6.3}$$

$$[g_i] \quad x_i^\alpha y_i^\beta - \lambda p_x = 0 \tag{6.4}$$

$$[\lambda_i] \quad p_x \omega_i^x + p_y \omega_i^y = p_x(x_i + g_i) + p_y y_i. \tag{6.5}$$

By the marginal rate of substitution conditions, $x_i = \alpha(g_i + g_j)$ and $y_i = \frac{\beta(g_i+g_j)p_x}{p_y}$. Substituting the two equations above into consumer i's budget constraint yields $p_x\omega_i^x + p_y\omega_i^y = p_x\alpha(g_i + g_j) + p_x\beta(g_i + g_j) + p_x g_i$, where consumer j's equation is symmetric to that of consumer i's with utility function coefficients γ and δ respectively. Normalizing p_y and solving the two equations simultaneously for g_i and g_j yields

$$g_i = \frac{p_x[\omega_i^x(1+\gamma+\delta) - \omega_j^x(\alpha+\beta)] - \omega_j^y(\alpha+\beta) + \omega_i^y(1+\gamma+\delta)}{p_x(1+\gamma+\beta+\alpha+\delta)}, \tag{6.6}$$

where g_j is symmetric to g_i. Then, solving for p_x^* using the first feasibility requirement gives

$$p_x^* = \frac{(\omega_i^y + \omega_j^y)(1+\alpha+\gamma)}{(\omega_i^x + \omega_j^x)(\beta+\delta)}, \tag{6.7}$$

which is neutral to income redistributions. The final allocation of x_i, y_i, g_i and G is then given by

$$g_i^* = \frac{\omega_j^x[\omega_i^y(\delta-\alpha) - \omega_j^y(\beta+\alpha)] + \omega_i^x[\omega_i^y(1+\gamma+\delta) + \omega_j^y(1-\beta+\gamma)]}{(\omega_i^y + \omega_j^y)(1+\alpha+\gamma)} \tag{6.8}$$

$$x_i^* = \frac{\alpha(\omega_i^x + \omega_j^x)}{1+\alpha+\gamma} \tag{6.9}$$

$$y_i^* = \frac{\delta(\omega_i^y + \omega_j^y)}{\beta+\delta} \tag{6.10}$$

$$G^* = \frac{\omega_i^x + \omega_j^x}{1+\alpha+\gamma}, \tag{6.11}$$

where g_j, x_j and y_j are symmetric to g_i, x_i and y_j respectively.

6.2 Appendix B

Government Provision on Private Contributions

Let consumer i's maximization problem be expressed as a Lagrange function

$$\mathcal{L}_i = x_i^\alpha y_i^\beta (g_i + g_j + 2t) + \lambda \left[p_x(\omega_i^x - t) + p_y \omega_i^y - p_x(x_i + g_i) - p_y y_i \right]. \tag{6.12}$$

It follows that the first order necessary conditions are

$$[x_i] \quad \frac{\alpha x_i^\alpha y_i^\beta (g_i + g_j + 2t)}{x_i} - \lambda p_x = 0 \tag{6.13}$$

$$[y_i] \quad \frac{\beta x_i^\alpha y_i^\beta (g_i + g_j + 2t)}{y_i} - \lambda p_y = 0 \tag{6.14}$$

$$[g_i] \quad x_i^\alpha y_i^\beta - \lambda p_x = 0 \tag{6.15}$$

$$[\lambda_i] \quad p_x(\omega_i^x - t) + p_y \omega_i^y = p_x(x_i + g_i) + p_y y_i. \tag{6.16}$$

By the marginal rate of substitution conditions, $x_i = \alpha(g_i + g_j + 2t)$ and $y_i = \frac{\beta(g_i+g_j+2t)p_x}{p_y}$. Substituting the two equations above into consumer i's budget constraint yields $p_x(\omega_i^x - t) + p_y\omega_i^y = p_x\alpha(g_i + g_j + 2t) + p_x\beta(g_i + g_j + 2t) + p_x g_i$, where consumer j's equation is symmetric to that of consumer i's with utility function coefficients γ and δ respectively. Normalizing p_y and solving the two equations simultaneously for g_i and g_j yields

$$g_i = \frac{p_x[\omega_i^x(1 + \gamma + \delta) - t\phi - \omega_j^x(\alpha + \beta)] - \omega_j^y(\alpha + \beta) + \omega_i^y(1 + \gamma + \delta)}{p_x\phi}, \tag{6.17}$$

where $\phi = 1 + \gamma + \beta + \alpha + \delta$ and g_j is symmetric to g_i. Then, solving for p_x^* using the first feasibility requirement gives $p_x^* = \frac{(\omega_i^y+\omega_j^y)(1+\alpha+\gamma)}{(\omega_i^x+\omega_j^x)(\beta+\delta)}$, which is neutral to income redistributions. Let G_{-t}^* be the total level of public good contributed by both consumers. Then,

$$G_{-t}^* = \frac{-2t(\alpha + \gamma) + (\omega_i^x + \omega_j^x) - 2t}{1 + \alpha + \gamma} \tag{6.18}$$

$$G^*_{-t} + 2t = \frac{\omega_i^x + \omega_j^x}{1 + \alpha + \gamma} \tag{6.19}$$

$$x_i^* = \frac{\alpha(\omega_i^x + \omega_j^x)}{1 + \alpha + \gamma} \tag{6.20}$$

$$y_i^* = \frac{\beta(\omega_i^y + \omega_j^y)}{\beta + \delta}, \tag{6.21}$$

where x_j and y_j are symmetric to x_i and y_i respectively.

6.3 Appendix C

Multiple Public Goods

Let consumer i's maximization problem be expressed as a Lagrange function

$$\mathcal{L}_i = x_i y_i + (g_i + g_j)(h_i + h_j) + \lambda \left[p_x \omega_i^x + p_y \omega_i^y - p_x(x_i + g_i) - p_y(y_i + h_i) \right]. \tag{6.22}$$

It follows that the first order necessary conditions are

$$[x_i] \quad y_i - \lambda p_x = 0 \tag{6.23}$$

$$[y_i] \quad x_i - \lambda p_y = 0 \tag{6.24}$$

$$[g_i] \quad h_i + h_j - \lambda p_x = 0 \tag{6.25}$$

$$[h_i] \quad g_i + g_j - \lambda p_y = 0 \tag{6.26}$$

$$[\lambda_i] \quad p_x \omega_i^x + p_y \omega_i^y = p_x(x_i + g_i) + p_y(y_i + h_i). \tag{6.27}$$

By the marginal rate of substitution conditions, $x_i = g_i + g_j$ and $y_i = \frac{p_x(g_i+g_j)}{p_y}$. Substituting the two equations above into consumer i's budget constraint yields $p_x \omega_i^x + p_y \omega_i^y = 3p_x g_i + 2p_x g_j + p_y h_i$. Since consumer j's budget constraint is symmetric to that of consumer i, there are four equations and four unknown variables (g_i, g_j, h_i, h_j):

$$g_i = \frac{p_x \omega_i^x + p_y \omega_i^y - 3p_y h_i - 2p_y h_j}{p_x} \tag{6.28}$$

$$g_j = \frac{p_x \omega_j^x + p_y \omega_j^y - 3p_y h_j - 2p_y h_i}{p_x} \tag{6.29}$$

$$h_i = \frac{p_x \omega_i^x + p_y \omega_i^y - 3p_x g_i - 2p_x g_j}{p_y} \tag{6.30}$$

$$h_j = \frac{p_x \omega_j^x + p_y \omega_j^y - 3p_x g_j - 2p_x g_i}{p_y}. \tag{6.31}$$

Solving these equations simultaneously yields

$$g_i = g_i \tag{6.32}$$

$$g_j = \frac{p_x(\omega_i^x + \omega_j^x) + p_y(\omega_i^y + \omega_j^y) - 6p_x g_i}{6p_x} \tag{6.33}$$

$$h_i = \frac{2(p_x\omega_i^x + p_y\omega_i^y) - (p_x\omega_j^x + p_y\omega_j^y) - 3p_x g_i}{3p_y} \tag{6.34}$$

$$h_j = \frac{p_x\omega_j^x + p_y\omega_j^y - (p_x\omega_i^x + p_y\omega_i^y) + 2p_x g_i}{2p_y}. \tag{6.35}$$

Consequently, the total amount of the public goods G and H are given by

$$G^* = \frac{p_x(\omega_i^x + \omega_j^x) + p_y(\omega_i^y + \omega_j^y)}{6p_x} \tag{6.36}$$

$$H^* = \frac{p_x(\omega_i^x + \omega_j^x) + p_y(\omega_i^y + \omega_j^y)}{6p_y}. \tag{6.37}$$

Since the first feasibility requirement states that $\omega_i^x + \omega_j^x = x_i + x_j + G$, solving for p_x^* yields

$$p_x^* = \frac{\omega_i^y + \omega_j^y}{\omega_i^x + \omega_j^x}, \tag{6.38}$$

which is neutral to income redistributions. Given that p_x^*, G, and H are neutral, it follows that the final allocation of x_i and y_i are neutral. Therefore, extended neutrality holds.

6.4 Appendix D

Leontief Production Function for the Public Good

Let consumer i's maximization problem be expressed as a Lagrange function

$$\mathcal{L}_i = x_i^{\alpha} y_i^{\beta} \left(\frac{\kappa(c_i + c_j)}{p_x + \kappa p_y} \right)^{\mu} + \lambda \left(p_x \omega_i^x + p_y \omega_i^y - p_x x_i - p_y y_i - c_i \right). \tag{6.39}$$

It follows that the first order necessary conditions are

$$[x_i] \quad \frac{\alpha x_i^{\alpha} y_i^{\beta} \left(\frac{\kappa(c_i+c_j)}{p_x+\kappa p_y} \right)^{\mu}}{x_i} - \lambda p_x = 0 \tag{6.40}$$

$$[y_i] \quad \frac{\beta x_i^{\alpha} y_i^{\beta} \left(\frac{\kappa(c_i+c_j)}{p_x+\kappa p_y} \right)^{\mu}}{y_i} - \lambda p_y = 0 \tag{6.41}$$

$$[g_i] \quad \frac{\mu x_i^{\alpha} y_i^{\beta} \left(\frac{\kappa(c_i+c_j)}{p_x+\kappa p_y} \right)^{\mu}}{c_i + c_j} - \lambda = 0 \tag{6.42}$$

$$[\lambda_i] \quad p_x \omega_i^x + p_y \omega_i^y = p_x x_i + p_y y_i + c_i. \tag{6.43}$$

By the marginal rate of substitution conditions, $x_i = \frac{\alpha(c_i+c_j)}{\mu p_x}$ and $y_i = \frac{\beta(c_i+c_j)}{\mu p_y}$. Substituting the two equations above into consumer i's budget constraint yields $p_x \omega_i^x + p_y \omega_i^y = \frac{(\alpha+\beta)(c_i+c_j)}{\mu} + c_i$. Since consumer j's budget constraint is symmetric to that of consumer i's, I can solve simultaneously for c_i and c_j in terms of prices and endowments to get

$$c_i^* = \frac{p_x[\omega_i^x(\delta + \gamma + \mu) - \omega_j^x(\alpha + \beta)] + \omega_i^y(\gamma + \delta + \mu) - \omega_j^y(\alpha + \beta)}{\alpha + \beta + \gamma + \delta + \mu}, \tag{6.44}$$

where c_j is symmetric to c_i. If I let $C = c_i + c_j$,

$$C^* = \frac{\mu[p_x(\omega_i^x + \omega_j^x) + (\omega_i^y + \omega_j^y)]}{\mu + \alpha + \beta + \gamma + \delta}, \tag{6.45}$$

which is immune to income redistributions. Next, I normalize p_y and use the first feasibility requirement to solve for p_x^*. To simplify, let

$$\phi = (\beta + \mu + \delta)^2 \left(\omega_i^x + \omega_j^x\right)^2 \mu^2) \tag{6.46}$$

$$\psi = 2\left(\omega_i^x + \omega_j^x\right)\left[\mu\left(-\delta - \alpha - \gamma - \beta - \mu\right) + \left(\beta + \delta\right)\left(\gamma + \alpha\right)\right](\omega_i^y + \omega_j^y)\kappa \tag{6.47}$$

$$\tau = (\alpha + \gamma + \mu)^2 \left(\omega_i^y + \omega_j^y\right)^2. \tag{6.48}$$

Subsequently,

$$p_x^* = \frac{(\phi + \psi + \tau)^{1/2} + (-\delta - \beta - \mu)(\omega_i^x + \omega_j^x)\kappa + (\gamma + \alpha + \mu)\left(\omega_i^y + \omega_j^y\right)}{2\left(\beta + \delta\right)\left(\omega_i^x + \omega_j^x\right)}, \tag{6.49}$$

which is also neutral to income redistributions. Given that p_x^* and C are neutral, it follows that the final allocation of x_i and y_i must be neutral. Hence, extended neutrality holds.

6.5 Appendix E

Sequential Games

I will show that the total level of public good supplied in a sequential game is the same in a simultaneous game. Using backward induction, the stage two problem is given by

$$\begin{aligned}
&\max_{x_i,y_i} \;\; u_i = x_i y_i (g_i^* + G_{-i}^*) \\
&\text{s.t. } \; p_x x_i + p_y y_i + p_x g_i \leq p_x \omega_i^x + p_y \omega_i^y \qquad (6.50)\\
&\text{where } x_i, y_i \geq 0.
\end{aligned}$$

Given G^*, the demand functions for x_i and y_i are given by $x_i = [p_x(\omega_i^x - g_i) + p_y \omega_i^y]/2p_x$ and $y_i = [p_x(\omega_i^x - g_i) + p_y \omega_i^y]/2p_y$. Normalizing p_y and solving for p_x^* I get $p_x^* = (\omega_i^y + \omega_j^y)/(\omega_i^x + \omega_j^x - G)$. Substituting p_x^* into the demand functions yields

$$x_i^* = \frac{(2\omega_i^y + \omega_j^y)(\omega_i^x - g_i) + \omega_i^y(\omega_j^x - g_j)}{2(\omega_i^y + \omega_j^y)} \qquad (6.51)$$

$$y_i^* = \frac{(2\omega_i^y + \omega_j^y)(\omega_i^x - g_i) + \omega_i^y(\omega_j^x - g_j)}{2(\omega_i^x + \omega_j^x - G)}. \qquad (6.52)$$

In the stage one problem, I substitute the two equations above into the consumer's utility function to get the indirect utility function given by

$$u_i^* = \frac{[(2\omega_i^y + \omega_j^y)(\omega_i^x - g_i) + \omega_i^y(\omega_j^x - g_j)]^2 (g_i + g_j)}{4(\omega_i^y + \omega_j^y)(\omega_i^x + \omega_j^x - g_i - g_j)}. \qquad (6.53)$$

Differentiating the indirect utility function with respect to g_i for each consumer i yields two best response functions. Using the best response functions to solve simultaneously for g_i and

g_j yields

$$g_i^* = \frac{\omega_i^x(7\omega_i^y + 5\omega_j^y) - \omega_j^x(2\omega_i^y + 4\omega_j^y)}{9(\omega_i^y + \omega_j^y)} \tag{6.54}$$

$$g_j^* = \frac{\omega_j^x(7\omega_j^y + 5\omega_i^y) - \omega_i^x(2\omega_j^y + 4\omega_i^y)}{9(\omega_i^y + \omega_j^y)}, \tag{6.55}$$

which maximize each consumer's utility.[1] Since $G = g_i^* + g_j^* = (\omega_i^x + \omega_j^x)/3$, I have established that the total amount of the public good supplied is the same in sequential and simultaneous games.

[1]Solving the best response functions yields three equilibrium allocations, but the remaining two not shown are minimum allocations.

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